(2)

Using the definition of correlation

$$C_{xy}[n] = \sum_{m} x[m]\, y[n+m]$$

let's take F.T. on both sides

$$G[k] = \sum_{n}\sum_{m} x[m]\, y[n+m]\, e^{-\frac{2\pi j n k}{N}}$$

let $n+m \rightarrow l$

$\therefore\ n = l - m$

$$= \sum_{l}\sum_{m} x[m]\, y[l]\, e^{-\frac{2\pi j (l-m) k}{N}}$$

$$= \sum_{l}\sum_{m} x[m]\, e^{\frac{2\pi j m k}{N}}\, y[l]\, e^{-\frac{2\pi j l k}{N}}$$

$$= \underbrace{\sum_{m} x[m]\, e^{\frac{2\pi j m k}{N}}}_{\text{conjugate}(X(k))}\ \underbrace{\sum_{l} y[l]\, e^{-\frac{2\pi j l k}{N}}}_{Y(k)}$$

$$\boxed{G[k] = Y(k)\, X(k)^{*}}$$

Now we can get C_{xy} by simply inverse Fourier transforming $G[k]$

$$\boxed{C_{xy}[n] = \sum_{k} Y(k) X(k)^{*}\, e^{\frac{2\pi j n k}{N}}}$$

5 (a)

$$S = \lim_{k \to 0} \frac{1 - \exp(-2\pi i k)}{1 - \exp\left(\frac{-2\pi i k}{N}\right)}$$

It's a 0/0 form of limit.

Applying L'Hospital's rule

$$\Rightarrow S = \lim_{k \to 0} \frac{-2\pi i}{\frac{-2\pi i}{N}}$$

or $\boxed{S = N}$

Q.5 (e)

Our window is

$$0.5 - 0.5\cos\left(\frac{2\pi x}{N}\right)$$
I ⋮ II

We know F.T. is a linear operator.

$\therefore$ F.T. of Window = F.T. (I) + F.T (II)

F.T. (I) = delta function at zero of height $\frac{N}{2}$

$$F(0.5)[k] = 0.5 \sum_n e^{\frac{-2\pi j k n}{N}}$$

↓ 0 for all k except k=0

$$F(k) = \delta[k]\frac{N}{2} \quad \text{or} \quad F[0] = \frac{N}{2} \qquad (1)$$

F.T. (II) = F.T. $\left\{0.5\times\frac{e^{\frac{2\pi x}{N}} + e^{\frac{-2\pi x}{N}}}{2}\right.$ ← cosine

or $$F(k) = 0.5\times\left(\sum_n \frac{e^{\frac{-2\pi n(k-1)}{N}}}{2} + \sum_n \frac{e^{\frac{-2\pi n(k+1)}{N}}}{2}\right)$$

↓ non zero @ k=1 ↓ non zero @ k=N-1

$$= \frac{1}{2}\times\left(\frac{N}{2}(k=1) + \frac{N}{2}(k=N-1)\right)$$

or $$F(k) = \frac{N}{4}\delta[k-1] + \frac{N}{4}\delta[k-N+1] \qquad (2)$$

From (1) & (2) → - sign because window is I - II

$$F(k) = \frac{N}{2}\delta[k] - \frac{N}{4}\delta[k-1] - \frac{N}{4}\delta[k-N+1] \qquad Q.E.D$$

Multiplication in time domain is equivalent to convolution in frequency domain.

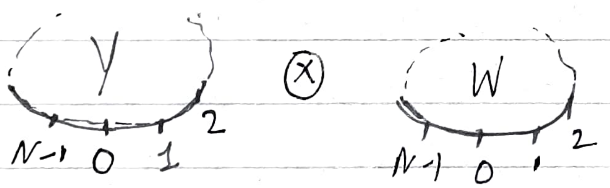
$\therefore$ We get windowed F.T. by convolving unwindowed F.T. with F.T. of window as follows

$$\underset{[0 \text{ to } N-1]}{C[k]} = \sum_{k'=0}^{N-1} \underbrace{Y[k']}_{\text{F.T. of signal}} \underbrace{W[k-k']}_{\text{F.T. of window}}$$

$\rightarrow$ Only 1 left & 1 right neighbour & the point itself all other zero.

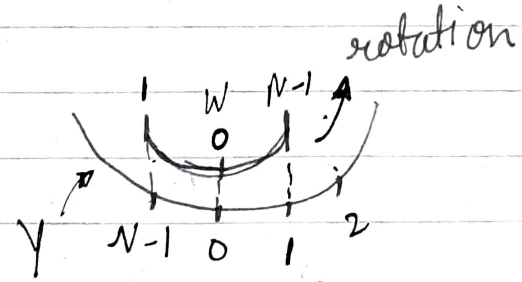
F.T. of window $= \left[\frac{N}{2}, -\frac{N}{4}, 0 \ldots 0, -\frac{N}{4}\right]$

Since convolutions in DFT regime are circular [to preserve output length] we can imagine above equation as



Since W is reversed before multiplication :-

$C[k] =$



Basically since only 3 elements are non-zero in W, we weigh each point by $\frac{N}{2}$ & its left & right neighbours by $-\frac{N}{4}$ & add.