Probability Theory and Random Processes (MA225)

Lecture Slides

Lecture 33 (November 14, 2019)

Poisson Thinning

Consider a Poisson process having rate λ. Suppose that each time an event occurs it is classified as either a type I or a type II event. Suppose further that each event is classified as a type I event with probability p or a type II event with probability $1 - p$, independently of all other events. Let $N_1(t)$ and $N_2(t)$ denote respectively the number of type I and type II events occurring in $[0, t]$. Note that $N(t) = N_1(t) + N_2(t)$.

Theorem: $N_1(\cdot)$ and $N_2(\cdot)$ are both Poisson processes with respective rates λp and $\lambda(1 - p)$. Furthermore, the two processes are independent.

Example

Example 1: Suppose non-negative offers to buy a house that you want to sell arrive according to a Poisson process with rate λ. Assume that each offer is the value of a CRV having PDF $f(\cdot)$ such that $f(x) \neq 0\,, \forall x > 0$ and $\int_0^\infty xf(x)dx > c/\lambda$. Once the offer is present to you must either accept it or reject it and wait for the next offer. Suppose you incur a loss at the rate c per unit time until the house is sold. Your objective is to maximize your expected net return which is equal to the amount you receive minus the total cost incurred. Suppose you employ the strategy of accepting the first offer that is greater than some specified value y. What is the best value of y? What is the maximal expected net return?

Cond Dist of Arrival Times

Theorem: Given that $N(t) = n$, the n arrival times $S_1, \ldots, S_n$ have the same distribution as the order statistics corresponding to n independent random variables uniformly distributed on the interval $(0, t)$.

Example 2: Suppose insurance claims arrive at an insurance firm according to a Poisson process with rate λ; the successive claim amounts are independent random variables having mean μ, and are independent of the claim arrival times. Let S_i and C_i denote, respectively, the time and the amount of the ith claim. Define

$$D(t) = \sum_{i=1}^{N(t)} e^{-\alpha S_i} C_i \, ,$$

where $\alpha > 0$ is the discount factor. Find $E(D(t))$.