Q.1

$$A^T N^{-1} A m = A^T N^{-1} d$$

Ignoring N for now.

$$A^T A m = A^T d$$

$$A = QR$$

$$R^T \underbrace{Q^T Q}_{I} R m = R^T Q^T d$$

$$\cancel{R^T} R m = \cancel{R^T} Q^T d$$

$$m = R^{-1} Q^T d$$

If $A \rightarrow n \times p$ dimension

$Q \rightarrow n \times p$ & $R \rightarrow p \times p$

If correlated N was present

$N \longrightarrow = L I L^T$

$$\Rightarrow \chi^2 = (d - Am)^T (LL^T)^{-1} (d - Am)$$

$$= (L^{-1}d - L^{-1}Am)^T (L^{-1}d - L^{-1}Am)$$

define $\tilde{d} = L^{-1}d$

$\tilde{A} = L^{-1}A$

& we have reduced problem to identity noise mat.

Q.3 $N = Q \Lambda Q^T$ (Eigendecomposition)

$$\chi^2 = (d - Am)^T N^{-1} (d - Am)$$

$$N^{-1} = (Q \Lambda Q^T)^{-1} = (\Lambda Q^T)^{-1} Q^{-1}$$

$$= Q^{T^{-1}} \Lambda^{-1} Q^{-1}$$

$$= Q \Lambda^{-1} Q^T \quad (Q^T = Q^{-1})$$

$$\chi^2 = (d - Am)^T Q \Lambda^{-1} Q^T (d - Am)$$

$$= (Q^T d - Q^T A m)^T \Lambda^{-1} (Q^T d - Q^T A m)$$

If we didn't have any model,

$$\chi^2 = (Q^T n)^T \Lambda^{-1} Q^T n$$

$\Lambda \rightarrow$ diagonal

$\leftrightarrow$ $(\therefore \tilde{N} = \Lambda)$

$\tilde{n} = Q^T n \rightarrow \boxed{\therefore n = Q \tilde{n}}$

$n \rightarrow$ True corr. N space

$\tilde{n} \downarrow$ diag $\tilde{N}$ space