

HS 224: Game Theory and Economics
BTech 4th Semester, End Semester Test
Total Marks 40, Time 180 minutes, Date: 7th May, 2019
[Answers should be accompanied by proper elaboration]

1. Recall the setup of the second-price and first-price sealed-bid auctions, and consider variants wherein *all bidders* pay the winning price.

(a) For two bidders second-price auction find all Nash equilibria of the game. [6]

[Hint: the game is like the *War of Attrition* game with a minor difference].

(b) For the two bidders first-price variant show that no Nash equilibrium exists. [2]

2. In a two player symmetric game with vNM preferences a **symmetric mixed strategy Nash equilibrium** is one which is a mixed strategy Nash equilibrium and α_i^* is the same for every player i . In the following game, there any symmetric mixed strategy Nash equilibrium? Is yes, what is it? [4]

		Player 2	
		X	Y
Player 1	X	0,0	1,1
	Y	1,1	0,0

3. Using iterated elimination of strictly dominated actions find all the pure and mixed strategy Nash equilibria of the following game. [6]

		Player 2		
		X	Y	Z
Player 1	A	0, 0	-1, 1	$-1, \frac{1}{3}$
	B	1, -1	-2, -2	0, -2
	C	$\frac{1}{2}, 0$	$-2, \frac{1}{3}$	2, 0

4. Consider the rivalry between Airbus and Boeing to develop a new commercial jet aircraft. Suppose Boeing is ahead in the development process and Airbus is considering whether to enter the competition. If Airbus stays out it earns zero profit, whereas Boeing enjoys a monopoly and earns a profit of \$1 billion. If Airbus decides to enter and develop the rival airplane, then Boeing has to decide whether to accommodate Airbus peaceably or to wage a price war. In the event of a peaceful competition, each firm will make a profit of \$300 million. If there is a price war, each will lose \$100 million because the price of airplanes will fall so low that neither firm will be able to recoup the development cost.
- (a) Specify the four components of this extensive game with perfect information and draw the game tree. [2+1]
- (b) What are the Nash Equilibria (or Equilibrium) of the game? [3]
- (c) What are the Subgame Perfect Equilibria (of Equilibrium) of the game? [2]

5. Find the Subgame Perfect Equilibrium (or Equilibria) of Stackelberg's duopoly game when $C_i(q_i) = q_i^2$ for $i = 1, 2$, and $P_d(Q) = \alpha - Q$ for $Q \leq \alpha$ (with $P_d(Q) = 0$ for $Q > \alpha$). Compare the equilibrium output outcome with the Nash equilibrium of Cournot's game under the same assumption. [5+3]
6. Two players Akshay and Bikash are playing the following game with a jar containing 100 balls. The players take their turns; Akshay goes first. Each time it is a player's turn, he takes between 1 and 10 balls out of the jar. The player whose move empties the jar wins.
- (a) If both players play optimally, who will win the game? Does this game have a first-mover advantage? Explain your reasoning. [4]
- (b) What are the optimal strategies for each player? [2]