

# Using Explanatory Variables

- Now let  $X_1$  be an indicator variable, which takes the value zero if an individual is on the standard drug, and unity if an individual is on the new drug.
- If  $x_{1i}$  is the value of  $X_1$  for the  $i^{\text{th}}$  individual in the study,  $i=1, 2, \dots, n$ , the hazard function for this individual can be written as

$$h_i(t) = e^{\beta x_{1i}} h_0(t)$$

- where  $x_i=1$  if the  $i^{\text{th}}$  individual is on the new treatment and  $x_i=0$  otherwise.
- This is the proportional hazards model for the comparison of two treatment groups.
- Let  $h_0(t)$  be the hazard function for an individual for whom the values of  $X_1$  zero.
  - The function  $h_0(t)$  is called the baseline hazard function.

# Using Exploratory Variables

- To get to the Cox Regression model we now generalize to the situation where there also other explanatory variables , such as
  - Gender
  - Age,.....
  - etc...

These are represented by variables

- Then the hazard of death at a particular time also depends on the values  $x_2, \dots, x_p$  of  $p$  explanatory variables,  $X_1, X_2, \dots, X_p$ .

$$\text{So } h_i(t) = e^{(\beta_1 * x_{1i} + \beta_2 * x_{2i} + \dots)} h_0(t)$$

- *Note:*
  - *These variables can be continuous, categorical, Binary.....*
  - *The values of these variables will be assumed to have been recorded at the time origin of the study.*

# The General form of the Proportions Hazard Model

- Since this model can be re-expressed in the format,

$$\text{Log}(h_i(t)/h_o(t)) = \text{Log}(\psi) = \beta_1 * x_{1i} + \beta_2 * x_{2i} + \dots$$

- The proportional hazards model may also be regarded as a linear model for the logarithm of the hazard ratio.
- It is a generalized linear model with the link function being the log function
- There are other possible forms for  $\psi$ , **but the choice  $\psi(x_i) = \exp(\beta'x_i)$**  leads to the most commonly used model for survival data.
  - Alternatives are the Weibull parametric model, the lognormal model, the Gompertz model
- Notice that there is no constant term in the linear component of the proportional hazards model.
  - If a constant term  $\beta_0$ , say, were included, the baseline hazard function could simply be rescaled by dividing  $h_o(t)$  by  $\exp(\beta_0)$ , and the constant term would cancel out.
  - Also we have made no assumptions concerning the actual form of the baseline hazard function  $h_o(t)$ .