Probability Theory and Random Processes (MA225)

Lecture Slides

Lecture 34 (November 15, 2019)

Example

Example 1: Suppose that items arrive at a processing plant in accordance with a Poisson process with rate λ. At a fixed time T, all items are dispatched from the system. The problem is to choose an intermediate time, $t \in (0, T)$, at which all items in the system are dispached, so as to minimize the total expected wait of all items.

Compound Poisson Process

Def: A stochastic process $\{X(t) : t \geq 0\}$ is said to be a compound Poisson process if it can be represented as

$$X(t) = \sum_{i=1}^{N(t)} Y_i,$$

where $\{N(t)\}$ is a Poisson process and Y_i's are i.i.d. random variables, also independent of $N(t)$.

Remark: If $N(t)$ is Poisson with rate λ, then $E(X(t)) = \lambda t E(Y_1)$ and $Var(X(t)) = \lambda t E(Y_1^2)$.

Example

Example 2: Suppose that buses arrive at a sporting event in accordance with a Poisson process, and suppose that the number of fans in each bus are independent and indentically distributed. Then $\{X(t) : t \geq 0\}$ is a compound Poisson process, where $X(t)$ denotes number of fans who have arrived by time t.

Example 3: Suppose customers leave a supermarket in accordance with a Poisson process. If Y_i, the amount spent by the ith customer for $i = 1, 2 \ldots$ are i.i.d., then $\{X(t)\}$ is a compound Poisson process, where $X(t)$ denotes the amount of money spent upto time t.

Example 4: Suppose that families migrate into a territory according to a Poisson process with rate $\lambda = 2$ per week. If the number of people in each family is independent and takes the values 1,2,3,4 with respective probabilities $1/6, 1/3, 1/3, 1/6$, then what is the expected value and variance of the number of individuals migrating into the territory during a fixed 5 week period.

Thank you all.
All the best for your End-Sem.