10/08/2024

4. MOVING CHARGES AND MAGNETISM

INTRODUCTION:

* A charge at rest produces electric field.
* A charge in motion produces both electric and magnetic field.
* In other words, a moving charge (current) creates a magnetic field in the space surrounding it.
* The relation ~~...~~ between electricity and magnetism was rediscovered and explained by Hans Christian ~~...~~ Oersted in 1820.

BIOT - SAVART LAW:

* According to the Biot - Savart Law, the ~~magnetic~~ magnitude of the magnetic field dB is

i) directly proportional to the current I through the conductor $\Rightarrow dB \propto I$

ii) directly proportional to the length dl of the current element $\Rightarrow dB \propto dl$

iii) inversely proportional to the square of the distance r of the point P from the current element $\Rightarrow dB \propto \frac{1}{r^2}$

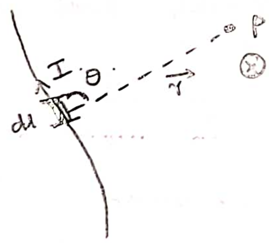
iv) directly proportional to $\sin\theta$ where θ is the angle between $\vec{dl}$ and $\vec{r}$. $\Rightarrow dB \propto \sin\theta$

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \times \frac{r}{r}$$

$$\boxed{\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3}}$$



⊗ → into the paper

* Here, μ_0 is called permeability of free space.
* Unit of magnetic field is tesla (T).
* ~~Unit~~ Value of μ_0 is equal to $4\pi \times 10^{-7}$ TmA^{-1}.
* The direction of $\vec{dB}$ is given by right hand screw rule.

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* Special Case : i) If $\theta = 0°$ [the point lies on the axis of current element], then $\sin\theta = 0$. $\Rightarrow dB = 0$

ii) If $\theta = 90°$ $\rightarrow \sin\theta = 1 \Rightarrow dB = \text{maximum}$:

* Thus, $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}$

→ If $I = 1\,A$, $\sin\theta = 1$, $r = 1\,m$, $dl = 1m$ [all are defined as unity]

$$dB = \frac{4\pi \times 10^{-7}}{4\pi} \times 1 = 10^{-7}\ T$$

→ Therefore, 1 tesla is 10^7 times the magnetic field produced by a conducting wire of length 1m and carrying current of 1 A at a distance of 1 m from it and perpendicular to it.

$$\frac{dB}{10^{-7}} = 1\,T \quad \rightarrow \quad 1\,T = 10^7 \times dB$$

BIOT-SAVART LAW VS COULOMB'S LAW :

* Similarities :

⇒ Both fields depend inversely on the square of the distance from the source to the point of observation.

⇒ Both are long range fields [when $r \longrightarrow \infty$, $B, E = 0$]

⇒ The principle of superposition to applicable to both fields.

⇒ Both the fields are linearly dependant on their sources.

[$dB \propto Idl$, $dE = dq$]. [power = 1]

* Dissimilarities / Points of Difference :

ELECTRIC FIELD	MAGNETIC FIELD
* It is produced by a scalar source (electric charge q).	* It is produced by a vector source (current element $Id\vec{l}$).
* The direction of electric field is along the displacement vector joining the source and the point.	* The direction of magnetic field is perpendicular to the plane containing the displacement vector $\vec{r}$ and the current element $Id\vec{l}$.
* It is a conservative field.	* It is a non-conservative field.
* The electric field is independent of angle.	* The magnetic field is proportional to $\sin\theta$ [θ is the angle between $Id\vec{l}$ and $\vec{r}$].

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RELATION BETWEEN μ_0, ε_0 AND c :

* $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2} \rightarrow \varepsilon_0 = \frac{1}{4\pi\times9\times10^9}$

* $\frac{\mu_0}{4\pi} = 10^{-7}\ TmA^{-1} \rightarrow \mu_0 = 4\pi\times10^{-7}$

* $\mu_0\varepsilon_0 = 4\pi\times10^{-7}\times\frac{1}{4\pi\times9\times10^9}$

$$= \frac{1}{9\times10^{16}} = \frac{1}{(3\times10^8)^2}$$

$$\boxed{\mu_0\varepsilon_0 = \frac{1}{c^2} \rightarrow c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}}$$

Note: $v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\cdot\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r}}$

$v = \frac{c}{\sqrt{\mu_r\varepsilon_r}} \Rightarrow \sqrt{\mu_r\varepsilon_r} = \frac{c}{v} =$ absolute refractive index.

μ_r = relative permeability $\quad \varepsilon_r$ = relative permittivity

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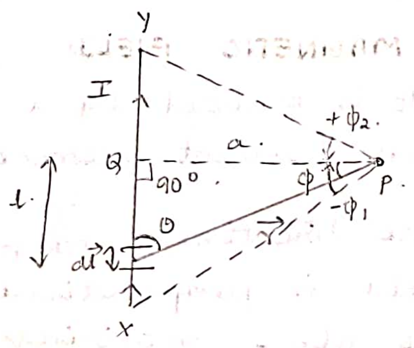
MAGNETIC FIELD DUE TO A LONG STRAIGHT CURRENT CARRYING CONDUCTOR: [FINITE WIRE]

* Consider a long straight conductor XY and a point P at a perpendicular distance 'a' from the wire.
* We wish to calculate the magnetic field at P due to XY.
* The current is XY is ~~I~~, I.



[Here, θ and r are variable for different position of $d\vec{l}$.]

[ϕ is also variable].

* Consider a current element $d\vec{l}$ at a distance l from the point Q.
* According to Biot Savart Law, the magnetic field at P is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

right ΔOQP

$$\theta + \phi + 90^\circ = 180^\circ$$

$$\theta = 90 - \phi$$

$$\sin\theta = \sin(90-\phi) = \cos\phi$$

$$\cos\phi = \frac{a}{r}$$

$$r = \frac{a}{\cos\phi} = a\sec\phi$$

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$\tan\phi = \frac{l}{a} \Rightarrow l = a\tan\phi$

$dl = a\sec^2\phi \, d\phi$

$B = \int dB = \int \frac{\mu_0 I}{4\pi} \times \frac{a\sec^2\phi \cos\phi \, d\phi}{a^2\sec^2\theta}$

$= \frac{\mu_0 I}{4\pi a} \int_{-\phi_1}^{\phi_2} \cos\phi \, d\phi = \frac{\mu_0 I}{4\pi a} [\sin\phi]_{-\phi_1}^{\phi_2}$

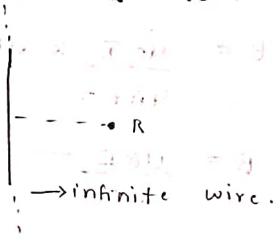
$= \frac{\mu_0 I}{4\pi a} [\sin\phi_1 - \sin(-\phi_2)]$

$$\boxed{B = \frac{\mu_0 I}{4\pi a} [\sin\phi_1 + \sin\phi_2]}$$

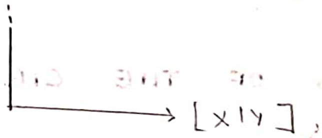
* Special case : i) XY is infinite. Then $\phi_1 = \phi_2 = 90°$.

$B = \frac{\mu_0 I}{4\pi a} [\sin 90° + \sin 90°]$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$



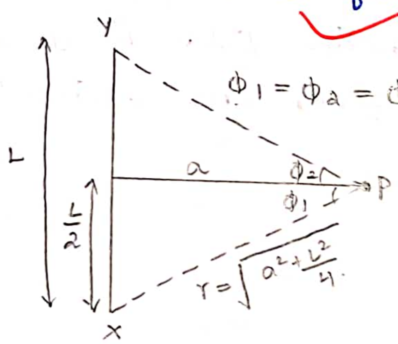
ii) XY is infinite but P lies opposite to one of then ends. Then $\phi_1 = 0$ and $\phi_2 = 90°$.



$B = \frac{\mu_0 I}{4\pi a} [\sin 90° + \sin 0°]$

$$\boxed{B = \frac{\mu_0 I}{4\pi a}}$$

iii) XY is finite and P lies in the midpoint.



$B = \frac{\mu_0 I}{4\pi a} [\sin\phi + \sin\phi]$

$= \frac{\mu_0 I}{4\pi a} \times 2 \times \frac{\frac{L}{2}}{\sqrt{a^2 + \frac{L^2}{4}}}$

$= \frac{\mu_0 I}{4\pi a} \left[\frac{L}{\sqrt{\frac{4a^2 + L^2}{4}}}\right]$

$$\boxed{B = \frac{\mu_0 I}{2\pi a} \left[\frac{L}{\sqrt{4a^2 + L^2}}\right]}$$

Note : The direction of magnetic field for straight wire are concentric circles with wire at the centre and in a plane perpendicular to the wire.

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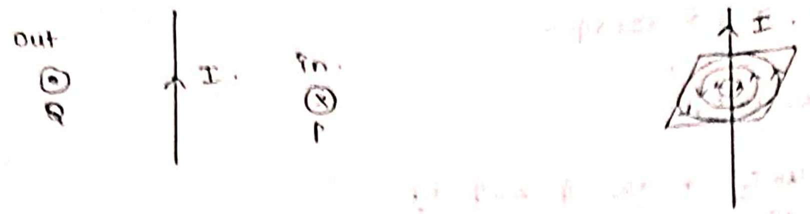


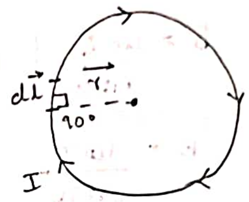
MAGNETIC FIELD AT THE CENTRE OF THE CIRCULAR CURRENT LOOP :

Consider a circular loop of radius (r) carrying current (I) :

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin\theta}{r^2} \qquad [\theta = 90^\circ \Rightarrow \sin 90^\circ = 1]$$

$$B = \int dB = \frac{\mu_0 I}{4\pi r^2} \int_0^{2\pi r} dl = \frac{\mu_0 I}{4\pi r^2} [l]_0^{2\pi r}$$

$$B = \frac{\mu_0 I}{4\pi r^2} \times 2\pi r$$



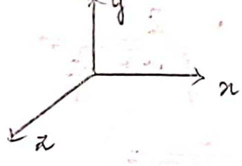
$$B = \frac{\mu_0 I}{2r}$$

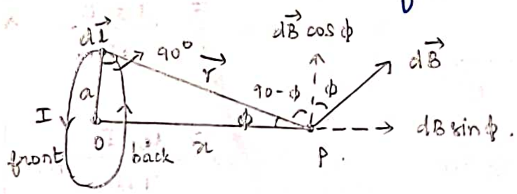
If there is a coil of N turns, then

$$\boxed{B = \frac{\mu_0 N I}{2r}}$$

MAGNETIC FIELD ON THE AXIS OF THE CIRCULAR CURRENT LOOP :

* Consider a circular loop of a wire of radius (a) and carrying current (I) as shown in the figure.
* We wish to calculate the magnetic field $\vec{B}$ at an axial point P at a distance (x) from the centre (O).





$r = \sqrt{a^2 + x^2}$

* Consider a current element $d\vec{l}$ at the top of the loop, it has an outward coming current.
* Let $\vec{r}$ be the position vector of point P with element $d\vec{l}$. Applying Biot - Savart Law :

$$dB = \frac{\mu_0 I dl \sin\theta}{4\pi r^2} \qquad [\theta = 90^\circ \Rightarrow \sin\theta = 1]$$

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$$\Rightarrow \boxed{dB = \frac{\mu_0 I dl}{4\pi r^2}}$$

* The $d\vec{B}$ can be resolved into two components :
  i) $dB \sin\phi$ along the axis
  ii) $dB\cos\phi$ perpendicular to the axis
* Due to symmetry for any 2 diametrically opposite elements the perpendicular components will be equal and opposite and will cancel out.
* Therefore, total magnetic field at point P in the direction OP is

$$B = \int dB \sin\phi$$

$$B = \int \frac{\mu_0 I dl}{4\pi r^2} \sin\phi$$ [r is constant at all loop points].

$$= \frac{\mu_0 I}{4\pi r^2} \cdot \frac{a}{r} \int_0^{2\pi a} dl.$$ $\left[\sin\phi = \frac{a}{r}\right]$

$$= \frac{\mu_0 I a}{4\pi r^3} [l]_0^{2\pi a}$$

$$= \frac{\mu_0 I a}{4\pi r^3} [2\pi a]$$

$$B = \frac{\mu_0 I a^2}{2 r^3}$$

$$B = \frac{\mu_0 I a^2}{2[(a^2+x^2)^{1/2}]^3}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}}$$

* If the coil has N turns, then

$$\Rightarrow \boxed{B = \frac{\mu_0 N I a^2}{2(a^2+x^2)^{3/2}}}$$

* The magnetic field is along (+ve) x axis even if the point lies in the opposite side of the loop.
* Special Case :
  i) At the centre of the loop : i.e., $x = 0$.

$$B = \frac{\mu_0 N I a^2}{2(a^2)^{3/2}}$$

$$= \frac{\mu_0 N I a^2}{2a^3}$$

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$\Rightarrow \boxed{B = \dfrac{\mu_0 N I}{2a}}$

2) At a point far away from the coil : i.e., $x >> a$

$\Rightarrow \boxed{B = \dfrac{\mu_0 N I a^2}{2x^3}}$

3) At a point on the axis is equal to the radius of the coil : i.e., $x = a$

$$B = \frac{\mu_0 N I a^2}{2(2a^2)^{3/2}}$$

$$B = \frac{\mu_0 N I a^2}{2^1\, 2^{3/2}\, a^3}$$

$\Rightarrow \boxed{B = \dfrac{\mu_0 N I}{2^{5/2} a}}$

CLOCK RULE :

* This rule gives the polarity of any face of a circular current loop.

1) Anticlockwise – North Pole
2) ~~Anti~~ Clockwise – South Pole

Note : i) Area of coil : Rectangle $= l \times b$

Square $= a^2$

Triangle $= \frac{1}{2} \times$ base $\times$ height

Circle $= \pi r^2$

ii) CGS unit of magnetic field is guass. (G)

$\Rightarrow 1\ G = 10^{-4}\ T$

iii) For semi-circular loop,

$$B = \frac{\mu_0 I}{4r} \quad \left[B = \frac{B \text{ of full loop}}{2} \right]$$

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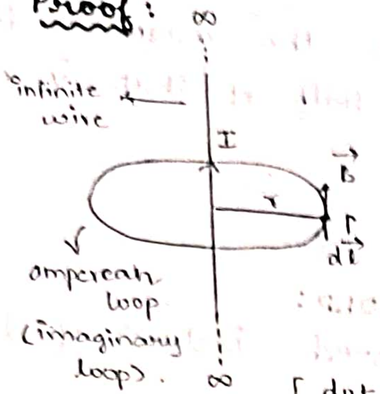
AMPERE'S CIRCUITAL LAW:

* It states that the line integral of the magnetic field around any closed path is equal to μ_0 times, the total current passing through the closed path.
* Mathematically, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$.

Proof:



We know that,

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint \vec{B} \cdot d\vec{l} = \int B\,dl \cos 0^\circ \quad [\text{dot product}].$$

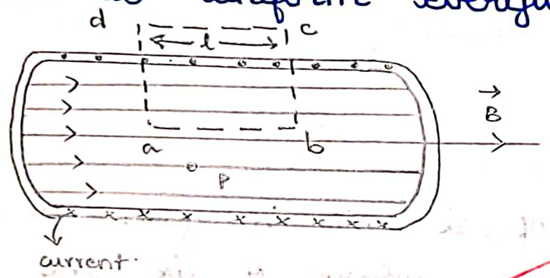
$$= \frac{\mu_0 I}{2\pi r} \int_0^{2\pi r} dl = \frac{\mu_0 I}{2\pi r} \times 2\pi r$$

[dot product] $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I}$

* The closed curve is called amperean loop which is a geometrical entity and not a real wire loop.

MAGNETIC FIELD INSIDE A SOLENOID:

* A solenoid means an insulated copper wire wound closely in the form of a helix.
* The magnetic field inside a closely wound long solenoid is uniform everywhere and zero outside it.



* Consider a rectangular closed path abcd.
* According to Ampere's Circuital law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times \text{total current enclosed by loop.}$$

* **abcd**: $\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \underbrace{\int_b^c \vec{B} \cdot d\vec{l}}_{\cos 90^\circ = 0} + \underbrace{\int_c^d \vec{B} \cdot d\vec{l}}_{cd \text{ is outside } B_{out} = 0} + \underbrace{\int_d^a \vec{B} \cdot d\vec{l}}_{\cos 90^\circ = 0}$

$$= B \int_a^b d\vec{l}$$

$$= B[l]_a^b = B[b-a]$$

$$\boxed{\oint \vec{B} \cdot d\vec{l} = Bl}$$

* $n \longrightarrow$ no. of turns per unit length

$\Rightarrow$ total number of turns in length $l = nl$

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→ Each wire = I

→ nl wire = Inl

Here, $n = \frac{N}{l}$

N = no. of turns

l = length of wire

* $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{tot}$.

$Bl = \mu_0 I n l$

$$\boxed{B = \mu_0 n I}$$

* It can be easily shown that the magnetic field at the ends of solenoid is just half of that in middle.

~~Bend = 1/2~~ $\boxed{B_{end} = \frac{1}{2} \mu_0 n I}$

MAGNETIC FIELD INSIDE A TORROID:

* Here, circumference is considered instead of length.

* $Bl = \mu_0 I_{total}$

$B \times 2\pi r = \mu_0 \times I n 2\pi r$

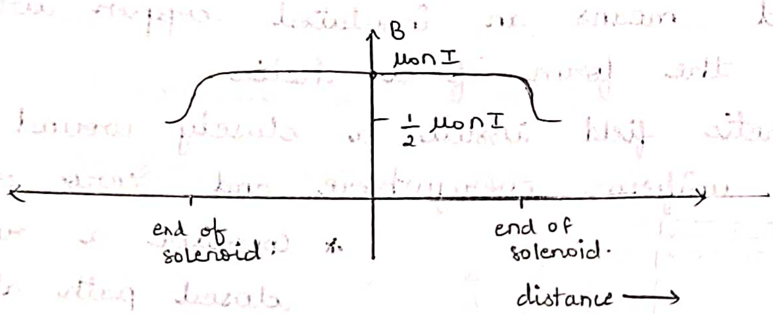
$$\boxed{B = \mu_0 n I}$$

Here, $n = \frac{N}{2\pi r}$

n = no. of turns

$2\pi r$ = circumference

GRAPH FOR SOLENOID:



Note: * When a segment of a coil is given the magnetic field $\boxed{B = \frac{\mu_0 I}{2a} \times \frac{\theta}{2\pi}}$ where θ is the angle subtended by the curved segment.

* If the point P lies on the wire or on the axis of the wire the magnetic field $\vec{B} = 0$.

FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD:

* The electric charges moving in a magnetic field experiences a force called Lorentz force.

* The force depends on:

i) $F \propto B$ (magnetic field)

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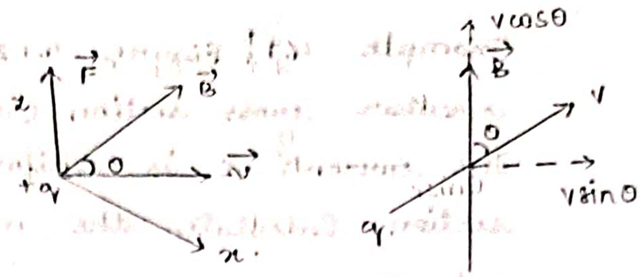
ii) $F \propto q$ (charge)
iii) $F \propto v\sin\theta$ (velocity).

$\therefore F \propto Bqv\sin\theta$

$F = kBqv\sin\theta$ [Here, k = 1]

$F = Bqv\sin\theta$

$\vec{F} = q(\vec{v} \times \vec{B})$



⇒ cos component is along $\vec{B}$, so it does not have any effect.

* The direction of Lorentz force for negative charges is opposite to the direction of $(\vec{v} \times \vec{B})$.

* Flemming's Left Hand Rule: If the thumb, index and middle finger are kept mutually perpendicular to each other, thumb shows the direction of Lorentz force, index shows the direction of magnetic field and middle finger shows the direction of current.

* Special case:

1) If $v = 0$ (charge is at rest), $F = 0$.

2) If $\theta = 0°$ (or) $180°$, charge is moving parallel or antiparallel to field, $F = 0$.

3) If $\theta = 90°$, charge is moving perpendicular to field, $\boxed{F_{max} = Bqv}$.

* Flemming's Left Hand Rule: Stretch the thumb and the first two fingers of the left hand mutually perpendicular to each other. If the forefinger points in the direction of magnetic field, the central finger points in the direction of current then the thumb gives the direction of force on the charged particle.

* Lorentz Force: The total force experienced by a charged particle moving in a region where both electric and magnetic fields are present is called lorentz force.

$\vec{F} = \vec{F_e} + \vec{F_m}$

$= q\vec{E} + q(\vec{v} \times \vec{B})$.

$$\boxed{\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]}$$

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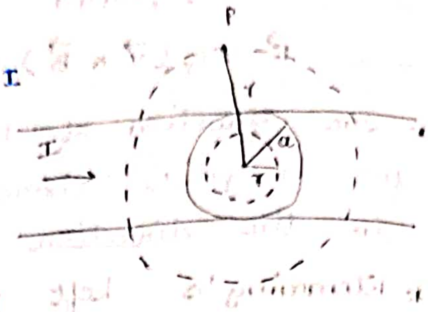
Example 4.8] Figure 4.18 shows a long straight wire of a circular cross section (radius a) carrying steady current I. The current I is uniformly distributed across this cross-section. Calculate the magnetic field in the region $r < a$ and $r > a$.



For the loop, current enclosed is I and length is $2\pi r$. (outside point).

$$\oint B\,dl = \mu_0 I \Rightarrow B[L]_0^{2\pi r} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow B \propto \frac{1}{r} \quad [r > a]$$

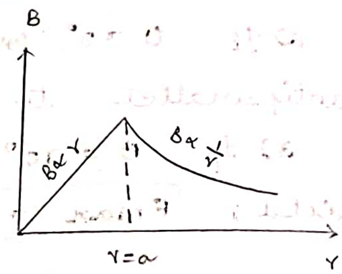
For an inside point,

For an area of $\pi a^2 \Rightarrow$ current is I.

For an area of $\pi r^2 \rightarrow$ current is I'.

$$I' = I\,\frac{\pi r^2}{\pi a^2} = \frac{I r^2}{a^2}$$

$$B(2\pi r) = \frac{\mu_0 I r^2}{a^2}$$

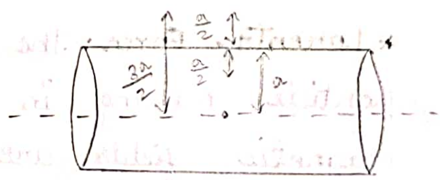


$$B = \frac{\mu_0 I r}{2\pi a^2}$$

$$\Rightarrow B \propto r \quad [r < a]$$

Q: Calculate the ratio of the magnetic field at a point $\frac{a}{2}$ above the surface of the wire to that at a point $\frac{a}{2}$ below its surface. What is the maximum value of the field of this wire?

For inside point,



$$B_i = \frac{\mu_0 I \frac{a}{2}}{2\pi a^2} = \frac{\mu_0 I}{4\pi a}$$

For outside point,

$$B_o = \frac{\mu_0 I}{2\pi\left(\frac{3a}{2}\right)} = \frac{\mu_0 I}{3\pi a}$$

$$\text{Ratio} = \frac{B_o}{B_i} = \frac{\cancel{\mu_0 I}}{3\pi \cancel{a}} \times \frac{4\pi \cancel{a}}{\cancel{\mu_0 I}}$$

$$\boxed{\frac{B_o}{B_i} = \frac{4}{3}}$$

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Magnetic field is maximum at $r = a$.

$$\boxed{B_{max} = \frac{\mu_0 I}{2\pi a}}$$

MOTION OF A CHARGED PARTICLE UNDER UNIFORM MAGNETIC FIELD:

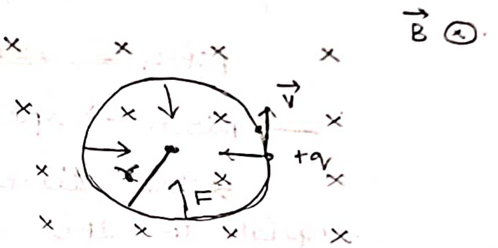
* When the initial velocity is parallel to the ~~fittd~~ magnetic field, $\theta = 0$

$$\vec{F} = q(\vec{v} \times \vec{B}) = 0.$$

⇒ Thus the parallel magnetic field does not exert any force on the moving charged particle and the charged particle will continue to move in the same path.

* When the initial velocity is perpendicular to the magnetic field, $\theta = 90°$

$$\vec{F} = qvB\sin 90° = qvB$$



⇒ The centripetal force is provided by the magnetic force.

Thus,

$$\frac{mv^{\cancel{2}}}{r} = q\cancel{v}B$$

$$\Rightarrow \boxed{r = \frac{mv}{Bq}}$$

$$\left[F_c = \text{centripetal force} = \frac{mv^2}{r}\right]$$

⇒ Time period: It is the time taken to complete one revolution. The time period of one revolution is

$$T = \frac{\text{distance (circumference)}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi m\cancel{v}}{\cancel{v}Bq}$$

$$\Rightarrow \boxed{T = \frac{2\pi m}{Bq}} \quad [T = \text{time period}]$$

Note: Clearly, time period is independent of v and r. If the particle moves faster, the radius is larger. Therefore the time taken is same.

⇒ Frequency: It is the reciprocal of time period.

$$f = \frac{1}{T}$$

$$\Rightarrow \boxed{f = \frac{Bq}{2\pi m}} \quad [f = \text{frequency}]$$

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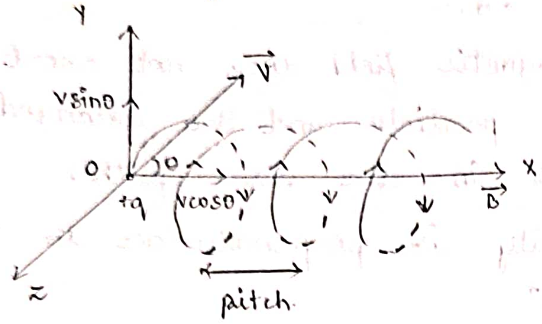
* When the initial velocity makes an angle θ with the field direction, The velocity can be resolved by two components - $v\cos\theta$ and $v\sin\theta$.

⇒ $v\cos\theta$ is $\parallel^{el}$ to $\vec{B}$, so the charged particle continues to move along field $v\cos\theta$. [$v\cos\theta = v_{\parallel}$]

⇒ $v\sin\theta$ is perpendicular component and is responsible for the circular path. [$v\sin\theta = v_{\perp}$]



$r = \frac{mv_{\perp}}{Bq}$

⇒ $\boxed{r = \frac{mv\sin\theta}{Bq}}$

$T = \frac{2\pi r}{v_{\perp}} = \frac{2\pi mv\sin\theta}{Bq\, v\sin\theta}$

⇒ $\boxed{T = \frac{2\pi m}{Bq}}$

~~pitch = distance~~

⟶ pitch = speed x time

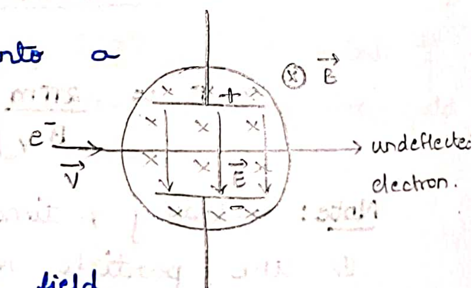
⇒ The time taken to cover linear distance is equal to the time taken to cover the circular path.

$\text{pitch} = v_{\parallel} \times T = v\cos\theta \times \frac{2\pi m}{Bq}$

⇒ $\boxed{\text{pitch} = \frac{2\pi m v\cos\theta}{Bq}}$

MOTION OF A CHARGE IN PERPENDICULAR ELECTRIC AND MAGNETIC FIELDS:

* ~~For~~ A beam of electrons enter into a region with cross (perpendicular) electric and magnetic fields.



* The electric field deflects the electron upwards and the magnetic field deflects the electron downwards,

* If Electric force = Magnetic force [⇒ electron is undeflected]

$$qE = qvB$$

$$eE = evB$$

$$E = vB$$

⇒ $\boxed{v = \frac{E}{B}}$ [v = velocity selector]

* Such an arrangement can be used to select charged

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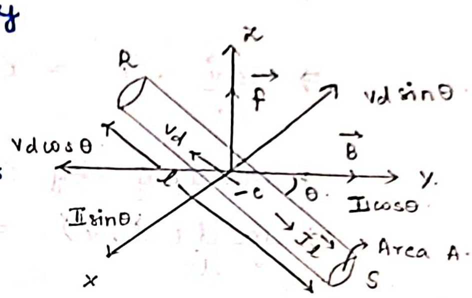
particle of particular velocity. This arrangement is called velocity selector or velocity filter.

FORCE ON A CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD:

* Consider a conductor RS of length l and area of cross section A carrying current I along the direction R to S at an angle θ with y-axis.
* The magnetic field is along the positive y-axis.
* The electrons drift towards R with a velocity, $\vec{v_d}$.
* Each electron experiences a magnetic lorentz force along positive x-axis which is given by

$$\vec{f} = -e(\vec{v_d} \times \vec{B})$$

[$\vec{f}$ = force due to 1 e^-]



* If n is the number of free electrons per unit volume, then total number of electrons in the conductor is

$$N = n \times \text{volume}$$
$$N = n \times Al$$

⇒ Here $I\cos\theta$ is along $\vec{B}$, hence has no effect.

⇒ As particle is (−ve), opp. dir., i.e. dir. of I is taken for v_d.

* Total force on the ~~dis~~ conductor due to the electrons is

$$\vec{F} = N\vec{f}$$
$$= nAl[-e(\vec{v_d} \times \vec{B})]$$
$$= -neAl(v_d \hat{v_d} \times \vec{B})$$
$$= neAv_d(-l\hat{v_d} \times \vec{B})$$
$$= neAv_d(l\hat{l} \times \vec{B})$$
$$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$$

[$I = neAv_d$]

[Here, $-\hat{v_d} = \hat{l}$]

* Magnitude of force $F = IlB\sin\theta$

⇒ Here, θ is the angle between $\vec{B}$ and (length $\vec{l}$) current (I).

* Special case:

1) If $\theta = 180°$ (or) $0°$, $F = 0$.

Thus a current carrying conductor placed parallel (or) anti-parallel to the direction of field does not experience any force.

2) If $\theta = 90°$, $F_{max} = IlB$

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WORK DONE BY A MAGNETIC FORCE ON A CHARGED PARTICLE IS ZERO:

* The magnetic force $\vec{F} = +q(\vec{v} \times \vec{B})$ always acts perpendicular to the velocity $(\vec{v})$ or the direction of motion of charge q.

$\Rightarrow \vec{F} \cdot \vec{v} = q(\vec{v} \times \vec{B}) \cdot \vec{v}$

[Angle btw $(\vec{v} \times \vec{B})$ and $\vec{v}$ is 90° → For dot product as $\theta = 90^\circ$, $\cos\theta = 0$]

$\Rightarrow \vec{F} \cdot \vec{v} = 0$

* According to Newton's Second Law,

$\Rightarrow \vec{F} = m\vec{a} = m\dfrac{d\vec{v}}{dt}$

{By Work Energy Theorem, change in K = W}

$\Rightarrow m\dfrac{d\vec{v}}{dt} \cdot \vec{v} = 0$

⇒ Multiply by 2 on both sides.

$\Rightarrow \dfrac{m}{2}\dfrac{d(\vec{v} \cdot \vec{v})}{dt} = 0 \Rightarrow \dfrac{m}{2}\dfrac{d}{dt}(v^2) = 0.$

$\Rightarrow \dfrac{d}{dt}\left(\dfrac{1}{2}mv^2\right) = 0$

{K = kinetic energy, W = work done}

$\Rightarrow \dfrac{dK}{dt} = 0. \quad \Rightarrow K = \text{constant}$

$\Rightarrow W = 0$

* Thus, a magnetic force does not change the kinetic energy of the charged particle.
* This indicates that the speed of particle does not change.
* Thus the work done on charged particle by the magnetic force is zero.

FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS:

* Consider two long parallel wires AB and CD carrying current I_1 and I_2.
* Let r be the separation between the two wires.
* The magnetic field due to I_1 on wire CD is

$\Rightarrow B_1 = \dfrac{\mu_0 I_1}{2\pi r}$

⇒ This field B_1 is perpendicular and inwards on wire CD carrying current I_2.

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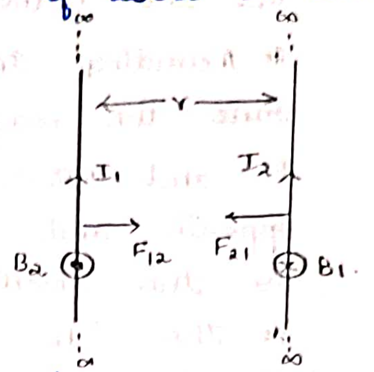
* Thus the force acting on length l of wire CD will be

$\Rightarrow F_{21} = I_2 B_1 l \quad [\theta = 90°]$

$\Rightarrow F_{21} = I_2 \frac{\mu_0 I_1}{2\pi r} l$



* Therefore, force per unit length

$\Rightarrow \frac{F}{l} = f = \frac{\mu_0 I_1 I_2}{2\pi r}$

* Similarly force on 1 due to 2 can be calculated using flemming's left hand rule.

$\Rightarrow \vec{F_{12}} = -\vec{F_{21}}$

Note: When the currents on the two wires are in
→ same direction = attractive force
→ opposite direction = repulsive force

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* Definition of Ampere: 1 Ampere is that value of steady current which on flowing in each of the two ~~infinitely~~ parallel infinitely long conductors of negligible cross section placed in vacuum at a distance of 1 m from each other produces between them a force of 2×10^{-7} N m^{-1} of their length.

$\Rightarrow I_1 = I_2 = 1$ A $\qquad \Rightarrow r = 1$ m.

$\Rightarrow f = \frac{\mu_0}{2\pi} \times \frac{1 \times 1}{1} = \frac{4\pi \times 10^{-7}}{2\pi}$

$\Rightarrow \boxed{f = 2 \times 10^{-7} \ Nm^{-1}}$

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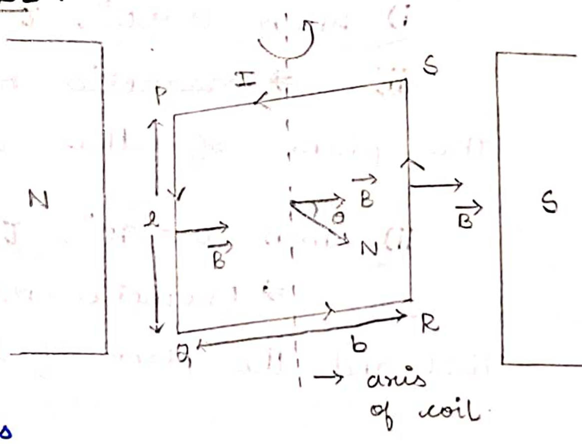
TORQUE EXPERIENCED BY A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD:

* Consider a rectangular loop PQRS suspended in a uniform magnetic field $\vec{B}$ with its axis perpendicular to the axis.



* Let I be the current flowing through the coil PQRS, l and b be the sides of the coil PQRS, A be the area of the PQRS $[A = lb]$ and θ be the angle between the magnetic field $\vec{B}$

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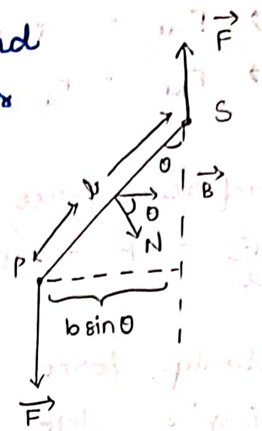
and the normal to the plane of the coil N.



* According to fleming's left hand rule the magnetic force on sides PS and QR are equal and opposite and act on same line. So the resultant is zero.

* The side PQ and RS experience an inward and outward force

$\vec{F} = Il B$

[N is radially outward from the plane of the coil].

and they form a couple which exerts a torque given by

$\tau = F \cdot \perp^{r}$ distance

$\tau = IlB \times b\sin\theta$

$\Rightarrow \boxed{\tau = IAB\sin\theta}$ $[A = lb]$

* For N number of turns,

$\tau = NIAB\sin\theta$

* But NIA = m (magnetic moment) of the loop.

$\boxed{\tau = mB\sin\theta}$

* In vector notation

$\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$

Note: The direction of $\vec{m}$ is given by the direction of $\vec{A}$.

*Special case:

i) When $\theta = 0^\circ$, $\tau = 0$.

$\Rightarrow$ Magnetic moment is parallel to field and the plane of the coil is perpendicular to field.

ii) When $\theta = 90^\circ$, $\tau = NIAB$ = maximum.

$\Rightarrow$ Magnetic moment is perpendicular to field and the plane of the coil is parallel to field.

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CURRENT LOOP AS A MAGNETIC DIPOLE:

The magnetic field due to a circular current loop at a point P on the axial line at a distance x from the centre of the loop of radius a ($x \gg a$) is

$\Rightarrow$ $$B = \frac{\mu_0 N I a^2}{2x^3}$$

$\Rightarrow$ Multiply and divide by $2\pi a^2$

$\Rightarrow$ $$B = \frac{\mu_0 N I \cancel{a^2}\, 2\pi a^2}{2 \times 2\pi \cancel{a^2} \times x^3}$$

$\Rightarrow$ $$B = \frac{\mu_0\, 2 N I A}{4\pi x^3}$$ $[A = \pi a^2]$ [A = area, a = radius]

* Comparing the electric field due to an electric dipole ~~on the~~ at a point on the axial line at a distance x ($x \gg a$) having dipole moment, $p = 2aq$ is

$$E_{axial} = \frac{2p}{4\pi \varepsilon_0 x^3}$$

* Since both B and E have the same distance dependence ($1/x^3$), this suggests that current loop behaves as a magnetic dipole having magnetic moment, $m = NIA$.

$$\boxed{\vec{m} = NI\vec{A}}$$

* SI unit of magnetic moment is Am^2.

MOVING COIL GALVANOMETER:

* A galvanometer is a device used to detect current in a circuit. The commonly used moving coil gavanometer is named so because its uses a current carrying coil that rotates (moves) in a magnetic field due to the torque acting on it.

* Principle:

$\Rightarrow$ A current carrying coil placed in a magnetic field experience a current depended torque, which tends to rotate the coil as the coil produces angular deflection.

* Construction:

$\Rightarrow$ As shown in the figure, a ~~uses~~ Weston (Pivoted (Hanging) type galvanometer consists of a rectangular coil of fine insulated copper wire wound on non-magnetic metallic (aluminium) frame and a soft iron core is placed inside the

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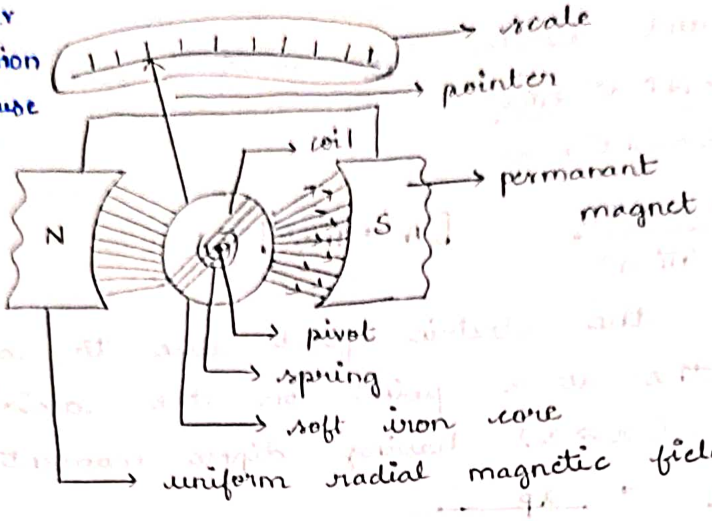
drum to increase the strength of radial magnetic field.

⇒ The motion of the coil is controlled by a pair hair springs of phosphor bronze.

⇒ The spring provides the restoring torque.

⇒ A light aluminium pointer attached to the coil measures its deflection on a suitable scale.

⇒ The phosphur bronze suspension is used because it has more tensile strength and small value of restoring torque per unit twist.



* **Theory and Working:**

⇒ Since, the field is radial, the plane of the coil always remains parallel to the field $\vec{B}$.

⇒ According to Fleming's Left Hand Rule, two forces equal and opposite act on a coil as a couple and exert a torque given by

$T = NIBA \sin\theta$ where $\theta = 90°$

$$\boxed{T_{max} = NIAB}$$

⇒ The elasticity of the spring provided a restoring torque such that

$T_{restoring} \propto \alpha$ [α - deflection]

$$\boxed{T_{restoring} = k\alpha}$$

where k is the torsion constant of the spring.

⇒ In equilibrium position

\# restoring torque = deflecting torque

\# $k\alpha = NIAB$

\# $\alpha = \frac{NIBA}{k}$ (or) $\alpha = \frac{NBA}{k} \cdot I$

\# $\boxed{\alpha \propto I}$

⇒ Thus, the deflection produced is proportional to the current flowing through the galvanometer.

$$I = \frac{k}{NBA}\alpha$$

$$\boxed{I = G\alpha}$$

⇒ The factor $G = \frac{k}{NBA}$ is called Galvanometer constant (or) figure of merit (or) current reduction factor.

FIGURE OF MERIT OF A GALVANOMETER :

It is defined as the current which produces a deflection of one scale division in the galvanometer.

$$G = \frac{k}{NBA} = \frac{I}{\alpha}$$

SENSITIVITY OF A GALVANOMETER :

If a large scale deflection is shown even for a small current (or) a small voltage then the galvanometer is sensitive.

(i) CURRENT SENSITIVITY : [I_s]

⇒ The deflection produced in the galvanometer when a unit current flows through it.

$$I_s = \frac{\alpha}{I} = \frac{NBA}{k}$$

⇒ Thus, I_s depends on :

a) no. of turns N
b) magnetic field B
c) area of the coil A
d) torsion constant K

(ii) VOLTAGE SENSITIVITY : [V_s]

⇒ It is deflection produced when a unit potential difference is applied across the ends of the galvanometer.

$$V_s = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NAB}{KR}$$

⇒ Thus, V_s depends on :

a) magnetic field B
b) area of the coil A

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c) Torsion constant k

Note: 1) If we increase the no. of turns N, the length of the wire increases and the resistance also increases. Thus, the voltage sensitivity does not depend on N.

2) If N increases, current sensitivity increases but voltage sensitivity remains constant.

ADVANTAGES OF MOVING COIL GALVANOMETER:

⇒ It can be made highly sensitive.

⇒ Since, a powerful radial field is used, the external fields do not affect the measurement.

⇒ Since, deflection is directly proportional to current, a linear scale can be used to measure the deflection.

⇒ The eddy currents produced in the metallic frame brings the coil to rest quickly.

DISADVANTAGES OF MOVING COIL GALVANOMETER:

⇒ The main disadvantage is that its sensitiveness cannot be changed at will.

⇒ All types of MCG are easily damaged by overloading as its hair spring is suspensional will burn out.

HW 4.2]

EXERCISES [Pg: 169]

Given: $I = 35\ A$

$r = 20\ cm = 0.2\ m.$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi} \times \frac{I}{r} \times 2$$

$$= \frac{10^{-7} \times 35 \times 2}{0.2}$$

$$\boxed{B = 35 \times 10^{-6}\ T}$$

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Q: A galvanometer with a coil of resistance 12 Ω shows full scale deflection for a current 2.5 mA. How will you convert into : (i) an ammeter of range 0 – 7.5A ?
(ii) a voltmeter of range 0 – 10 V ?

Given: $I_g = 2.5 \times 10^{-3}$ A , $R_G = 12\ \Omega$

(i) Range = 0 – 7.5 ⇒ I = 7.5 A

$$R_S = \left(\frac{I_g}{I - I_g}\right) R_G = \left(\frac{2.5 \times 10^{-3}}{7.5 - 2.5 \times 10^{-3}}\right) 12 = \frac{2.5 \times 10^{-3} \times 12}{7.4975}$$

(approximating)

$$= \frac{1}{3} \times 10^{-3} \times 12 = 4 \times 10^{-3}\ \Omega$$

$$\boxed{R_S = 4\ m\Omega}$$

Thus by connecting 4 mΩ in parallel with galvanometer, it can be converted into an ammeter of required range.

(ii) Range = 0 – 10 ⇒ V = 10 V

$$R = \frac{V}{I_g} - R_G = \frac{10}{2.5 \times 10^{-3}} - 12$$

$$= 4 \times 10^{3} - 12 = 4000 - 12$$

$$R = 3988\ \Omega$$

Thus by connecting a resistance of 3988 Ω in series with the galvanometer, we connect it into a voltmeter of required range.

Q: An ammeter of resistance 0.80 Ω can measure current upto 1 A. What must be the shunt to measure current upto 5 A? What is the combined resistance of ammeter and shunt ?

Given: $R_G = 0.80\ \Omega$, $I_g = 1$ A , $I = 5$A

Here, the ammeter is considered as galvanometer.

$$R_S = \left(\frac{I_g}{I - I_g}\right) R_G = \left(\frac{1}{5-1}\right) \times 0.8 = \frac{1}{4} \times 0.8$$

$$\boxed{R_S = 0.2\ \Omega}$$

$$\frac{1}{R_A} = \frac{1}{R_G} + \frac{1}{R_S} = \frac{1}{0.8} + \frac{1}{0.2} = \frac{0.2 + 0.8}{0.8 \times 0.2}$$

$$R_A = \frac{0.16}{1}$$

$$\boxed{R_A = 0.16\ \Omega}$$

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EXERCISES [Pg : 169]

4. 5] Given: $I = 8\ A$, Angle between I and B $(\theta) = 30^\circ$

$B = 0.15\ T$

$F = IlB\sin\theta$

$\frac{F}{l} = IB\sin\theta = IB\sin 30^\circ = 8 \times 0.15 \times \frac{1}{2}$

$\boxed{\frac{F}{l} = 0.6\ N\,m^{-1}}$

4. 6] Given: $B = 0.27\ T$ [uniform field due to solenoid]

A wire of $l = 3 \times 10^{-2}\ m$ is kept perpendicular to the magnetic field B.

$\theta = 90^\circ$.

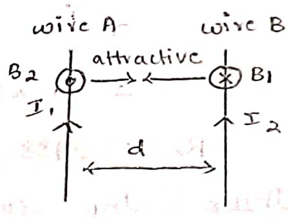
$I = 10\ A$.

$F = IlB\sin\theta = IlB\sin 90^\circ = 10 \times 3 \times 10^{-2} \times 0.27 \times 1$

$\boxed{F = 8.1 \times 10^{-2}\ N}$

4. 7] Given: $I_1 = 8\ A$, $I_2 = 5\ A$; $d = 4 \times 10^{-2}\ m$

$l = 10\ cm = 10 \times 10^{-2}\ m$



$f = \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$F = \frac{\mu_0 I_1 I_2}{2\pi d} \times l$ $\left[\frac{\mu_0}{4\pi} = 10^{-7} \Rightarrow \frac{\mu_0}{2\pi} = 2 \times 10^{-7}\right]$

$= \frac{2 \times 10^{-7} \times 8 \times 5 \times 10 \times 10^{-2}}{4 \times 10^{-2}}$

$= 200 \times 10^{-7}\ N$

$F = 2 \times 10^{-5}\ N$

The force between the two wires is attractive as both the currents are in the same direction.

4.8] Given: $l = 80 \times 10^{-2}\ m$, no. of layers = 5

Each layer has 400 turns.

$N = 5 \times 400 = 2000$

$n = \frac{N}{l} = \frac{2000}{80 \times 10^{-2}} \Rightarrow \boxed{n = 2500\ m^{-1}}$

$d = 1.8 \times 10^{-2}\ m$, $r = 0.9 \times 10^{-2}\ m$

$I = 8\ A$

$B = \mu_0 n I = 4\pi \times 10^{-7} \times 2500 \times 8$

$[\mu_0 = 4\pi \times 10^{-7}]$

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$B = 12.56 \times 200 \times 100 \times 10^{-7}$

$B = 12512 \times 10^{-5}$

$B = 2.512 \times 10^{-2}$

$\boxed{B = 0.02512 \text{ T}}$

4.9] <u>Given</u>: $l = 10 \times 10^{-2}$ m, $N = 20$, $I = 12$ A

$B = 0.8$ T, $\theta = 30°$

$\tau = NIBA \sin\theta = 20 \times 12 \times 0.8 \times \sin 30° \times (10 \times 10^{-2})^2$

$= 20 \times 12 \times 0.8 \times \frac{1}{2} \times 10^{-2} = 16 \times 12 \times 10^{-2} \times \frac{1}{2}$

$= 96 \times 10^{-2}$

$\boxed{\tau = 0.96 \text{ Nm}}$

HW 4.1] <u>Given</u>: $N = 100$, $r = 8 \times 10^{-2}$ m, $I = 0.40$ A

$B = \frac{\mu_0 N I}{2r} = \frac{4\pi \times 10^{-7} \times 10\cancel{0} \times \cancel{0.40}}{\cancel{2} \times \cancel{8} \times 10^{-2}}$

$\boxed{B = 31.4 \times 10^{-5} \text{ T}}$

4.10] (a) current sensitivity $(I_s) = \frac{\alpha}{I} = \frac{NBA}{k}$

<u>Given</u>: $k_1 = k_2$

$\frac{I_{s2}}{I_{s1}} = \frac{\frac{N_2 B_2 A_2}{\cancel{k_2}}}{\frac{N_1 B_1 A_1}{\cancel{k_1}}} = \frac{\cancel{42} \times \cancel{0.5} \times \cancel{1.8} \times 10^{-3}}{30 \times \cancel{0.25} \times \cancel{3.6} \times 10^{-3}}$

$\boxed{\frac{I_{s2}}{I_{s1}} = 1.4}$

∴ Ratio of current sensitivity is 1.4.

(b) $V_s = \cancel{\frac{NBA}{k}} \frac{NBA}{KR} = \frac{I_s}{R}$

$\frac{V_{s2}}{V_{s1}} = \frac{I_{s2}/R_2}{I_{s1}/R_1} = \frac{I_{s2}}{I_{s1}} \times \frac{R_1}{R_2} = 1.4 \times \frac{10}{14}$

$\boxed{\frac{V_{s2}}{V_{s1}} = 1.1}$

∴ Ratio of voltage sensitivity is 1.1.

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4.11] The perpendicular magnetic field exerts a force on the electron perpendicular to its path. This force continuously deflects the electron from its path and makes it move along a circular path.

Magnetic force = centripetal force

$$qvB\sin\theta = \frac{mv^2}{r} \quad [\theta = 90^\circ \Rightarrow \sin 90^\circ = 1]$$

$$e\cancel{v}B(1) = \frac{mv^{\cancel{2}}}{r}$$

$$r = \frac{mv}{eB} = \frac{9.1\times10^{-31} \times 4.8\times10^{6}}{1.6\times10^{-19}\times6.5\times10^{-4}}$$

$$r = 4.2\times10^{-31+6+19+4}$$

$$r = 4.2\times10^{-2}$$

$$\boxed{r = 4.2\ \text{cm}}$$

HW 4.3] Given: $I = 50\,A$, $r = 2.5\,m$ [upwards - out of the plane of paper]

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\cancel{4\pi}\times10^{-7}\times\cancel{50}}{\cancel{2\pi}\times\cancel{2.5}}$$

$$\boxed{B = 4\times10^{-6}\ T}$$

Using ~~Fleming's Left~~ Right Hand Thumb Rule, the direction of the field is found to be upwards.

HW 4.4] Given: $I = 90\,A$, $r = 1.5\,m$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\cancel{4\pi}\times10^{-7}\times\cancel{90}}{\cancel{2\pi}\times\cancel{1.5}}$$

$$\boxed{B = 12\times10^{-6}\ T}$$

Using Right Hand Thumb Rule, the direction of the field is found to be towards south.

HW 4.12]

$$f = \frac{Bq}{2\pi m} = \frac{Be}{2\pi m} = \frac{6.5\times10^{-4}\times1.6\times10^{-19}}{2\times3.14\times9.1\times10^{-31}}$$

$$f = 18.2\times10^{6}\ Hz$$

$$\boxed{F = 18\ MHz}$$

From the formula used, it can be said that frequency is independent of the speed of the electron.

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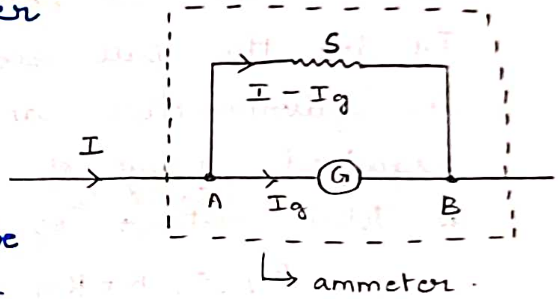
CONVERSION OF GALVANOMETER INTO AN AMMETER :

* The use of ammeter in a circuit must not alter the current. Thus, it should have very low resistance (ideal ammeter has zero resistance).
* To convert the galvanometer into an ammeter a small resistance called shunt resistance is ~~decreased~~, ~~the value of shunt~~ connected in parallel to the galvanometer.
* Thus, the equivalent resistance is decreased.
* The value of shunt resistance depends on the range of current required to be measured (0 - I).
* Let R_G be the resistance of galvanometer, I_g be the current with which galvanometer gives full scale deflection and 0 - I be the required current range of ammeter, R_S be the shunt resistance and $I - I_g$ be the current through the shunt.



* Since, Galvanometer and Shunt are connected in parallel,

$\Rightarrow$ PD across galvanometer = PD across shunt

$$\Rightarrow \quad I_g R_G = (I - I_g) R_S$$

$$\Rightarrow \quad I_g R_G + I_g R_S = I R_S$$

$$\boxed{I = I_g \left(\frac{R_S + R_G}{R_S} \right)}$$

$$\Rightarrow \quad \text{Also,} \quad \boxed{R_S = \left(\frac{I_g}{I - I_g} \right) R_G}$$

* Equivalent Resistance of Ammeter :

$$\frac{1}{R_A} = \frac{1}{R_S} + \frac{1}{R_G} = \frac{R_G + R_S}{R_S R_G}$$

$$\boxed{R_A = \frac{R_S R_G}{R_S + R_G}} < R_S$$

* Shunt : A shunt is a low resistance which is connected in parallel to the galvanometer to protect it from strong currents.

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* **Uses of Shunt:**

i) To prevent galvanometer from being damaged due to large current.

ii) To convert galvanometer into ammeter.

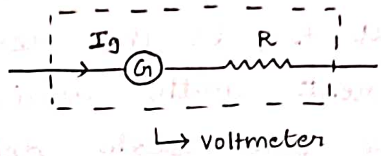
iii) To increase the range of ammeter.

CONVERSION OF GALVANOMETER INTO A VOLTMETER:

* An ideal voltmeter should have infinite resistance.

* Thus, high resistance R is connected in series to get very high resistance of voltmeter.

* Let R_G be the resistance of galvanometer, R be the high resistance which restricts the current to safe limit I_g, I_g be the full scale deflection in galvanometer and 0-V be the required range of voltmeter.



* Total ~~voltage~~ resistance of voltmeter

$$R_V = R + R_G \gg R_G$$

* By Ohm's Law,

$$\Rightarrow I_g = \frac{V}{R_V} = \frac{V}{R + R_G}$$

$$\Rightarrow \boxed{V = I_g (R + R_G)} \quad \text{or}$$

$$\Rightarrow \frac{V}{I_g} = R + R_G$$

$$\Rightarrow \boxed{R = \frac{V}{I_g} - R_G}$$

EXERCISES [Pg: 169]:

4.13] (a) Given: $N = 30$, $I = 6A$, $r = 8 \times 10^{-2}$ m

$B = 1T$, $\theta = 60°$

$\tau = NIAB\sin\theta$ = magnitude of counter torque.

$= 30 \times 6 \times \frac{22}{7} \times (8 \times 10^{-2})^2 \times 1 \times \sin 60°$

$= 180 \times \frac{22}{7} \times 8 \times 8 \times 10^{-4} \times \frac{1.732}{2} = 180 \times 11 \times 16 \times 10^{-4}$

$\tau = 31680 \times 10^{-4}$

$\boxed{\tau = 3.168 \text{ Nm}}$

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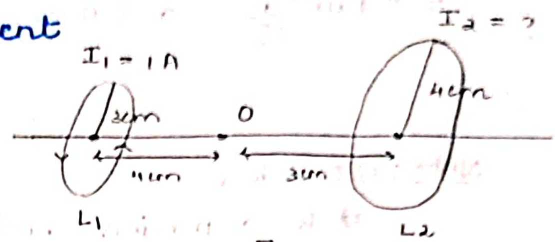
(b) No, the answer would not change because the above formula of torque is true for a planar loop of any shape.

PRACTICE QUESTIONS:

Q: Two coaxial circular loops L_1 and L_2 of radii 3cm and 4 cm are placed as shown. What should be the magnitude and direction of current in the loop L_2 so that the net magnetic field at point O is zero.



Ans] Magnetic field due to current I_1 in L_1 is equal to magnetic field due to current I_2 in L_2.

$$B = \frac{\mu_0 I N a^2}{2(a^2+x^2)^{3/2}} \qquad [\text{ N is not given } \Rightarrow N = 1]$$

$$|\vec{B_1}| = |\vec{B_2}|$$

$$\frac{\mu_0 I_1 a_1^2}{\cancel{2}(a_1^2+x_1^2)^{3/2}} = \frac{\mu_0 I_2 a_2^2}{\cancel{2}(a_2^2+x_2^2)^3}$$

~~1 × (9 × 10~~

$$\frac{1 \times (3\times10^{-2})^2}{((3\times10^{-2})^2 + (4\times10^{-2})^2)^{3/2}} = \frac{I_2 (4\times10^{-2})^2}{((4\times10^{-2})^2 + (3\times10^{-2})^2)^{3/2}}$$

$$\frac{9\times\cancel{10^{-4}}}{\cancel{(9+16)^{3/2}}\ \cancel{(10^{-4})^{3/2}}} = \frac{I_2 \times 16 \times \cancel{10^{-4}}}{\cancel{(9+16)^{3/2}}\ \cancel{(10^{-4})^{3/2}}}$$

$$\boxed{I_2 = \frac{9}{16}\text{ A} = 0.56\text{ A}}$$

For the net magnetic field at point O to be zero, the direction of current in loop L_2 should be opposite to that of loop L_1.

Q: Two identical coils P and Q each of radius R are lying in perpendicular planes such that they have a common centre. Find the magnitude and direction of the magnetic ~~magnitude~~ field at the common centre of the two coils, if they carry currents equal to I and $\sqrt{3}$I, respectively.

ns] $B_P = \dfrac{\mu_0 I}{2R}$ $\qquad B_Q = \dfrac{\mu_0 \sqrt{3} I}{2R}$

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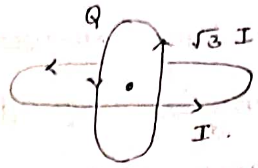
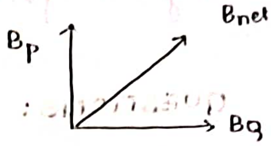
$$B_{net} = \sqrt{\left(\frac{\mu_0 I}{2R}\right)^2 + \left(\frac{\mu_0 \sqrt{3} I}{2R}\right)^2}$$



$$= \frac{\mu_0 I}{2R} \sqrt{1+3}$$

$B_{net} = \sqrt{B_p^2 + B_q^2}$



$$= \frac{\mu_0 I}{2R} \times 2$$

$$\boxed{B_{net} = \frac{\mu_0 I}{R}}$$

$$\tan\theta = \frac{B_p}{B_q} = \frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I \sqrt{3}}{2R}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\frac{1}{\sqrt{3}} \Rightarrow \boxed{\theta = 30^\circ = \frac{\pi}{6}}$$

<u>Note</u>: 1) $\tau = k\alpha$ [$\alpha \rightarrow$ deflection $\rightarrow$ degree, radians]

$\Rightarrow k \rightarrow$ torsion constant

<u>Unit</u>: $Nm\ deg^{-1}$

2)