Probability Theory and Random Processes (MA225)

Lecture Slides

Lecture 16 (September 09, 2019)

Let (X, Y) be a random vector.
Def: $Var(X|Y) = h(Y)$ where $h(y) = E((X - E(X|Y))^2|Y = y)$
$= E(X^2|Y = y) - (E(X|Y = y))^2$.

Theorem: $Var(X) = E(Var(X|Y)) + Var(E(X|Y))$.

Example 1: Let $X_0, X_1, X_2, \dots, X_n$ be a sequence of i.i.d. RVs with mean μ and variance σ^2. Let $N \sim Bin(n, p)$, independent of $\{X_i\}$.

Define $S = \sum_{i=0}^{N} X_i$. Find $Var(S)$.

Computing Probability by Conditioning

$$P(E) = \begin{cases} \displaystyle\sum_y P(E|Y=y)P(Y=y) & \text{for} \quad Y \text{ discrete} \\ \displaystyle\int_{-\infty}^{\infty} P(E|Y=y)f_Y(y)dy & \text{for} \quad Y \text{ continuous}. \end{cases}$$

Example 2: Let X and Y be independent CRVs having PDFs f_X and f_Y, respectively. Compute $P(X < Y)$.

Example 3: Let X and Y be i.i.d. CRVs having common PDF f. Then $P(X < Y) = P(X > Y) = 0.5$. And $P(X = Y) = 0$.

Example 4: Suppose X and Y are two independent RVs, either discrete or continuous. What is the distribution of $X + Y$?