HS 224: Game Theory and Economics
B Tech, 4th Semester, Mid-Semester Test
Total Marks: 40
Date: 26 February, 2019
(Provide adequate explanations for your answers)

1. Define weakly dominated actions and strict Nash equilibirum. Can weakly dominated actions be played in a strict Nash equilibrium? Explain. [4+4]

2. In the film "A Beautiful Mind," John Nash and three of his friends find themselves faced with a dilemma while at a bar. There are four brunettes and a single blonde available for them to approach. Each young man wants to approach and win the attention of one of the young women. The payoff to each of winning the blonde is 10; the payoff of winning a brunette is 5; the payoff from ending up with no girl is zero. The catch is that if two or more young men go for the blonde, she rejects all of them, and then the brunettes also reject the men because they don't want to be second choice. Thus each player gets a payoff of 10 only if he is the sole suitor for the blonde.

   (i) Show the (three-dimensional) payoff matrix in which there are 3 young men (3 brunettes, 1 blonde are also there but they are not active players). Find all the Nash equilibria of this game. [3+3]
   (ii) Without the use of a matrix, find all the Nash equilibria where 4 young men are there (as well as 4 brunettes, 1 blonde). [3]
   (iii) Can the outcome specified in the movie as the Nash equilibrium of the game – that all of the young men choose to go for brunettes – even really be a Nash equilibrium of the game? [1]

3. Two people are engaged in a joint project. If each person i puts in the effort x_i, a non-negative number equal to at most 1, which costs her $c(x_i)$, the outcome of the project is worth $f(x_1, x_2)$. The worth of the project is split equally between the two people, regardless of their effort levels. Formulate this situation as a strategic game. Find the Nash equilibrium/equilibria of the game $f(x_1, x_2) = 4x_1x_2$ and $c(x_i) = x_i$ for $i = 1, 2$. Is there a pair of effort levels that yields both players higher payoffs than the Nash equilibrium effort levels? [2+4+2]

4. Consider a variant of the Cournot duopoly model with linear demand function,
   $Q = \alpha - P$ for $\alpha \geq P$,
   $= 0$, otherwise.
   The cost functions are of the form, $C_i(q_i) = c.(q_i)^2$, with $\alpha > c > 0$. Find the Nash equilibrium of the model (find the equilibrium quantity, price and profit). [8]

5. Suppose in the Bertrand duopoly model firm 1 maximises its market share subject to not making a loss. Firm 2 maximises profit. Other standard assumptions as discussed in the class hold. Find the best response functions and the Nash equilibrium/ equilibria. [6]