Heat Transfer: Convection

Forced Convection in Internal Flow – Developing versus Fully Developed Flows and the Caloric Mean Temperature

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Learning goals

Forced convection in internal flows:

- Knowledge of the essential differences between external and internal flows
- Understanding of the hydrodynamic and thermal **inlet behavior**
- Ability to calculate the **caloric mean temperature**
- Ability to calculate the local temperatures and heat fluxes as well as the average heat transfer coefficient

Classifications according to flow condition

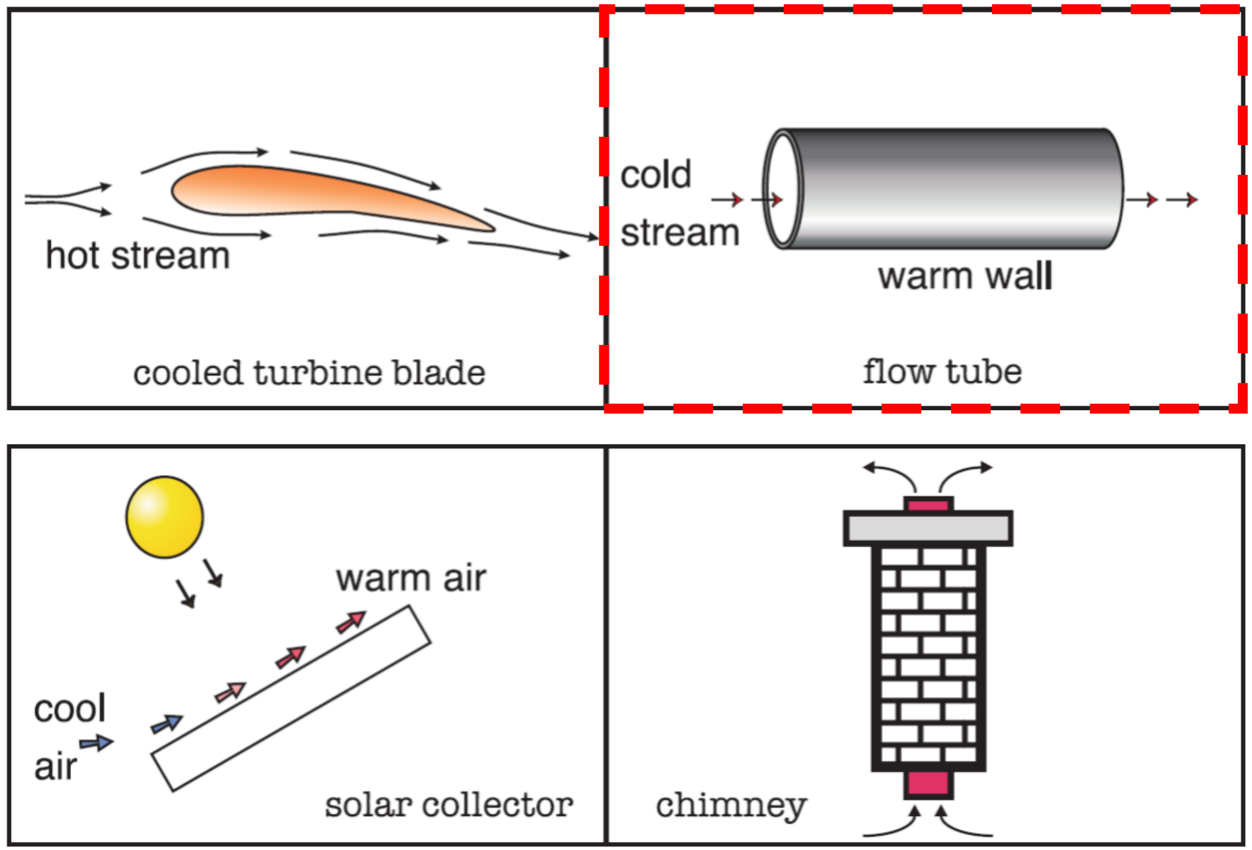
Forced convection:

- Driven by externally generated movement of the fluid/object

Free convection:

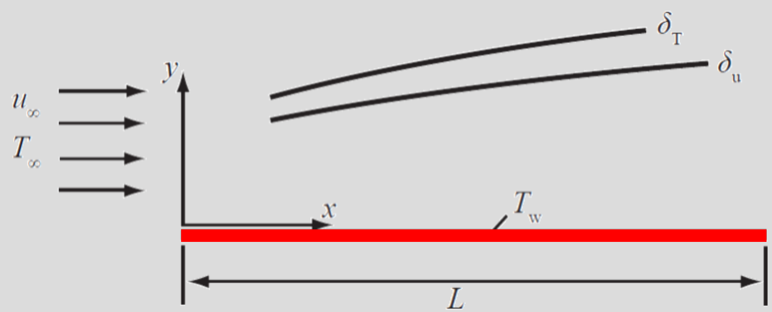
- Inherently driven due to heat transfer (density differences)

External | **Internal**



Difference between external flow and internal flow

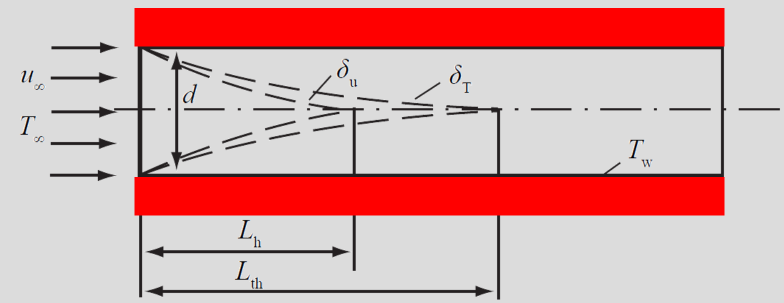
External flow:



- u_∞ and T_∞ remain constant.
- Continuous growth of the two boundary layers.
- Turbulent transition at x_{crit} even for very slow flows.
- Heat transfer at isothermal wall:

$$\dot{Q} = \alpha A(T_W - T_\infty)$$

Internal flow:



- Mean velocity $\bar{u}$ constant due to mass conservation.
- Merging of boundary layers, then constant velocity profile.
- Laminar or turbulent flow throughout the channel depending on the flow velocity.
- Caloric average temperature $\bar{T}$ increases and approaches T_W.
- Heat transfer with **logarithmic mean temperature difference**.

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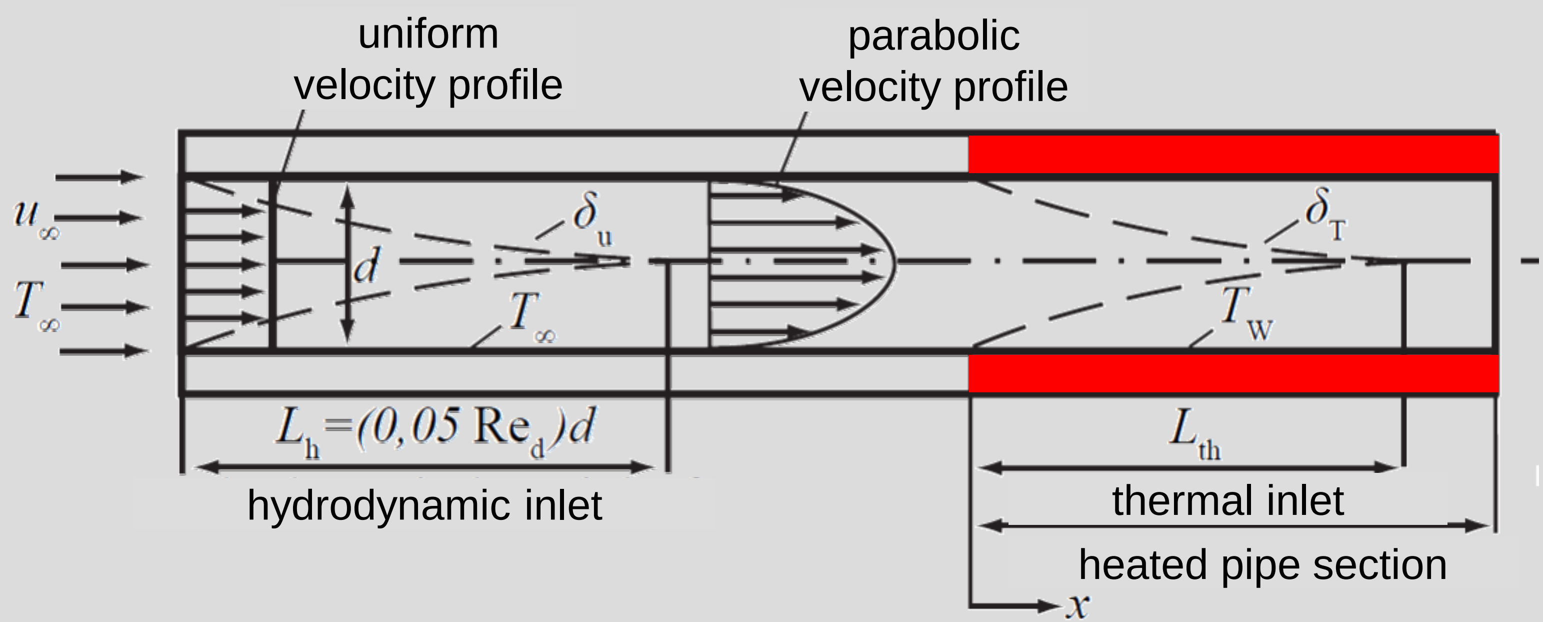
Classification of pipe flows

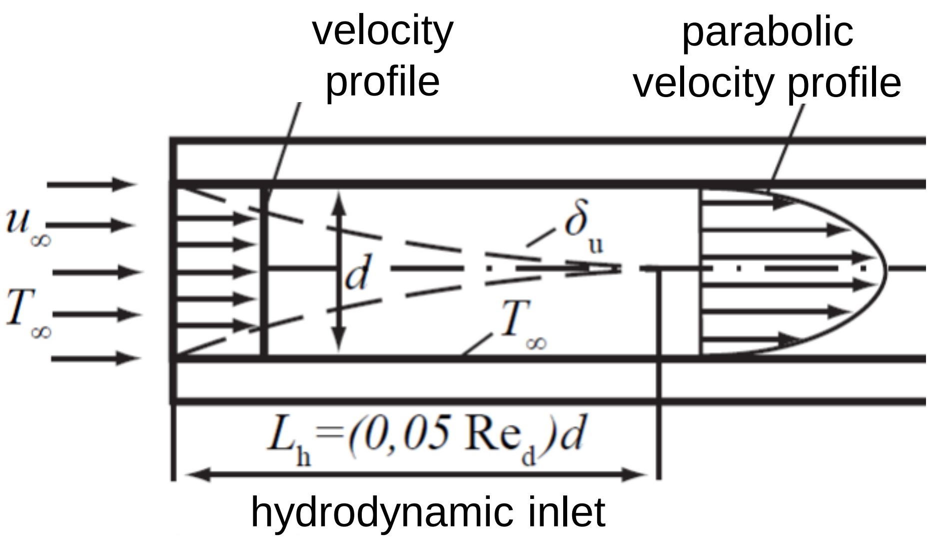
Inlet flow vs. fully developed flow for internal flow geometries:



Hydrodynamic inlet length **Thermal inlet length**

- With the inlet length, the boundary layer grows until reaching the channel center. Shear stress and heat transfer rate decrease drastically in the inlet section.
- In fully developed region, the boundary layers are fully formed. Shear stress approaches a constant value.

Hydrodynamically fully developed flow



Hydrodynamically fully developed:

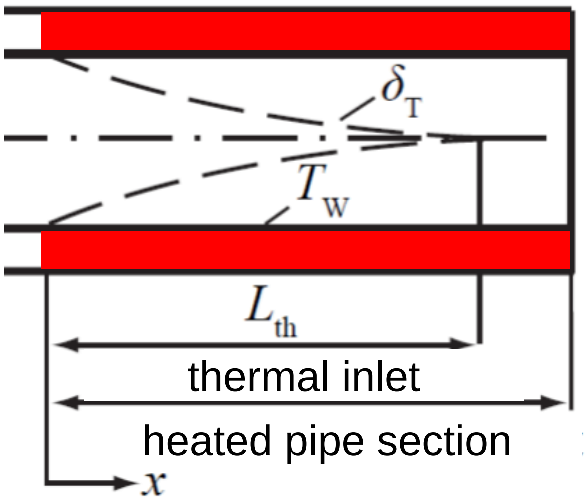
- Velocity profile invariant in flow direction:

$$\frac{\partial u(r,x)}{\partial x} = 0$$

- Wall shear stress does no longer depend on spatial position:

$$\tau_w = \mu \left.\frac{\partial u}{\partial r}\right|_{r=R} \neq f(x)$$

Thermally fully developed flow



Thermally fully developed:

- "Temperature" profile in independent of x :
  - → **But**: Temperature continuously rises due to the wall-side heat flux in axial direction. Temperature is not fixed at the wall unlike the velocity.
  - → **Aim**: Define a temperature profile such that the heat transfer coefficient becomes independent of x, consistent with the behavior of wall shear stress.

Solution:

Wall temperature

$$\alpha = \frac{\dot{q}''}{(T_W - T_m)}$$

Mean (reference) temperature

In order for the heat transfer coefficient to be constant at constant heat flow, the mean temperature must increase in the same way as the wall temperature.

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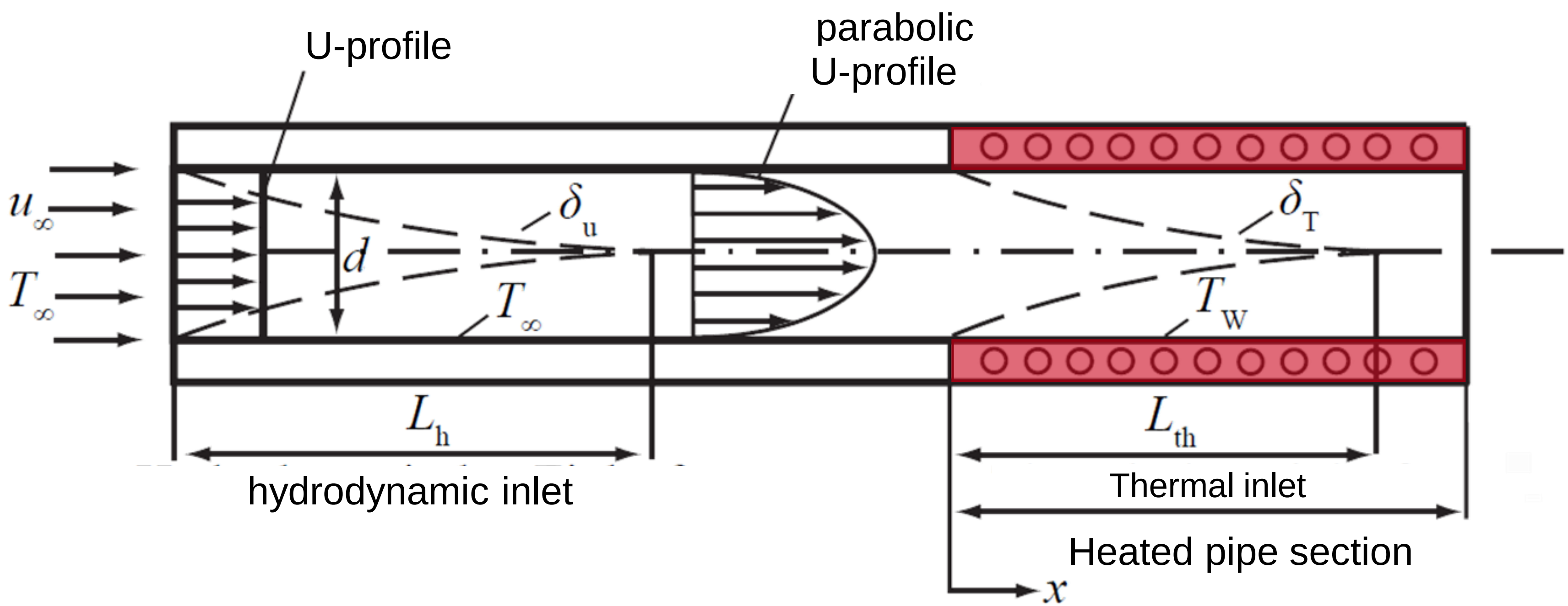
Pipe flow with hydrodynamically formed flow at the beginning of the heated/cooled pipe section and isothermal surface



	laminar	turbulent
Hydrodynamic entry length	$L_h \approx 0.05\, Re_d d$	$L_h \approx 10d$
Thermal entry length	$L_{th} \approx 0.05\, Re_d Pr d$	$L_h \approx 10d$

Average temperature: Mean caloric temperature

Mean caloric temperature:

- Defined by the **flow of enthalpy** through a cross-sectional area

Enthalpy flow through a cross section:

$$\dot{\mathrm{H}} = \int_A \rho c_p T(r) u(r) dA$$

$$dA = 2\pi r dr \text{ (pipe)}$$

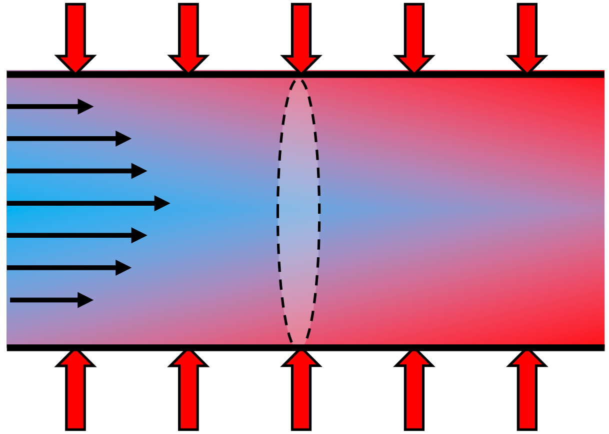
Definition of the mean temperature:

$$\dot{\mathrm{m}} c_p T_\mathrm{m} \equiv \dot{\mathrm{H}} = \int_A \rho c_p T(r) u(r) dA$$

$$T_\mathrm{m} \equiv \frac{1}{\dot{\mathrm{m}} c_p} \int_A \rho c_p T(r) u(r) dA$$

$$T_\mathrm{m} \equiv \frac{1}{\rho U A} \int_A \rho T(r) u(r) dA$$

U = mean flow velocity



How to determine axial temperature profile in the pipe and outlet temperature?

Development of energy balance:

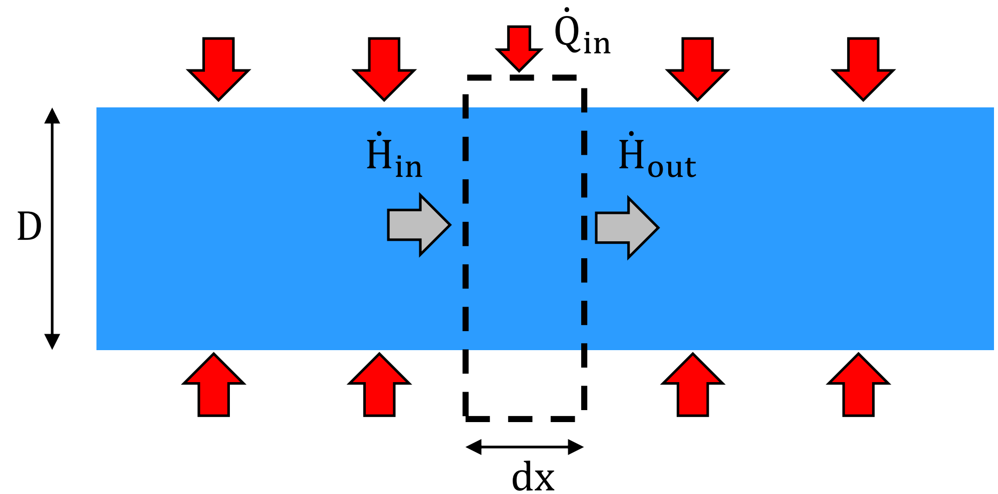
- Develop local energy balance for the temperature profile

→ **Energy balance**: $0 = \dot{H}_{\mathrm{in}} - \dot{H}_{\mathrm{out}} + \dot{Q}_{\mathrm{in}}$

$$\dot{\mathrm{H}}_{\mathrm{in}} = \dot{m} c_p T_{\mathrm{m}}(x)$$

$$\dot{\mathrm{H}}_{\mathrm{out}} = \dot{m} c_p T_{\mathrm{m}}(x + dx)$$

$$\dot{\mathrm{Q}}_{\mathrm{in}} = \dot{q}'' A = \dot{q}'' \pi D dx$$



Differential equation:

$$0 = -\dot{m} c_p \frac{\partial T_{\mathrm{m}}(x)}{\partial x} + \dot{q}'' \pi D$$

$$\frac{\partial T_{\mathrm{m}}(x)}{\partial x} = \frac{\dot{q}'' \pi D}{\underbrace{\dot{m} c_p}_{\text{constant}}}$$

Linear increase in temperature

Change of mean temperature in pipe flow with constant heat flux

Differential equation:

$$0 = -\dot{m}c_p \frac{\partial T_\mathrm{m}(x)}{\partial x} + \dot{q}''\pi D$$

Boundary condition:

$$T_\mathrm{m}(x = 0) = T_\mathrm{in} \rightarrow C = T_\mathrm{in}$$

Outlet temperature:

$$T_\mathrm{m}(x = L) = T_\mathrm{out} \rightarrow T_\mathrm{out} = \frac{\dot{q}''\pi D}{\dot{m}c_p} L + T_\mathrm{in} \quad (\pi D L = A_\mathrm{surface})$$

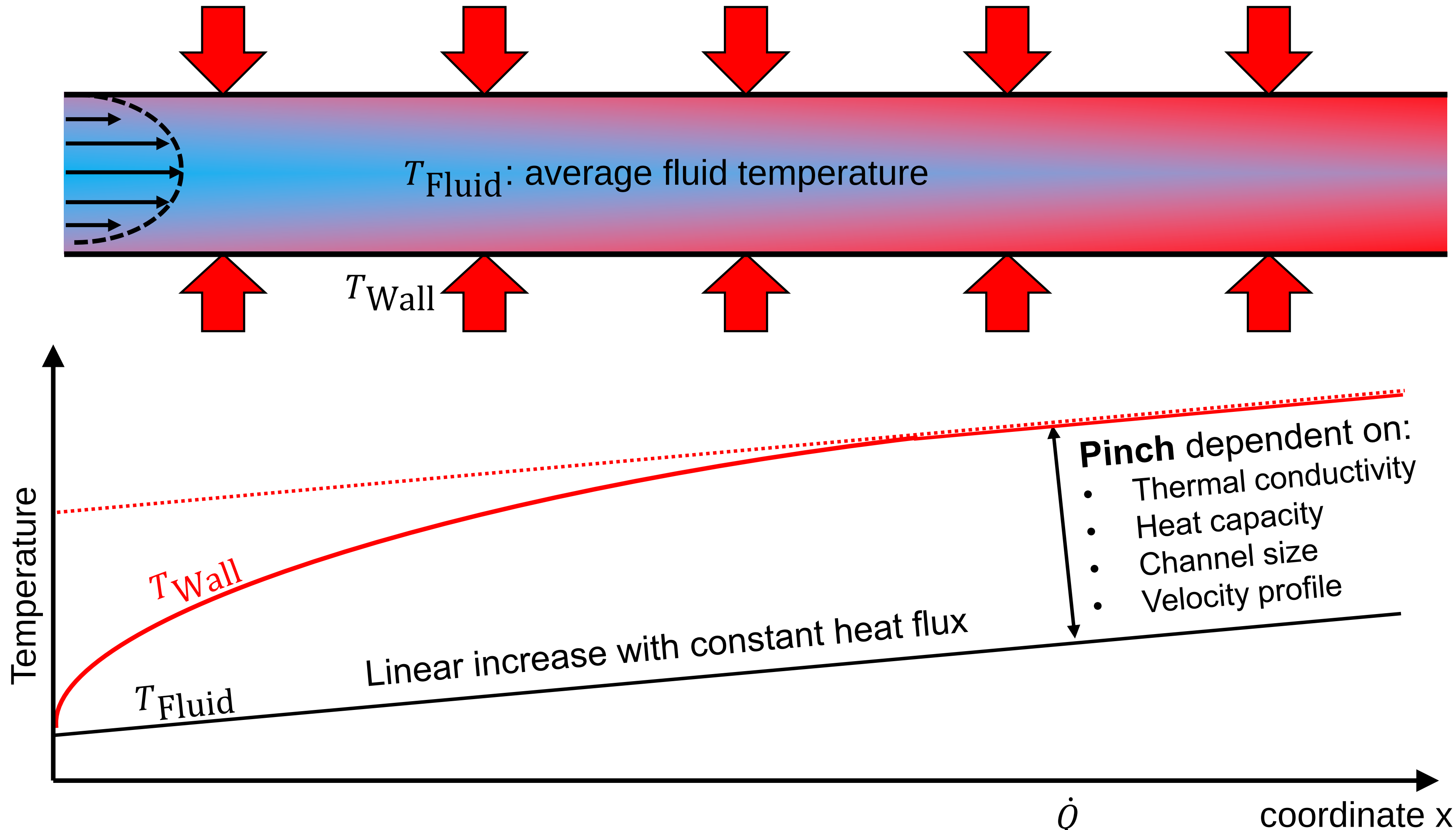
Wall temperature:

$$\dot{q}_\mathrm{w}'' = \alpha(T_\mathrm{w} - T_\mathrm{m}) \quad \rightarrow T_\mathrm{w} = T_\mathrm{m} + \frac{\dot{q}_w''}{\alpha}$$

How to determine the heat transfer coefficient α?

Content of the next video!!

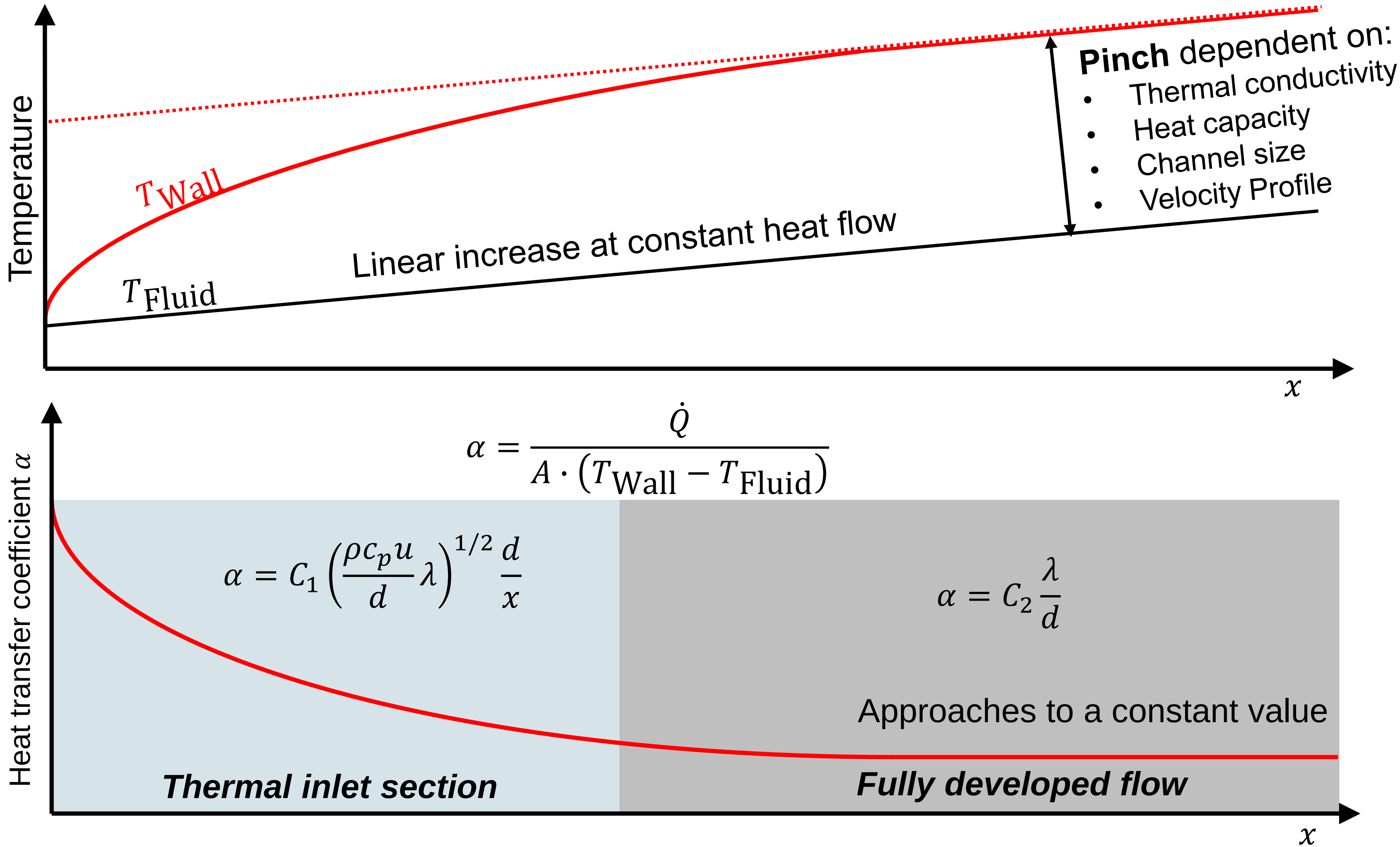
Change of mean temperature in pipe flow with constant heat flux



$$\dot{Q} = \alpha \cdot A \cdot \left(T_{\text{Wall}} - T_{\text{Fluid}}\right)$$

$$\alpha = \frac{\dot{Q}}{A \cdot \left(T_{\text{Wall}} - T_{\text{Fluid}}\right)}$$

Change of mean temperature in pipe flow with constant heat flux



Comprehension questions

What are the differences between external and internal flows?

What is the meaning of the hydrodynamic and thermal entrance length?

What is the meaning of the caloric mean temperature and how can it be calculated?

How does the local heat transfer coefficient change inside a laminar pipe flow?