Chapter 3
Probability of an Event

Suppose we have the Sample space throwing 2 coins, $S = \{ HH , HT , TH , TT \}$, The event E is the Event of receiving at last one head. so event E can be expressed ay as .

$$E = \{ HH , HT , TH \}$$

The probability of the event E is expressed by the possibility of receiving at least one head . which occurs 3 times out 4 times (sample space points) .

The probability of 3 is then = $^3/_4 = 0.75$.

The probability is expressed by number between and including 0 and 1 . The probability of the event E has the following properties .

- $0 \le P (E) \le 1$
- $P (S) = 1$
- $P (\emptyset) = 0$

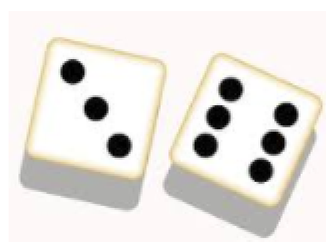
3 .1 Calculating Probability

The probability of the event E is calculated by

$$P(E) = \frac{n\,(E)}{n\,(\,s\,)} = \frac{n(E)}{N} = \frac{no.of\ sample\ points\ in\ E}{no\ .of\ sample\ Points\ in\ S}$$

Example: In tossing two dice experiment what is the probability of getting 6 in both dices

Sol : The sample space for two dice is 6 x 6 = 36 sample points only single sample point has 6 in both dice .



$$S = \{r, y | 1 \le x \le 6, 1 \le y \le 6\}$$

$$E = \{(6,6)\}$$

$$p(E) = \frac{n\,(E)}{N\,(S)} = \frac{1}{36} = 0.027$$

بمعني أنه في حالة رمي زهرتين فإن الحصول على 6 في كلا الأوجه يحتمل بنسبة 0.027 مما تعني انه بمعدل الف رمية يوجد احتمال وجود 27 مرة بالشكل (6 . 6)

Example: A jar of 13 candies (حلوى) , 6 of which are mint taste, 4 are toffee, and 3 are chocolate if a person selects two candies at a time. what is the probability of

$$\frac{M \quad T \quad C}{6 \quad 4 \quad 3}$$

- Getting 2 mints. *m* event .
- Getting 2 toffee or 2 chocolate .

Solution: Selecting any two out of has the following no . of selections .

$$\binom{13}{2} = \frac{13\,!}{(\,13 - 2\,)!\ \times 2!} = \frac{13 \times \cancel{2} * 6 \times \cancel{11}!}{\cancel{11}!\ \times \cancel{2!}} = 13\ \times 6 = 78$$

- Selecting 2 mints has the following Ways .

$$\binom{6}{2} = \frac{6!}{(6-2)! \times 2!} = \frac{\cancel{36} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}!} = 15$$

$$P(m) = \frac{n(m)}{n(S)} = \frac{15}{78} = 0.19$$

- Call getting toffee event T and getting chocolate C, Selecting 2 toffee has the following ways .

$$\binom{4}{2} = \frac{4!}{(4-2)! \times 2!} = \frac{\cancel{34} \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2}!} = 6$$

selecting two chocolate out of 3 has the following ways.

$$\binom{3}{2} = \frac{3!}{2! \times (3-2)} = \frac{\cancel{3} \times 2 \times \cancel{2!}}{\cancel{2!} \times \cancel{1}!} = 3$$

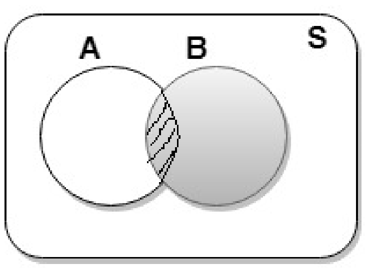
number of ways of selecting toffee or chocolate = number of ways select toffee + number of ways of selecting choc .

= 6+3 = 9

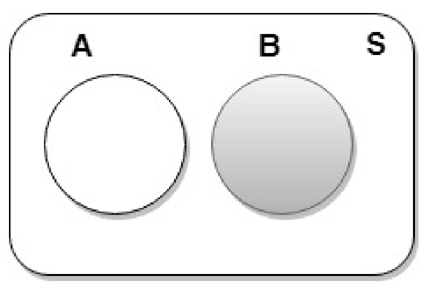
$$P(T\ U\ C) = \frac{n(T\ U\ C)}{n(S)} = \frac{9}{78}$$

3 .2 Additive Rule

An event probability can be calculated considering its sub. events. suppose there are two events A , B as show in figure .



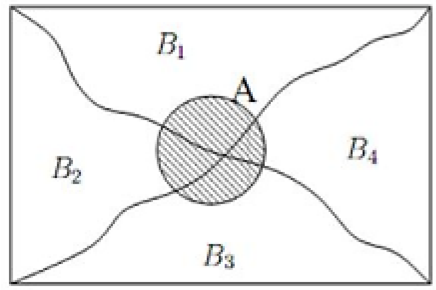
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$P(A \cup B) = P(A) + P(B)$$

A,B disjoint

If the two events intersected, they have $A \cap B$ part repeated twice . So, part of them is removed when calculating $P\ (A \cup B\ )$.

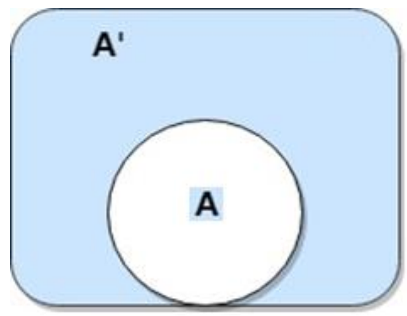


If the two events are mutually exclusive, i.e. $A \cap B = \emptyset$ then the probability of $A \cup B$ is simply the probability of the addition of the probability of sub events A , B

If the Sample space can be divided to B1, B2, B3, B4 events then.

$P\ (B1 \cup B2 \cup B3 \cup B4\ ) = P\ (B1) + P(B2) + P(B3) + P(B4) = P(S) = 1$

This is called partitioning. We also can conclude that if the event A and its complement A' are only found in the space then



$P(A) + P(A') = 1$
$P(A) = 1 - P(A')$

De Morgan theory:

$\overline{(A \cup B\ )} = A' \cap B'$

$\overline{(A \cap B\ )} = A' \cup B'$

So, $P(C \cup D\ ) = 1 - \overline{P\ (C \cup D\ )}$
$= 1 - P\ (C' \cap D'\ )$
$= 1 - P\ (C')\ P\ (D'\ )$

<u>Example:</u> The probability that Ahmed passes mathematics is $^2/_3$, the probability that he passes English is $^4/_9$. If the probability that he passes both courses is $^1/_4$. What is the probability that he will pass at least one course

<u>Solution :</u>

$P(M) = {}^2/_3\ , P\ (E) = {}^4/_9$

$p\ (M \cap E\ ) = {}^1/_4$

$P(M \cup E\ ) = P(M) + P(E) - P(M \cap E)$

$= {}^2/_3 + {}^4/_4 - {}^1/_4 = \frac{31}{36}$

Example: If the probabilities that a car service shop receives 3,4,5,6,7,8 or more cars in a work day is 0.12, 0.19, 0.28,0.24,0.10 and 0.07 respectively. what is the probability that the shop receives at least 5 cars in the next work day ?

solution **:** let E be the event that at least 5 cars are received now $P(E) = 1 - P(E')$, where E' is the event that less than 5 cars are received

$P(E') = P\ (\ getting\ 3\ cars\ ) + P\ (\ getting\ 4\ cars\ )$

$P(E') = 0.12 + 0.19 = 0.31$

So $P(E) = 1 - 0.31 = 0.69$

Example: A manufacturer is producing computer cables with length 2000 mm +- 10mm which is acceptable. Cables greater than 2010 and less than 1990 are not acceptable . and the cable is called defective, if the probability if getting acceptable cable is 0.99 .

- What is the probability of getting too large cable, i.e. cables with length more than 2010.
- What is the probability that a randomly selected cable is larger than 1990 .

Solution: Let us define the events and name them accordingly, the events are as follow

M is the event of producing acceptable cables.

L is the event that the cable is large.

T is the event that the cable is small.

- $P(M) = 0.99, P(M) + P(T) + P(L) = 1$

  $P(T) = P(L) = \frac{1-p(M)}{2} = \frac{1-0.99}{2} = 0.005$
- We want to calculate.

  $P(M) + P(L) = 1 - P(T) = 1 - 0.005 = 0.995$

3.3: Conditional Probability

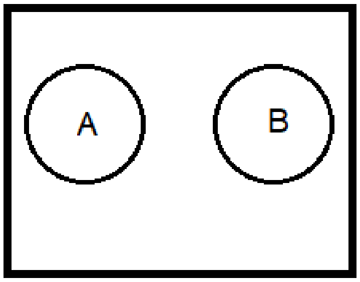
Any system is complex with multiple events. to calculate the probability of an event We shall consider the corresponding events in the same environments. There are two important relations between events.

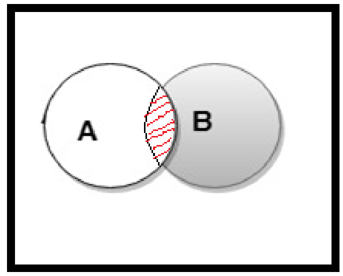
- Dependent events.
- Independent events.

Dependent events such as speed of the car and no# of accidents. Independent events such as person's height and the number of family members or the # of accidents of cars and # of Aeroplan accidents, they are not related but still they can occur at the same time. Disjoint events are the events that never occur together. Here, we consider only the dependent/independent events.

The probability that B occurs given that A occurs is called conditional probability and denoted by P (B | A), pronounced as “probability of B occurs given that A occurs.

We have two cases shown in the figure.





$P\ (\ B|A\ ) = 0$

A, B disjoint

$In\ case, P\ (\ B|A\ ) = P\ (\ B\ )$

A, B Independent

$P\ (\ B|A\ ) = \frac{P\ (A \cap\ B\ )}{P\ (A)}$

A, B dependent

<u>Note</u>**:** if it is proved that P (B | A) = P (B) then we are sure that A,B are Independent .

<u>Example:</u> In basket analysis the buyer history is shown in the table, Tid refers to the receipt id.

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

If B is event of buying a Beer and D is the event of buying diaper, find P(D), P(B ∩ D), P(B|D) and P(D|B).

<u>Solution</u>: P(D) means the probability of buying Diaper. The byer bought Diaper 4 times out of 5 transactions. So, P(D)=4/5=0.8

P(B ∩ D) means getting the probability of buying both Beer and Diaper in the same time. Bear and Diaper appeared 3 times out of total of 5 times (5 is the number of receipts). So P(B ∩ D)=3/5=0.6.

P(B|D) is the probability of buying Beer given that the buyer already bought a Diaper. The buyer bought Diaper 4 times out of which she bought Beer 3 times. So, P(B|D)=3/4=0.75.

P(D|B) is the probability of buying Diaper given that the buyer already bought a Beer. The buyer bought Beer 3 times out of which she bought Diaper 3 times. So, P(D|B)=3/3=1.0

We can easily verify that $P\ (\ B|D\ ) = \frac{P\ (D \cap B\ )}{P\ (D)} = \frac{3/5}{4/5} = \frac{3}{4} = 0.75$

<u>Hint</u>: P(B|D) will be equal to P(B) only if the Diaper appeared in all transactions. In this case we can say that B and D are independent events. In other words, buying Diaper has no effect on buying Beer. In addition, buying Milk (M) and buying Beer B are two disjoint events.

Example**:** The probability that a flight arrives in time is P(A) = 0.82. The probability that it departed in time is P (D) = 0.83. The probability that it departed in time and arrives in time is $P\ (D\ \cap A\ ) = 0.78$ Find,

- The prob. that it arrives in time given that it departed in time.
- The prob. that it departed on time given that it had arrived in time.

Solution: this an example of dependent events.

- $P\ \ (A|D) = \frac{P\ (A \cap D\ )}{P(D)} = \frac{0.78}{0.83} = 0.94$
- $P\ \ (D|A) = \frac{P\ (A \cap D\ )}{P(A)} = \frac{0.78}{0.82} = 0.95$

Which is different in both cases, $P\ (D|A)$ is higher simply because if it arrived in time there is higher prob. that it departed in time.

Example: Consider an experiment in which 2 cards are drawn in succession from an ordinary deck with replacement the events are defined as .

- The first card is an ace.
- The second card is spade .

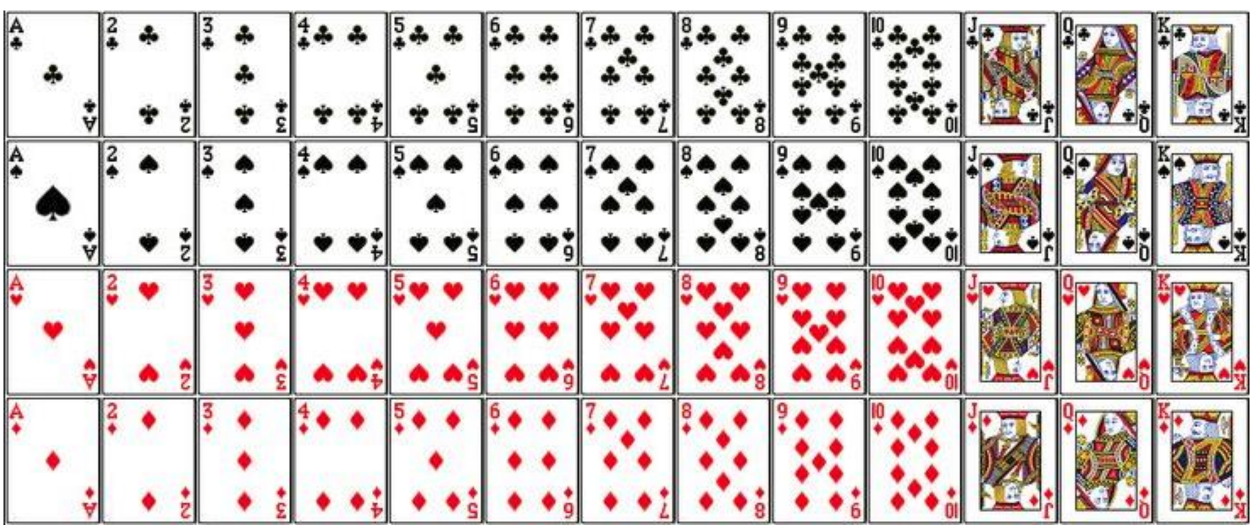
Find $P\ (B|A)\quad and \quad P\ (A|B)$

Solution**:** this is an example of independent events since replacement forces the first event to be independent from the second event.

There are 4 aces in 52 cards and 13 spades.

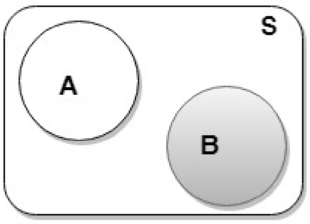
$P\ (B|A) = P\ (\ B\ ) \rightarrow independant\ , P\ (B|A) = \frac{13}{52} = \frac{1}{4}$

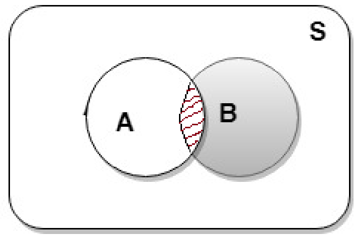
$P\ (A|B) = P\ (A) \rightarrow mdependant.\ , P\ (A|B) = \frac{4}{52} = \frac{1}{13}$



3.4 Multiplicative Rules

There are three cases for many events either they are disjoint, dependent or independent.





$P\ (A \cap B\ ) = 0$

A, B disjoint

$P\ (B|A) = P\ (B),$ (1)

For A,B independent substitute 1 In 3

$P\ (\ A \cap B\ ) = P\ (\ B\ )P\ (\ A\ )$

A, B independent

$$P\ (B|A) = \frac{P(A \cap B)}{P(A)}, \qquad (2)$$

$$P(A \cap B) = P(B|A)\ P\ (A), \qquad (3)$$

$$P\ (\ A \cap B\ ) = P\ (\ A\ )\ P\ (B|A)$$

A, B dependent

Example: Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. if 2 fuses are taken from the box in succession without replacing the first, what is the probability that both fuses are defective?

Solution: Let A be the event that the first fuse is defective and B the second fuse is defective.

$P\ (\ A\ \cap B\ )$ is the event that A ocoures, then B occurs after A occurred.

The probability of the first fuse is defective is

$P\ (A) = \frac{5}{20} = \frac{1}{4}\ ,\ P\ (B) = \frac{4}{19}$

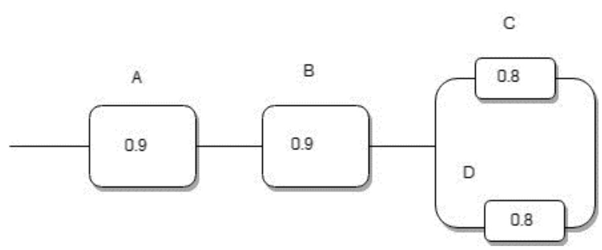
The probability of the second is defective given that the first fuse was defective is

$P(A \cap B) = P(A)P(B) \rightarrow independant$

$= \left(\frac{1}{4}\right)\left(\frac{4}{19}\right) = \frac{1}{19}$

Example: The entire system works, if A , B work together with either C o r D . give the probability of the following cases.

- The entire system works.
- The component C doesn't work given that the entire system works



A, B , C and D work independently.

Solution: A , B Serial C, D parallel

- $P\big(A \cap B \cap (C \cup D)\big) = P(A)P(B)P(C \cup D)$

$$= P(A)P(B)\big(1 - P\,(C' \cap D')\big)$$
$$= P(A)P(B)\big(1 - P\,(C')\,P\,(D')\big)$$
$$= 0.9\ \times 0.9 \times (\,1 - 0.2\ \times 0.2\,) = 0.7776$$

- $P = \frac{P\,(\,system\ works\ but\ C\ not\ work)}{P\,(\,system\ Works\,)}$

$$= \frac{P\,(\,A\ \cap B \cap C' \cap D)}{P(\,sys.works)} = \frac{0.9 \times 0.9 \times 0.2 \times 0.8}{0.7776} = 0.1667\ \#$$

Notice that**:**

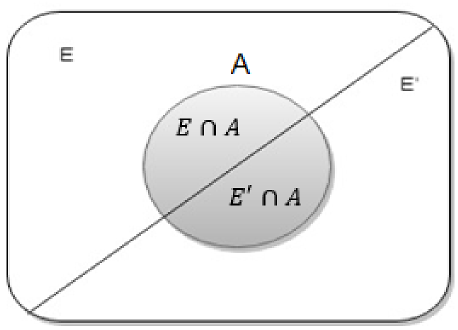
$$P\,(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P\,(A'|B) = \frac{P(A' \cap B)}{P(B)}$$

$P\,(C'|SYS) = \frac{P(C' \cap SYS\,)}{P\,(SYS)}$, Where SYS is the event that the entire system works

3 .5 Bayes Rule

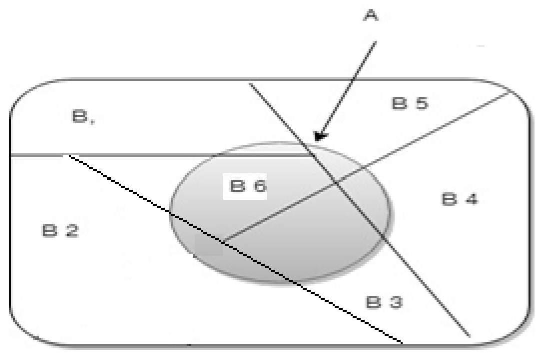
We can write A as a union of two independent events $E\ \cap A$, $E' \cap A$. Which is depicted in the following figure. Hence, A = $(E\ \cap A\,)\ \cup (\,E' \cap A\,)$



P (A) $= P[\,(\,E \cap A\,) \cup (E' \cap A\,)]$

$$= P(\,E\ \cap A\,) + P\,(\,E' \cap A\,)$$
$$= P\,(E)P\,(A|E) + P\,(\,E')P\,(A|E') \qquad , (1)$$

An extension of (1) is called the “Theorem of total probability” as shown in the figure



P (A) $= P\,[(\,B_1 \cap A) \cup (B_2\ \cap A\,) \cup \ldots \qquad (\,B_k \cap A\,)]$

$= P\ (\,B_1)\ P(A|B_1) + P(B_2\,)\ P(A|B_2\,) + \cdots \qquad P(\,B_k)\ P(A|B_k)$

$= \sum_{i=1}^{k} P(\,Bi)\ P(A|Bi)$, and that is the "Rule of elimination", , (2)

Example**:** In a certain assembly plant, machines B1, B2 and B3 make 30%, 45% and 25%, respectively of products. it is known that 2%, 3% and 2% respectively for each machine are defective. What is the probability of that a selected random product is defective.

Solution: Consider the following events:

A: The product is defective.

B1: The product is made by machine 1

B2: The product is made by machine 2

B3: The product is made by machine 3

Using the total probability theorem,

$$P\ (A) = P\ (B_1)(\ P\ (A|B_1) + P(B_2)P\ (A|B_2) + P\ (B_3)P\ (A|B_3)$$

$P\ (\ B_1)P\ (A|B_1) = 0.3\ \times 0.02 = 0.006$

$P\ (\ B_2)P\ (A|B_2) = 0.45\ \times 0.03 = 0.0135$

$P\ (\ B_3)P\ (A|B_3) = 0.25\ \times 0.02 = 0.005$

Hence, $P(A) = 0.0006 + 0.0135 + 0.005 = 0.0245$

Suppose a random product is selected, what is the probability that it is made by machine B_1? We are not asking for the probability of getting defective product as solved by rule of domination, instead we ask for the source machine that produced that defective product. Bayes' rule solves such type of problems it is known that,

$P(B_r|A) = \frac{P(\,B_r \cap A)}{P(A)}$, substituting the P(A) from equation (2)

$P(B_r|A) = \frac{P(\,B_r \cap A)}{\sum_{i=1}^{k} P(\,B_i \cap A)}$, we know that $P(B\ \cap A\,) = P(B)\ P(A|B)$

$= \frac{P\ (\,B_r\,)P\ (A|B_r)}{\sum_{i=1}^{k} P\ (\,B_i)P\ (A|B_i)}$, Bayes' Rule

Example: Referring to example 1, if a product was chosen randomly and found to defective; what is the probability that it was made by machine B3?
Answer: Using Bayes' rule

$$P\ (B_3|A) = \frac{P\ (B_3)P\ (A|B_3)}{P\ (\ B_1)P\ (A|B_1) + P\ (\ B_2\ )P\ (A|B_2) + P\ (\ B_3)P\ (A|B_3)}$$

$$= \frac{0.005}{0.006 + 0.0135 + 0.005} = \frac{0.005}{0.0245} = \frac{10}{49}\ \#$$

Exercises3

Question1: Let $S = \{E1, E2, \ldots, E6\}$; $A = \{E1, E3, E5\}$; $B = \{E1, E3\}$; $C = \{E1, E2, E3\}$; $D = \{E1, E6\}$. Suppose that all elementary events are equally likely.

(i) What does it mean that all elementary events are equally likely?
(ii) Use the complementation rule to find $P(A^c)$.
(iii) Find $P(A|B)$ and $P(B|A)$
(iv) Find $P(D)$ and $P(D|C)$
(v) Are A and B independent? Are C and D independent? *(Hint, check to see if $P(A|B)=P(A)$, and also check to see if $P(D|C) = P(D)$)*

Question 2: Suppose S={1, 2, 3, 4, 5, 6} A={1, 2, 3, 4, 5 } B={3, 4, 5, 6 }
Find Probability of $A \cap B$ assuming each sample point has equal probability of 1/6

Question 3: A coin is tossed twice. What is the probability that at least 1 head occurs?

Question 4: A spinner with five equally likely outcomes is spun. The outcomes are 1, 2, 3, 4 and 5.

b) What is the probability of getting a two?
c) What is the probability of getting an even number?

Question5: Khalid passes Math with probability ¼ and passes English with probability 2/3, if he passes both courses with probability ¾ what is the probability that he

a) pass at least one course
b) pass Math and fail English
c) fail both courses

Question6: complete the following formulas

a) P(B| A) = ………………, P(A) > 0, where A, B are events, P is the probability
b) P(S) = ……………… where S is the sample space and P is the probability
c) if P(A|B) = P(B|A) then both events A, B are said to be ………………
d) the distribution of random variable X has the following property $\sum_{all\ x} f(x) = \ldots\ldots\ldots\ldots$

Question7: Use event relationships to fill in the blanks in table below. Show your answers under the table.

P(A)	*P(B)*	*Conditions for Events for A and B*	*P(A ∩ B)*	*P(A ∪ B)*	*P(A | B)*
0.3	0.4	………..	0.12	………..	………..
0.3	0.4	………..	………..	0.7	………..
0.1	0.5	Mutually exclusive	………..	………..	………..
0.2	0.5	Independent	………..	………..	………..

Question8: If each coded item in a catalog begins with 3 distinct letters followed by 4 distinct nonzero digits, find the probability of randomly selecting one of these coded items with the first letter a vowel and the last digit odd.

Question9: It is common in many industrial areas to use a filling machine to fill boxes full of product. This occurs in the food industry as well as other areas in which the product is used in the home, for example, detergent. These machines are not perfect, and indeed they may A, fill to specification, B, underfill, and C, overfill. Generally, the practice of underfilling is that which one hopes to avoid. Let P(B) = 0.002 while P(A) = 0.980**.**

(a) Give P(C).

(b) What is the probability that the machine does not underfill?

Question10: Complete the following list of equations

- In case of two dependent events, $P(M \cup E) = P(M) + P(E) -$ …………
- The conditional probability of two dependent events $P(B|A) =$ ………………
- $P(A) + P(A') =$ ………………
- In case of 3 independent events A, B, and C the probability $P(M \cup E \cup F) =$ ……………………………………………=1
- The multiplicative rule in case of dependent events $P(A \cap B) =$ **………………………………..**

Multiple Choice Questions:

1. Two events, A and B, are said to be independent if:
   a. P(A U B) = P(A).P(B)
   b. P(A U B) = P(A) + P(B)
   c. P(A | B) = P(B)
   d. P(B | A) = P(A)

2. If P(A) = 0.8, P(B) = 0.3 and P(A|B) = 0.6, what is $P(A \cap B)$?
   a. 0.18
   b. 0.24
   c. 0.03
   d. 0.30

3) $P(A \cap B) = \emptyset$ represents:
   a) Independent events.
   b) Mutually exclusive events.
   c) Conditional events.
   d) Dependent events.

4) The following formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ represents:
   a. the conditional probability.

b. the additive rule.
c. independence.
d. the multiplication rule.

5) Suppose that a sample space S consists of four simple events: A, B, C, and D. That is $S = \{A, B, C, D\}$. If $P(A) = .4$, $P(B) = .1$, $P(C) = .2$, what is $P(D)$?

a. 0.7
b. 0.1
c. 0.3
d. 1

6) An event A will occur with probability 0.5. An event B will occur with probability 0.6. The probability that both A and B will occur is 0.1. The conditional probability of A given B is

A) Cannot be determined from the information given.
B) is 0.167.
C) is 0.200.
D) is 0.833.

7) If three fair coins are tossed, what is the probability of getting at least two heads?
[1] 2/3 [2] 1/2 [3] 3/8 [4] 1/8

8) If two events (both with probability greater than 0) are mutually exclusive, then:
A. They also must be independent.
B. They also could be independent.
C. They cannot be independent.

9) suppose that the probability of event A is 0.2 and the probability of event B is 0.4. Also, suppose that the two events are independent. Then $P(A|B)$ is:
A. $P(A)=0.2$
B. $P(A)/P(B)=0.2/0.4=1/2$
C. $P(A)\times P(B)=(0.2)(0.4)=0.08$
D. None of the above.

10) Consider the two events A, B with: $P(A) = 0.5, P(B) = 0.7$ and $P(A \cap B) = 0.3$, find the probability $P(A \cup B)$
a- 0.14 b- 0.76 c- 0.9 d- none

11) Two events A, B with $P(A) = 0.3$, $P(B) = 0.6$ and $P(B|A) = 0.6$, The conditional probability $P(A|B)$ is equal to:
a- 0.3 b- 0.6 c- 0.2 d- 0.12

12) Suppose that the probability of event A is 0.4 and the probability of event B is 0.3, suppose A, B are independent, then $P(A \cap B) =$ …..
a- 0.4 b-0.3 c- 0.7 d- 0.12

Answers:

Question1: Let $S = \{E1, E2, \ldots, E6\}$; $A = \{E1, E3, E5\}$; $B = \{E1, E2, E3\}$; $C = \{E2, E4, E6\}$; $D = \{E6\}$. Suppose that all elementary events are equally likely.

(i) What does it mean that all elementary events are equally likely?

(ii) Use the complementation rule to find $P(A^c)$.

(iii) Find $P(A|B)$ and $P(B|A)$

(iv) Find $P(D)$ and $P(D|C)$

(v) Are A and B independent? Are C and D independent?

(vi) Find $P(A \cap B)$ and $P(A \cup B)$.

Question 1

i) this means every event has equal probability

so $P(E_1) = P(E_2) = \cdots = P(E6) = \frac{1}{6}$

ii) $P(A') = 1 - P(A)$

$= 1 - P(E_1) + P(E_3) + P(E_5)$

$= 1 - \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1 - \frac{3}{6} = \frac{6}{6} - \frac{3}{6}$

$= \frac{1}{2} = 0.5$

iii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(E_1) + P(E_3)}{P(E_1) + P(E_2) + P(E_3)}$

$= \frac{2/6}{3/6} = \frac{2}{6} \times \frac{6}{3} = \frac{2}{3}$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(E_1) + P(E_3)}{P(E_1) + P(E_3) + P(E_5)} = \frac{2/6}{3/6}$

$= \frac{2}{3}$

iv) $P(D) = P(E_6) = \frac{1}{6}$

$P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{P(E_6)}{P(E_3) + P(E_4) + P(E_6)}$

$= \frac{1/6}{3/6} = \frac{6}{3} \times \frac{1}{6} = \frac{1}{3} = 0.33$

v) ~~A~~ $P(A|B) \stackrel{?}{=} P(A)$

$P(A) = P(A') = \frac{1}{2}$

A, B dependant $\rightarrow$ $P(A|B) = \frac{2}{3}$

$P(D|C) \stackrel{?}{=} P(D)$

C, D dependant $\rightarrow$ $\frac{1}{5}$ $\qquad \frac{1}{6}$

vi) ~~Fin~~ $P(A \cap B)$

$= P(E_1) + P(E_3) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

$P(A \cup B) = P(E_1) + P(E_2) + P(E_3) + P(E_5)$

$= \frac{4}{6} = \frac{2}{3}$

Question6: complete the following formulas

a) P(B| A) = ..., P(A) > 0, where A, B are events, P is the probability

b) P(S) = ... where S is the sample space and P is the probability

c) if P(A|B) = P(B|A) then both events A, B are said to be ....

d) the distribution of random variable X has the following property $\sum_{all\ x} f(x) =$ ....

Question 6

a) $P(B|A) = \frac{P(A \cap B)}{P(A)} \quad P(A) > 0$

b) $P(S) = 1$

c) If $P(A|B) = P(B|A)$, A, B is said equal

or $P(A) = P(B)$

d) $\sum_{x} f(x) = 1$ for all values of x

Question8: If each coded item in a catalog begins with 3 distinct letters followed by 4 distinct nonzero digits, find the probability of randomly selecting one of these coded items with the first letter a vowel and the last digit odd.

Question 8

letters four digits.

Vowels: a, e, i, o, u

odd: 1, 3, 5, 7, 9

$5 \times 25 \times 24 \times 8 \times 7 \times 6 \times 5$

$$P(E) = \frac{5 \times 25 \times 24 \times 8 \times 7 \times 6 \times 5}{26 \times 25 \times 24 \times 9 \times 8 \times 7 \times 6}$$

$= 0.0855$

Question10: Complete the following list of equations

- In case of two dependent events, P(M∪E)= P(M) + P(E) − ....
- The conditional probability of two dependent events *P(B|A)* = ...
- P(A) + P(A′) = ...
- In case of 3 independent events A, B, and C the probability P(M∪E∪F)= ....=1
- The multiplicative rule in case of dependent events P(A ∩ B)= ....

Question 10

- $P(M \cap E)$
- $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$
- $P(A) + P(A') = 1$
- $P(S) = 1$
- $P(A \cap B) = P(B|A)\,P(A)$

The following is an example showing that two events can be independent and at the same time they are intersecting

$S = \{1, 2, 3, 4, 5, 6\}$

$C = \{1, 2, 5\} \rightarrow \} = S$

$D = \{1, 5\}$

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{1/6}{2/6} = \frac{1}{6} \times \frac{6}{2} = \frac{1}{2}$$

$P(C) = \frac{1}{2} \Rightarrow P(C|D) = P(C)$

C, D Independant

$$P(D|C) = \frac{P(C \cap D)}{P(C)} = \frac{1/6}{1/2} = \frac{1}{6} \times \frac{2}{1} = \frac{2}{6} = \frac{1}{3}$$

$P(D) = \frac{2}{6} = \frac{1}{3}$ $\qquad P(D|C) = P(D)$

C, D Independant.