

Say the true value at the point we're stepping is:

$$\rightarrow y(x+h)$$

It can be expressed in 2 ways (ignored)

$$y(x+h) = y_1 + \frac{h^5}{5} \phi(x) + O(h^6) \quad - (1)$$

[  $\phi$  is some func. of  $x$  ]

[  $y_1$  is estimate from RK4, with stepsize  $h$  ]

$$y(x+h) = y_2 + 2\left(\frac{h}{2}\right)^5 \phi(x) + O(h^6) \quad - (2)$$

[  $y_2$  is estimate of  $y(x+h)$  for 2 steps of RK4 with stepsize  $h/2$ . Each step has an error of  $O((h/2)^5)$ , hence factor 2 ]

Solving (1) & (2), we get

$$y(x+h) = \frac{16y_2 - y_1}{15}$$

No. of steps

Method 1 uses 4 ~~steps~~ <sup>func evals</sup> (usual RK4) per step

Method 2 uses 11 ~~steps~~ <sup>func evals</sup> (1 RK4 of  $h$ , 2 RK4 of  $h/2$ ) per step

So if no. of evals is to be same

4 evals

8 evals.

$$4 \times n_{\text{step1}} = 11 \times n_{\text{step2}}$$

but first step can be reused in (2)  
so  $8 + 4 - 1 = 11$