

Comparing Exponential Distribution with CLT

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In this project we will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. For this assignment `lambda` will be 0.2 for all of the simulations. We will investigate the distribution of averages of 40 exponentials for 1000 simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should

1. Show the sample mean and compare it to the theoretical mean of the distribution.

```
lambda = 0.2
n = 40
sim = 1:1000
set.seed(12345)
sampleMeans <- data.frame(x = sapply(sim, function(x) {mean(rexp(n, lambda))}))
head(sampleMeans)
```

```
##      x
## 1 5.554
## 2 4.671
## 3 4.256
## 4 4.614
## 5 5.304
## 6 3.627
```

```
sampleMean <- mean(sampleMeans$x)
theoreticalMean <- 1/lambda
sampleMean
```

```
## [1] 4.972
```

```
theoreticalMean
```

```
## [1] 5
```

2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the

distribution.

```
variance <- sd(sampleMeans$x)
theoreticalVariance <- (1/lambda)/sqrt(40)
variance
```

```
## [1] 0.7716
```

```
theoreticalVariance
```

```
## [1] 0.7906
```

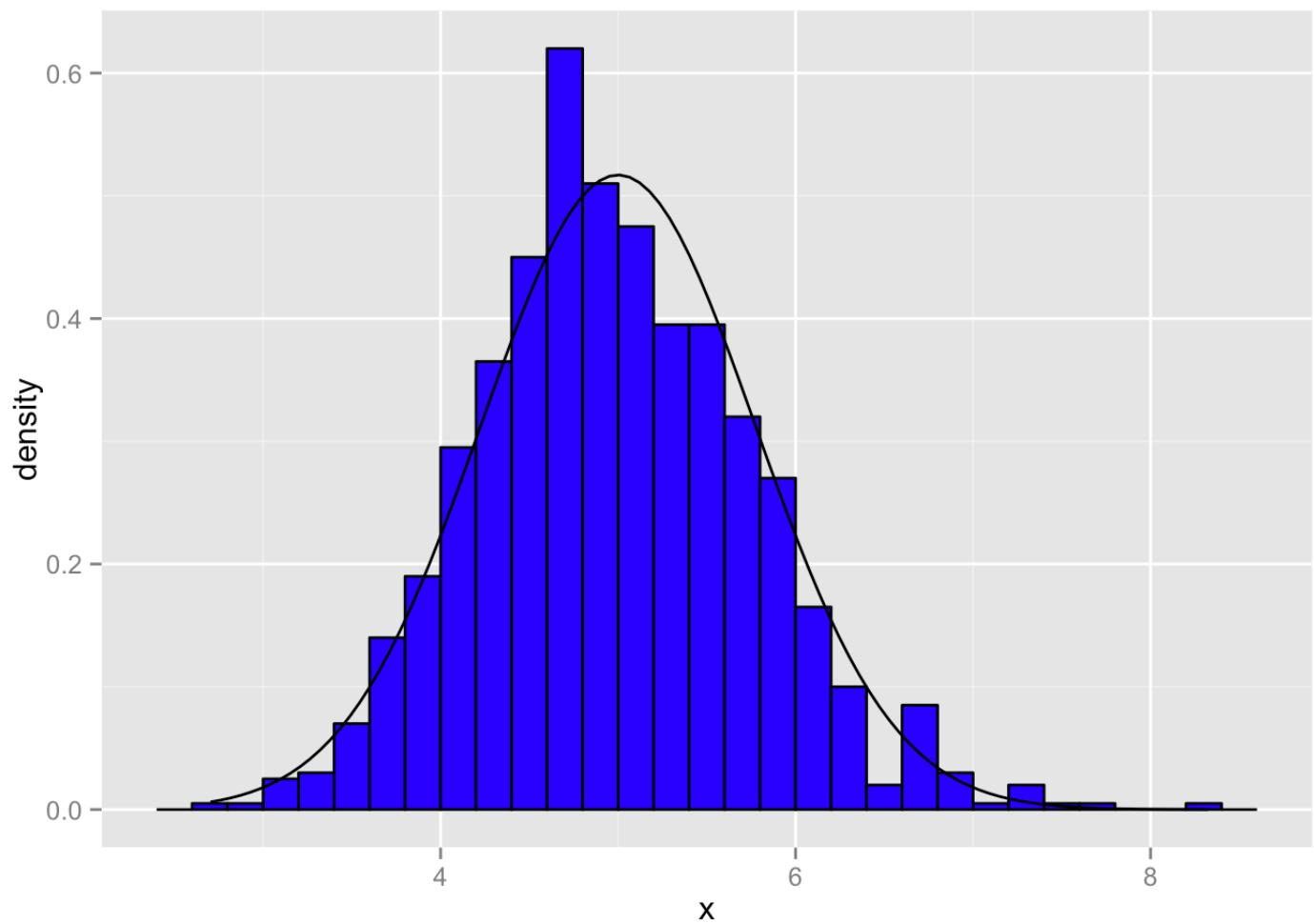
Sample mean: 4.972 Theoretical mean: 5

Variance of the distribution: 0.7716 Theoretical Variance : 0.7906

3. Show that the distribution is approximately normal.

Below is a histogram plot of the means of the 1000 simulations of $\text{rexp}(n, \lambda)$. It is overlaid with a normal distribution with mean 5 and variance 0.7716.

```
library(ggplot2)
ggplot(data = sampleMeans, aes(x = x)) +
  geom_histogram(aes(y=..density..), fill = I('blue'),
                 binwidth = 0.20, color = I('black')) +
  stat_function(fun = dnorm, arg = list(mean = theoreticalMean, sd = variance))
```



The distribution of our simulations appears normal.

Focus on the difference between the distribution of a large collection of random exponentials and the distribution of a large collection of averages of 40 exponentials.

```
sampleMean + c(-1,1)*1.96*variance/sqrt(nrow(sampleMeans))
```

```
## [1] 4.924 5.020
```