Probability Theory and Random Processes (MA225)

Lecture Slides

Lecture 17 (September 12, 2019)

Def: Let (X, Y) be a random vector. Then

$$E\left(h(X,\ Y) | (X,\ Y) \in A\right) = \frac{E(h(X,\ Y) I_A(X,\ Y))}{P\left((X,\ Y) \in A\right)}.$$

Example 1: $X \sim Exp(1)$. Find $E\left(X | X \geq 2\right)$.

Example 2: $(X,\ Y)$ is uniform on unit square. Find $E\left(X | X + Y > 1\right)$.

Some Interesting Problems

Example 3: At a party n men throw their hats into the center of a room. The hats are mixed up and each men randomly selects one. Those who are choosing their own hats depart, while the others put their selected hats in the center of the room, mix them up, and then reselect. Also, suppose that this process continues until each individual has his own hat. Find $E(R)$, where R is the number of rounds that are necessary.

Some Interesting Problems

Example 4: Start with n strings, which of course have 2n ends. Then randomly pair the ends and tie together each pair. (Therefore you join each of the n randomly chosen pairs.) Let L be the number of resulting loops. Compute $E(L)$.

Some Interesting Problems

Example 5: Suppose you are typing with your eyes closed. Let T be the number of hits required to write the word TECHNICHE. Find $E(T)$.

Some Interesting Problems

Example 6: Let $U_1, U_2, \ldots$ be a sequence of i.i.d. uniform(0,1) random variables. Define

$$N = \min\{n \geq 2 : U_n > U_{n-1}\}$$

and

$$M = \min\{n \geq 1 : U_1 + U_2 + \ldots + U_n > 1\}.$$

Find $E(N)$ and $E(M)$.

See Ross, *Introduction to Probability Models*, 11th Edition, Page 124.