Tuesday

23/04/2024

3. CURRENT ELECTRICITY

CURRENT ELECTRICITY:

* The study of charges in motion is called current electricity (or) electrodynamics.
* The study of charges at rest is called electrostatics.

ELECTRIC CURRENT:

* The flow of electric charges through a conductor constitutes an electric current.

$$I = \frac{\Delta Q}{\Delta t} \Rightarrow \lim_{\Delta t \to 0} \frac{dQ}{dt}$$ [instantaneous current → at a particular instant (small amount time)]

⇒ For direct current (DC) (Steady current) ⇒ $I = \frac{Q}{T}$

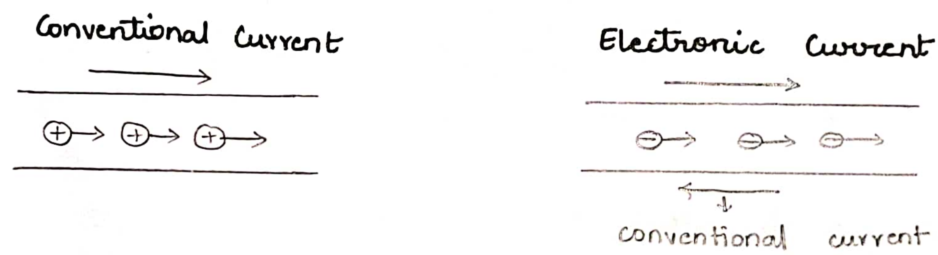
⇒ For alternating current (AC) ⇒ $I = \frac{dQ}{dt}$ [changes continuously]

* Current that exists for a short duration is called transient current.
* SI unit of current is ampere (A).

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

* Smaller units of current are mA and μA. [nA → not possible]

CONVENTIONAL AND ELECTRONIC CURRENT:

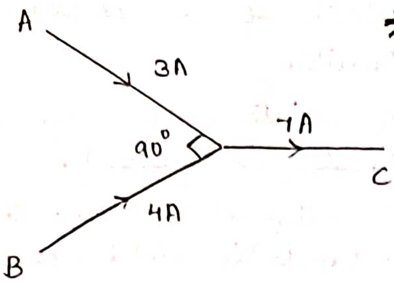


* The direction of positive charges is taken as the direction of current.
* It is opp. to the direction of electrons.
* Conventional current is the movement of +ve charge and electronic current is the movement of electrons.
* Conventional and electronic currents are opp. to

3/04/2024

each other.

ELECTRIC CURRENT IS A SCALAR QUANTITY:



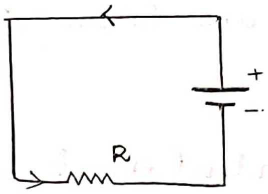
→ The current in wire OC is 7 A which is scalar addition (3A + 4A) instead of 5 A, which is vector addition ($\sqrt{3^2+4^2}$).

→ Although current has both magnitude and direction, it does not obey the laws of vector addition.

→ Thus, it is a scalar quantity.

MAINTANENCE OF STEADY CURRENT:

* To maintain steady current through the conductor, some ~~et~~ external device must do work at a steady rate to take +ve charge from lower potential to higher potential. Such a potential is a source of electromotive force (emf).



source of emf
(charge pump)

* ELECTRO MOTIVE FORCE [EMF]:

It is defined as the work done by the source in taking a unit +ve charge from lower to higher potential.

$$\text{emf} = \frac{\text{work done}}{\text{charge}}$$

→ The term emf is misleading because it is not a force at all. It is a special case of potential difference.

→ It's SI unit is volt (V).

→ It is a scalar quantity.

* ELECTROMOTIVE ~~EM~~ FORCE AND POTENTIAL DIFFERENCE:

ELECTROMOTIVE FORCE	POTENTIAL DIFFERENCE
→ It is the work done by a source in taking a unit charge once round the complete circuit.	→ It is the amount of work done in taking a unit charge from one point of a circuit to another.

3/04/2024

ELECTROMOTIVE FORCE	POTENTIAL DIFFERENCE
⇒ It is equal to the maximum potential difference between the two terminals of a source when it is in an open circuit.	⇒ Potential difference may exist between any two points of a closed circuit.
⇒ It exists even when the circuit is not closed.	⇒ It exists only when the circuit is closed.
⇒ It has non-electrostatic origin.	⇒ It originates from the electrostatic field set up by the charges accumulated on the the two terminals of the source.
⇒ It is a cause. When emf is applied in a circuit, potential difference is caused.	⇒ It is an effect.
⇒ It is equal to the sum of potential differences across all the components of a circuit including p.d. required to send current through the cell itself.	⇒ Every circuit component has its own potential difference across its ends.
⇒ It is independent of: i) resistance of the circuit ii) internal resistance of the cell iii) current in the circuit.	⇒ It depends on: i) resistance ii) internal resistance of the cell iii) current in the circuit.

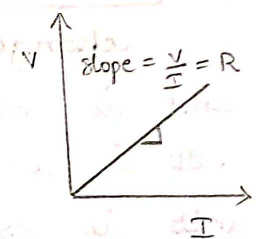
OHM'S LAW:

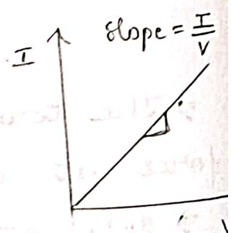
* The current flowing through a conductor is directly proportional to the potential difference applied across its ends provided ~~that~~ the ~~phys~~ temperature and other physical conditions remain unchanged.
* Current is an independant quantity but voltage is a dependant quantity.

$$V \propto I$$

$$V = IR$$

$$\boxed{R = \frac{V}{I}}$$





$$1\ \text{ohm}(\Omega) = \frac{1\ \text{volt}}{1\ \text{ampere}} = \frac{1V}{1A}$$

* Ohm is the unit of resistance.

$$\left.\begin{array}{l} R \propto l \\ R \propto \frac{1}{A} \end{array}\right\} \longrightarrow R \propto \frac{l}{A} \Rightarrow \boxed{R = \frac{\rho l}{A}}$$

24/09/2024

* Factors affecting resistance :

⇒ $R \propto l$ (length)

⇒ $R \propto \frac{1}{A}$ (area)

⇒ It depends on the nature of the material.

Ex: Nichrome wire is having 60 times more resistance than copper wire for the same length and area of cross section.

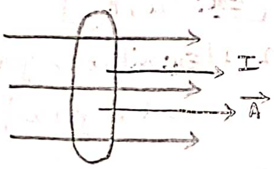
* Combining all factors

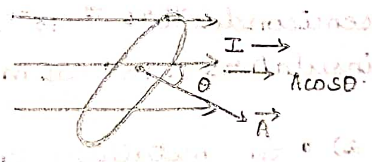
$$R \propto \frac{l}{A} \Rightarrow \boxed{R = \frac{\rho l}{A}}$$ [ρ = resistivity / specific resistance]

* It is defined as the resistance of the conductor having unit length and unit area of cross - section.
* SI unit of ρ is Ωm.
* Resistance is equal to the ratio of the potential difference applied across its ends to the current flowing through it.

CURRENT DENSITY: [j]

* It is the amount of charge flowing per second through a unit area held normal to the direction of the flow of charge at that point.





$$j = \frac{I}{A\cos\theta}$$

$$I = jA\cos\theta$$

$$\boxed{I = \vec{j} \cdot \vec{A}}$$

* $\vec{j}$ is used to denote current density.
* It is a vector quantity.
* It's unit is $A m^{-2}$.

CONDUCTANCE [G] :

$$\boxed{G = \frac{1}{R}}$$

* SI unit of conductance is Ω^{-1}, mho, siemens (S).

24/04/2024

CONDUCTIVITY [σ]:

$$\boxed{\sigma = \frac{1}{\rho}}$$

* SI units of conductivity are mho m^{-1}, $ohm^{-1} m^{-1}$, $S m^{-1}$.

OHM'S LAW IN VECTOR FORM:

* We know that,

$V = El$

$I = jA$

$V = IR$

$El = jAR$

$E = j\frac{RA}{l}$ $\quad \left[\rho = \frac{RA}{l}\right]$

$$\boxed{E = \rho j}$$

* In vector form,

$\vec{E} = \rho \vec{j}$

$\vec{j} = \frac{1}{\rho}\vec{E}$ $\quad \left[\sigma = \frac{1}{\rho}\right]$

$$\boxed{\vec{j} = \sigma \vec{E}}$$

* This equation is ohm's law in vector form.
* It is equivalent to scalar form $V = IR$.

Note: 1) Resistivity for:

- conductors = $10^{-8}\,\Omega m$ to $10^{-6}\,\Omega m$ [Ex: Cu, Ag]
- semiconductors = $10^{-6}\,\Omega m$ to $10^{4}\,\Omega m$ [Ex: silicon, germanium]
- insulators $\geq 10^{4}\,\Omega m$ [Ex: rubber, wood, glass]

2)
- In metallic conductors, e^- are the charge carriers. (-ve charge) (solids)
- In electrolytic solutions, ions (+ve and -ve) are the charge carriers. (liquids)
- In gases, +ve and -ve ions and e^- are the charge carriers.
- In vacuum tubes, free e^- emitted by the heated cathode act as charge carriers.

~~EXERCISES [Pg:~~

MECHANISM OF CURRENT FLOW IN A CONDUCTOR:

* Electrons are in a state of continuous random motion due to thermal energy about $10^5\ ms^{-1}$
* However, they are distributed randomly.

24/04/2024

* Therefore, average velocity of N electrons,

$$\Rightarrow \vec{v} = \frac{\vec{u_1} + \vec{u_2} + \vec{u_3} + \dots \vec{u_N}}{N} = 0$$

* In the presence of electric field $\vec{E}$, each electron experiences a force $-e\vec{E}$.
* Therefore,

$$\Rightarrow \vec{a} = \frac{F}{m} = \frac{-e\vec{E}}{m}$$

$$\Rightarrow \boxed{\vec{a} = \frac{-e\vec{E}}{m}}$$

* Thus, the average velocity $\vec{V_a}$ of all N electrons will be

$$\Rightarrow \vec{V_a} = \frac{\vec{V_1} + \vec{V_2} + \vec{V_3} + \dots + \vec{V_N}}{N}$$

$$\Rightarrow \vec{V_a} = \frac{(\vec{u_1} + \vec{a}\tau_1) + (\vec{u_2} + a\vec{\tau_2}) + \dots + (\vec{u_N} + a\vec{\tau_n})}{N}$$

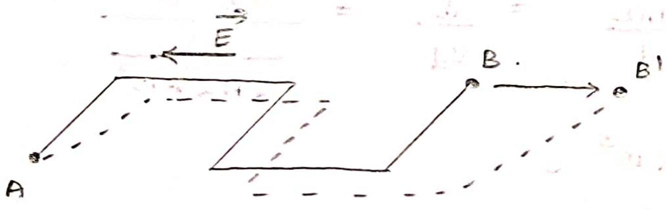
$$\Rightarrow \vec{V_a} = \frac{u_1 + u_2 + u_3 + \dots + u_N}{N} + \frac{\vec{a}(\tau_1 + \tau_2 + \tau_3 + \dots + \tau_N)}{N}$$

$$\Rightarrow \vec{V_a} = \vec{a} \cdot \vec{\tau}$$

$$\Rightarrow \boxed{\vec{V_a} = \frac{-e\vec{E}}{m} \cdot \tau}$$

* Here, the average time between two successive collisions is called relaxation time.
* $\vec{V_a}$ is called drift velocity and is defined as the average velocity gained by free e^- of a conductor in the opposite direction of the external electric field.

Note: For most conductors, the relaxation time is of the order 10^{-14} s.



24/04/2024

EXERCISES [Pg: 105]:

3.2] $\mathcal{E} = 10\,V$, $r = 3\,\Omega$, $I = 0.5\,A$

Terminal voltage $(V) = \mathcal{E} - Ir$

$V = 10 - (0.5)(3)$

$= 10 - 1.5$

$\boxed{V = 8.5\ V}$

Resistance $(R) = \frac{V}{I} = \frac{8.5}{0.5}$

$\boxed{R = 17\ \Omega}$

3.4] $l = 15\,m$, $A = 6 \times 10^{-7}\,m^2$, $R = 5\,\Omega$

$R = \frac{\rho l}{A}$

$\rho = \frac{RA}{l} = \frac{\cancel{5}^{1} \times \cancel{6}^{2} \times 10^{-7}}{\cancel{15}\ \cancel{3}\ 1}$

$\boxed{\rho = 2 \times 10^{-7}\ \Omega m}$

3.9] $n = 8.5 \times 10^{+28}\ m^{-3}$

$l = 3\,m$

$A = 2 \times 10^{-6}\ m^2$

$I = 3.0\ A$

$e = 1.6 \times 10^{-19}\ C$

$I = neAv_d$

$v_d = \frac{I}{neA} = \frac{3}{8.5 \times 10^{+28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-6}}$

$v_d = \frac{3}{27.2 \times 10^{3}}\ m/s$

~~Vd/ +/~~

$t = \frac{\text{distance}}{\text{speed}} = \frac{l}{v_d} = \frac{\cancel{3}^{1}}{\frac{\cancel{3}^{1}}{27.2 \times 10^{3}}}$

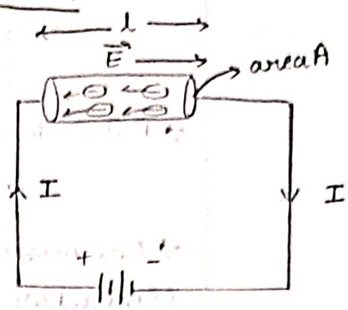
$t = 27.2 \times 10^{3}\ s$

$\boxed{t = 2.7 \times 10^{4}\ s}$

24/03/2024

Imp

RELATION BETWEEN CURRENT AND DRIFT VELOCITY:



* Let n be the number of electrons per unit volume.

$$n = \frac{N}{Al}$$

$$q = Ne = nAle$$

* Time taken by the e^- to move through the wire is :

$$\boxed{t = \frac{\text{distance}}{\text{speed}} = \frac{l}{v_d}}$$

[v_d = drift velocity].

* We know that,

$$I = \frac{q}{t} = \frac{neA\cancel{l}}{\cancel{l}/v_d}$$

$$\boxed{I = neAv_d}$$

[n = electronic density]

* current density ($\vec{j}$),

$$\vec{j} = \frac{I}{A}$$

$$\boxed{\vec{j} = ne\vec{v}_d}$$

* This equation is valid for all both positive and negative values of charge q.

DEDUCTION OF OHM'S LAW:

* We know that,

$$v_d = \frac{eE}{m} \cdot \tau$$

$$v_d = \frac{eV}{ml}\tau$$

$$\therefore V = \frac{v_d ml}{e\tau}$$

* We know that,

$$I = neAv_d$$

$$\frac{V}{I} = \frac{v_d ml}{e\tau} \times \frac{1}{neAv_d}$$

$$\frac{V}{I} = \frac{ml}{ne^2A\tau}$$

* At a fixed temperature, the quantities n, l, e, τ and A, all have constant values for a given conductor. $\Rightarrow \frac{V}{I}$ = constant, R

24/04/2024

→ This proves Ohm's law for a conductor and here,

$$\boxed{R = \frac{ml}{ne^2 A \tau}}$$

→ But $R = \rho \frac{l}{A}$ ⇒ $\boxed{\rho = \frac{m}{ne^2 \tau}}$

→ Obviously, ρ is independant of the dimensions of the conductor (l, A) but is dependant on two parameters

1) no. of free electrons per unit volume $\left[\rho \propto \frac{1}{n}\right]$

2) the ~~relat~~ relaxation time $\left[\rho \propto \frac{1}{\tau}\right]$

Note: 1) 'n' implies electron density. It's unit is m^{-3}.

2) N has no units.

Imp

RELATION BETWEEN $\vec{j}$, σ and $\vec{E}$:

$$v_d = \frac{eE}{m}\tau$$

$$j = nev_d$$

$$j = ne \cdot \frac{eE}{m} \cdot \tau$$

$$j = \frac{ne^2 \tau E}{m}$$

$$j = \frac{1}{\rho} E \qquad \left[\frac{1}{\rho} = \sigma\right]$$

$$\boxed{\vec{j} = \sigma \vec{E}} \qquad [\sigma = \text{conductivity}]$$

Note: 1) Collisions are the basic cause of resistance.

2) Larger the number of collisions per second, smaller is the relaxation time, larger will be the resistivity

3) The cause of instantaneous current: although the drift speed is 1 mm per sec, the bulb lights up as soon as we turn on the switch, because e^- are present everywhere, the electric field provides a potential difference, almost as the speed of light, hence e^- in every part of the circuit begins to drift and current begins to flow almost immediately.

24/04/2024

MOBILITY OF CHARGE CARRIER:

* It is the drift velocity acquired per unit field by the charge carrier.

$$\mu = \frac{v_d}{E} = \frac{\text{drift velocity}}{\text{electric field}}$$

* SI unit of mobility is $m^2/Vs \Rightarrow m^2 V^{-1} s^{-1}$.

$$\mu = \frac{eE\tau}{mE}$$

$$\boxed{\mu_e = \frac{e\tau}{m}}$$

* For any charge, $\mu = \frac{q}{m}\tau$.

Imp

TEMPERATURE DEPENDANCE ON RESISTIVITY:

$$\rho = \frac{m}{ne^2\tau}$$

1) Metals : As temperature increases, the thermal speed of free electrons increases, consequently, the free electrons collide more frequently. This means, the relaxation time (τ) decreases. Moreover, the number density (n) is almost independent of temperature for metals. Hence the resistivity of a metal ($\rho \propto \frac{1}{\tau}$) increases, and conductivity decreases with increase in temperature.

$$\alpha = \frac{\rho - \rho_0}{\rho_0} \times \frac{1}{(T - T_0)} \quad \left[\alpha = \text{temperature coefficient of } \cancel{\text{metals}} \text{ resistivity} \right]$$

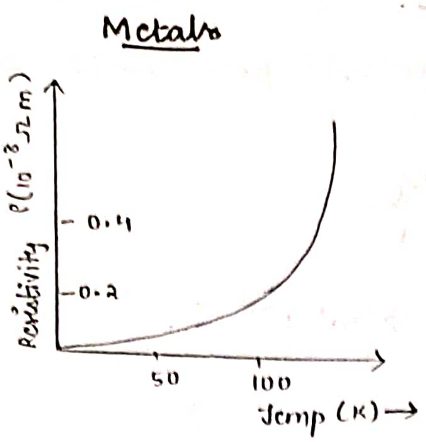
$$\alpha = \frac{1}{\rho_0}\left[\frac{d\rho}{dT}\right]$$

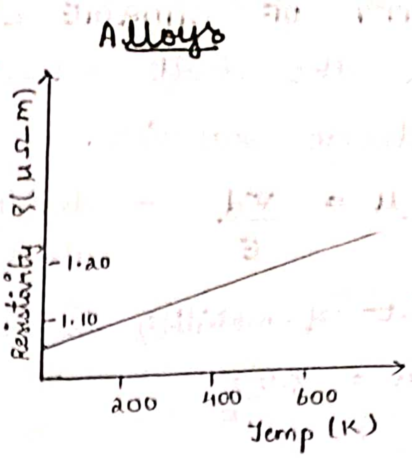
⇒ Thus, the temperature coefficient of resistivity (α) may be defined as the increase in resistivity per unit resistivity per degree rise in temperature.

⇒ α is positive for alloys and metals but α value is low for alloys ~~is low~~ when compared to metals.

⇒ That means, when temp. increases, resistivity increases rapidly in metals while resistivity almost remains constant in alloys.

24/04/2023





→ $\alpha = \frac{\rho - \rho_0}{\rho_0}\left(\frac{1}{T - T_0}\right)$

→ $\alpha(T - T_0) = \frac{\rho}{\rho_0} - 1$

→ $1 + \alpha(T - T_0) = \frac{\rho}{\rho_0}$

→ $\boxed{\rho = \rho_0(1 + \alpha(T - T_0))}$

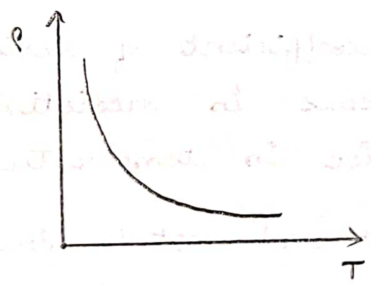
→ |||ly $R = \frac{\rho l}{A} \Rightarrow R \propto \rho$

→ $\boxed{R = R_0(1 + \alpha(T - T_0))}$

2) Semiconductors and insulators : The number density of free electrons increases exponentially with increase in temperature. Thus, resistivity decreases exponentially ~~with~~ and conductivity increases with increase in temp.

⇒ α is negative for both semiconductors and insulators.

⇒ As temp increases, semiconductivity decreases.



3) Electrolytes : For electrolytes, as temp. increases interionic attraction decreases and viscosity also decreases. The ions move more freely. Thus, resistivity decreases and conductivity increases with increase in temp.

⇒ α is negative for electrolytes.

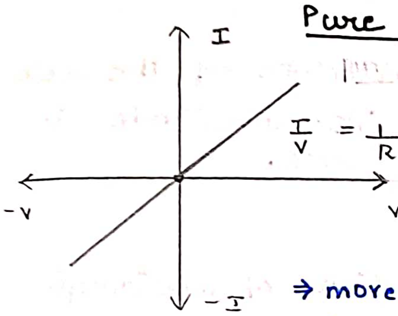
25/04/2024

Note: In an alloy, Ex: nichrome, Ni^{2+} and Cr^{3+} ions have different charge size. They occupy random locations. Therefore, e^- passes through a very random medium and is very frequently. Hence, collision increases, ~~relation~~ relaxation time decreases and resistivity increases.

LIMITATION OF OHM'S LAW:

* Ohm's law is obeyed by many substances under certain conditions but it is not a fundamental law of nature.
* Conductors which obey Ohm's law are called as Ohmic conductors.
* The V-I graph for Ohmic conductors is a straight line passing through the origin.

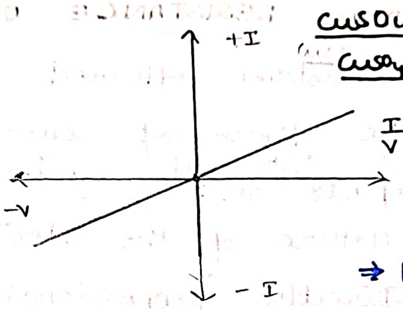
Pure metals



$\frac{I}{V} = \frac{1}{R} =$ slope

⇒ more slope, less resistance

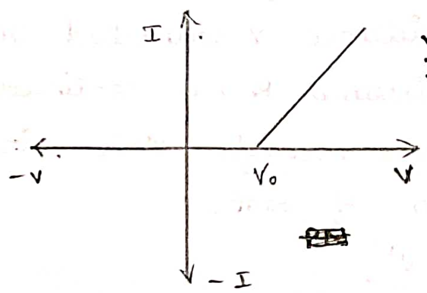
$CuSO_4$ solution with Cu electrodes



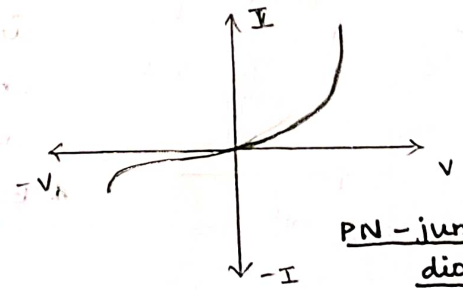
$\frac{I}{V} = \frac{1}{R} =$ slope

⇒ less slope, more resistance

* Conductors which do not obey Ohm's law are called ~~Ohm~~ non-ohmic conductors.
* The resistance of such conductors is not constant.
* Non-Ohmic conditions may be one of the following types:
  1) The straight line VI graph does not pass through the origin.
  2) VI relation is non-linear.
  3) VI relation depends on the sign of V.
  4) VI relation is non-unique.

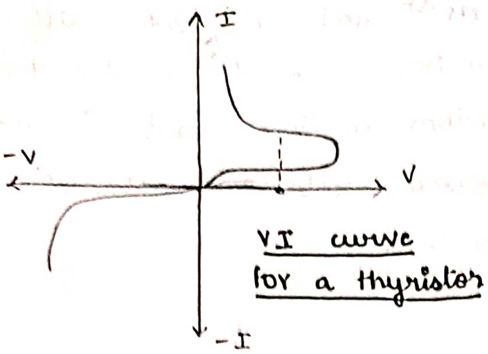


VI water Voltameter

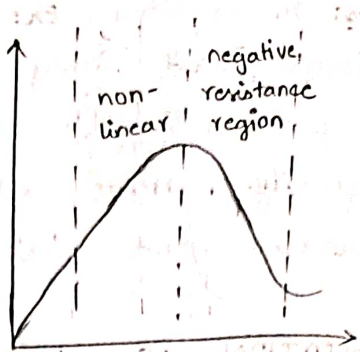


PN-junction diode

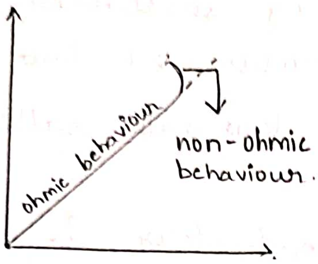
24/04/2024



VI curve for a thyristor



VI graph for GaAs

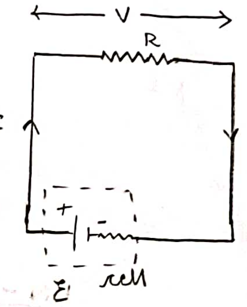


INTERNAL RESISTANCE OF A CELL:

* The resistance offered by the electrolyte of the cell to the flow of current between its electrode is called internal resistance of a cell.
* It depends on:
  1) Nature of the electrolyte.
  2) Directly proportional to concentration of electrolyte
  3) Directly proportional to distance between two electrodes
  4) Inversely proportional to the common area of the electrodes immersed in the electrolyte.
  5) Inversely proportional to the temperature of electrolyte

Note: The internal resistance of a freshly prepared cell is usually low but it increases over time.

RELATION BETWEEN INTERNAL RESISTANCE, EMF AND TERMINAL POTENTIAL DIFFERENCE OF A CELL:



* Consider a cell of emf ε and internal resistance r connected to external resistance R, a continuous current flows through this circuit.
* By definition of emf,

$$\varepsilon = V + V'$$

* By Ohm's law

25/04/2024

$[V = IR]$ $\quad$ $[V' = Ir]$

$\Rightarrow \mathcal{E} = IR + Ir$

$\Rightarrow \mathcal{E} = I[R + r]$

$\Rightarrow \boxed{I = \dfrac{\mathcal{E}}{R+r}}$

$\Rightarrow \boxed{V = \left(\dfrac{\mathcal{E}}{R+r}\right) I}$

Also, $V = \mathcal{E} - V'$

$\Rightarrow \boxed{V = \mathcal{E} - Ir}$

$\Rightarrow Ir = \mathcal{E} - V$

$\Rightarrow r = \dfrac{\mathcal{E} - V}{I}$

$\Rightarrow \boxed{r = \left(\dfrac{\mathcal{E} - V}{V}\right) . R}$

* Special cases:

i) When cell is open, $I = 0$, $V_{open} = \mathcal{E}$

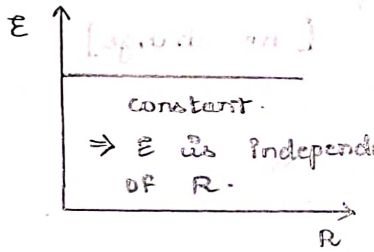
Thus, emf = P.D. when no current is drawn.

ii) When $I \neq 0$, $V < \mathcal{E}$.

~~Note~~ Note: while charging a battery, $V = \mathcal{E} + IR$ $[V > \mathcal{E}]$.

CHARACTERISTIC CURVES FOR A CELL:

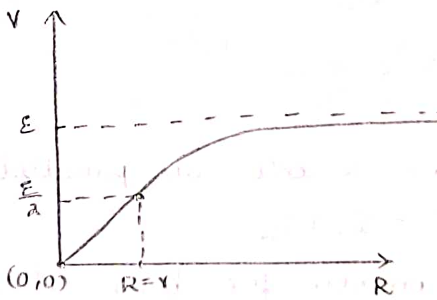
1) $\mathcal{E}$ vs R graph:



2) V vs R graph:

$V = \left(\dfrac{\mathcal{E}}{R+r}\right) R = \left(\dfrac{\mathcal{E}}{\frac{R+r}{R}}\right)$

$V = \dfrac{\mathcal{E}}{1 + \frac{r}{R}}$



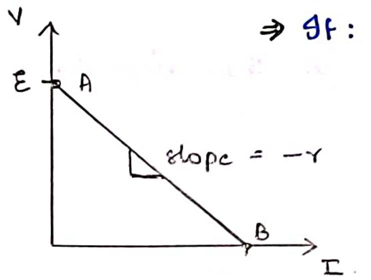
$\Rightarrow$ If 1) $R = 0, V = 0$

2) $R = r$, $V = \dfrac{\mathcal{E}}{2}$

3) $R = \infty$, $V = \mathcal{E}$

$\Rightarrow$ When $R = \infty$ (i.e. very very high), the circuit behaves like an open circuit and current does not flow. Thus, $V = \mathcal{E}$.

3) V vs I graph:



$\Rightarrow$ If: $I_A = 0$, $V = \mathcal{E}$

$\mathcal{E} = I_B r$, $V = 0$

$\boxed{r = \dfrac{\mathcal{E}}{I_B}}$

$V = \mathcal{E} - rI$

$[y = c + mx] \Rightarrow [m = \text{slope}]$

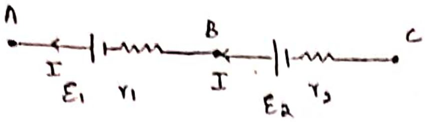
25/04/2024

$r = -$ slope of the VI graph

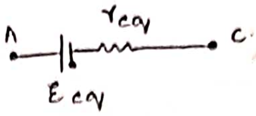
Imp.

COMBINATION OF CELLS:

1) SERIES:



$\equiv$



$V_{AB} = V_A - V_B = \mathcal{E}_1 - Ir_1$

$V_{BC} = V_B - V_C = \mathcal{E}_2 - Ir_2$

$V_{AC} = V_A - V_B + V_B - V_C$

$= \mathcal{E}_1 - Ir_1 + \mathcal{E}_2 - Ir_2$

$V_{AC} = V_A - V_C = \mathcal{E}_1 + \mathcal{E}_2 - I(r_1 + r_2)$

$\mathcal{E}_{eq} - Ir_{eq} = \mathcal{E}_1 + \mathcal{E}_2 - I(r_1 + r_2).$

$\Rightarrow \mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2$ $\Rightarrow r_{eq} = r_1 + r_2$

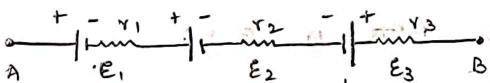
We can extend this rule for n-cells in series.

$\Rightarrow \mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 + \ldots\ldots + \mathcal{E}_n$

$\Rightarrow r_{eq} = r_1 + r_2 + \ldots\ldots\ldots + r_n.$

Note: If the cell is connected in the reverse order, then its emf is taken as negative and r_{eq} remains the same.

Ex:

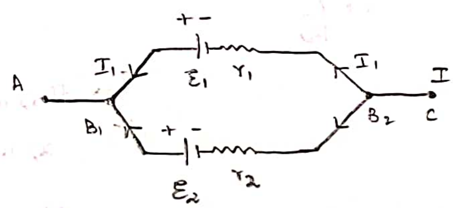


connected in reverse manner

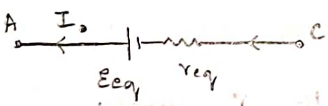
$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_3$
[$-$ve $\mathcal{E}_3$]

$r_{eq} = r_1 + r_2 + r_3$
[no change]

2) PARALLEL:



$\equiv$



* Consider 2 cells in parallel as shown in the figure.

$I = I_1 + I_2$

* V is common for both cells.

$V = \mathcal{E}_1 - I_1 r_1$

$V = \mathcal{E}_2 - I_2 r_2$

26/04/2024

* Therefore, $I_1 = \frac{E_1 - V}{r_1}$ $\quad I_2 = \frac{E_2 - V}{r_2}$

$\Rightarrow I = I_1 + I_2$

$= \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2} = \frac{E_1 r_2 - V r_2 + E_2 r_1 - V r_1}{r_1 r_2}$

$= \frac{E_2 r_1 + E_1 r_2 - (r_1 + r_2)V}{r_1 r_2}$

$\Rightarrow \frac{E_{eq} - V}{r_{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} - \frac{r_1 + r_2}{r_1 r_2} V$

On comparing

$\Rightarrow \frac{E_{eq}}{r_{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2}$

$\Rightarrow \frac{E_{eq}}{r_{eq}} = \frac{E_1 r_2}{r_1 r_2} + \frac{E_2 r_1}{r_1 r_2}$

$\Rightarrow \frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$

$\Rightarrow \frac{1}{r_{eq}} = \frac{r_1 + r_2}{r_1 r_2}$

$\Rightarrow \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$

For 2 cells:

$\rightarrow r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$

$\rightarrow \frac{E_{eq}}{r_{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2}$

$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} \times \frac{r_1 r_2}{r_1 + r_2}$

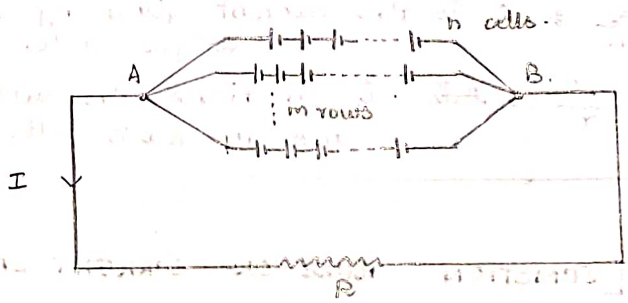
$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

* The above equations can be extended for n cells.

$\Rightarrow \frac{E_{eq}}{E}$ $\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \dots + \frac{E_3}{r_3}$

$\Rightarrow \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$

3) MIXED GROUPING IN CELLS:



* Current (I) = Total emf / Total resistance

* Each row: n cells

$\rightarrow E_{eq} = E + E + \dots + E$

$E_{eq} = nE$

$\rightarrow r_{eq} = r + r + \dots + r$

$r_{eq} = nr$

* For m rows: $\rightarrow E_{eq} = nE$ [potential is common in parallel]

$\frac{1}{r_{eq}} = \frac{1}{nr} + \frac{1}{nr} + \dots + \frac{1}{nr} = \frac{m}{nr}$

$\rightarrow r_{eq} = \frac{nr}{m}$

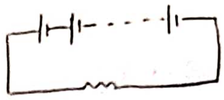
* $I = \frac{E_{eq}}{r_{eq} + R} = \frac{nE}{\frac{nr}{m} + R} = \frac{nE}{\frac{nr + mR}{m}}$

$\boxed{I = \frac{mnE}{nr + mR}}$

$n \Rightarrow$ each row.
$m \Rightarrow$ rows

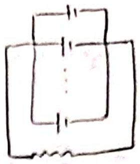
27/04/2024

Case 1] If only one row is present, $m = 1$.



$\Rightarrow I = \frac{nE}{R + nr}$

Case 2] If only one cell is present in each row, $n = 1$.



$\Rightarrow I = \frac{mE}{mR + r}$

→ If $m = 1$, $I = \frac{nE}{R + nr}$,

i) If $R >> r$, $I = \frac{nE}{R} = n\left(\frac{E}{R}\right)$

- I is n times the current (E/R) than can be drawn from one cell.

ii) If $R << nr$, $I = \frac{nE}{nr} = \frac{E}{r}$

- This is the current given by a single cell.

★Thus, when the external resistance is much higher than the total internal resistance, the cells should be connected in series to get maximum current.

If $n = 1$, $I = \frac{mE}{mR + r} = \frac{E}{R + \frac{r}{m}}$

i) If $R >> \frac{r}{m}$, $I = \frac{E}{R}$ ⇒ I is the current given by a single cell.

ii) If $R << \frac{r}{m}$, $I = \frac{mE}{r}$ ⇒ This is m times the current due to each cell.

★Thus, when external resistance is much smaller than the net internal resistance, the cells should be connected in parallel connection to get maximum current.

KIRCHHOFF'S LAW:

1) KIRCHHOFF'S FIRST LAW [JUNCTION RULE OR CURRENT LAW]:

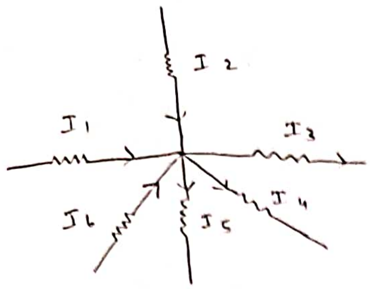
*In an electric circuit, the algebraic sum of currents ~~~~ at any junction is zero.

*[OR] The sum of currents entering a junction is equal to the sum of currents leaving that junction.

Mathematically, $\Sigma I = 0$.

Note: Current entering is +ve and current leaving is -ve.

17/04/2024



$I_1 + I_2 + I_6 = I_3 + I_4 + I_5$

$I_1 + I_2 + I_6 - I_3 - I_4 - I_5 = 0$

Incoming current = Outgoing current

* Justification: This is based on the law of conservation of charge. Charges cannot accumulate or originate at any point of the circuit. Therefore, for a steady current charges flowing towards the junction must be equal to charges flowing away from that junction.

2) KIRCHHOFF'S SECOND LAW [LOOP RULE OR VOLTAGE LAW]:

* Around any closed loop of a network, the algebraic sum of changes in potential ~~ma~~ must be zero.
* [OR] The algebraic sum of the emfs in any loop of a circuit is equal to the sum of the products of currents and resistances in it.

*Mathematically, $\Rightarrow \Sigma \Delta V = 0$

$\Rightarrow \Sigma \mathcal{E} = \Sigma IR$.

Note: 1) We can take any direction (clockwise or anticlockwise) as the direction of traversal. [In the direction of loop taken].

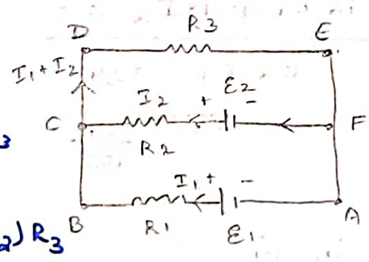
2) If direction of traversal is from − to + across the battery, then the emf is taken as +ve and vice versa.

3) If current and direction of traversal are in the same direction, then current − resistance product is taken as (+ve) and vice versa.

Loop ABCFA: $\mathcal{E}_1 - \mathcal{E}_2 = I_1 R_1 - I_2 R_2$

Loop CDEFC: $\mathcal{E}_2 = I_2 R_2 + (I_1 + I_2) R_3$

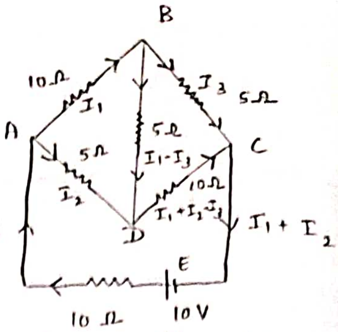
Loop ABCDEFA: $\mathcal{E}_1 = I_1 R_1 + (I_1 + I_2) R_3$



* Justification: This law is based on law of conservation of energy. ~~As~~ As the electrostatic force is a conservative force, the work done by it along any closed path is zero.

27/04/2024

EXERCISES [Pg: 105]:



3.7] $I = I_1 + I_2$

loop ADCEA:

$$10 = 10I + 5I_2 + 10(I_1 + I_2 - I_3)$$

$$10 = 10I_1 + 10I_2 + 5I_2 + 10I_1 + 10I_2 - 10I_3$$

$$[10 = 20I_1 + 25I_2 - 10I_3] \div 5$$

$$4I_1 + 5I_2 - 2I_3 = 2 \quad \text{— (1)}$$

loop ABDA:

$$0 = 10I_1 + 5(I_1 - I_2) - 5I_2$$

$$0 = 10I_1 + 5I_1 - 5I_3 - 5I_2$$

$$[15I_1 - 5I_2 - 5I_3 = 0] \div 5$$

$$3I_1 - I_2 - I_3 = 0 \quad \text{— (2)}$$

loop BCDB:

$$0 = 5I_3 - 10(I_1 + I_2 - I_3) - 5(I_1 - I_3)$$

$$[-15I_1 - 10I_2 + 20I_3 = 0] \div 5$$

$$-3I_1 - 2I_2 + 4I_3 = 0 \quad \text{— (3)}$$

(1) × 3 ⇒ $8I_1 + 10I_2 - 4I_3 = 4$

(3) × 1 ⇒ $-3I_1 - 2I_2 + 4I_3 = 0$

$$5I_1 + 8I_2 = 4.$$

(2) × 4 ⇒ $12I_1 - 4I_2 - 4I_3 = 0$

(3) × 1 ⇒ $-3I_1 - 2I_2 + 4I_3 = 0$

$$9I_1 - 6I_2 = 0$$

(4) × 6 ⇒ $30I_1 + 48I_2 = 24$

(5) × 8 ⇒ $72I_1 - 48I_2 = 0$

$$102I_1 = 24$$

$$I_1 = \frac{24}{102} \Rightarrow \boxed{I_1 = \frac{4}{17} \text{ A}}$$

sub I_1 in (4)

$$5\left(\frac{4}{17}\right) + 8I_2 = 4$$

$$\frac{20}{17} + 8I_2 = 4$$

$$8I_2 = 4 - \frac{20}{17} = \frac{68 - 20}{17} = \frac{48}{17}$$

27/04/2024

$$I_2 = \frac{48}{17 \times 8}$$

$$\boxed{I_2 = \frac{6}{17} \text{ A}}$$

Sub. I_1 and I_2 in (2)

$$3\left(\frac{4}{17}\right) - \frac{6}{17} - I_3 = 0$$

$$\frac{12-6}{17} - I_3 = 0.$$

$$\boxed{I_3 = \frac{6}{17} \text{ A}}$$

∴ current in each branch is :

⇒ $I_{AB} = I_1 = \frac{4}{17}$ A

⇒ $I_{BC} = I_3 = \frac{6}{17}$ A

⇒ $I_{AD} = I_2 = \frac{6}{17}$ A

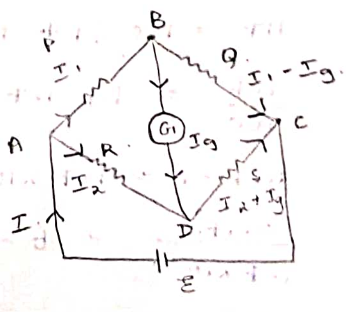
⇒ $I_{BD} = I_1 - I_3 = \frac{4-6}{17} = -\frac{2}{17}$ A [direction of current is wrong]

⇒ $I_{CD} = I_1 + I_2 - I_3 = \frac{4}{17}$ A

⇒ $I_{CA} = I_1 + I_2 = \frac{10}{17}$ A

WHEATSTONE'S BRIDGE :

* It is an arrangement of four resistances used to determine one of which resistances quickly and accurately in terms of the remaining three resistances.
* Resistance can also be calculated using V and I but the value will not be accurate as some amount of current passes through voltmeter as well.
* An ideal ammeter has zero resistance and an ideal voltmeter has infinite resistance.
* Applying Kirchhoff's law.



→ loop ABDA :

$$0 = I_1 P + I_g G - I_2 R \quad \text{—— (1)}$$

⇒ loop BCDB :

$$0 = (I_1 - I_g)Q - (I_2 + I_g)S - I_g G \quad \text{—— (2)}$$

29/04/2024

→ For null deflection in galvanometer, $I_g = 0$.

→ Sub $I_g = 0$ in ① and ②

① → $I_1 P = I_2 R$ — ③

② → $I_1 Q = I_2 S$ — ④

$$\frac{③}{④} \Rightarrow \boxed{\frac{P}{Q} = \frac{R}{S}}$$

→ This proves the condition for the balanced wheat stone bridge.

* Advantages :

→ It is a null method, thus, the internal resistance of the cell and the resistance of the galvanometer do not affect the null point.

→ As it does not involve any measurement of current and potential differences, the resistances of ammeter and voltmeter do not affect the measurements.

→ The unknown resistance can be measured to a very high degree of accuracy by increasing the ratio of resistances in the arms P and Q.

→ Note: i) Important application of wheat stone bridge is meter bridge.

ii) If we interchange the galvanometer and battery, the balance condition still remains the same.

EXERCISES [Pg: 105] :

3.1] $I = \frac{\varepsilon}{R+r}$

$I_{max} \Rightarrow R = 0$.

$I_{max} = \frac{\varepsilon}{r} = \frac{12}{0.4}$

$\boxed{I_{max} = 30\,A}$

3.3] $t_0 = 27\,°C$, $R_0 = 100\,\Omega$, $t = ?$, $R = 117\,\Omega$, $\alpha = 1.70 \times 10^{-4}\,°C^{-1}$

$R = R_0(1 + \alpha(t - t_0))$

$117 = 100(1 + 1.70 \times 10^{-4}(t - 27))$

$\frac{117}{100} = 1 + 1.7 \times 10^{-4}(t - 27)$

$1.17 - 1 = 1.7 \times 10^{-4}(t - 27)$

$\frac{0.17}{1.7 \times 10^{-4}} = t - 27$

29/04/2024

$$t = 10^{-2+4+1} + 27$$

$$t = 1000 + 27$$

$$\boxed{t = 1027\ ^\circ C}$$

3.5] $t_0 = 27.5^\circ C$, $R_0 = 2.1\,\Omega$, $t = 100^\circ C$, $R = 2.7\,\Omega$

$$\alpha = \frac{\rho - \rho_0}{\rho_0}\left(\frac{1}{t - t_0}\right)$$

$$\alpha = \frac{R - R_0}{R_0}\left(\frac{1}{t - t_0}\right) = \frac{2.7 - 2.1}{2.1} \times \left(\frac{1}{100 - 27.5}\right)$$

$$= \frac{\cancel{0.6}}{2.1} \times \frac{1}{\cancel{72.5}} = \frac{10^{-1}}{24.368}$$

0.1, 12.08

$$= 0.00394\ ^\circ C^{-1}$$

$$\boxed{\alpha = 3.94 \times 10^{-3}\ ^\circ C^{-1}}$$

3.6] $V = 230\ V$, $I_0 = 3.2\ A$, $I = 2.8\ A$, $t_0 = 27^\circ C$, $\alpha = 1.7 \times 10^{-4}\ ^\circ C^{-1}$

$$R = R_0 (1 + \alpha(t - t_0))$$

$$\frac{V}{I} = \frac{V}{I_0}(1 + \alpha(t - t_0))$$

$$\frac{I_0}{I} = 1 + \alpha(t - t_0)$$

$$\frac{3.2}{2.8} = 1 + 1.7 \times 10^{-4}(t - 27)$$

$$\frac{8}{7} - 1 = 1.7 \times 10^{-4}(t - 27)$$

$$\frac{1}{7} \cdot \frac{1}{1.7 \times 10^{-4}} = t - 27$$

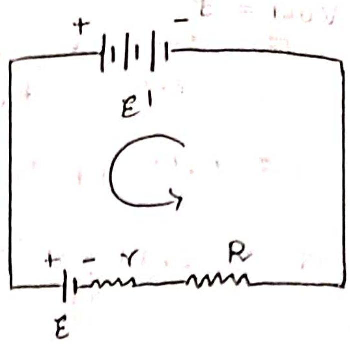
$$\frac{10000}{7 \times 1.7} + 27 = t$$

$$t = 840.34 + 27$$

$$\boxed{t = 867.34\ ^\circ C}$$

3.8] $\varepsilon' = 120\ V$, $\varepsilon = 8\ V$,

$r = 0.5\ \Omega$, $R = 15.5\ \Omega$



Applying Kirchoff's law,

$$\varepsilon' - \varepsilon = I(r + R)$$

$$120 - 8 = I(0.5 + 15.5)$$

$$112 = I(16)$$

$$I = \frac{112}{16}$$

$$\boxed{I = 7A}$$

Now, $V = \varepsilon + IR$ for charging

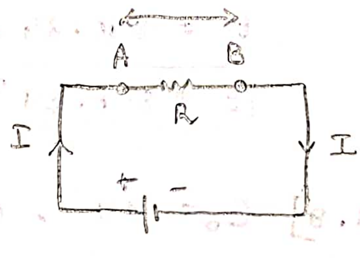
$$V = 8 + 7(0.5)$$

$$= 8 + 3.5$$

$$\boxed{V = 11.5\ V}$$

ELECTRICAL ENERGY AND POWER:

* The current flows from A to B and the charges move from A (higher potential) to B (lower potential). Thus the change in potential energy is



$$\Rightarrow \Delta U = U_f - U_i$$

$$\Rightarrow \Delta U = qV_B - qV_A = q(V_B - V_A)$$

$$\Rightarrow \Delta U = -qV \Rightarrow \Delta K = -\Delta U$$

$$\Rightarrow \Delta K = qV = ItV$$

* This potential energy must be converted to increase in kinetic energy but due to collision, the speed of electron ~~should~~ remains constant and the loss of energy is in the form of heat.

$$\Rightarrow H = VIt = I^2Rt = \frac{V^2}{R}t \quad [\text{Joule's law}]$$

* Power is the rate at which electrical energy is consumed.

$$\Rightarrow P = \frac{H}{t}$$

20/04/2024

$$\Rightarrow P = VI = I^2R = \frac{V^2}{R}$$

* In transmission of electricity, power dissipated in the cable :

$$\Rightarrow P_c = I^2 R_c$$

$$\Rightarrow P_c = \frac{P^2}{V^2} R_c$$

* Thus by increasing the voltage of the transmission, the power loss can be drastically reduced.