

BRANDS USING BANZHAF POWER AND SHAPLEY-
SHUBIK POWER INDEX

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M.C.Sc. (THESIS)

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**COMPARING VOTING POWER FOR HANDSET
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BY

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ABSTRACT

Voting systems produce an aggregated result of the individual preferences of the voters. One type of voting system in which the voters or the alternatives may be treated unequally is a weighted voting system. Each participant has a specified number of votes, called his or her weight. A power index gives a way to measure the share of power that each participant in a voting system. This system decides which brand is the most popularity of different kind of handset brands. In current time, the popularity and favorite handset is which company or which is, is decided. This system is also ranked the number of vote with plurality method. This system is applied to Banzhaf power and Shapley-Shubik Power to get powerful position by using C# programming language.

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CHAPTER 1

INTRODUCTION

1.1 Introduction of the System

Voting as a method for a group (electorate, meeting, part of society, country) to transfer and combine their will, preferences or motions of individuals represented by the vote. These weighted values can be aggregated in different ways fulfilling the basic attributes of the voting system. Scoring voting systems assign a special value to each position in a preferential sequence. This system is applied to popular handsets for voting with a manual voting. Voters can choose preferences brand (handset).

This system calculates the voting power for most preference handsets. This system compares the voting power using Banzhaf power and Shapley-Shubik power index.

Voting is a well-known technique used to combine decisions of peer experts. It has wide application in many domains. Voting systems work for various purposes and in various domains. This system is applied for the popular and famous handset brands. Six types of brand is used in this system for more than 70 players or voters. Weighted voting systems are used for corporation or organization in which the players of the organization are not equal.

A weighted voting system takes into account discrepancies among voters by assigning each voter a weight, which corresponds to the number of votes that he holds. It seems logical then, that assigning weights to the members of the board is based directly on the number of shares that they hold.

In general, a weighted voting system refers to a yes-no voting

system in which the voters are deciding on a single issue. Each voter, or player, $v_1, v_2, v_3, \dots, v_n$ (where n is the number of voters in the system) is assigned a weight, w_i (where w_i refers to the number of votes given to voter i). The quota is the minimum number of votes required to "pass" an issue. The notation used to describe a system with n voters and quota q is $[q; w_1, w_2, \dots, w_n]$. There are several methods for measuring the power of a voter in a weighted voting system including, most notably, the Shapley-Shubik, and Banzhaf, Johnston indices.

1.2 Objectives of the System

The objectives of this system are:

- To study the weighted voting system
- To know the Banzhaf power index and Shapley-Shubik power index methods
- To calculate voting power and compare power results using the Banzhaf power and Shapley-Shubik Power of a voter for brands (handsets)
- To know the smallest and largest power of popular brands (handsets)

1.3 Motivation of the System

The goal of a voting system, for creating this system, is that group of people is evaluating a finite set of possible alternatives; these alternatives could correspond to political candidates, possible verdicts in a trial, amounts of money to spend on national defense, nominees for an award, or any other set of options in a decision. The people involved wish to produce a single group ranking that orders the alternatives from best to worst, and that in some sense reflects the collective opinion of the group.

This system implements the popularity brands using weight voting and power of Shapley-Shubik index and the Banzhaf index for popular handset brands.

1.4 Organization of the System

The system is organized into five sections. Chapter 1 includes introduction of the system, the main objectives of about the system and motivation of the system. Chapter 2 describes theoretical background that includes history of voting system, weighted voting systems, voting system, rank-order voting, areas of application of voting power, etc. Chapter 3 discusses power index for handset brands. Chapter 4 explains the system design and the implementation. Finally, Chapter 5 presents the main conclusion, limitations and further extension items.

CHAPTER 2

BACKGROUND THEORY

2.1 Voter Satisfaction and Electoral Systems

Since the onset of the current wave of democratization, there has been a growing interest in researching the institutional factors underlying citizen support for democracy. This has also, in part, reflected a renewed scholarly interest in seeking answers to the questions of whether and how institutions ‘matter’ [5]—in this instance, with regard to the theme of democratic stability.

Electoral system design may be perceived as important by academic scholars and electoral engineers, but what tangible evidence is there of an electoral system actually making a difference to democratic stability? Whether and how electoral systems can affect levels of voter satisfaction with democracy is therefore unresolved; work is required, among other things, on how best to operationalize the dependent variable.

The studies to date have treated electoral systems solely in terms of their vote-aggregation outcomes, with attention focused on representation in the microcosmic sense in terms of who gets elected and the representation of social and ethnic groups—in short, the age-old issue of ‘proportionality’. It is universally accepted in the electoral systems literature that the two most important features of an electoral system affecting aggregate proportionality are district magnitude (the number of politicians elected in a district) and electoral formula (the counting rule determining how votes are translated into seats). But there is a third feature of electoral systems, ballot structure, and while its effects on proportionality are negligible, there is no disputing that it has an effect both on voters, in the sense of determining the nature and extent of choice

available to them on polling day, and on politicians, who are cognizant of the effect on voters and react accordingly.

2.1.1 Ballot Structure

The main distinction in electoral system ballot structures is between categorical and ordinal systems, the latter allowing voters greater choice in determining the fate of individual candidates. At one extreme are non-preferential systems, such as closed list, in which the voter makes a simple categorical choice between parties. At the other extreme are preferential systems, such as the single transferable vote (STV) system, in which the voter can rank order all the candidates (from all parties) on a ballot paper. There is a range of preferential systems, which vary in terms of the degree of choice given to voters. Other prominent members of this category include: the cumulative vote, the limited vote, open list, ~~panache~~, and the alternative vote (AV; also referred to as preferential voting or instant run-off voting).

These systems share in common the characteristic that the voters are given much greater freedom in completing the ballot paper, either in terms of making multiple marks against several candidates, or in some cases being able to rank-order the candidates. There can be little doubt that this feature of voter choice has important implications for how voters cast their ballot (not least on the degree of effort those bothering to vote are required to make in the polling station), how parties and candidates campaign, and how politicians represent their voters. It is easy to see how this might have an impact on the attitudes of voters to the political system. The connection between ballot structure characteristics of electoral systems and voter attitudes to democracy derives from at least three mechanisms, one originating directly from the voting act itself, another based more indirectly on the relationship between politicians and

their voters, and a third referring to the ideological tendency within the party political system. In the first instance, there is the argument, usually propounded by supporters of preferential systems like STV, that a principal strength is the maximization of voter choice. Even among more sober scholarly treatments, there is stress on how systems like STV give ‘voters greater choice and makes possible ballot splitting to express highly differentiated preferences’.

In a context in which voters are being given more choice in the electoral act it could be argued that this should result in a greater sense of efficacy on the part of voters, and more say in electing their representatives.

Following on from this, a second perspective draws attention to the linkage between politicians and voters.

There is general agreement that ballot structure affects the representative role of politicians, and studies based on surveys of politicians have demonstrated how electoral systems that are characterized by a candidate-orientation in politics and high degrees of preferential voting tend to produce greater attention to personal vote chasing by politicians and the maintenance of close links with their electorates. There are grounds for expecting a more positive attitude by voters towards their elected representatives in such systems (and by extension towards the wider political system) than might be the case in those electoral contexts in which politicians are devoting most attention to their internal electorates [5, 12].

2.1.2 Preferential Voting and Candidate-Centeredness

When the focus is on ballot structure, electoral systems can be differentiated in terms of the *nature* of the vote choice—party-based versus candidate-based votes—and also the *extent* of the vote choice—in

terms of degrees of ordinary, or preferential voting (Farrell 2001). Systems characterized by candidate-based voting and high degrees of ordinary—which we shall refer to hereinafter as ‘preferential’ systems—encourage a greater emphasis by candidates on cultivating personal votes, as opposed to party-centered electoral systems, where the fate of the candidate is determined largely by the support for their parties.

Three main characteristics are *Ballot*, *Vote* and *District*, in which the higher scores across these components are indicative of a candidate-centered preferential system and lower scores of a party-centered categorical system [2].

The *Ballot* component is designed to measure the degree of party versus voter control over the ballot placement of candidates, revealing the extent to which the party leadership (and/or electorate) can exercise influence over the party’s candidates. The lower the ballot control, the greater the incentive for candidates to place emphasis on their personal reputation.

Voters may not disturb the order of the candidate list (closed list PR); Voters may disturb the order of the candidate list (open list PR); Ballot access requires first surviving a preliminary round of popular voting (the French runoff system); Ballot access nearly unrestricted (STV).

The *Votes* component, as outlined by Carey and Shugart (1995), distinguishes between single-vote list and nominal systems, with systems of ‘multiple votes’ (i.e. ordinal systems) comprising an intermediate category. According to Carey and Shugart, ‘the value to legislative candidates of personal reputation’ is highest in nominal systems (1995: 42). They provide very little indication of how they might distinguish the different multi-candidate systems, but the following extract is revealing:

When multiple votes are cast, personal reputation is not as overwhelmingly important relative to party reputation as when all candidates are competing simultaneously for the same indivisible support of each voter. When multiple votes are cast simultaneously, the candidates from one party can run as a bloc, rather than running against each other [3].

In the case of the *District* component our coding is identical to Shugart's. This component takes account of an earlier argument developed about how the effect of district magnitude (M) can vary depending on the nature of the ballot structure. In systems where voters cast party-based votes, they find that the personal reputation of the candidate declines in significance as M rises, whereas in systems characterized by candidate-based (nominal) votes, as M rises and candidates face more inter-party and intra-party competitors, the incentives for personal vote chasing increases. This component is coded as follows:

- 1 $M > 1$, with *Vote*=1;
- 2 $M = 1$;
- 3 $M > 1$, with *Vote*>1 and *Ballot*>1.

2.1.3 Preferential Systems and Voter Satisfaction with Democracy

With the proliferation of democracy across the world following the collapse of communism, much scholarly attention has been devoted to studying citizens' satisfaction with democracy. Starting with [11], scholars have made a distinction between satisfaction with the day-to-day operation of a government, and the diffuse support for values, norms and belief which underpin the political system. More recently, [13] has identified five objects of support—support for the political community,

the principles, performance and institutions of the regime, and political actors—and shown that citizens clearly differentiate between each of them. Others have found similar results though critics have questioned both the concept and meaning of the measures used.

2.2 Introduction of Weighted Voting Systems

Voting is often used to decide yes or no questions. Legislatures vote on bills, stockholders vote on resolutions presented by the board of directors of a corporation, and juries vote to acquit or convict a defendant. The theorem of Kenneth May quoted says that majority rule is the only system with the following properties:

1. All voters are treated equally.
2. Both alternatives are treated equally.
3. If you vote “no,” and “yes” wins, then “yes” would still win if you switched your vote to “yes,” provided that no other voters switched their votes.
4. A tie cannot occur unless there is an even number of voters.

There are many situations in which one or more of these properties are not valid. Some systems where the voters appear to be unequal in power actually have all of the properties required by May’s theorem. Any student of politics will attest that not all legislators are equally powerful. The theorem of Kenneth shall find two measures of voting power that apply when voters are not treated equally or alternatives are not treated equally, or both: the Shapley–Shubik power index, and the Banzhaf power index. The Banzhaf power index is an accurate measure of power when there is no spectrum of opinion. For example, if each voter decides which way to vote by tossing a coin, the Banzhaf power index will indicate each voter’s share of power. The Shapley–Shubik index is

appropriate in a process where measures are crafted so as to attract enough votes to win [6,14].

One type of voting system in which the voters or the alternatives may be treated unequally is a **weighted voting system**. Each participant has a specified number of votes, called his or her **weight**. “If my voting weight is more than yours, then I might have more power than you to influence the outcome, and certainly I won’t have less”. In any voting system, there must be a criterion for deciding whether “yes” or “no” has won. In a weighted voting system, this is done by specifying a number called the **quota**. If the sum of the weights of all the voters who favor a motion is equal to the quota, or exceeds it, then “yes” wins. Otherwise, “no” wins. The quota must be greater than half of the total weight of all the voters, to avoid situations where contradictory motions can pass, and it cannot be greater than the total weight, or no motion would ever pass.

2.3 Notation for Weighted Voting Systems

To describe a weighted voting system, you must specify the voting weights w_1, w_2, \dots, w_n of the participants, and the quota, q . The following notation is a shorthand way of making these specifications: $[q: w_1, w_2, \dots, w_n]$. The weighted voting system $[51: 40, 60]$ describes a voting system in which there are two voters, with voting weights 40 and 60, and the quota is 51 [6, 14].

2.3.1 A Dictator

Suppose there is one voter, D , who has all of the power. A motion will pass if and only if D is in favor, and it doesn’t matter how the other participants vote. Most weighted voting systems that they will consider do not have a **dictator**, but if there is one, his or her voting weight must

be equal to or more than the quota. The system [51: 40, 60] has a dictator because the weight-60 voter can pass any motion that she wants.

2.3.2 Dummy Voters

A voting system may include some participants—called **dummy voters**—whose votes don't count. For example, the U.S. Congress has a nonvoting delegate who represents the District of Columbia. If a voting system has a dictator, all of the participants except the dictator are dummy voters. In the voting system [8 : 5, 3, 1], the weight-1 voter is a dummy, because a motion will pass only if it has the support of the weight-5 and weight-3 voters, and then the additional 1 vote is not needed. For another example, consider [51: 26, 26, 26, 22]. The voter with weight 22 is not needed when two of the other voters combine to support a motion; they have enough weight to pass the motion without her. If she joins forces with just one of the other voters, their total weight, 48, is not enough to win. Thus, the weight 22 voter is a dummy [6, 14].

2.3.3 Three More Three-Voter Systems

By adjusting the quota, the distribution of power in a weighted voting system can be altered. They have seen that the weight-1 voter in [8 : 5, 3, 1] is a dummy, but by increasing the quota to 9 they obtain a system in which the power is equally distributed—unanimous support is required to pass a motion in [9 : 5, 3, 1]. The weight-1 voter is also not a dummy in [6: 5, 3, 1] because he can join the weight-5 vote to pass a motion, even if the weight-3 voter opposes. Finally, consider [51: 49, 48, 3]. Although it looks as if the weight-3 voter will have relatively little power, and may even be a dummy, in fact she has the same voting power as the other two voters. Any two of the three voters in this system can

pass a measure.

2.3.4 Veto Power

A voter whose vote is necessary to pass any motion is said to have **veto power**. For example, in the system [6: 5, 3, 1], the weight-5 voter has veto power because the other two voters do not have enough combined weight to pass a motion. A dictator always has veto power, and it is possible for more than one voter to have veto power as well. In a criminal trial, each juror has veto power. In the system [8: 5, 3, 1] the voters with weights 5 and 3 each have veto power. A voter has veto power if no issue can pass without his or her vote. A voter with veto power is a one-person blocking coalition [6, 14].

2.3.5 Coalition

The set of participants in a voting system favor or oppose a given motion. A coalition may be empty (if, for example, the voting body unanimously favors a motion, the opposing coalition is empty), it may contain some but not all voters, or it may consist of all the voters. **Losing coalition** a coalition does not have the voting power to get its way.

Minimal winning coalition A winning coalition will become losing if any member defects. Each member is a critical voter. A **winning coalition** favors a motion, and has enough votes to pass it. A **blocking coalition** opposes a measure, and has the votes to defeat it. For example, in a dictatorship, a coalition in favor of a motion is a winning coalition if and only if the dictator belongs to it. Similarly, a coalition opposing a measure is a blocking coalition if and only if it includes the dictator. In a winning or blocking coalition, there may be some voters whose votes are necessary to win. If any one of these voters should switch to the other

side, the coalition would not have the votes it needs to have its way: It would become a **losing coalition**. These voters are called **critical voters** in the coalition.

2.4 Several Strategies for Voting

There are several strategies for voting including: majority, weighted voting, plurality, instant runoff voting, threshold voting, and the more general weighted k-out-of-n systems. To use a voting schema in any application domain, they have to understand the various tradeoffs and parameters and how they impact the correctness, reliability, and confidence in the final decision made by the voting system [9].

It ranks everything. It ranks movies, bands and songs, racing cars, politicians, professors, sports teams and the plays of the day. It asks voters to declare their favorite or to rank the alternatives first, second, and third.

There are four preferential voting methods:

- Plurality method
- Borda count method
- Plurality with elimination method
- Pairwise comparison method

2.4.1 The Plurality Method

The first-past-the-post voting method is one of several plurality voting systems. It is also known as the 'winner-takes-all', or 'simple plurality'. A first-past-the-post (abbreviated FPTP or FPP) election is one that is won by the candidate with more votes than any other(s). First-past-the-post voting methods can be used for single and multiple member elections. In a single member election, the candidate with the highest

number, not necessarily a majority, of votes is elected. In a multiple member first-past-the-post ballot, the first number of candidates, in order of highest vote, corresponding to the number of positions to be filled is elected.

By far, the most commonly used election procedure in the United States is simple plurality rule. Each eligible voter casts one vote and the winner is the candidate that receives the most votes. In terms borrowed from horse racing, it is sometimes called a “first past the post” procedure. The winner need only have more support than the second-placed competitor.

2.4.2 The Borda Count Method

The Borda count, which they label BC, is a method of combining the rankings of many individual voters. Suppose there are k alternatives. The voters rank the alternatives and assign integer scores to them, beginning with 0 (for the worst) stepping up in integer units to $k - 1$ (for the best). The rankings assigned by the voters are summed and the alternative with the largest Borda count is the winner. The Borda count generates a complete, transitive social ordering of the alternatives [7].

2.4.3 Sequential Pairwise Comparisons

The critics of the Bowl Championship Series often contend that college football should adopt a tournament format to select a national champion. A tournament is used in many other college sports as well as professional baseball, basketball and football. Would a tournament solve the controversy over “who's number one?” There are good reasons to be skeptical. A tournament might increase advertising revenue, but they doubt it would put an end to questions about whether the best team was

actually ranked number one at the end [7].

Like the Borda count, the majority rule tournament format can produce some truly bizarre outcomes. The tournament never rejects a Condorcet winner, if there is one, but interesting things can still happen. Perhaps the most well-known is the “dominated winner paradox”. It is possible that the winner of a tournament can be *unanimously defeated* by another alternative. They mean to say not just that the tournament picks the second-best or third-best, but rather, that the tournament winner is *unanimously considered inferior*.

2.4.4 Condorcet Methods: The Round Robin Tournament

On the basis of the preceding analysis, the reader should believe that the following claims are correct.

1. If there is a Condorcet Winner, then a single-elimination tournament format will select that alternative.
2. If there is no Condorcet Winner, the tournament winner is determined by the pairings of the alternatives.

The presiding officer of a town council might exercise the power to set the agenda to advantage some community groups over others. There's a charming essay about an economist and a lawyer who studied social choice theory and then used it to horns waggle the voters in a club that purchased airplanes for recreational use [8].

To address this problem, Condorcet's approach was to search for a method of voting that will give a meaningful result when all of the pairwise comparisons are considered. Condorcet's suggested that they collect enough information from voters so that we can hold (what is now called) a “round robin” tournament. In a round robin, each alternative faces each of the others in a head-to-head competition. The **Condorcet**

Criterion states that if there is a Condorcet winner-one alternative can defeat each of the others head-on-then it should win. If there is no Condorcet winner, then problem is to find a way to summarize the pairwise information and choose or shape the results into a ranking. Duncan Black, a pioneer of modern social choice research, suggested “The reasons may not seem so overwhelmingly convincing, but we are moving in a region where all considerations are tenuous and fine-spun; and the claims of the Condorcet criterion to rightness seem to us much stronger than those of any other” [1]. In the time since Condorcet, many different schemes have been proposed to summarize the outcomes of pairwise comparisons. Black suggested the use of a Borda Count. They consider just a few of the many interesting proposals.

2.5 The Banzhaf Power Index

While the Shapley–Shubik power index is based on a count of permutations in which a voter is pivotal, the Banzhaf index is based on a count of coalitions in which a voter is *critical*. A **coalition** is a set of voters who are prepared to vote for, or to oppose, a motion. A voter’s **Banzhaf power index** is the number of distinct winning or blocking coalitions in which his or her vote is critical.

To determine the Banzhaf power index of a voter A , they must count all possible winning and blocking coalitions of which A is a member and casts a critical vote. The weight of a winning coalition must be q or more, where q is the quota. A blocking coalition must be large enough to deny the “yes” voters the q votes they need to win. If the total weight of all the voters is n , then the weight of the blocking coalition has to be more than $n - q$. Assuming that all weights are integers, this means that the weight of a blocking coalition must be at least $n - q + 1$. To

determine which voters are critical in a given winning or blocking coalition, the following principle is useful.

A winning coalition with total weight w has $w - q$ extra votes. A blocking coalition with total weight w has $2(w - q + 1)$ extra votes. The critical voters are those whose weight is more than the coalition's extra votes. These are the voters that the coalition can't afford to lose.

2.5.1 Critical Voters

Consider a committee of three members, A , B , and C . The chairperson of the committee, A , has two votes, while B and C each have one. The quota is three, and this voting system is $[3: 2, 1, 1]$. The coalition $\{A, B, C\}$ is a winning coalition because it has all four votes. Suppose that A decides to leave the coalition.

2.5.2 Winning/ Blocking Duality

The number of winning coalitions in which a given voter is critical is equal to the number of blocking coalitions in which the same voter is critical. To understand why winning/ blocking duality works, consider a voter, A , who is critical in a winning coalition, C . The voters who are not in the coalition C are voting "no" but do not have enough votes to block. However, if A changes her vote to "no," then C will be a losing coalition because A was a critical voter. When joined by A , the "no" voters form a blocking coalition, with A as a critical voter. Thus, there is a one-to-one correspondence between winning coalitions in which A is a critical voter and blocking coalitions with A as a critical voter. By the winning/ blocking duality principle, we can determine a voter's Banzhaf power index by doubling the number of winning coalitions in which he or she is a critical voter—this will account for the blocking coalitions.

2.6 The Shapley–Shubik Power Index

The **Shapley–Shubik power index** of each voter is computed by counting the number of permutations in which he or she is pivotal, then dividing it by the total number of permutations. Thus, if we consider each permutation to belong to the voter who is pivotal, each voter's Shapley–Shubik index is his or her share of the permutations.

For voting systems with no more than four voters, the Shapley–Shubik power index may be calculated by making a list of all the voting permutations and identifying the pivotal voter in each. Listing all of the permutations is the brute force way of calculating the Shapley–Shubik power index [14].

If all the voters have the same voting weight, you don't need to make a list of all the permutations, because each has the same share of power. If there are 100 voters, each with 1 vote, the Shapley–Shubik index of each is $1/100$. If all but one or two of the voters have equal power, they can still calculate the Shapley–Shubik power index of each without making a list of permutations. It uses two principles to do this:

1. Voters with the same voting weight have the same Shapley–Shubik power index.
2. The sum of the Shapley–Shubik power indices of all the voters is 1.

2.6.1 Permutation

A **permutation** of voters is an ordering of all of the voters in a voting system: A specific ordering from first to last of the elements of a set; for example, an ordering of the participants in a voting system.

2.6.2 Pivotal Voter

The first voter in a permutation who, when joined by those coming

before him or her, would have enough voting weight to win is the **pivotal voter** in the permutation. Each permutation has exactly one pivotal voter.

2.6.3 Factorial

If there are n voters, the number of permutations is called the **factorial** of n and is denoted $n!$. There is a simple formula for $n!$: For a positive whole number n ,

$$n! = n * (n-1) * (n-2) * \dots * 2 * 1, \text{ and} \quad (2.1)$$

$$0! = 1. \quad (2.2)$$

The number of permutations of n voters (or n distinct objects) is called *n-factorial*, or in symbols, $n!$. Because the empty coalition can be ordered in only one way, $0! = 1$. When n is a positive whole number, $n!$ is equal to the product of all the integers from 1 up to n . If n has more than one digit, then $n!$ is a pretty big number: $10!$ is more than three million, and $1000!$ has 2568 digits.

2.7 Voting System

Voting system means, a method of casting and processing votes that functions wholly or partly by use of electromechanical, or electronic apparatus or by use of paper ballots and includes, but is not limited to, the procedures for casting and processing votes and the programs, operating manuals, tabulating cards, printouts and other software necessary for the system's operation.

A voting system consists of a configuration of specific hardware and software components, procedures and expendable supplies; configured into a system which allows votes to be cast and tabulated. No single component of a voting system, such as a precinct tabulation device, meets the definition of a voting system. Sufficient components must be

assembled to create a configuration, which will allow the system as a whole to meet all the requirements described for a voting system in this publication [8].

2.8 The Random Voting Model

The random voting model is empirically inappropriate for election ~~systems~~. We devote some space to this model, however, because it is standard in the voting power literature. In addition, this simplified model has some interesting features. Under random voting, all 2^n vote configurations are equally likely, and so the power of voter i is simply $2^{(n-1)}$ times the number of configurations of the other $n - 1$ voters for which voter i is decisive (and counting semi-decisive configurations, in which votes are exactly tied, as 1/2). Voting power calculations can thus be seen as combinatorial [8, 10].

2.9 Application Areas of Voting Power

There are a variety of voting situations in which votes are not simply tallied, and so voting power is a nontrivial concept. This section focuses on weighted majority voting (as in the example in the first paragraph of Section 1) and the closely-related setting of two-level voting, where the representatives casting the weighted vote are themselves elected by majority vote in separate districts. This is the structure of the U.S. Electoral College and, implicitly, the European council of ministers (where each minister represents an individual country whose government is itself democratically elected).

The author shall also consider bloc voting, in which subsets of the electorate can voluntarily form binding coalitions, so that all the votes for a ~~winning~~ are assigned to the party that wins a majority of the vote within the

(Recall that we assume two parties and no abstention throughout.)

The blocs can themselves be nested. The author assumes that tied elections at all levels are decided by coin flips. Voting power measures have been developed in the statistics, mathematics, political science, and legal literatures for over 50 years.

2.3.1 Voting Power

Voting power can and has been defined in a variety of ways [4, 7]; in this paper the author shall use the definition based on the probability that a vote affects the outcome of the election. Consider an electoral system with n voters. The author uses the notation i for an individual voter, $v_i = \pm 1$ for his or her vote, $v = (v_1, \dots, v_n)$ for the entire vector of votes, and $R = R(v) = \pm 1$ for the rule that aggregates the n individual votes into a single outcome. It is possible for R to be stochastic (because of possible ties or, even more generally, because of possible errors in vote counting). The author shall assume various distributions on v and rules $R(v)$ which then together induce distributions on R [6].

The probability that the change of the individual vote v_i will change the outcome of the election is,

$$\text{power}_i = \text{Power of voter } i = \Pr(R = +1/v_i = +1) + \Pr(R = +1/v_i = -1) \quad (2.3)$$

If your voting power is zero, then changing your vote from -1 to +1 has no effect on the probability of either candidate winning.

2.3.2 Electronic Voting

Electronic voting protocols should respect some basic properties of elections, namely accuracy, democracy, privacy and verifiability [10]. One of those properties, democracy, states that “each eligible voter is

allowed to vote and to vote at most once". This property is a normal requirement in most electoral processes but is far from being axiomatic. In fact, there are some scenarios where the votes from some persons have, or should have, a different weight than the other votes. For instance, some communities, such as the associates of a football club, can have a weight somehow proportional to duration of the membership; or a member from an administration board can have a different weight for solving draw situations. Thus, voting weights are a useful, real-life form of differentiating participants in voting processes. However, as of today we have no knowledge of electronic voting systems with support for weighted votes [4].

REVS is a fault-tolerant electronic voting system designed for voting through the Internet. It uses replication as the basic mechanism to tolerate system failures in communications, servers and voters' applications. Furthermore, it also tolerates failures in the correct behavior of the several entities running the voting protocol: neither voters nor servers, until a certain level of collusion, can interfere with the correct behavior of the system without notice. Supporting weighted votes means that, when voting, a voter's vote is worth w votes. To implement this service in REVS, several requirements were considered.

The basic requirement was to minimize the modifications on the protocol of REVS, in order to reduce the probability of introducing new vulnerabilities in the final protocol. Other more specific requirements were the following:

- Scalability: support for an arbitrarily large set of available weights and support of arbitrarily high weights;
- Efficiency: the performance of the system should not be notably disturbed when extensively using different weights;
- Usability: for a voter it should be equal to vote the "traditional"

- way or using a weight attribute; and
- Anonymity: a weighted vote, or a set of w votes provided by a voter with weight w , should not provide any hint about the voter who cast it.

The final solution respects all these requirements except the last one, anonymity: in some particular cases, a voter can be linked to his vote. In fact, when there is a single voter allowed to cast a vote with a specific weight, or a single voter that actually used a specific weight, the voter can be linked to his vote. This problem was not solved in the protocol presented in this paper because we didn't find a fairly easy solution that does not interfere too much with any of the other requirements.

Furthermore, this anonymity problem already appears in paper-based voting systems, where voters with different weights use bulletins of different colors or cast their votes in different ballot boxes, one per each weight [12].

2.10.1 Computer Algorithms for Voting Power Analysis

These pages provide access to computer software for voting power analysis that you can run over the internet from your web browser. The programs calculate voting power indices for weighted voting bodies in which the members or parties possess different numbers of votes and decisions are taken by qualified majority voting. In such a system of qualified majority voting a decision is taken when the total number of votes cast in favor of a particular action equals or exceeds the quota. Many real world organizations use such systems for example the World Bank, EU Council, the US presidential Electoral College, corporations and political parties. There are therefore many important applications for which this web page may be a useful analytical resource [5].

In a weighted voting body, a member's power is not related to its weight in a simple way. Certainly it is not generally true that a member's power is represented by its weight: "weighted voting doesn't work" in the words of a well-known journal article on the topic. In fact, a member's power is a property of the whole voting body and depends on all other members' weights and the decision rule, and hence in order to analyze a member's power it is necessary to investigate all the possible outcomes of votes that could occur. It can only be found with a lot of computational effort. The computer programs that can be run from this web page enable this to be done easily. Various algorithms are provided to deal with different aspects.

The algorithms here compute the so called "classical" power indices of Shapley and Shubik and Banzhaf based on different coalition models. They also provide the indices - closely related to the Banzhaf index - due to Penrose (also known as the absolute Banzhaf index) and Coleman (the power to initiate action and the power to prevent actions which are useful when the quota exceeds half the total weight). Other power indices have been defined and used but they are outside the scope of this web page and are not calculated currently [14].

2.11 Related Works

Like markets, voting systems also serve to aggregate information across a group, and as a result, it's hard to draw a perfectly clear dividing line between these two kinds of institutions. But there are definite distinctions between the respective settings in which they are typically applied [3]. A first important distinction is that voting is generally used in situations where a group of people is expressly trying to reach a single decision that in a sense will speak for the group. When a population votes

on a set of candidates or ballot initiatives, a legislative body votes on whether to pass a bill, a jury votes on a verdict in a trial, a prize committee votes on the recipient of an award, or a group of critics votes on the top movies of the past century, the resulting decision is a single outcome that stands for the group, and has some kind of binding effect going forward. In contrast, markets synthesize the opinions of a group more indirectly, as investors' beliefs are conveyed implicitly through their transactions in the market — choosing how much to invest or to bet, choosing whether to buy or not to buy, and so forth. The overt goal of the market is to enable these transactions, rather than any broader synthesis or group decision that might in fact arise from the transactions in aggregate.

There are other important distinctions as well. A simple but important one is that the choices in a market are often numerical in nature (how much money to transact in various ways), and the synthesis that takes place generally involves arithmetic on these quantities — weighted averages and other measures. Many of the key applications of voting, on the other hand, take place in situations where there's no natural way to "average" the preferences of individuals — since the preferences are over different people, different policy decisions, or different options under a largely subjective criterion. Indeed, much of the richness of the theory of voting comes from precisely this attempt to combine preferences in the absence of simple metaphors like averaging. The notion of voting encompasses a broad class of methods for reaching a group decision.

The voting application that occurs immediately is that which employs the usual tabulation/mixing center approach to provide anonymity. In this setting, the protocols of this paper offer important advantages. They are much more efficient, and allow the mixing centers to be completely independent of the authorities who hold some share of

the key necessary to decrypt ballots.

Perhaps, however, a more valuable and exciting application of the new protocol is for creating “anonymous credentials”. A member of an authorized group, identified only by a set of DSA, or Diffie-Hellman public keys, can authenticate group membership, and/or sign in a onetime way, without revealing his/ her individual identity. This leads to a novel solution to the voting problem that is universally verifiable, but does not require any special set of “authorities” in order to tabulate [8]. It also offers a better privacy model to the voter, and speeds tabulation enormously since ballots do not need to be encrypted/ decrypted. In effect, instead of mixing encrypted vote cypher texts *after* ballots have been received at the vote collection center, voter credentials are mixed *before* the start of the election.

assumes that each member's votes carries equal weight. A weighted voting system is characterized by three things: the players, the weights and the quota [9, 10].

The voters are the players (P), players are a persons which carries the number of votes he controls. The quota (q) is the minimum number of votes required to pass a motion.

3.3. Parameters in Weighted Voting

Weight. The number of votes each player has.

Motion. A yes/no vote.

Quota. The minimum number of votes needed to pass a motion.

$$\text{Quota} = \frac{\text{Total number of weights}}{2} + 1 \quad (3.1)$$

The notation used to describe a system with n players and quota q is (P, W, V, q) . We will typically refer to the system as \mathcal{S} .

CHAPTER 3

POWER INDEX

3.1 Weighted Voting Systems

Weighted voting are mathematical models, used to analyze situations where voters with variable voting weight vote in favor of or against a decision. They have been applied in various political and economic organizations. The calculation of voting powers in a weighted voting has been extensively researched in the last few years. Weighted voting systems are voting systems based on the idea that not all voters are equal. Instead, it can be desirable to recognize differences by giving voters different amounts of say (weights) concerning the outcome of an election. This is in contrast to normal parliamentary procedure, which assumes that each member's vote carries equal weight. A weighted voting system is characterized by three things: the players, the weights and the quota [9, 14].

The voters are the players (P_1, P_2, \dots, P_N). A player's weight (w) is the number of votes he controls. The quota (q) is the minimum number of votes required to pass a motion.

3.1.1 Parameters in Weighted Voting

Weight: The number of votes each player has.

Motion: A yes/no vote.

Quota: The minimum number of votes needed to pass a motion.

$$\text{Quota} = \frac{\text{Total number of weights}}{2} + 1 \quad (3.1)$$

The notation used to describe a system with n players and quota q is $[q: w_1, w_2, \dots, w_n]$ where typically $w_1 \geq w_2 \geq \dots \geq w_n$.

Coalition: A nonempty set of players in a weighted voting system. For n players, the possible number of coalitions is $2^n - 1$.

Veto Power: A player has veto power if that player can prevent any motion from passing by voting no.

Dummy: A player with no power.

Dictator: A player whose power is greater than or equal to the quota.

Critical Player: A player in a coalition is a critical player for that coalition if the coalition needs that voter to win.

Permutation: A permutation is an ordering of all of the players in a voting system.

Sequential Coalition: is one in which the players are listed in the order that they entered the coalition.

Pivotal Player: A player in a sequential coalition who changes the coalition from a losing to a winning one.

3.2 Sample Data Sets for Case Study

Table 3.1 shows the sample data set for handset brands. In this table, six types of handset brands are used to vote by 30 voters. These handset brands are Samsung, HTC, Huawei, Sony, Nokia and Lenovo. In this dataset, the system defines maximum number and minimum number for one brand. For each brand, the maximum vote number is 5 and minimum vote number is 1.

Table 3.1 Sample Data Sets for Handset Brands

Voters	Samsung	HTC	Huawei	Sony	Nokia	Lenovo
1	4	5	3	3	1	2
2	5	4	4	3	2	1
3	4	3	2	5	5	1
4	1	1	5	4	1	3
5	5	5	4	2	1	3

Voters	Samsung	HTC	Huawei	Sony	Nokia	Lenovo
6	1	3	2	5	2	5
7	4	2	2	1	1	5
8	2	2	4	4	3	2
9	5	5	1	4	5	1
10	5	3	3	3	5	4
11	3	3	1	2	2	4
12	3	5	3	3	5	4
13	5	5	3	4	5	3
14	4	3	5	1	1	2
15	5	3	2	4	1	1
16	5	4	4	5	1	1
17	5	4	3	5	1	1
18	2	2	5	5	3	1
19	4	5	4	3	3	3
20	5	3	2	3	4	2
21	5	2	2	5	2	2
22	5	3	3	4	5	3
23	4	2	4	5	1	2
24	3	3	4	4	5	1
25	5	5	5	2	3	3
26	2	4	1	1	1	1
27	4	5	5	3	3	1
28	5	4	3	5	2	1
29	4	5	3	4	2	4
30	5	3	4	4	2	2

Table 3.2 Notations for Handset Brands and MaxCount

Brands	Notations	MaxCount
Samsung	P1 or A	14

Brands	Notations	MaxCount
HTC	P2 or B	9
Sony	P3 or C	8
Nokia	P4 or D	7
Hauwei	P5 or E	5
Lenovo	P6 or F	2

From table 3.2, the system calculates the quota.

$$\text{Quota} = ((14+9+8+7+5+2) / 2) + 1 = 45/2 = 24 [24 : 14, 9, 8, 7, 5, 2]$$

3.2.1 Calculating Power Index using Banzhaf Power Index

To calculate the Banzhaf power index of a given voting system:

1. Make a list of the winning and blocking coalitions.
2. Use the extra-votes principle to identify the critical voters in each coalition.

A voter's Banzhaf power index is then the number of coalitions in which he or she appears as a critical voter. A voter's Banzhaf power index is the number of distinct winning or blocking coalitions in which his or her vote is critical.

- Step 1. List all winning coalitions.
- Step 2. Determine the critical players in each winning coalition.
- Step 3. Count how many times a particular player P_i is critical and call this number B_i .
- Step 4. Count the total number of times players are critical and call this number T .

The Banzhaf power index of player P_i is the fraction B_i/T . The Banzhaf power distribution is the complete list of all players. Banzhaf

power indexes (which always sums to 1). P₁: P₂: P₃: P_n: Banzhaf Power: The Banzhaf power of a voter equals the number of winning coalitions in which the voter is critical. Total Banzhaf Power (TBP): The sum of Banzhaf powers of all voters in a weighted voting system.

$$\text{Banzhaf Power Index} = \frac{\text{Voter's Banzhaf Power}}{\text{Total Banzhaf Power}} \quad (3.2)$$

$$BI(v_i) = \frac{TBP(v_i)}{TBP(v_i) + \dots + TBP(v_n)} = \frac{TBP(v_i)}{\sum_{j=1}^n TBP(v_j)} \quad (3.3)$$

The possible number of coalitions is $2^n - 1 = 2^6 - 1 = 63$. The following are number of possible coalitions:

Quota = [24 : 14, 9, 8, 7, 5, 2] [24 : P₁, P₂, P₃, P₄, P₅, P₆]
 Coalition : {P₁} = 14, {P₂} = 9, {P₃} = 8, {P₄} = 7, {P₅} = 5, {P₆} = 2,
 {P₁, P₂} = 23, {P₁, P₃} = 22, {P₁, P₄} = 21, {P₁, P₅} = 19, {P₁, P₆} = 16,
 {P₂, P₃} = 17, {P₂, P₄} = 16, {P₂, P₅} = 14, {P₂, P₆} = 11, {P₃, P₄} = 15,
 {P₃, P₅} = 13, {P₃, P₆} = 10, {P₄, P₅} = 12, {P₄, P₆} = 9, {P₅, P₆} = 7,
 {P₁, P₂, P₃} = 31, {P₁, P₂, P₄} = 30, {P₁, P₂, P₅} = 28, {P₁, P₂, P₆} = 25,
 {P₁, P₃, P₄} = 29, {P₁, P₃, P₅} = 27, {P₁, P₃, P₆} = 24, {P₁, P₄, P₅} = 26,
 {P₁, P₄, P₆} = 23, {P₁, P₅, P₆} = 21, {P₂, P₃, P₄} = 24, {P₂, P₃, P₅} = 22,
 {P₂, P₃, P₆} = 19, {P₂, P₄, P₅} = 21, {P₂, P₄, P₆} = 18, {P₂, P₅, P₆} = 16,
 {P₃, P₄, P₅} = 20, {P₃, P₄, P₆} = 17, {P₃, P₅, P₆} = 15, {P₄, P₅, P₆} = 14,
 {P₁, P₂, P₃, P₄} = 38, {P₁, P₂, P₃, P₅} = 36, {P₁, P₂, P₃, P₆} = 33
 {P₁, P₂, P₄, P₅} = 35 {P₁, P₂, P₄, P₆} = 32 {P₁, P₃, P₅, P₆} = 30,
 {P₁, P₃, P₄, P₅} = 34, {P₁, P₃, P₄, P₆} = 31, {P₁, P₃, P₅, P₆} = 29,
 {P₁, P₄, P₅, P₆} = 28, {P₁, P₄, P₅, P₆} = 28, {P₂, P₃, P₄, P₅} = 29,
 {P₂, P₃, P₄, P₆} = 26, {P₂, P₃, P₅, P₆} = 24, {P₂, P₄, P₅, P₆} = 23,
 {P₃, P₄, P₅, P₆} = 22, {P₁, P₂, P₃, P₄, P₅} = 43, {P₁, P₂, P₃, P₄, P₆} = 40,
 {P₁, P₂, P₃, P₅, P₆} = 36, {P₁, P₂, P₃, P₅, P₆} = 38, {P₁, P₂, P₄, P₅, P₆} = 37,
 {P₂, P₃, P₄, P₅, P₆} = 31, {P₁, P₂, P₃, P₄, P₅, P₆} = 45.

3.2.2 Coalition Table

A table of all the possible coalitions for a given weighted voting system, together with the weight of each coalition, and whether it is winning or losing, will be essential to determining power. Being systematic in constructing the coalition table is the only way to make sure you do not forget to list any of the coalitions. Table 3.3 is the example of coalition table and Table 3.4 is the selection of winning coalition.

Table 3.3 Coalition Table

Coalition Table		
Coalition	Weight	Winning or Losing

Table 3.4 Winning Coalition Table

Coalition	Weight	Winning
{P1,P2,P3}	31	Winning
{P1,P2,P4}	30	Winning
{P1,P2,P5}	28	Winning
{P1,P2,P6}	25	Winning
{P1,P3,P4}	29	Winning
{P1,P3,P5}	27	Winning
{P1,P3,P6}	24	Winning
{P1,P4,P5}	26	Winning
{P2,P3,P4}	24	Winning
{P1,P2,P3,P4}	38	Winning
{P1,P2,P3,P5}	36	Winning
{P1,P2,P3,P6}	33	Winning
{P1,P2,P4,P5}	35	Winning
{P1,P2,P4,P6}	32	Winning
{P1,P2,P5,P6}	30	Winning
{P1,P3,P4,P5}	34	Winning
{P1,P3,P4,P6}	31	Winning

Coalition	Weight	Winning
{ <u>P1</u> ,P3,P5,P6}	29	Winning
{ <u>P1</u> ,P4,P5,P6}	28	Winning
{P2,P3,P4,P5}	29	Winning
{P2,P3,P4,P6}	26	Winning
{P2,P3,P5,P6}	24	Winning
{P1,P2,P3,P4,P5}	43	Winning
{P1,P2,P3,P4,P6}	40	Winning
{ <u>P1</u> ,P3,P4,P5,P6}	36	Winning
{P1,P2,P3,P5,P6}	38	Winning
{ P1,P2,P4,P5,P6}	37	Winning
{P2,P3,P4,P5,P6}	31	Winning
{P1,P2,P3,P4,P5,P6}	45	Winning

The above table shows the winning coalition according to the sum of weight and critical players are underlined. Critical players are counted from table 3.4.

Critical player (Banzhaf Power): $P1 = 19$, $P2 = 11$, $P3 = 11$, $P4 = 7$, $P5 = 5$, $P6 = 3$

Total Banzhaf Power : $19 + 11 + 11 + 7 + 5 + 3 = 56$

Banzhaf Power Index (BPI) : Samsung = $19/56 = 34\%$

HTC = $11/56 = 20\%$

Sony = $11/56 = 20\%$

Nokia = $7/56 = 13\%$

Huawei = $5/56 = 9\%$

Lenovo = $3/56 = 5\%$

3.2.3 Calculating Power Index using Shapley-Shubik Power Index

The Shapley-Shubik power index of each player is computed by counting the number of permutations, then dividing it by the total number

of permutations. The first voter in a permutation would have enough voting weight to win is the pivotal player in the permutation. Each permutation has exactly one pivotal voter. The Shapley-Shubik power index counts how likely a player is to be pivotal. If there are n players, the number of permutations is called the factorial of n and is denoted $n!$. There is a simple formula for $n!$:

List all sequential coalitions. In each sequential coalition, determine the pivotal player. Count up how many times each player is pivotal. Convert these counts to fractions or decimals by dividing by the total number of sequential coalitions [14].

$$\text{Quota} = [24 : 14, 9, 8, 7, 5, 2][24 : A, B, C, D, E, F]$$

Sequential coalitions: $6! = 720$

Calculate Permutations, Weights and Pivot.

Table 3.5 Calculation of Permutation, Weight and Pivot

No.	Permutation	Weights	Pivot
1	A B C D E F	14 23 31 38 43 45	C
2	A B C D F E	14 23 31 38 40 45	C
3	A B C E D F	14 23 31 36 43 45	C
4	A B C E F D	14 23 31 36 38 45	C
5	A B C F D E	14 23 31 33 40 45	C
:	:	:	:
543	E C D B A F	5 13 20 29 43 45	B
:	:	:	:
720	F E D C B A	2 7 14 22 31 45	B

The following are the pivotal times counted from table 3.4.

A is pivotal 252 times,

B is pivotal 144 times,

C is pivotal 144 times,

D is pivotal 84 times,

E is pivotal 60 times,

F is pivotal 36 times.

Power index for Samsung is $252/720 = 35\%$

Power index for HTC is $144/720 = 20\%$

Power index for Sony is $144/720 = 20\%$

Power index for Nokia is $84/720 = 12\%$

Power index for Huawei is $60/720 = 8\%$

Power index for Lenovo is $36/720 = 5\%$

3.2.4 Comparison Result

Table 3.6 is the comparison result of power index by using Banzhaf Power Index and Shapley-Shubik Power Index.

Table 3.6 Comparison Result of Power Index

Handset Brands	Power Index	
	Banzhaf Power Index	Shapley-Shubik Power Index
Samsung	34%	35%
HTC	20%	20%
Sony	20%	20%
Nokia	13%	12%
Huawei	9%	8%
Lenovo	5%	5%

According to the above table, Samsung is the most popular handset brand with 34% by Banzhaf Power Index and 35% by Shapley-Shubik Power Index. So, Shapley-Shubik Power Index is better than Banzhaf Power Index which gives more percentage result.

CHAPTER 4

SYSTEM DESIGN AND IMPLEMENTATION

4.1 System Design

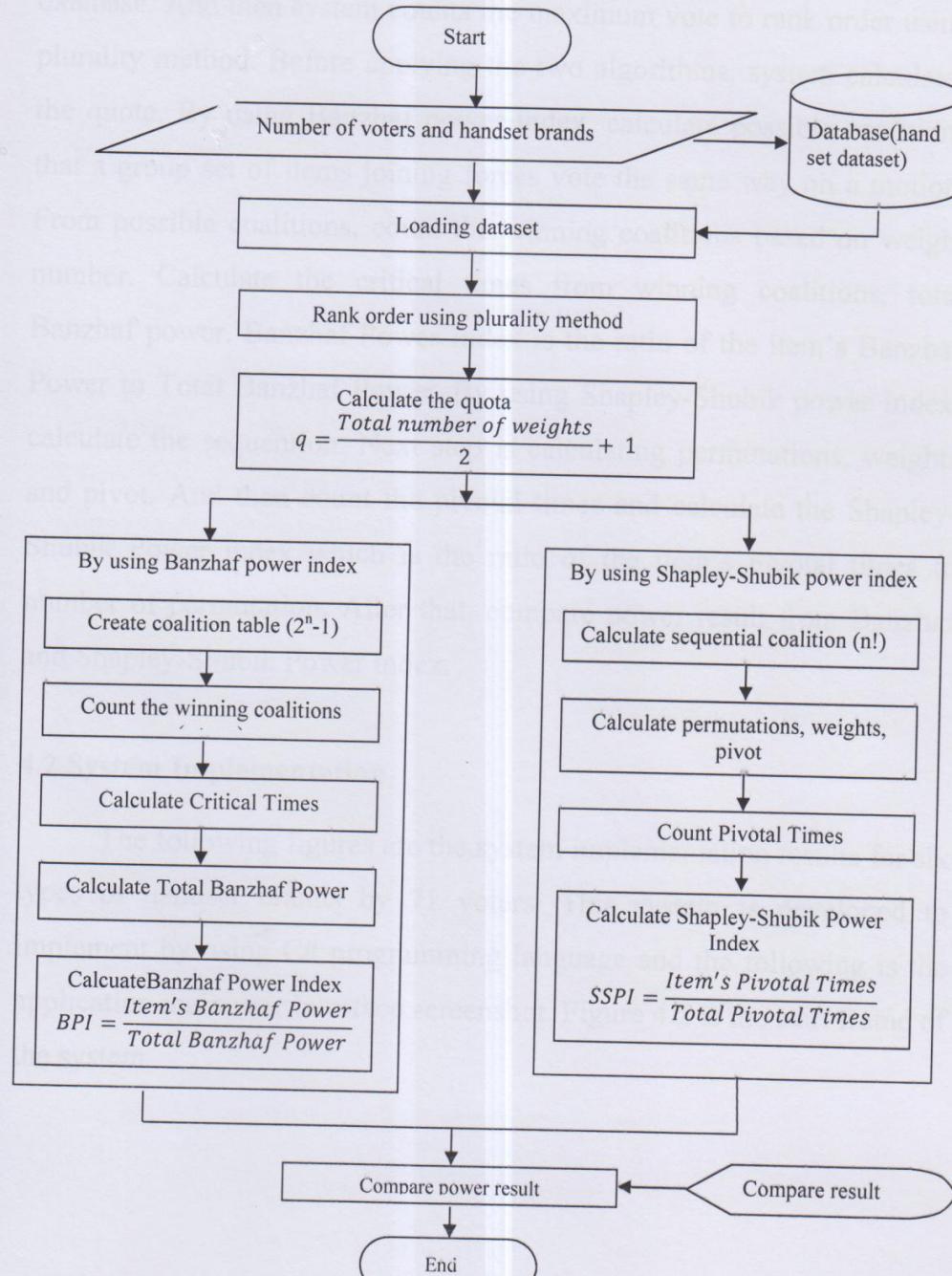


Figure 4.1 System Flow Diagram

The system flow diagram is illustrated in Figure 4.1. This system describes the flow of the voting power for handset brands by using Banzhaf Power Index and Shapley-Shubik Power Index. Firstly, this system enters the vote number for each handset and save them to database. And then system counts the maximum vote to rank order using plurality method. Before applying the two algorithms, system calculates the quota. By using Banzhaf power index, calculate possible coalitions that a group set of items joining forces vote the same way on a motion. From possible coalitions, count the winning coalitions based on weight number. Calculate the critical times from winning coalitions, total Banzhaf power. Banzhaf Power Index is the ratio of the item's Banzhaf Power to Total Banzhaf Power. By using Shapley-Shubik power index, calculate the sequention. Next step is calculating permutations, weights and pivot. And then count the pivotal times and calculate the Shapley-Shubik Power index which is the ratio of the item's pivotal times to number of permutation. After that, compare power result from Banzhaf and Shapley-Shubik Power index.

4.2 System Implementation

The following figures are the system implementation results for six types of handset brands by 71 voters. This system is developed to implement by using C# programming language and the following is the application main user interface screenshot. Figure 4.2 is the start frame of the system.

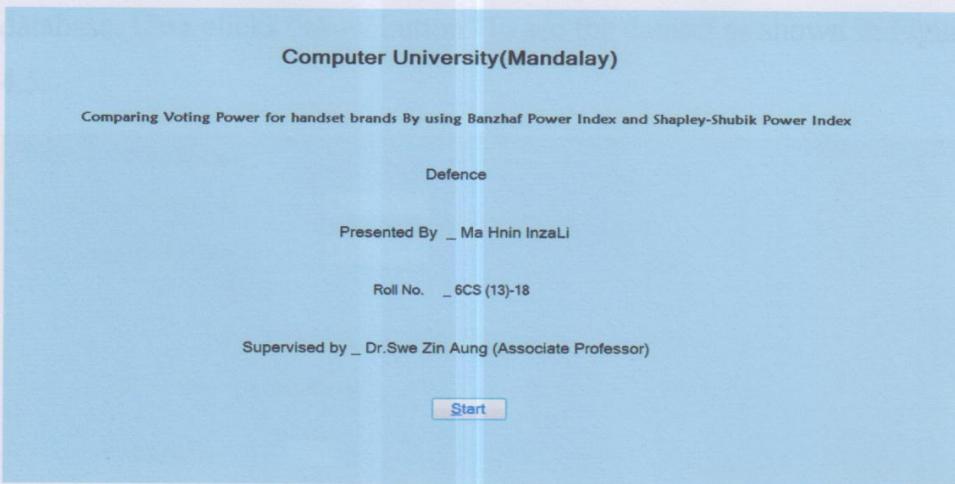


Figure 4.2 Start Frame

4.2.1 Main Frame

Main Frame of the system is as shown in Figure 4.3. Main Frame includes six buttons to calculate the power index. When user clicks “Data Input” button, Figure 4.4 appears.

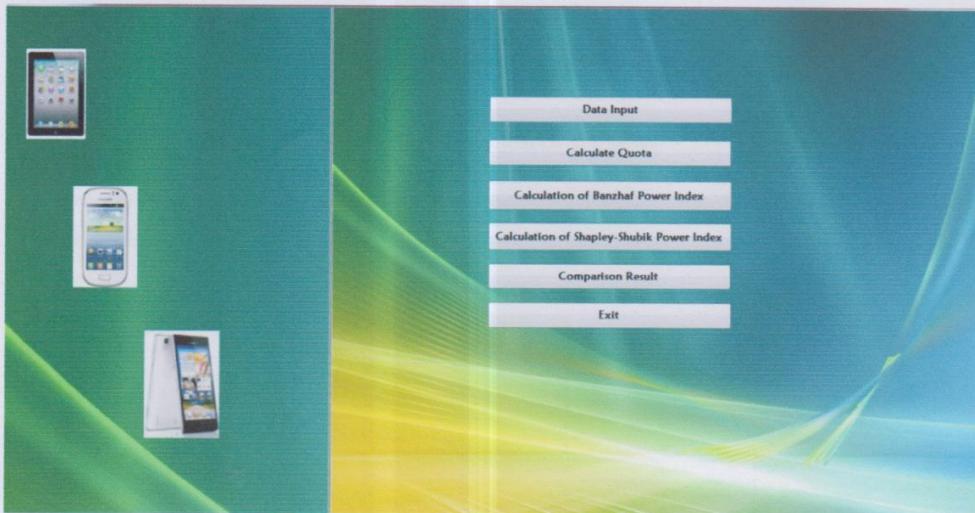


Figure 4.3 Main Frame

4.2.2 Data Input for Handset Brands

In Figure 4.4, min-max voting number is defined to enter vote in the first part. The vote number is entered by the players in the second part. If save button is clicked, the dataset from textbox will be saved to

database. User clicks “show button” to see the dataset as shown in Figure 4.5.

The screenshot shows a web-based interface for managing投票 (voting) data. At the top, there's a header "Define Min-Max Voting Number". Below it, two input fields are displayed: "Maximum vote for one brand = 5" and "Minimum vote for one brand = 1". The main area is titled "Brand Name Enter Vote" and lists several brands with corresponding input fields for voting. The brands listed are Samsung, HTC, Hauwei, Sony, Nokia, and Lenovo. To the right of each brand name is a small input field. To the right of the input fields for Samsung through Lenovo is a "Save" button. To the right of the Lenovo input field is a "Show" button. In the bottom right corner of the main area, there is a "Back" button. The entire interface has a light blue background.

Figure 4.4 Data Input for Handset Brands

4.2.3 Dataset with Handset Brands and Voters

The screenshot displays a dataset in a table format. The columns represent different handset brands: Voters, Samsung, HTC, Huawei, Sony, Nokia, and Lenovo. The rows are numbered from 1 to 24. Each row contains a voter ID and their respective votes for each brand. The data shows varying levels of support for different brands across the 24 voters. A "Back" button is located in the bottom right corner of the table area. The table has a light gray background.

Voters	Samsung	HTC	Huawei	Sony	Nokia	Lenovo
1	4	5	3	3	1	2
2	5	4	4	3	2	1
3	4	3	2	5	5	1
4	1	1	5	4	1	3
5	5	5	4	2	1	3
6	1	3	2	5	2	5
7	4	2	2	1	1	5
8	2	2	4	4	3	2
9	5	5	1	4	5	1
10	5	3	3	3	5	4
11	3	3	1	2	2	4
12	3	5	3	3	5	4
13	5	5	3	4	5	3
14	4	3	5	1	1	2
15	5	3	2	4	1	1
16	5	4	4	5	1	1
17	5	4	3	5	1	1
18	2	2	5	5	1	1
19	4	5	4	3	3	3
20	5	3	2	3	4	2
21	5	2	2	5	2	2
22	5	3	3	4	5	3
23	4	2	4	5	1	2
24	3	3	4	4	5	1

Figure 4.5 Dataset with Handset Brands and Voters

Figure 4.5 is the dataset with handset brands and voters. If user clicks “Back” button, Figure 4.3 is shown. Then if the “Calculate Quota” is clicked to go to the next step, Figure 4.6 is shown.

4.2.4 Calculation of Quota

Calculation of Quota

Alpha1	Alpha2	HandsetName	HMaxCount
P1	A	Samsung	32
P2	B	Huawei	23
P4	D	Sony	12
P3	C	HTC	12
P5	E	Nokia	11
P6	F	Lenovo	6
*			

No: of brands (Handset) =

No: of Voters =

Quota = $((32+23+12+12+11+6)/2)+1=(96.0/2)+1=49$

The notation used to describe a system with n players and quota q is [50:32,23,12,12,11,6] where typically $W_1 > W_2 > \dots > W_n$

[Back](#)

Figure 4.6 Calculation of Quota

In Figure 4.6, the table displays maximum number of count for each item by using plurality method. This system calculates the quota by using maximum number of count from table. If user clicks “Back” button, Figure 4.3 is shown again.

4.2.5 Creating Coalition Table

After calculating the quota, the following Figure 4.7 creates a coalition table including winning or losing. If show power table button is clicked, the power index results are shown in Figure 4.8.

FillBy

For Banzhaf Power Index

Number of Items (n) = 6
 $n = 6$
Number of Coalitions = $2^n - 1 = 63$
Coalition Table For [49 : 32, 23, 12, 11, 6, 12,]

Coalition	Winning_or_Losing
{P1}	Losing
{P2}	Losing
{P5}	Losing
{P6}	Losing
{P3}	Losing
{P1,P2}	Winning
{P1,P4}	Losing
{P1,P5}	Losing
{P1,P6}	Losing
{P1,P3}	Losing
{P2,P4}	Losing
{P2,P5}	Losing
{P2,P6}	Losing
{P2,P3}	Losing
{P4,P5}	Losing
{P4,P6}	Losing
{P4,P3}	Losing
{P5,P6}	Losing
{P5,P3}	Losing

[Show Power Table](#)

[Back](#)

Figure 4.7 Create Coalition Table

4.2.6 Winning Coalition Table

FillBy

Winning Coalition Table

Coalition	Weight	Critical_Player
{P1,P2}	55	P1,P2
{P1,P2,P4}	67	P1,P2
{P1,P2,P5}	67	P1,P2
{P1,P2,P6}	66	P1,P2
{P1,P2,P3}	61	P1,P2
{P1,P4,P5}	56	P1,P4,P5
{P1,P4,P6}	55	P1,P4,P6
{P1,P4,P3}	50	P1,P4,P3
{P1,P5,P6}	55	P1,P5,P6
{P1,P5,P3}	50	P1,P5,P3
{P1,P6,P3}	49	P1,P6,P3
{P1,P2,P4,P5}	79	P1
{P1,P2,P4,P6}	78	P1
{P1,P2,P4,P3}	73	P1
{P1,P2,P5,P6}	78	P1
{P1,P2,P5,P3}	73	P1
{P1,P2,P6,P3}	72	P1
{P1,P4,P5,P6}	67	P1
{P1,P4,P5,P3}	62	P1
{P1,P5,P6,P3}	61	P1

[Show Critical Time](#)

[Back](#)

Figure 4.8 Winning Coalition Table

Winning coalition table is shown in Figure 4.8 and critical times table is shown in Figure 4.9. If the user clicks the “Show Percentage” button in Figure 4.9, power indexes are shown in Figure 4.10.

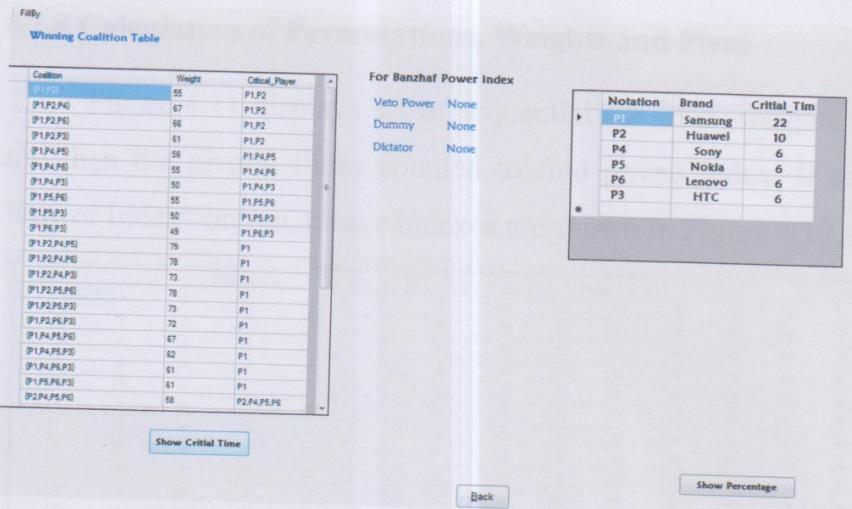


Figure 4.9 Winning Coalition and Critical_Times Tables

4.2.7 Result of Page Using Banzhaf Power Index

Figure 4.10 is the power index result by using Banzhaf Power Index. Banzhaf Power Index is the ratio of the voter's Banzhaf Power to total critical times.

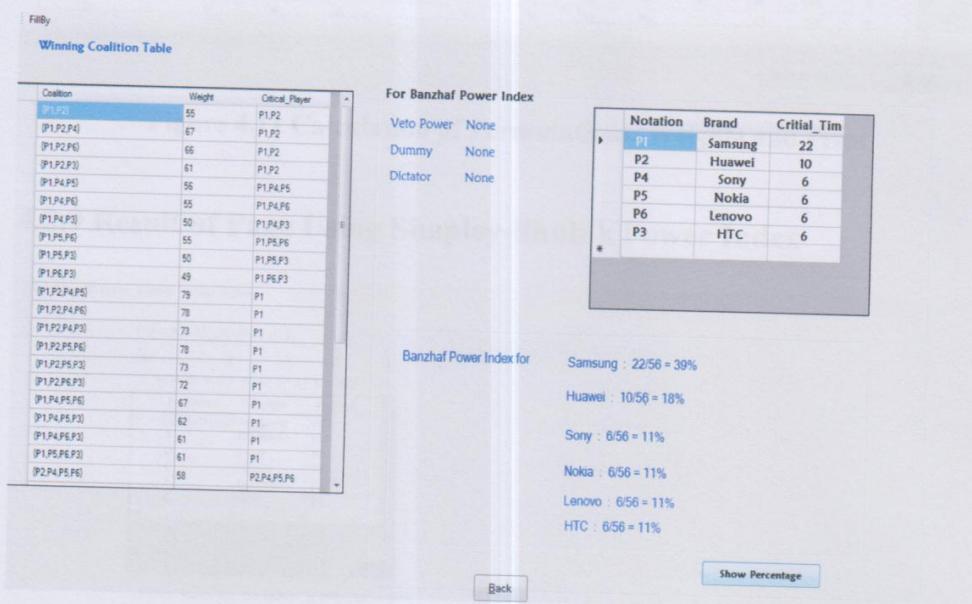


Figure 4.10 Result of Page Using Banzhaf Power Index

4.2.8 Calculation of Permutations, Weights and Pivot

Figure 4.11 displays set of sequential coalition, weights and pivot and then the pivotal times counted to find power index. If user clicks “Power Index” button, power indexes are shown in Figure 4.12.

Per_ID	Permutations						Weight					Pivot
1	A	B	C	D	E	F	14	23	31	38	43	45
2	A	B	C	D	F	E	14	23	31	38	40	45
3	A	B	C	E	D	F	14	23	31	36	43	45
4	A	B	C	E	F	D	14	23	31	36	38	45
5	A	B	C	F	D	E	14	23	31	33	40	45
6	A	B	C	F	E	D	14	23	31	33	38	45
7	A	B	D	C	F	E	14	23	30	38	40	45
8	A	B	D	C	E	F	14	23	30	38	43	45
9	A	B	D	E	C	F	14	23	30	25	43	45
10	A	B	D	E	F	C	14	23	30	35	37	45
11	A	B	D	F	C	E	14	23	30	32	40	45
12	A	B	D	F	E	C	14	23	30	32	37	45
13	A	B	E	C	D	F	14	23	28	36	43	45
14	A	B	E	C	F	D	14	23	28	36	38	45
15	A	B	E	D	C	F	14	23	28	35	43	45
16	A	B	E	D	F	C	14	23	28	35	37	45
17	A	B	E	F	C	D	14	23	28	30	38	45
18	A	B	E	F	D	C	14	23	28	30	37	45
19	A	B	F	C	E	D	14	23	25	33	38	45
20	A	B	F	C	D	E	14	23	25	33	40	45
21	A	B	F	D	C	E	14	23	25	32	40	45
22	A	B	F	D	E	C	14	23	25	32	37	45
23	A	B	F	E	C	D	14	23	25	30	38	45
24	A	B	F	E	D	C	14	23	25	30	37	45
25	A	C	B	D	E	F	14	22	31	38	43	45
26	A	C	B	D	F	E	14	22	31	38	40	45

Power Index

Back

Figure 4.11 Calculation of Permutations, Weights and Pivot

4.2.9 Result of Page Using Shapley-Shubik Power Index

By Using Shapely_Shubik Power Index

[49 : 32 ,23 ,12 ,11 ,6 ,12]
[49 : A ,B ,D ,E ,F ,C]
 $n! = 6! = 6 * 5 * 4 * 3 * 2 * 1 = 720$

Notation	HName	Tcount
A	Samsung	287
B	Huawei	144
D	Sony	72
E	Nokia	73
F	Lenovo	72
C	HTC	72

Power Index for 'Samsung' is 287/720 = 40%
Power Index for 'Huawei' is 144/720 = 20%
Power Index for 'Sony' is 72/720 = 10%
Power Index for 'Nokia' is 73/720 = 10%
Power Index for 'Lenovo' is 72/720 = 10%
Power Index for 'HTC' is 72/720 = 10%

Back

Figure 4.12 Result of Page Using Shapley-Shubik Power Index

After processing the permutation, this system counts the pivotal times for each brand. Figure 4.11 is the power index of the page by using Shapley-Shubik Power Index.

4.2.10 Comparing Result of Page

By using Banzhaf Power Index for Handset Brands	By using Shapley_Shubik Power Index for Handset Brands
Power index for Samsung = 22/56 = 39%	Power index for Samsung 287/720 = 40%
Power index for Huawei = 10/56 = 18%	Power index for Huawei 144/720 = 20%
Power index for Sony = 6/56 = 11%	Power index for Sony 72/720 = 10%
Power index for Nokia = 6/56 = 11%	Power index for Nokia 73/720 = 10%
Power index for Lenovo = 6/56 = 11%	Power index for Lenovo 72/720 = 10%
Power Index for HTC = 6/56 = 11%	Power index for HTC 72/720 = 10%

[Show Chart](#)

Figure 4.13 Comparing Result of Page

This page is shown in comparison result from Banzhaf Index and Shapley-Shubik Index. If user clicks the show chart button, Figure 4.14 is shown.

4.2.11 Comparison Result by Chart

Figure 4.14 is the comparison result showing chart. In this Figure, power index result for most popular handset brands is compared by using Banzhaf Power and Shapley-Shubik Power Index. Shapley-Shubik Power index gives more accurate power index result than Banzhaf Power index. So, Shapley-Shubik Power index is the best.

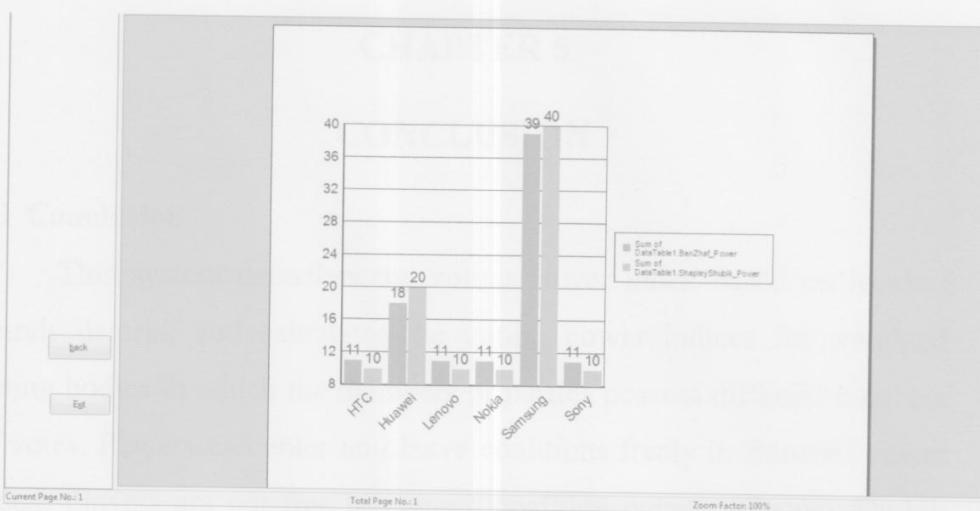


Figure 4.14 Comparison Result by Chart

The final part of the system is the comparison result by chart. This part shows the comparison results of the total power for most popular brands in the market. In this part, there is no specific depiction. But it's presented in a graphical chart form, so that user can easily understand the results. The chart shows the total power for most popular brands in the market. It is a bar chart with three bars for each brand. The first bar is light gray and the second bar is dark gray. The third bar is medium gray. The chart is titled "Comparison Result by Chart". The Y-axis represents the total power, ranging from 8 to 40. The X-axis lists the brands: HTC, Huawei, Lenovo, Nokia, Samsung, and Sony. The data is as follows:

For HTC, the total power is 11 (BanDha_Power) and 10 (ShapeyShah_Power). For Huawei, the total power is 18 (BanDha_Power) and 20 (ShapeyShah_Power). For Lenovo, the total power is 11 (BanDha_Power) and 10 (ShapeyShah_Power). For Nokia, the total power is 11 (BanDha_Power) and 10 (ShapeyShah_Power). For Samsung, the total power is 39 (BanDha_Power) and 40 (ShapeyShah_Power). For Sony, the total power is 11 (BanDha_Power) and 10 (ShapeyShah_Power). Further work is needed to develop more of the visualizations and comparisons with existing data to determine and compare the results. To understand the functioning of this part, the following steps are taken in this work for various companies and their total power. The comparison

CHAPTER 5

CONCLUSION

5.1 Conclusion

This system describes the voting power index based on handset brands dataset, and calculates the voting power indices for weighted voting bodies in which the members or parties possess different numbers of votes. Players can enter and leave coalitions freely in Banzhaf power index. Players are not free to leave a coalition once in Shapley-Shubik power index. It is more accurate to measure a player's power using either the Banzhaf power index or the Shapley-Shubik power index.

The two power indexes often come up with different measures of power for most popular brand yet neither one is necessarily a more accurate depiction. Banzhaf power index and Shapley-Shubik power show final results with percentage. Banzhaf power index gives a popularity brand with 39% and Shapley-Shubik index gives a popularity brand with 40% by 71 voters. So, Shapley-Shubik power index is better than Banzhaf power index which gives more percentage result.

5.2 Limitation and Further Extension

This system applies plurality method to rank order. This system compares the two power index methods, Banzhaf power index and Shapley-Shubik power index. It is further possible to extend the different types of ranking method for voting and can calculate voting power for the different application in this system.

Further work is needed to develop models of individual voters in a way consistent with available data on elections and voting, and to understand the implications of these models for voting power. Voting systems work for various purposes and in various domains. The analysis

of such systems is useful in understanding their theory of operation and the underlying assumptions made by several researchers to simplify their deployment. Theoretical and experimental analysis of the behavior of voting schemas is thus an important research topic. There have been several investigations on the theoretical analysis of voting systems.

Many real world organizations use such systems for example the IMF, World Bank, EU Council, corporations and political parties. Although power indexes are often considered as mathematical definitions, they ultimately depend on statistical models of voting. Voting is a general technique that finds application and acceptance in many domains including software systems.

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