Probability Theory and Random Processes (MA225)

Lecture Slides

Lecture 22 (September 30, 2019)

Theorem: Let $\boldsymbol{X} \sim N_2(\boldsymbol{\mu}, \Sigma)$ be such that Σ is invertible, then

1. for all $y \in \mathbb{R}$, the conditional PDF of X given $Y = y$ is given by

$$f_{X|Y}(x|y) = \frac{1}{\sigma_{x|y}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - \mu_{x|y}}{\sigma_{x|y}}\right)^2\right] \quad \text{for } x \in \mathbb{R},$$

   where $\mu_{x|y} = \mu_x + \rho\frac{\sigma_x}{\sigma_y}(y - \mu_y)$ and $\sigma_{x|y} = \sigma_x^2(1 - \rho^2)$.

2. $E(X|Y = y) = \mu_{x|y} = \mu_x + \rho\frac{\sigma_x}{\sigma_y}(y - \mu_y)$ for all $y \in \mathbb{R}$.

Theorem: Let $X_1, X_2, \ldots X_n$ be i.i.d. $N(0,1)$ random variables. Then $\sum_{i=1}^n X_i^2 \sim Gamma(n/2, 1/2) \equiv \chi_n^2$.

Theorem: Let $X_1, X_2, \ldots X_n$ be i.i.d. $N(\mu, \sigma^2)$ random variables. Then $\overline{X} \sim N(\mu, \sigma^2/n)$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$, and $\overline{X}$ and S^2 are independently distributed. Here $\overline{X} = \frac{1}{n}\sum_{i=1}^n X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X})^2$.