# Chapter 2
# Counting Sample Events

## 2.1 Sample Space and Events

The experiment is done on specific population to extract the samples and get the possible outcomes.

Random experiment: It is an experiment with a predictable outcome.

Example: to select randomly two students and check if they are smoker or non-smoker we have the following possibilities.

- The first student is non – smoker the second is non-smoker too.
- The first student is smoker the second is non – smoker.
- The first student is non – smoker the second is smoker .
- The first student is smoker, the second is smoker too .

If we define smoker by S and Non-smoker by N, We have the following sample points NN, NS , SN, SS.

The set of all possible outcomes is called sample space S.

$$S = \{NN\ , NS, SN, SS\}$$

Example: Get the sample space of tossing coin

Ans: We have two sample points for a coin either image / writing also called head / tail

$$S = \{H\ , T\ \}$$

Example: find the sample space of tossing a die.

Ans: we have six faces of the dice, so the sample space is

$$S = \{1\ , 2\ , 3\ , 4\ , 5\ , 6\}$$

### 2.1.1 Event

The event is a subset of S which is defined with a condition on S. For tossing a die, we have the event of getting an even number $E_1$

$$E_1 = \{\ 2\ , 4\ , 6\ \}$$

Here $E_1 \ \subseteq S$

Example**:** what is the sample space of selecting 3 items from a manufacturing line. given that each item can be classified as either Defective D or Non – Defective N.

What the event of:

- Getting at least two defective items.
- Getting at most one defective item.
- Getting 3 defectives.

Answer: 3 items with 2 possibilities each, we get the following sample Space:

$$S = \{DDD , DDN , DND , DNN , NDD , NDN , NND , NNN\}$$

We call the first event A, the second event B, the third event C.

$A = \{DDD , DDN , DND , NDD\}$

$B = \{DNN , NDN , NND , NNN\}$

$C = \{DDD \}$

$A, B, C \; are \; \subseteq S$

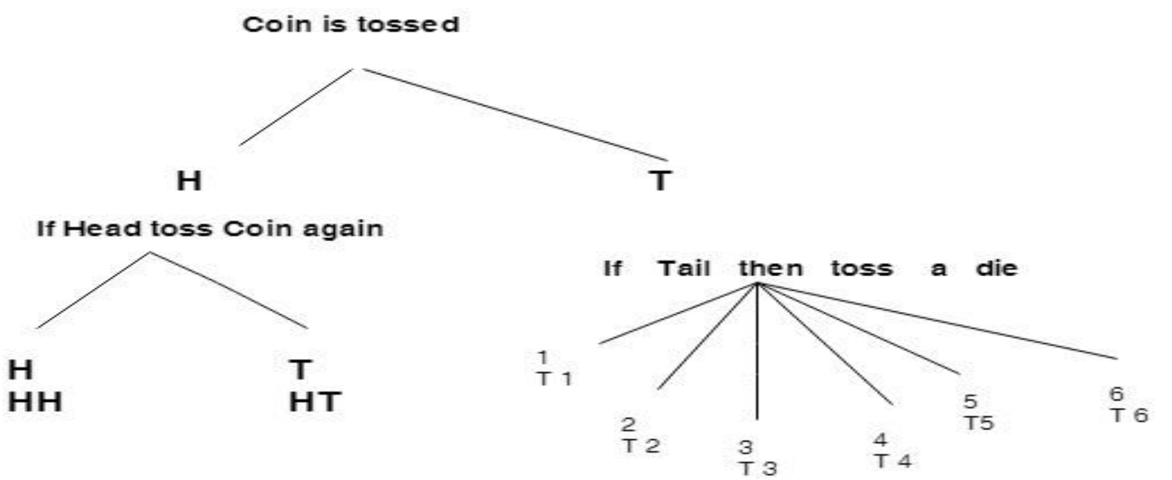
Example**:** An exponent consists of tossing a coin and then throw Adie if tail occurs in the coin otherwise the coin is tossed another time find the sample space and the event of getting at least one tail .

Ans : The sample space $S = \{HH , HT , T1 , T2 , T3 , T4 , T5 , T6\}$

The event of getting at least one tail

$$Et = \{, HT , T1 , T2 , T3 , T4 , T5 , T6\}$$

$Et = \emptyset$ If the event is not possible such as getting – 1 is case of tossing die




**2.1.2 Rule Method:** Events can be expressed with the use of rule method. It is difficult to write all the prime numbers from 1 to 1000 in a set. If the number of items is huge we have to use the rule method.

Example**:** Use rule method to write the set of all cities in the world .

$$E = \{x \quad | x \; is \; a \; city \; in \; the \; world \}$$

All non – smoker students in Taibah University.

$$E = \{B : B \; is \; non - smooking \; student \; in \; taibah \; university\}$$

## 2 . 2 – Operations on Events

### 2.2.1 Complement

The complement of an event A is the elements found in S but not found in A .

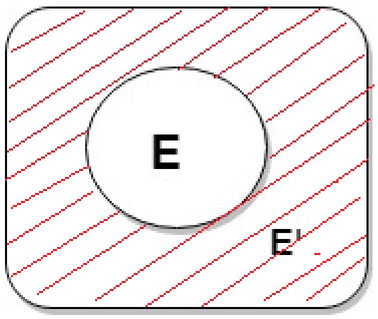
Example 1**:** $S = \{1 , 2 , 3 , 4 , 5 , \; 6\}$

E is the event getting and odd number

$$E = \{1, 3, 5\}$$

Then E complement is $E' = \{2, 4, 6\}$, Note that E and E' has no elements in common .

The following Venn – Diagram shows the complement of event E .




$S = \{1,2,3,4,5,6\}$

$E = \{1,3,5,\}$

$E' = \{2,4,6,\}$

**2.2.2 Intersection**

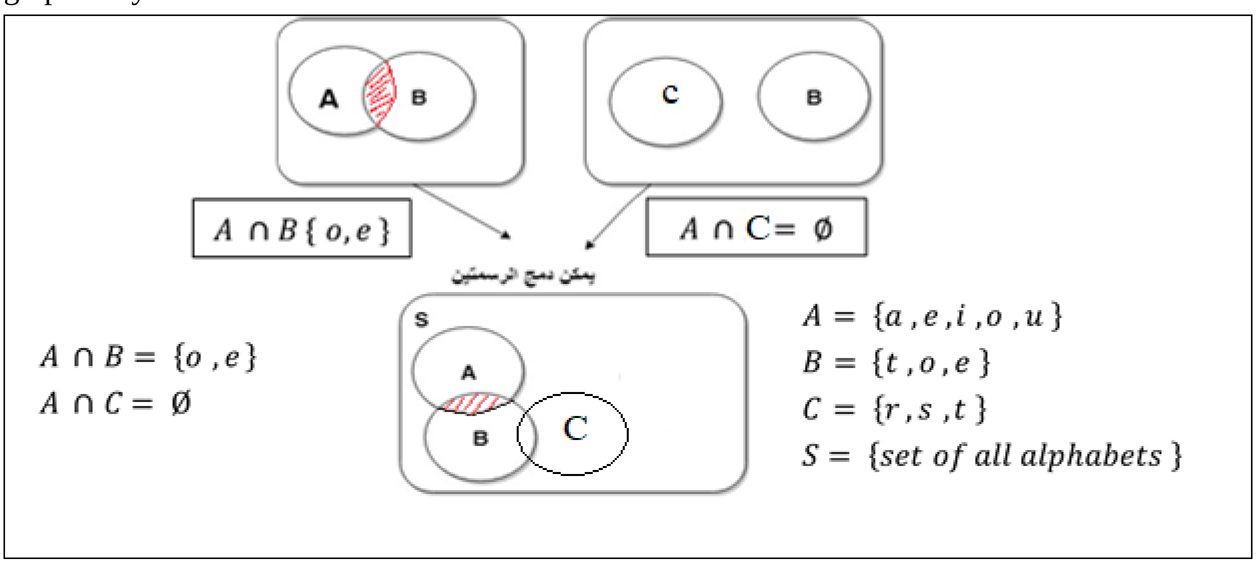
The interaction of two events A, B denoted by $A \cap B$ contains the elements which are common to both A and B.

<u>Example 2</u> **:** Suppose $A = \{a, e, i, o, u\}$, $B = \{t, o, e\}$, $C = \{r, s, t\}$

$find\ A \cap B, A \cap C$

<u>Ans</u>**:** $A \cap B = \{o, e\}$

$A \cap C = \{\emptyset\}$, Using the following Venn Diagram, we can see how intersection is represented graphically




**2.2.3 Union**

The union of two events A, B denoted as $A \cup B$ is the set of all elements in A, B without repetition.

Example 3: Suppose $A = \{a, r, s, t, u\}, R = \{L, m, n\}$ and $B = \{s, t, w, z, k\}, C = \{s, t, u\}$

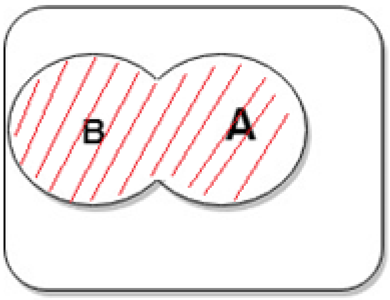
Find $A \cup B, A \cup C, A \cup R$

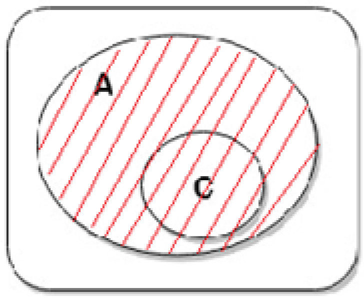
Ans:

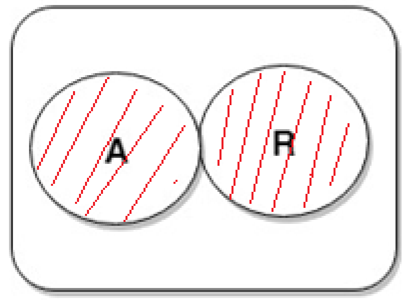
$A \cup B = \{a, r, r\, s, t, u, w, z, k\}$

$A \cup C = \{a, r, s, t, u\} = A$

$A \cup R = \{a, r, s, t, u, l, m, n\}$




| $A \cup B$<br>يوجد قاسم مشترك | $A \cup C$<br>A تقع داخل C | $A \cup R$<br>لا يوجد قاسم |
|---|---|---|

Example: if $m = \{x \mid 2 < x < g\}$

$$n = \{y \mid 5 < y < 12\}$$

$$m \cup n = \{z \mid 2 < z \mid 2\}$$

## MATLAB (A)

To create random toss of coin we use

$\gg$ **a =′ Ht′**

$\gg$ **a ((rand (1.1) < 0.5) + 1)**

The output should be randomly H or T. If we want to create sequence of heads and tails

$\gg$ **a ((rand (1.4) < 0.5) + 1)**

The output get randomly four consecutive H and T such as THTT.

To get randomly 3 items each item is defective or non – defective (N or D)

$\gg$ **a =′ ND′**

$\gg$ **a = ((rand (1.3) < 0.5) + 1)**

The output should be randomly 3 items defective or non-defective Such as NND or NNN or DND etc.

To create toss of die use the following command

$\gg$ **unidrnd (b)**

The output should be random value from 1 to 6 to simulate many tosses for example 10 tosses

$\gg$ **unidrnd ( 6, 1, 10 )**

The output should be something like 6 6 3 1 2 3 4 2 4 5

Operations on Events: For two sets a , b , the intersection is found as

$\gg$ **a = [100 10 50 50 90 ]**

$\gg$ **b = [2 100 10 ]**

$\gg$ **intersect ( a , b )**

The output should be 10, 100

For the union of two sets use

$\gg$ **union ( s , b )**

The output is 2 10 50 90 100

another example.

$\gg$ **A = {'cat','dog,'pig','bird'}**

$\gg$ **B = {'cat','fish','horse','bird'}**

$\gg$ **Interset ( A , B )**

$\gg$ **union ( A, B )**

For the complement of two sets use

$\gg$ **setdiff ( a, b )**

The output should be 50, 90

The elements found in a and not found in b However.

$\gg$ **setdiff ( b , a )**

The output should be 2

The elements found in b and not in a .

## 2.3 – Counting Techniques

Counting techniques are used to estimate the possible number of sample points in a random experiment. Some experiments produce thousands of cases which can't be listed one by one .

we talk about the following counting techniques.

- Multiplication rule .
- Permutations . تباديل
- Combinations . توافيق

### 2.3.1 – Multiplication rule

If experiment produces n1 possible cases and another experiment produces n2 possible case then we have a total of n1 x n2 possible cases.

Example: Tossing two dice produces 36 ways. Because n1 = 6 ways and n2 = 6 ways , and hence the two dice produce $6\ x\ 6 = 36$ points or ways .

Example: Car license plate has two digits and three English positions, how many plates can be produced?

Ans**:**

| | | |
|---|---|---|
| 2 | 3 | A: C: R |

P1 (Plate position #1) Can be done with 10 possible numbers.

P2 Can be done with 10 possible numbers.

P3 Can be done with 26 possible letters.

P4 and p5 also can be done with 26 possible letters, a total of $= 10\ \ x\ 10\ x\ 26x\ 26x26$ license plates.

Example: An ATM machine is hacked by using a card with 4 digits secret number, how many trials the hacker can do to guess the secure number if he knows the following .

- The numbers have repeated digits.
- The numbers has no repeated digits.
- The numbers starts with 19 and the last digits are not similar .

Ans**:**

- The number has repeated digits the four digits are guessed among the 10 digits ( 0 ,1,2,3,4,5,6,7,8,9,10 ). Guesses = 10x10x10x10 = 10000 guesses .
- No repeated digits, So if the first digit can be tried in 10 ways, the second digit on be tried in 10 – 1 or 9 ways and so on. Guesses = 10 x9x8x7 = 5040
- Starts with 19 and no the last digits are not similar but including 1 and 9. guesses = 1 X 1 X 10 X 9 = 90 guesses .

Example 5 : How many 3 digits even numbers can be formed from the digits 3,4,6, 7. if it is known that no digit repetitions .

Ans **:**

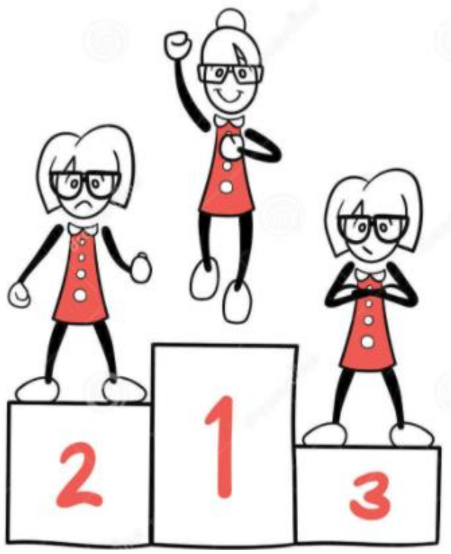
| | | |
|---|---|---|
| | | **4 or 6** |
| **3** | **2** | **2** |

A total of 3x2x2 = 4x3=12 possible even numbers can be formed from 3,4,6,4 without repetition.

### 2 .3 . 2 : permutation

The possible arrangements of all or part of a set of objects. It is applied when objects are “Ordinal”. In other words, when order matters. For example three persons Mona, Hoda and Amira in a winning race .

**التباديل:** تستخدم في حالة وجود مجموعة من العناصر، عندما يتم تغيير ترتيبهم يظهر شكل جديد. ففي حالة السباق إذا كان لدينا ثلاث متسابقين فإن تغيير الترتيب يعني تغيير الفائز. وهذا شكل مختلف تماما.




We have 6 possible assignments of the persons. So if we use the first letters of the names MHA, we have 6 possible winning arrangements. MHA, MAH, AMH, AHM, HAM, HMA. The order matters.

Example : how many possible arrangements of permuting the letters x,y,z .

Ans :

We have $(x, y, z), (x, z, y), (y, z, x), (y, x, z), (z, x, y), (z. y. x\ )$ 6 possible cases .

The same result can be obtained by using the multiplication rule or 3x2x1 = 6 possible cases. Mathematically, permutation can be calculated by using factorial

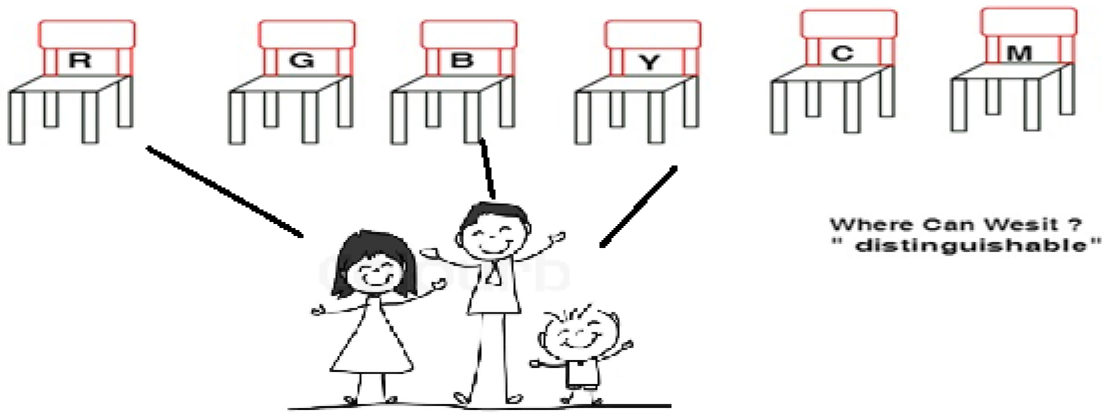
$$\boxed{\# \, permutations = n!}$$

the permutation of n distinct objects taken r at a time is ( كمثال إذا كان عدد المتسابقين 6 وعدد والميداليات تكون ثلاثة ميداليات فقط)

$$\boxed{nPr = \frac{n!}{(\,n - r\,)!}}$$

Example 2 : How many possible ways of assigning mother, father and son on 6 possible colored chairs .

Ans :




Father , mother and the son need only 3 chairs out of 6 , they can set in the following possible ways

$$6P3 = \frac{6!}{(6-3)} = \frac{6x5x4x\cancel{3!}}{\cancel{3!}}$$

$$= 6\ x\ 5\ x4 = 20\ x\ 6 = 120\ ways\ .$$

Example 3 **:** How many ways a president and vice president can be chosen out of 5 students ( V,W,X,Y,Z ) .

- If there is no restrictions.
- If V will serve only if and only if he is a president.
- If W and X will be together or not at all.
- If Y and Z will not serve together.

Ans :

- No restriction means $\frac{5!}{(5-2)!} = \frac{5\ x\ 4x\ \cancel{3!}}{\cancel{3!}} = 20\ cases$
- V want to be president, So we have the possible 4 selections VW, VX, VY, VZ or (n – 1 ) selection . in addition to any other two except V or $\frac{4!}{(4-2)!} = \frac{4x3x2!}{2!} = 12$ a total of 4 + 12 = 16 cases.

- We have two cases that W, X appears together which are WX and XW, so we have 2 cases in addition to selecting 2 out of the remaining (n – 2) students (v, y, z ) or $\frac{3!}{(3-2)!} = 3! = 6\ ways$ . A total of 2 + 6 = Cases .
- Y can serve (n – 1) times if he is president. but in only another case when Z is Vice – President he will not serve as Vice in (n – 2) times. Y will not work if Z is the president. a total of $2\ x\ (n-2) = 2\ x\ (5-2) = 6$, Z can serve the same times of Y which is 6 times and also the other cases are possible if Y and Z are not selected (N – 2) a total of, $6 + 6 + \frac{3!}{(3-2)!} = 6 + 6 + 6 = 18\ times$

In Case Some of the things to be permuted are similar. for example if axx is given and we are asked to make three letter words using them we get axx, xax , xxa which are 3 generally .

$\left(\frac{n}{n1\ ,n\ 2\ ,n\ 3 \ldots nr} = \frac{n\,!}{n1\,!\times\, n2\,!\times\, n3! \times \ldots nr\,!}\right)$, Where $n1 + n2 + n3 + \cdots \ldots \ldots \ldots + nr = n$

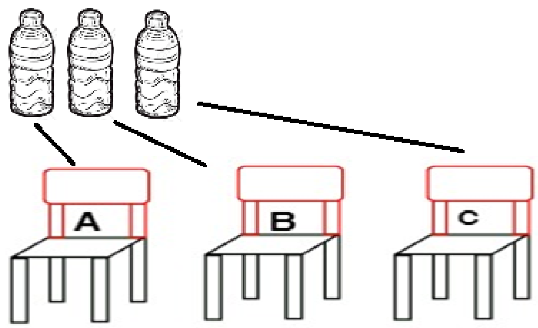
In the ‘a x x’ example we get , $\left(\frac{3}{1.2}\right) = \frac{3!}{1!x\ 2\,!} = 3$

<u>Example</u>: 10 students are to be assigned to two triple and two double rooms How many ways they can be selected .

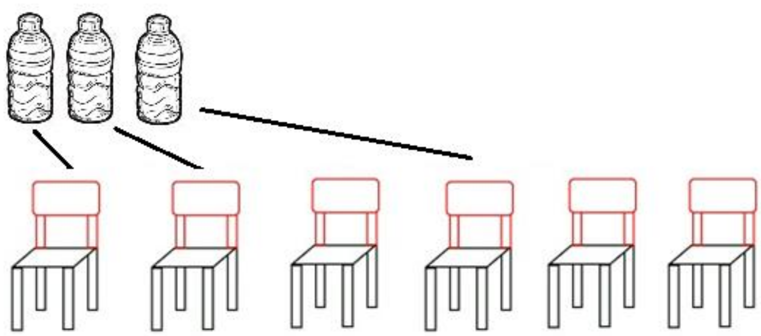
$$\left(\frac{10}{3\ ,3\ ,2\ ,2}\right) = \frac{10\ !}{3!\ \times 3!\ \times 2!\ \times 2!} = \frac{10\ \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!\ \times 3! \times 2 \times 2} = 90\ \times 56$$
$$= 25200\ ways$$

**2.3.3: Combinations**

In many problems we are interested to find how many ways of selecting r objects out of n objects without regard to order. for example if we have 3 indistinguishable water bottles to be put on the three chairs we would have only single permutation .




If we changed the position of one bottle to another chair, that will not affect the permutation it will remain single combination .



If there are six chairs and 3 bottles, the resetting permutation would be.

$\frac{6\,!}{3!(\ 6-3\ )!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3\,!} = 20\ ways$

Compared to permutation, the combination ways are less than permutations. Generally.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad r = 0\,,1\,,2\,,00000\,,n$$

<u>Example:</u> we have 10 operation rooms, 4 patients. How many ways can we assign the 4 patients to the rooms?

<u>Ans</u>: $n = 10 \qquad r = 4$

$$\binom{10}{4} = \frac{10!}{4!\,(10-4)!} = \frac{10\,!}{4!\times 6!} = 210\ Ways$$

## MATLAB (B)

Permutations can be done with.

$\gg$ **a = [ 10 5 6 ]**

$\gg$ **perms (a)**

These will result in permuting all the numbers 3 at a time, to get the number of permutation We use the factorial function.

$\gg$ **factorial ( 3 )**

To select 3 objects out of six we used the following formula

$\gg$ **n = 6; r = 3;**

$\gg$ **npr = factorial(n)/factorial (n − r)**

In combinations, we use similar formula

$\gg$ **n = 10; r = 4 ;**

$\gg$ **nCr = factorial(n)/foctorial(r) ∗ factorial (n − r )**

# Exercises2

**Question 1:** Suppose S={1, 2, 3, 4, 5, 6} A={1, 2, 3, 4, 5 } B={3, 4, 5, 6 }, C={6}

1. Find A' or the complement of A.
2. A ∪ B
3. A ∩ B, draw the Venn diagram
4. A ∩ C, draw the Venn diagram

**Question 2:** Count the possible ways for the following cases

1. A pair of dice is thrown once.
2. Three dices and two coins are thrown once.
3. Permutation of a, b, c three letters at a time.
4. Permutation of a, b, c two letters at a time.
5. Combinations of a,b,c,d,e,f three letters at a time.

**Question 3**: Suppose S = {0,1,2,3,4,5,6,7,8,9} and A ={0,2,4,6,8}, B = {1,3,5,7,9}, C = {2,3,4,5}, and D = {1,6, 7} Find

i. A ∪ C, draw the Venn diagram
ii. A ∩ B, draw the Venn diagram
iii. Find C' or the complement of C.
iv. C' ∩ D, draw the Venn diagram

**Question 4**: How many even three-digit numbers can be formed from the digits 0, 1, 2 and 4 if each digit can be used only once.

**Question 5**: How many different letter arrangements can be made from the letters of the word GOOGLE

**Question 6:** if S={0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, and A= {0, 2, 4, 6, 8}, B={ 1, 3, 5, 7, 9}, C={2, 3, 4, 5} and D= {1, 6, 7}, list the elements of the sets corresponding to the following events, draw Venn diagram for each event:

a) A U C  b) A ∩B  c) $C^c$  d) ($C^c$ ∩ D) U B  e) (S ∩ C)$^c$

**Question 7**: Suppose that the checking of errors in transmission of CPU binary numbers is done with even parity. It is needed to transfer 101101 binary string.

a) How many permutations are expected to get Negative binary string (i.e. binary string without errors).
b) How many permutations are expected to get False Negative binary string (i.e. binary string with correct parity but has errors).

**Multiple Choice Questions:**

1) The difference between a permutation and a combination is:

a) In a permutation order is important and in a combination it is not.
b) In a permutation order is not important and in a combination it is important.

c) In both the order is important.
d) Non of above

2) ${}_5P_3 =$

a) 5 b)20 c)10 d)60

3) How many words consisting of 3 letters that can be construct from a x x ?
a) 1 b) 2 c) 3 d) none

4) If we have 7 equal–priority operations and only 3 operating rooms are available, in how many ways can we choose the 3 patients to be operated on first?
a) 15 b) 35 c) 25 d) 21

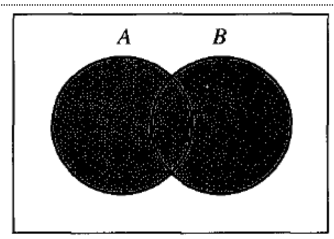
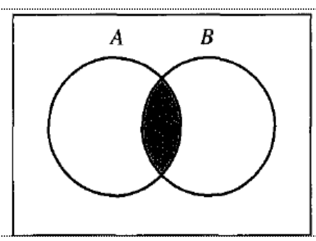
5) If an experiment consists of throwing a coin and then drawing a letter at random from the English alphabet, how many points are there in the sample space?
a) 52 b) 56 c) 156 d) 28

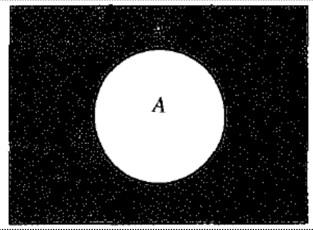
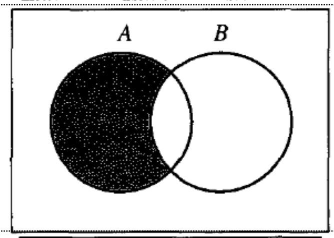
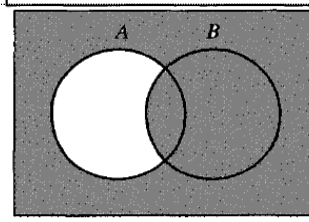
6) In 3 tosses of a coin which of following equals the event "getting two heads"?
[1] = {THH,HTH,HHT,HHH}
[2] = {THH,HTH,HHT}
[3] = {HTH,THH}
[4]={HHH,HHT,HTH,HHT,THH,THT,TTH,TTT}

7) In how many ways can a true-false test consisting of 9 questions be answered:
[1] 9 [2] 2 [3]18 [4] 2^9

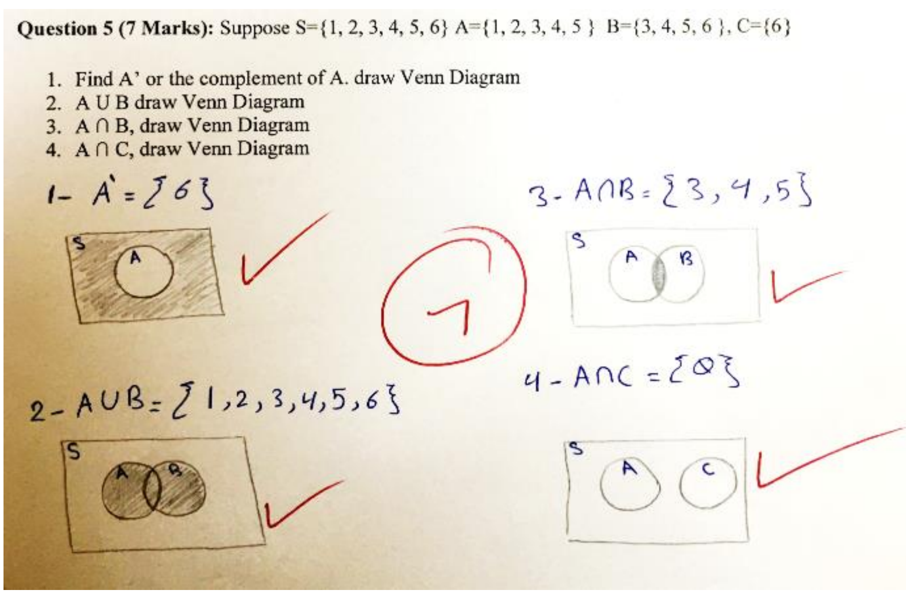
8) A president and treasure are to be chosen from student club consisting of 50 people. How many different choices of officer are possible if A will serve only if he president:
[1] 2450 [2] 2352 [3] 2401 [4] 49

**Connection Question:** Connect each set with the corresponding Venn diagram

| | *Answer* | | *Venn Diagram* |
|---|---|---|---|
| (........) | $\bar{A} \cup B$ | (a) |  |
| (........) | $A \cup B$ | (b) |  |

| | | | |
|---|---|---|---|
| (........) | $A \cap \overline{B}$ | (c) |  |
| (........) | $A \cap B$ | (d) |  |
| (........) | $\overline{A}$ | (e) |  |

**Answers to selected questions**



**Question 5 (7 Marks):** Suppose S={1, 2, 3, 4, 5, 6} A={1, 2, 3, 4, 5 } B={3, 4, 5, 6 }, C={6}

1. Find A' or the complement of A. draw Venn Diagram
2. A U B draw Venn Diagram
3. A ∩ B, draw Venn Diagram
4. A ∩ C, draw Venn Diagram

1- A' = {6}

3- A∩B = {3, 4, 5}

7

4- A∩C = {∅}

2- AUB = {1, 2, 3, 4, 5, 6}



**Question 7**: Suppose that the checking of errors in transmission of CPU binary numbers is done with even parity. It is needed to transfer 101101 binary string.

a) How many permutations are expected to get Negative binary string (i.e. binary string without errors).

b) How many permutations are expected to get False Negative binary string (i.e. binary string with correct parity but has errors).

Answer:

a) There is only one possible combination which is the exact string 101101, so the answer is 1.

b) Here, the string is of correct even parity if it has two 1's or four 1's or six 1'

for two 1's ~~we can~~ they can appear at any position in the string, selecting 2 out of 6

$$\binom{6}{2} = \frac{6!}{4! \times 2!} = \frac{{}^{3}\cancel{6} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}} = 15$$

selecting 4 out of 6

$$\binom{6}{4} = \frac{6!}{4! \times 1!} = \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!} \times 1} = 30$$

One combination out of the 30 combinations is True Negative, so we have a total of

$$15 + 30 - 1 = 15 + 29 = 44$$

another combination $\binom{6}{6} = 1$

so the total is $44 + 1 = 45$