Tuesday

19/03/2024 — 2. ELECTROSTATIC POTENTIAL AND CAPACITANCE

INTRODUCTION:

* The region around a charge can be described in two ways:
  (i) Electric field ($\vec{E}$) - vector quantity
  (ii) Electric potential (V) - scalar quantity
* Both of these quantities are characteristic properties of any point around a charge (field), and they are inter-related.

ELECTRIC POTENTIAL: [for only one point]

* The potential at infinity is taken to be zero.
* Electric potential at a point in an electric field is the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic forces.

$$V = \frac{W}{q_0} = \frac{\text{Work done}}{\text{charge}}$$

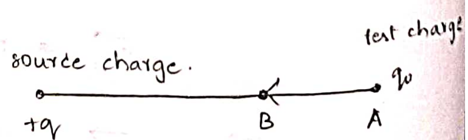
* It is a scalar quantity. [does not depend on direction].
* It's SI unit is volt (V).
* Define 1 volt: If 1 joule of work is done in moving a unit positive charge of 1 ~~idk~~ coulomb from infinity to a point in an electric field againts the electrostatic forces, then the potential at that point is said to be 1 volt.

POTENTIAL DIFFERENCE: [for potential between two points].

* When a test charge q_0 is moved from A to B, a work W_{AB} has to be done in moving against the repulsive force exerted by charge $+q$, then

$$V = V_B - V_A = \frac{W_{AB}}{q_0}.$$

* It is a scalar quantity.
* It's SI unit is volt (V).



Note: 1) The potential at infinity is zero for any charge.
2) For potential difference, two points are considered within the electric field.

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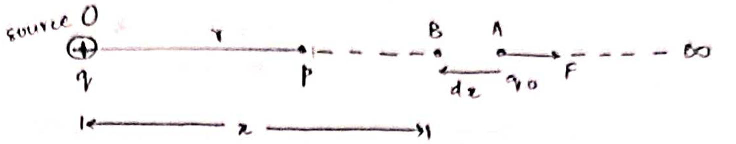
ELECTRIC POTENTIAL DUE TO POINT CHARGE:

* Consider a positive point charge q placed at the origin O. We wish to calculate its electric potential at a point P at a distance r from it.
* By coulomb's law, the force on q_0 is :



$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}$$

* The small work done in moving q_0 from A to B through a dx against the force is :

$$dW = \vec{F}\cdot d\vec{x} = Fdx\cos 180^\circ$$

$$dW = -Fdx$$

* Total work done,

$$\left[\int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$W = \int dW = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}\cdot dx$$

$$= -\frac{qq_0}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}dx = -\frac{qq_0}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \cancel{\frac{1}{\infty}}^{0}\right]$$

$$W = \frac{qq_0}{4\pi\varepsilon_0 r}$$

* The potential difference is

$$V = \frac{W}{q_0} = \frac{qq_0}{4\pi\varepsilon_0 r}\times\frac{1}{q_0}$$

$$\boxed{V = \frac{q}{4\pi\varepsilon_0 r}}$$

* Clearly, $V \propto \frac{1}{r}$.

* The potential due to a point charge is spherically symmetric and does not depend on the direction of the point with respect to the charge.
* Moreover, the potential at infinity is zero.

VARIATION OF ELECTRIC POTENTIAL AND ELECTRIC FIELD DUE TO A POINT CHARGE AT A DISTANCE 'r':

* The decrease in electric field is greater than that of electric potential but both the magnitudes are same

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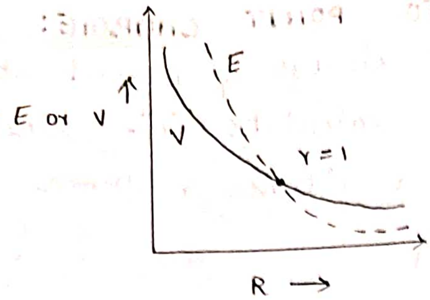
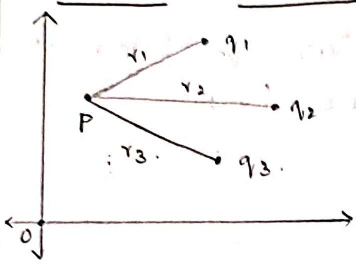


when $r = 1$.

$E \propto \frac{1}{r^2}$ $\quad V \propto \frac{1}{r}$

⇒ decreases faster

ELECTRIC POTENTIAL DUE TO A SYSTEM OF CHARGES:



$$V = V_1 + V_2 + V_3 + \ldots\ldots + V_n$$

$$= \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \ldots\ldots + \frac{kq_n}{r_n}$$

$$\boxed{V = k \sum_{i=1}^{N} \frac{q_i}{r_i}}$$ ⇒ For discrete charge distribution.

* For continuous charge distribution:

$$V_{continuous} = \frac{1}{4\pi\varepsilon_0}\left[\int_L \frac{\lambda dL}{|\vec{r}-\vec{r_i}|} + \int_S \frac{\sigma ds}{|\vec{r}-\vec{r_i}|} + \int_V \frac{\rho dV}{|\vec{r}-\vec{r_i}|}\right]$$

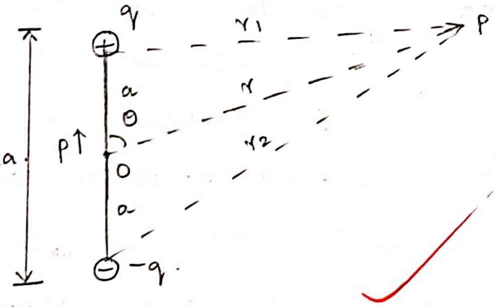
⇒ Here, $|\vec{r}-\vec{r_i}|$ represents magnitude of displacement vector. $[dq = \lambda dL = \sigma ds = \rho dV]$.

* For general charge distribution:

$$V = V_{cont} + V_{dis}$$

ELECTRIC POTENTIAL DUE TO A DIPOLE:

Here, the formula is same for all the points as it is a scalar quantity and direction is not considered.



θ = angle between $\vec{p}$ and $\vec{r}$.

* Consider an electric dipole $-q$ and $+q$ separated by distance $2a$.
* Let P be a point at a distance r from centre O making an θ with dipole moment $\vec{p}$.
* We wish to calculate the potential at P.

$$V = V_1 + V_2 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$

$$V = \frac{kq}{r_1} + \frac{k(-q)}{r_2}$$

$$\boxed{V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

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* By cosine law of triangles,

$$\left[r_2^2 = r^2 + a^2 + \cancel{2ar\cos\theta} \quad \cancel{r_1^2 = r^2 + a^2 -} \right]$$

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$

$$r_2^2 = r^2 + a^2 + 2ar\cos\theta$$

* We take $r >> a$, then,

$$r_1^2 = r^2\left[1 + \left(\frac{a}{r}\right)^2 - \frac{2a}{r}\cos\theta\right]$$

→ The term $\frac{a^2}{r^2}$ can be neglected.

→ Taking square root.

$$\therefore r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{1/2}$$

→ Taking reciprocal.

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2}$$

→ Since $\frac{a}{r}$ is very small, we can expand the term binomially and neglect the higher powers. $[(1+x)^n \cong 1 + nx]$

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{1}{2}\cdot\frac{2a}{r}\cos\theta\right) \qquad [\text{Here, } n = -1/2]$$

$$\boxed{\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)}$$

$|||^{ly}$, $$\boxed{\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q \quad 2a\cos\theta}{4\pi\varepsilon_0 \quad r^2} \qquad [p = 2aq]$$

$$\boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}} \times \frac{r}{r} \longrightarrow \text{scalar form} \qquad \left[V \propto \frac{1}{r^2}\right]$$

$$\boxed{V = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}} \quad [\text{dot product}] \longrightarrow \text{vector form.} \qquad \left[V = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^3}\right]$$

* Special cases:

1] $\theta = 0°$ ⇒ axial point ⇒ $V_{ax} = \frac{q}{4\pi\varepsilon_0 r^2}$ $[\cos 0° = 1]$

↓ point is near +ve charge.

2] $\theta = 180°$ ⇒ axial point ⇒ $V_{ax} = \frac{-q}{4\pi\varepsilon_0 r^2}$ $[\cos 180° = -1]$

↓ point is near -ve charge.

3] $\theta = 90°$ ⇒ equitorial point ⇒ $V_{eq} = 0$ $[\cos 90° = 0]$

Note: The potential due to a dipole is cylindrically

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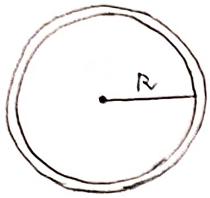
symmetric about the dipole axis. Therefore, the potential at any point on the equitorial plane is zero.

ELECTRIC POTENTIAL DUE TO A UNIFORMLY CHARGED THIN SPHERICAL SHELL:



(i) $r > R$

$V = \frac{kQ}{r}$

(ii) $r = R$

$V = \frac{kQ}{R}$

(iii) $r < R$

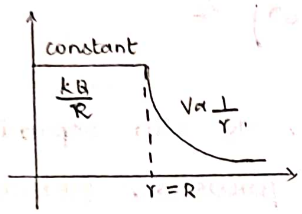
$V = \frac{kQ}{R}$

→ constant at all points inside shell

⇒ $E = 0$ → vector - cancel ou

⇒ $V \neq 0$ → scalar - added.

Graph:

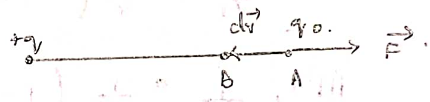


RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL:

$W = \vec{F} \cdot \vec{dr} = F dr \cos 180^\circ$

$= - F dr$

$= - q_0 E dr$ — ①



$W = (V_B - V_A) q_0$

$= q_0 dV$ — ②

Equating ① and ②

$-q_0 E dr = q_0 dV$

$$\boxed{E = \frac{-dV}{dr}}$$

* The quantity $\frac{dV}{dr}$ is rate of change of potential with distance and is called potential gradient.
* Thus, electric field at any point is equal to negative of potential gradient.

Note: 1) Electric field is in that direction in which the potential decrease is steepest.

2) The magnitude of EF is equal to potential gradient and normal to the equipotential surface at that point.

3) Also, $dV = - \vec{E} \vec{dr}$

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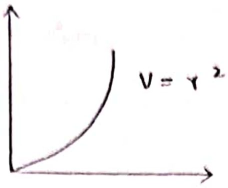
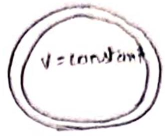
$$\boxed{V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r}}$$

Ex:





V = constant = 0

$V \propto -\int r^{-2}\, dr$

$V \propto -\frac{r^{-2+1}}{-2+1} = \frac{-r^{-1}}{-1} = \frac{1}{r}$

$E \propto \frac{d}{dr}(r^2) = 0$

$E \propto 2r \Rightarrow E \propto r$

$E = \frac{d(\text{constant})}{dr} = 0$

* The SI unit of electric field can also be written as V/m.

EQUIPOTENTIAL SURFACE AND ITS PROPERTIES:

* Any surface that has same electric potential at every point on it is called an equipotential surface

* Properties:

i) No work is done in moving a test charge over an equipotential surface.

$W = (V_B - V_A)\, q_0$

For equipotential surface, $V_B = V_B$.

$\therefore W = 0$.

ii) Two equipotential surfaces cannot intersect each other.

iii) Equipotential surfaces are closer together in the region of strong field and far apart in the region of weak field.

We know that $E = -\frac{dV}{dr}$

If $dV =$ ~~const~~ constant.

Then $dr \propto \frac{1}{E}$.

iv) Electric field is always normal to the equipotential surface at every point.

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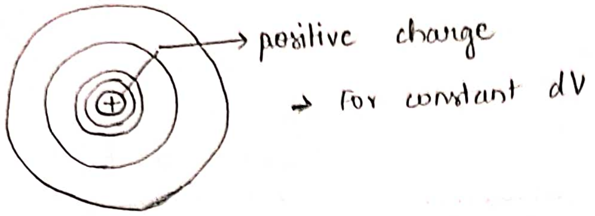
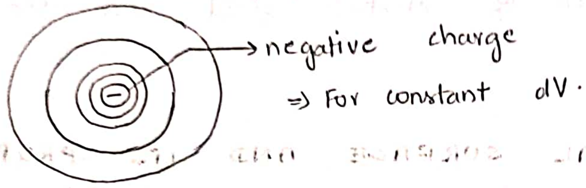
EQUIPOTENTIAL SURFACES FOR DIFFERENT CHARE SYSTEMS:

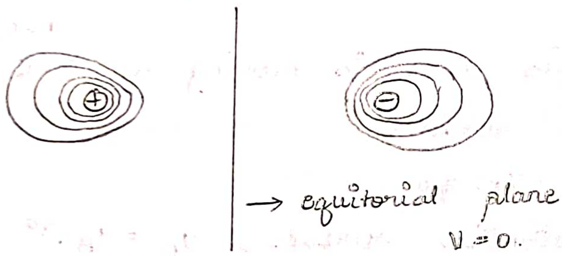
i) POSITIVE CHARGE:



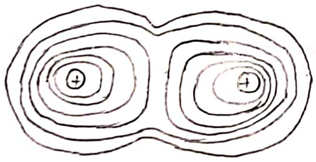
ii) NEGATIVE CHARGE:



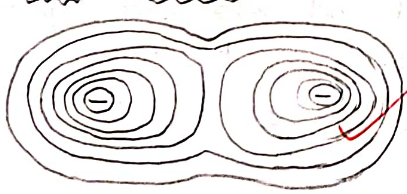
iii) ELECTRIC DIPOLE:



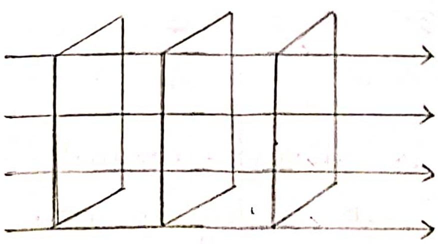
iv) TWO EQUAL POSITIVE CHARGES:



v) TWO EQUAL NEGATIVE CHARGES:



vi) UNIFORM ELECTRIC FIELD:



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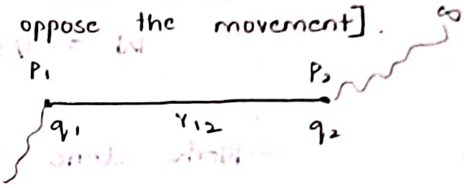
ELECTROSTATIC POTENTIAL ENERGY :

* It may be defined as the amount of work done in assembling the charges at their locations by bringing them in from infinity.
* It is also known as energy of association.
* For two point charges,

→ Let W_1 be the work done to bring q_1 from infinity to point P_1.

$\therefore$ $W_1 = 0$ [As there is no 2nd charge to oppose the movement].



→ Let W_2 be the work done to bring q_2 from infinity to point P_2.

$\therefore$ $W_2 = (\text{potential due to } q_1)\, q_2$

~~(potential due to q_2)~~

$W_2 = Vq_2$

$W_2 = \dfrac{kq_1}{r_{12}} q_2$

⇒ As electrostatic force is conserved, the path followed by charge to come to a fixed point is not considered.

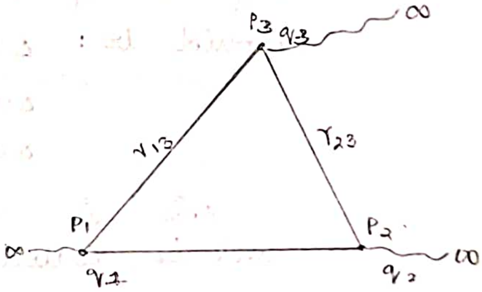
⇒ Work and energy are scalar quantities.

→ Total potential energy,

$U = W_1 + W_2 = 0 + \dfrac{kq_1q_2}{r_{12}}$

$$\boxed{U = \frac{kq_1q_2}{r_{12}}}$$

* For three point charges,



→ We know that, $W_1 = 0$

$W_2 = \dfrac{kq_1q_2}{r_{12}}$

$W_3 = (\text{potential due to } q_1)\, q_3 + (\text{potential due to } q_2)\, q_3$

$W_3 = \dfrac{kq_1}{r_{13}} q_3 + \dfrac{kq_2}{r_{23}} q_3$

→ Therefore, total potential energy,

$U = W_1 + W_2 + W_3$

$$\boxed{U = 0 + \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}}$$

* For 'n' point charges,

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{\text{all pairs} \\ i \neq j}} \frac{q_iq_j}{r_{ij}}}$$

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POTENTIAL ENERGY IN AN EXTERNAL FIELD:

i) For single charge:

$$U = W_1 = V(\vec{r_1})\, q_1$$

ii) For two point charges:

$\vec{r_1}$ and $\vec{r_2}$ vectors are the position vectors of the charges q_1 and q_2. The potential at these points due to the external field is $V(\vec{r_1}) + V(\vec{r_2})$.

→ Work done to bring q_1 at P_1:

$$W = V(\vec{r_1})\, q_1$$

→ Work done to bring q_2 at P_2:

$$W_2 = V(\vec{r_2})\, q_2 + \frac{k q_1 q_2}{r_{12}}$$

→ Total potential energy:

$$U = V(\vec{r_1})\, q_1 + V(\vec{r_2})\, q_2 + \frac{k q_1 q_2}{r_{12}}$$

* The SI unit of potential energy is joule. It is a scalar quantity.
* Suppose, an electron is moved through a potential difference of 1 V, then its change in potential energy would be: $\Delta U = q \Delta V = 1e \cdot 1V$

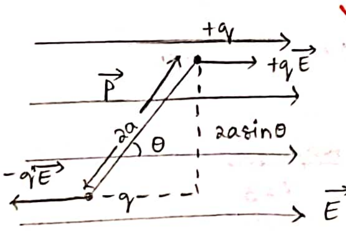
$$\Delta U = 1eV$$

$$\Delta U = 1.6 \times 10^{-19}\ J.$$

→ This is a commonly used unit of energy in atomic physics and is called electron Volt (eV).

Note: The energy of dissociation = −(potential energy).

POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM ELECTRIC FIE



* Consider an electric dipole placed in field $\vec{E}$ with its dipole moment $\vec{P}$ making an angle θ with the field.
* Torque (τ) = $pE\sin\theta$
* Work done to rotate dipole by small angle $d\theta$ against torque is:

$$dW = \tau\, d\theta$$

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* Total work done to rotate from θ_1 to θ_2 is :

$$W = \int dW = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

{If θ_1 is not given, take $\theta_1 = 90°$.}

$$W = pE [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$W = pE[-\cos\theta_2 + \cos\theta_1]$$

$$W = pE[\cos\theta_1 - \cos\theta_2]$$

∴ This work done is stored ~~as~~ as potential energy of the dipole.

$$\boxed{U = W = pE[\cos\theta_1 - \cos\theta_2]}$$

* If initially dipole is $\perp^r$ $[\theta_1 = 90°]$ and rotated to angle θ $[\theta_2 = \theta]$, then

$$U = pE[\cancel{\cos 90°}^{0} - \cos\theta] \quad [\cos 90° = 1]$$

$$U = -pE\cos\theta$$

$$\boxed{U = -\vec{p} \cdot \vec{E}}$$

Note: The potential energy of a dipole in an external field, when rotated
⇒ from stable ($\theta_1 = 0°$ to unstable ($\theta_2 = 180°$) equilibrium
$U = pE(1-(-1))$
$= +2pE$
⇒ from unstable to stable
$U = -2pE$

* Special cases : [For $U = -pE\cos\theta$]

i) Position of stable equilibrium : If $\theta = 0°$, $U = -pE$ (minimum energy).

ii) Position of zero energy : If $\theta = 90°$, $U = 0$.

iii) Position of unstable equilibrium : If $\theta = 180°$, $U = +pE$ (maximum energy).

* $\tau = 0°$ ⇒ equillibrium

→ $\theta = 0°$, $U = -PE$ → minimum energy
⇒ stable equilibrium.

→ $\theta = 180°$, $U = PE$ → maximum energy
⇒ unstable equillibrium.

CONDUCTORS AND INSULATORS :

* Substances which allow large scale physical movement of charges through them in an external field are called conductors.

Ex: Ag, Cu, Al, graphite, human body, acids, alkalis, etc...

* Substances which do not allow physical movement of charge in external field are called insulators.

Ex: glass, wood, diamond, mica, wax, distilled water, ebonite, etc...

* The rubbed insulators are able to retain charges, so, they are called dielectrics.

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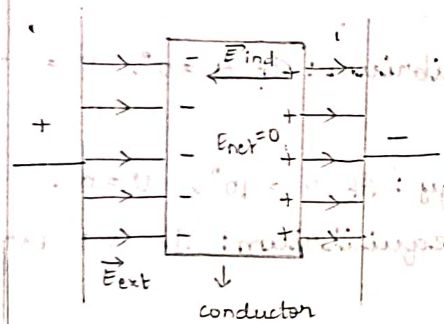
* Rubbed conductors couldn't retain charges (immediately drained out) and are called non-electrics.

FREE AND BOUND CHARGES:

* In metallic conductors, outer electrons are loosely bound and get detached easily; these free electrons drift in opposite direction of electric field.
* Positive nucleus and inner electrons are fixed and are called bound charges.
* In electrolytic conductors, both +ve and -ve ~~ch~~ ions are charge carriers.
* In insulators, electrons are tightly bound to the nucleus and are called bound charges.

BEHAVIOUR OF CONDUCTORS IN ELECTROSTATIC FIELD:

* Net electrostatic field is zero in the interior of a conductor.

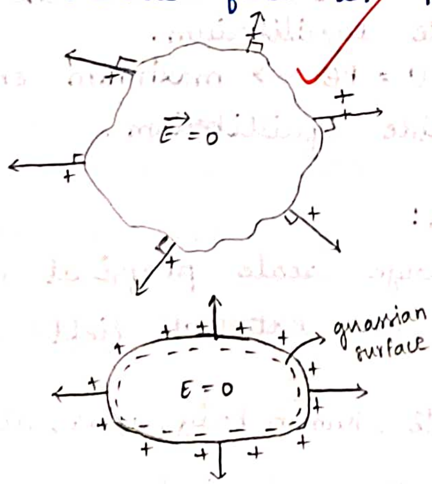


* $\vec{E}_{ind}$ → due to induced charges [$\vec{E}$ induced] → equal and opposite to external field

$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{ind}$

$\boxed{\vec{E}_{net} = 0}$

* Just outside the surface of a charged conductor, the electric field is normal to the surface.



* The net charge in the interior of a conductor is 0 and any excess charge resides at its surface.

$\phi = \int E ds = 0$

$\frac{q}{\varepsilon_0} = 0$

$\boxed{q_{en} = 0}$

* Potential is constant within and on the surface of the conductor.

⇒ $E = -\frac{dv}{dr}$

⇒ But inside, $E = 0$

⇒ and E has no tangential component on the surface.

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$\Rightarrow E_{tangential} = 0.$

$\Rightarrow$ Thus, $\frac{dV}{dr} = 0$

$$\boxed{V = \text{constant}}$$

$\therefore$ The surface of a conductor is an equipotential surface.

* Electric field at the surface of a charged conductor is proportional to the surface charge density.

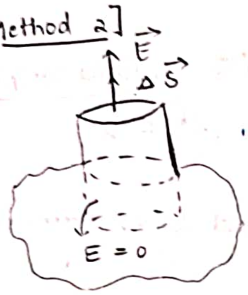
Method 1]

$$\sigma = \frac{q}{\text{Area}} = \frac{q}{4\pi r^2}$$

Area $= 4\pi r^2$ spherical surface

$$E = \frac{kq}{R^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0} \text{ [constant]}.$$

Method 2]



$$\Delta Q = \sigma \Delta S.$$

$$\Delta\phi = \frac{q_{en}}{\varepsilon_0} = \frac{\sigma\Delta S}{\varepsilon_0} \quad -①$$

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} \quad [\theta = 0°]$$

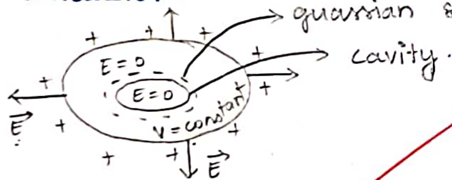
$$\Delta\Phi = E\Delta S \quad -②$$

Equating ① and ②

$$E\Delta S = \frac{q\Delta S}{\varepsilon_0}$$

$$\boxed{E = \frac{q}{\varepsilon_0}}$$

* Electric field is zero in the cavity of a hollow charged conductor.



ELECTROSTATIC SHIELDING:

* The phenomenon of making a region free from any electric field is called electrostatic shielding.

* **Applications:**

i) In thunderstorm with lightening, it is safe to sit inside a car rather than under a tree or open ground because the metal surface/body provides electrostatic shielding.

ii) Sensitive components of electronic devices are protected ~~by placing~~ from external electric field by placing metal shields around them [Faraday's cage].

ELECTRICAL CAPACITANCE OF A CONDUCTOR:

* The electrical capacitance of a conductor is the measure of its ability to hold electric charge.

* If charge Q is added to a conductor, then its potential increases by V.

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$$Q \propto V$$

$$Q = CV$$

Here, C is called capacitance.

$\therefore$ Capacitance = $\dfrac{\text{Charge}}{\text{Potential}}$

→ Units : CV^{-1} (or) F

$1 \text{ farad} = \dfrac{1 \text{ coulomb}}{1 \text{ volt}}$

* The capacitance depends on:
  - → size and shape of a capacitor
  - → nature of the surrounding medium
  - → presence of other conductors in its neighbourhood.

→ Practical Units: $1 mF = 10^{-3} F$, $1 \mu F = 10^{-6} F$, $1 pF = 10^{-12} F$, $1 nF = 10^{-9} F$.

The SI unit of capacitance is farad (F), named in the honour of Michael Faraday.

* Dimensions of farad :

$$[F] = [C][V]^{-1}$$

$$= [AT]\left[\frac{ML^2T^{-2}}{AT}\right]^{-1}$$

$$= A^2T^2M^{-1}L^{-2}T^2$$

$$[F] = [M^{-1}L^{-2}A^2T^4]$$

C = charge = current × time = AT

V = potential = $\dfrac{\text{work done}}{\text{charge}}$ = $ML^2T^{-3}A^{-1}$

SPHERICAL CAPACITOR:

* The potential of a spherical conductor on the surface

$$V = \frac{kQ}{R}$$

but $C = \dfrac{Q}{V} = \dfrac{R}{k} \Rightarrow \boxed{C = 4\pi\varepsilon_0 R}$

* Clearly the capacitance of a spherical conductor is proportional to its radius.

$1F = \dfrac{R}{9\times10^9} \Rightarrow R = 9\times10^9 \text{ m} = 9\times10^6 \text{ km}$

* If $C = 1F$, $R = 9\times10^9 = 9\times10^6$ km $\left[R = \dfrac{C}{4\pi\varepsilon_0}\right]$. This radius is 10500 times the radius of earth. Therefore,

i) 1 F is a very large value of capacitance.

ii) It is not possible to have single isolated conductor of very large capacitance.

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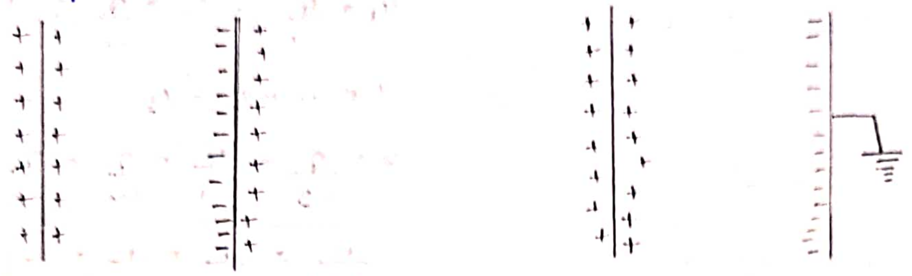
PRINCIPLE OF CAPACITANCE:

* The capacitance of an insulated conductor is considerably increased when we place an earthed connected conductor near it. Such a system of 2 conductors is called ~~a capitor~~ capacitor.

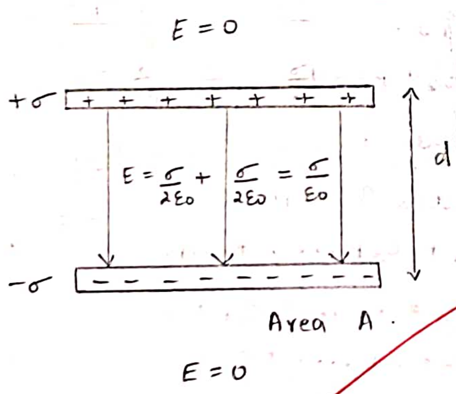


induced

* The symbol of capacitor is ⊣⊢ for fixed and ⊣⊢ for variable. [two parallel lines]

PARALLEL PLATE CAPACITOR:

* It is the most simplests and widely used capacitor.
* It consists of two largely plane parallel conducting plates separated by a small distance.
* Let A be the common area of capacitor, d be the distance between the two plates, $\pm\sigma$ be the surface charge density, ~~± be~~ $\pm Q = \pm \sigma A$ be the total charge on each plate.



$E = \frac{\sigma}{\varepsilon_0}$ but $E = -\frac{dV}{dr}$

For uniform E,

$E = \frac{V}{d}$

$V = Ed = \frac{\sigma d}{\varepsilon_0}$

$C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma d}{\varepsilon_0}}$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

* Capacitance depends on:
  1) $C \propto A$
  2) ~~$C \propto d$~~ $C \propto \frac{1}{d}$
  3) $C \propto \varepsilon$ (Nature of medium btw plates)

* If the space between the plates is fully filled with a medium, then

$C' = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$

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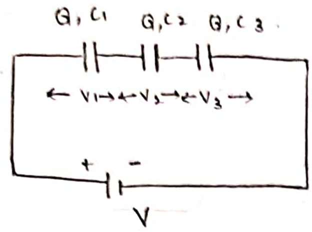
$C' = \varepsilon_r C$ [ε_r = dielectric constant]

$\therefore$ The capacitance increases by ε_r times.

~~CAPACITOR~~ COMBINATION OF CAPACITORS:

1) CAPACITORS IN SERIES:



Q = constant.

$$V_1 = \frac{Q}{C_1} \; ; \; V_2 = \frac{Q}{C_2} \; ; \; V_3 = \frac{Q}{C_3}$$

$$V = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For n- capacitors

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots + \frac{1}{C_n}$$

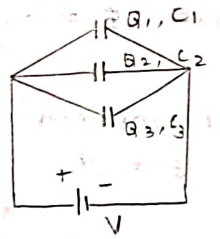
If $C_1 = C_2 = C_3 = \ldots\ldots = C_n = C$

$$\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \ldots\ldots + \frac{1}{C}$$

$$\frac{1}{C_S} = \frac{n}{C}$$

$$\boxed{C_S = \frac{C}{n}}$$

2) CAPACITORS IN PARALLEL:



V = constant.

$$Q = Q_1 + Q_2 + Q_3.$$

$$C_p V = C_1 V + C_2 V + C_3 V$$

$$\boxed{C_p = C_1 + C_2 + C_3}$$

For n- capacitors

$$C_p = C_1 + C_2 + C_3 + \ldots\ldots + C_n.$$

If $C_1 = C_2 = C_3 = \ldots\ldots = C_n = C$

$$C_p = C + C + C + C + \ldots\ldots + C$$

$$\boxed{C_p = nC}$$

SERIES	PARALLEL
* The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the sum of the individual capacitancies.	* The equivalent capacitance is equal to the sum of the individual capacitances.
* Charge on each capacitor is constant.	* Potential difference across each capacitor is constant.

Continuation in the next page → → → → → →

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SERIES	PARALLEL
* The potential difference across each capacitor is inversely proportional to its capacitance.	* The charge on each capacitor is directly proportional to its ~~capitance~~. capacitance.
* The equivalent capacitance is smaller than the smallest individual capacitance.	* The equivalent capacitance will be larger than the largest individual capacitance.

Note: For n-capacitors, $\frac{C_P}{C_S} = \frac{nC}{n/C} = n^2 : 1$

ENERGY STORED IN A CAPACITOR:

* The work done in charging the capacitor is stored as its electrical potential energy.
* Consider plates 1 and 2 having Q' and $-Q'$ charge, potential difference $V' = \frac{Q'}{C}$.

* Suppose, a small additional charge dQ' is transferred from plate 2 to 1, then the work done will be,

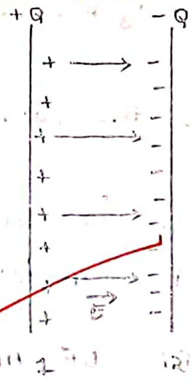

diagram 2

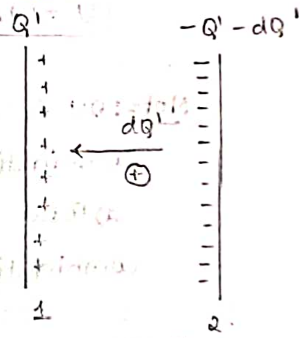

diagram 1

$$dW = V'\,dQ'$$

Therefore, the total work done in tranferring charge Q from 2 to 1 will be,

$$W = \int dW = \int V' dQ'$$

$$W = \int_0^Q \frac{Q'dQ'}{C} \qquad \left[\because V' = \frac{Q'}{C}\right]$$

$$W = \frac{1}{C}\left[\frac{(Q')^2}{2}\right]_0^Q \qquad \left[\int x\,dx = \frac{x^2}{2}\right]$$

$$W = \frac{1}{2C}Q^2 \qquad [\text{Here, } C \text{ is constant}].$$

$$\boxed{U = \frac{Q^2}{2C}}$$ [Here, work done is stored as potential energy]

[We know that $Q = CV$]

$$\boxed{U = \frac{1}{2}CV^2}$$

$$\boxed{U = \frac{1}{2}QV}$$

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ENERGY STORED IN SERIES COMBINATION:

$$U = \frac{1}{2}\frac{Q^2}{C_S}$$

Q = constant

$$U = \frac{Q^2}{2}\left[\frac{1}{C_S}\right] = \frac{Q^2}{2}\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\dots\right]$$

$$U = \frac{Q^2}{2C_1}+\frac{Q^2}{2C_2}+\frac{Q^2}{2C_3}+\dots$$

$$\boxed{U = U_1 + U_2 + U_3 + \dots}$$

ENERGY STORED IN PARALLEL COMBINATION:

$$U = \frac{1}{2}C_p V^2$$

V = constant

$$U = \frac{V^2}{2}\left[C_1 + C_2 + C_3 + \dots\right]$$

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + \dots$$

$$\boxed{U = U_1 + U_2 + U_3 + \dots}$$

Note: 1) U is always the same irrespective of the type of combination.

2) Total energy is additive both in series and parallel combination of capacitors.

ENERGY DENSITY OF AN ELECTRIC FIELD:

* The presence of an electric field implies stored energy.
* For a capacitor, $C = \frac{\varepsilon_0 A}{d}$, $E = \frac{\sigma}{\varepsilon_0}$, $U = \frac{1}{2}\frac{Q^2}{C}$

$Q = \sigma A$.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{\sigma^2 A^2}{\varepsilon_0 \frac{A}{d}} = \frac{1}{2}\frac{\sigma^2 A^2}{\varepsilon_0 A}\frac{d\,\varepsilon_0}{\varepsilon_0}$$

$$U = \frac{1}{2}\varepsilon_0 E^2 Ad \quad \left[E = \frac{\sigma}{\varepsilon_0}\right]$$

$$\frac{U}{Ad} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2 \quad [\text{Volume} = \text{Area} \times d = Ad$$

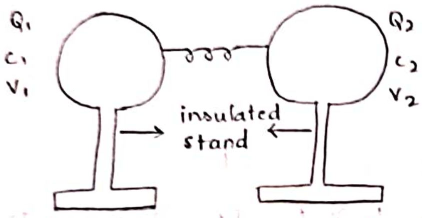
$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2} \quad [u = \text{energy density}]$$

⇒ This equation is applicable for all electric field.

⇒ This equation gives the energy density of an electric field, although we have derived for parallel plate capacitor it is true for all electric field due to any charge configuration.

REDISTRIBUTION OF CHARGES:

* Consider two insulated ~~conducting~~ conductors of capacitance C_1 and C_2 having charges Q_1 and Q_2 and potential V_1 and V_2, respectively.



$Q_1 = C_1 V_1$ $\qquad$ $Q_2 = C_2 V_2$

{Here, it is a convention that +ve charges flow.}

* The positive charge will flow from conductor at ~~potential~~ higher potential to lower potential until the potentials become equal.
* Thus, the common potential.

$$V = \frac{\text{Total charge}}{\text{Total capacitance}}$$

$$V = \frac{Q_1 + Q_2}{C_1 + C_2}$$

$$\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

* After restribution, the charge on both conductors will be Q_1' and Q_2' respectively.

$Q_1' = C_1 V$ $\qquad$ $Q_2' = C_2 V$

$$\frac{Q_1'}{Q_2'} = \frac{C_1 V}{C_2 V}$$

$$\boxed{\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}}$$

* The charges are in the ratio of their capacitances after redistribution

LOSS OF ENERGY IN REDISTRIBUTION OF CHARGES:

{change = final − initial
loss = initial − final}

$$U_{initial} = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

$$U_{final} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2}(C_1 + C_2) V^2$$

$$= \frac{1}{2} \cancel{(C_1+C_2)} \frac{(C_1V_1 + C_2V_2)^2}{(C_1+C_2)^{\cancel{2}}}$$

$$= \frac{1}{2} \frac{C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2}{(C_1+C_2)}$$

$$U_{loss} = U_{initial} - U_{final} = U_i - U_f$$

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$$U_{loss} = \frac{1}{2}\left[C_1V_1^2 + C_2V_2^2 - \left(\frac{C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2}{C_1+C_2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{C_1V_1^2(C_1+C_2) + C_2V_2^2(C_1+C_2) - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1+C_2}\right]$$

$$= \frac{1}{2(C_1+C_2)}\left[\cancel{C_1^2V_1^2} + C_1C_2V_1^2 + C_2C_1V^2 + \cancel{C_2^2V_2^2} - \cancel{C_1^2V_1^2} - \cancel{C_2^2V_2^2} - 2C_1C_2V_1V_2\right]$$

$$= \frac{C_1C_2}{2(C_1+C_2)}\left[V_1^2 + V_2^2 - 2V_1V_2\right]$$

$$\boxed{U_{loss} = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2}$$

⇒ Capacitance [C_1 and C_2] is always positive.
This is always +ve, whether $V_1 > V_2$ or $V_2 > V_1$, there is always some loss of potential energy in the form of heat due to the flow of charges in connecting wires.

DIELECTRICS AND THEIR POLARISATION:

* A dielectric is essentially an insulator which can be polarised through small localised displacement of its charges.

  Ex: glass, wax, wood, water, plastic, rubber, etc....

* Polar molecules: centre of mass of +ve and -ve charges do not coincide. They have dipole moment.

  Ex: HCl, NH_3, CO, etc...

  H^+ — Cl^-

  $\vec{P}$ ⇒ dipole moment

* Non-polar molecules: Centre of mass of +ve and -ve charges coincide. They do not have dipole moment.

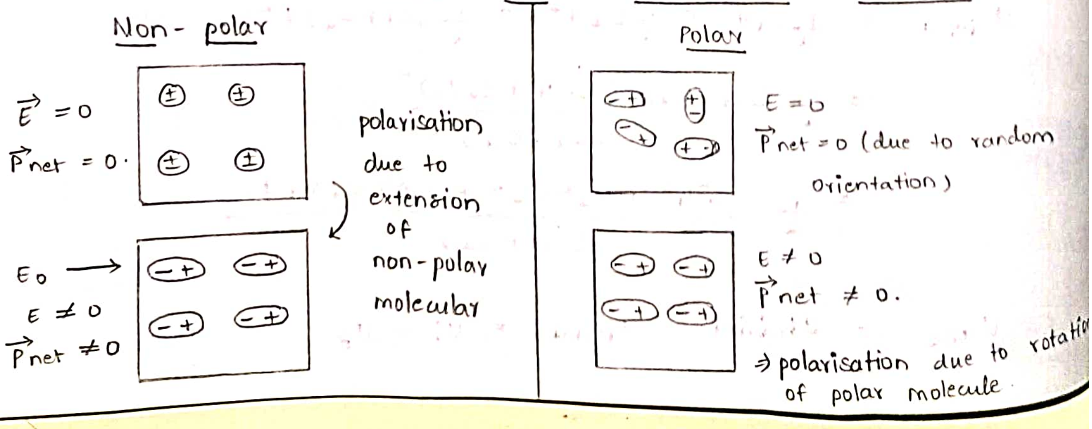
  Ex: H_2, N_2, N_2, CO_2, etc....

  H — H

  $\vec{P} = 0$.

HW. 2.3, 2.4, 2.6, 2.7, 2.5, 2.10.

POLARISATION OF POLAR AND NON-POLAR DIELECTRIC:



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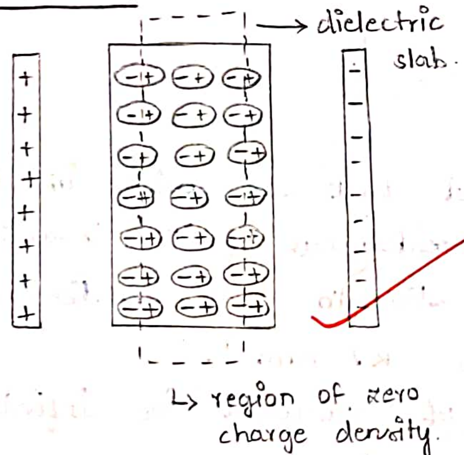
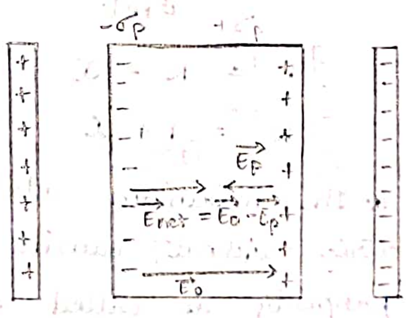
NON - POLAR	POLAR
* When $E = 0$, the individual dipole moment is 0, thus $\vec{p}_{net} = 0$.	* When $E = 0$ individual dipole dipole moment is present but they are oriented randomly due to thermal agitation, thus $\vec{p}_{net} = 0$.
* When $E \neq 0$, dipole is induced creating a net dipole moment. Thus, $\vec{p}_{net} \neq 0$.	* When $E \neq 0$, dipoles are rotated due to external potential energy creating net dipole moment along the direction of external field. Therefore, $\vec{p}_{net} \neq 0$.

⇒ Hence, both polar and non-polar dielectrics develop a net dipole moment in the presence of an electric field.

⇒ This fact is called polarisation of dielectric.

REDUCTION OF ELECTRIC FIELD BY POLARISATION OF THE DIELECTRIC:





* Reduced field → $\vec{E}_{net} = \vec{E_0} - \vec{E_p}$

$$K = \frac{\epsilon}{\epsilon_0} \qquad \left[E_0 = \frac{\sigma}{\epsilon_0} \qquad E_1 = \frac{\sigma}{\epsilon} \right]$$

$$\boxed{K = \frac{\epsilon}{\epsilon_0} = \frac{\vec{E_0}}{E_{net}} = \frac{\vec{E_0}}{\vec{E_0} - \vec{E_p}}}$$

POLARISATION DENSITY:

* It is the net dipole moment per unit volume.

$$P = \frac{\text{net dipole moment}}{\text{Volume}}$$

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$$P = \frac{Q_p d}{Ad} = \frac{Q_p}{A}$$

$$\boxed{P = \sigma_p}$$

ELECTRIC SUSCEPTIBILITY:

* If the $\vec{E}$ field $\vec{E}$ is not large, then polarisation $\vec{P}$ is proportional to resultant field.

$$\vec{P} \propto \vec{E}_{net}$$

$$\boxed{\vec{P} = \varepsilon_0 \chi \vec{E}_{net}} \quad [\chi = \text{chi}]$$

RELATION BETWEEN KAPPA AND CHI:

$$\vec{E}_{net} = \vec{E}_0 - \vec{E}_p$$

$$= \vec{E}_0 - \frac{\sigma_p}{\varepsilon_0}$$

$$= \vec{E}_0 - \frac{\vec{P}}{\varepsilon_0} \quad [\because \vec{P} = \sigma_p]$$

$$\vec{E}_{net} = \vec{E}_0 - \chi \vec{E}_{net} \quad [\vec{P} = \varepsilon_0 \chi \vec{E}_{net}]$$

Divide by $\vec{E}_{net}$

$$1 = \frac{\vec{E}_0}{\vec{E}_{net}} - \chi$$

$$1 = K - \chi \quad \left[\because K = \frac{\vec{E}_0}{\vec{E}_{net}}\right]$$

$$\boxed{K = 1 + \chi}$$

* The maximum electric field that can exist in dielectric without causing the breakdown of its insulating property is called dielectric strength of the material.
* It's practical unit is $kV\ mm^{-1}$.
* The dielectric strength of vacuum is infinity.

CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A DIELECTRIC SLAB:

* The capacitance of a parallel plate capacitor

$$C_0 = \frac{\varepsilon_0 A}{d}$$

* The electric field

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

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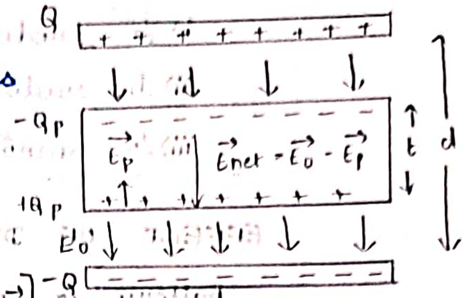
* A dielectric slab of thickness $t < d$ is placed between the plates.
* E_0 polarises the dielectric and produces induced field $\vec{E_p}$.



$$V = Ed \quad \left[\because E = \frac{-dV}{dr} \right]$$

$$V = \vec{E_0}(d-t) + \vec{E_0} - \vec{E_p}\,(t)$$

$$V = \vec{E_0}(d-t) + \frac{E_0}{K}t \quad \left[K = \frac{\vec{E_0}}{\vec{E}_{net}} \Rightarrow \vec{E}_{net} = \frac{\vec{E_0}}{K} \right]$$

$Q_p \rightarrow Q$ polarisation.

$$C = \frac{Q}{\cancel{\frac{E_0 Q}{A\varepsilon_0}}\left[d-t-\frac{t}{K}\right]} = \frac{Q}{\frac{Q}{A\varepsilon_0}\left[d-t-\frac{t}{K}\right]} \qquad \left[C = \frac{Q}{V}\right]$$

$$\left[E = \frac{Q}{A\varepsilon_0}\right]$$

$$C = \frac{A\varepsilon_0}{\left[(d-t)+\frac{t}{K}\right]}$$

* The capacitance increases when dielectric slab is inserted.
* Special case:

If $d = t$

$$C = \frac{\varepsilon_0 A}{d}K$$

$$\boxed{C = KC_0}$$

CAPACITANCE OF PARALLEL PLATE CONDUCTOR / CAPACITOR ~~FOR~~ WITH CONDUCTING SLAB:

For a conductor, $\vec{E}_{net} = 0$

$$\boxed{V = E_0(d-t)} + 0$$

$$C = \frac{Q}{V} = \frac{Q}{E_0(d-t)}$$

$$C = \frac{Q}{\frac{Q}{A\varepsilon_0}[d-t]}$$

$$\boxed{C = \frac{\varepsilon_0 A}{(d-t)}}$$

$$C = \frac{\varepsilon_0 A}{(d-t)} \times \frac{d}{d}$$

$$\boxed{C = \frac{d}{(d-t)}C_0}$$

* Clearly $C > C_0$, C increases by a factor $\frac{d}{(d-t)}$.

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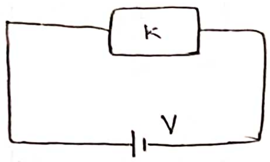
USES OF CAPACITORS:

i) To produce electric fields of desired patterns.
ii) In radio circuits for tuning
iii) To smoothen rectified current [filter circuits].

EFFECT OF DIELECTRIC ON VARIOUS PARAMETERS:

Battery is connected



1) $V = \text{constant}$

2) $C = kC_0$ (increases).

3) $Q = CV = kC_0V$

$Q = kQ_0$ (increases).

4) $E = \dfrac{V}{d} = \text{constant}$

5) $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}kC_0V^2$

$U = kU_0$

Battery is disconnected.

1) $Q = \text{constant}$

2) $C = kC_0$ (increases).

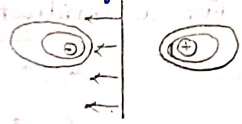
3) $V = \dfrac{Q}{C} = \dfrac{Q}{kC} = \dfrac{V_0}{k}$ (decrease

4) $E = \dfrac{V}{d} = \dfrac{V_0}{kd} = \dfrac{E_0}{K}$

5) $U = \dfrac{Q^2}{2C} = \dfrac{Q^2}{2kC_0} = \dfrac{U_0}{K}$

EXERCISES: [Pg: 79]

2.3] (a) The equipotential surface is present as a plane normal to AB located at the midpoint of the dipole.



(b) At every point the the equitorial plane, the direction of the electric ~~pot~~ field is normal to the plane.

2.4] (a) The electric field inside a spherical conductor is zero. $[\vec{E} = 0]$

(b) $\vec{E}_{\cancel{outside}\ surface} = \dfrac{kq}{R^2} = \dfrac{9\times10^9 \times 1.6\times10^{-7}}{(12\times10^{-2})^2} = 10^5\ NC^{-1}$

(c) $\vec{E}_{outside} = \dfrac{kq}{r^2} = \dfrac{9\times10^9\times1.6\times10^{-7}}{(18\times10^{-2})^2} = 4.4\times10^4\ NC^{-1}.$

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2.5] When distance is halved $[d \rightarrow d/2]$ and kept in a medium of dielectric constant 6 $[\varepsilon_0 \rightarrow 6\varepsilon_0]$

$\cancel{C'} = \cancel{\frac{\varepsilon_m A'}{d'}} = \cancel{6\varepsilon_0}$ $\quad C' = \frac{\varepsilon_m A'}{d'} = \frac{6\varepsilon_0 A}{d/2}$

$\left[\varepsilon_r = \frac{\varepsilon_m}{\varepsilon_0} \Rightarrow 6 = \frac{\varepsilon_m}{\varepsilon_0} \Rightarrow \varepsilon_m = 6\varepsilon_0 \right]$

$C' = 12 \frac{\varepsilon_0 A}{d} = 12 \times 8$

$\boxed{C' = 96 \text{ pF}}$

2.6] (a) $\frac{1}{C_S} = \frac{n}{C} = \frac{3}{9} = \frac{1}{3}$

$C_S = 3$ pF

(b) $V = \frac{Q}{C} \times 3 \Rightarrow Q = \frac{VC}{3} = \frac{120 \times 9^{3}}{3_1} = 360$ C

$V_{\text{across each capacitor}} = \frac{Q}{C} = \frac{360}{9} = 40$ V

Method 2] Divide total V by 3 as all 3 capacitors have same capacitance.

2.7] (a) $C_p = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9$ pF

(b) $Q_1 = VC_1 = 100 \times 2 = 200$ pC.

$Q_2 = VC_2 = 100 \times 3 = 300$ pC

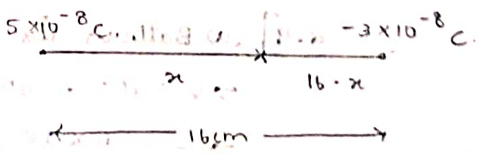
$Q_3 = VC_3 = 100 \times 4 = 400$ pC

2.10] $U = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$.

$\boxed{U = 1.5 \times 10^{-8} \text{ J}}$

2.1] $V = V_1 + V_2 = 0$

$\frac{kq_1}{r_1} + \frac{kq_2}{r_2} = 0$



$k\left[\frac{5 \times 10^{-8}}{x} - \frac{3 \times 10^{-8}}{16 - x}\right] = 0$

$\frac{5 \times \cancel{10^{-8}}}{x} = \frac{3 \times \cancel{10^{-8}}}{16 - x}$

$5(16 - x) = 3x$

$80 - 5x = 3x$

$80 = 8x \Rightarrow \boxed{x = 10 \text{ cm}}$

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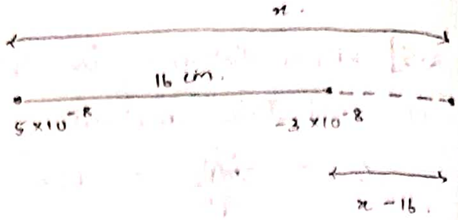
Also, $V = V_1 + V_2 = 0$



$$\frac{5\times10^{-8}}{x} = \frac{+3\times10^{-8}}{x-16}$$

$$5(x-16) = 3x$$

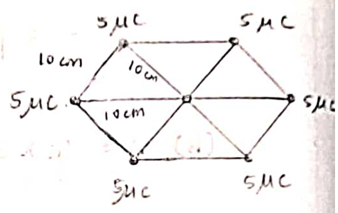
$$5x - 80 = 3x$$

$$2x = 80$$

$$x = 40\ cm$$

∴ At points 10 cm and 40 cm from the positive charge, the electric potential is zero.

2.2] $q = 5\mu C$, $r = 10$ cm



$$V = \frac{6kq}{r} = \frac{6\times9\times10^{9}\times5\times10^{-6}}{10\times10^{-2}}$$

$$= 270\times10^{+9-6+2-1}$$

$$= 270\times10^{4}$$

$$\boxed{V = 2.7\times10^{6}\ V}$$

→ Each triangle is equilateral triangle

2.8] $A = 6\times10^{-3}\ m^2$, $d = 3\times10^{-3}\ m$.

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$

$$C = 17.708\times10^{-12}\ F$$

$$\boxed{C = 17.708\ pF}$$

$V = 100\ C$

$$Q = CV = 17.708\times10^{-12}\times100$$

$$= 17.708\times10^{-10}$$

$$= 1.77\times10^{-9}\ C$$

$$\boxed{Q = 1.77\ nC = 1.8\ nC}$$

∴ Charge on each plate is 1.77 nC.

2.9]

(a) Battery is connected.	(b) Battery is not connected.
→ $V = 100\ V$ (constant).	→ $Q = 1.8\ nC$ (constant).
→ $C = 6\times18\ pF$ $\boxed{C = 108\ pF}$	→ $C = kC_0 = 6\times18\ pF$ $\boxed{C = 108\ pF}$
→ $Q = CV = kQ_0$ $= 6\times1.8\ nC$ $\boxed{Q = 10.8\ nC}$	→ $Q = V = \frac{V_0}{K} = \frac{100}{6}$ $\boxed{V = 16.67\ V}$

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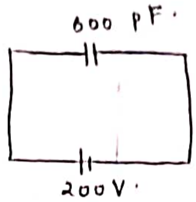
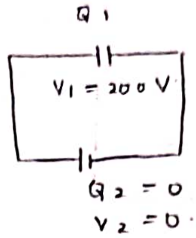
2.11] $C_1 = C_2 = 600\,pF$

$V_1 = 200\,V$

$V_2 = 0$





$$\text{loss} = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2}\frac{C}{2}(V_1)^2$$

$$= \frac{1}{2} \times \frac{600 \times 10^{-12}}{2} \times 200 \times 200$$

$$\boxed{\text{loss} = .6 \times 10^{-6}\ J}$$

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