DATA ANALYSIS CHALLENGES IN GRAVITATIONAL WAVE ASTRONOMY

Soumya D. Mohanty

UTRGV

Plan

- Brief introduction to General Relativity
- Brief review of gravitational wave detections
  - Highlight the critical role of data analysis
- Address the main types of data analysis methods
- Discuss some of the open challenges in data analysis
  - Emphasis on areas where new algorithms are required
  - Less emphasis on the issues involved in analyzing real GW data
  - Highlight some of the work I have done

Gravitation

Einstein's special theory of relativity (1905): Information cannot be transferred faster than the speed of light

Newtonian gravity $F = \frac{GM_1M_2}{r^2}$

- Postulates instantaneous action at a distance which is incompatible with the finite speed of light

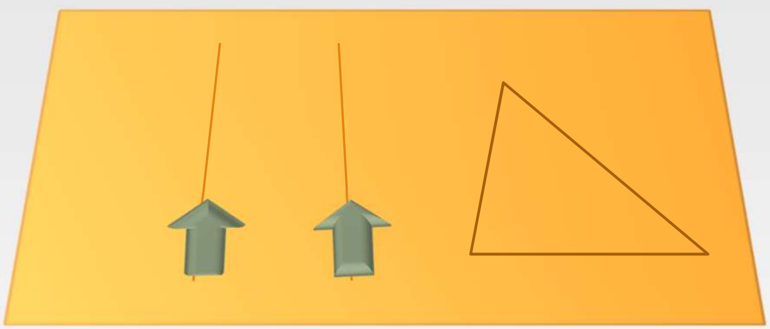
Correct explanation: Einstein's General Theory of Relativity

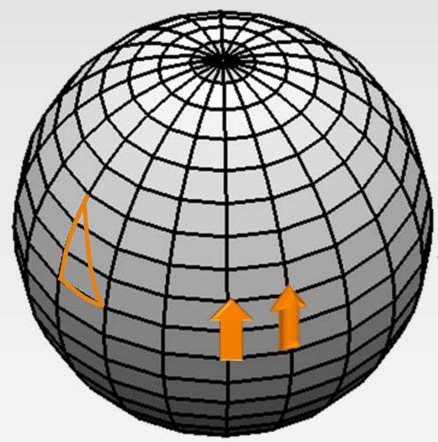
General Theory of Relativity (GR) (1915): Gravity is a manifestation of curvature in space-time geometry

GEOMETRY

FLAT Geometry

CURVED Geometry





"Straight" lines in curved geometry: paths of shortest distance (e.g., great circles on a sphere)

- Parallel vectors stay parallel when displaced along their own instantaneous direction
- Angles of a triangle add up to 180°

- Parallel vectors do not stay parallel when displaced along their own instantaneous direction
- Angles of a triangle do not add up to 180°

Space-time geometry

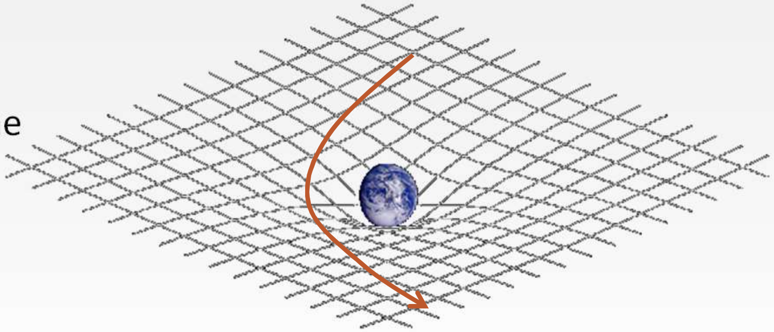
"Straight" line path for curved space-time geometry

Change to a different coordinate system to better see the "straightness"

Gravity is not a force but the effect of a curved space-time

Matter influences space-time geometry

Space-time geometry guides the motion of matter



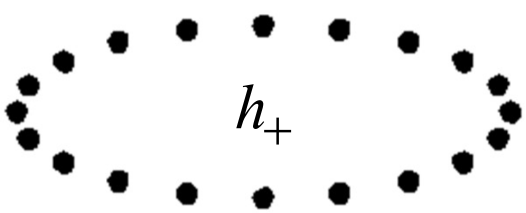
Gravitational waves

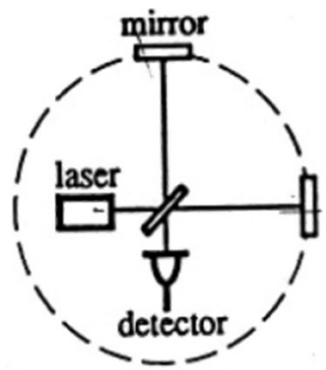
Gravitational waves: change in space-time geometry propagating as a wave

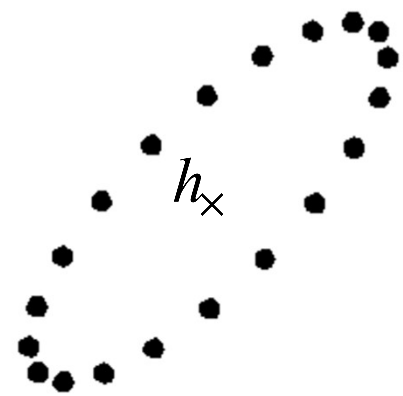
- Perturbation $\mathbf{h}(x)$ in the metric tensor $\mathbf{g}(x)$
- Plane wave in TT gauge (moving along $\hat{k}$, $kx = k_\mu x^\mu$):

$$\mathbf{h}(x) = h_+(kx)\mathbf{e}_+(\hat{k}) + \mathrm{h}_\times(kx)\mathbf{e}_\times(\hat{k})$$

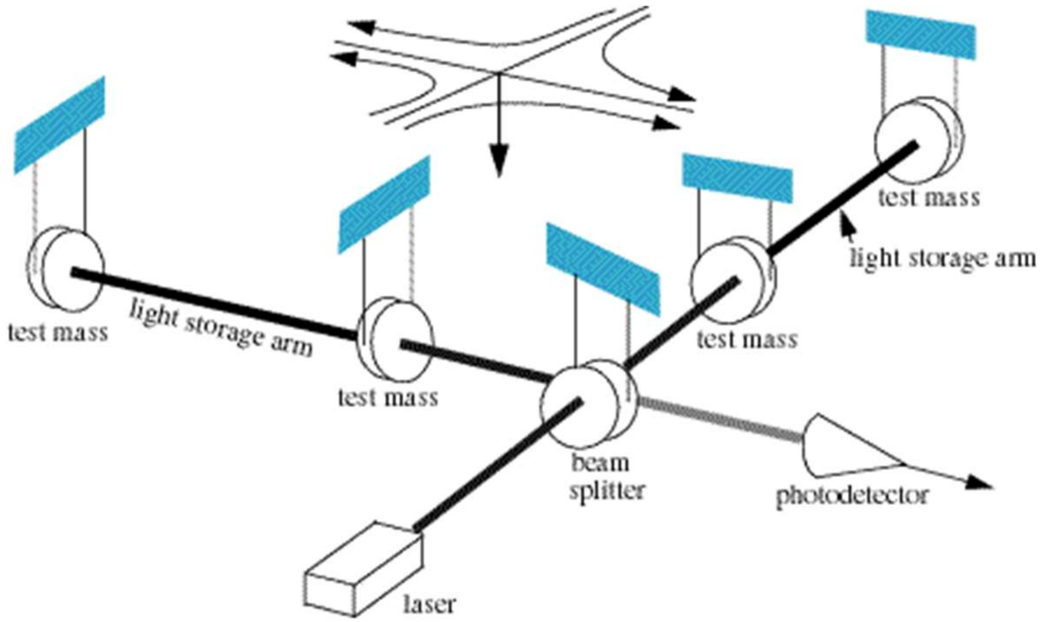
- h_+ and $h_\times$: GW polarizations







Interferometric GW detector

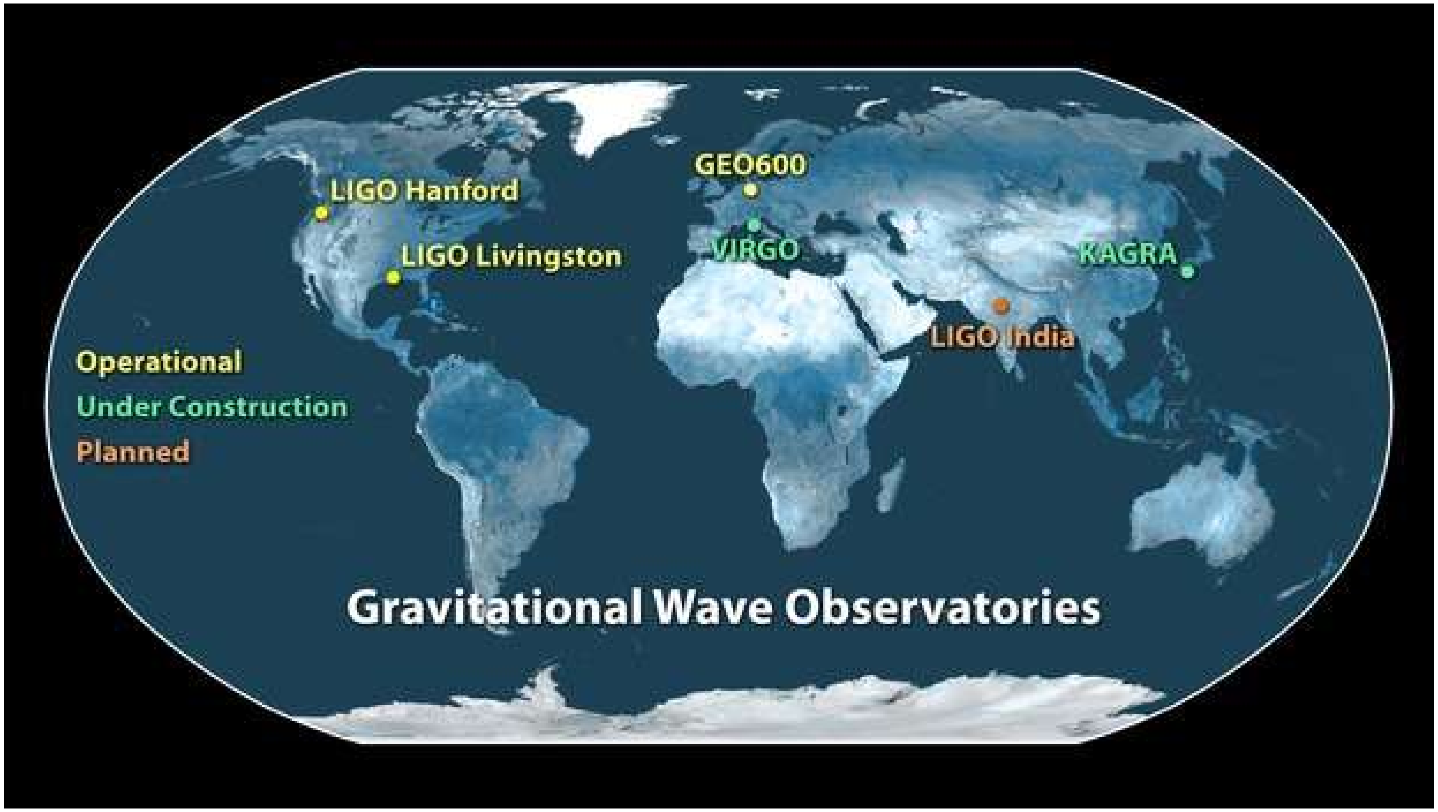


- The most sensitive instruments today are laser interferometers
- Fabry-Perot optical cavities
- Increasing the number of photons reduces photon counting noise $\Rightarrow$ better measurement of fringe displacement

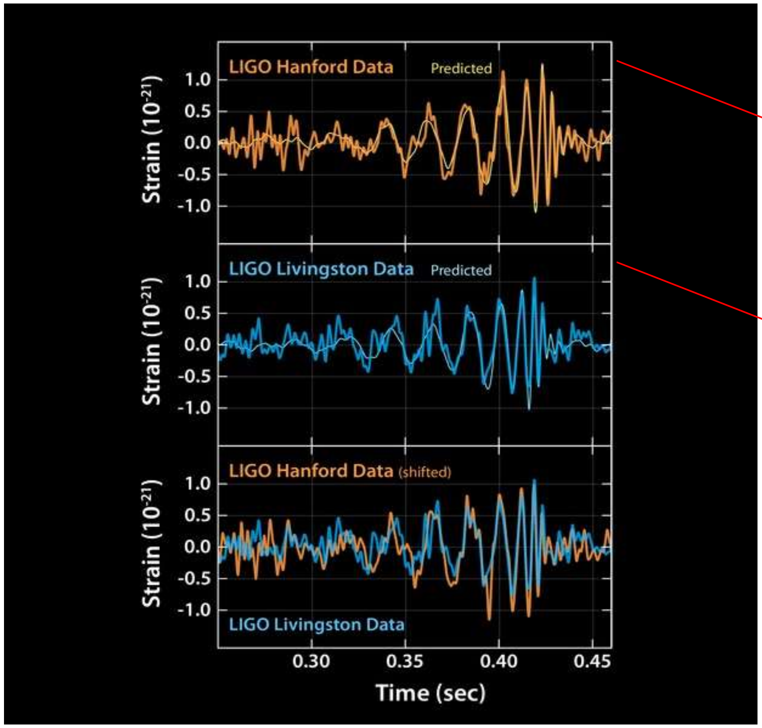
Hanford, WA

LIGO Detectors

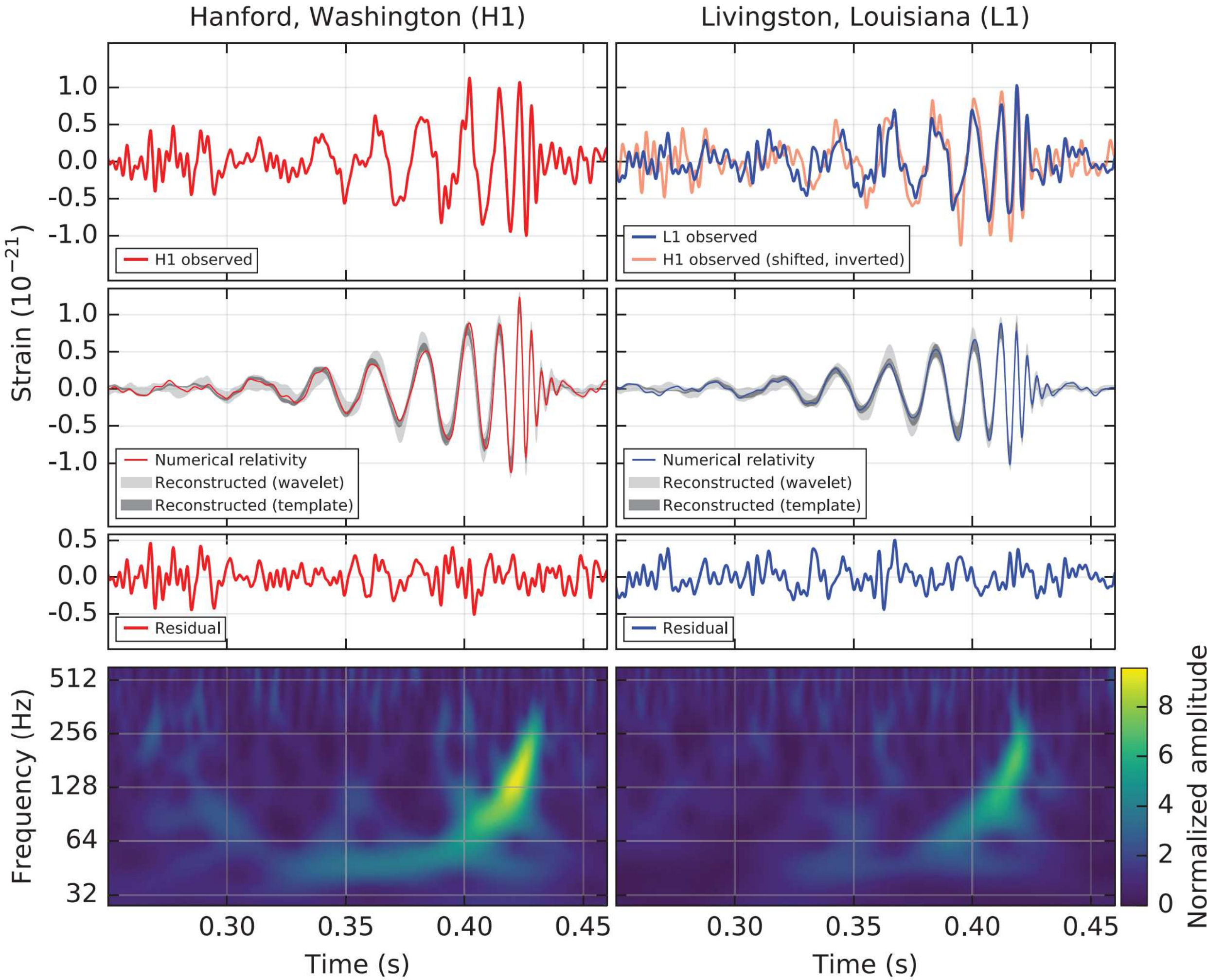
Worldwide GW detector network



GW150914: GW Astronomy launched!



Phys. Rev. Lett. 116, 061102 (2016)



Whitened Data: Noise + Signal

- Requires removal of instrumental features

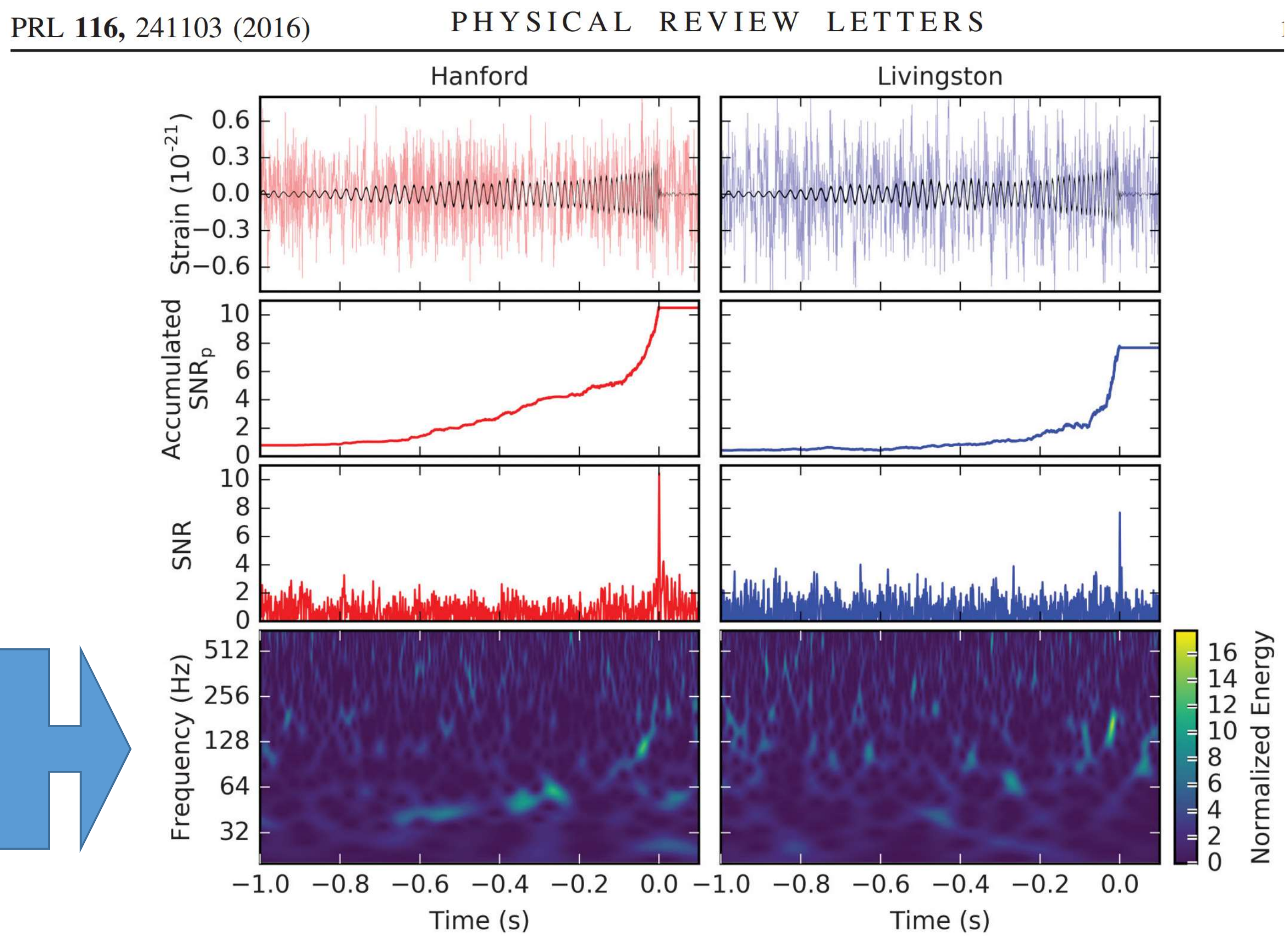
Best fit signal

- Parametric
- Non-parametric

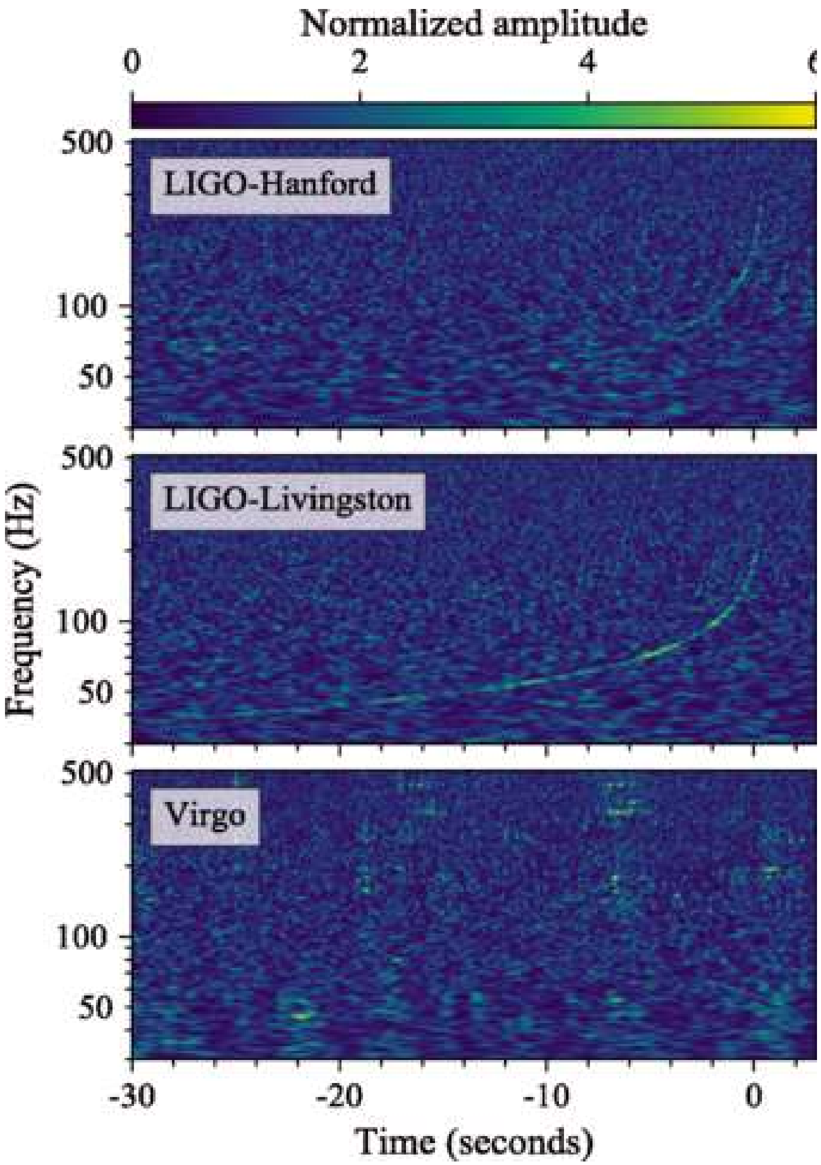
Residual = Data-Fit = noise

Time-frequency plot of data (Q-transform)

GW 151226

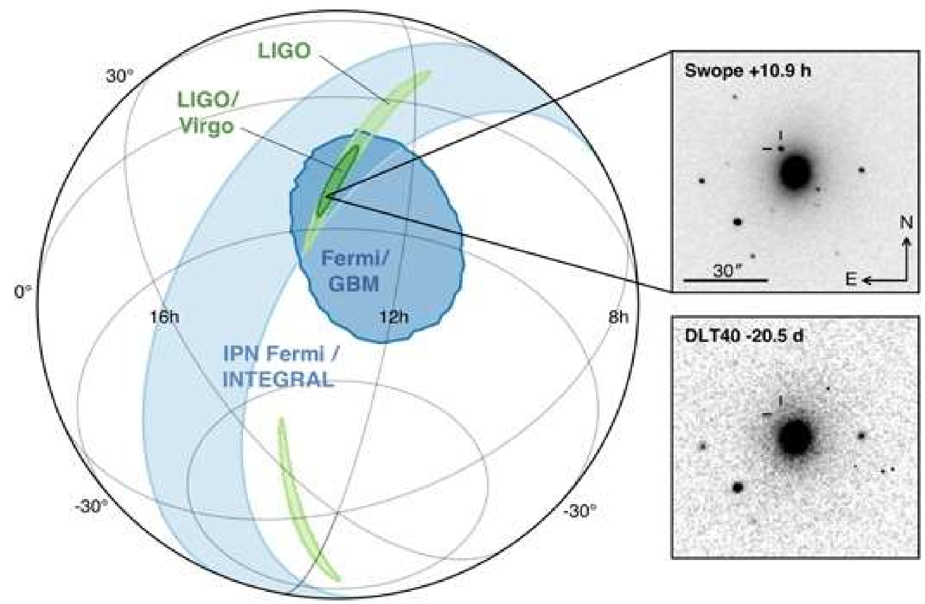


Signal not obvious in time-frequency plots; Reqires matched filtering



GW170817: Double Neutron Star inspiral

- “Golden Binary”
- Detection of a Kilonova in the sky region determined from LIGO-Virgo data
- Required optimal combination of data from a network of detectors → Coherent Network Analysis



Coherent network analysis

$$\begin{pmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{pmatrix} = \begin{pmatrix} F_{+,1}(\theta,\phi)B(\tau_1(\theta,\phi)) & F_{\times,1}(\theta,\phi)B(\tau_1(\theta,\phi)) \\ \vdots & \vdots \\ F_{+,N}(\theta,\phi)B(\tau_N(\theta,\phi)) & F_{\times,N}(\theta,\phi)B(\tau_N(\theta,\phi)) \end{pmatrix} \begin{pmatrix} h_+(t) \\ h_\times(t) \end{pmatrix} + \begin{pmatrix} n_1(t) \\ \vdots \\ n_N(t) \end{pmatrix}$$

Responses of GW detectors

(θ, φ) : source position
B(τ)[ ]: Time delay
$F_{+,\times}$: Antenna response

Noise

$$\mathbf{x}(t) = \mathbf{A}(\theta,\phi)\mathbf{h}(t) + \mathbf{n}(t)$$

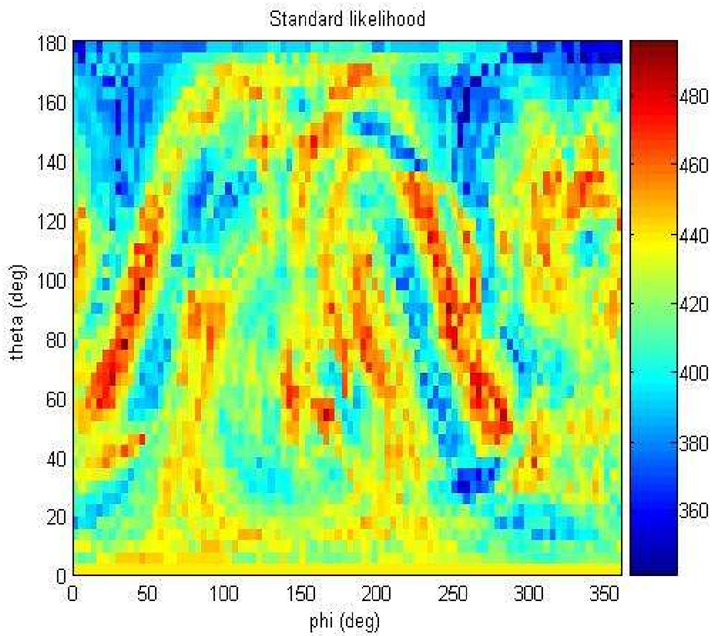
$$\mathbf{h}_{ML}(t) = \mathbf{M}\mathbf{A}^{\mathrm{T}}(\theta_{ML},\phi_{ML})\mathbf{x}(t)$$

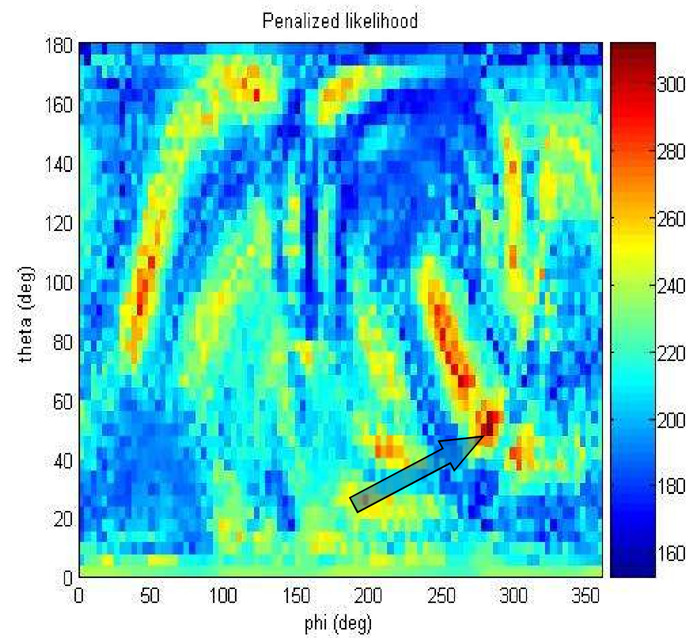
$$\mathbf{M} = \left(\mathbf{A}^{\mathrm{T}}(\theta,\phi)\mathbf{A}(\theta,\phi)\right)^{-1}$$

But the problem is ill-posed! **A**(θ,φ) can become rank-deficient

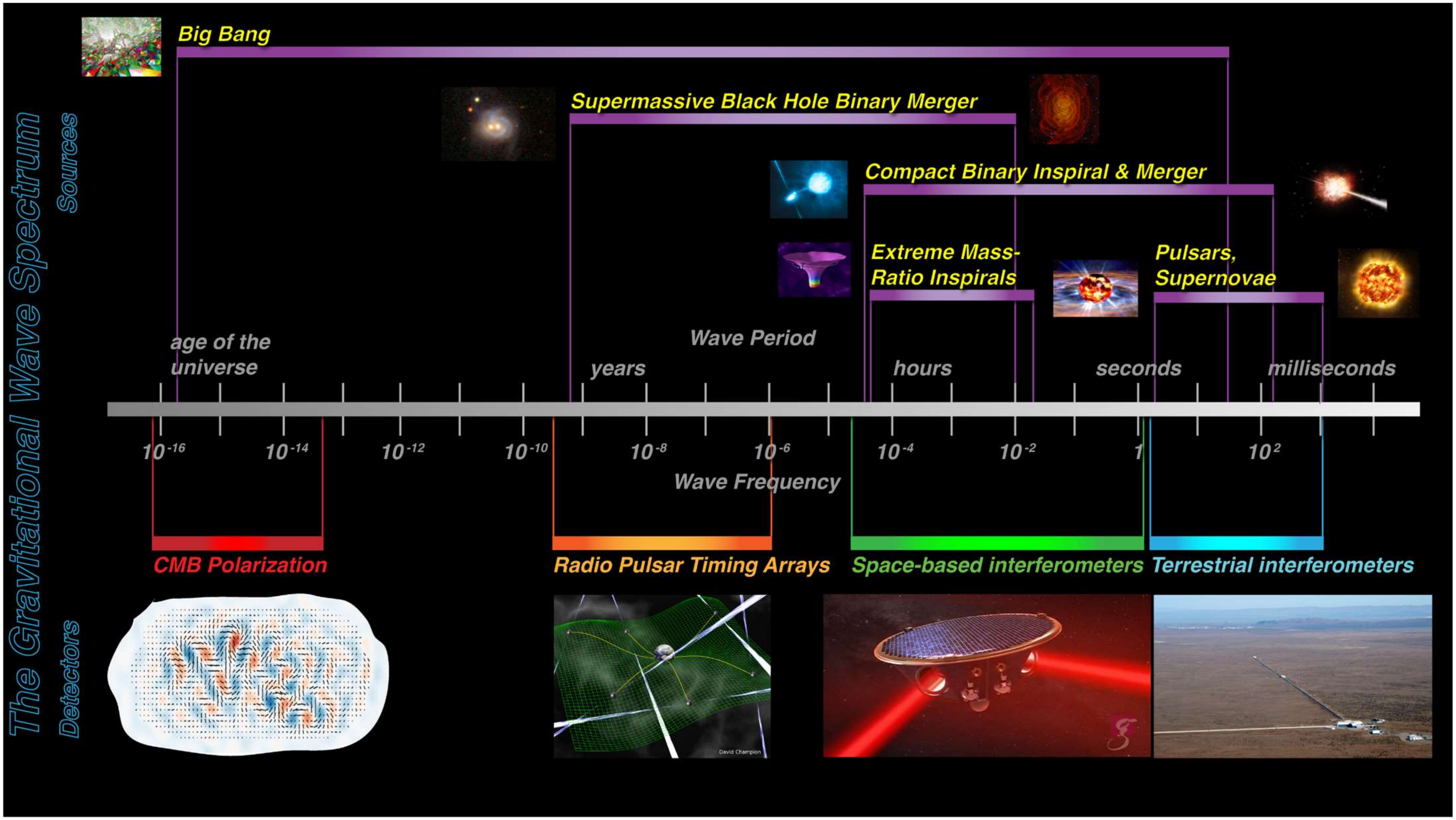
Regularization required

Demonstration: LIGO and GEO600





- Constrained likelihood
- Klimenko, Mohanty, Rakhmanov, Mitselmakher, PRD, 2005⟶ Basis of the Coherent WaveBurst pipeline → First pipeline to detect GW150914
- The choice of regulators is still not a settled issue


The Gravitational Wave Spectrum
Sources
Detectors
Big Bang
Supermassive Black Hole Binary Merger
Compact Binary Inspiral & Merger
Extreme Mass-Ratio Inspirals
Pulsars, Supernovae
Wave Period
age of the universe
years
hours
seconds
milliseconds
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
10^{2}
Wave Frequency
CMB Polarization
Radio Pulsar Timing Arrays
Space-based interferometers
Terrestrial interferometers

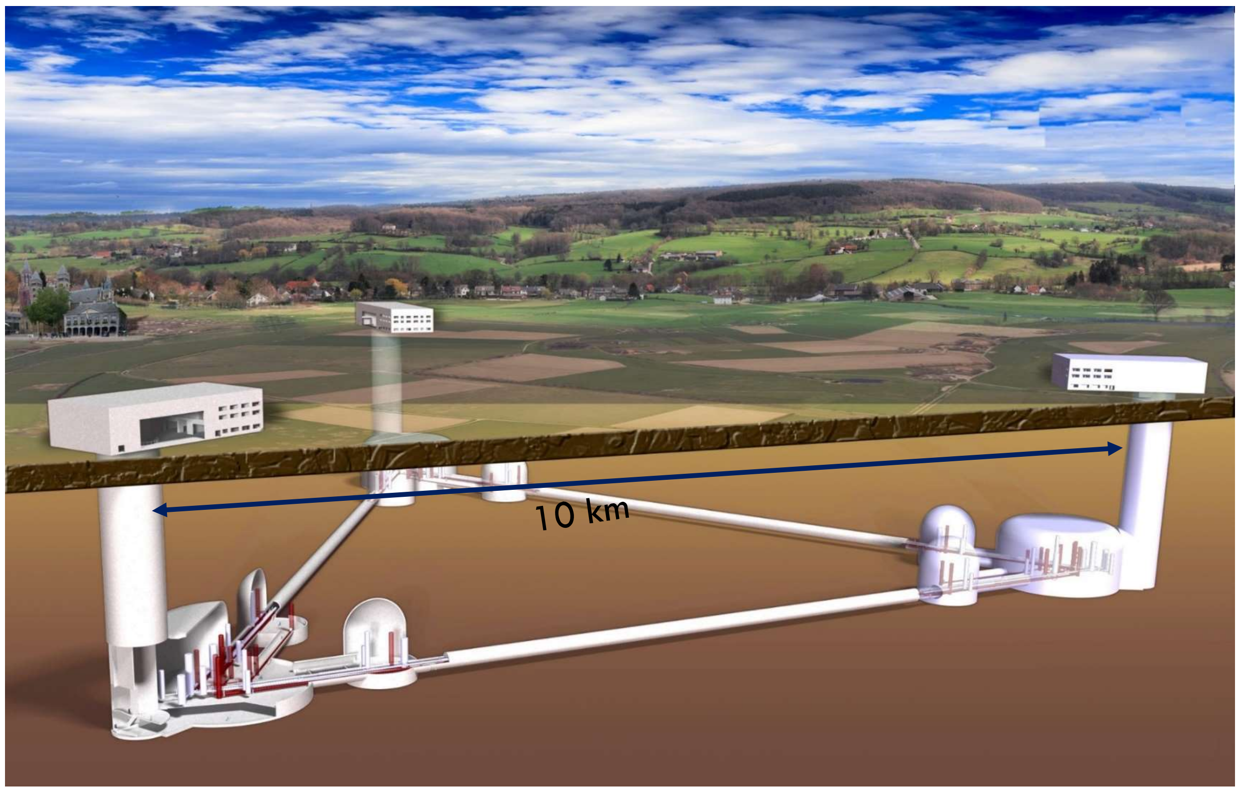
THIRD GENERATION GW DETECTORS

Einstein Telescope

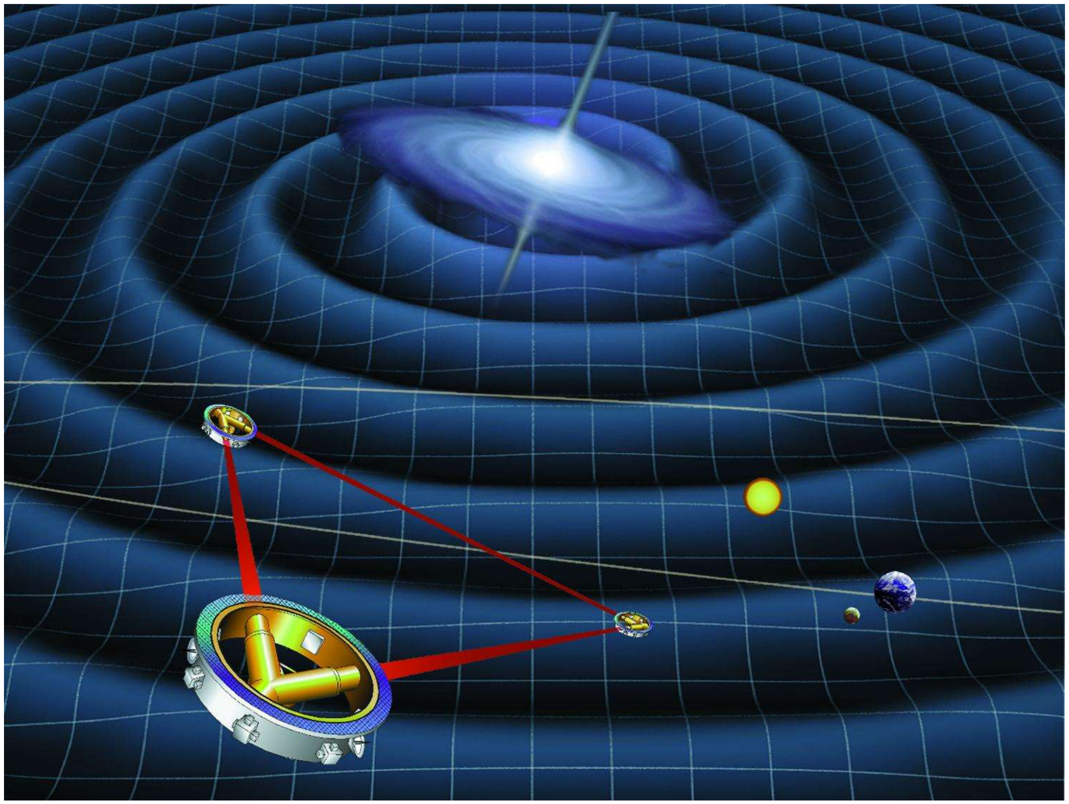
- Design study only
- 10 km arms
- 3 detectors on one site
- Underground: reduce seismic noise
- Cryogenic mirrors
- 10 times better sensitivity than 2nd generation detectors

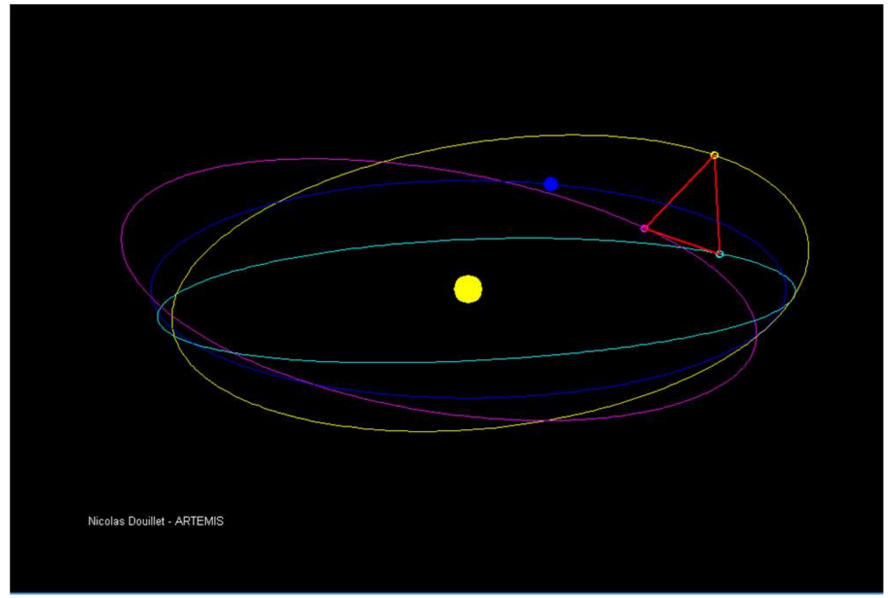
Cosmic explorer

- 40 km arm length
- LIGO-T1500290



LISA





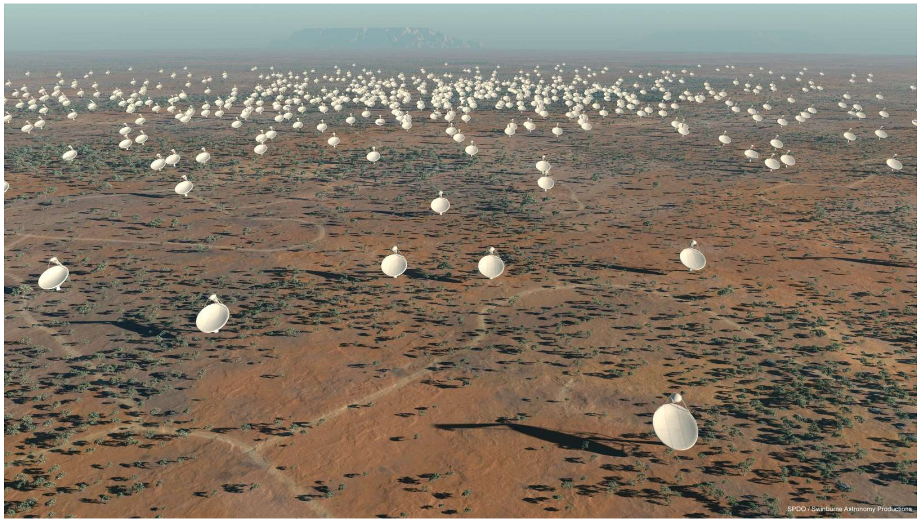
- Multiple co-located Interferometers with 2.5 million km arms
- Satellites exchanging laser beams while following Keplerian orbits
- Monitoring the position of freely falling proof masses

Pulsar timing array (PTA) projects

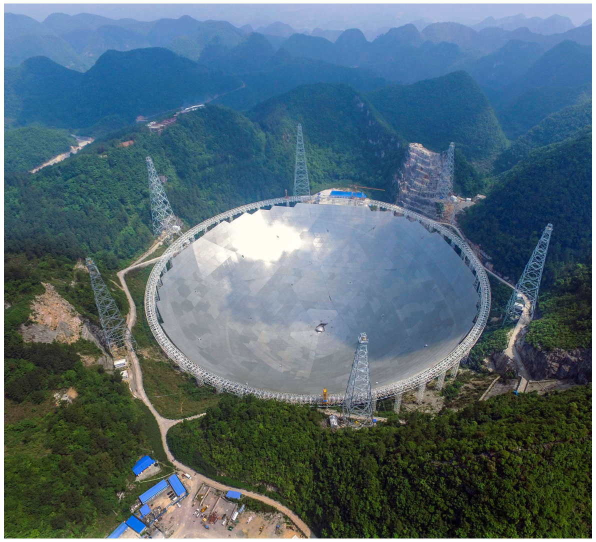
- NanoGrav
- EPTA
- PPTA

IPTA

Next Gen Radio Telescopes



- Square Kilometer Array (SKA) will find about 6000 Millisecond Pulsars (Smits et al, 2009)
- PTA with ≈ 1000 well-timed pulsars!



- FAST will discover ≈ 500 Millisecond pulsars
- Mostly in the Northern hemisphere
- Complementary to SKA coverage

- Wang, Mohanty, PRL, 2017
  - An SKA-era PTA will see $10^{10} M_{\odot}$ (redshifted chirp mass) Supermassive Black Hole Binary out to a redshift of 28
- Combining the PTAs will lead to better source direction estimates

Data analysis challenges

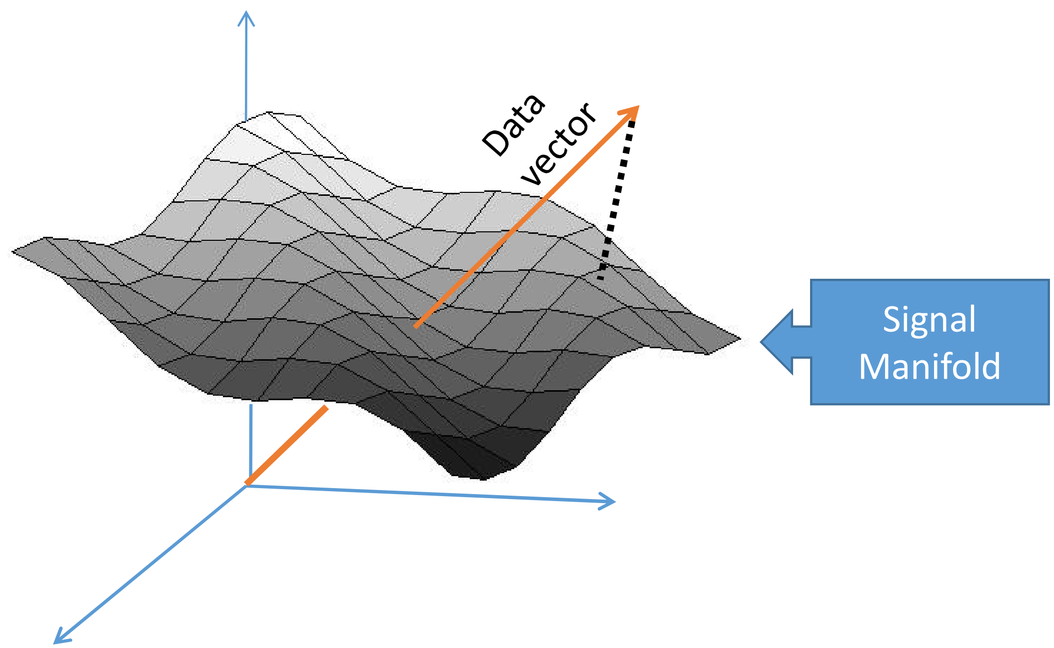
Data Analysis in GW astronomy

- Data Analysis Methods
  - **Parametric**
    GW signal waveform can be predicted theoretically
    Binary inspiral
    Binary Black Hole merger
    Continuous Wave (deformed Neutron star)
  - **Non-parametric** (unmodeled signal)
    Details of the GW signal unknown
    Core-collapse Supernovae
    Unanticipated sources
  - **Stochastic**
    GW signal is a noise process
    - Parametric
    - Non-Parametric
  - **Data conditioning and Characterization**
    Trapping and Removing Instrumental features

Parametric methods

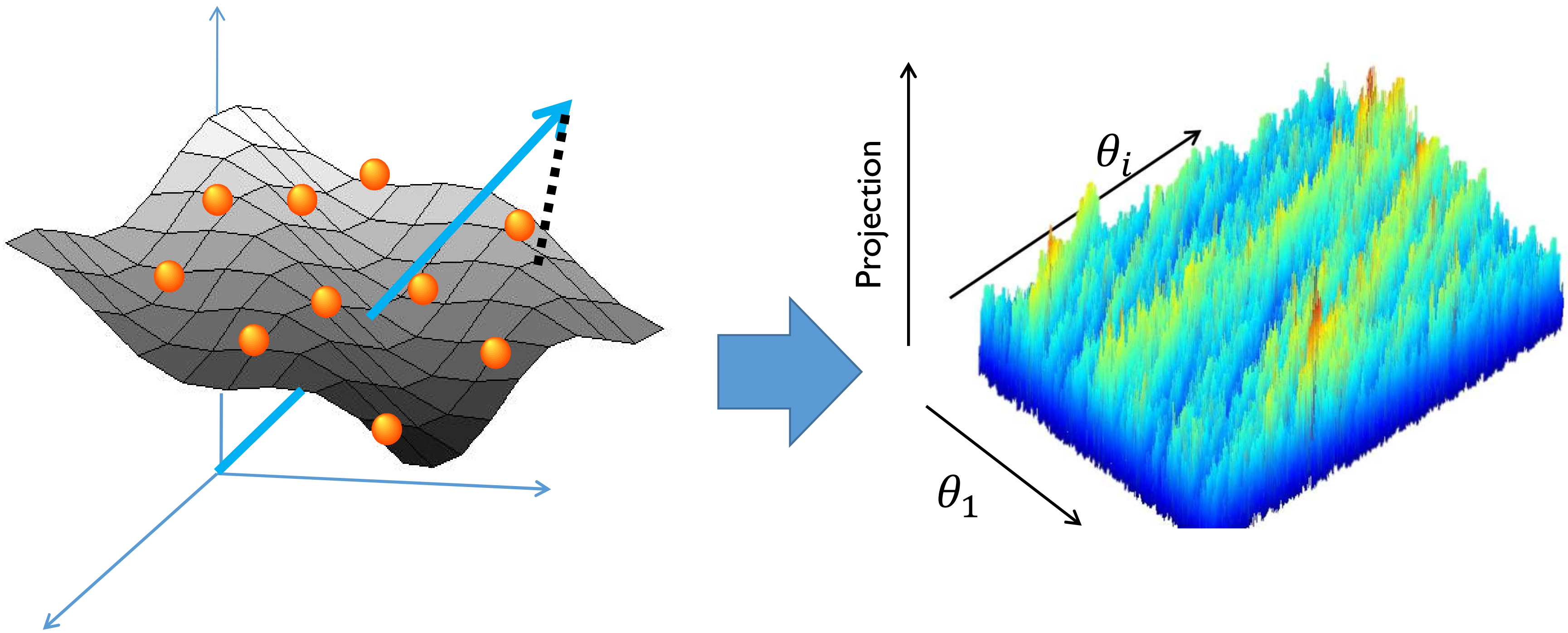
Geometric Formulation

- Parameterized waveform family $s(t; \Theta)$
- Data $x = s(\Theta) + n$
  - $x \in R^N$
  - $s(\Theta)$: signal manifold embedded in R^N
- Best fit signal parameters: projection of the data vector on the signal manifold



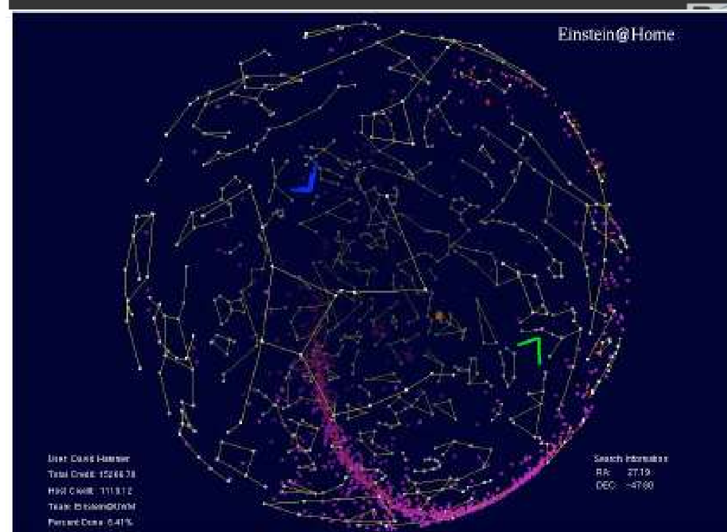
Grid Based Search

- Put a sufficiently dense grid of points to find the projection
- Ad hoc criterion used: threshold on the distance of the data from the manifold



Computational challenges

- Very high computational cost of grid search in most cases
  - Continuous wave search, 1 year integration time, requires 10^4 Tflops
- Actual search is limited to suboptimal schemes
  - Einstein@home: 100,000 computers distributed worldwide
- Current binary inspiral searches do not use coherent network analysis: 25% reduction in detection volume (McLeod et al, PRD, 2016)
- Sub-optimal schemes have lower sensitivity
- High computational cost increases latency between signal arrival and announcement of detection
- Computational cost amplified by the need to estimate background rate: The same algorithm is run on a much larger volume of time-shifted data

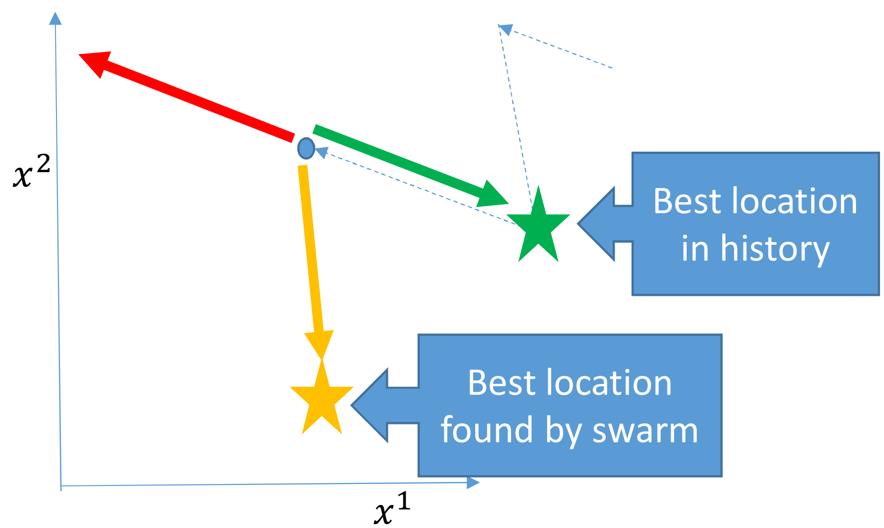


Alternatives to grid search needed

- Stochastic optimizers: MCMC, Genetic Algorithms, Differential Evolution,…
- Particle Swarm Optimization (Kennedy, Eberhart, 1995) seems to be the best option
  - Wang, Mohanty, PRD, 2010: First application to GW data analysis
  - Wang, Mohanty, Jenet, ApJ, 2014, 2015: Application to PTA data analysis
  - Weerathunga, Mohanty, PRD, 2017
    - Coherent network analysis for binary inspirals: Number of search points reduced by a factor of 10

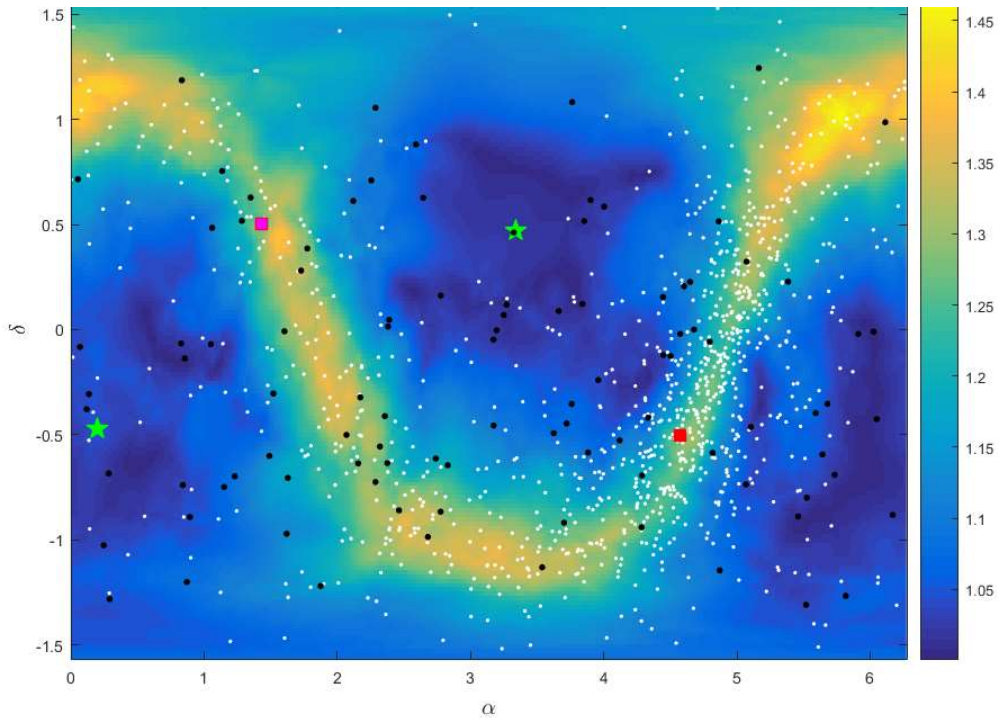
PSO is inspired by flocking behavior

$$v_i[k+1] = w\, v_i[k] + c_1 r_1 (p_i\,[k] - x_i\,[k]) + c_2 r_2 (g[k] - x_i\,[k])$$



Mulit-source resolution

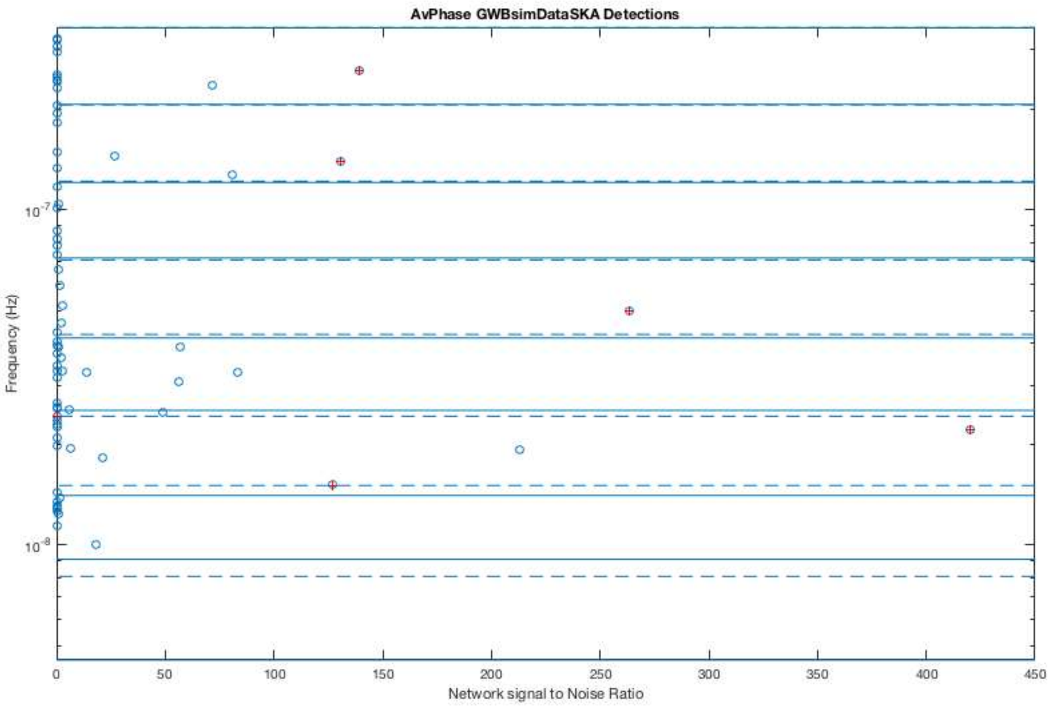
Large-scale PTA



Large-scale PTA and multiple Supermassive Black Hole Binaries

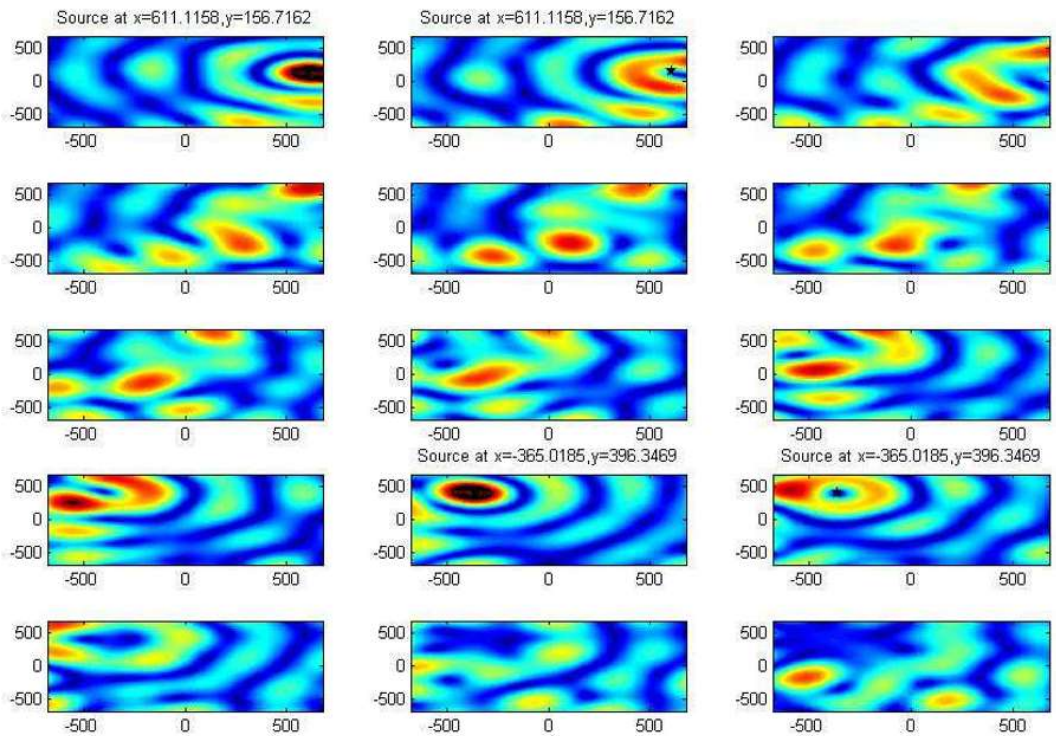
- The large distance reach of future PTAs will lead to multiple signals present simultaneously
- Number of parameters grows in proportion to number of sources assumed to be present
- Grid-based search of the associated parameter space is impossible
- Similar problem in the case of LISA
- Will be a problem for 3rd generation terrestrial detectors also

Source confusion



- Methods assuming a single source can fail badly when multiple sources have comparable strengths
- Iterative source subtraction?
- Simultaneous fit of multiple sources?
- Higher order problem: Mix of different types of GW sources

Imaging the GW sky

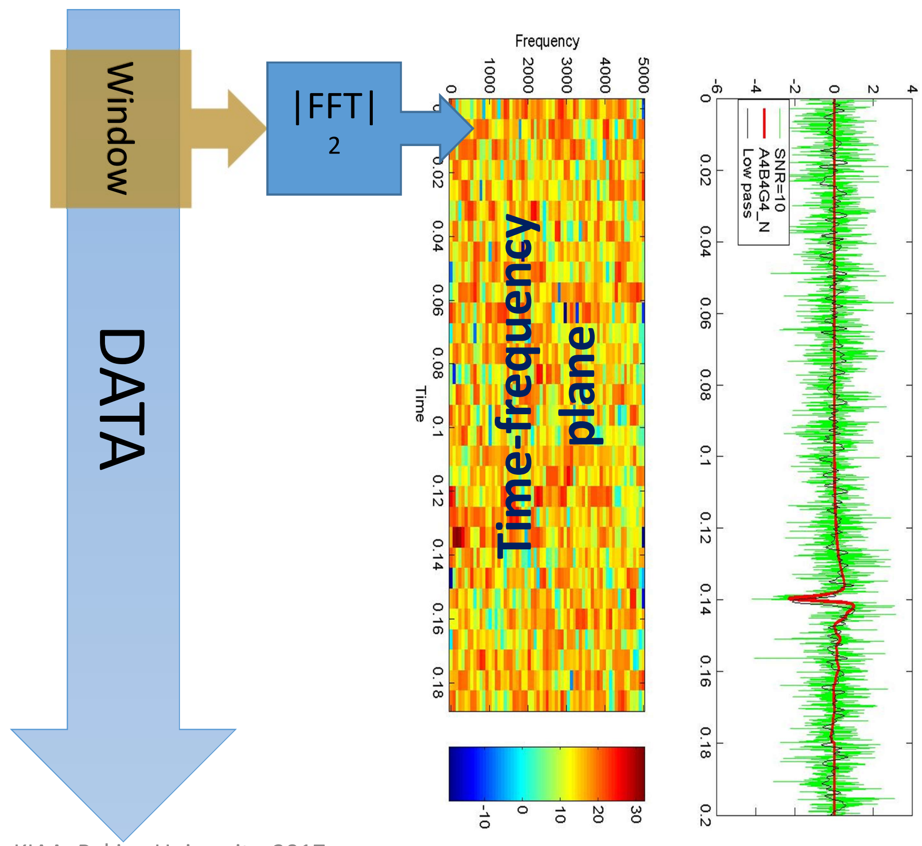


- Is the parametric approach the best for multiple sources?
- How can we make an image of the GW sky?
- Mohanty, Nayak, PRD, 2005: Tomographic method
  - LISA response is the Radon Transform of the GW emission from the entire sky
  - The projections arise from doppler modulation of GW signals due to LISA's motion
- Gair et al, PRD, 2014: Mapping stochastic background

Non-parametric methods

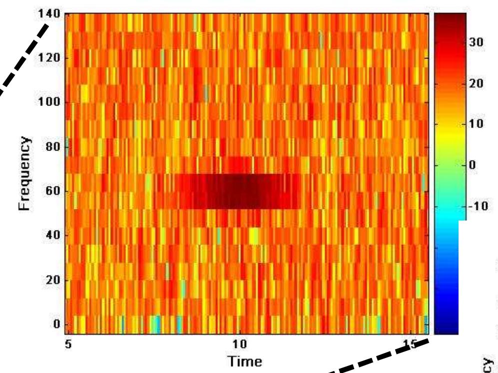
Time-frequency methods

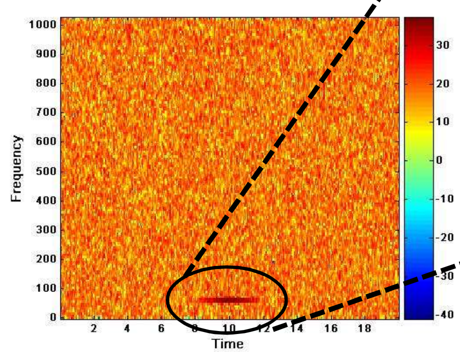
The simplest time-frequency decomposition of data is the spectrogram (Gabor transform)



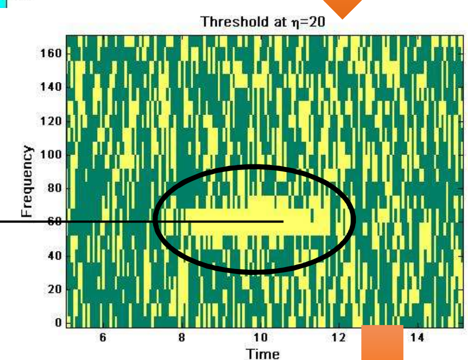
TF clustering

Assume that a simply connected region of the time-frequency plane is occupied by a signal





•Apply a threshold η: set pixels whose value is $< \eta$ to zero

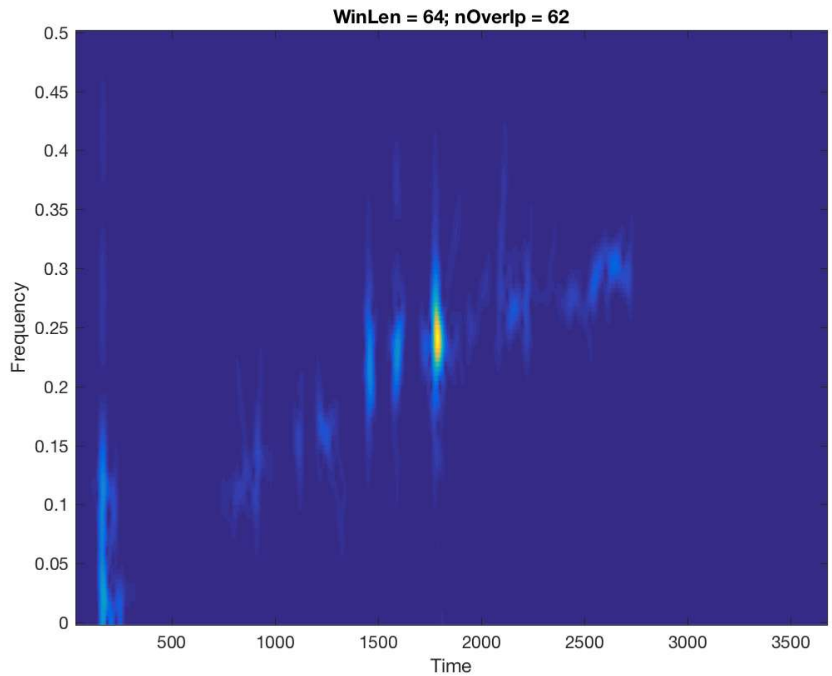


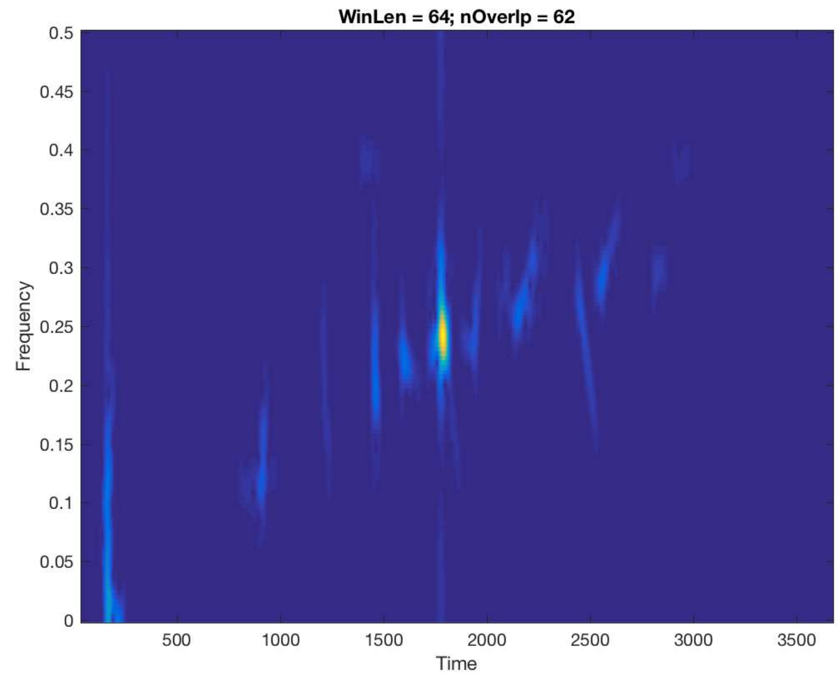
Cluster
Find groups of pixels that are “connected” to each other
•A binary metric: distance 0 or 1

Retain only clusters above a certain threshold size

Further threshold on average value of pixels in the cluster

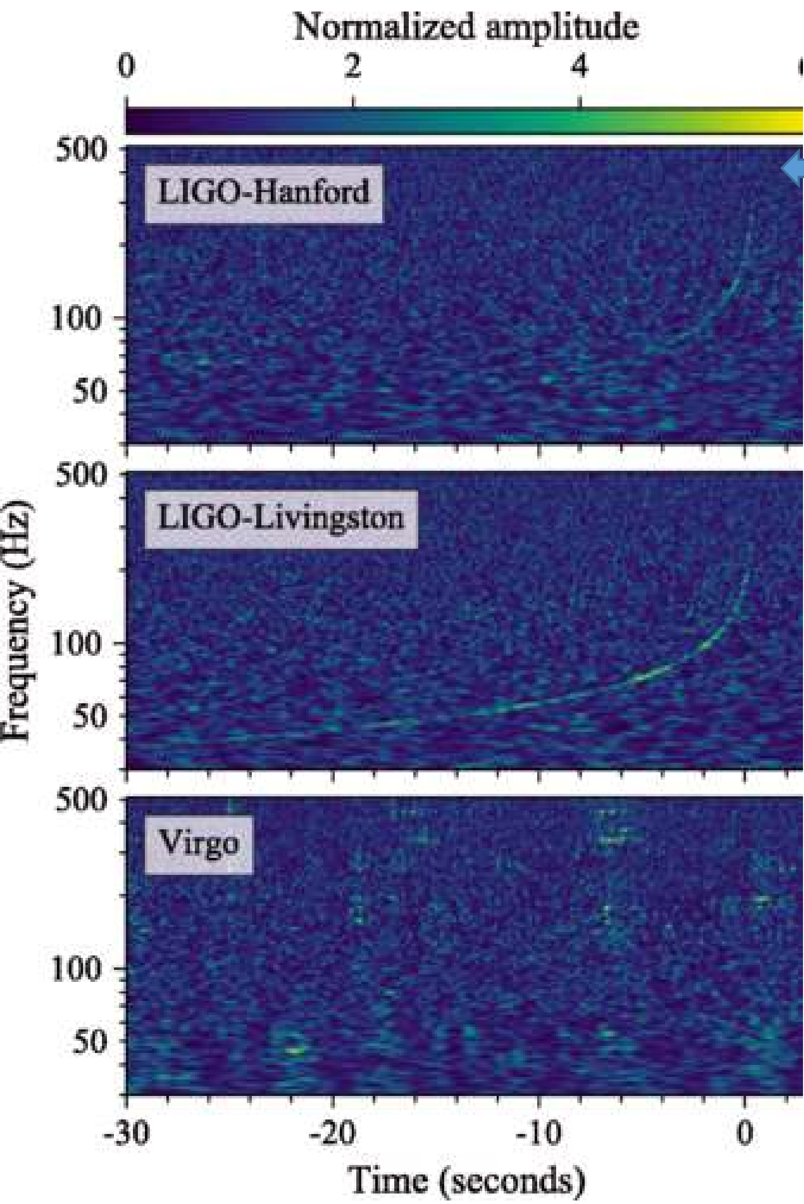
The challenge of Core-collapse Supernovae





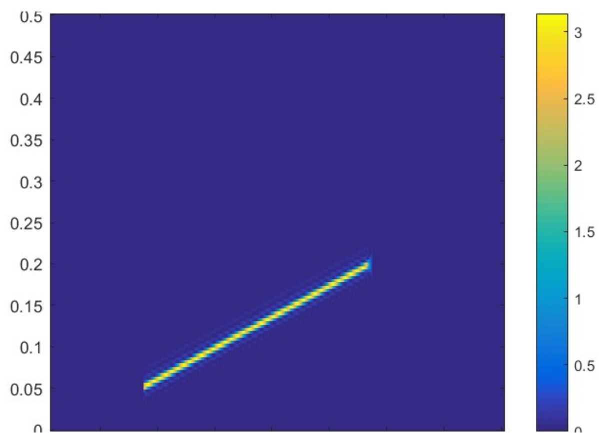
Sparsity regularization: $\min_{\alpha}(\|y - \alpha X\|_2^2 + \lambda\|\alpha\|_1)$

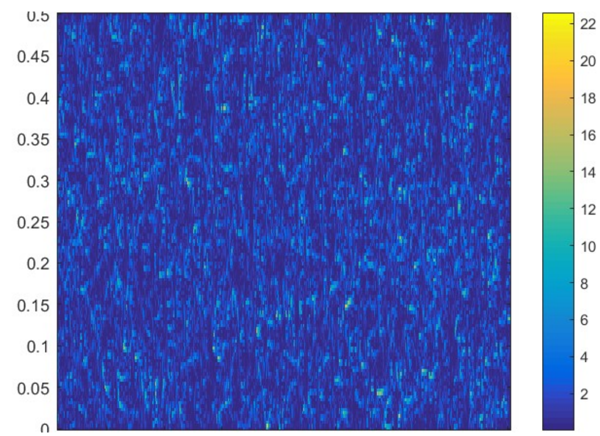
Mohanty, in progress



Is there a post-merger GW signal?

Unmodeled chirps

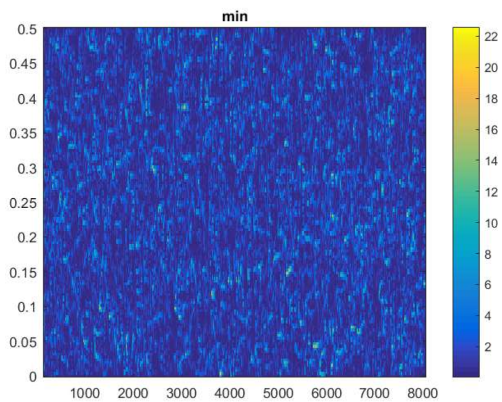




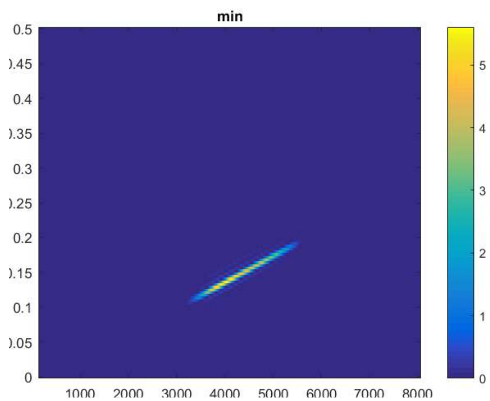
- Even simple chirps can disappear in the noise in a time-frequency transform
  - Example: Matched filtering SNR=15 linear chirp in white noise

Spline-based method

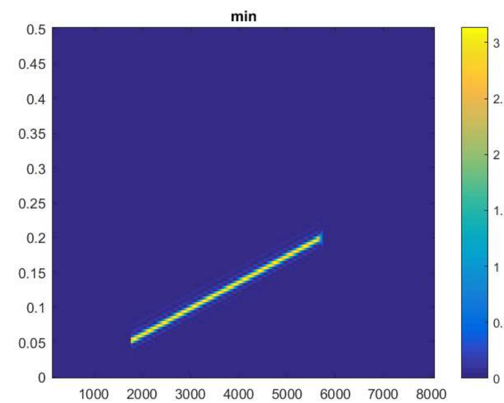
Mohanty, PRD, 2017



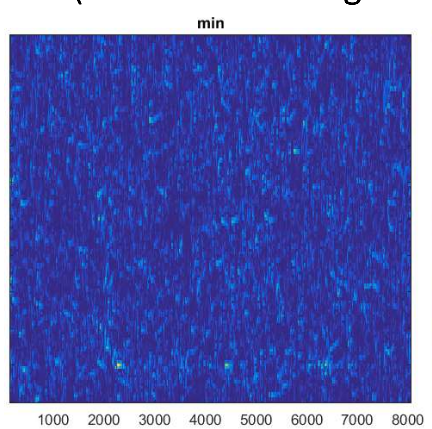
Data (Matched filtering SNR=15)

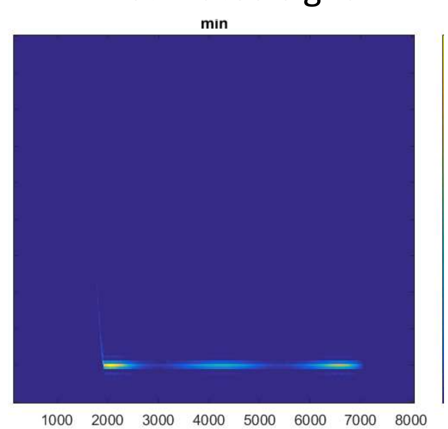


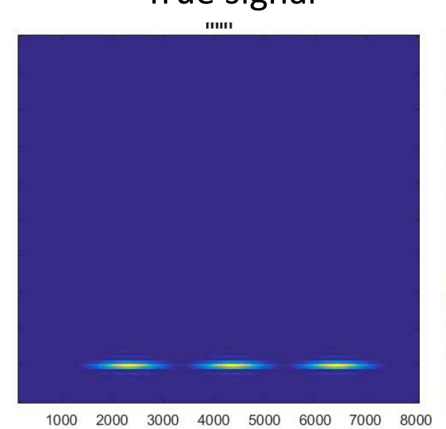
Estimated signal



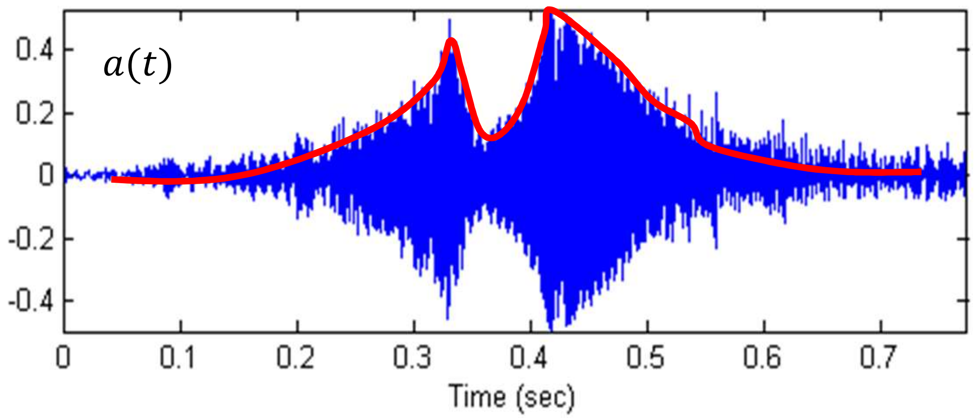
True signal

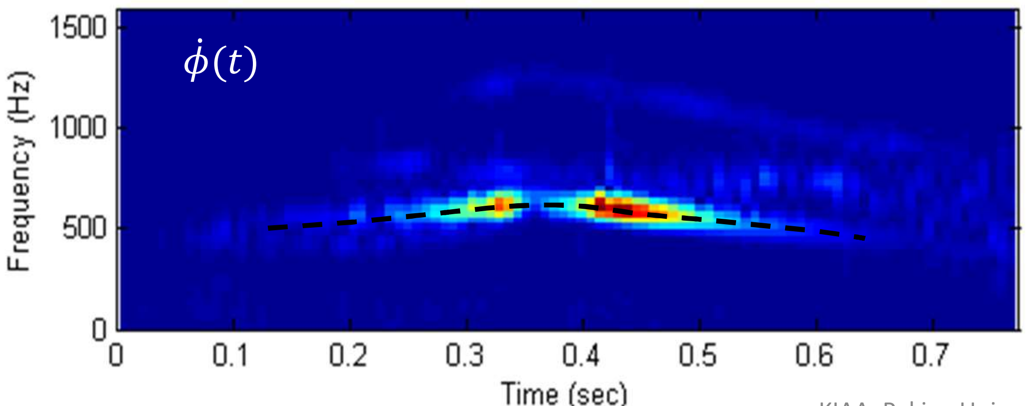






SEECR: Spline Enabled Effective Chirp Regression





- Uses splines to represent the amplitude envelope and instantaneous frequency of a chirp
- Use PSO to optimize the spline breakpoints
- Plus: Penalized spline; Generalized Cross Validation; model selection etc.

Summary

- Many data analysis challenges have been solved but many remain in GW astronomy
- Addressing them is critical to the success of GW astronomy
  - Improved sensitivity
  - Improved parameter estimation
- Highly multi-disciplinary effort
  - GW theory, Astrophysics, Statistics, Computing, Optimization, …
- New approaches required!

Thank you!