<u>**Question Bank**</u>

Semester: B.Tech 2nd **Branch: Common to all Branches**
Subject: Mathematics-II **Course Code: A000212(014)**

Unit-I
<u>**MULTIVARIABLE CALCULUS (Integration)**</u>

2 marks Questions

1) Evaluate $\int_1^2\int_1^3 xy^2 dxdy$.
2) Evaluate $\int_0^4\int_0^{x^2} e^{y/x} dxdy$.
3) State Stoke's theorem.
4) State Gauss Divergence theorem.
5) State Green's theorem.

4 marks Questions

1) Evaluate $\int_{-1}^1\int_0^z\int_{x-z}^{x+z}(x+y+z)dxdydz$.
2) Calculate the volume of the solid bounded by the planes x = 0, y = 0, x+y+z = 1 and z = 0.
3) If $\vec{F} = 3xy\hat{\imath} - (y^2)\hat{\jmath}$, evaluate $\int_C \vec{F}\cdot d\vec{R}$, where C is the curve $y = 2x^2$ in the *xy*-plane from (*0*,*0*) to (*1*,*2*).
4) If $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - xz)\hat{\jmath} + (z^2 - xy)\hat{k}$, then find its scalar potential $\emptyset$.
5) Apply Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\int_C xdy - ydx$.

8 marks Questions

1) Evaluate $\int_0^1\int_0^{\sqrt{1+x^2}}\frac{dxdy}{1+x^2+y^2}$.
2) Evaluate $\int_1^e\int_1^{\log y}\int_1^{e^x}\log z\ dzdxdy$
3) Evaluate by changing the order of integration $\int_0^a\int_{\sqrt{ax}}^a\frac{y^2dxdy}{\sqrt{y^4-a^2x^2}}$.
4) Evaluate by changing the order of integration $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$.
5) Evaluate by changing the order of integration $\int_0^\infty\int_0^x xe^{-x^2/y} dydx$.
6) Show that the area enclosed by the parabola $y^2 = 4ax \ and \ x^2 = 4ay$ is $\frac{16}{3}a^2$.
7) Find the area included between the parabola $y = 4x - x^2$ and the line *y* = *x*.
8) Find the volume bounded by the cylinder $x^2+y^2 = 4$ and the planes $y + z = 4 \ and \ z = 0$.
9) Find the volume enclosed by the cylinders $x^2+y^2 = 2ax \ and \ z^2 = 2ax$.
10) Compute the line integral $\int y^2dx - x^2dy$, about the triangle whose vertices are (1,0), (0,1) and (-1,0).
11) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along:
    a. The straight line from (0,0,0) to (2,1,3).
    b. The curve defined by $x^2 = 4y, 3x^3 = 8z$ from *x*=0 to *x*=2.
12) Verify the Green's theorem for: $\int_C [(3x - 8y^2)dx + (4y - 6xy)dy]$, where C is boundary of the region bounded by lines $x = 0$, $y = 0$ and $x + y = 1$.
13) Verify Stoke's theorem for the vector field $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken round the rectangle bounded by $x = \pm a, y = 0, y = b$.
14) Evaluate $\int_S \vec{F}\cdot d\vec{S}$, where $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ and the surface S is the region bounded by $x^2 + y^2 = 4,\ z = 0 \ and \ z = 3$.

15) Evaluate by Stoke's theorem $\int_C (yzdx + zxdy + xydz)$ where C is the curve$x^2 + y^2 = 1, z = y^2$.

Unit –II

First Order Ordinary Differential Equation

2 Marks Questions:

1) Define Exact Differential Equation.
2) State necessary condition for a differential equation M dx + N dy=0 to be exact.
3) Define Integrating factor.
4) Write Clairaut's equation.
5) What is the general solution of $y = px + ap(1 - p)$.

4 Marks Questions:

1) Examine the Differential Equation $\frac{dy}{dx} = -\frac{\cos y}{y^2 - x\sin y}$ for Exactness.
2) Find the general solution of the equation $p = \sin(y - xp)$.
3) Solve $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2}$=0
4) Find the Integrating Factor of the differential equation $(ylogy)dx + (x - logy)\, dy = 0$
5) Solve the following equation
   $(\sec x \tan x \tan y - e^x)dx + \sec x \sec^2 y dy = 0$

8 Marks Questions:

1) Solve the following differential equations,
   (i) $\left(xy^2 - e^{1/x^3}\right)dx - x^2y\, dy = 0$
   (ii) $y(xy + 2x^2y^3)dx + x(xy - x^2y^2)dy = 0$
   (iii) $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$
   (iv) $x^4 \frac{dy}{dx} + x^3y + cosec\, xy = 0$
   (v) $(xy \sin xy + \cos xy)y\, dx + (xy \sin xy - \cos xy)x\, dy = 0$
   (vi) $(x^4e^x - 2m\, xy^2)dx + 2m\, x^2y\, dy = 0$
   (vii) $(1 + xy)y\, dx + (1 - xy)x\, dy = 0$
2) Solve the following differential equations,
   (i) $p^2 + 2py\, cotx = y^2$
   $(ii) p^3 + 2x\, p^2 - y^2p^2 - 2x\, y^2p = 0$
   (iii) $y = x\left[p + \sqrt{1 + p^2}\right]$
3) Solve the following differential equations,
   (i) $y = x + a\, tan^{-1}p$
   (ii) $y = xp^2 + p$
   (iiii) $y + px = x^4p^2$
4) Solve the following differential equations,
   (i) $p^3 - 4xy\, p + 8\, y^2 = 0$
   (ii) $p = \tan\left(x - \frac{p}{1+p^2}\right)$

UNIT – III

ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDER

2 Marks Questions:

1. Define linear differential equation.
2. Explain Method of variation of parameters.
3. Define Cauchy's and Legendre's linear differential equation.
4. Define Ordinary and Singular point.
5. Solve $\frac{d^2y}{dx^2}+y=0.$

4 Marks Questions:

1. Solve $\frac{d^2y}{dx^2}+2y=x^2e^{3x}+e^x\cos 2x.$
2. Solve $\frac{d^2y}{dx^2}-4y=x\sinh x.$
3. Solve using variation of parameters $\frac{d^2y}{dx^2}+y=\sec x.$
4. Solve the equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log_e x.$
5. Solve in series the equation $\frac{d^2y}{dx^2}+xy=0.$

8 Marks Questions:

1. Solve the equation
$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$
2. Solve the equation $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin\{\log(1+x)\}.$
3. Solve using variation of parameters $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}.$
4. Solve using variation of parameters $\frac{d^2y}{dx^2}+4y=\tan 2x.$
5. Solve using variation of parameters $\frac{d^2y}{dx^2}+y=x\sin x.$
6. Solve the equation $(D-2)^2=8\left(e^{2x}+\sin 2x+x^2\right).$
7. Solve the equation $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10(x+\frac{1}{x}).$
8. Solve the equation $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^{x^x}.$

9. Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{e^x}$.

10. Solve the differential equation $\left(D^2-1\right)y=x\sin 3x+\cos x.$

11. Solve in series the equation $9x(1-x)\frac{d^2y}{dx^2}-12\frac{dy}{dx}+4y=0.$

12. Solve in series the equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}-xy=0.$

13. Obtain the series solution of the equation $x(1-x)\frac{d^2y}{dx^2}-(1+3x)\frac{dy}{dx}-y=0.$

14. Solve in series $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0.$

15. Solve in series $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+xy=0.$

UNIT IV
COMPLEX VARIABLE-DIFFERENTIATION

2 Marks Questions.

1. Separate exp(z^2) into real and imaginary parts.
2. Find the general value of log($-i$)
3. Define analytic function.
4. Find the real and imaginary parts of sinz.
5. Write the relations between circular and hyperbolic functions.

4 Marks Questions.

1. Find the general and principal values of i^i
2. If $\sin(\alpha + i\beta) = x + iy$, prove that:

   $$\frac{x^2}{\sin^2\alpha} - \frac{y^2}{\cos^2\alpha} = 1.$$

3. If f(z) = u + iv be an analytic function in some region of z plane, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
4. Prove that $\cosh^2 x - \sinh^2 x = 1$
5. Determine the analytic function whose real part is $\log\sqrt{x^2 + y^2}$

8 Marks Questions

1. If $u = \log\tan(\pi/4 + \theta/2)$, prove that

   (i) tanh u/2 = tan θ/2 (ii) $\theta = -i\log\tan\left(\frac{\pi}{4} + \frac{iu}{2}\right)$

2. If $\tan(\theta + i\phi) = e^{i\alpha}$, Show that

   $$\theta = \left(n + \frac{1}{2}\right)\frac{\pi}{2} \quad \text{and} \quad \phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$

3. Reduce $\tan^{-1}(\cos\theta + i\sin\theta)$ to the form a+ib. Hence show that,

   $$\tan^{-1}\left(e^{i\theta}\right) = \frac{n\pi}{2} + \frac{\pi}{4} - \frac{i}{2}\log\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

4. If $\sin^{-1}(u + iv) = \alpha + i\beta$ prove that $\sin^2\alpha$ and $\cosh^2\beta$ are the roots of the equation
   $x^2 - x\left(1 + u^2 + v^2\right) + u^2 = 0$

5. Show that $\tan^{-1} z = \frac{i}{2}\log\left(\frac{i+z}{i-z}\right)$

6. If $i^{i^{i^{\cdots\cdots}}} = A + iB$, Prove that $\tan\frac{\pi A}{2} = \frac{B}{A}$ and $A^2 + B^2 = e^{-\pi B}$

7. If $\tan(\theta + i\phi) = \tan\alpha + i\sec\alpha$, prove that

$$e^{2\phi} = \pm\cot\frac{\alpha}{2} \text{ and } 2\theta = \left(\mathrm{n} + \frac{1}{2}\right)\pi + \alpha$$

8. Prove that $\tan\left[i\log\left(\frac{\mathrm{a - ib}}{\mathrm{a + ib}}\right)\right] = \frac{\mathrm{2ab}}{\mathrm{a^2 - b^2}}$

9. If tan (x + iy) = sin (u + iv), prove that

$$\frac{\sin 2x}{\sinh 2y} = \frac{\tan u}{\tanh v}$$

10. Prove that the function f(z) defined by

$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ $(z \neq 0)$, $f(0) = 0$ is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

11. If $\omega = \phi + i\psi$ represents the complex potential for an electric field and

$\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function ϕ.

12. If the potential function is $\log(x^2 + y^2)$, find the flux function and the complex potential function.

13. Find the analytic function f(z) = u + iv , if $u - v = (x - y)(x^2 + 4xy + y^2)$.

14. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$

15. Find the analytic function f(z) = u + iv, whose imaginary part is given by

$v = e^x (x \sin y + y \cos y)$.

UNIT-V
Complex Variable – Integration

2 Marks Questions:

1. State Cauchy's Theorem
2. Evaluate $\int \frac{e^{z}}{(z-3)^{2}} dz$ at $|z|=2$
3. Define Isolated Singularity.
4. Singularity of $e^{z^{-1}}$ at z=0 is the type ………………………..
5. Find the zeros of $f(z)=\left(\frac{z+1}{z^2+1}\right)^3$

4 Marks Questions:

1: Evaluate $\int_C \frac{\sin^2 z}{(z-\pi/6)^3} dz$ where c is the circle $|z| = 1$ Using Cauchy Integral Formula.

2. Evaluate $\int_C z^2 dz$ where C is the Straight line joining the origin O to the point p(2,1) on the complex plane.

3. Evaluate $\int_C |z|\, dz$ where C is the straight line z=-i to z=i.

4. Evaluate $\int_C \tan z dz$ where C is the circle $|z| = 2$

5. Evaluate using Cauchy's integral formula $\int_C \frac{z}{z^2-3z+2} dz$ where C is $|z-2| = \frac{1}{2}$

8 Marks Questions:

1. **Expand** $f(z)=\frac{1}{(z-1)(z-2)}$ in the region:

   (a) $|z| \prec 1$ (b) $1 \prec |z| \prec 2$ (c) $|z| \succ 2$

2. Find the Laurent's Series expansion of $f(z)=\frac{7z-2}{(z+1)z(z-2)}$ in the region $1 \prec |z+1| \prec 3$

3. Find the Laurent's Series expansion of $f(z)=\frac{1}{z^2 \sinh z}$ and evaluate $\int_C \frac{dz}{z^2 \sinh z}$ where C is the circle $|z-1|=2$

4. Find Laurent's series expansion of $f(z)=\frac{e^{2z}}{(z-1)^2}$ about z=1

5. Find Laurent's series expansion of $f(z)=\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 \prec |z+2| \prec 5$

6. Evaluate $\int_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)}dz$ where C is the circle $|z|=3$

7. Evaluate $\int_C \frac{z-3}{z^2+2z+5}dz$ where C is the Circle

(i) $|z|=1$

(ii) $|z+1-i|=2$

(iii) $|z+1+i|=2$

8. Evaluate $\int_C \frac{3z^2+2}{(z-1)(z^2+9)}dz$ where C: $|z-2|=2$

9. If $F(z)=\int_C \frac{4z^2+z+5}{z-3}dz$ where C is the ellipse $\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1$ Find the value of (a) F(3.5)

(b) F(i) F'(-1) and F''(-i)

10. Find the Residue of $F(z)=\int_C \frac{z^3}{(z-1)^4(z-2)(z-3)}dz$ at its poles and hence evaluate $\int_C F(z)dz$ where C is the circle $|z|=2.5$

11. Show that $\int_0^{2\pi} \frac{d\theta}{25-24\cos\theta}=\frac{2\pi}{7}$

12. By integrating around unit circle ,Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}d\theta$

13. Show that $\int_0^{2\pi} \frac{d\theta}{1-2p\sin\theta+p^2}=\frac{2\pi}{1-p^2}$ $(0 \prec p \prec 1)$

14. Apply the Calculus of residue to prove that $\int_0^{2\pi} \frac{a\,d\theta}{a^2+\sin^2\theta}=\frac{2\pi}{\sqrt{1+a^2}}$ $a \succ 0$

15. Apply the Calculus of residue to prove that $\int_0^{\pi} \frac{d\theta}{17-8\cos\theta}=\frac{\pi}{15}$