HS 224: Game Theory and Economics
BTech 4th Semester, End Semester Test
Total Marks 40, Time 180 minutes
Date: 3rd May, 2017
[*Answers should be accompanied by proper elaboration*]

1. [First-price all-pay auction] Two cities, city *A* and city *B*, are competing to host the next Olympic games. The more a city spends on its infrastructure, architecture, transport, etc. the greater are its chances to win the bid to host Olympics. Valuations for hosting the games are given by v_A and v_B ($v_A > v_B > 0$), which are known to both parties. If x_A and x_B are the spendings done by the cities, probability of city *i*'s winning the auction is given by,

$$p_i = \begin{cases} 1, if\ x_i > x_j \\ \frac{1}{2}, if\ x_i = x_j \\ 0, if\ x_i < x_j \end{cases}$$

Here *i, j* = *A*, *B* ($i \neq j$) and $p_j = 1 - p_i$.

(i) Model the auction as a strategic game. [2]

(ii) Construct the best response functions and find the equilibria in pure strategy. [Hint: by writing down the payoff functions you should know that the game is different from the 'war of attrition' game.] [4]

2. Do mixed strategies $\alpha_1 = (\frac{2}{3}, \frac{1}{3})$, $\alpha_2 = (0, \frac{1}{2}, \frac{1}{2})$ constitute a mixed strategy Nash equilibrium of the following game? [4]

		2		
		L	M	R
1	T	2,2	0,3	1,2
	B	3,1	1,0	0,2

3. In the following game,

(i) Which action, if any, is strictly dominated? Find *all* mixed strategies which dominate it. [2]

(ii) Find *all* Nash equilibria of the game. [4]

(iii) Find payoff of players in the proper mixed strategy equilibria (if they exist). [2]

		2	
		L	R
1	T	1,2	2,5
	M	3,2	0,0
	B	0,1	4,2

4. Arman and Bikas take turns in removing matchsticks from a pile. They start with 21 matchsticks, and Arman goes first. On each turn, each player may remove one, two, three or four matchsticks. The player to remove the last matchstick wins the game.

(i) Suppose there are only six matchsticks left and it is Bikas's turn. What move should Bikas make to guarantee himself victory? Explain your reasoning. [2]

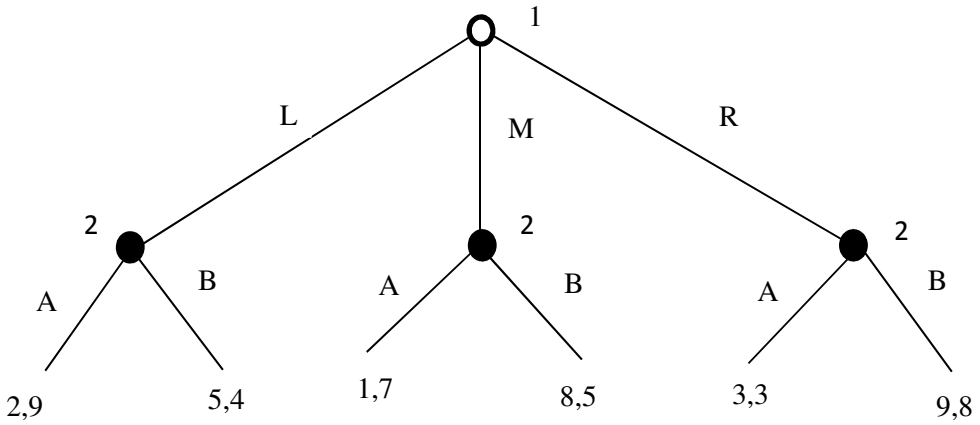
(ii) Suppose there are 12 matchsticks left and it is Bikas's turn. What move should Bikas make to guarantee himself victory? [2]

(iii) Now start from the beginning of the game. If both players play optimally who will win? Is there a first mover advantage? [2]

(iv) What are the optimal strategies for each player (Subgame Perfect Nash Equilibrium strategies)? [2]

5. Consider a market with two firms. The firms compete by choosing a quantity to produce, but this competition is sequential: firm 1 first chooses a quantity to produce q_1 and given this observed choice, firm 2 then chooses a quantity to produce q_2: The inverse demand function for this market is $P(Q) = 30 - 2Q$; where P is the unit price and Q is the total quantity in the market: $Q = q_1+q_2$. The per unit cost of production is $c_1 = 4$ for firm 1 and $c_2 = 2$ for firm 2. Solve for the Subgame Perfect Nash equilibrium of this game (the quantity choices of both firms). Which firm earns a higher profit? [4+2]

6.



(i) Find by backward induction the Subgame Perfect Nash Equilibrium of the game above. [2]

(ii) Verify that the SPNE strategies constitute a Nash equilibrium of the strategic form game. [3]

(iii) Identify all the pure strategy Nash equilibria of this game. [3]