Probability Theory and Random Processes (MA225)

Lecture Slides

Lecture 28 (November 01, 2019)

Example

Example 1:

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Example 2:

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/2 \end{bmatrix}$$

Example

Example 3: Consider a simple random walk: $S = \{0, \pm 1, \pm 2, \ldots\}$, $p_{i,\,i-1} = p = 1 - p_{i,\,i+1}$.

1. The chain is irreducible.
2. If $p \neq 1/2$, the state 0 is transient.
3. If $p = 1/2$, the state 0 is recurrent.

$[n! \sim n^{n+1/2} e^{-n} \sqrt{2\pi}\ ]$

Period

Def: The period of a state i is defined by the greatest common divisor of all integers $n \geq 1$ for which $p_{ii}^{(n)} > 0$, *i.e.*,

$$d(i) = \begin{cases} \gcd\left\{n \geq 1 : p_{ii}^{(n)} > 0\right\} & \text{if } \left\{n \geq 1 : p_{ii}^{(n)} > 0\right\} \neq \phi \\ 0 & \text{if } \left\{n \geq 1 : p_{ii}^{(n)} > 0\right\} = \phi. \end{cases}$$

Example 4: $S = \{0, \pm 1, \pm 2, \ldots\}$. $p_{i,\,i+1} = a$, $p_{i,\,i-1} = b$, $p_{ii} = c$, where $a + b + c = 1$, $a > 0$, $b > 0$, $c \geq 0$.

Theorem: If $i \leftrightarrow j$, then $d(i) = d(j)$.