Instructions

Ensure that you get a Bubble Sheet which has # 151 printed as your Exam Copy #.

- Check your name: E1
- Check your roll #: 151

Section T

TQ 1 For any two events A and B, if $P(A \cup B) = P(A) + P(B)$ then A and B are mutually exclusive.

A True B False

TQ 2 For any random variable X, $\mathrm{Var}(X) \leq \mathrm{E}(X)$.

A True B False

TQ 3 Let S be a sample space on which a probability function P is defined. Then, there exists atleast one outcome $s \in S$ such that $P(s) \neq 0$.

A True B False

TQ 4 Let E and F be any two independent events such that $P(F) \neq 0, 1$. Then, $P(E|F) = P(E|F^c)$.

A True B False

TQ 5 If f and g are probability density functions of two (continuous) random variables, then their product fg is a probability density function.

A True B False

TQ 6 If two events are both independent and mutually exclusive, then at least one of them has a zero probability of occurrence.

A True B False

TQ 7 Let E and F be any two independent events such that $P(F) \neq 0$. Then, $P(E|F) = P(E^c|F)$.

A True B False

TQ 8 On any sample space with at least two outcomes, there are infinitely many probability functions.

A True B False

Section S

SQ 9 Let X be a random variable. For any two real numbers α and β, $\mathrm{Var}(\alpha X + \beta)$ equals

A $\alpha^2\mathrm{Var}(X)$

B $\alpha^2\mathrm{Var}(X)+\beta^2$

C $\alpha(\mathrm{Var}(X))^2 + \beta^2$

D $\mathrm{Var}(X)$

E $\alpha(\mathrm{Var}(X))^2 + \beta$

F $\alpha^2(\mathrm{Var}(X))^2$

G $\alpha\mathrm{Var}(X)+\beta$

H $\alpha(\mathrm{Var}(X))^2$

I $\alpha\mathrm{Var}(X)$

SQ 10 The random variable X has probability density function

$$f(x) = \begin{cases} ax + bx^2 & \text{if } 0 < x < 1 \\ 0 & \text{if otherwise} \end{cases}$$

for some real numbers a and b. If $E(X) = 0.6$, what is $\mathrm{Var}(X)$?

A .0066 B .0216 C .066 D .06 E .36 F .66 G 6.6

SQ 11 A geometric random variable X satisfies $3125 \cdot P(X = 14) = 243 \cdot P(X = 9)$. Then, $E(X)$ is

[A] $\frac{243}{3125}$ [B] $\frac{2}{5}$ [C] $\frac{3}{5}$ [D] 1 [E] $1\frac{1}{3}$ [F] 1.5 [G] 2.5 [H] $\frac{3125}{243}$

SQ 12 A binomial random variable X with parameters n and p satisfies $E(X) = 3$ and $\mathrm{Var}(X)$=2. Then (n, p) is

[A] $(6, \frac{2}{3})$ [B] $(9, \frac{1}{3})$ [C] $(12, \frac{1}{4})$ [D] $(9, \frac{2}{3})$ [E] $(6, \frac{1}{2})$ [F] $(2, \frac{1}{3})$

SQ 13 A cancer diagnosis test is 95% accurate both on those who have and do not have the disease. Assume 0.4% of the population has cancer. Suppose the test result is positive for cancer on Ram. What is the probability that Ram has cancer?

[A] $\frac{19}{2843}$ [B] $\frac{19}{1472}$ [C] $\frac{19}{681}$ [D] $\frac{19}{268}$ [E] $\frac{19}{129}$ [F] $\frac{19}{43}$ [G] $\frac{19}{20}$

SQ 14 Let X be a Poisson random variable with parameter λ. Then, $E(X^3)$ equals

[A] $e^{-\lambda}\lambda^3$
[B] $\lambda^3 + 3\lambda^2 + \lambda$
[C] $e^{-\lambda}\left(\lambda^3 + \lambda\right)$
[D] λ
[E] $\lambda^3 + \lambda$
[F] $e^{-\lambda}\left(\lambda^3 + 2\lambda^2 + \lambda\right)$
[G] λ^3
[H] $\lambda^3 + 2\lambda^2 + \lambda$
[I] $e^{-\lambda}\left(\lambda^3 + 3\lambda^2 + \lambda\right)$

SQ 15 Let X be a continuous random variable such that its cumulative distribution function $F : \mathbb{R} \to (0, 1)$ is bijective. Then, the expected value of $F(X)$ is

[A] 0 [B] $\frac{1}{e}$ [C] $\frac{1}{\sqrt{2}}$ [D] 0.5 [E] 1 [F] $\sqrt{2}$ [G] 2 [H] e

SQ 16 Two symmetric dice have both had two of their sides painted black, two other sides painted red, one of the remaining sides painted yellow and the last side painted white. When this pair of dice is rolled, what is the probability that they both land on the same color?

[A] $\frac{3}{36}$ [B] $\frac{6}{36}$ [C] $\frac{9}{36}$ [D] $\frac{10}{36}$ [E] $\frac{12}{36}$ [F] $\frac{18}{36}$ [G] $\frac{24}{36}$ [H] $\frac{30}{36}$

SECTION M

►►MQ 17 Let X be a normal random variable with mean μ and variance σ^2. Then, which of the following are necessarily TRUE?

[A] $X - \mu$ has mean 0 and variance σ^2
[B] $X - \mu^2$ has mean 0 and variance σ^2
[C] $\frac{X-\mu}{\sigma}$ is a uniform random variable over $[-1, 1]$
[D] $\frac{X-\mu}{\sigma}$ has mean 0 and variance 1
[E] For any two real numbers a and b, $aX + b$ has mean $a\mu + b$ and variance $a^2\sigma^2$
[F] $\frac{X-\mu}{\sigma^2}$ has mean 0 and variance 1

►►MQ 18 Let A, B and C be any three events in a sample space S. Which of the following are necessarily TRUE?

[A] If $P(A \cap B \cap C) = P(A)P(B)P(C)$, then A, B and C are independent
[B] If A, B and C are independent, then $P(A \cap B) = P(A)P(B)$
[C] If A and B are independent, then $P(A \cap B) = P(A)P(B)$
[D] If $P(A \cap B) = P(A)P(B)$, then A and B are independent
[E] If A, B and C are independent, then $P(A \cap B \cap C) = P(A)P(B)P(C)$
[F] If $P(A \cup B \cup C) = P(A) + P(B) + P(C)$, then A, B and C are mutually exclusive

►►MQ 19 Which of the following are necessarily TRUE?

- A If X is an exponential random variable, then for any real numbers s and t, $P(X > s+t|X > t) = P(X > s)$
- B If X is a geometric random variable, then for any two real numbers $a \leq b$, $P(a \leq X \leq b) = P(-b \leq X \leq -a)$
- C If X is a normal random variable, then for any two real numbers $a \leq b$, $P(a \leq X \leq b) = P(-b \leq X \leq -a)$
- D If X is a geometric random variable, then for any natural numbers n and k, $P(X = n+k|X > n) = P(X = k)$

►►MQ 20 Let A and B be two sets randomly picked (with replacement) from the collection of all subsets of $T = \{1, 2, 3, 4, 5\}$. Which of the following are TRUE?

- A $P(A^c \subset B) = \frac{343}{3125}$
- B $P(A \subset B^c) = \frac{1}{32}$
- C $P(A^c \subset B) = \frac{343}{1024}$
- D $P(A \cup B = T) = \frac{343}{1024}$
- E $P(A \subset B) = \frac{32}{343}$
- F $P(A \cap B = \emptyset) = \frac{343}{1024}$
- G $P(A \subset B^c) = \frac{343}{1024}$
- H $P(A \cup B = T) = \frac{343}{3125}$
- I $P(A \subset B) = \frac{343}{1024}$
- J $P(A \cap B = \emptyset) = \frac{1}{343}$

SECTION J

JQ 21 (Jackpot: 10 marks for full solution, NO partial credit.) Prove Waring's Theorem: Let $A_1, A_2, \ldots, A_n$ be a collection of $n(\in \mathbb{N})$ events. For a $k \in \{1, 2, \ldots, n\}$, let Θ_k be the event that exactly k of the given n events occur. Prove

$$P(\Theta_k) = \sum_{i=0}^{n-k} (-1)^i \binom{k+i}{k} S_{k+i}, \qquad \text{where} \qquad S_j = \sum_{i_1 < i_2 < \cdots < i_j} P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_j})$$