

Approximate Entropy (ApEn) and Sample Entropy (SampEn)

In Time Series Analysis:

Both **Approximate Entropy (ApEn)** and **Sample Entropy (SampEn)** measure the *complexity* or *unpredictability* of a time series. They help us understand how regular or random the data pattern is, which relates directly to forecastability.

1. Approximate Entropy (ApEn)

- Measures the likelihood that patterns in data remain similar on the next comparison.
- **Small ApEn:** Time series is more regular and predictable → **Forecastability is possible.**
- **Large ApEn:** Time series is irregular or random → **Forecasting is difficult.**

Formula:

$$\text{ApEn}(m, r, N) = \Phi_m(r) - \Phi_{m+1}(r)$$

where:

- m = length of compared runs
- r = filtering level (tolerance)
- N = length of the time series

2. Sample Entropy (SampEn)

- Improved version of ApEn that avoids self-matching and is less dependent on data length.
- **Small SampEn:** Time series is regular and has repeating patterns → **Forecastability is possible.**
- **Large SampEn:** Time series is highly irregular → **Forecasting is difficult.**

Formula:

$$\text{SampEn}(m, r, N) = -\ln [A / B]$$

where:

- A = number of $(m+1)$ -length matching patterns
- B = number of m -length matching patterns
- m = embedding dimension
- r = tolerance
- N = data length

Summary:

Both ApEn and SampEn quantify time series complexity. Smaller values indicate higher regularity and stronger forecastability, while larger values suggest randomness and weak predictability.