For rotationally sym. paraboloid

$$z - z_0 = a(x - x_0)^2 + a(y - y_0)^2$$

or

$$z = a(x-x_0)^2 + a(y-y_0)^2 + z_0$$

$$= ax^2 - 2axx_0 + x_0^2 + ay^2 - 2ayy_0 + y_0^2 + z_0$$

$$= \underbrace{a}_{m_3}\underbrace{(x^2+y^2)}_{t_3} - \underbrace{2ax_0}_{m_2}\underbrace{x}_{t_2} - \underbrace{2ay_0}_{m_1}\underbrace{y}_{t_1} + \underbrace{x_0^2+y_0^2+z_0}_{m_0}$$

$$= \sum_{i=0}^{3} m_i t_i \quad \{t_0 = 1\}$$

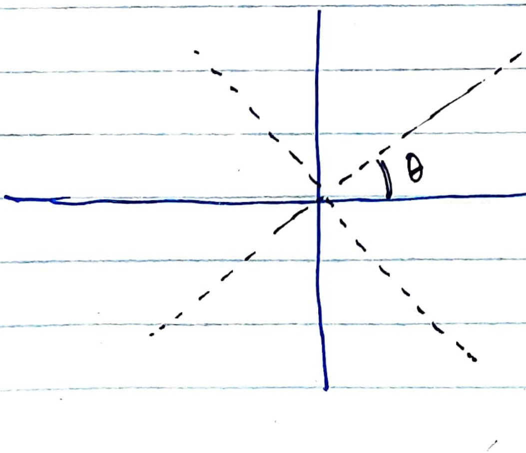
linear problem in m. We can write in matrix form as:-

$$Z = A\,m$$

A is (475, 4) matrix, m is (4, 1) matrix.

In case of non-circular paraboloid

$$z - z_0 = a(x' - x_0)^2 + b(y' - y_0)^2$$



Princ. Axes of paraboloid at an angle θ w.r.t. to our coord. system.

Let original coords be x', y'

In our axes (observed) coords will be related to x', y' as:-

$$x' = \cos\theta\, x + \sin\theta\, y$$
$$y' = -\sin\theta\, x + \cos\theta\, y$$

For a parabola

$$y = \frac{1}{4f} x^2$$

where f is the focus.

If written in the form

$$y = a x^2$$

$$f = \frac{1}{4a} \quad -(1)$$

Let $f = f(a)$

Using Taylor expansion around a

$$f(a+\delta a) = f(a) + \delta a \, f'(a) + O(\delta a^2)$$

$$\Rightarrow |f(a+\delta a) - f(a)| \approx |\delta f| \approx |f'(a)\, \delta a|$$

from (1), $f'(a) = \frac{-1}{4a^2}$

$$\Rightarrow |\delta f| = \left|\frac{\delta a}{4a^2}\right| \longrightarrow \text{obtained } 0.51 \text{ mm}$$

(output.txt)

δa is error in a, obtained from $\text{diag}\{(A^T N^{-1} A)^{-1}\}$
[first element is a]
as per my code.