Advanced Statistical Modeling

Non-parametric models - Generalized Additive Models and Semiparametric models

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When we use Generalized nonparametric multiple regression models, the nonparametric estimation of the smooth function θ and the smooth regression function m by maximum local likelihood can suffer the effects of the curse of dimensionality, that is, in high dimensional spaces the neighborhood of any point x does not contain enough observed data. To overcome this problem a solution is to use Generalized Addittive Models (GAM).

The GAM are nonparametric regression models less flexible than the generalized multiple nonparametric regression models, but have the advantage of being able to be estimated even when the number of explanatory variables is high. The main assumption with this kind of models is that the nonparametric link functions $g_j(x_j)$ are combined additively to produce the nonparametric p-dimensional regression function.

Furthermore, some modifications can be applied to the link function $g_j(x_j)$, allowing these functions to be linear or combining the effect of several explanatory variables. The resulting model is known as Semiparametric model and we will use it to explain the response variable.

Generalized additive models and semiparametric models

In this exercise we are going to use the `countries` dataset, which contains development indicators of the different countries. We are going to build different Generalized Addittive Models to explain the difference in life expectancy between men and women (le.fm) using the infant mortality rate (inf.mort) and life expectancy at birth (life.exp) as explanatory variables. Additionally, we will build semiparametric models applying spline based smooths over the explanatory variables. At last, we will use analysis of variance (ANOVA) to compare the semiparametric models and choose the best one. For GAM models the goal is minimize the residual deviance (goodness of fit) and minimize the degrees of freedom.

Bacause the variable `le.fm` always takes non-negative values, except for one country we will take 0 as minimum value and define this new variable as `le.fm.0`

```
countries <- read.table(file="countries.txt",head=T,row.names=2,dec=",")
attach(countries)
le.fm.0 <- pmax(0,le.fm)
```

We will fit the following local Poisson regression models using the `gam` function from package `mgcv`

- `le.fm.0 ~ inf.mort + life.exp`
- `le.fm.0 ~ s(inf.mort) + life.exp`
- `le.fm.0 ~ inf.mort + s(life.exp)`
- `le.fm.0 ~ s(inf.mort) + s(life.exp)`
- `le.fm.0 ~ s(inf.mort,life.exp)`

```
gam1.0 <- gam(le.fm.0 ~ inf.mort + life.exp, data=countries,family=poisson)
gam1.1 <- gam(le.fm.0 ~ s(inf.mort) + life.exp, data=countries,family=poisson)
gam1.2 <- gam(le.fm.0 ~ inf.mort + s(life.exp), data=countries,family=poisson)
gam1.3 <- gam(le.fm.0 ~ s(inf.mort) + s(life.exp), data=countries,family=poisson)
gam1.4 <- gam(le.fm.0 ~ s(inf.mort,life.exp), data=countries,family=poisson)
```

Now we will compare the models using ANOVA. GAM models are directly comparable with GLMs and therefore, we can use classical tests based on model deviance (Chi-squared or F tests) to compare the models. In this excercise we will use Chi-squared test.

```r
anova(gam1.0,gam1.1,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ inf.mort + life.exp
## Model 2: le.fm.0 ~ s(inf.mort) + life.exp
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    129.00     90.315
## 2    128.12     88.953 0.87956   1.3615   0.2108
```

```r
anova(gam1.0,gam1.2,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ inf.mort + life.exp
## Model 2: le.fm.0 ~ inf.mort + s(life.exp)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    129.00     90.315
## 2    125.47     85.032 3.5262    5.283    0.206
```

```r
anova(gam1.0,gam1.3,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ inf.mort + life.exp
## Model 2: le.fm.0 ~ s(inf.mort) + s(life.exp)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    129.00     90.315
## 2    125.06     83.564 3.9404   6.7507   0.1449
```

```r
anova(gam1.0,gam1.4,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ inf.mort + life.exp
## Model 2: le.fm.0 ~ s(inf.mort, life.exp)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    129.00     90.315
## 2    127.23     87.668 1.7742   2.6467   0.2263
```

```r
anova(gam1.1,gam1.2,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ s(inf.mort) + life.exp
## Model 2: le.fm.0 ~ inf.mort + s(life.exp)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    128.12     88.953
## 2    125.47     85.032 2.6466   3.9216   0.2213
```

```r
anova(gam1.1,gam1.3,test="Chisq")
```

```
## Analysis of Deviance Table
##
```

```
## Model 1: le.fm.0 ~ s(inf.mort) + life.exp
## Model 2: le.fm.0 ~ s(inf.mort) + s(life.exp)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    128.12     88.953
## 2    125.06     83.564 3.0608   5.3892   0.1511
```

```r
anova(gam1.1,gam1.4,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ s(inf.mort) + life.exp
## Model 2: le.fm.0 ~ s(inf.mort, life.exp)
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    128.12     88.953
## 2    127.23     87.668 0.89466   1.2852   0.2273
```

```r
anova(gam1.2,gam1.3,test="Chisq") # .
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ inf.mort + s(life.exp)
## Model 2: le.fm.0 ~ s(inf.mort) + s(life.exp)
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    125.47     85.032
## 2    125.06     83.564 0.41415   1.4676  0.08323 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(gam1.2,gam1.4,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ inf.mort + s(life.exp)
## Model 2: le.fm.0 ~ s(inf.mort, life.exp)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    125.47     85.032
## 2    127.23     87.668 -1.752  -2.6364   0.2237
```

```r
anova(gam1.3,gam1.4,test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: le.fm.0 ~ s(inf.mort) + s(life.exp)
## Model 2: le.fm.0 ~ s(inf.mort, life.exp)
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    125.06     83.564
## 2    127.23     87.668 -2.1661   -4.104   0.1466
```

Comparing the base model $le.fm.0 \sim inf.mort + life.exp$ (gam1.0) with the rest of them we do not see a significant reduction in the residual deviance and therefore, none of the alternatives represent an improvement over the base model. The same situation happens in all the cases, with the exception of the model gam1.3, with which, the deviance is reduced 1.4676 points with 0.1 level of significance, obtaining a residual deviance of 83.564.

Hirsutism dataset

In this exercise we are going to use the `Hirsutism` dataset, which contains information about female patients who suffer from this condition. We will build semiparametric models to predict the Ferriman-Gallwey score value at the end of the treatment (12 months) (`FGm12`), using the variables that has been measured at the begining of the clinical trial. These variables are the baseline hirsutism level at the begining of the trial (`FGm0`), the `Treatment`, systolic blood pressure (`SysPres`), diastolic blood pressure (`DiaPres`), `weight` and `height`.

```
hirsutism <- read.table("hirsutism.dat",header=T,sep = "\t")
hirsutism <- hirsutism[complete.cases(hirsutism),]
attach(hirsutism)
# plot(hirsutism[,c(1,2,5,6,7,8,9)])
```

First we are going to build the base model using all the variables as they are and then build semiparametric models to find the better one.

```
# Base model
m.base <- gam(FGm12~FGm0 + SysPres + DiaPres + weight + height, data = hirsutism)
summary(m.base)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
##
## Parametric coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 17.294894  14.809288   1.168  0.24614
## FGm0         0.513919   0.174413   2.947  0.00415 **
## SysPres     -0.078184   0.054332  -1.439  0.15382
## DiaPres      0.006665   0.074166   0.090  0.92860
## weight       0.016958   0.045012   0.377  0.70730
## height      -6.446993   9.175256  -0.703  0.48419
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## R-sq.(adj) =  0.0819   Deviance explained = 13.3%
## GCV = 26.835  Scale est. = 25.065    n = 91
```

From the summary we can see that the only significant variable is the baseline hirsutism level (FGm0), hence, we will build semiparametrics models applyting smooths to the rest of the variables. Intutively, we will group the systolic and diastolic blood presures and`height` and `weight` varibles.

```
m1 <- gam(FGm12~FGm0 + SysPres + s(DiaPres) + weight + height, data = hirsutism)
summary(m1)
m2 <- gam(FGm12~FGm0 + s(SysPres) + DiaPres + weight + height, data = hirsutism)
summary(m2)
m3 <- gam(FGm12~FGm0 + s(SysPres, DiaPres) + weight + height, data = hirsutism)
summary(m3)
m4 <- gam(FGm12~FGm0 + s(SysPres) + s(DiaPres) + weight + height, data = hirsutism)
summary(m4)
m5 <- gam(FGm12~FGm0 + SysPres + DiaPres + s(weight) + height, data = hirsutism) #mejore p-value
```

```r
summary(m5)
m6 <- gam(FGm12~FGm0 + SysPres + DiaPres + weight + s(height), data = hirsutism)
summary(m6)
m7 <- gam(FGm12~FGm0 + SysPres + DiaPres + s(weight, height), data = hirsutism) # mejor R-sq y GCV
summary(m7)
m8 <- gam(FGm12~FGm0 + SysPres + DiaPres + s(weight) + s(height), data = hirsutism)
summary(m8)
m9 <- gam(FGm12~FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height), data = hirsutism)
summary(m9)
```

Having fit all the semiparametrics models we are going to use anova to compare each of them

```r
anova(m.base, m1, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    84.235     2104.3 0.76531    26.22   0.2301
```

```r
anova(m.base, m2, test = "Chisq") # ***
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
##   Resid. Df Resid. Dev         Df   Deviance Pr(>Chi)
## 1        85     2130.6
## 2        85     2130.6 4.0653e-10 5.3496e-09 4.55e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m.base, m3, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    84.277     2111.4 0.72258    19.15   0.2779
```

```r
anova(m.base, m4, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    84.235     2104.3 0.76531    26.22   0.2301
```

```r
anova(m.base, m5, test = "Chisq")
```

```
## Analysis of Deviance Table
```

```
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    83.897     2081.4 1.1029   49.194   0.1784
```

```r
anova(m.base, m6, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1     85.00     2130.6
## 2     81.11     2007.8 3.8905   122.77   0.2719
```

```r
anova(m.base, m7, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
##   Resid. Df Resid. Dev    Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    83.204     2058.5 1.796   72.076   0.1974
```

```r
anova(m.base, m8, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    80.008     1960.0 4.9916   170.54   0.2139
```

```r
anova(m.base, m9, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    80.008     1960.0 4.9916   170.54   0.2139
```

```r
anova(m2, m1, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    84.235     2104.3 0.76531    26.22   0.2301
```

```r
anova(m2, m3, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    84.277     2111.4 0.72258    19.15   0.2779
```

```r
anova(m2, m4, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    84.235     2104.3 0.76531    26.22   0.2301
```

```r
anova(m2, m5, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    83.897     2081.4 1.1029   49.194   0.1784
```

```r
anova(m2, m6, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1     85.00     2130.6
## 2     81.11     2007.8 3.8905   122.77   0.2719
```

```r
anova(m2, m7, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
##   Resid. Df Resid. Dev    Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    83.204     2058.5 1.796   72.076   0.1974
```

```r
anova(m2, m8, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
```

```
## 1    85.000     2130.6
## 2    80.008     1960.0 4.9916   170.54   0.2139
```

```r
anova(m2, m9, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    85.000     2130.6
## 2    80.008     1960.0 4.9916   170.54   0.2139
```

```r
anova(m3, m1, test = "Chisq") # *
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev       Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    84.235     2104.3 0.042726   7.0697  0.03208 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m3, m4, test = "Chisq") # *
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev       Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    84.235     2104.3 0.042726   7.0697  0.03208 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m3, m5, test = "Chisq") # .
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    83.897     2081.4 0.38033   30.044  0.09407 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m3, m6, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    81.110     2007.8 3.1679   103.62   0.2593
```

```r
anova(m3, m7, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    83.204     2058.5 1.0734   52.926   0.1552
```

```r
anova(m3, m8, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
##   Resid. Df Resid. Dev    Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    80.008     1960.0 4.269   151.39   0.2042
```

```r
anova(m3, m9, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev    Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    80.008     1960.0 4.269   151.39   0.2042
```

```r
anova(m4, m1, test = "Chisq") # ***
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev          Df    Deviance  Pr(>Chi)
## 1    84.235     2104.3
## 2    84.235     2104.3 -2.1214e-10 -7.9081e-10 2.576e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m4, m5, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    84.235     2104.3
## 2    83.897     2081.4 0.33761   22.974   0.1085
```

```r
anova(m4, m6, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
```

```
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    84.235     2104.3
## 2    81.110     2007.8 3.1252   96.553    0.285
```

```r
anova(m4, m7, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    84.235     2104.3
## 2    83.204     2058.5 1.0307   45.856   0.1779
```

```r
anova(m4, m8, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    84.235     2104.3
## 2    80.008     1960.0 4.2263   144.32   0.2226
```

```r
anova(m4, m9, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    84.235     2104.3
## 2    80.008     1960.0 4.2263   144.32   0.2226
```

```r
anova(m5, m1, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev       Df Deviance Pr(>Chi)
## 1    83.897     2081.4
## 2    84.235     2104.3 -0.33761  -22.974   0.1085
```

```r
anova(m5, m6, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    83.897     2081.4
## 2    81.110     2007.8 2.7876   73.579    0.354
```

```r
anova(m5, m7, test = "Chisq")
```

```
## Analysis of Deviance Table
```

```
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    83.897     2081.4
## 2    83.204     2058.5 0.69306   22.882   0.2297
```

```r
anova(m5, m8, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    83.897     2081.4
## 2    80.008     1960.0 3.8886   121.35   0.2698
```

```r
anova(m5, m9, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    83.897     2081.4
## 2    80.008     1960.0 3.8886   121.35   0.2698
```

```r
anova(m6, m1, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    81.110     2007.8
## 2    84.235     2104.3 -3.1252  -96.553    0.285
```

```r
anova(m6, m7, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    81.110     2007.8
## 2    83.204     2058.5 -2.0945  -50.697   0.3748
```

```r
anova(m6, m8, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    81.110     2007.8
## 2    80.008     1960.0 1.1011    47.77   0.1792
```

```r
anova(m6, m9, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + s(height)
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    81.110     2007.8
## 2    80.008     1960.0 1.1011    47.77   0.1792
```

```r
anova(m7, m1, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    83.204     2058.5
## 2    84.235     2104.3 -1.0307  -45.856   0.1779
```

```r
anova(m7, m8, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    83.204     2058.5
## 2    80.008     1960.0 3.1956   98.467    0.279
```

```r
anova(m7, m9, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight, height)
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev     Df Deviance Pr(>Chi)
## 1    83.204     2058.5
## 2    80.008     1960.0 3.1956   98.467    0.279
```

```r
anova(m8, m1, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    80.008     1960.0
## 2    84.235     2104.3 -4.2263  -144.32   0.2226
```

```r
anova(m8, m9, test = "Chisq") # ***
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev          Df    Deviance  Pr(>Chi)
```

```
## 1    80.008       1960
## 2    80.008       1960 -3.0059e-08 -1.2206e-06 2.542e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m9, m1, test = "Chisq")
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    80.008     1960.0
## 2    84.235     2104.3 -4.2263  -144.32   0.2226
```

We saw that 6 models were significantly different from the others. So we will inspect the residual deviance to choose the best one.

```r
anova(m.base, m2, test = "Chisq") # ***
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + DiaPres + weight + height
##   Resid. Df Resid. Dev         Df   Deviance Pr(>Chi)
## 1        85     2130.6
## 2        85     2130.6 4.0653e-10 5.3496e-09 4.55e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m3, m1, test = "Chisq") # *
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev       Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    84.235     2104.3 0.042726   7.0697  0.03208 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m3, m4, test = "Chisq") # *
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev       Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    84.235     2104.3 0.042726   7.0697  0.03208 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m3, m5, test = "Chisq") # .
```

```
## Analysis of Deviance Table
##
```

```
## Model 1: FGm12 ~ FGm0 + s(SysPres, DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + height
##   Resid. Df Resid. Dev      Df Deviance Pr(>Chi)
## 1    84.277     2111.4
## 2    83.897     2081.4 0.38033   30.044  0.09407 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m4, m1, test = "Chisq") # ***
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + weight + height
## Model 2: FGm12 ~ FGm0 + SysPres + s(DiaPres) + weight + height
##   Resid. Df Resid. Dev          Df    Deviance  Pr(>Chi)
## 1    84.235     2104.3
## 2    84.235     2104.3 -2.1214e-10 -7.9081e-10 2.576e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m8, m9, test = "Chisq") # ***
```

```
## Analysis of Deviance Table
##
## Model 1: FGm12 ~ FGm0 + SysPres + DiaPres + s(weight) + s(height)
## Model 2: FGm12 ~ FGm0 + s(SysPres) + s(DiaPres) + s(weight) + s(height)
##   Resid. Df Resid. Dev          Df    Deviance  Pr(>Chi)
## 1    80.008       1960
## 2    80.008       1960 -3.0059e-08 -1.2206e-06 2.542e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Looking at the models we can see that the ones that minimize the residual deviance are the m8 and m9. They have in common that both apply smooth functions over the `height` and `weight` variables, separately. However, because the m8 is simpler, we will choose it as the best model.