Intégration

$$(1) \quad \frac{\partial \omega}{\partial t} = F(\omega) + \alpha \omega$$

On pose $\tilde{\omega}(t) = \omega e^{-\alpha t}$

$$\frac{\partial \tilde{\omega}}{\partial t} = -\alpha e^{-\alpha t} \omega + e^{-\alpha t} \frac{\partial \omega}{\partial t}$$

$$\Rightarrow \frac{\partial \omega}{\partial t} = \alpha e^{+\alpha t} \tilde{\omega} + e^{\alpha t} \frac{\partial \tilde{\omega}}{\partial t}$$

$$(1) \rightarrow \quad e^{\alpha t} \frac{\partial \tilde{\omega}}{\partial t} + \cancel{\alpha e^{+\alpha t} \tilde{\omega}} = F(\omega) + \cancel{\alpha \tilde{\omega} e^{+\alpha t}}$$

$$e^{\alpha t} \frac{\partial \tilde{\omega}}{\partial t} = F(\omega) \rightarrow \frac{\partial \tilde{\omega}}{\partial t} = e^{-\alpha t} F(\omega)$$

$$\tilde{\omega}(t+dt) - \tilde{\omega}(t) = \int_t^{t+dt} e^{-\alpha t'} F(\omega(t'))\, dt'$$

$$\omega(t+dt) e^{-\alpha(t+dt)} - \omega(t) e^{-\alpha t} = \int_t^{t+dt} e^{-\alpha t'} F(\omega(t'))\, dt'$$

$t'' = t' - t$

$$\omega(t+dt) e^{-\alpha dt} - \omega(t) = \int_0^{dt} e^{-\alpha t''} F(\omega(t+t''))\, dt''$$

$$\omega(t+dt) = \omega(t) \times e^{\alpha dt} + e^{\alpha dt} \left\{ \int_0^{dt} e^{-\alpha t''} F(\omega(t+t''))\, dt'' \right\}$$

le calcul de l'intégrale dépend du schéma utilisé

$$\omega(t+dt) = \omega(t) \cdot e^{\alpha dt} + e^{\alpha dt} \left\{ 3\, e^{-\alpha \times (-dt)} F(\omega(t-dt)) - e^{-\alpha \times 0} F(\omega(t+0)) \right\}$$

$\hookrightarrow$ schéma décentré amont d'ordre 2

$$\omega(t+dt) = \omega(t)\, e^{\alpha dt} + \left(\frac{e^{\alpha dt}}{2}\right)^2 \left\{ 3 \times F(\omega(t)) - F(\omega(t-dt)) \right\}$$