Lect 11

Adder and Multiplier

CS221: Digital Design

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Outline

- Combinational Block
- Adder, Substractor, BCD Adder
- Efficient : Adder Design
  - RCA, CS_kA, CS_lA, CLA
- Binary Multiplier
  - Array, Sequential, Booth
- Floating Point

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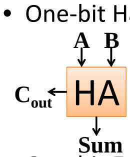
Adding Two One-bit Operands

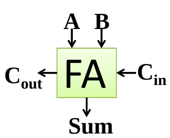
- One-bit Half Adder:



Sum = A ⊕ B
Cout = A.B

A	B	Sum	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

- One-bit Full Adder:

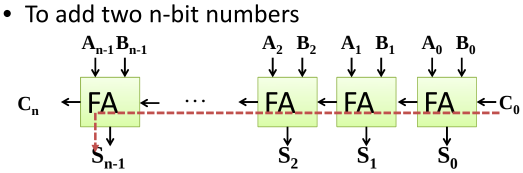


Sum = A ⊕ B ⊕ Cin
Cout = A.B + B.Cin + A.Cin

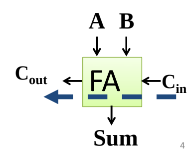
C_{in}	A	B	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

N-Bit Ripple-Carry Adder: Series of FA Cells

- To add two n-bit numbers



- **Adder delay = Tc * n**
- Tc = (C_{in} to C_{out} delay) of a FA
- **Adder Area: N*A_{FA}**



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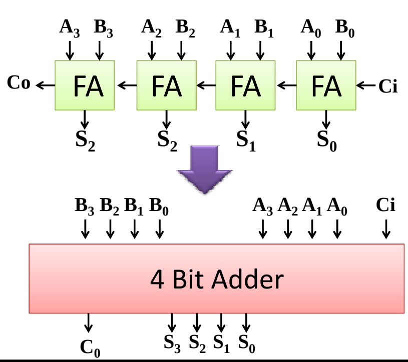
4 bit Binary Adder



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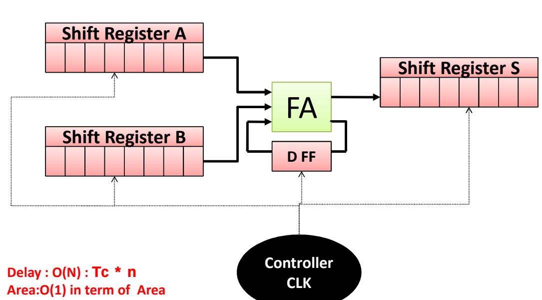
4 bit Binary Adder: Serial

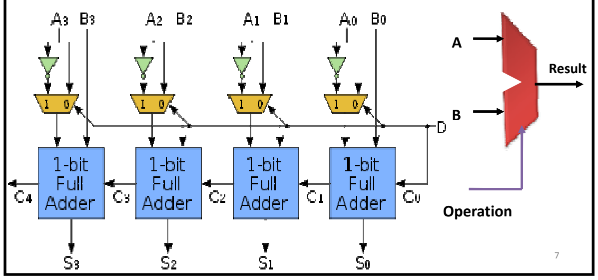


Delay : O(N) : Tc * n
Area:O(1) in term of Area of FA : 1*A_{FA}

Shift Reg. PIPO
Right Shift

Adder/Substractor

- C= B-A=B+(-A)=B+ (A^b+1), A^b is complement of A
- D is control bit: D=0/1 operation is add/sub



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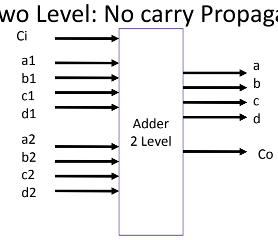
Binary Adder (Two Level)

- Treat as 9 input & 5 output functions
- Generate Truth Table for each outputs
- Solve each function using KMAP/QM Method
- Only Two Level: No carry Propagation



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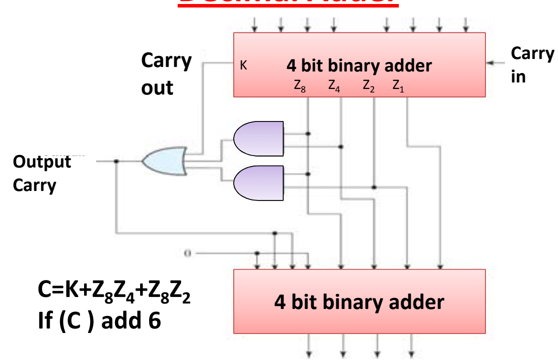
Decimal Adder

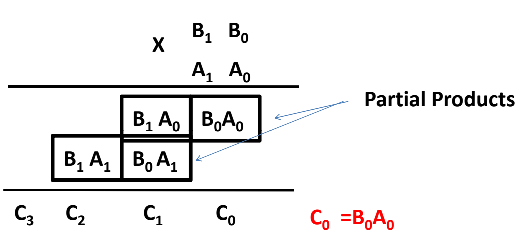
- Decimal numbers are represented with BCD code.
- When two BCD digits A and B are added
  - if A+B<10 result is a valid BCD digit
  - if A+B>9 result will not be valid BCD digit. 'It must be corrected by adding 6 to the result
- If A+B >9 add 6 to solve this issue

Decimal Adder



$C=K+Z_8Z_4+Z_8Z_2$
If (C) add 6

Binary Multiplier: 2 Bits



$C_0 = B_0A_0$
$C_1 = B_0A_1 + B_1A_0$
$C_2 = B_1A_1$ +Carry of C_1
C_3 = Carry of C_2

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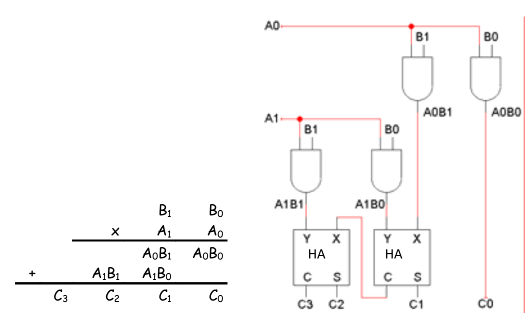
Binary Multiplier: 2 Bits



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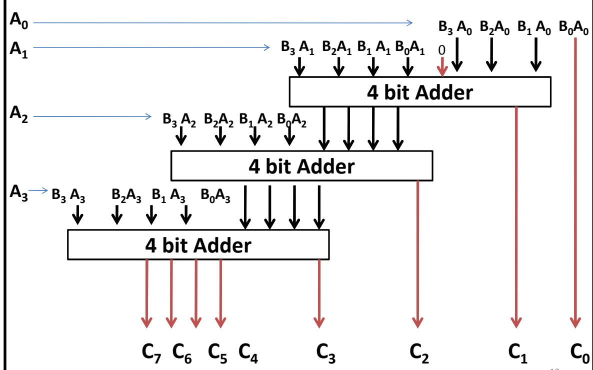
Binary Multiplier: 4 Bits



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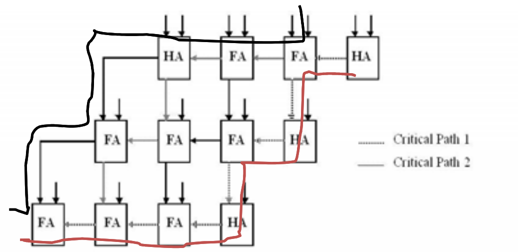
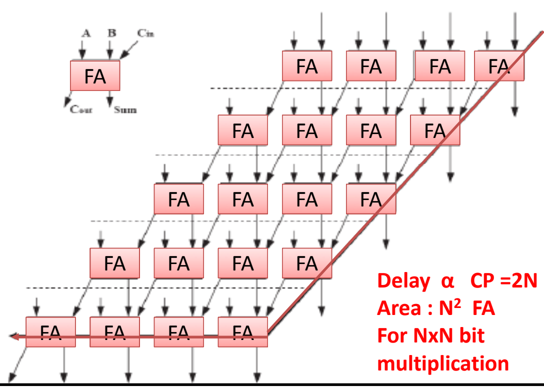
Area & Delay (Critical Path): Multiplier



Delay α CP =2N
Area : N^2 FA
For NxN bit multiplication

Area & Delay (Critical Path): Multiplier



Delay α CP =2N
Area : N^2 FA
For NxN bit multiplication

Efficient Adder Design

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Adder Universal Use

- Adder : A = B + C
- Substractor: A = B + (-C), 2's complement
- Compare : C = A> B ? 1 : 0 , (A-B > 0) ? 1 : 0
  - Special case of compare with 0
- Multiply
- Divide
- Mod
- Floating point: Add/sub/mul...

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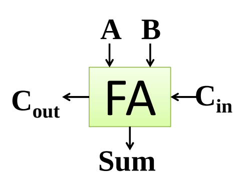
Adding Two One-bit Operands

- One-bit Full adder



Sum = A ⊕ B ⊕ Cin
Cout = A.B + B.Cin + A.Cin

Cin	A	B	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

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Addition of Two N-Bit numbers

$$x + y + c_{in} = 2^n c_{out} + s$$

The solution:

$$s = (x + y + c_{in}) \bmod 2^n$$

$$c_{out} = 1 \text{ if } (x + y + c_{in}) \geq 2^n \text{ else } 0$$

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Example

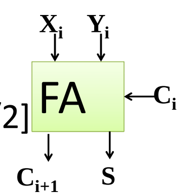
- 011110 + 101101 = 1 (x 2^6)+ 001011
- X=30, Y=45
- 30 + 45 = 75= 2^6 x 1 + 11
- Solution
  - S= (30+45+0) % 2^6=11
  - Cout= 1 if (30+45+0 >= 2^6) else 0 = 1

Primitive module FA

$$x_i + y_i + c_i = 2\ c_{i+1} + s_i$$

with solution

- $s_i = (x_i + y_i + c_i) \bmod 2$
- $c_{i+1} = \text{floor}\ [\ (x_i + y_i + c_i)/2]$



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N-Bit Ripple-Carry Adder: Series of FA Cells

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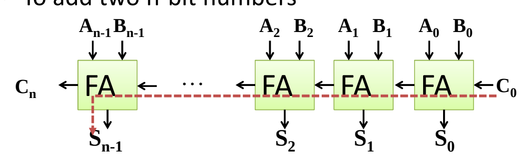

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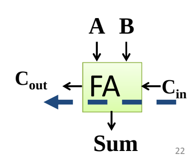

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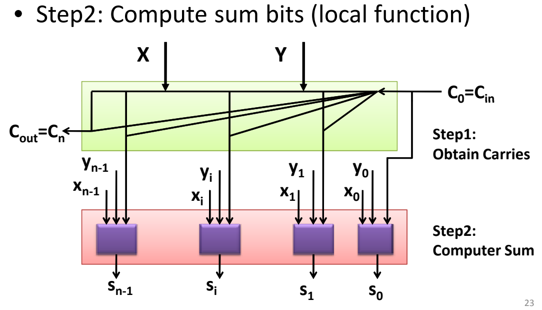
Adder Schemes

- Step1: Obtain carries
  - (Carry at i depends on j < i), Non-trivial to do fast
- Step2: Compute sum bits (local function)



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Mathematically: C_i & S_i

- C_i =FuncC (x_{i-1}, ..,x_0, y_{i-1}, ..,y_0, c_{in})
- S_i =FuncS (x_i, y_i, c_i)

  = (x_i + y_i + c_i) mod 2

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