

For a linear, first-order decay reaction:

$$\frac{dn}{dt} = -kn \quad (1)$$

where k is rate-constant.

It can be easily related to half-life $t_{1/2}$ as

$$k = \frac{\ln(2)}{t_{1/2}}$$

\therefore solution of (1) is $N = N_0 e^{-kt}$

Assuming U_{238} goes directly to lead Pb_{206}

$$\frac{dn_u}{dt} = -k_u n_u$$

$$\Rightarrow n_u = n_0 e^{-k_u t}$$

$$\Rightarrow n_{pb} = n_0 (1 - e^{-k_u t})$$

$$\frac{dn_{pb}}{dt} = k_u n_u$$

$$\frac{n_{pb}}{n_u} = \frac{n_0 (1 - e^{-k_u t})}{n_0 e^{-k_u t}} = \boxed{\frac{e^{k_u t} - 1}{1}}$$

exponential increase.

$$k_u = \frac{\ln(2)}{t_{1/2} U_{238}}$$

↓
MATCHES
EXACTLY
in plot