Chapter 5
Some Discrete and Continuous Distributions

5.1 Discrete Distributions

In this section, we study the following discrete probability distributions.

- Discrete uniform distribution.
- Binomial distribution.
- Hypergeometric Distribution
- Poisson Distribution

5.1.1 Discrete Uniform Distribution

Assume random variable X with values $x1, x2, x3, x4 \ldots \ldots$ with equal probabilities, then X has the discrete uniform distribution given by .

$$f\ (x) = p(X = x) = \begin{cases} 1/_k \ ,x_1\ ,x_2 \ldots\ldots\ldots\ldots x_k \\ 0\ ,elsewhere \end{cases}$$

k is the distribution parameter, We can easily get the mean and variance of this distribution this distribution can be written in the following table.

x	x_1 x_2 x_3 … … … x_k
F(x)	$1/_k$ $1/_k$ $1/_k$ … … … . $1/_k$

$$\mu = \sum_x x\ f\ (x) = \sum_{i=1}^{k} x_i \frac{1}{k} = \frac{1}{k}\sum_{i=1}^{k} x_i$$

$$\partial^2 = \sum_{i=1}^{k}(x_i - \mu)^2\ f\ (x) = \sum_{i=1}^{k}(x_i - \mu)^2\ \frac{1}{k} = \frac{1}{k}\sum_{i=1}^{k}(x_i - \mu)^2$$

Example1 : For tossing a balanced die the sample space is $S = \{\ 1,2,3,4,5,6\}$

$$p(x_1) = p(1) = \frac{1}{6}, p(x_2) = p\ (x_3) = p(x_4) = p\ (x_5) = p\ (x_6) = \frac{1}{6}$$

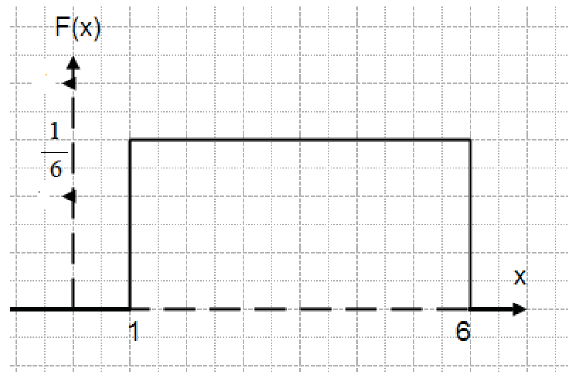
$$\mu = \frac{1}{k}\sum_{i=1}^{k} x_i = \frac{1}{6} \times (\ 1 + 2 + 3 + 4 + 5 + 6) = 3.5$$

$$\partial = \frac{1}{k}\sum_{i=1}^{k}(x_i - \mu)^2$$

$$= \frac{1}{6}\ [\ (\ 1 - 3.5)^2 + (2 - 3.5)^2 + (3 - 3.5)^2 + (4 - 3.5)^2 + (5 - 3.5)^2 + (6 - 3.5)^2] = \frac{35}{12}$$

The resulting distribution is as follow

x	1	2	3	4	5	6
F(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



5.1.2 Binomial Distribution

Bernoulli trial: is an experiment with only two possible outcomes, namely Success, & Failure

P (Success)= p, P (Failure) = 1 – p = q

As an example, tossing a coin is a Bernoulli experiment with p (H) = 0.5 and P (T) = 1-0.5 = 0.5, repeating the above trials n times is called Bernoulli Process.

Binomial Random Variable: Consider the random variable x: is the number of successes in the n trials in Bernoulli process.

The random variable x has a binomial distribution with parameter n (number of trials) and P (probability of success as

$$f(x) = b\,(x; n, p) = \begin{cases} \binom{n}{x} P^x\,(1-p)^{n-x} & ,x=0\,,1\,,2\ldots n \\ 0\,, & otherwise \end{cases}$$

In a table it can be written as

x	0	1	……..	n – 1	n	Total
F(x)	$\binom{n}{0} P^0\,(1-p)^{n-0}$ $= (1-p)^n$	$\binom{n}{1}\; P^1\,(1-p)^{n-1}$	……..	$\binom{n}{n-1} P^{n-1}\,(1-p)^0$	p^n	1

The mean & variance can be calculated as

$$\mu = np$$
$$\partial^2 = n\,p\,(1\,-p\,)$$

Example 1: Suppose that a chips box has 25% of the bags are winner bags, the child's mom lets him select three bags of the box. The child want to know the probability of getting no winner bags, one winner bag, two winner bags and three winner bags (i.e find the distribution)	

Solution: x is the number of winner bags. We need to find the probability distribution of x.

Probability of success = $= p = \frac{1}{4} = 0.25$

Probability of failure = $1 \; - p = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$

Number of trials n = 3, Then $f(x) = b\,(x; 3, \frac{1}{4}) = \begin{cases} \binom{3}{x}\left(\frac{1}{4}\right)^{x}\left(1-\frac{1}{4}\right)^{3-x} & x=0,1,2,3 \\ 0 & ,otherwise \end{cases}$

$$f(0) = b\left(0; 3, \frac{1}{4}\right) = \binom{3}{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{3} = \frac{27}{64}$$

$$f(1) = b\left(1; 3, \frac{1}{4}\right) = \binom{3}{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{2} = 3 \times \frac{1}{4} \times \frac{9}{16} = \frac{27}{64}$$

$$f(2) = b\left(2; 3, \frac{1}{4}\right) = \binom{3}{2}\left(\frac{1}{4}\right)^{2}\left(\frac{3}{4}\right)^{1} = 3 \times \frac{1}{16} \times \frac{3}{4} = \frac{9}{64}$$

$$f(3) = b\left(3; 3, \frac{1}{4}\right) = \binom{3}{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{0} = \frac{3!}{3! \times 0!} \times \frac{1}{64} \times 1 = 1 \times \frac{1}{64} \times 1 = \frac{1}{64}$$

$$\mu = np = 3 \times \frac{1}{4} = \frac{3}{4}$$

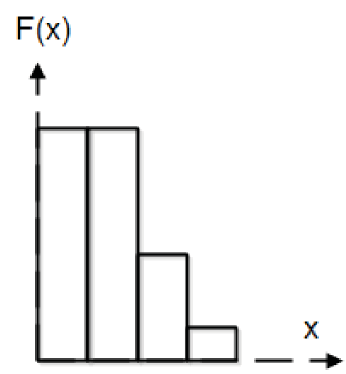
$$\delta^2 = np\,(1-p) = 3 \times \frac{1}{4} \times \frac{3}{4} = \frac{9}{16}$$

<u>Example 2</u>**:** For the previous example find,

- Probability of getting at least two winner bags.
- Probability of getting at most two winner bags.

Solution**:** The previous example distributions is .

x	**0**	**1**	**2**	**3**
F(x)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$



- $p(x \geq 2) = P\,(x = 2) + P\,(x = 3) = \frac{9}{64} + \frac{1}{64} = \frac{10}{64}$
- $p(x \leq 2) = P\,(x = 0) + P\,(x = 1) + P(x = 2) = \frac{27}{64} + \frac{27}{64} + \frac{9}{64} = \frac{63}{64}$

5.1.3 Hypergeometric Distribution

The hypergeometric distribution is used to calculate probabilities when sampling is done without replacement. For example, suppose selecting a card from deck of 52 cards, then selecting a second card without replacing the first and so on. suppose there is a population with 2 types of elements success & failure.

N: is the population size

K: number of elements of the (success) type in the population.

N – K: number of elements of the failure type in the population. A Sample of size n elements is selected at random from the population, X is the probability of getting x elements in the sample of size n

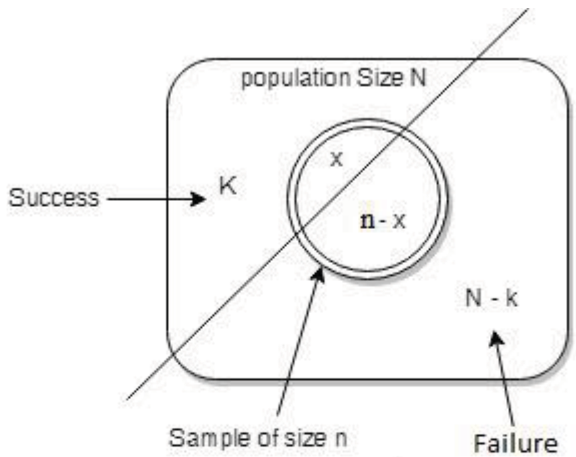
the probability distribution of X is hypergeometric and is obtained by;

$$f(x) = h\,(x, N, n\,, k\,) = \begin{cases} \dfrac{\binom{k}{x} \times \binom{N-k}{n-x}}{\binom{N}{n}}, & x=0,1,2,\ldots\ldots n \\ 0 & otherwise \end{cases}$$

The hypergeometric distribution's mean and variance is respectively calculated by;

$$\mu = \frac{nk}{N}$$

$$\partial^2 = \sqrt{\frac{nk\,(\,N-k\,)(\,N-n)}{N^2\,(\,N-1)}}$$



Example 1: Suppose you sampled 3 cards of a deck of 52 cards without replacement. what is the probability that exactly two of the drawn cards will be aces (4 of 52 are aces in the deck).

Solution: In this example, K = 4 because there are four aces in the deck, x = 2 because the problem asks about the probabilities of getting two aces , N = 52 is the entire cards . n=3 because 3 Cards are Sampled.

$$f(x) = \frac{\binom{4}{2}\binom{52-4}{3-2}}{\binom{52}{3}} = \frac{\frac{41}{2!\,2!} \cdot \frac{48!}{47!\,1\,!}}{\frac{52!}{49!\,3!}} = 0.013$$

Use MATLAB to find the remaining values of x, https://octave-online.net/. The following function is used :

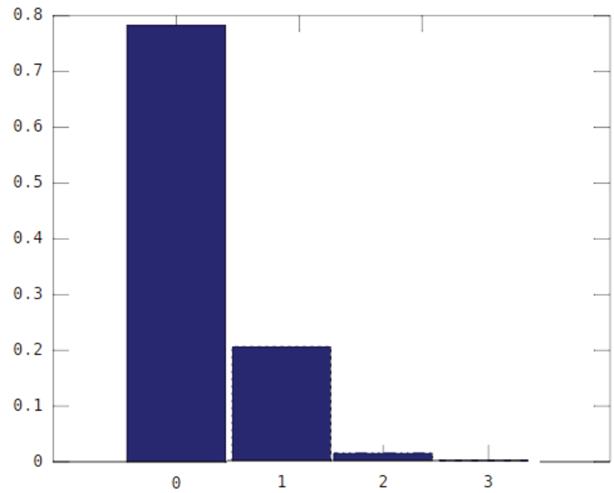
```
p = hygepdf(x values,N,K,n)
```

For the above example the following is used

```
>> x=0:3;
>> p = hygepdf(x,52,4,3)
>> bar(x,p)
```

The resulting distribution is shown in the figure

x	0	1	2	3
F(x)	0.782	0.204	0.013	0.001



<u>Example 2</u>: A chips box contains 40 chips bags, the child selects 5 bags randomly (without replacement) what is the probability of getting exactly one winning bag if the child knows that the entire box contains only 3 winning bags.

<u>Solution</u> : let x the number of winning bags in the sample .

$$N = 40,\ \ K = 3, \quad n = 5\,,\ \ x = 1$$

$$f(x) = P\ (X = x\ ) = \begin{cases} \frac{\binom{3}{1} \times \binom{37}{5-1}}{\binom{40}{5}} \\ 0 \quad otherwie \end{cases}$$

$$f(x) = \frac{\frac{3!}{1! \times 2!} \quad \frac{37!}{4!\,.\,33!}}{\frac{40!}{5! \times 35!}} = 0.3011$$

5.2 Continuous Distributions

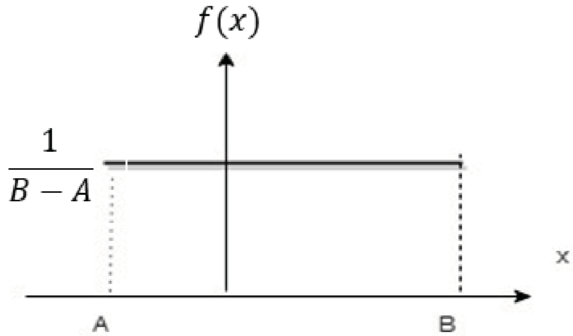
We study the following continuous probability distributions

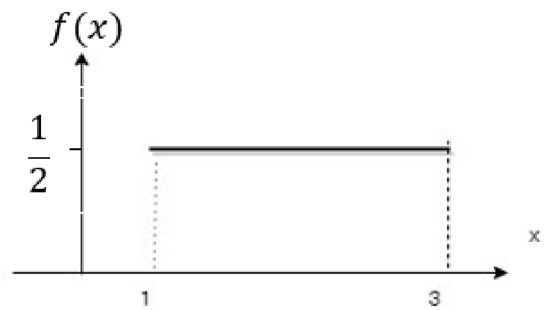
- Continuous uniform distribution.
- Normal distribution.

5.2.1 Continuous Uniform Distribution

The probability density function (PDF) of the continuous uniform random variable X on the interval [A, B] is given by :

$$f(x) = f\ (x, A\ , B\ ) = \begin{cases} \frac{1}{B-A} & A \le x \le B \\ 0\ , & elsewhere \end{cases}$$





The mean and variance of the uniform continuous distribution is calculated by

$$\mu = \frac{A+B}{2} \text{ is the distribution mean.}$$

$$\partial^2 = \frac{(B-A)^2}{12} \text{ is the distribution variance.}$$

<u>Example1</u>: suppose that for a certain company, the conference time, X, has a uniform distribution on the interval [0, 4] hours .

- What is the probability density function of X?
- What is the probability that any conference. lasts at least 2 hours.

Solution**:**

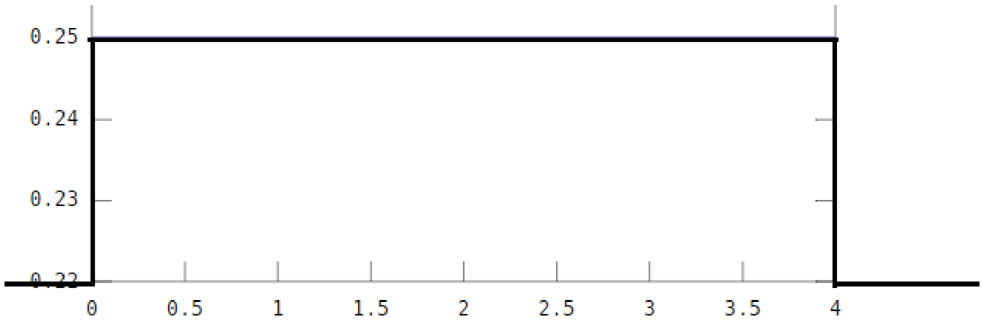
- $f(x) = \begin{cases} \frac{1}{4-0} = \frac{1}{4} & 0 \le x \le 4 \\ 0\ , & elsewhere \end{cases}$
- $P\ (\ x \ge 2\ ) = \int_2^4 f\ (x)dx = \int_2^4 \frac{1}{4}\ dx$

$$= \frac{x}{4}\ \Big|_{x=2}^{x=4} = \frac{1}{4}\ (\ 4-2\ ) = \frac{1}{4} \times 2 = \frac{2}{4} = \frac{1}{2}$$

MATLAB is used to plot the Probability Density Function (PDF), go to https://octave-online.net/

```
>> Y = unifpdf(0:4,0,4)
>> figure;
>> stairs(x,Y,'r','LineWidth',2);
```

```
>> hold on;
>> ylim([0 1]);
>> hold off;
```



5.2.2 Normal Distribution

Normal Distribution can describe data with central value μ and a fluctuation value called standard deviation. most of the data came around the mean for example the student height has μ = 174 cm and ∂ = 20 most of the height are near (above or below) 174 cm. the distribution shape is bell shaped. The probability density function (PDF) of the random variable X with normal distribution is given by.

$$f\ (x\ ) = \frac{1}{\partial\sqrt{2\,\pi}}\ e^{-\frac{1}{2}\left(\frac{x-\mu}{\partial}\right)^2}, \text{where } \pi = 3.14 \text{ and } e = 2.71$$

The mean of normal distribution, E (x) = μ .

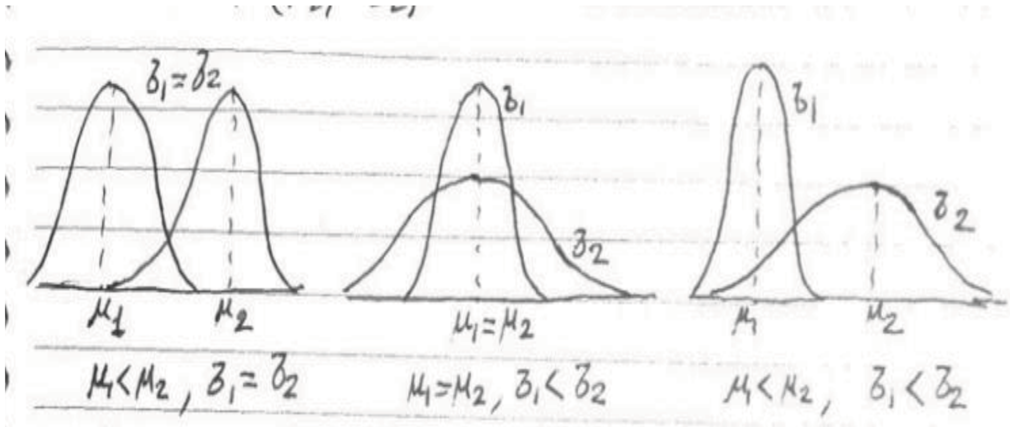
The variance of the normal distribution Var (x) = ∂^2

Two variables μ and = ∂^2 control the shape of the normal curve, suppose there are two distributions.

- $N\ (\mu_1, \partial_1)$
- $N\ (\mu_2, \partial_2)$

The properties of the normal curve are:

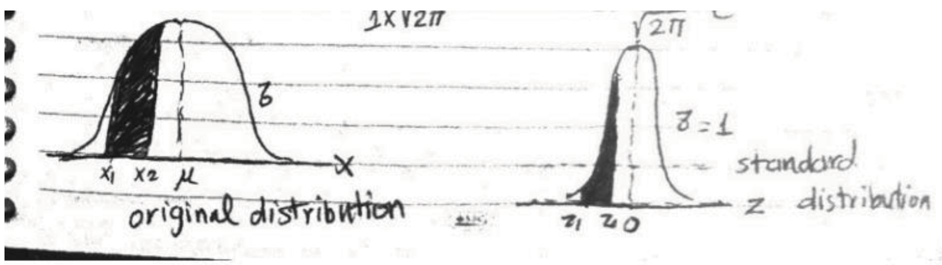
1. f(x) is symmetric about the mean μ .
2. f (x) has two points of inflection at $x = \mu \pm\ \partial$.
3. The total area under the curve of f (x) = 1 .
4. The highest point of the curve or f (x) at mean μ .



<u>Standard Normal Distribution</u>: Normal distribution with $\mu = 0 \; and \; \partial^2 = 1$ is called standard normal distribution and is denoted by N (0,1), We can get the standard Normal distribution by substituting $x = z \, , \mu = 0 \, , \partial^2 = 1$, we then get

$$f\,(x\,) = \frac{1}{\partial\sqrt{2\,\pi}}\; e^{-\frac{1}{2}\left(\frac{x-\mu}{\partial}\right)^2}$$

$$f\,(x\,) = \frac{1}{1\times\sqrt{2\,\pi}}\; e^{-\frac{1}{2}\left(\frac{z-0}{1}\right)^2} = \frac{1}{\sqrt{2\,\pi}}\; e^{-\frac{1}{2}z^2}$$



We can use the formula $z = \frac{x-\mu}{\partial}$, to convert between the original and standard distributions.

The standard Normal distribution Cumulative Probability Distribution (CDF) has one of the following forms.

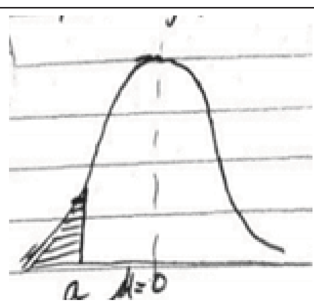
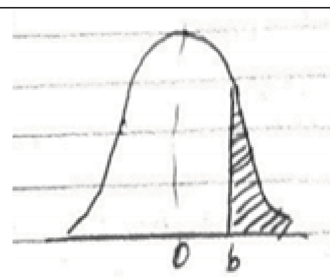
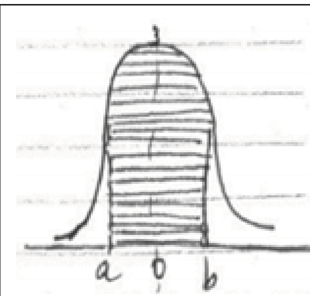
		
$P(\,z \le a\,)$ $\int_{-\infty}^{a} \frac{1}{\sqrt{2\,\pi}}\; e^{-\frac{1}{2}z^2}\, dz$	$P(z \ge b\,) = 1 - p(z \le b\,)$ $1 - \int_{-\infty}^{b} \frac{1}{\sqrt{2\,\pi}}\; e^{-\frac{1}{2}z^2}\, dz$	$P\,(a \le z \le b) = P(z \le b) - P(z \le a\,)$ $\int_{a}^{b} \frac{1}{\sqrt{2\,\pi}}\; e^{-\frac{1}{2}z^2}\, dz$

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

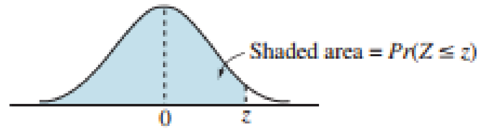


TABLE 1

Standard normal curve areas

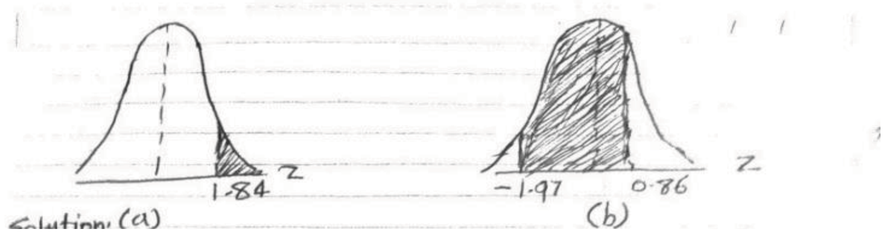
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Example 1**:** Given a standard normal distribution find the area under the curve that lies:

- To the right of $z = 1.84$.
- Between $z = -1.97$ and $z = 0.86$ as shown in the figure.

Solution**:**

- Normally, we must calculate $\int_{-\infty}^{1.84} \frac{1}{\sqrt{2\,\pi}}\; e^{-\frac{1}{2}z^2}\, dz$ but instead we can use the table A.3 to get the resulting value of the integration.
  Because the area to the right is needed, we have to get area to the left which is the CDF from table A.3 in figure (a). Then we subtract the CDF from 1 to get the right area. The area under curve to the right of Z = 1.84 is equal 1 – 0.9671 = 0.0329.
- The area in figure (b) between z = - 1.97 and z = 0.86 is equal to the area to the left of z 0.86 minus the area to the left of z = - 1.97. From table A.3 We find the desired area to be 0.8051 – 0.0244 = 0.7807.



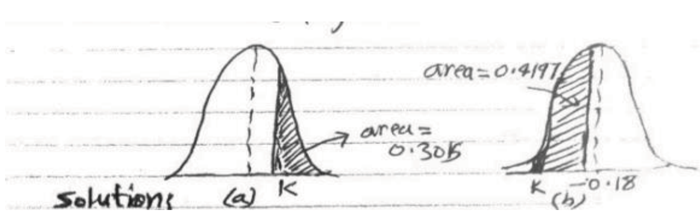
MATLAB can be used to solve the example above

```
>> left_area=normcdf(1.84)
 left_area =  0.96712
>> right_area=1-left_area
 right_area =  0.032884
```

```
>> a_left=normcdf(-1.97)
 a_left =  0.024419
>> b_left=normcdf(0.86)
 b_left =  0.80511
>> area=b_left-a_left
area =  0.78069
```

Example 2: given a standard normal distribution find the value of k such that:

- $P\,(z\; > k) = 0.3015$
- $P\;(\,k\; \leq z\; \leq\; -0.18\,) = 0.4197$, as shown in the figures.



Solution:

- In figure (a) we see that the value leaving an area of 0.3015 to the right must then leave an area of 0.6985 to the left. Then from table A.3 it follows that k= 0.52

- From table A.3 we note the total area to the left of – 0.18 is equal to 0.4286 , in figure (b) we see that the area between k and - 0.18 is 0.4197 so that the area to the left of K must be 0.4286 – 0.4197 = 0.0089 . Hence, from table A.3 We have k = - 2.37.

<u>Example 3</u>**:** Given a random variable X having a normal distribution with $\mu = 50$ and $\partial = 10$. Find the probability that X assumes a value between 45 and 62.

<u>Solution</u>**:**

The Z values corresponds to $x_1 = 45$, $x_2 = 62$ are

$$z_1 = \frac{45 - 50}{10} = -0.5 \qquad , z_2 = \frac{62 - 50}{10} = 1.2$$

Therefore

$$p\,(45 < x < 62) = p\,(-0.5 < z < 1.2) = P(z < 1.2) - P\,(z < -0.5)$$
$$= 0.8849 - 0.3085 = 0.5764$$

<u>Example 4</u>**:** Given normal distribution with $\mu = 40$ and $\partial = 6$ find the value of x that has .

- 45% of the area to the left.
- 14% of the area to the right.

<u>Solution</u> **:**

- We require a z value that leaves an area of 0.45 to the left , from table A . 3 We find $p\,(z < -0.13) = 0.45$ so that the desired z value is 0.13 Hence;
  $x = z\partial + \mu = -0.13 \times 6 + 40 = 39.22$
- We require z value that leaves an area of 1-0.14 = 0.86 to the left . form table A.3 we find that $P(z < 1.08) = 0.86$ so that the desired z value is 1.08 and $x = z\partial + \mu = 6 * 1.08 + 40 = 46.78$

Exercises 6

Discrete

1. A chips box contains 20 chips bags, the child selects 3 bags randomly (without replacement) what is the probability of getting exactly one winning bag if the child knows that the entire box contains only 2 winning bags.
2. Suppose that a chips box has 75% of the bags are winner bags. Child selects three bags from the box. Find the probability of getting at least two winner bags.

Continuous

Question1: Given a standard normal dist., find the area under the curve that lies:(*Use the attached A.3 Table*)

(a) to the right of $z = 1.84$.

(b) Between $z = 1.97$ and $z = 0.86$

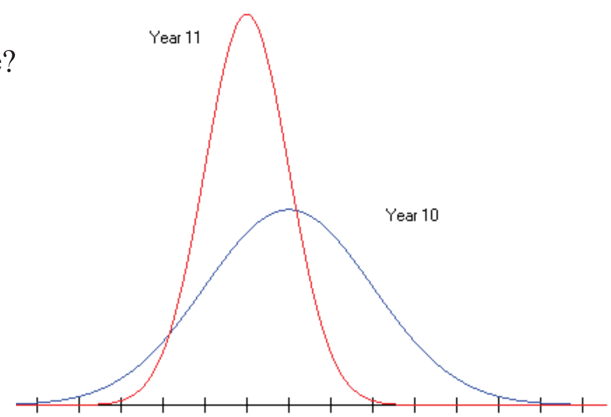
Question2: Suppose that, for a certain company, the conference time, X, has a uniform distribution on the interval [0,6] (hours).

(a) What is the probability density function of X?

(b)]What is the probability that any conference lasts at least 2 hours?

Multiple Choices Questions

1) The diagram shows two normal distribution curves, the scores achieved on an assignment by a group of Year 11 students, and the scores achieved on the same assignment by a group of Year 10 students.



Which one of the following sets of statements is true?

A) The mean score for the Year 11 students is higher than the mean score for the Year 10 students, but the variance of Year 11 more than the Year 10 marks.

B) The mean score for the Year 11 students is higher than the mean score for the Year 10 students, and the variance of Year 11 marks and Year 10 marks are equally variable.

C) The mean score for the Year 11 students is higher than the mean score for the Year 10 students, but the variance of Year 11 less than the Year 10 marks.

D) The mean score for the Year 11 students is lower than the mean score for the Year 10 students, but the variance of Year 11 less than the Year 10 marks.

2) Let x be a normal random variable with a mean of 50 and a standard deviation of 3.

A z score was calculated for x, and the z score is -1.2. What is the value of x?

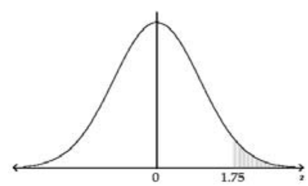
A. 53.6 B. 0.8849 C. 0.1151 D. 46.4

3) If X is a normally distributed random variable with a mean of 80 and a standard deviation of 12, then the $P(X \leq 68)$ is

A. .1587 B. .0000 C. .6587 D. .8413

4) The area of the following shaded area is:

(a) 0.0228 (b) 0.9599 (c) 0.0401 (d) 0.0668



5) If the continuous random variable X is uniformly distributed over the interval [15,20] then the mean of X is:

a. 17.5

b. 15

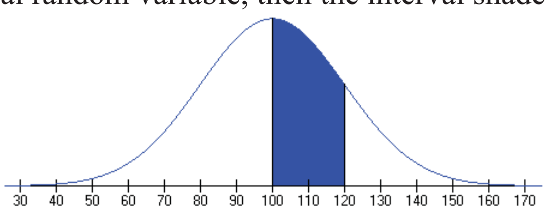
c. 25.3

d. 35

6) If X is normally distributed random variable with mean 100 and standard deviation 20, and Z is the standard norm +

+

al random variable, then the interval shaded in the diagram below can be written as:



A) $P(Z < 1)$

B) $P(Z > 100)$

C) $P(Z < 120)$

D) $P(100 < Z < 120)$