Monday

26/02/2024

1. ELECTRIC CHARGES AND FIELDS

WHAT IS CHARGE ?

* It is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.
* It is a scalar property / quantity.
* Its SI unit is coulomb (C). 1 coulomb consists of 6.28×10^{18} number of electrons. [electrons are the basis of charge].
* Two kinds of charges:
  1) positive charge - loss of electrons
  2) negative charge - gain of electrons

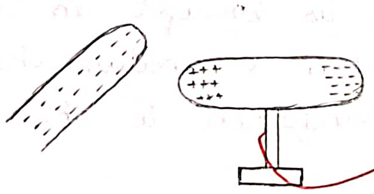
METHODS OF CHARGING :

1) CHARGING BY FRICTION :

* Friction provides some energy to the object which helps in the loss/gain of electrons based on whether the object is electropositive/ electronegative, respectively.
* When two substances are rubbed against each other, charges are developed on both substances due to transfer of electrons by friction.

2) CHARGING BY INDUCTION :

* It is the phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at the farther end.



Note: 1) Substances through which electric charges can flow are called conductors.

2) Substances through which electric charges cannot flow are called insulators.

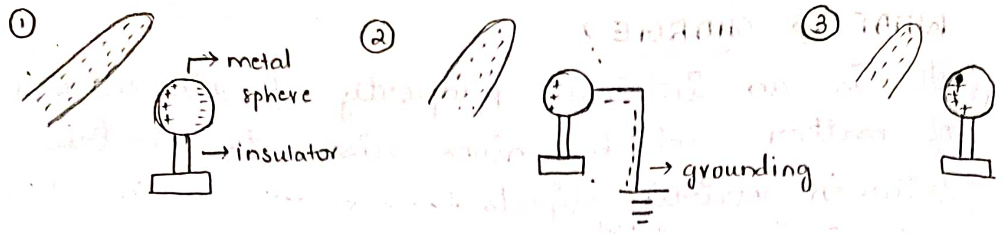
* EARTHING AND SAFETY :

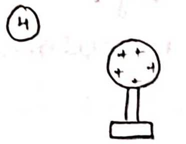
⇒ The process in which a body shares its charges with earth is called grounding / earthing.

* CHARGING OF A SPHERE BY INDUCTION :

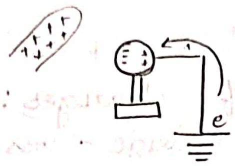
⇒ Earth always has zero voltage.

27/02/2024





In case +vely charged rod is used,



① Hold a metal sphere on an insulating stand. Bring a negatively charged plastic rod near it. The free electrons are repelled. Thus, the near end/ closer end becomes positively charged due to deficit of electrons.

② The far end is connected to the ground. Thus, free electrons flow to the ground.

③ When the sphere is disconnected from the earth, the positive charge remains held due to the force of attraction between the external charge.

④ When the plastic rod is removed, the positive charge spreads uniformly on the sphere.

3) CHARGING BY CONDUCTION:

* If a charged body is brought in contact with an uncharged body, ~~then a similar charge~~, then a similar charge is transferred to the uncharged body.

BASIC PROPERTIES OF CHARGE:

1) CONSERVATION OF CHARGE:

* The law of conservation of charge states that:

(i) The total charge of an isolated system remains constant.

(ii) The electric charges can neither be created nor destroyed. They can only be transferred from one body to another.

Note: The total charge of the entire universe is constant.

Ex: 1) $NaCl \rightleftharpoons Na^+ + Cl^-$ [net charge = 0]

[mass and charge are conserved]

28/02/2024

2) $^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$ + Energy

[nucleus is attacked]

3) $e^+ + e^- \longrightarrow 2\gamma$ - rays

positron (anti-particle) electron (particle)

[electromagnetic radiation - has energy]

⇒ This process is called annihilation.

4) γ rays $\longrightarrow e^+ + e^-$

⇒ This process is called pair production.

→ In 1, both mass and charge are conserved but in 2, 3 and 4, charge is conserved but ~~charge~~ mass is not conserved.

2) QUANTIZATION OF CHARGE :

⇒ The quantization of a physical quantity means that it cannot vary continuously to have any arbitrary (random) value but it can change discontinuously to take any one of only a discrete set of values.

⇒ The minimum amount by which a physical quantity can change is called its quantum.

⇒ The electric charges occur in discrete amounts instead of continous amounts.

⇒ The quantization of electric charge means that the total charge q is an integral multiple of a basic quantum of charge (e).

$$q = ne, \quad n = 0, \pm 1, \pm 2, \ldots$$

3) ADDITIVITY OF CHARGE :

⇒ It means that the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.

$$q = q_1 + q_2 + q_3 + \ldots + q_n$$

ELECTRIC CHARGE V/S MASS :

ELECTRIC CHARGE	MASS
* It can be positive, negative or zero.	* Mass is always positive.
* Charge is always quantized.	* Quantization of mass has

28/02/2024

ELECTRIC CHARGE	MASS
$q = ne$	not yet established.
* Charge of the body does not depend on speed.	* Mass of the body increases with speed.
* Charge is strictly conserved.	* Mass is not conserved by itself as some of the mass may get changed into energy or vice versa.
* Electrostatic forces between two charges may be attractive or repulsive.	* Gravitation forces between two masses are always attractive.
* A charged body always possess some mass.	* A body possessing mass may not have net charge.

EFFECT OF SPEED ON MASS AND ON CHARGE:

* According to the special theory of relativity, the mass of the body increases with speed in accordance with the relation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

m_0 = rest mass/initial mass.
v = speed of m_0.
c = speed of light

* In contrast to mass, the charge on the body remains constant and does not change with speed.

Q: 1 C (or) charge of 1 electron is bigger?

$Q = ne.$

$1\,C = n \times 1.6 \times 10^{-19}\,C.$

$n = \frac{1}{1.6 \times 10^{-19}}$

$= 0.625 \times 10^{+19}$

$n = 6.25 \times 10^{+18}$ electrons.

$\therefore$ 1 C is bigger than $1e^-$.

Intext:

Q: How much positive and negative charge is there in a cup of water?

H_2O

$2 \times 1p + 8p = 10\,p$

$2 \times 1e + 8e = 10\,e^-$

1 cup = 250 ml

$1\,gm^3 = 250$ ml

28/02/2024

No. of molecules in 250 g of H_2O $\Big\}$ $= \frac{250\,g}{18\,g} \times 6.022 \times 10^{23}$

$= 8.36 \times 10^{24}$ molecules.

Total no. of protons or electrons $\Big\}$ $= 8.36 \times 10^{24} \times 10$

$= 8.36 \times 10^{25}$

Total positive or negative charge in Q $\Big\}$ $= ne$

$= 8.36 \times 10^{25} \times 1.6 \times 10^{-19}\ C$

$= 1.34 \times 10^{7}\ C$

Intext: Q: If a body gives out 10^9 electrons every second, how much time is required to get a total charge of 1C from it?

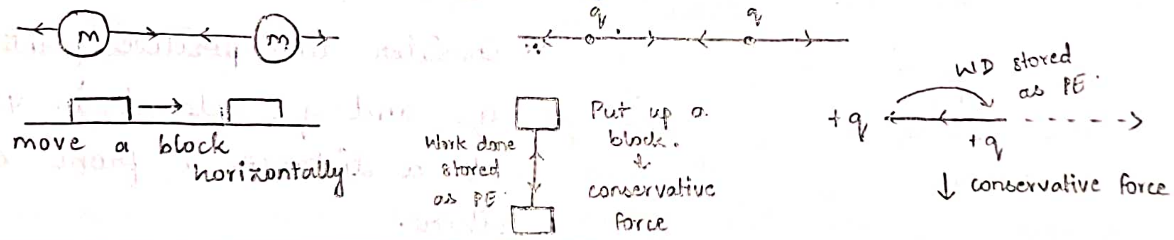
Time required to get a charge of 1C $= \frac{6.25 \times 10^{18}}{10^9} = 6.25 \times 10^{9}\ s.$

$\boxed{t = 198.18 \text{ years}}$

This shows how large is 1C as the unit of charge.

CENTRAL FORCES:

* Gravitational and electrostatic forces act between or against the line joining the centre of masses.

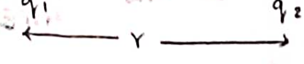


COULOMB LAW FOR ELECTRIC FORCE: [INVERSE SQUARE LAW]

* The law states that the forces of attraction / repulsion between two stationary point charges is directly proportional to the product of the magnitude of two charges and inversely proportional to the square of the distance between them.
* This force acts along the line joining the two forces.

$F \propto q_1 q_2$ $\quad F \propto \frac{1}{r^2}$

$F \propto \frac{q_1 q_2}{r^2} \Rightarrow \boxed{F = \frac{k q_1 q_2}{r^2}}$ [k = electrostatic force constant]



$k = \frac{1}{4\pi\varepsilon_0}$ $\quad \left\{ \varepsilon_0 = \text{permissivity of free space / vacuum} = 8.854 \times 10^{-12}\ C^2 N^{-1} m^{-2} \right\}$

$k = \frac{1}{4 \times 3.14 \times 8.854 \times 10^{-2}} \simeq 9 \times 10^{9}\ Nm^2 C^{-2}$

$\boxed{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$

28/02/2024

UNIT OF CHARGE: (i) SI UNIT:

* One coulomb is that amount of charge that repels an equal and similar charge with a force of 9×10^9 N when placed in vacuum at a distance of 1 m from it.

$$F = 9 \times 10^9 \times \frac{(1)^2}{(1)^2}$$

$$\boxed{F = 9 \times 10^9 \text{ N}}$$

(ii) CGS UNIT:

* One electro static unit (1 esu), also called as 1 statcoulomb, was the CGS unit of charge.

$\Rightarrow$ 1 coulomb $= 3 \times 10^9$ statcoulomb

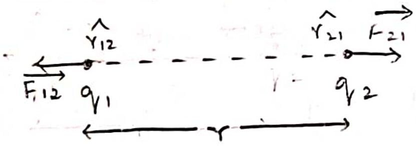
$$\boxed{1\text{ C} = 3 \times 10^9 \text{ esu of charge}}$$

* One electro magnetic unit of charge (1 emu) is also known as 1 abcoulomb.

$\Rightarrow$ 1 coulomb $= \frac{1}{10}$ abcoulomb

$$\boxed{1\text{ C} = \frac{1}{10} \text{ emu of charge}}$$

COULOMB'S LAW IN VECTOR FORM:



* Consider two positive point charges q_1 and q_2 placed in vacuum at a distance r from each other.

$F_{12} \rightarrow$ Force on 1 due to 2

$F_{21} \rightarrow$ Force on 2 due to 1.

$$\left[|\vec{r}_{21}| = |\vec{r}_{12}| = r \quad \vec{r}_{21} = -\vec{r}_{12} \right]$$

$$\left[\hat{r} = \frac{\vec{r}}{|r|} \right]$$

* They repel each other.
* In vector form, Coulomb's law can be represented as:

★ $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$

★ $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$

★ $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \left(\frac{\vec{r}_{21}}{|r_{21}|} \right) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21}$

★ $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12}$

* Importance of Vector Form:

$\Rightarrow$ Since $\hat{r}_{12} = -\hat{r}_{21}$, $\vec{F}_{12} = -\vec{F}_{21}$

This means that the two charges exert equal and

29/02/2024

opposite forces on each other. So coulombian forces obey Newton's third law of motion.

⇒ As the coulombian forces act along the line joining the two charges, they are called central forces.

* **Limitations of Coulomb's Law:**

⇒ The electric charges must be at rest (stationery).

⇒ The electric charges must be point charges, i.e., the extension of charges must be much smaller than the separation of the two charges.

⇒ The separation between the charges must be greater than 10^{-15} m (nuclear size) because for distances less than 10^{-15} m, the strong nuclear force dominates over the electrostatic force.

DIELECTRIC CONSTANT / RELATIVE PERMITIVITY:

* The ratio of permitivity of the medium to the permitivity of free space is called relative permitivity (or) dielectric constant of the given medium.

$$\varepsilon_r \text{ (or) } K = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{vacuum}}{F_{medium}}$$ [K = Kappa]

* ε_r is dimensionless.

Q: The electrostatic force of repulsion between two positively charged ions carrying equal charges is 3.7×10^{-9} N. When they are separated by a distance of 5 Å, how many electrons are missing from each ion?

q ——— 5×10^{-10} m ——— q

Given: $F = 3.7 \times 10^{-9}$ N

$r = 5$ Å $= 5 \times 10^{-10}$ m.

To find: $n = ?$

Formula: $F = k \frac{q_1 q_2}{r^2}$

$q = ne$

$$3.7 \times 10^{-9} = 9 \times 10^{9} \times \frac{q^2}{(5 \times 10^{-2})^2}$$

$$q^2 = \frac{3.7 \times 10^{-9} \times (5 \times 10^{-10})^2}{9 \times 10^{9}}$$

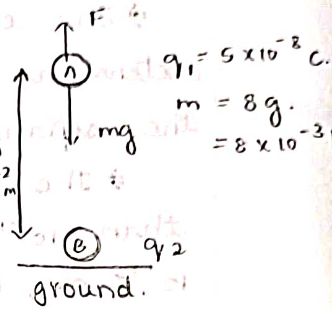
$$q = \frac{5}{3} \times 10^{-10} \times 10^{-9} \times \sqrt{3.7}$$

01|03|2024

$$n = \frac{q}{e} = \frac{\frac{5}{3} \times 10^{-19} \times \sqrt{3.7}}{1.6 \times 10^{-19}}$$

$\boxed{n = 2}$ [n is always an integer]

Q: A free pitball of A of 8 g carries a positive charge of 5×10^{-8} C. What must be the nature and magnitude of charge that should be given to a 2nd pitball B fixed 5 cm below the former ball so that the upper ball is stationary?



$F = mg$

$$\frac{kq_1q_2}{r^2} = mg$$

$$\frac{9 \times 10^{+9} \times 5 \times 10^{-8} \times q_2}{(5 \times 10^{-2})^2} = 8 \times 10^{-3} \times 10$$

$$q_2 = \frac{80 \times 10^{-3} \times 25 \times 10^{-4}}{9 \times 10^{9} \times 5 \times 10^{-8}}$$

$= 4.5 \times 10^{-8}$ C $= 0.45 \times 10^{-6}$ C

$\boxed{q_2 = 0.45\ \mu C}$ ⇒ positive charge (nature)

Q: A particle of mass m and carrying ~~charg~~ charge $-q_1$ is moving around a charge $+q_2$ ~~is moving along a circular path~~ of radius r. Prove that the period of ~~req~~ revolution of charge $-q_1$ about $+q_2$ is given by : $T = \sqrt{\dfrac{16\pi^3 \varepsilon_0 m r^3}{q_1 q_2}}$

$$v = \frac{2\pi r}{T}$$

$$\frac{mv^2}{r} = \frac{kq_1q_2}{r^2} \Rightarrow \frac{m}{r}\frac{4\pi^2 r^2}{T^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$T = \sqrt{\frac{16\pi\varepsilon_0 m r^3}{q_1q_2}}$$

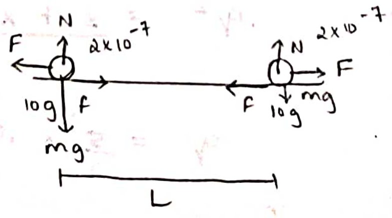
Q: Two identical charged particles each having a mass 10g and charge 2×10^{-7} C are placed on a horizontal table with a separation 'L' between them such that they stay in limited equilibrium. If the coefficient of friction between them is 0.25, find the value of 'L'.

$f_{max} = \mu_s N$

$N = mg$

$F = f_{max}$

$$\frac{kq^2}{L^2} = \mu_s mg$$



02/03/2024

$$L^2 = \frac{kq^2}{\mu_s mg} \Rightarrow L = \sqrt{\frac{9\times10^9 \times (2\times10^{-7})^2 \times 4}{1 \times 10 \times 10^{-2} \times 10}}$$ [Here, $0.25 = \frac{1}{4}$]

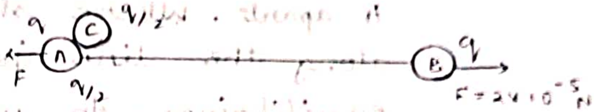
$$L = \frac{3\times2\times10^{-7} \times 2\times10^{-6}}{10}$$

$$L = 12\times10^{-2}\ m$$

(or) $\boxed{L = 12\ cm}$

Q: 2 similarly equally charged identical metal spheres A and B repel each other with a force 2×10^{-5} N. A third identical uncharged sphere C is touched to A and placed at the midpoint of A and B. Calculate the net electrostatic force on C.

$\vec{F} = \frac{kq^2}{r^2} = 2\times10^{-5}$ N.

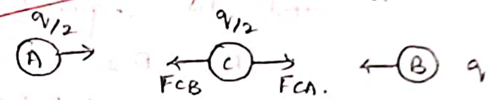


$\vec{F}_{net} = \vec{F}_{CA} + \vec{F}_{CB}$

$$= k\frac{\frac{q}{2}\frac{q}{2}}{\left(\frac{r}{2}\right)^2}(\hat{i}) + \frac{k\frac{q}{2}q}{\left(\frac{r}{2}\right)^2}(-\hat{i})$$

Here, charge is divided equally so A and C have equal charge of q/2

→ formula → $q = \frac{q_1+q_2}{2}$.



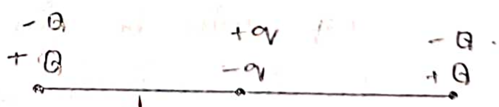
$$= \frac{kq^2}{r^2}[\hat{i} + 2(-\hat{i})]$$

$$= -\frac{kq^2}{r^2}\hat{i}$$

$F_{net} = 2\times10^{-5}$ N along BC.

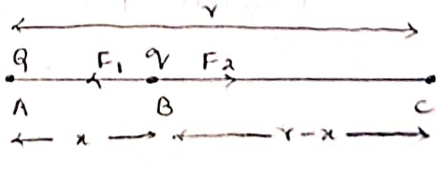
Q: Two identical charges Q each are kept at a distance r from each other. A third charge q is placed on the line joining the two charges such that all three charges are in equilibrium. What is the magnitude, nature and position of the charge q?

The charge q must be opposite to that Q in order to attain equilibrium of all three charges. Thus, it means that the net charge on Q should be zero.



For equilibrium, outer charges have same sign and middle charge has different sign.

To find the position of small Q,



$F_1 = F_2$

$$\frac{kqQ}{(x)^2} = \frac{kqQ}{(r-x)^2}$$

05/03/2024

$(r - x)^2 = x^2$

$r - x = x$

$r = 2x$

$\Rightarrow x = \frac{r}{2}$

To find the magnitude of q:
The net force on Q must be zero

$F_{QQ} = F_{Qq}$

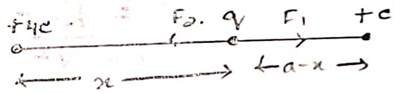
$\frac{kQ^2}{r^2} = \frac{kQq}{(\frac{r}{2})^2} \Rightarrow \frac{Q}{r^2} = \frac{q}{r^2} \times 4$

$\Rightarrow \boxed{q = \frac{-Q}{4}}$

Q: Two point charges +4e and e are fixed at a distance A apart. Where should a 3rd charge q be placed along the line joining it so that it may be in equilibrium. In which case is the equilibrium stable and in which case, is it unstable?

For equilibrium of q the force due to +4e and +e must be equal and opposite

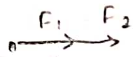
$F_1 = F_2$



$\frac{k(4e)q}{x^2} = \frac{k(e)q}{(a-x)^2}$

$\frac{4}{x^2} = \frac{1}{(a-x)^2}$

$4(a-x)^2 = x^2$



$2(a-x) = \pm x$

$2a - 2x = +x \qquad 2a - 2x = -x$

$\boxed{x = \frac{2a}{3}} \qquad x = 2a$

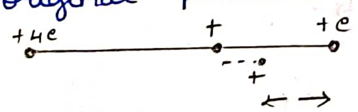
$\therefore x = 2a$ is not possible.

When kept at 2a distance, the particle will experience only repulsive force on the same side

Both +q and -q at $x = \frac{2a}{3}$ will be in equilibrium but +q will be in stable equilibrium where -q will be in unstable equilibrium.

$\Rightarrow$ Stable Equilibrium: will come back to its original position when displaced.

$\Rightarrow$ Unstable Equilibrium: will not come back to its original position when displaced.



repulsive force, comes back to original position.

attractive force, does not come back to original position.

05/02/2024

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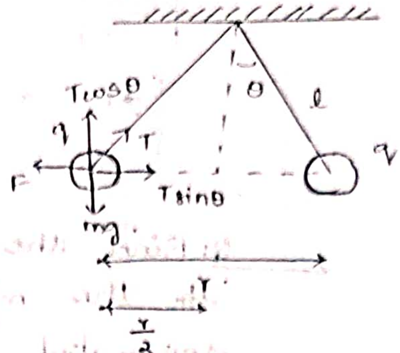
Example 1.8 (SL Aurora)]

$T\sin\theta = F \Rightarrow \cancel{T\sin\theta = mg}$ — ①

$\cancel{T\cos\theta = \frac{kq^2}{r^2}}$ $\quad T\cos\theta = mg$ — ①

$T\sin\theta = \frac{kq^2}{r^2}$

$T\sin\theta = \frac{kq^2}{(2l\sin\theta)^2}$ — ② $[r = 2l\sin\theta]$



$\left[\begin{array}{l}\sin\theta = \frac{r/2}{l} \\ r = 2l\sin\theta\end{array}\right]$

$\frac{②}{①} = \frac{T\sin\theta}{T\cos\theta} = \frac{kq^2}{(4l^2\sin^2\theta)\,mg}$

$\boxed{q^2 = 4mg\,l^2\sin^2\theta\tan\theta\;4\pi\varepsilon_0}$

ELECTROSTATIC AND GRAVITATIONAL FORCES:

* SIMILARITIES:

⇒ Both forces obey the inverse square law $\left[F \propto \frac{1}{r^2}\right]$.

⇒ Both forces are proportional to the product of masses or charges.

⇒ Both forces are conservative forces.

⇒ Both forces are central forces.

⇒ Both forces can operate in vacuum.

* DISSIMILARITIES:

⇒ Gravitational force is attractive whereas electrostatic force may be attractive or repulsive.

⇒ Electrostatic force are much stronger than gravitational force.

⇒ Gravitational force has larger range than electro-static force.

⇒ Gravitational force does not depend on the medium while electrostatic force depends on the medium. $[\varepsilon_0/\varepsilon]$

Intext:

Q: Compare the strength of electrostatic and gravitational forces for a proton and electron.

only magnitude

$\left|\frac{F_e}{F_G}\right| = \frac{ke^2/r^2}{Gm_pm_e/r^2} = \frac{ke^2}{Gm_pm_e}$ e and p have same charge in magnitude.

$\left|\frac{F_e}{F_G}\right| = \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\,(1.6\times10^{-27})\,(9.1\times10^{-31})}$

$\left|\frac{F_e}{F_G}\right| = \frac{ke^2}{Gm_pm_e}$ ⇒ no units.

05/03/2024

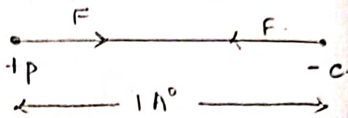
$$\left|\frac{F_e}{F_G}\right| = 22.7 \times 10^{40}$$

$$\Rightarrow F_e = 2.27 \times 10^{39} F_G$$

Q: Find the acceleration of proton and electron due to the mutual coulombic attraction when separated by $1 A^\circ$ distance.

$$F = \frac{ke^2}{r^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(10^{-10})^2}$$

$$F = 2.3 \times 10^{-8}\ N$$



$1 A^\circ = 10^{-10} m.$

$$a_e = \frac{F}{m_e} = \frac{2.3 \times 10^{-8}}{9.1 \times 10^{-31}}$$

$$a_e = 2.5 \times 10^{22}\ ms^{-2}$$

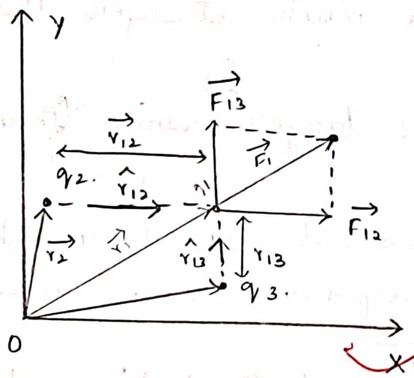
$$a_p = \frac{F}{m_p} = \frac{2.3 \times 10^{-8}}{1.6 \times 10^{-27}}$$

$$a_p = 1.5 \times 10^{19}\ ms^{-2}$$

FORCES BETWEEN MULTIPLE CHARGES (SUPERPOSITION PRINCIPLE):

⇒ The principle of superposition states that when a number of charges are interacting the total force on a given charge is the vector sum of the forces exerted on it due to all other charges.

⇒ The force between two charges is not affected by the presence of other charges.



⇒ According to Coulomb's law, the force on q_1 due to q_2 is $\vec{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r_{12}}$.

$$\vec{r_{12}} = \vec{r_1} - \vec{r_2}$$

$$\vec{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2} \cdot \frac{\vec{r_1} - \vec{r_2}}{|\vec{r_1} - \vec{r_2}|}$$

$$\vec{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^3} \cdot \vec{r_1} - \vec{r_2}$$

Similarly, $\vec{F_{13}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{|\vec{r_1} - \vec{r_3}|^3} \cdot \vec{r_1} - \vec{r_3}$

∴ The resultant force on q due to N charges:

05/03/2024

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \ldots\ldots + \vec{F_{1N}}$$

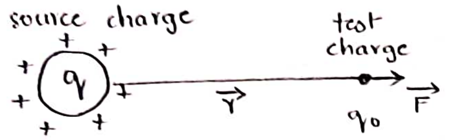
$$\vec{F_1} = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^{N} \frac{q_i}{|\vec{r_1} - \vec{r_i}|^3} \; \vec{r_1} - \vec{r_i}$$

$$\vec{F_2} = \frac{q_2}{4\pi\epsilon_0} \sum_{\substack{i=1 \\ i\neq 2}}^{N} \frac{q_i}{|\vec{r_2} - \vec{r_i}|^3} \cdot \vec{r_i} - \vec{r_i}$$

ELECTRIC FIELD: [also called electric field intensity].

* It is assumed that the charge q produces an electrical environment in the surrounding space called electric field.

[If source charge ⇒ +ve ⇒ $\vec{E}$ = outward q_0
⇒ −ve ⇒ $\vec{E}$ = toward q_0]



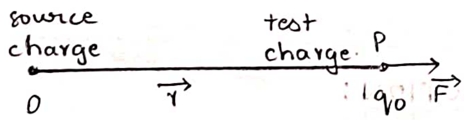
F = force on test charge due to / exerted by source charge.

* Electric field is said to exist at a point if a force of electrical origin is exerted on a stationary charge placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$ [$\vec{E}$ = electric field due to source charge.
q_0 = test charge (always +ve)]

* Quantitatively, electric field is defined as the force experienced by a unit positive test charge placed at that point without disturbing the position of the source charge.
* SI unit of electric field = N/C.
* Dimensional formula = $[MLT^{-3}A^{-1}]$

ELECTRIC FIELD DUE TO A POINT CHARGE:



* According to Coulomb's law,

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} \hat{r}$$

But $$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2 q_0} \cdot \hat{r}$$

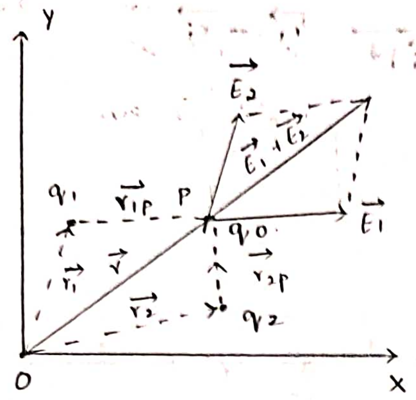
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cdot \hat{r}$$

* Clearly, $\vec{E} \propto \frac{1}{r^2}$

* Such a field is called radial field / spherically symmetric field in a homogenous medium.

06/02/2024

ELECTRIC FIELD DUE TO A SYSTEM OF POINT CHARGES:



* We wish to determine the electric field at point P whose position vector is $\vec{r}$. The force on test charge due to q_1 is:

$$\vec{F}_1 = \frac{kq_1q_0}{r_{1p}^2}\hat{r}_{1p}$$

$$\vec{E} = \frac{\vec{F}_1}{q_0}$$

$$\vec{E}_1 = \frac{kq_1}{r_{1p}^2}\hat{r}_{1p}$$

III^{ly}, $\vec{E}_2 = \frac{kq_2}{r_{2p}^2}\hat{r}_{2p}$

∴ The net electric field at P will be

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

For N charges,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots\ldots + \vec{E}_N$$

$$\vec{E} = \frac{kq_1}{r_{1p}^2}\hat{r}_{1p} + \frac{kq_2}{r_{2p}^2}\hat{r}_{2p} + \ldots\ldots + \frac{kq_N}{r_{Np}^2}\hat{r}_{Np}$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_{ip}^2}\hat{r}_{ip}}$$ ⇒ in terms of displacement vector

* We can also write electric field in terms of position vectors.

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r}-\vec{r}_i|^3}\left(\vec{r}-\vec{r}_i\right)}$$ ⇒ in terms of position vector (discrete system).

CONTINUOUS CHARGE DISTRIBUTION:

* If the charge is spread in a ~~...~~ region in a continuous manner, we call it as continuous charge distribution.

* 3 types :

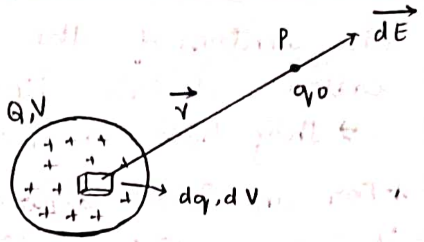
(i) VOLUME CHARGE DISTRIBUTION:

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

06/03/2024

$$\vec{E}_V = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho\, dV}{r^2} \hat{r}$$

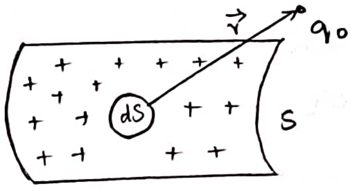


[Charge density $= \rho = \frac{dq}{dV}$

$dq = \rho\, dV$]

Here, ρ is ~~vol char~~ volume charge density. It's SI unit $= Cm^{-3}$.

(ii) SURFACE CHARGE DISTRIBUTION :



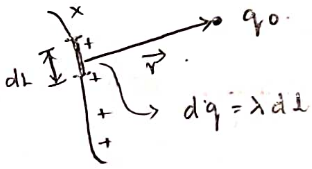
$dq = \sigma ds$.

$$\vec{E}_S = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma . ds}{r^2} \hat{r}$$

Here, σ is surface charge density.

It's unit [SI] $= Cm^{-2}$.

(iii) LINEAR CHARGE DISTRIBUTION :



$$\vec{E}_L = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda\, dL}{r^2} \hat{r}$$

Here, λ is linear charge density.

It's unit is Cm^{-1}.

* If all three charge distributions are given, then

$$\vec{E}_{continuous} = \vec{E}_V + \vec{E}_S + \vec{E}_L .$$

* General charge distribution :

$$\vec{E}_{total} = \vec{E}_{continuous} + \vec{E}_{discrete} .$$

$$\vec{E}_{total} = \frac{1}{4\pi\varepsilon_0} \left[\int_V \frac{\rho\, dV}{r^2} \hat{r} + \int_S \frac{\sigma\, ds}{r^2} \hat{r} + \int_L \frac{\lambda\, dL}{r^2} \hat{r} + \sum_{i=1}^{N} \frac{q_i}{r_{ip}^2} \hat{r}_{ip} \right]$$

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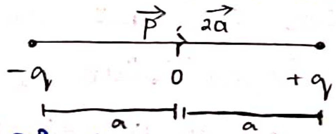
ELECTRIC DIPOLE :

* A pair of equal and opposite charges separated by a small distance is called an electric dipole.



⇒ -ve charge is kept first

⇒ direction is taken from -ve to +ve charge.

* Dipole moment = magnitude of either charge × a vector drawn from -ve to +ve charge

$$\vec{p} = q\, 2\vec{a}$$

* Examples of electric dipoles : HCl, CH_3COOH, H_2O, C_2H_5OH, etc

⇒ If the centre of the +ve charges does not coincide with

07/03/2024

the centres of the -ve charges, such moleculer are called electric dipoles.

→ They have a permanant dipole moment.

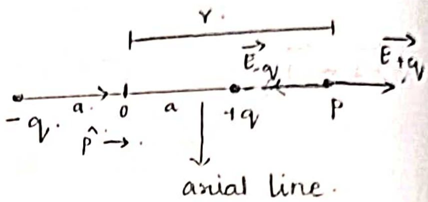
* For an ideal dipole $2a \to 0$ and $q \to \infty$ such that it has a finite dipole moment.

* They are also called as point dipole / short dipole.

Here, for axial and equitorial points, there are 2 diff. formulas and no formula for other points because it is a vector quantity and direction is considered.

ELECTRIC FIELD AT AN AXIAL POINT DUE TO AN ELECTRIC DIPOLE:

* Consider an electric dipole with charge $-q$ and $+q$ separated by distance $2a$ in vacuum. Let P be a point on the axial line at a distance r from the centre O near the charge $+q$.



$$\vec{E}_{axial} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$\vec{E}_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}(-\hat{p})$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2+a^2+2ar-r^2-a^2+2ar}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2.2ar}{(r^2-a^2)^2}\hat{p}$$

$$\boxed{\vec{E}_{ax} = \frac{2pr}{4\pi\varepsilon_0(r^2-a^2)^2}\hat{p}}$$

Here; $p = q2a$.

For short dipole, $r \gg a$, a^2 can be neglected.

$$\vec{E}_{ax} = \frac{2p}{4\pi\varepsilon_0 r^3}\hat{p}$$

* When P is placed between $-q$ and $+q$ (between the dipole), the direction of its electric field is opposite to that the dipole and when P is outside on either side of the dipole, electric field direction of P is same as the direction of dipole moment.

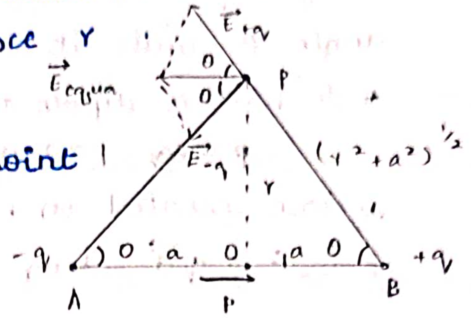
Clearly, electric field at any axial point of the dipole acts along the dipole axis from negative to positive charge; i.e., in the direction of the dipole moment $\vec{p}$.

28/03/2024

ELECTRIC FIELD AT AN EQUITORIAL POINT OF A DIPOLE :

* Consider an electric dipole of charge $-q$ and $+q$, separated by $2a$ placed in vacuum.

Let P be a point at a distance r from the centre O on the equitorial line. Electric field at point P due to $+q$ is :



$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2} \text{ along BP}$$

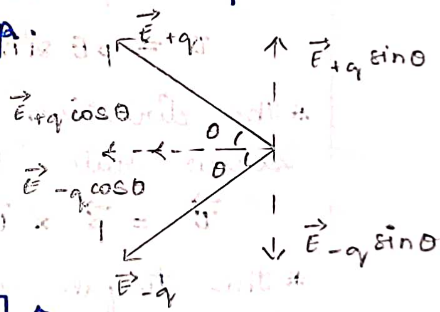
+ve charge = outwards
−ve charge = towards

Electric field at point P due to $-q$ is :

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2} \text{ along PA}$$

⇒ Here, $-q\,+q \parallel \vec{E}_{equa}$

Thus, the magnitudes of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ vector are equal. Then the components of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ perpendicular to the dipole axes will cancel out and the components parallel to dipole axes will add up.



$$\vec{E}_{equa} = E_{+q}\cos\theta(-\hat{p}) + E_{-q}\cos\theta(-\hat{p})$$

$$= -\cos\theta\left[E_{+q} + E_{-q}\right]\hat{p}$$

$$= \frac{-a}{\sqrt{r^2+a^2}}\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2}\right]\hat{p}$$

$$= \frac{-2aq}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}\hat{p}$$

$$\left[\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{a}{\sqrt{r^2+a^2}}\right]$$

$$\boxed{\vec{E}_{equa} = \frac{-p}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}\hat{p}} \quad [p = 2aq]$$

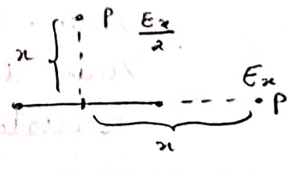
* For a short dipole, $r \gg a$, we can neglect a^2.

$$\vec{E}_{equa} = \frac{-p}{4\pi\varepsilon_0 r^3}$$

* Clearly, the direction of electric field at any point on the equitorial line of the dipole will be anti-parallel to the dipole moment $\vec{p}$.

For a short dipole, (for the same distance)

$$\vec{E}_{ax} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3} \qquad \vec{E}_{eq} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$$



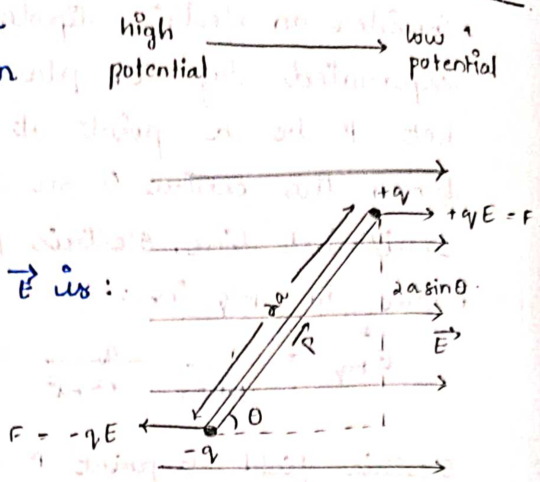
$$\Rightarrow |\vec{E}_{ax}| = 2|\vec{E}_{eq}| \qquad [p \times \hat{p} \Rightarrow \vec{p}]$$

08/03/2024

TORQUE ON A DIPOLE IN A UNIFORM ELECTRIC FIELD:

* The dipole is placed in a uniform electric field at an angle θ with it.
* It has a dipole moment:

  ~~p = 2aq~~ $p = 2aq$



* Force exerted on $+q$ due to $\vec{E}$ is:

  $F = q\vec{E}$ along $\vec{E}$

* Force exerted on $-q$ due to $\vec{E}$ is:

  $F = -q\vec{E}$ opposite to $\vec{E}$

* Torque is equal to one of the forces × perpendicular distance between the two forces.

$$\tau = F\,2a\sin\theta$$
$$= qE\,2a\sin\theta$$
$$\tau = pE\sin\theta \quad [\because p = 2aq]$$

* The direction of torque is given by right hand ~~thumb~~ screw rule. Therefore,

  $\vec{\tau} = \vec{p} \times \vec{E}$ [here, it is always $\vec{p} \times \vec{E}$, not $\vec{E} \times \vec{p}$].

* The torque will be zero if the dipole is parallel or antiparallel to the electric field.

  $\tau = 0$, if $\theta = 0°$ (or) $\theta = 180°$

* The torque will be max, if the dipole is perpendicular to the electric field.

  $\tau = max$, if $\theta = 90°$ $\Rightarrow$ $\tau_{max} = pE$ if $\theta = 90°$

Note: i) The dipole will not experience a net force but it will experience a non-zero torque when kept in a uniform electric field. Thus, the motion of the dipole is oscillatory in this case.

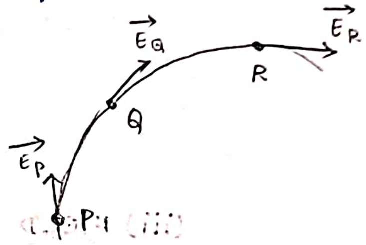
ii) **Dipole in Non-uniform electric field:** In a non-uniform field, a dipole experiences different forces on each ~~of~~ charge. The net force on the dipole is not zero. Thus, dipole experiences non-zero force and non-zero torque making the motion of the dipole oscillatory and translatory in this case.

11/03/2024

ELECTRIC FIELD LINES:

* Michael Faraday introduced the concept of line of force to visualise the nature of electric and magnetic fields.
* An electric line of force may be defined as the curve along which a small positive charge would tend to move when free to do so in an electric field and the tangent to which at any point gives the direction of electric field at that point.

Note: The line of force do not really exists. They are imaginery curves but the field which they represent is real.



▲ Electric field line/ line of force.

* PROPERTIES OF LINES OF FORCE:

(i) The lines of force is a continuous, smooth curve without any breaks.

(ii) They start at +ve charge and end at -ve charge. If a single charge is present, it will start and end at infinity.

(iii) The tangent to a line of force at any point gives the direction of the electric field at that point.

(iv) No two lines of force can intersect at one point because at the point of intersection, two tangents will represent two different directions of electric field at the same point which is not possible.

(v) The relative closeness gives a measure of the strength of the electric field in any region.

(a) Close together – strong field

(b) Far apart – weak field

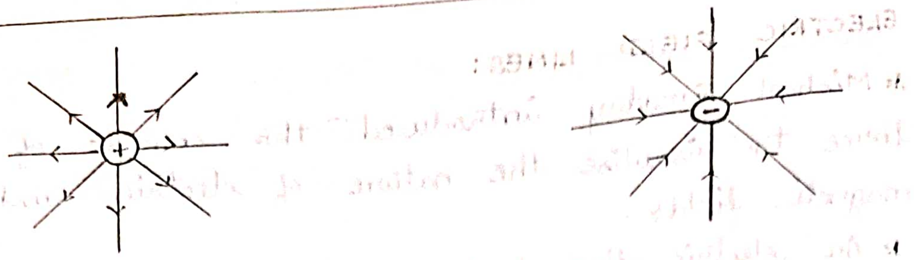
(c) Parallel and equally spaced – uniform electric field.

FIELD LINES FOR DIFFERENT CHARGED CONDUCTORS:

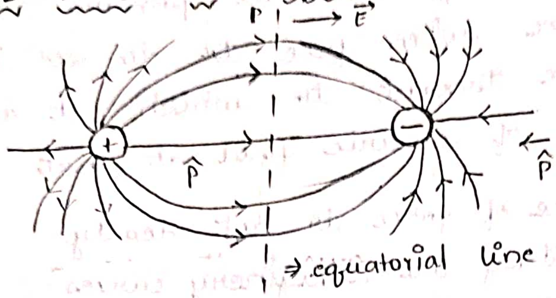
(i) FIELD LINES OF POINT CHARGES:

[diagram in next page].

11/08/2024



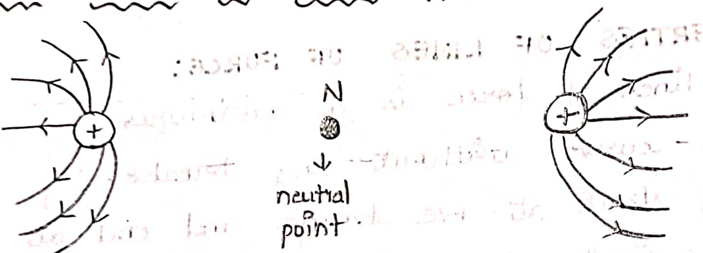
(ii) FIELD LINE OF EQUAL AND OPPOSITE CHARGE:



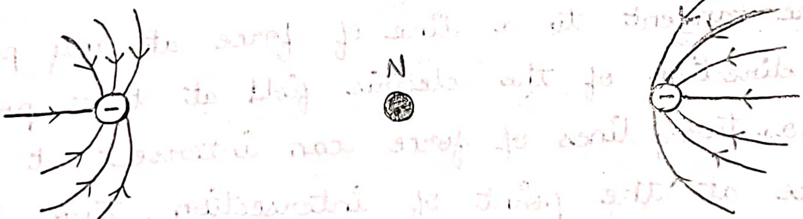
* at equatorial point, direction of electric field is opp. to direction of dipole moment.

$\hat{p}$ ⇒ dipole moment

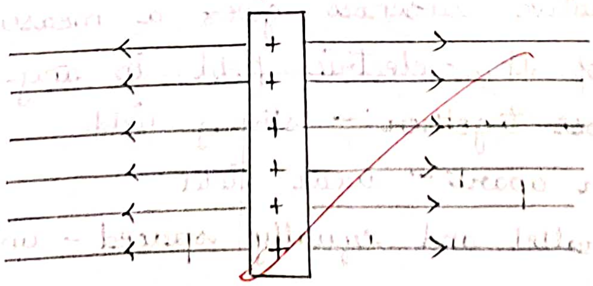
(iii) FIELD LINES OF EQUAL AND LIKE CHARGES:



* Here, the electric field at N is zero and it is called as the neutral point.

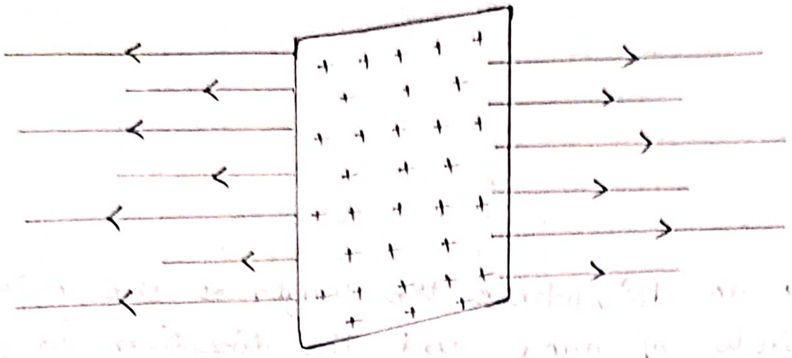


(iv) FIELD LINES OF INFINITE LINE OF POSITIVE CHARGE



11/03/2024

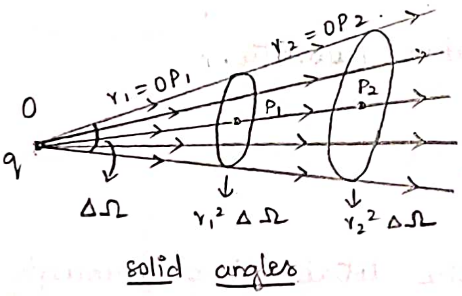
(iv) FIELD LINE OF INFINITE PLANE OF POSITIVE CHARGE:



Note: Electric field strength is proportional to density of field lines of force.

⇒ Density refers to number of field lines per unit area.

CONSISTENCY OF THE INVERSE SQUARE LAW WITH THE ELECTRIC FIELD LINES:



density at P_1

$$= \frac{n}{r_1^2\,\Delta\Omega}$$

density at P_2

$$= \frac{n}{r_2^2\,\Delta\Omega}$$

Electric field ∝ density of lines of force

$$\frac{E_1}{E_2} = \frac{\frac{n}{r_1^2\,\Delta\Omega}}{\frac{n}{r_2^2\,\Delta\Omega}} = \frac{r_2^2}{r_1^2}$$

$$\Rightarrow \boxed{E \propto \frac{1}{r^2}}$$

Here, n and $\Delta\Omega$ are constant for both area elements.

ELECTRIC FLUX:

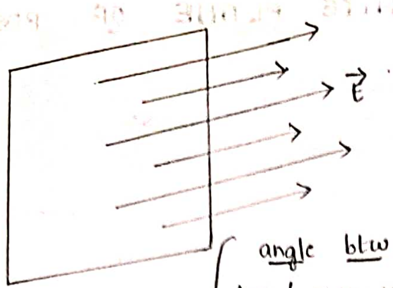
* The electric flux through a given area held inside an electric field is the measure of the total number of electric lines of force passing normally through that area.

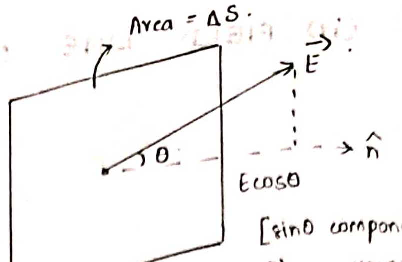
* $\Delta\phi_E = E\cos\theta \cdot \Delta S$ [Here, angle btw $\vec{E}$ and $\Delta\vec{S} = 0°$]

$\Delta\phi_E = \vec{E} \cdot \Delta\vec{S}$ [dot product of electric field and area vector = flux (scalar)]

Note: We come across many situations where we need to know the direction of the surface area. Thus, we define area

12/03/2024





[$\sin\theta$ components of $\vec{E}$ go upwards and hence, do not pass through the area]

angle b/w:
Area of area vector = 90°
Area of Electric field = 90°

vector as $d\vec{s}$, where the length of the vector represents magnitude of area and the direction is given by a normal pointing outwards from the area.

$$d\vec{s} = ds\ \hat{n}$$

* We can divide the surface into small areal elements. Then, total flux:

$$\phi_E = \sum_{i=1}^{N} \vec{E_i} \cdot \vec{\Delta S_i}$$

* If the number of elements are infinitely large. ($N \to \infty$ and $\Delta S \to 0$), then,

$$\phi_E = \lim_{\substack{N\to\infty \\ \Delta S \to 0}} \sum_{i=1}^{N} \vec{E_i} \cdot \vec{\Delta S_i} = \oint_S \vec{E} \cdot d\vec{s}$$

[$\oint_S \Rightarrow$ surface integral for closed objects (upper and lower limits are same).]

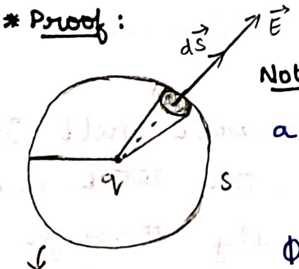
* Electric ~~field~~ flux is a scalar quantity.
* Its SI unit Nm^2C^{-1}.

GAUSS'S THEOREM:

* Gauss theorem states that the total flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the closed surface.

$$\phi_E = \oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$$

* **Proof:**



Spherical Gaussian surface.

Proving Gauss's Law using: Coulomb's Law:

Note: Any ~~the~~ hypothetical closed surface enclosing a charge is called gaussian surface.

Total flux through surface S is:

$$\phi_E = \int d\phi_E = \int \vec{E} \cdot d\vec{s}$$
$$= \int E\, ds \cos 0°$$
$$= E \int ds$$
$$= E \times 4\pi r^2$$

13/02/2024

From Coulomb's Law,

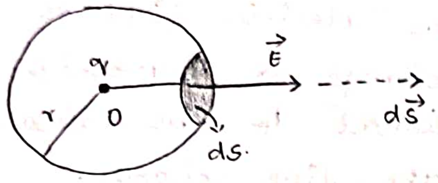
$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\phi = \frac{q}{4\pi\epsilon_0 r^2} \times 4\pi r^2$$

$$\boxed{\phi = \frac{q}{\epsilon_0}}$$

The Gaussian surface should either include or exclude charges. It cannot be drawn on a charge. For questions, charges on the boundary of the Gaussian surfaces should not be considered.

Proving Coulomb's Law using: Gauss's Law:



Total flux

$$\Rightarrow \phi_E = \int \vec{E} \cdot d\vec{s}$$

$$= \int E ds = E \int ds$$

$$= E\, 4\pi r^2$$

From Gauss's Law,

$$\phi_E = \frac{q}{\epsilon_0}$$

$$\Rightarrow \frac{q}{\epsilon_0} = E\, 4\pi r^2$$

$$\Rightarrow E = \frac{q}{4\pi r^2 \epsilon_0}$$

$$[F = q_0 E]$$

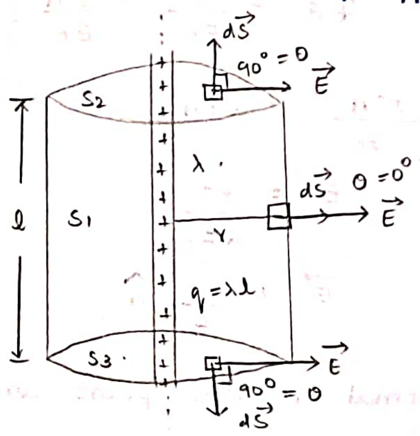
$$\Rightarrow \boxed{F = \frac{q q_0}{4\pi\epsilon_0 r^2}}$$

* APPLICATIONS OF GAUSS' LAW:

(i) Field due to an infinitely long charged wire:

⇒ Consider an infinitely long straight wire having a uniform linear charge density ($\lambda = Cm^{-1}$).

⇒ We wish to calculate the electric field at a point P at a distance r from the line of charge.



Total flux through Gaussian surface,

$$\phi_E = \oint \vec{E} \cdot d\vec{s}$$

$$= \int E ds_1 \cos 0° + \int E ds_2 \cos 90° + \int E ds_3 \cos 90°$$

$$\phi_E = E \int ds_1$$

$$= E\, 2\pi r l$$

By Gauss's Law,

13/03/2024

$$\phi_E = \frac{q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

Here ; λ = linear charge density

$$\lambda = \frac{q}{l} = Cm^{-1}$$

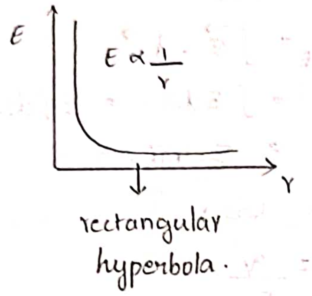
$$q = \lambda l$$

$$\frac{\lambda \cancel{l}}{\epsilon_0} = E\, 2\pi r \cancel{l}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi \epsilon_0 r}\hat{r}}$$

⇒ The direction of electric field is radially outwards.



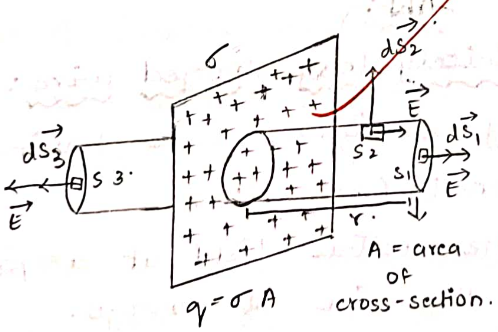
⇒ Thus, the electric field of a line charge is inversely proportional to the distance from the line charge.

(ii) Field due to a uniformly charged infinitely plane sheet :

⇒ Consider a thin, infinite plane sheet of uniform surface charge density ($\sigma = Cm^{-2}$).

⇒ We wish to determine the electric field at point P at a distance r from the sheet.

⇒ Consider a Gaussian surface of length 2r and cross sectional area A.



Total flux through Gaussian surface,

$$\phi_E = \oint \vec{E} \cdot \vec{ds}$$

$$= EA + 0 + EA$$

⇒ As the flux is outwards at S_1 and S_3, we are adding it. (+ve flux). If they are entering the surface, it is added (-ve flux). If it is entering at S_1 and leaving at S_2, it is subracted.

$$\phi_E = 2EA$$

By Gauss's Law,

$$\phi_E = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$\frac{\sigma A}{\epsilon_0} = 2EA$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

Here, σ = surface charge density

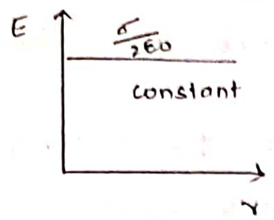
$$\sigma = \frac{q}{A} = Cm^{-2}$$

$$q = \sigma A$$

⇒ Here, $\hat{n}$ is the unit vector normal to the plane and

13/03/2024

going away from it.



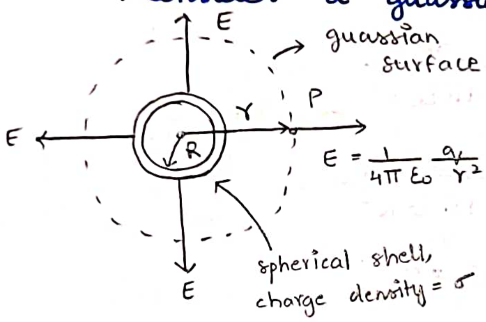
→ Clearly, E is independent of r.

(iii) Field due to a thin uniformly charged spherical shell:

→ Consider, a thin spherical shell of radius R and uniform surface charge density (σ).

(a) Point P lies outside the shell:

→ Consider a guassian surface of radius r.



Flux through the guassian surface:

$$\phi_E = E \times 4\pi r^2$$

By gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}}$$

→ Hence, for points outside the shell, the field due to shell is as if entire charge of the shell is concentrated at its centre.

(b) Point P lies on the shell:

→ Consider a guassian surface such that it just encloses the charged shell.

Total flux : $\phi_E = E \times 4\pi R^2$

By gauss's law,

$$\phi_E = \frac{q}{\varepsilon_0}$$

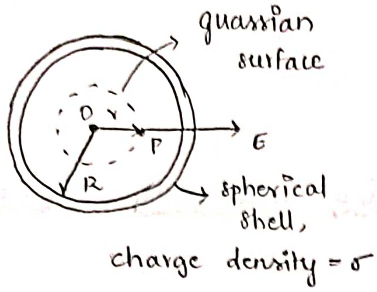
$$\therefore E = \frac{q}{4\pi\varepsilon_0 R^2}$$

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{r}}$$

13/03/2024

(c) Point P lies inside the shell:

→ Consider a guassian surface of radius r inside the shell.



Total flux: $\phi_E = E \times 4\pi r^2$

By gauss's law,

$$\phi_E = \frac{q_{enclosed}}{\varepsilon_0} = 0$$

$$E \times 4\pi r^2 = 0$$

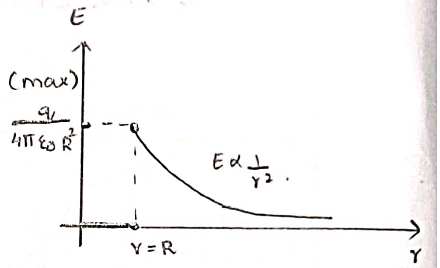
$$\boxed{E = 0}$$

→ Hence, the electric field due to charged shell is zero at all points inside the shell.

Graph: For i) $r > R \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$

ii) $r = R \Rightarrow E = \frac{q}{4\pi\varepsilon_0 R^2}$

iii) $r < R \Rightarrow E = 0$



Note: (i) Negative flux: When the field lines enter the surface, the flux is said to be negative.

⇒ Here, the angle between electric field and area vector is 180°. (cos 180° = −1).

(ii) If the flux is zero through a surface, it does not necessarily mean that there are no charges inside the surfaces. It means that the net charge is zero.

(iii) If a surface has non-zero flux, it does not necessarily mean that there is charge inside the surface. It may also mean that the surface is kept in a non-uniform electric field (resulting in imaginary charge).

(iv) If there are charges inside and outside the guassian surface, then the electric field is due to all the charges present inside and outside the surface but the term q of Gauss's law, however, only represents the total charge enclosed by the surface.

$$\underset{\text{enclosed}}{\phi} = \oint \underset{\text{all}}{\vec{E}}\, d\vec{s} = \frac{q_{enclosed}}{\varepsilon_0}$$

enclosed

13/03/2024

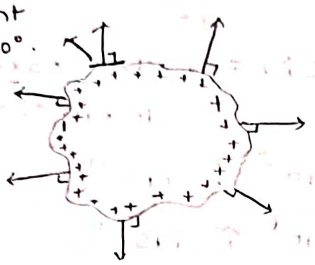
(v) A body gets charged by transfer of electrons. Thus, the mass of the body may increase or decrease in the process of charging.

(vi) Charges cannot reside inside a conductor.

⇒ If a charge is kept inside a conductor, it will move and reach the surface of the conductor.

⇒ The electric field lines are always perpendicular to the surface of the conductor.

draw tangent and find 90°.



(vii) Electric field lines have the tendency to contract lengthwise and expand laterally. The reason is that due to attractive and repulsive forces between the charges.

ELECTRIC FIELD DUE TO AN INSULATING SOLID SPHERE :

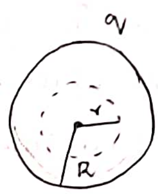
⇒ Consider an insulating solid sphere of radius R and uniform volume charge density ($\rho = Cm^{-3}$).

⇒ If point lies outside the sphere, $r > R$ ⇒ $E = \frac{q}{4\pi\epsilon_0 r^2}$.

⇒ If the point lies on the sphere, $r = R$ ⇒ $E = \frac{q}{4\pi\epsilon_0 R^2}$.

⇒ If the point lies inside the sphere, $r < R$.

* Consider a guassian surface of radius 'r' inside the surface.



Volume charge density $= \rho = \frac{q}{\frac{4}{3}\pi R^3}$

The charge enclosed in the guassian surface :

$$q' = \rho \times V' = \frac{q}{\frac{4\pi R^3}{3}} \times \frac{4\pi r^3}{3}$$

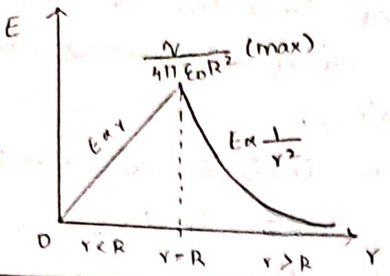
$$q' = q\frac{r^3}{R^3}$$

$$\phi = E\,4\pi r^2$$

By Gauss's law, $\phi = \frac{q'}{\epsilon_0} = \frac{q r^3}{\epsilon_0 R^3}$

$$\frac{q r^3}{\epsilon_0 R^3} = E 4\pi r^2 \Rightarrow \boxed{E = \frac{q}{4\pi\epsilon_0}\frac{r}{R^3}}$$

15/03/2024

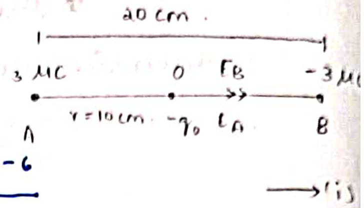


EXERCISES [Pg : 42] :

Q:1.8] (a) $\vec{E}_{net} = \vec{E}_A + \vec{E}_B$

$$= \frac{kq}{r^2}(\hat{i}) + \frac{kq}{r^2}(\hat{i})$$

$$= \frac{2kq}{r^2}(\hat{i}) = \frac{2 \times 9 \times 10^9 \times 3 \times 10^{-6}}{(10 \times 10^{-2})^2}$$



$$= 54 \times 10^5 \; \hat{i} \; NC^{-1}$$

$$\boxed{\vec{E}_{net} = 5.4 \times 10^6 \; \hat{i} \; NC^{-1}}$$

(b) $q_0 = -1.5 \times 10^{-9}$ C

$$\vec{F} = q_0 E = 1.5 \times 10^{-9} C \times 5.4 \times 10^6 \; (-\hat{i})$$

$$\boxed{F = -8.1 \times 10^{-3} \; \hat{i} \; N}$$

$\therefore$ $E_{net} = 5.4 \times 10^6$ along OB and

$F = 8.1 \times 10^{-3}$ N along OA.

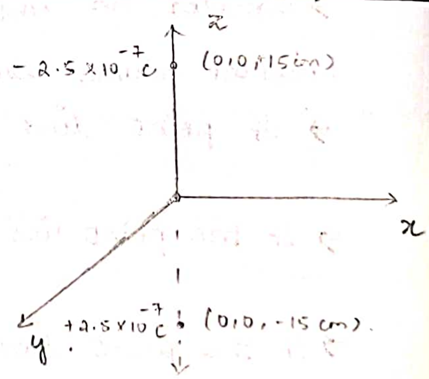
Q:1.9] $q_{net} = 0$ [Total charge $= q_A + q_B$]

$$\vec{p} = q \cdot 2\vec{a}$$

$$= 2.5 \times 10^{-7} \times 30 \times 10^{-2} \; (-\hat{k})$$

$$= -7.5 \times 10^{-9} \; \hat{k}$$

$$= -7.5 \times 10^{-8} \; Cm$$



is the negative z-axis

Q: 1.13] Particles 1 and 2 are negative because they are deflected towards positive plate. Particle 3 is positive because it is deflected towards negative plate.

Acceleration on charge q due to electric field E in y direction :

$$a = \frac{F}{m} \Rightarrow a = \frac{qE}{m}.$$

$\therefore$ deflection of charged particle in time t is y direction is :

15/03/2024

$$h = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2}\frac{qE}{m}t^2$$

The electric field and time t are constant for all three particles. Thus,

$$h \propto \frac{q}{m}$$

∴ The particle with high $\frac{q}{m}$ ratio will be the one having maximum deflection.

∴ Thus, particle 3 has maximum $\frac{q}{m}$ ratio.

Q: 1.1] $F = \frac{kq_1q_2}{r^2}$

$$= \frac{9\times10^9 \times 2\times10^{-7}\times3\times10^{-7}}{(30\times10^{-2})^2}$$

$$= 6\times10^{-3}\ N$$

Q: 1.2] (a) $F = \frac{kq_1q_2}{r^2} \Rightarrow r = \sqrt{\frac{kq_1q_2}{F}}$

$$r = \sqrt{\frac{0.4\times10^{-6}\times0.8\times10^{-6}\times9\times10^9}{0.2}} = \sqrt{144\times10^{-4}}$$

$$r = 12\times10^{-2} = 0.12\ m$$

(b) As the charges have opposite charges, the force on second charge due to first charge will also be 0.2 N.

Q: 1.3] $\frac{Ke^2}{Gm_em_p} = \frac{Nm^2C^{-2}\cdot C^2}{Nm^2kg^{-2}\cdot kg\cdot kg} = [M^0L^0T^0]$

$$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times9.1\times10^{-31}\times1.67\times10^{-27}}$$

$$\frac{Ke^2}{Gm_em_p} = 2.3\times10^{39}$$

∴ It is the ratio between electrostatic force and gravitational force between proton and electron.

Q: 1.4] (a) * The "electric charge of a body is quantised" means that only integral number of electrons can be transferred from one body to another.

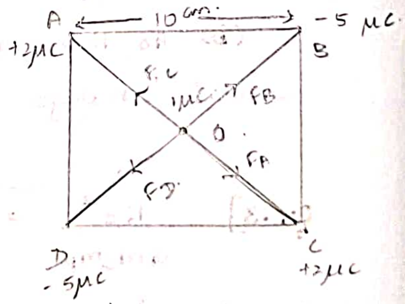
15/03/2024

* Charges cannot be transferred in fractions.
* Hence, the total charge possessed by a body is only an integral multiple of electric charge.

(b) In case of large-scale or macroscopic objects, the charge which is used is comparatively too huge to the magnitude of the electric charge. Hence, on a macroscopic level, the quantisation of charge is of no use. Therefore, it is ignored and the electric charge is considered to be continuous.

Q: 1.5] When two bodies are rubbed against each other, a charge is developed on both bodies. These charges are equal but opposite in nature. This phenomenon of inducing a charge is known as charging by friction. The net charge on both the bodies is zero. When we rub a glass rod with a silk cloth, a charge with opposite magnitude is generated in both the magnitudes. This phenomenon is in consistence with the law of conservation of energy.

Q: 1.6] Here, the forces due to q_A and q_C will cancel out each other and forces due to q_B and q_D cancel out each other as their magnitudes are equal but in different direction.



Q: 1.7] (a) When a charge is placed in an electrostatic field, it experiences a continuous force. Therefore, an electrostatic field line is a continuous curve. A charge moves continuously and does not jump from one point to another. Thus, it cannot have a sudden break.

(b) If two field lines cross each other at a point, there will be two tangents showing two different directions for the electric field at that point.

15/03/2024

Therefore, two field lines can never cross each other

Q: 1.10] $\vec{\tau} = \vec{p} \times \vec{E} = pE \sin\theta$

$= 4 \times 10^{-7} \times 5 \times 10^{4} \times \sin 30$

$= 20 \times 10^{-5}$

$\vec{\tau} = 10^{-4}$ Nm.

Q: 1.11] i) $q = ne$

$n = \frac{q}{e} = \frac{-3 \times 10^{-7}}{-1.6 \times 10^{-19}} = 1.87 \times 10^{12}$ electrons

$\therefore$ 1.87×10^{12} electrons are transferred from wool to polythene.

ii) Yes, mass is transferred as electron, which as some mass is transferred from wool to polythene.

Q: 1.12] (a) $F = \frac{kq_1q_2}{r^2} = \frac{9 \times 10^{9} \times (6.5 \times 10^{-7})^2}{(50 \times 10^{-2})^2}$

$F = 1.52 \times 10^{-2}$ N

(b) If the charge is doubled, $q' = 1.3 \times 10^{-6}$ C.

and the radius is halfed, $r' = 25 \times 10^{-2}$ m.

$F' = \frac{9 \times 10^{9} \times (1.3 \times 10^{-6})^2}{(25 \times 10^{-2})^2}$

$F' = 0.243$ N

Q: 1.18] $\phi = \frac{q}{\varepsilon_0} = \frac{2 \times 10^{-6}}{8.854 \times 10^{-12}}$

$\phi = 2.26 \times 10^{5}$ Nm^2 C^{-1}

Q: 1.19] (a) Electric flux does not depend on the size of the Guassian surface but only on the charged enclosed. Thus, if the radius of the Gaussian surface is doubled, the flux would still be $-1 \times 10^{+3}$ Nm^{2}C^{-1}.

(b) $\phi = \frac{q}{\varepsilon_0} \Rightarrow q = \phi \times \varepsilon_0 = -1 \times 10^{+3} \times 8.854 \times 10^{-12}$

$\Rightarrow q = 8.854 \times 10^{-9}$ C

Q: 1.22] $\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0} \Rightarrow \lambda = \vec{E}\, 2\pi r \varepsilon_0 = 9 \times 10^{4} \times 2 \times 3.14 \times 2 \times 10^{-2} \times 8.854 \times 10^{-12}$

$\Rightarrow \lambda = 10\ \mu C/m$

18/03/2024

Q: 1.14] $\vec{E} = 3 \times 10^3 \hat{i} \; NC^{-1}$

$a = 10$ cm

$a^2 = 100 \text{ cm}^2 = 100 \times (10^{-2})^2 = 100 \times 10^{-4}$

$\vec{S} = 10^{-2} \; m^2 \; \hat{i}$

(a) $\phi = \vec{E} \cdot \vec{S} = 3 \times 10^3 \times 10^{-2} \; (\hat{i} \cdot \hat{i})$.

$\boxed{\phi = 30 \; Nm^2 C^{-1}}$

(b) $\theta = 60°$

$\phi = \vec{E} \cdot \vec{S} = ES\cos\theta$

$= 3 \times 10^3 \times 10^{-2} \times \cos 60°$

$\boxed{\phi = 15 \; Nm^2 C^{-1}}$

Q: 1.15] The net flux through the cube is zero because the flux entering on one face (left face) and the flux leaving the other face are equal. (Negative flux = Positive flux).

Q: 1.16] (a) $\phi = 8 \times 10^3 \; Nm^2/C$

Therefore, $q_{enclosed} = \phi \times \varepsilon_0$ [from Gauss's Law]

$= 8 \times 10^3 \times 8.854 \times 10^{-12}$

$= 70.8 \times 10^{-9}$ C

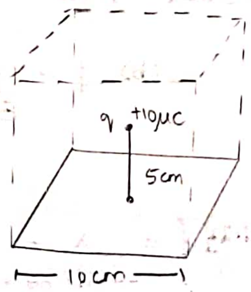
$= 70.8$ nC

$= 0.071 \; \mu C$.

(b) No, ~~The net cha~~ we cannot say that there are no charges at all inside the box. We can only say that the net charge inside the charge is zero.

Q: 1.17] We can imagine the square as a face of a cube with edge 10 cm and with a charge $+10 \mu C$ placed at its centre.

Total flux shared by 6 faces will be $\frac{q}{\varepsilon_0}$.



Therefore, flux due to each face will be $\frac{q}{6\varepsilon_0}$.

19/03/2024

$$\phi = \frac{q}{6\varepsilon_0} \times \frac{4\pi}{4\pi} = \frac{10\times10^{-6}\times 9\times10^{9}\times4\times3.14}{6}$$

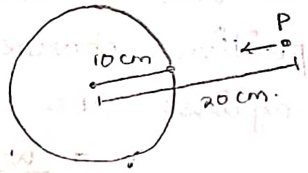
$$= \cancel{665.2\times10^{3}} \quad 188.4\times10^{3}$$

$$\boxed{\phi = 1.884\times10^{5}\ Nm^2C^{-1}}$$

Note:

Charge location	Total flux	Flux at one face
centre	[no face is shared] $\frac{q}{\varepsilon_0}$	$\frac{q}{6\varepsilon_0}$ [for all 6 faces]
face	[1 face is shared] $\frac{q}{2\varepsilon_0}$	$\frac{q}{10\varepsilon_0}$ [for 5 faces]
edge	[2 faces are shared] $\frac{q}{4\varepsilon_0}$	$\frac{q}{16\varepsilon_0}$ [for 4 faces]
vertex	[3 faces are shared] $\frac{q}{8\varepsilon_0}$	$\frac{q}{24\varepsilon_0}$ [for 3 faces].

Q:1.20] Electric field due to all points outside the conducting sphere is:
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

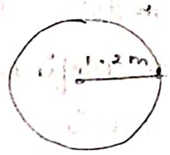


$$q = \frac{Er^2}{\frac{1}{4\pi\varepsilon_0}} = \frac{1.5\times10^{3}\times0.2\times0.2}{9\times10^{9}}$$

$$= 0.00667\times10^{-6}$$

$$\boxed{q = -6.67\ nC}$$ [$-ve \Rightarrow \vec{E}$ field is inwards].

Q:1.21] $\sigma = 80\times10^{-6}\ C$, $r = 1.2\ m$



(a) $q = \sigma\, 4\pi r^2$

$$= 80\times10^{-6}\times4\times3.14\times1.2\times1.2$$

$$\boxed{q = 1.45\times10^{-3}\ C}$$

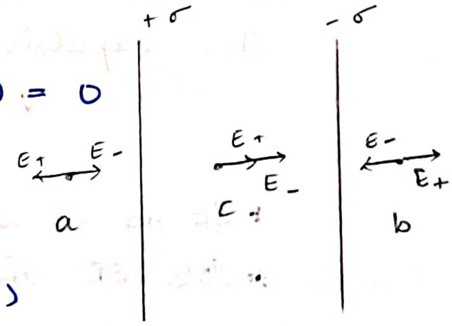
(b) $\phi = \frac{q}{\varepsilon_0}\frac{\times 4\pi}{\times 4\pi}$

$$= 1.4\times10^{-3}\times9\times10^{9}\times4\times3.14$$

$$\boxed{\phi = 1.6\times10^{8}\ Nm^2C^{-1}}$$

Q:1.23] (a) $\vec{E_a} = \vec{E_+} + \vec{E_-} = \frac{\sigma}{2\varepsilon_0}(-\hat{\imath}) + \frac{\sigma}{2\varepsilon_0}(\hat{\imath}) = 0.$

$$\vec{E_a} = 0$$



(b) $\vec{E_b} = \vec{E_+} + \vec{E_-} = \frac{\sigma}{2\varepsilon_0}(\hat{\imath}) + \frac{\sigma}{2\varepsilon_0}(-\hat{\imath}) = 0$

$$\vec{E_b} = 0$$

(c) $\vec{E_c} = \vec{E_+} + \vec{E_-} = \frac{\sigma}{2\varepsilon_0}(\hat{\imath}) + \frac{\sigma}{2\varepsilon_0}(\hat{\imath})$

$$= \frac{\sigma}{\varepsilon_0}\hat{\imath} = \frac{17\times10^{-22}}{8.854\times10^{-12}}$$

$$\boxed{\vec{E_c} = 1.92\times10^{-10}\ \hat{\imath}\ NC^{-1}}$$