

PROBABILITY THEORY AND RANDOM PROCESSES (MA225)

LECTURE SLIDES

Lecture 08 (August 09, 2019)

Transformation of RV

- ① If X is a random variable then $Y = g(X)$ is a random variable where $g : \mathbb{R} \rightarrow \mathbb{R}$.
- ② Our aim is to find the distribution (CDF/PMF/PDF) of $Y = g(X)$ for a known distribution of X .
- ③ There are mainly three techniques.

Technique 1

Example 1: Let the random variable X has the following PDF:

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find the distribution of $Y = [X]$.

Example 2: Let the random variable X has the following PDF:

$$f(x) = \begin{cases} \frac{|x|}{2} & \text{if } -1 < x < 1 \\ \frac{x}{3} & \text{if } 1 \leq x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find the distribution of $Y = X^2$.

Technique 2 for DRV

Example 1: Let the random variable X has the following PMF:

$$f(x) = \begin{cases} \frac{1}{7} & \text{if } x = -2, -1, 0, 1 \\ \frac{3}{14} & \text{if } x = 2, 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the distribution of $Y = X^2$.

Theorem: Let X be a DRV with PMF $f_X(\cdot)$ and support S_X . Let $g : \mathbb{R} \rightarrow \mathbb{R}$ and $Y = g(X)$. Then Y is a DRV with support $S_Y = \{g(x) : x \in S_X\}$ and PMF

$$f_Y(y) = \begin{cases} \sum_{x \in A_y} f_X(x) & \text{if } y \in S_Y \\ 0 & \text{otherwise,} \end{cases}$$

where $A_y = \{x \in S_X : g(x) = y\}$.

Example 1: $X \sim \text{Bin}(n, p)$. Find the distribution of $Y = n - X$.

Technique 2 for CRV

Theorem: Let X be a CRV with PDF $f_X(\cdot)$ and support S_X , which is an interval. Let $g : S_X \rightarrow \mathbb{R}$ be a differentiable function and either $g'(x) < 0$ for all $x \in S_X$ or $g'(x) > 0$ for all $x \in S_X$. Then the RV $Y = g(X)$ is a CRV with PDF

$$f_Y(y) = \begin{cases} f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right| & \text{for } y \in g(S_X) \\ 0 & \text{otherwise.} \end{cases}$$

Example 1: Let $X \sim U(0, 1)$, then $Y = -\ln X \sim \text{Exp}(1)$.

Example 2: Let $X \sim \text{Exp}(1)$, then find the distribution of $Y = X^2$.

Example 3: Let $X \sim N(0, 1)$, then find the distribution of $Y = X^2$.