Model Solution

Problem Set 03

(MA225)

$\boxed{Q1(a)}$

$P(X=1) = F_X(1) - F_X(1-) = \frac{1}{2} - \frac{1}{2} = 0.$

$P(1<X<2) = F_X(2-) - F_X(1) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$

$P(1 \le X < 2) = F_X(2-) - F_X(1-) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$

$P(1<X\le 2) = F_X(2) - F_X(1) = 1 - \frac{1}{2} = \frac{1}{2}.$

$P(1\le X \le 2) = F_X(2) - F_X(1-) = 1 - \frac{1}{2} = \frac{1}{2}.$

$P(X \ge 1) = 1 - F_X(1-) = 1 - \frac{1}{2} = \frac{1}{2}.$ $\square$

$\boxed{Q1(b)}$ The set of discontinuity points of F_X is $\{2\}$.
As $P(X=2) = F_X(2) - F_X(2-) = 1 - 2/3 = 1/3 \neq 1$, X is not a discrete random variable. $\square$

$\boxed{Q1(c)}$ As the CDF $F_X(\cdot)$ is not continuous, X is not a continuous random variable. $\square$

$\boxed{Q2(a)}$ As $f_X(\cdot)$ is a PDF,

$$\int_{-0.5}^{1/2} (k - |x|)\,dx = 1 \Rightarrow k = 5/4. \quad \square$$

$\boxed{Q2(b)}$

$$P(X<0) = \int_{-1/2}^{0} \left(\frac{5}{4} - |x|\right) dx = \int_{-1/2}^{0} \left(\frac{5}{4} + x\right) dx = \left[\frac{5}{4}x + \frac{x^2}{2}\right]_{-1/2}^{0}$$

$$= \frac{1}{2}.$$

$$\therefore P(X\le 0) = \int_{-1/2}^{0}\left(\frac{5}{4}-|x|\right)dx = \frac{1}{2}.$$

$$P(0< X \le \tfrac{1}{4}) = \int_{0}^{1/4}\left(\frac{5}{4}-|x|\right)dx = \left[\frac{5}{4}x-\frac{x^2}{2}\right]_0^{1/4} = \frac{9}{32}.$$

$$P(0\le X < \tfrac{1}{4}) = \int_{0}^{1/4}\left(\frac{5}{4}-|x|\right)dx = \frac{9}{32}$$

$$P(-\tfrac{1}{2}\le X \le \tfrac{1}{4}) = \int_{-1/2}^{1/4} f_X(x)\,dx = \int_{-1/2}^{1/4}\left(\frac{5}{4}-|x|\right)dx$$

$$= \int_{-1/2}^{0}\left(\frac{5}{4}+x\right)dx + \int_{0}^{1/4}\left(\frac{5}{4}-x\right)dx$$

$$= \frac{1}{2}+\frac{9}{32} = \frac{25}{32}. \qquad \square.$$

$\boxed{Q2(c)}$

$$P(X>\tfrac{1}{4} \mid |X| > \tfrac{2}{5}) = \frac{P(X>\frac{2}{5})}{P(|X|>2/5)}$$

$$= \frac{\int_{2/5}^{1/2}(k-x)\,dx}{\int_{-1/2}^{-2/5}(k+x)\,dx + \int_{2/5}^{1/2}(k-x)\,dx}$$

$$= \frac{\left[\frac{5}{4}x-\frac{x^2}{2}\right]_{2/5}^{1/2}}{\left[\frac{5}{4}x+\frac{x^2}{2}\right]_{-1/2}^{-2/5} + \left[\frac{5}{4}x-\frac{x^2}{2}\right]_{2/5}^{1/2}}$$

$$= \frac{2/25}{2/25+2/25} = \frac{1}{2}.$$

$$\cancel{P\left(\frac{1}{8} < X < \frac{1}{2} \,\middle|\, \frac{1}{10} < X < \frac{1}{5}\right) = \frac{P\left(\frac{1}{8} < X < \frac{1}{5}\right)}{P\left(\frac{1}{10} < X < \frac{1}{5}\right)}}$$

$$P\left(\frac{1}{10} < X < 1 \,\middle|\, \frac{1}{10} < X < \frac{1}{5}\right) = \frac{P\left(\frac{1}{10} < X < \frac{1}{5}\right)}{P\left(\frac{1}{10} < X < \frac{1}{5}\right)} = 1. \quad \square$$

$\boxed{\text{Q2(d)}}$ The CDF of X is

$$F_X(x) = \begin{cases} 0 & \text{if } x < -\frac{1}{2} \\ \displaystyle\int_{-1/2}^{x} \left(\frac{5}{4} + t\right) dt & \text{if } -\frac{1}{2} \leq x < 0 \\ \displaystyle\int_{-1/2}^{0} \left(\frac{5}{4} + t\right) dt + \int_{0}^{x} \left(\frac{5}{4} - t\right) dt & \text{if } 0 \leq x < \frac{1}{2} \\ 1 & \text{if } x \geq \frac{1}{2} \end{cases}$$

$$= \begin{cases} 0 & \text{if } x < -\frac{1}{2} \\ \dfrac{2x^2 + 5x + 2}{4} & \text{if } -\frac{1}{2} \leq x < 0 \\ \dfrac{-2x^2 + 5x + 2}{4} & \text{if } 0 \leq x < \frac{1}{2} \\ 1 & \text{if } x \geq \frac{1}{2} \end{cases}$$

$\square$.

$\boxed{Q3}$ The PMF of $Y_1 = X^2$ is

$$f_{Y_1}(y) = \begin{cases} \binom{n}{\sqrt{y}} p^{\sqrt{y}} (1-p)^{n-\sqrt{y}} & \text{if } y = 0, 1, 4, 9, \ldots, n^2 \\ 0 & \text{o.w.} \end{cases}$$

The PMF of $Y_2 = \sqrt{X}$ is

$$f_{Y_2}(y) = \begin{cases} \binom{n}{y^2} p^{y^2} (1-p)^{n-y^2} & \text{if } y = 0, 1, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{n}. \\ 0 & \text{o.w.} \end{cases}$$

$\square$.

$\boxed{Q4}$ The CDF of $Y = \frac{X}{X+1}$ is

$$F_Y(y) = P\left(\frac{X}{X+1} \le y\right)$$

$$= P(X \le y + Xy) \quad \text{as } P(X+1>0) = 1.$$

$$\cancel{= P\left(X \le \frac{y}{1-y}\right)}$$

$$= P(X(1-y) \le y)$$

$$= \begin{cases} P\left(X \le \frac{y}{1-y}\right) & \text{if } y < 1 \\ 1 & \text{if } y \ge 1 \end{cases}$$

$$= \begin{cases} 0 & \text{if } y < 0. \\ P\left(X \le \frac{y}{1-y}\right) & \text{if } 0 \le y < 1 \\ 1 & \text{if } y \ge 1 \end{cases}$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ \displaystyle\sum_{x=0}^{\left[\frac{y}{1-y}\right]} \frac{1}{2}\left(\frac{2}{3}\right)^x & \text{if } 0 \le y < 1 \\ 1 & \text{if } y \ge 1 \end{cases}$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ \frac{3}{2}\left(1 - \left(\frac{2}{3}\right)^{\left[\frac{y}{1-y}\right]+1}\right) & \text{if } 0 \le y < 1 \\ 1 & \text{if } y \ge 1 \end{cases}$$

The PMF of Y is

$$g_Y(y) = \begin{cases} \frac{1}{2}\left(\frac{2}{3}\right)^{\frac{y}{1-y}} & \text{if } y = 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots \\ 0 & \text{o.w.} \end{cases}$$

$\square$.

$\boxed{Q5}$ The CDF of $Y = X^+$ is

$$F_{X^+}(x) = P(X^+ \le x) = \begin{cases} 0 & \text{if } x < 0 \\ P(\max\{X, 0\} \le x) & \text{if } x \ge 0 \end{cases}$$

For $x \ge 0$, $F_{X^+}(x) = P(X \le x) = F_X(x)$.

Hence $F_{X^+}(x) = \begin{cases} 0 & \text{if } x < 0 \\ F_X(x) & \text{if } x \ge 0. \end{cases}$

Now $F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \int_{-1}^{x} \frac{1+t}{2}\,dt & \text{if } -1 \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$

$$= \begin{cases} 0 & \text{if } x < -1 \\ \frac{(x+1)^2}{4} & \text{if } -1 \le x < 1 \\ 1 & \text{o.w.} \end{cases}$$

Hence the CDF of X^+ is

$$F_{X^+}(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{(x+1)^2}{4} & \text{if } 0 \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

At $x = 0$, $F_{X^+}(\cdot)$ is discontinuous, hence X^+ is not a CRV and it does not have a PDF.

The CDF of $X^- = \text{Max}\{-X, 0\}$ is

$$F_{X^-}(x) = P(X^- \le x)$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ P(-X \le x) & \text{if } x \ge 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ P(X \ge -x) & \text{if } x \ge 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ 1 - F_X(-x) & \text{if } x \geq 0 \end{cases}$$

$$\cancel{= \begin{cases} 0 & \text{if } x < 0 \\ 1 - \frac{(x+1)^2}{4} & \text{if } 0 \leq x < 1 \end{cases}}$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ 1 - \frac{(1-x)^2}{4} & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

As $F_{X^-}(\cdot)$ has a jump at $x = 0$, X^- is not CRV.

The CDF of $Y_1 = |X|$ is

$$F_{Y_1}(y) = P(|X| \leq y)$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ P(-y \leq X \leq y) & \text{if } y \geq 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ F_X(y) - F_X(-y) & \text{if } y \geq 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ y & \text{if } 0 \leq y < 1 \\ 1 & \text{if } y \geq 1 \end{cases}$$

It is clear that $f(y) = \begin{cases} 1 & \text{if } 0 < y < 1 \\ 0 & \text{o.w} \end{cases}$

The CDF of $Y_2 = X^2$ is

$$F_{X_2}(y) = P(X^2 \le y)$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ P(-\sqrt{y} \le X \le \sqrt{y}) & \text{if } y \ge 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ F_X(\sqrt{y}) - F_X(-\sqrt{y}) & \text{if } y \ge 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } y < 0 \\ \sqrt{y} & \text{if } 0 \le y < 1 \\ 1 & \text{if } y > 1 \end{cases}$$

It ~~to~~ is clear that

$$f_{Y_1}(y) = \begin{cases} \frac{1}{2\sqrt{y}} & \text{if } 0 < y < 1 \\ 0 & \text{o.w.} \end{cases}$$

is a PDF corresponding to $F_{Y_2}(\cdot)$. $\square$.

$\boxed{Q6}$ Let X be the price money obtained if ~~the~~ Q1 is answered first.

Let Y be the price money obtained if Q2 is answered first.

Here $P(X=100) = 0.8 \times 0.5 = 0.4$

$P(X=300) = 0.8 \times 0.5 = 0.4$

$P(X=0) = 0.2$

Hence $E(X) = 0 \times 0.2 + 100 \times 0.4 + 300 \times 0.4$

$= 160.$

$P(Y=0) = 0.5,$

$P(Y=200) = 0.5 \times 0.2 = 0.1$

$P(Y=300) = 0.5 \times 0.8 = 0.4$

Hence $E(Y) = 0 \times 0.5 + 200 \times 0.1 + 300 \times 0.4$

$= 140.$

Hence one should answer Q1 first to maximize the expected value of total prize money. $\square$

$\boxed{Q7}$

Let x be the RV that denotes the number of items produced by the factory in a week.

Here $E(x) = 500.$

Now $P(X > 1000) = P(|x| > 1000) \leq \frac{E(|x|)}{1000} = \frac{1}{2}$ Using Markov Inequality $\square$.

as $\cancel{E(x) = E(|x|)}$ $P(X<0) = 0.$

The variance of X is $Var(X) = 100$

$P(400 < X < 600) = P(-100 < X - 500 < 100)$

$= P(|X-500| < 100)$

$= 1 - P(|X - 500| \geq 100)$

$\geq 1 - \frac{100}{100^2}$ [using Chebyshev's Ineq.] $\square$

$= 0.99.$

Q8 Let X denote the number of throws to obtain a 6.

Then PMF of X is

$$f_X(x) = \begin{cases} \frac{1}{6}\left(\frac{5}{6}\right)^{x-1} & x = 1, 2, \ldots \\ 0 & \text{o.w.} \end{cases}$$

Now $$\sum_{x=1}^{\infty} |x| f_X(x) = \sum_{x=1}^{\infty} x f_X(x) = \frac{1}{6}\sum_{x=1}^{\infty} x\left(\frac{5}{6}\right)^{x-1}$$

The partial sum of the series, S_n, can be expressed as

$$6S_n = 1 + 2\left(\frac{5}{6}\right)^1 + \cdots + n\left(\frac{5}{6}\right)^{n-1}$$

$$\frac{5}{6}6S_n = \left(\frac{5}{6}\right)^1 + \cdots + (n-1)\left(\frac{5}{6}\right)^{n-1} + n\left(\frac{5}{6}\right)^n$$

$$S_n = \frac{1-(5/6)^n}{1-5/6} - n\left(\frac{5}{6}\right)^n$$

$$= 6\left\{1-\left(\frac{5}{6}\right)^n\right\} - n\left(\frac{5}{6}\right)^n$$

Now $\lim_{n\to\infty} S_n = 6.$

Hence $\sum_{x=1}^{\infty} |x| f_X(x)\,dx < \infty$ and $E(X) = 6.$ $\square$

Q9 Let X denote the time (in hour) to get the class.

Here $P\left(X = \frac{2}{5}\right) = 0.6$ and $P\left(X = \frac{2}{30}\right) = 0.4.$

Hence $E(X) = \frac{2}{5}\times 0.6 + \frac{2}{30}\times 0.4 = \frac{4}{15}$ hours $\square$

$\boxed{Q10}$

$\boxed{Q10(a)}$

$$P\left((X - E(X))^2 \geq 0\right) = 1$$

$$\Rightarrow \quad S_{(X-E(X))^2} \subset [0, \infty)$$

$$\Rightarrow \quad Var(X) = E\left[(X - E(X))^2\right] \geq 0. \qquad \square$$

$\boxed{Q10(b)}$ $Var(X) \geq 0 \Rightarrow E(X^2) - (E(X))^2 \geq 0 \Rightarrow E(X^2) \geq (E(X))^2 \; \square$

$\boxed{Q10(c)}$

$$Var(a+bX) = E\left[(a+bX - E(a+bX))^2\right]$$

$$= E\left[(a+bX - a - bE(X))^2\right]$$

$$= b^2 E\left[(X - E(X))^2\right] \qquad \square.$$

$$= b^2 \, Var(X).$$

$\boxed{Q11}$ The CDF of X is

$$F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ P(X \leq x) & \text{if } 1 \leq x < N \\ 1 & \text{if } x \geq N. \end{cases}$$

Now for $1 \leq x < N$,

$$P(X \leq x) = P(\text{all of } n \text{ drawn tickets are } \leq x)$$

$$= \left(\frac{[x]}{N}\right)^n$$

Hence

$$F_X(x) = \begin{cases} 0 & \text{if} \quad x < 1 \\ \left(\frac{[x]}{N}\right)^n & \text{if} \quad 1 \le x < N \\ 1 & \text{if} \quad x \ge N. \end{cases}$$

The ~~di~~, set of discontinuity of $F_X(\cdot)$ is

$$\{1, 2, \ldots, N\}.$$

For $x \in \{1, 2, \ldots, N\}$,

$$P(X = x) = F_X(x) - F_X(x-)$$

$$= \left(\frac{x}{N}\right)^n - \left(\frac{x-1}{N}\right)^n$$

As $\sum_{x=1}^{N} P(X = x) = 1$, X is a DRV with PMF

$$f_X(x) = \begin{cases} \dfrac{x^n - (x-1)^n}{N^n} & \text{if} \quad x = 1, 2, \ldots, N \\ 0 & \text{o.w.} \end{cases}$$

$$\Rightarrow E(X) = \sum_{x=1}^{N} x \times \frac{x^n - (x-1)^n}{N^n}.$$

$$= \frac{1}{N^n}\left\{\sum_{x=1}^{N} x^{n+1} - \sum_{x=1}^{N} x(x-1)^n\right\}$$

$$= \frac{1}{N^n}\sum_{x=1}^{N}\left\{x^{n+1} - (x-1)^{n+1} - (x-1)^n\right\}$$

$$= \frac{1}{N^n}\left\{N^{n+1} - \sum_{x=1}^{N-1} x^n\right\}$$

$$= N - \frac{1}{N^n}\sum_{x=1}^{N-1} x^n \qquad \square.$$

12. $$EX = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^1 x \cdot \frac{x}{2}\,dx + \int_1^2 x \cdot \frac{1}{2}\,dx + \int_2^3 x \cdot \frac{(3-x)}{2}\,dx$$

$$= \frac{1}{6} + \frac{3}{4} + \frac{7}{12} = \frac{2+9+7}{12} = \frac{18}{12} = \frac{3}{2}.$$

$$EX^2 = \int_0^1 x^2 \cdot \frac{x}{2}\,dx + \int_1^2 x^2 \cdot \frac{1}{2}\,dx + \int_2^3 x^2 \frac{(3-x)}{2}\,dx$$

$$= \frac{1}{8} + \frac{7}{6} + \frac{11}{8} = \frac{3+28+33}{24} = \frac{\cancel{64}^{8}}{\cancel{24}_{3}} = \frac{8}{3}.$$

Thus $EY = E[X^2 - 5X + 3]$

$$= EX^2 - 5EX + 3 = \frac{8}{3} - \frac{15}{2} + 3$$

$$= \frac{16-45+18}{6} = -\frac{11}{6}.$$

13. $$EX = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_{-\infty}^{\infty} (x-\mu+\mu) f_X(x)\,dx$$

$$= \int_{-\infty}^{\infty} (x-\mu) f_X(x)\,dx + \mu$$

$$= \int_{-\infty}^{\infty} x f_X(x+\mu)\,dx + \mu$$

$$= 0$$ [$\because$ the first integrand is an odd function and EX exists].

14. Let Y denote the score. Then

$$P(Y=4) = P\left(0 \le X < \frac{1}{\sqrt{3}}\right)$$

$$P(Y=3) = P\left(\frac{1}{\sqrt{3}} \le X < 1\right)$$

$$P(Y=2) = P\left(1 < X \le \sqrt{3}\right)$$

Thus $P(Y=4) = \frac{2}{\pi}\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{1+x^2}\,dx = \frac{2}{\pi} \tan^{-1} x \Big|_0^{1/\sqrt{3}}$

$$= \frac{2}{\pi} \cdot \frac{\pi}{6} = \frac{1}{3}.$$

$$P(Y=3) = \frac{2}{\pi}\int_{\frac{1}{\sqrt{3}}}^{1} \frac{1}{1+x^2}\,dx = \frac{2}{\pi}\left[\frac{\pi}{4} - \frac{\pi}{6}\right]$$

$$= \frac{2}{\pi} \cdot \frac{\pi}{12} = \frac{1}{6}.$$

$$P(Y=2) = \frac{2}{\pi}\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx = \frac{2}{\pi}\left(\frac{\pi}{3} - \frac{\pi}{4}\right]$$

$$= \frac{1}{6}.$$

Thus expected score is $EY = \frac{4}{3} + \frac{3}{6} + \frac{2}{6}$

$$= \frac{13}{6}.$$

15.

$$E|X|^\alpha = \int_{-\infty}^{\infty} |x|^\alpha f_X(x)\,dx$$

$$= \int_{|x|<1} |x|^\alpha f_X(x)\,dx + \int_{|x|\geq 1} |x|^\alpha f_X(x)\,dx$$

$$\leq \int_{|x|<1} f_X(x)\,dx + \int_{|x|\geq 1} |x|^\beta f_X(x)\,dx \quad [\because \alpha \leq \beta]$$

$$\leq \int_{-\infty}^{\infty} f_X(x)\,dx + \int_{-\infty}^{\infty} |x|^\beta f_X(x)\,dx$$

$$= 1 + E|X|^\beta < \infty.$$

16.

If $X \sim Poi(\lambda)$, then $E[e^{tX}] = \sum_{k=0}^{\infty} e^{tk} e^{-\lambda} \frac{\lambda^k}{k!}$

$$= e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}.$$

If $X \sim U(a,b)$, then $E(e^{tX}) = \frac{1}{b-a} \int_a^b e^{tx}\,dx$

$$= \frac{e^{tb} - e^{ta}}{t(b-a)}.$$

17. Trivial.

18. Consider the RV, X', having PMF,

$$f_{X'}(-1) = \frac{1}{8}, \quad f_{X'}(1) = \frac{1}{4}, \quad f_{X'}(2) = \frac{1}{8},$$
$$f_{X'}(3) = \frac{1}{2}, \quad f_{X'}(x) = 0 \text{ elsewhere}.$$

Then it is easy to see that X' has the given MGF. So by uniqueness theorem of MGF, X has the same distribution at X'.

19. Observe that the given MGF is the MGF of $U(-2, 1)$ RV. Thus

$$P(Y \le x) = P(X^2 \le x) = P(-\sqrt{x} \le X \le \sqrt{x})$$

where $X \sim U(-2,1)$ and $x \geq 0$.

$$P(Y \le x) = 0 \quad \text{for } x < 0.$$

Thus

$$
\begin{aligned}
F_Y(x) &= 0 && \text{for } x < 0 \\
&= \frac{1}{3}\int_{-\sqrt{x}}^{\sqrt{x}} dt && \text{for } 0 \le x < 1 \\
&= \frac{2\sqrt{x}}{3} && \text{for } 0 \le x < 1 \\
&= \frac{1}{3}\int_{-\sqrt{x}}^{1} dt && \text{for } 1 \le x < 4 \\
&= \frac{1+\sqrt{x}}{3} && \text{for } 1 \le x < 4 \\
&= 1 && \text{for } x \geq 4.
\end{aligned}
$$

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$$P(X \geq a) = P(e^{tX} \geq e^{ta}) \quad \text{for } 0 < t < h$$

$$\le \frac{E[e^{tX}]}{e^{ta}} = e^{-ta} M(t) \quad \text{[Markov Inequality}$$

$$P(X \le a) = P(e^{tX} \geq e^{ta}) \quad \left[\text{For } -h<t<0,\ e^{tx} \text{ is decreasing}\right]$$

$$\leq \frac{E[e^{tX}]}{e^{ta}} = e^{-ta} M(t). \quad \left[\text{Markov Ineq.}\right]$$

21. $$P(X \geq 2\mu) = P(|X| \geq 2\mu) \quad [\because P(X \leq 0) = 0]$$

$$\leq \frac{E|X|}{2\mu} = \frac{EX}{2\mu} = \frac{1}{2} \quad \left(\because P(X \leq 0) = 0,\ EX = E|X|\right]$$

22. $$P(-2 < X < 8) = P(-5 < X-3 < 5)$$

$$= P(|X-3| < 5) = 1 - P(|X-3| \geq 5)$$

$$\leq 1 - \frac{\operatorname{Var} X}{25} = 1 - \frac{13-9}{25} = 1 - \frac{4}{25}$$

$$= \frac{21}{25}.$$