CSE 6242 / CX 4242
Apr 3, 2014

Text Analytics (Text Mining)

LSI (uses SVD), Visualization

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Some lectures are partly based on materials by Professors Guy Lebanon, Jeffrey Heer, John Stasko, Christos Faloutsos, Le Song

Singular Value Decomposition (SVD):
Motivation

Problem #1:

Text - LSI uses SVD find “concepts”

Problem #2:

Compression / dimensionality reduction

SVD - Motivation

Problem #1: text - LSI: find "concepts"

term / document	data	information	retrieval	brain	lung
CS-TR1	1	1	1	0	0
CS-TR2	2	2	2	0	0
CS-TR3	1	1	1	0	0
CS-TR4	5	5	5	0	0
MED-TR1	0	0	0	2	2
MED-TR2	0	0	0	3	3
MED-TR3	0	0	0	1	1

SVD - Motivation

Customer-product, for recommendation system:

$$
\begin{array}{r c}
 & \begin{array}{ccccc} \text{bread} & \text{lettuce} & \text{tomatos} & \text{beef} & \text{chicken} \end{array} \\
\begin{array}{r} \\ \\ \text{vegetarians} \\ \\ \\ \text{meat eaters} \\ \\ \end{array} &
\left[\begin{array}{ccccc}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{array}\right]
\end{array}
$$

SVD - Motivation

- problem #2: compress / reduce dimensionality

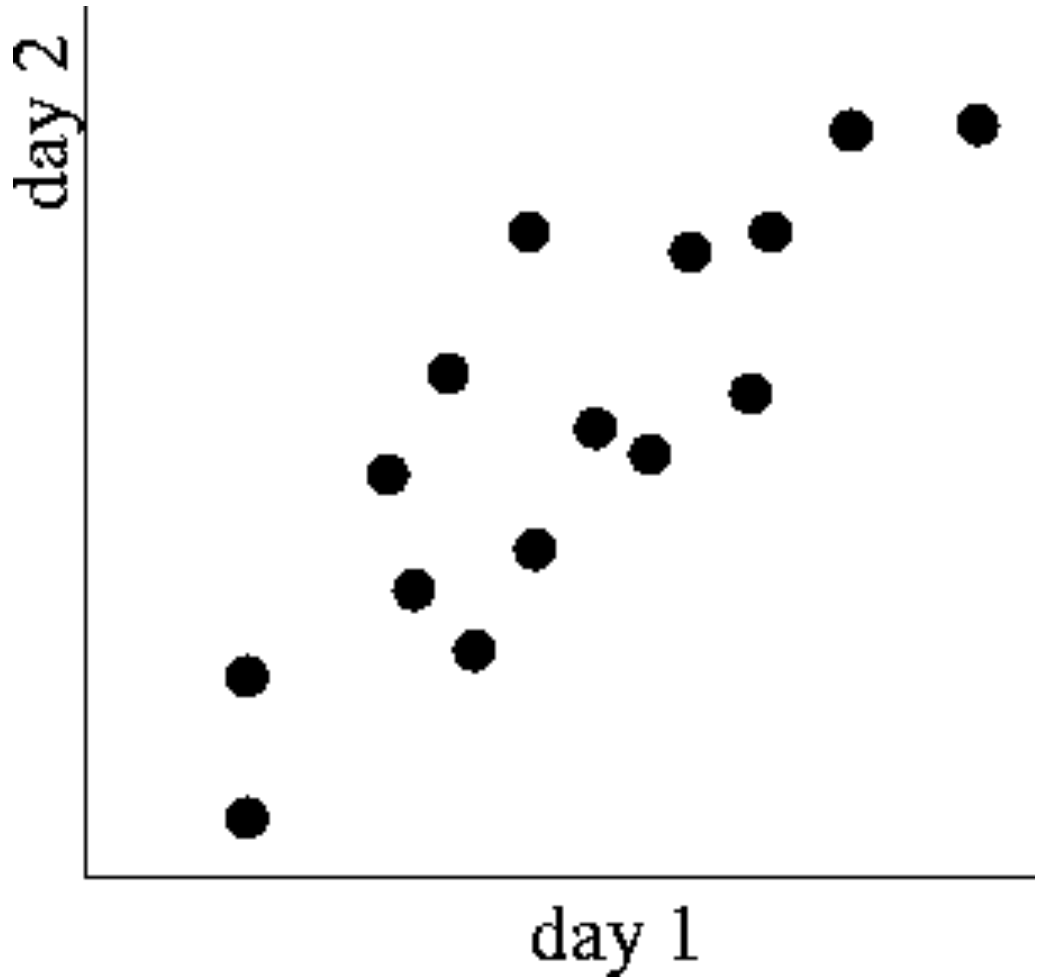
Problem - Specification

~10^6 rows; ~10^3 columns; no updates;

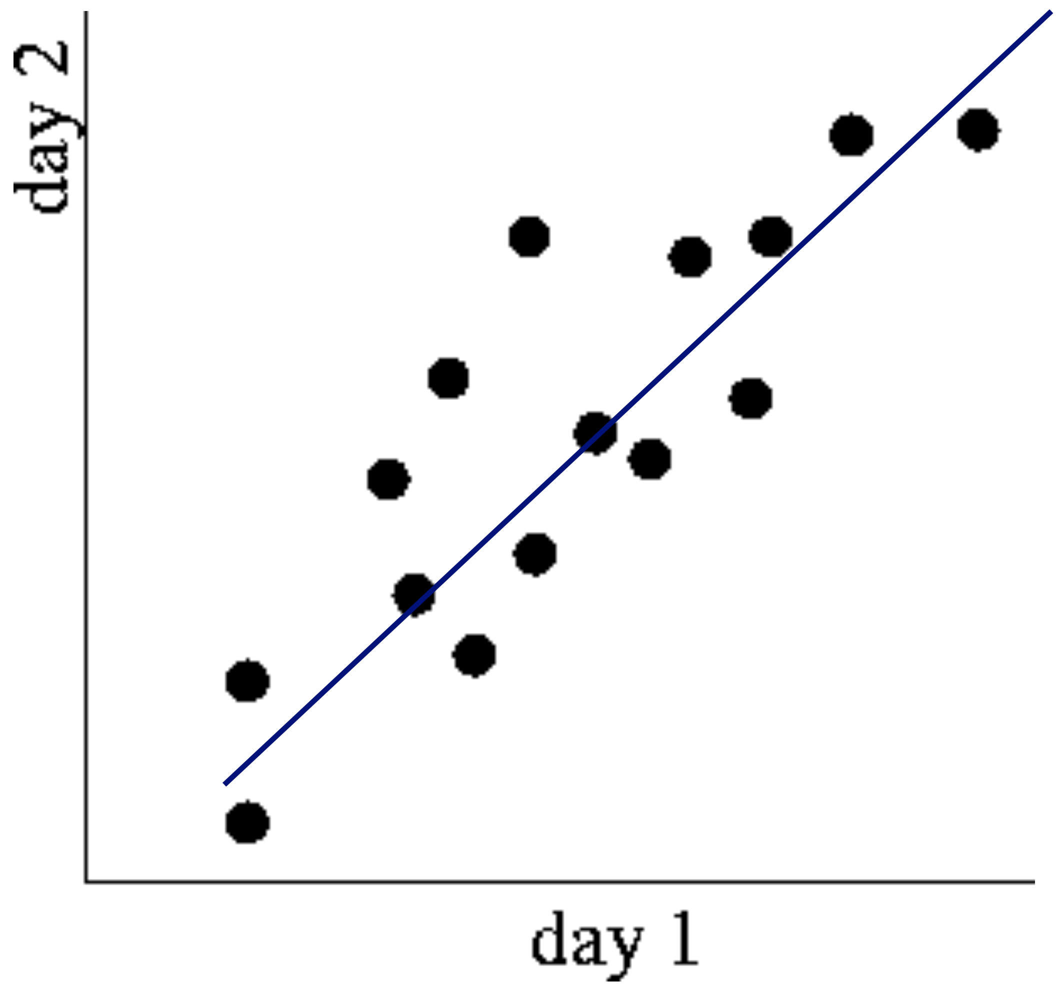
Random access to any cell(s); small error: OK

day customer	We 7/10/96	Th 7/11/96	Fr 7/12/96	Sa 7/13/96	Su 7/14/96
ABC Inc.	1	1	1	0	0
DEF Ltd.	2	2	2	0	0
GHI Inc.	1	1	1	0	0
KLM Co.	5	5	5	0	0
Smith	0	0	0	2	2
Johnson	0	0	0	3	3
Thompson	0	0	0	1	1

SVD - Motivation



SVD - Motivation



SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

3×2 $\quad$ 2×1

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \text{x} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$3 \text{ x } 2 \qquad 2 \text{ x } 1 \qquad 3 \text{ x } 1$

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \text{ x } \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ \\ \\ \end{bmatrix}$$

$$\mathit{3} \text{ x } \mathit{2} \qquad \mathit{2} \text{ x } \mathit{1} \qquad \mathit{3} \text{ x } \mathit{1}$$

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \mathrm{x} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}$$

3×2 $\quad$ 2×1 $\quad$ 3×1

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

SVD - Definition

$$\boxed{\mathbf{A}_{[\mathbf{n} \times \mathbf{m}]} = \mathbf{U}_{[\mathbf{n} \times \mathbf{r}]} \, \Lambda_{[\,\mathbf{r} \times \mathbf{r}]} \, (\mathbf{V}_{[\mathbf{m} \times \mathbf{r}]})^{T}}$$

A: n x m matrix
e.g., n documents, m terms

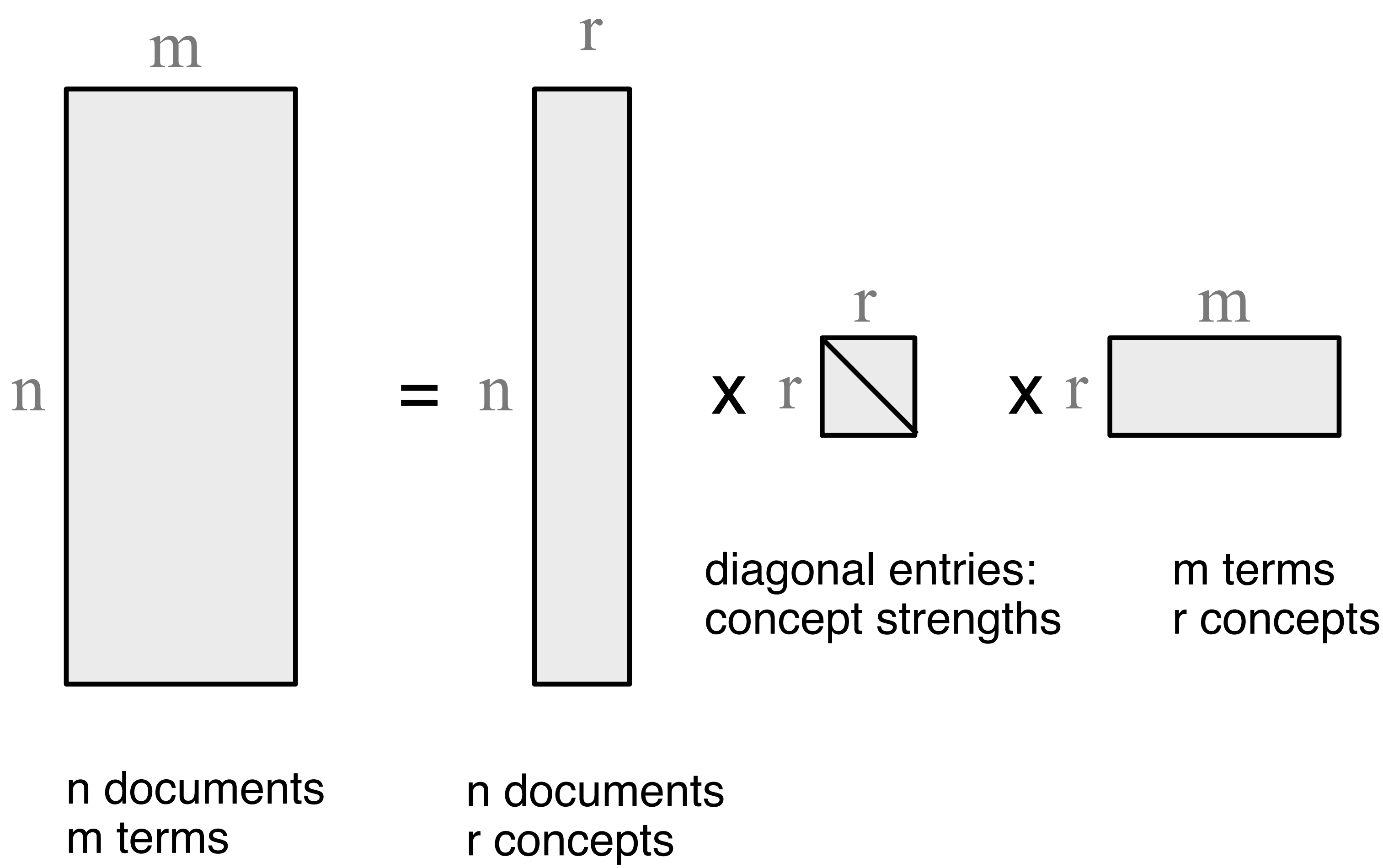
U: n x r matrix
e.g., n documents, r concepts

Λ: r x r diagonal matrix
r : rank of the matrix; strength of each 'concept'

V: m x r matrix
e.g., m terms, r concepts

SVD - Definition

$$\boxed{A_{[n \times m]} = U_{[n \times r]}\, \Lambda_{[r \times r]}\, (V_{[m \times r]})^T}$$



SVD - Properties

THEOREM [Press+92]:

always possible to decompose matrix $\mathbf{A}$ into

$\mathbf{A} = \mathbf{U}\,\Lambda\,\mathbf{V}^T$

$\mathbf{U}, \Lambda, \mathbf{V}$: unique, most of the time

$\mathbf{U}, \mathbf{V}$: column orthonormal

i.e., columns are unit vectors, orthogonal to each other

$\mathbf{U}^T\,\mathbf{U} = \mathbf{I}$

$\mathbf{V}^T\,\mathbf{V} = \mathbf{I}$

($\mathbf{I}$: identity matrix)

Λ: diagonal matrix with non-negative diagonal entires, sorted in decreasing order

SVD - Example

$\mathbf{A} = \mathbf{U}\ \Lambda\ \mathbf{V}^T$ - example:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \\ \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\ \boldsymbol{\Lambda}\ \mathbf{V}^T$ - example:

CS-concept

MD-concept

$$
\begin{array}{c c}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \\ \end{array} &
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}
\end{array}
=
\begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix}
\text{ x }
\begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix}
\text{ x }
\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\, \mathbf{\Lambda}\, \mathbf{V}^T$ - example:

doc-to-concept
similarity matrix

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \\ \\ \text{CS} \\ \\ \\ \text{MD} \\ \\ \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\begin{array}{c}
\begin{array}{cc} \text{CS-concept} & \text{MD-concept} \end{array} \\
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\end{array}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\ \Lambda\ \mathbf{V}^T$ - example:

‘strength’ of CS-concept

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \\ \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\,\Lambda\,\mathbf{V}^T$ - example:

term-to-concept similarity matrix

CS-concept

$$\begin{array}{cc} & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\ \begin{array}{c} \\ \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \mathrm{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \mathrm{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\ \boldsymbol{\Lambda}\ \mathbf{V}^T$ - example:

term-to-concept similarity matrix

CS-concept

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \\ \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\mathrm{x}
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\mathrm{x}
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

- $\mathbf{U}$: document-to-concept similarity matrix
- $\mathbf{V}$: term-to-concept sim. matrix
- Λ: its diagonal elements: ‘strength’ of each concept

SVD – Interpretation #1

'documents', 'terms' and 'concepts':

Q: if $\mathbf{A}$ is the document-to-term matrix, what is $\mathbf{A}^T \mathbf{A}$?

A:

Q: $\mathbf{A}\,\mathbf{A}^T$?

A:

SVD – Interpretation #1

'documents', 'terms' and 'concepts':

Q: if $\mathbf{A}$ is the document-to-term matrix, what is $\mathbf{A}^T \mathbf{A}$?

A: term-to-term ([m x m]) similarity matrix

Q: $\mathbf{A} \mathbf{A}^T$?

A: document-to-document ([n x n]) similarity matrix

SVD properties

- $\mathbf{V}$ are the eigenvectors of the *covariance matrix* $\mathbf{A}^\mathsf{T}\mathbf{A}$

$$\mathbf{X}^\mathsf{T}\mathbf{X} = \left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\mathsf{T}\right)^\mathsf{T}\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\mathsf{T}\right) = \mathbf{V}\boldsymbol{\Sigma}^2\mathbf{V}^\mathsf{T}$$

- $\mathbf{U}$ are the eigenvectors of the *Gram (inner-product) matrix* $\mathbf{A}\mathbf{A}^\mathsf{T}$

$$\mathbf{X}\mathbf{X}^\mathsf{T} = \left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\mathsf{T}\right)\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\mathsf{T}\right)^\mathsf{T} = \mathbf{U}\boldsymbol{\Sigma}^2\mathbf{U}^\mathsf{T}$$

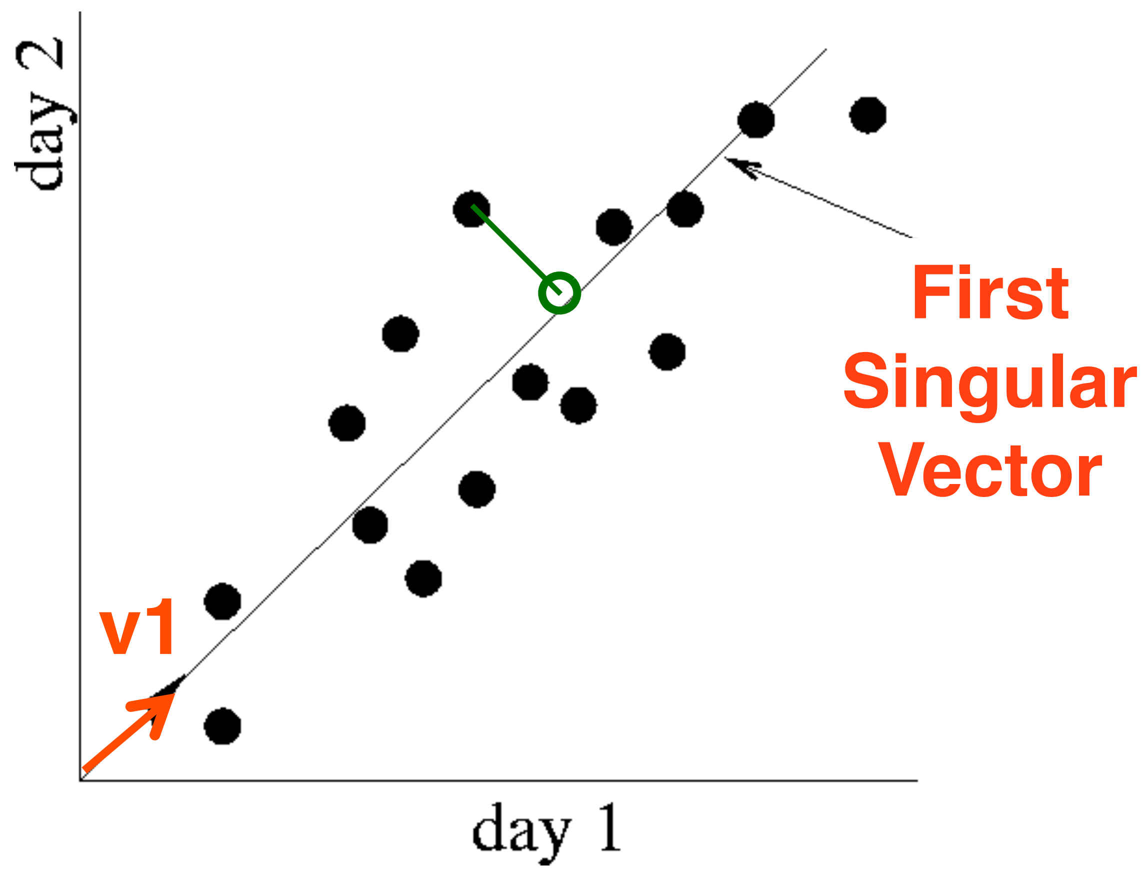
Thus, SVD is closely related to PCA, and can be numerically more stable.
For more info, see:

http://math.stackexchange.com/questions/3869/what-is-the-intuitive-relationship-between-svd-and-pca
Ian T. Jolliffe, *Principal Component Analysis* (2nd ed), Springer, 2002.
Gilbert Strang, *Linear Algebra and Its Applications* (4th ed), Brooks Cole, 2005.

SVD - Interpretation #2

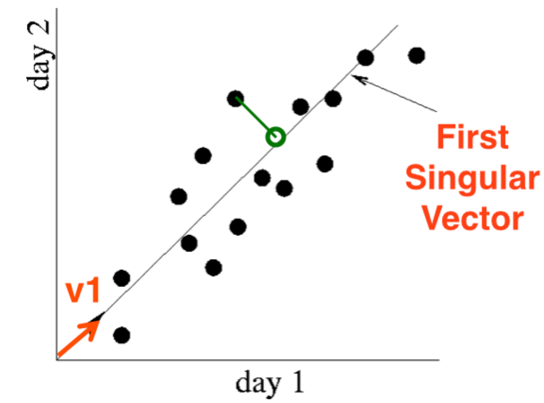
best axis to project on

('best' = min sum of squares of projection errors)



min RMS error

SVD - Interpretation #2



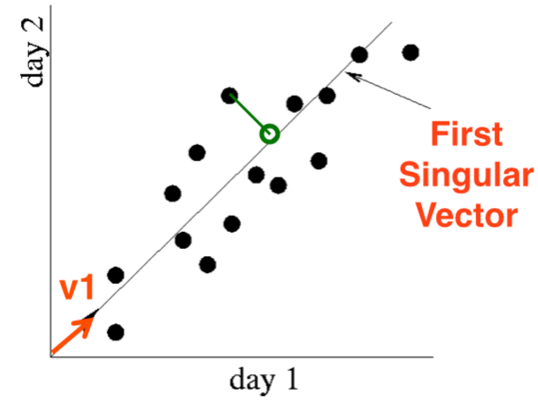
- $\mathbf{A} = \mathbf{U}\ \Lambda\ \mathbf{V}^{T}$ - example:

variance ('spread') on the v1 axis

v1

$$
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\mathbf{x}
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\mathbf{x}
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Interpretation #2



- $\mathbf{A} = \mathbf{U}\, \mathbf{\Lambda}\, \mathbf{V}^T$ - example:
  - $\mathbf{U}\,\mathbf{\Lambda}$ gives the **coordinates** of the points in the projection axis

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}
=
\begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix}
\times
\begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix}
\times
\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}
$$

SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \mathrm{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \mathrm{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?
- A: set the smallest singular values to zero:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & \cancel{5.29} \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 & 0 \\ 0 & \cancel{0} \end{bmatrix} \text{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 & \cancel{0} \\ 0.36 & \cancel{0} \\ 0.18 & \cancel{0} \\ 0.90 & \cancel{0} \\ 0 & \cancel{0.53} \\ 0 & \cancel{0.80} \\ 0 & \cancel{0.27} \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & \cancel{0} \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0.71} & \cancel{0.71} \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 \\ 0.36 \\ 0.18 \\ 0.90 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 \end{bmatrix} \text{x}$$

$$\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & \multicolumn{2}{c}{\multirow{3}{*}{$\boxed{\begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix}}$}} \\ 0 & 0 & 0 & & \\ 0 & 0 & 0 & & \end{bmatrix}$$

SVD - Interpretation #3

- finds non-zero ‘blobs’ in a data matrix

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}
$$

SVD - Interpretation #3

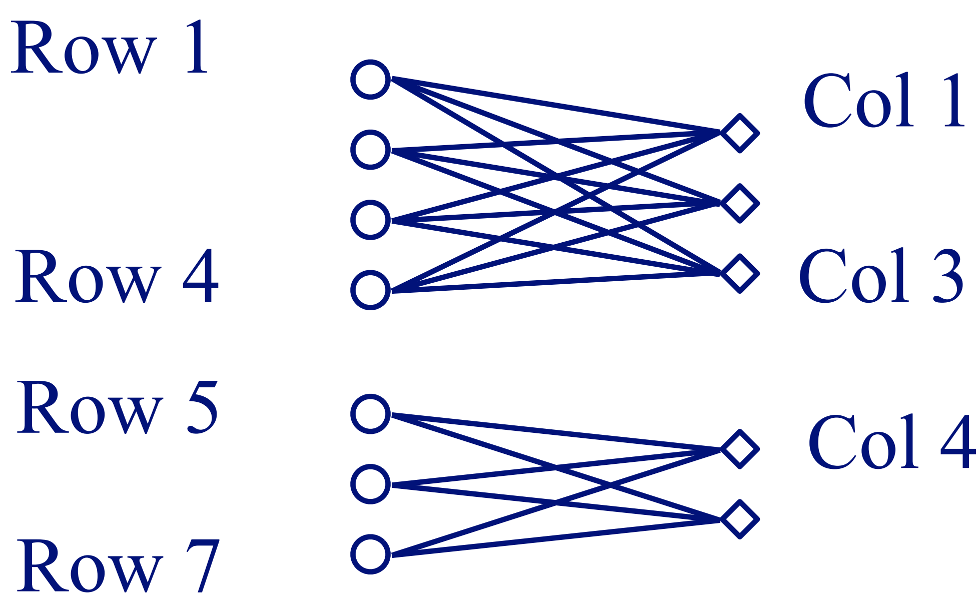
- finds non-zero 'blobs' in a data matrix

$$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #3

- finds non-zero 'blobs' in a data matrix =
- 'communities' (bi-partite cores, here)

$$
\left[\begin{array}{ccc|cc}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
\hline
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{array}\right]
$$



SVD algorithm

- Numerical Recipes in C (free)

SVD - Interpretation #3

- Drill: find the SVD, 'by inspection'!
- Q: rank = ??

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} ?? \end{bmatrix} \text{ x } \begin{bmatrix} ?? \end{bmatrix} \text{ x } \begin{bmatrix} ?? \end{bmatrix}$$

SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/ cols)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} | & | \\ ?? & ?? \\ | & | \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} \text{———} ?? \text{———} \\ \text{———} ?? \text{———} \end{bmatrix}$$

SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/ cols)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \text{x} \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \text{x}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

orthogonal??

SVD - Interpretation #3

- column vectors: are orthogonal - but not unit vectors:

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 0 & 1/\mathrm{sqrt}(2) \\ 0 & 1/\mathrm{sqrt}(2) \end{bmatrix} \mathrm{x} \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \mathrm{x}
$$

$$
\begin{bmatrix} 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 0 & 0 \\ 0 & 0 & 0 & 1/\mathrm{sqrt}(2) & 1/\mathrm{sqrt}(2) \end{bmatrix}
$$

SVD - Interpretation #3

- and the singular values are:

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 0 & 1/\mathrm{sqrt}(2) \\ 0 & 1/\mathrm{sqrt}(2) \end{bmatrix} \mathrm{x} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \mathrm{x}
$$

$$
\begin{bmatrix} 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 0 & 0 \\ 0 & 0 & 0 & 1/\mathrm{sqrt}(2) & 1/\mathrm{sqrt}(2) \end{bmatrix}
$$

SVD - Interpretation #3

- Q: How to check we are correct?

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/sqrt(3) & 0 \\ 1/sqrt(3) & 0 \\ 1/sqrt(3) & 0 \\ 0 & 1/sqrt(2) \\ 0 & 1/sqrt(2) \end{bmatrix} \text{x} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \text{x}
$$

$$
\begin{bmatrix} 1/sqrt(3) & 1/sqrt(3) & 1/sqrt(3) & 0 & 0 \\ 0 & 0 & 0 & 1/sqrt(2) & 1/sqrt(2) \end{bmatrix}
$$

SVD - Interpretation #3

- A: SVD properties:
  - matrix product should give back matrix $\mathbf{A}$
  - matrix $\mathbf{U}$ should be column-orthonormal, i.e., columns should be unit vectors, orthogonal to each other
  - ditto for matrix $\mathbf{V}$
  - matrix Λ should be diagonal, with non-negative values

SVD - Complexity

O(n*m*m) or O(n*n*m) (whichever is less)

Faster version, if just want singular values

or if we want first k singular vectors

or if the matrix is sparse [Berry]

No need to write your own!

Available in most linear algebra packages (LINPACK, matlab, Splus/R, mathematica ...)

References

- Berry, Michael: http://www.cs.utk.edu/~lsi/
- Fukunaga, K. (1990). Introduction to Statistical Pattern Recognition, Academic Press.
- Press, W. H., S. A. Teukolsky, et al. (1992). Numerical Recipes in C, Cambridge University Press.

Case study - LSI

Q1: How to do queries with LSI?

Q2: multi-lingual IR (english query, on spanish text?)

Case study - LSI

Q1: How to do queries with LSI?

Problem: Eg., find documents with 'data'

$$
\begin{array}{c|c}
 & \text{data} \quad \text{inf.} \quad \text{retrieval} \quad \text{brain} \quad \text{lung} \\
\begin{array}{c} \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \\ \end{array} &
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}
\end{array}
=
\begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix}
\times
\begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix}
\times
\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}
$$

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into ‘concept space’ – how?

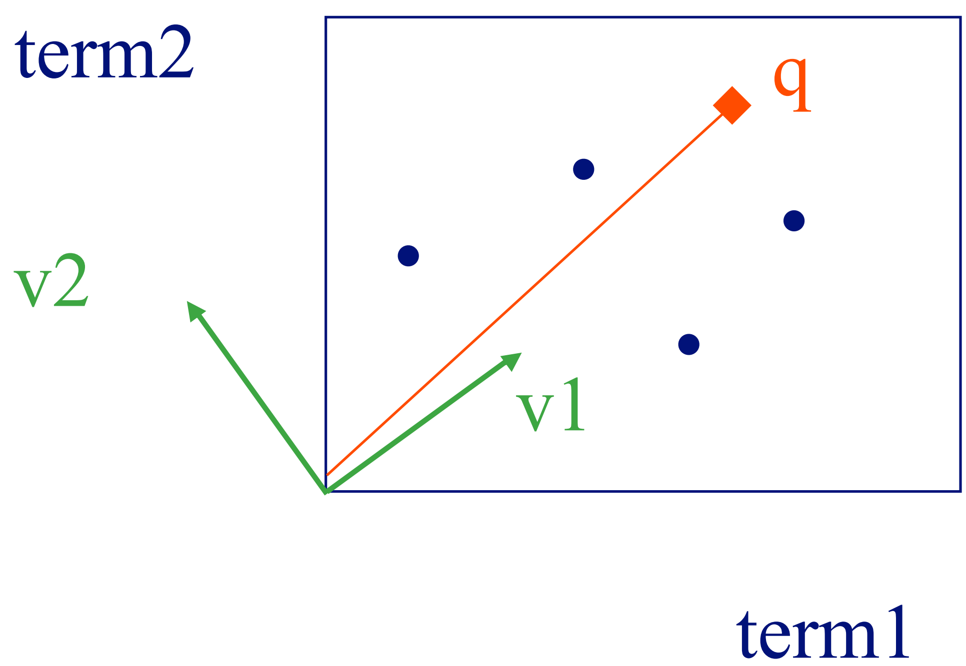
$$
\begin{array}{c}
\\
\text{CS} \\
\\
\text{MD}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into ‘concept space’ – how?

$$q= \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$



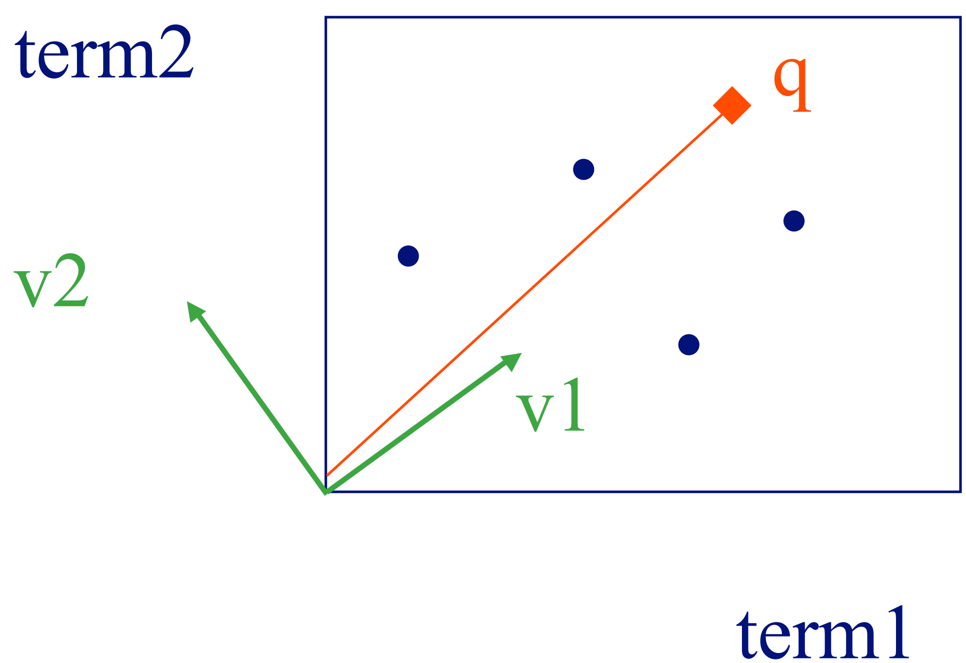
Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into ‘concept space’ – how?

$$q = \begin{array}{ccccc} \text{data} & \text{inf. retrieval} & \text{brain} & \text{lung} & \\ \end{array}$$

$$q = \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array}\right]$$



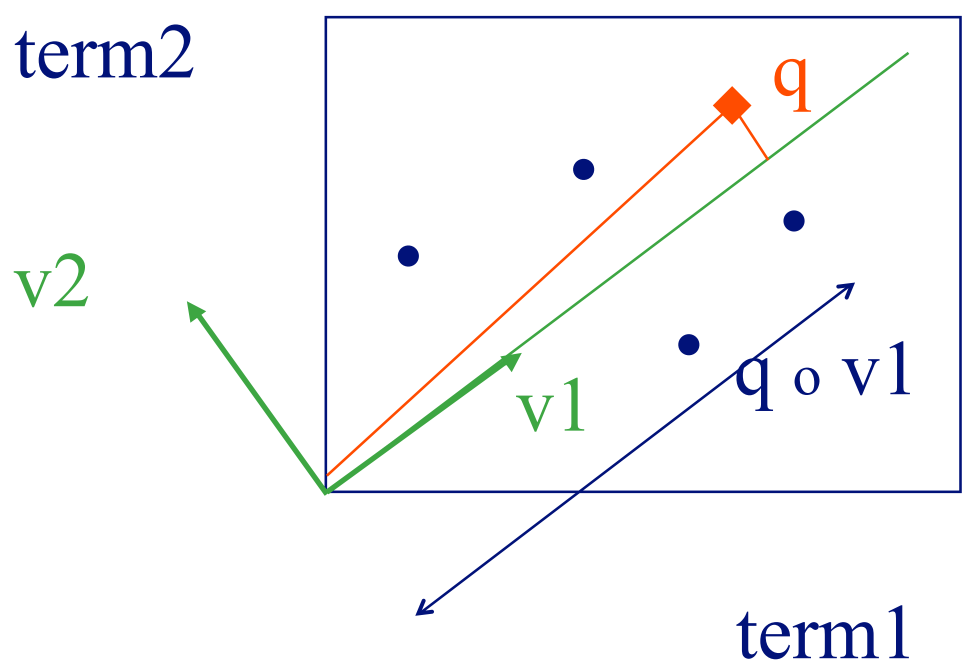
A: inner product
(cosine similarity)
with each ‘concept’ vector v_i

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into 'concept space' – how?

$$q = \begin{array}{c} \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



A: inner product (cosine similarity) with each 'concept' vector v_i

Case study - LSI

compactly, we have:

$$q\,\mathbf{V} = q_{concept}$$

Eg:

CS-concept

$$q = \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \end{array}$$

$$q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.58 & 0 \\ 0.58 & 0 \\ 0.58 & 0 \\ 0 & 0.71 \\ 0 & 0.71 \end{bmatrix} = \begin{bmatrix} 0.58 & 0 \end{bmatrix}$$

term-to-concept similarities

Case study - LSI

Drill: how would the document (‘information’, ‘retrieval’) be handled by LSI?

Case study - LSI

Drill: how would the document ('information', 'retrieval') be handled by LSI? A: SAME:

$d_{concept} = d\,\mathbf{V}$

Eg:

$$
\begin{array}{cccccc}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\
d= & \left[\begin{matrix} 0 & 1 & 1 & 0 & 0 \end{matrix}\right.
\end{array}
\left.\right]
\underbrace{\begin{bmatrix} 0.58 & 0 \\ 0.58 & 0 \\ 0.58 & 0 \\ 0 & 0.71 \\ 0 & 0.71 \end{bmatrix}}_{\text{term-to-concept similarities}}
= \overset{\text{CS-concept}}{\begin{bmatrix} 1.16 & 0 \end{bmatrix}}
$$

Case study - LSI

Observation: document ('information', 'retrieval') will be retrieved by query ('data'), although it does not contain 'data'!!

$$
\begin{array}{ccccccc}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} & \\
d= & [\,0 & 1 & 1 & 0 & 0\,] & \dashrightarrow [\,1.16 \quad 0\,] \\
q= & [\,1 & 0 & 0 & 0 & 0\,] & \dashrightarrow [\,0.58 \quad 0\,]
\end{array}
$$

CS-concept

Case study - LSI

Q1: How to do queries with LSI?

Q2: multi-lingual IR (english query, on spanish text?)

Case study - LSI

- Problem:
  - given many documents, translated to both languages (eg., English and Spanish)
  - answer queries across languages

Case study - LSI

- Solution: ~ LSI

$$\begin{array}{c} \\ \\ \\ \mathrm{CS} \\ \\ \\ \mathrm{MD} \\ \\ \end{array}
\begin{array}{ccccc:ccccc}
\text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} & \text{datos} & \text{informacion} & & & \\
\hline
1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 & 1 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 & 5 & 5 & 4 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 2 & 3 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1
\end{array}$$

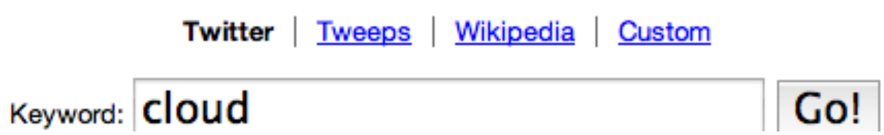
Switch Gear to **Text Visualization**

What comes up to your mind?

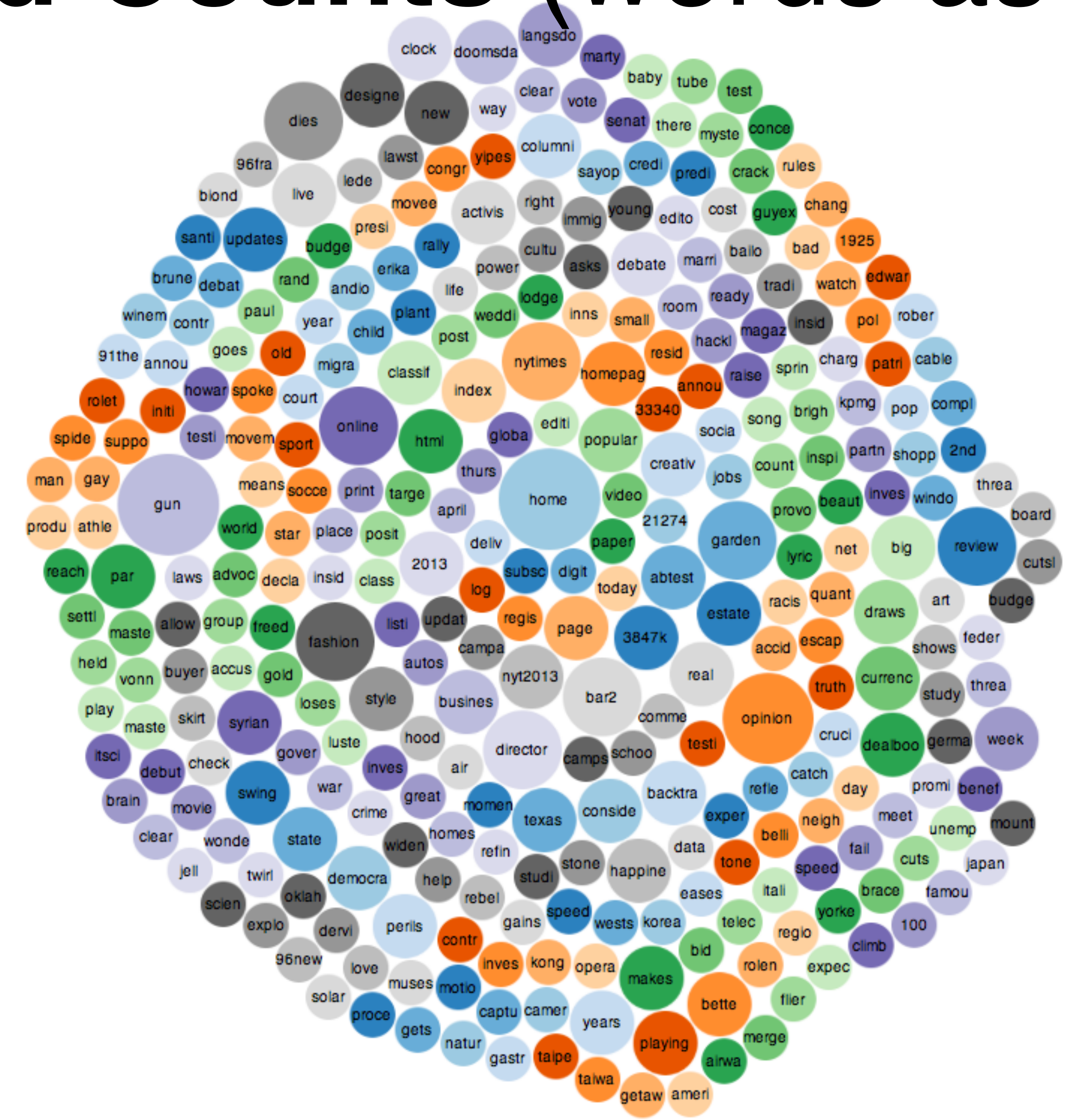
What visualization have you seen before?

Word/Tag Cloud (still popular?)

http://www.wordle.net



Word Counts (words as bubbles)



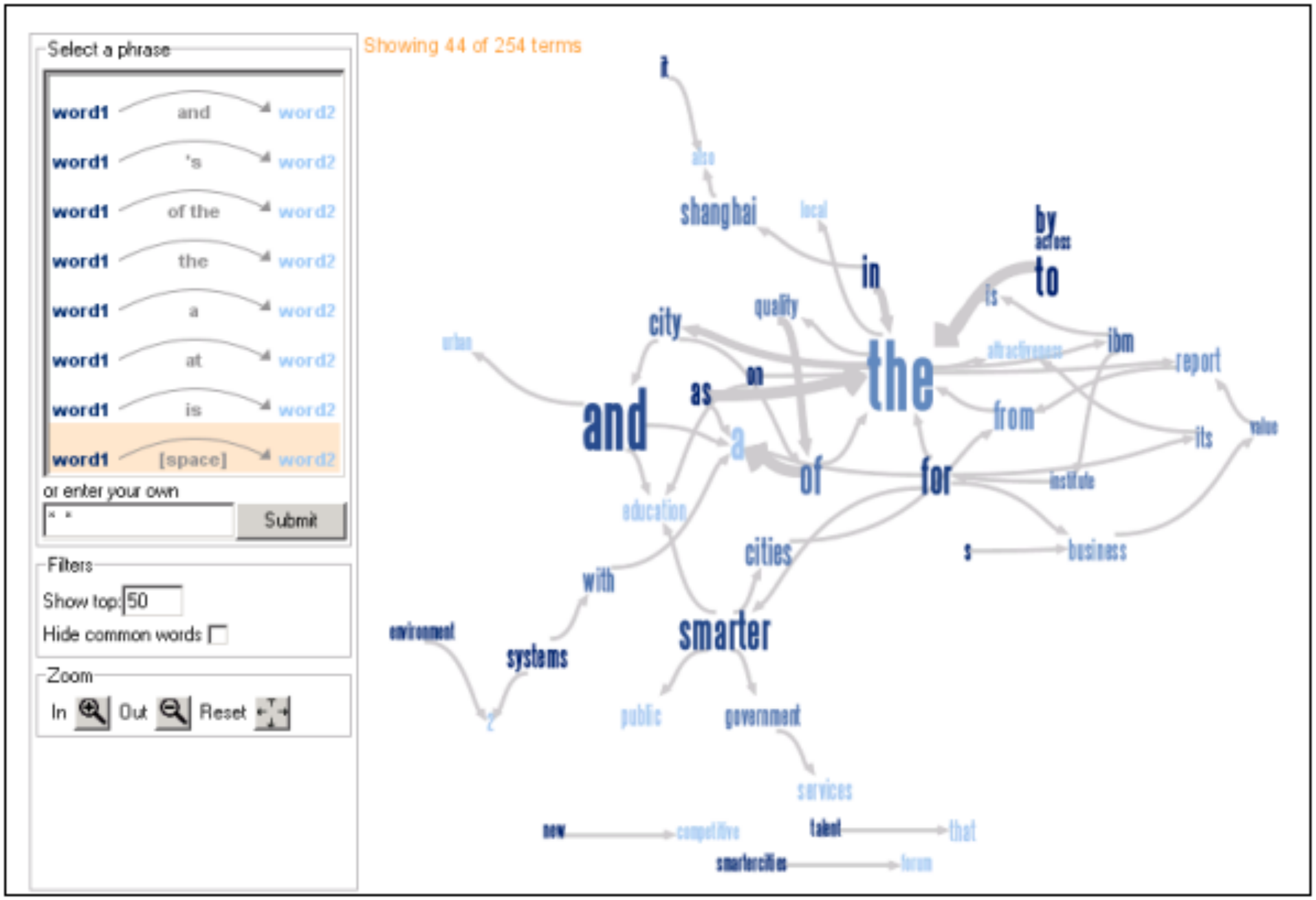
http://www.infocaptor.com/bubble-my-page

Word Tree

http://www.jasondavies.com/wordtree/

Phrase Net

Visualize pairs of words that satisfy a particular pattern, e.g., X and Y



http://www-958.ibm.com/software/data/cognos/manyeyes/page/Phrase_Net.html