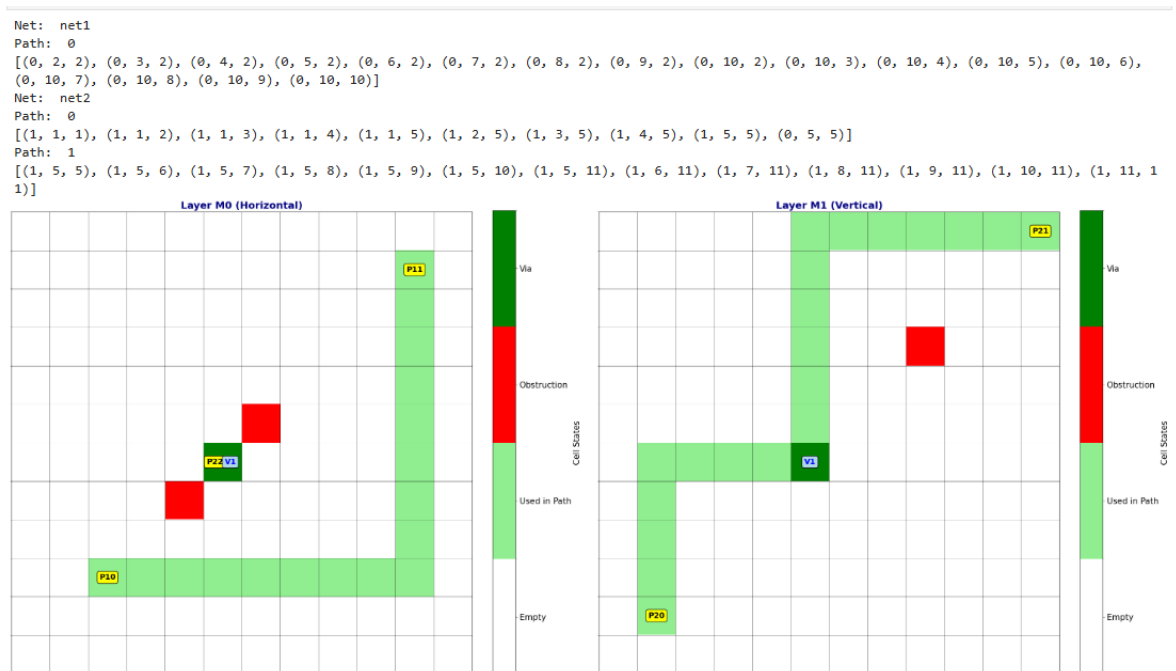**TEST CASE 1**:

12, 12, 5, 2

OBS (0, 4, 4)

OBS (0, 6, 6)

OBS (1, 8, 8)

net1 (0, 2, 2) (0, 10, 10)

net2 (1, 1, 1) (1, 11, 11) (0, 5, 5)

```
Net:  net1
Path:  0
[(0, 2, 2), (0, 3, 2), (0, 4, 2), (0, 5, 2), (0, 6, 2), (0, 7, 2), (0, 8, 2), (0, 9, 2), (0, 10, 2), (0, 10, 3), (0, 10, 4), (0, 10, 5), (0, 10, 6),
(0, 10, 7), (0, 10, 8), (0, 10, 9), (0, 10, 10)]
Net:  net2
Path:  0
[(1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 1, 4), (1, 1, 5), (1, 2, 5), (1, 3, 5), (1, 4, 5), (1, 5, 5), (0, 5, 5)]
Path:  1
[(1, 5, 5), (1, 5, 6), (1, 5, 7), (1, 5, 8), (1, 5, 9), (1, 5, 10), (1, 5, 11), (1, 6, 11), (1, 7, 11), (1, 8, 11), (1, 9, 11), (1, 10, 11), (1, 11, 1
1)]
```


NET 1

The Raw Path:
[(0, 2, 2), (0, 3, 2), (0, 4, 2), (0, 5, 2), (0, 6, 2), (0, 7, 2), (0, 8, 2), (0, 9, 2), (0, 10, 2), (0, 10, 3), (0, 10, 4), (0, 10, 5), (0, 10, 6), (0, 10, 7), (0, 10, 8), (0, 10, 9), (0, 10, 10)]

Tracing:
This path originates from (0, 2, 2) on **Layer M0** and traverses horizontally along the second row (z = 2) from column 3 to 10. After reaching (0, 10, 2), the path moves vertically up to (0, 10, 10) on the same column (x = 10) in **Layer M0**.

The path avoids obstructions (red cells) and follows a clear horizontal route in Layer M0 and terminates at the far-right end of Layer M0.

NET 2

Raw Paths:
Path 0: [(1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 1, 4), (1, 1, 5)]
Path 1: [(1, 5, 5), (1, 5, 6), (1, 5, 7), (1, 5, 8), (1, 5, 9), (1, 5, 10), (1, 5, 11), (1, 6, 11), (1, 7, 11), (1, 8, 11), (1, 9, 11), (1, 10, 11), (1, 11, 11)]

Tracing:

Path 0 begins at (1, 1, 1) and traverses vertically through rows ($z = 1$ to $z = 5$) in **Layer M1**. It is a straightforward vertical path.

Path 1 starts at (1, 5, 5) in **Layer M1** and moves horizontally across columns ($x = 5$ to $x = 11$) on row $z = 11$.

The path integrates a via to connect Layer M0 to Layer M1.

Path 1 avoids obstructions and uses the clear cells along the upper horizontal grid in Layer M1.
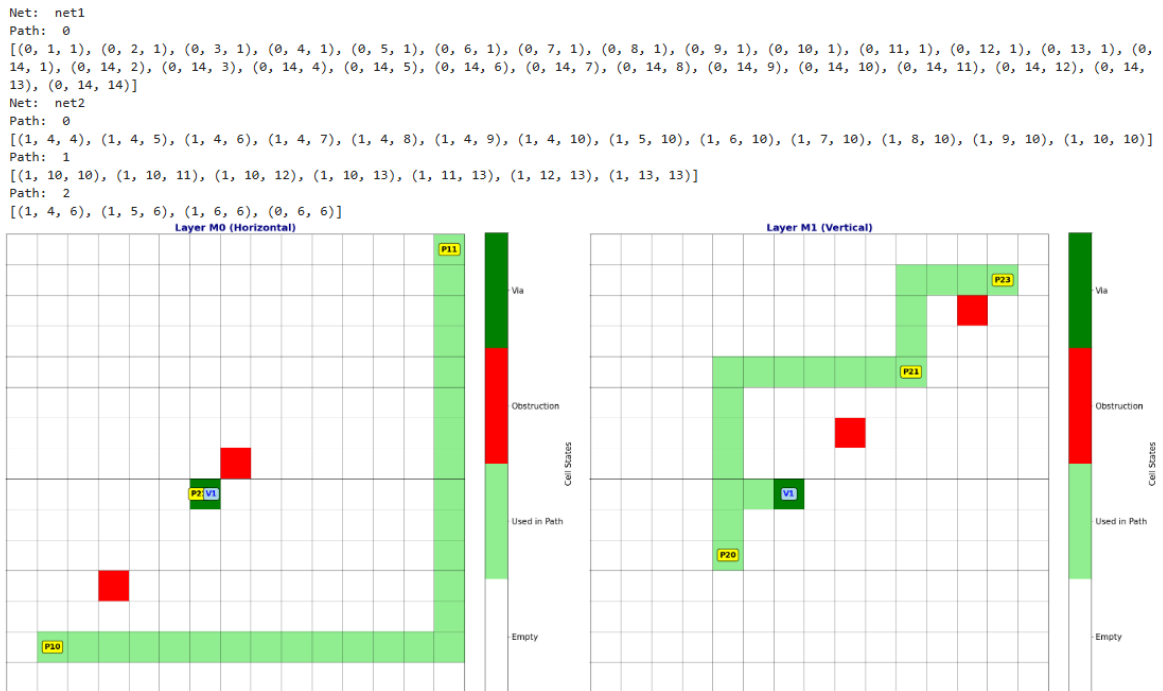
TEST CASE 2:

15, 15, 5, 3

OBS (0, 7, 7)

OBS (0, 3, 3)

OBS (1, 12, 12)

OBS (1, 8, 8)

net1 (0, 1, 1) (0, 14, 14)

net2 (1, 4, 4) (1, 10, 10) (0, 6, 6) (1, 13, 13)

```
Net:  net1
Path:  0
[(0, 1, 1), (0, 2, 1), (0, 3, 1), (0, 4, 1), (0, 5, 1), (0, 6, 1), (0, 7, 1), (0, 8, 1), (0, 9, 1), (0, 10, 1), (0, 11, 1), (0, 12, 1), (0, 13, 1), (0,
14, 1), (0, 14, 2), (0, 14, 3), (0, 14, 4), (0, 14, 5), (0, 14, 6), (0, 14, 7), (0, 14, 8), (0, 14, 9), (0, 14, 10), (0, 14, 11), (0, 14, 12), (0, 14,
13), (0, 14, 14)]
Net:  net2
Path:  0
[(1, 4, 4), (1, 4, 5), (1, 4, 6), (1, 4, 7), (1, 4, 8), (1, 4, 9), (1, 4, 10), (1, 5, 10), (1, 6, 10), (1, 7, 10), (1, 8, 10), (1, 9, 10), (1, 10, 10)]
Path:  1
[(1, 10, 10), (1, 10, 11), (1, 10, 12), (1, 10, 13), (1, 11, 13), (1, 12, 13), (1, 13, 13)]
Path:  2
[(1, 4, 6), (1, 5, 6), (1, 6, 6), (0, 6, 6)]
```

NET 1

Raw Path:
[(0, 4, 4), (0, 4, 6), (1, 4, 6), (1, 5, 6), (1, 6, 6), (0, 6, 6)]

Tracing:
The path starts from (0, 4, 4) in **Layer M0**, moves horizontally to (0, 4, 6), and then transitions vertically through a via to (1, 4, 6) in **Layer M1**. From there, it proceeds horizontally within Layer M1 (z = 6) before descending back to (0, 6, 6) in **Layer M0**.

The route efficiently uses vias to connect the two layers while avoiding obstructions.

NET 2:

Raw Paths:
Path 0: [(1, 10, 10), (1, 10, 11), (1, 10, 12), (1, 10, 13), (1, 11, 13), (1, 12, 13), (1, 13, 13)]
Path 1: [(0, 6, 6), (0, 7, 6), (0, 8, 6), (0, 9, 6), (0, 10, 6)]

Tracing:

Path 0 in Layer M1 starts at (1, 10, 10) and moves vertically (z = 10 to z = 13) along column x = 10. It then turns horizontally from (1, 11, 13) to (1, 13, 13) on row z = 13.

Path 1 in Layer M0 starts at (0, 6, 6) and traverses horizontally across columns (x = 6 to x = 10) on row z = 6.

Path 0 transitions smoothly to the vertical grid and uses a clear route without obstructions.

Path 1 complements this by operating entirely within Layer M0 on a separate segment of the grid.
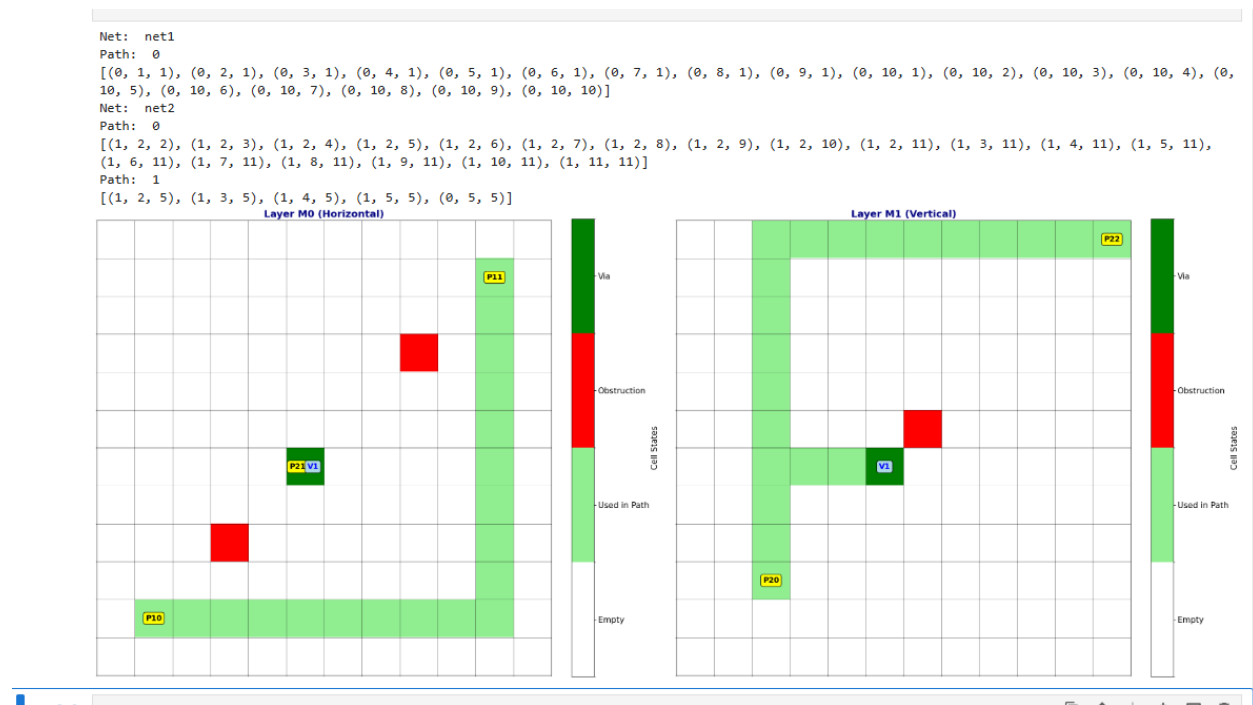
TEST CASE 3:

12, 12, 10, 5

OBS (0, 3, 3)

OBS (1, 6, 6)

OBS (0, 8, 8)

net1 (0, 1, 1) (0, 10, 10)

net2 (1, 2, 2) (0, 5, 5) (1, 11, 11)

NET 1

Raw Path:
[(0, 1, 1), (0, 2, 1), (0, 3, 1), (0, 4, 1), (0, 5, 1), (0, 6, 1), (0, 7, 1), (0, 8, 1), (0, 9, 1), (0, 10, 1), (0, 10, 2), (0, 10, 3), (0, 10, 4), (0, 10, 5), (0, 10, 6), (0, 10, 7), (0, 10, 8), (0, 10, 9), (0, 10, 10)]

Tracing:
The path originates from (0, 1, 1) in **Layer M0** and proceeds horizontally through row z = 1 until column 10. At (0, 10, 1), it turns vertically and moves through the columns (z = 2 to z = 10), ending at (0, 10, 10).

The path uses a single layer (M0) and does not require via connections.

It avoids obstructions while following a straightforward route.

NET 2

Raw Paths:

Path 0: [(1, 2, 2), (1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 2, 6), (1, 2, 7), (1, 2, 8), (1, 2, 9), (1, 2, 10), (1, 2, 11)]

Path 1: [(1, 2, 5), (1, 3, 5), (1, 4, 5), (1, 5, 5), (0, 5, 5)]

Tracing:

Path 0 begins at (1, 2, 2) in **Layer M1** and travels vertically across rows ($z = 2$ to $z = 11$) on column $x = 2$.

Path 1 starts at (1, 2, 5) in Layer M1, moves horizontally through columns ($x = 2$ to $x = 5$), and then transitions to Layer M0 at (0, 5, 5) using a via.

The grid highlights a clear distinction between vertical and horizontal paths.

Via placement ensures seamless layer transitions without obstructions.
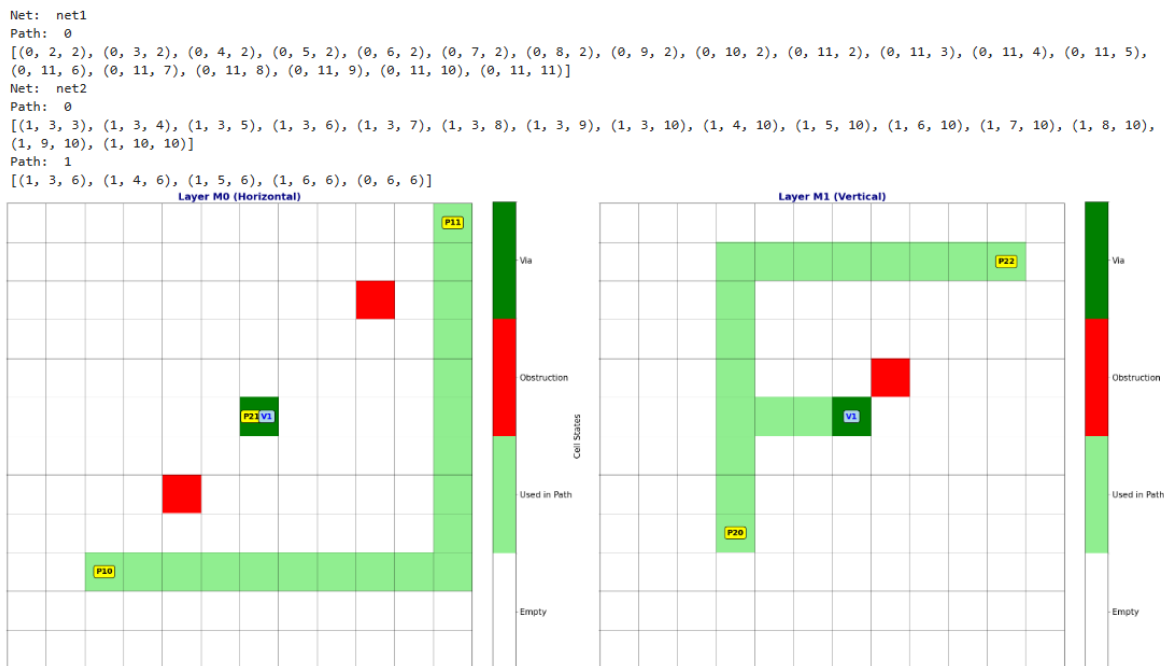
TEST CASE 4:

12, 12, 10, 5

OBS (0, 4, 4)

OBS (1, 7, 7)

OBS (0, 9, 9)

net1 (0, 2, 2) (0, 11, 11)

net2 (1, 3, 3) (0, 6, 6) (1, 10, 10)

```
Net:  net1
Path:  0
[(0, 2, 2), (0, 3, 2), (0, 4, 2), (0, 5, 2), (0, 6, 2), (0, 7, 2), (0, 8, 2), (0, 9, 2), (0, 10, 2), (0, 11, 2), (0, 11, 3), (0, 11, 4), (0, 11, 5),
(0, 11, 6), (0, 11, 7), (0, 11, 8), (0, 11, 9), (0, 11, 10), (0, 11, 11)]
Net:  net2
Path:  0
[(1, 3, 3), (1, 3, 4), (1, 3, 5), (1, 3, 6), (1, 3, 7), (1, 3, 8), (1, 3, 9), (1, 3, 10), (1, 4, 10), (1, 5, 10), (1, 6, 10), (1, 7, 10), (1, 8, 10),
(1, 9, 10), (1, 10, 10)]
Path:  1
[(1, 3, 6), (1, 4, 6), (1, 5, 6), (1, 6, 6), (0, 6, 6)]
```


Layer M0 (Horizontal)   Layer M1 (Vertical)

NET 1

Raw Path:
[(0, 2, 2), (0, 3, 2), (0, 4, 2), (0, 5, 2), (0, 6, 2), (0, 7, 2), (0, 8, 2), (0, 9, 2), (0, 10, 2), (0, 11, 2),
(0, 11, 3), (0, 11, 4), (0, 11, 5), (0, 11, 6), (0, 11, 7), (0, 11, 8), (0, 11, 9), (0, 11, 10), (0, 11, 11)]

Tracing:

The path begins at **(0, 2, 2)** in **Layer M0 (Horizontal).**

It moves **horizontally** along row **z = 2**, traversing columns **x = 2 to 11**.

At **(0, 11, 2)**, the path **turns vertically** and continues upward through rows **z = 3 to z = 11** on column **x = 11**, and finally concludes at **(0, 11, 11).**

The path exclusively uses **Layer M0**, requiring no via transitions.

It avoids obstructions and efficiently follows a direct route.

NET 2

Raw Paths:

Path 0: [(1, 3, 3), (1, 3, 4), (1, 3, 5), (1, 3, 6), (1, 3, 7), (1, 3, 8), (1, 3, 9), (1, 3, 10), (1, 4, 10), (1, 5, 10), (1, 6, 10), (1, 7, 10), (1, 8, 10), (1, 9, 10), (1, 10, 10)]

Path 1: [(1, 3, 6), (1, 4, 6), (1, 5, 6), (1, 6, 6), (0, 6, 6)]

Tracing:

Path 0:

Starts at **(1, 3, 3)** in **Layer M1 (Vertical)**.

Moves **vertically downward** along column **x = 3**, traversing rows **z = 3 to z = 10**.

At **(1, 3, 10)**, the path turns **horizontally rightward** along row **z = 10**, crossing columns **x = 3 to x = 10**.

Ends at **(1, 10, 10)**.

Path 1:

Begins at **(1, 3, 6)** in **Layer M1 (Vertical)**.

Moves **horizontally rightward** along row **z = 6**, crossing columns **x = 3 to x = 6**.

At **(1, 6, 6)**, the path transitions via **via (V1)** to **Layer M0**.

The path concludes at **(0, 6, 6)** on **Layer M0 (Horizontal)**.

Path 0 exclusively uses **Layer M1**, avoiding obstructions and maintaining a smooth trajectory.

Path 1 utilizes a **via (V1)** for seamless layer transition from **Layer M1** to **Layer M0**, ensuring connectivity despite spanning multiple layers.