

Question 1

In [5]:

```
# Some pointers
# Numpy matrices are strictly 2-dimensional, while numpy arrays (ndarrays) are N-dimensional.
# Matrix objects are a subclass of ndarray, so they inherit all the attributes and methods of ndarrays.
```

In [14]:

```
# 1.Creating matrix M and its transpose
import numpy as np

m = np.array([[1,1],[2,4],[3,9],[4,16]])

t = m.T

print(f"M:\n{m}\n")
print(f"transpose(M):\n{t}")
```

M:

```
[[ 1  1]
 [ 2  4]
 [ 3  9]
 [ 4 16]]
```

transpose(M):

```
[[ 1  2  3  4]
 [ 1  4  9 16]]
```

In [15]:

```
# 2. Finding MMt and MtM
```

```
import numpy as np
m = np.array([[1,1],[2,4],[3,9],[4,16]])
t = m.T

MMt = m.dot(t)
MtM = t.dot(m)

print(f"MMt:\n{MMt}\n")
print(f"MtM:\n{MtM}")
```

```
MMt:
[[ 2  6 12 20]
 [ 6 20 42 72]
 [12 42 90 156]
 [20 72 156 272]]
```

```
MtM:
[[ 30 100]
 [100 354]]
```

In []:

```
# 3. Eigen pairs of MtM
```

In [1]:

```
# 4. Eigen values and vectors of MMt

# The function la.eig returns a tuple (eigvals,eigvecs)
# where eigvals is a 1D NumPy array of complex numbers giving the eigenvalues of , and
# eigvecs is a 2D NumPy array with the corresponding eigenvectors in the columns

import scipy.linalg as la
import numpy as np
m = np.array([[1,1],[2,4],[3,9],[4,16]])
t = m.T

MMt = m.dot(t)

eigen = la.eig(MMt)
print(f"Eigen Values:\n{eigen[0].real}\n")

eigenvecs = eigen[1]
print(f"Eigen Vectors:\n")
for i in range(len(eigenvecs)):
    print(eigenvecs[:,i])
```

Eigen Values:

```
[ 3.82378570e+02  1.62142978e+00 -7.45088994e-15  1.29844423e-16]
```

Eigen Vectors:

```
[-0.06315773 -0.22470839 -0.484652  -0.84298854]
[ 0.54109638  0.65339536  0.33689693 -0.40839891]
[-0.45663653  0.72081202 -0.50444616  0.13208775]
[ 0.42599675  0.27225124 -0.7889984   0.34912399]
```

Question 2

In [20]:

```
# 1. Create Matrix M and find its transpose
```

```
import numpy as np
```

```
m = np.array([[1,2,3],[3,4,5],[5,4,3],[0,2,4], [1,3,5]])
```

```
t = m.T
```

```
print(f"M:\n{m}\n")
```

```
print(f"transpose(M):\n{t}")
```

M:

```
[[1 2 3]
 [3 4 5]
 [5 4 3]
 [0 2 4]
 [1 3 5]]
```

transpose(M):

```
[[1 3 5 0 1]
 [2 4 4 2 3]
 [3 5 3 4 5]]
```

In [21]:

```
# 2. Compute matrices MtM and MMt
```

```
import numpy as np
```

```
m = np.array([[1,2,3],[3,4,5],[5,4,3],[0,2,4], [1,3,5]])
```

```
t = m.T
```

```
MMt = m.dot(t)
```

```
MtM = t.dot(m)
```

```
print(f"MMt:\n{MMt}\n")
```

```
print(f"MtM:\n{MtM}")
```

MMt:

```
[[14 26 22 16 22]
 [26 50 46 28 40]
 [22 46 50 20 32]
 [16 28 20 20 26]
 [22 40 32 26 35]]
```

MtM:

```
[[36 37 38]
 [37 49 61]
 [38 61 84]]
```

In [32]:

```
# 3. Find eigen values and vectors

import numpy as np
import scipy.linalg as la

m = np.array([[1,2,3],[3,4,5],[5,4,3],[0,2,4], [1,3,5]])
t = m.T

MMt = m.dot(t)
MtM = t.dot(m)

eigen1 = la.eig(MMt)
print("For MMt:\n")
print(f"Eigen Values:\n{eigen1[0].real}\n") #.real is to get rid of the complex nature
eigenvecs = eigen1[1]
print(f"Eigen Vectors:\n")
for i in range(len(eigenvecs)):
    print(f">> {eigenvecs[:,i]}")

eigen2 = la.eig(MtM)
print("\n\nFor MtM:\n")
print(f"Eigen Values:\n{eigen2[0].real}\n")
eigenvecs = eigen2[1]
print(f"Eigen Vectors:\n")
for i in range(len(eigenvecs)):
    print(f">> {eigenvecs[:,i]}")
```

For MMt:

Eigen Values:

```
[ 1.53566996e+02 -1.03322028e-14  1.54330035e+01  2.54653026e-15
 -3.61063094e-15]
```

Eigen Vectors:

```
>> [0.29769568 0.57050856 0.52074297 0.32257847 0.45898491]
>> [ 0.94131607 -0.17481584 -0.04034212 -0.18826321 -0.21515796]
>> [-0.15906393  0.0332003  0.73585663 -0.5103921 -0.41425998]
>> [ 0.12508859 -0.45318832  0.32553276  0.72000366 -0.39318742]
>> [ 0.07520849 -0.07287035 -0.10566284 -0.72571726  0.67171677]
```

For MtM:

Eigen Values:

```
[1.53566996e+02 1.54330035e+01 4.80589926e-15]
```

Eigen Vectors:

```
>> [-0.40928285 -0.56345932 -0.7176358 ]
>> [-0.81597848 -0.12588456  0.56420935]
>> [ 0.40824829 -0.81649658  0.40824829]
```

In []:

```
# COMPUTATION OF SVD : STEPS  
#  $A_{n \times p} = U_{n \times n} S_{n \times p} V_{p \times p}^T$ 
```

In [8]:

```

import numpy as np
import scipy.linalg as la

m = np.array([[1,2,3],[3,4,5],[5,4,3],[0,2,4], [1,3,5]])
#m = np.array([[2,4],[1,3],[0,0],[0,0]])
#m = np.array([[1,0,1,0], [0,1,0,1]])
t = m.T

MMt = m.dot(t)

MtM = t.dot(m)

# Step 1
e1 = la.eig(MMt)
e2 = la.eig(MtM)
vals1 = []
vals2 = []
ind1 = []
ind2 = []
for i in range(len(e1[0].real)):
    a = format(e1[0].real[i], 'f')
    if '0.0' not in a:
        vals1.append(e1[0].real[i])
        ind1.append(i)
for i in range(len(e2[0].real)):
    a = format(e2[0].real[i], 'f')
    if '0.0' not in a:
        vals2.append(e2[0].real[i])
        ind2.append(i)

U = e1[1][:,ind1]
V = e2[1].T[:,ind2]
singulars = [i*0.5 for i in vals1]
'''
print(vals1,"\n\n")
print(ind1)
print(ind2)
print(U)
print(V)
print(singulars)
'''
S = np.diagflat(singulars)

print("U: \n", U)
print("\nS: \n",S)
print("\nV: \n",V)
print("\ntranspose(V):\n", V.T)

```

```
U:  
[[ 0.29769568 -0.15906393]  
 [ 0.57050856  0.0332003 ]  
 [ 0.52074297  0.73585663]  
 [ 0.32257847 -0.5103921 ]  
 [ 0.45898491 -0.41425998]]
```

```
S:  
[[12.39221516  0.      ]  
 [ 0.          3.92848616]]
```

```
V:  
[[-0.40928285 -0.56345932]  
 [-0.81597848 -0.12588456]  
 [ 0.40824829 -0.81649658]]
```

```
transpose(V):  
[[-0.40928285 -0.81597848  0.40824829]  
 [-0.56345932 -0.12588456 -0.81649658]]
```

In []:

In []: