Heat Transfer

Natural Convection in External Flow

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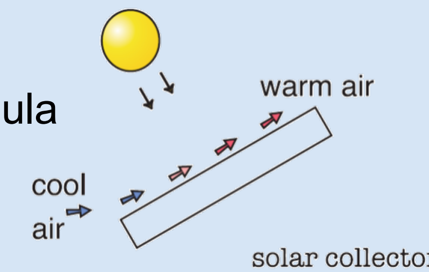
Learning Goals

- Natural convection in External Flow
  - ➢ Knowledge of the correlations given in the reader and on the formula sheet for cases of natural convection



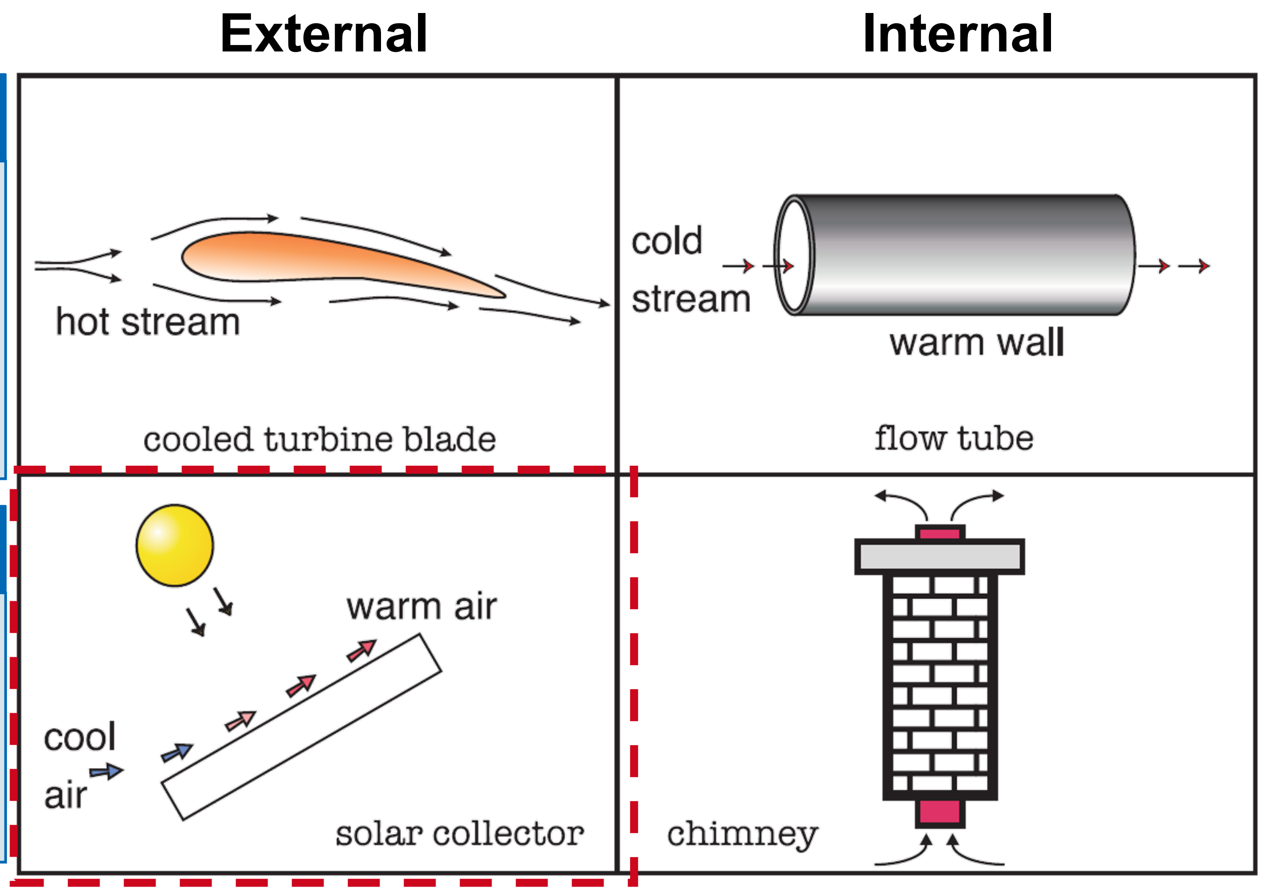
Classifications according to flow regime

Forced Convection

- Driven by externally generated movement of the fluid/object

Free Convection

- Inherently driven due to heat transfer (density differences)



Important dimensionless numbers

Nusselt number:	$Nu = \frac{\alpha L}{\lambda}$	The Nusselt number is the dimensionless heat transfer coefficient
Prandtl number:	$Pr = \frac{\eta c_p}{\lambda}$	The Prandtl number compares the momentum transport due to friction with the heat transport due to conduction
Reynolds number:	$Re = \frac{\rho u_\infty L}{\lambda}$	The Reynolds number gives the ratio of inertial forces to viscous forces
Grashof number:	$Gr = \frac{\rho^2 g \beta (T_\mathrm{W} - T_\mathrm{fl}) L^3}{\eta^2}$	The Grashof number describes the relationship between the buoyancy forces of a fluid and the acting viscous forces

Correlation function for natural convection

General form

Natural Convection: $Nu = Nu(Gr, Pr)$

Applicability criteria

- Geometry
- Flow regime
- Thermal boundary conditions

Material properties

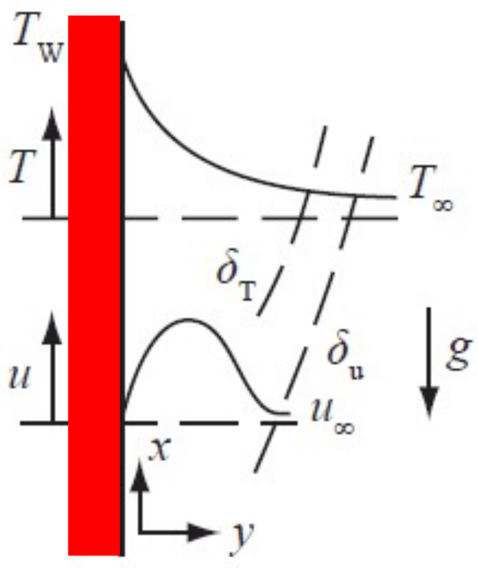
Substance properties used in dimensionless numbers at temperature:

$$T_{\mathrm{Prop}} = \frac{T_W + T_\infty}{2}$$

Exception: Isobaric expansion coefficient β for (perfect) ideal gases:

$$\beta = \frac{1}{T_\infty}$$

Vertical heated plate with laminar boundary layer flow and isothermal surface



Local Heat Transfer:

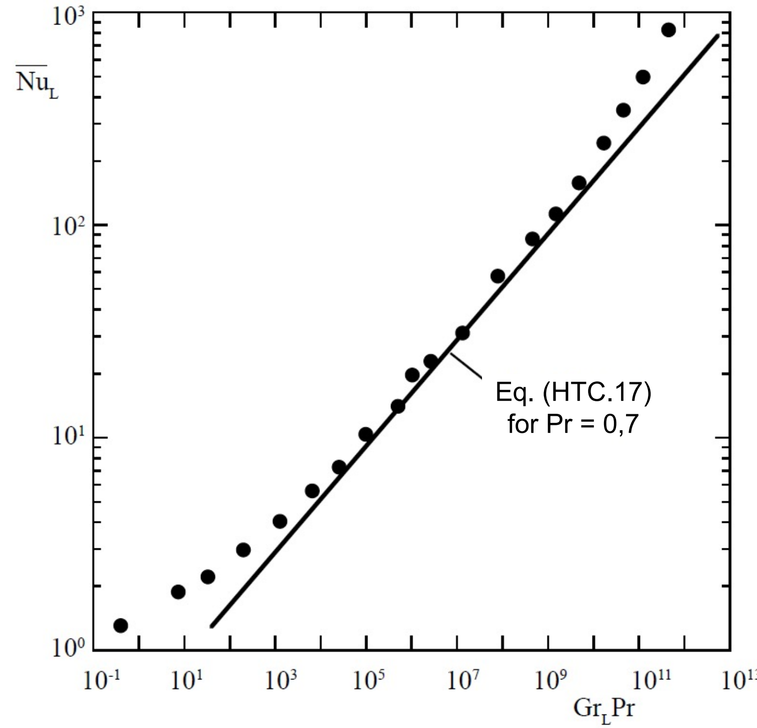
$$\mathrm{Nu}_x = 0{,}508 \left(\frac{\mathrm{Pr}}{0{,}952 + \mathrm{Pr}} \right)^{1/4} (\mathrm{Gr}_x \mathrm{Pr})^{1/4} \qquad \text{(HTC.16)}$$

Vertical heated plate with laminar boundary layer flow and isothermal surface

Average heat transfer:

$$\overline{\mathrm{Nu}_{\mathrm{L}}} = C\,(\mathrm{Gr_L Pr})^{\frac{1}{4}} \qquad \text{(HTC.17)}$$

$$\text{for} \quad \mathrm{GrPr} < \mathrm{Gr_{L,crit}Pr}$$



with the constant C, dependent on the Prandtl number

Pr	0,003	0,01	0,03	0,72	1	2	10	100	1000	∞
C	0,182	0,242	0,305	0,516	0,535	0,568	0,620	0,653	0,665	0,670

Vertical heated plate with laminar boundary layer flow and constant heat flow

Constant heat flow:

$$\mathrm{Nu_x} = 0{,}60\left(\mathrm{Gr_x^*Pr}\right)^{\frac{1}{5}} \qquad \text{(HTC.18)}$$

$$\text{for} \quad 10^5 < \mathrm{Gr}_x^* < 10^{11}$$

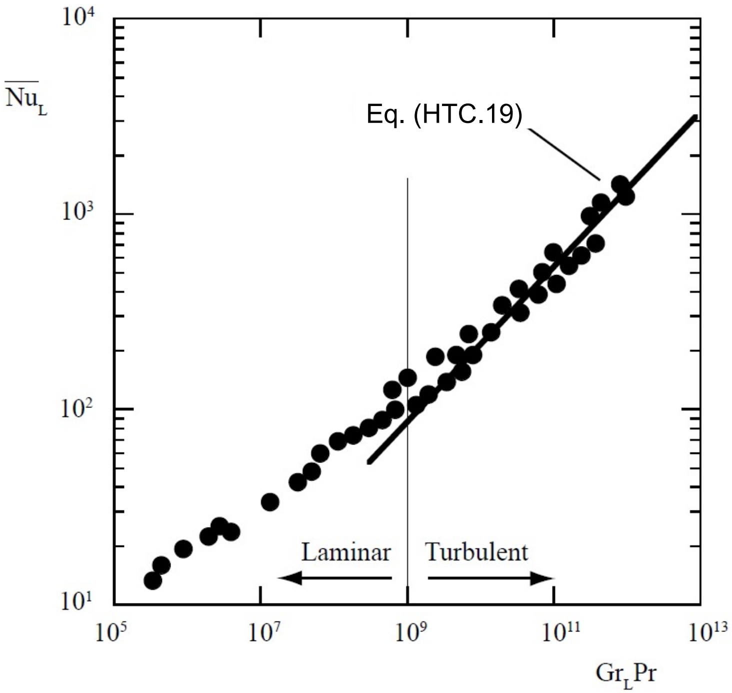
(Note: Since the heat flux is given as a boundary condition, for simplicity, a modified Grashof number $\mathrm{Gr_x^*}$ shall be defined $\mathrm{Gr_x^*} \equiv \mathrm{Gr_x Nu_x} = \frac{\rho^2 g \beta \dot{q}_W'' x^4}{\lambda \eta^2}$)

$$Gr_x Nu_x = \frac{\rho^2 g \beta (T_\mathrm{W} - T_\mathrm{fl}) x^3}{\eta^2} \frac{\dot{q}_W''}{(T_\mathrm{W} - T_\mathrm{fl})} \frac{x}{\lambda}$$

Constant Wall temperature:

$$\overline{\mathrm{Nu_L}} = C\left(\mathrm{Gr_L Pr}\right)^{\frac{1}{4}} \qquad \text{(HTC.17)}$$

Vertical heated plate with turbulent boundary layer flow and isothermal surface



$$\overline{\mathrm{Nu}}_L = 0{,}13\,(\mathrm{Gr}_L Pr)^{1/3} \qquad \text{(HTC.19)}$$

$$\text{for} \quad 10^9 < (\mathrm{Gr}_L \mathrm{Pr}) < 10^{12}$$

(Note: For turbulent boundary layer flow the exponent $\frac{1}{3}$ of the Grashof number makes the heat transfer coefficient independent of the height of the plate)

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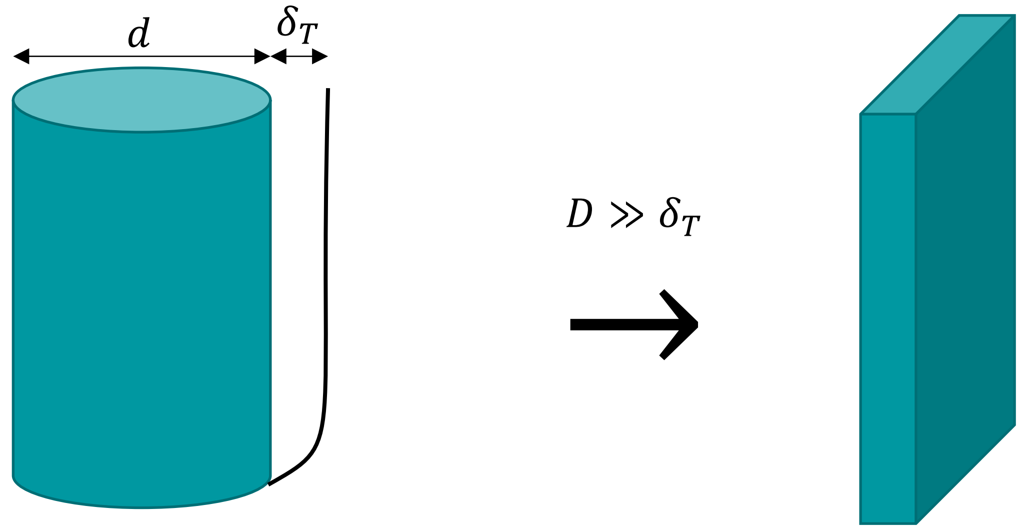
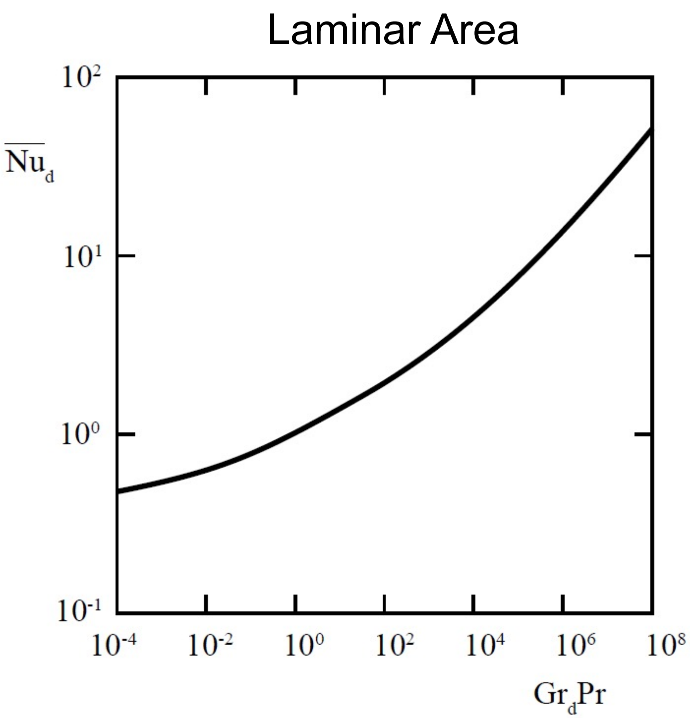
Vertical Cylinder under the influence of laminar and turbulent boundary layer flows



As long as the diameter of the cylinder is significantly higher than the developed boundary layer thickness, the relationships valid for the vertical plate can be applied to this case. The limiting factor is approximately $\frac{d}{L} > 35 \cdot \mathrm{Gr_L}^{-\frac{1}{4}}$. The characteristic geometrical length is the length of the cylinder.

Horizontal Cylinders with isothermal surface under the influence of laminar and turbulent flows



natural convection around large horizontal cylinders
...isumi, T., J. Heat and Mass Tra...

- for the **laminar** range $10^4 < \mathrm{Gr_d Pr} < 10^9$

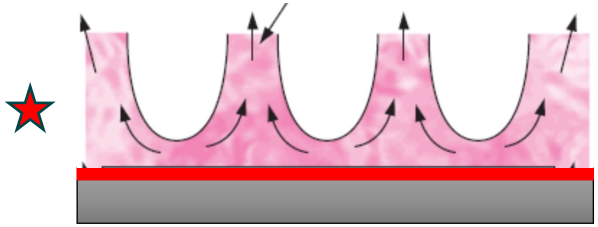
$$\overline{\mathrm{Nu}}_\mathrm{d} = 0{,}53\,(\mathrm{Gr_d Pr})^{\frac{1}{4}} \tag{HTC.20}$$

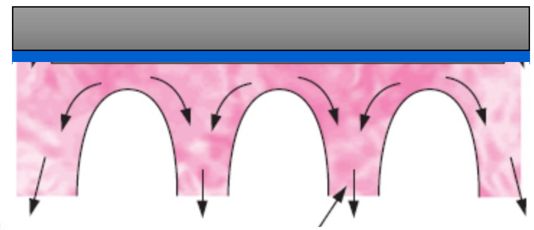
- for the **turbulent** range $10^9 < \mathrm{Gr_d Pr} < 10^{12}$

$$\overline{\mathrm{Nu}}_\mathrm{d} = 0{,}13\,(\mathrm{Gr_d Pr})^{\frac{1}{3}} \tag{HTC.21}$$

with the approximation relations according to Bayley et al. (1972)

Horizontal plates in laminar and turbulent flow with isothermal surface (heated upper side or cooled lower side)





Temperature given

- **laminar** range $2 \cdot 10^4 < \mathrm{Gr_L Pr} < 8 \cdot 10^6$

$$\overline{\mathrm{Nu_L}} = 0{,}54\,(\mathrm{Gr_L Pr})^{\frac{1}{4}} \qquad \text{(HTC.22a)}$$

- **turbulent** range $8 \cdot 10^6 < \mathrm{Gr}_L\mathrm{Pr} < 10^{11}$

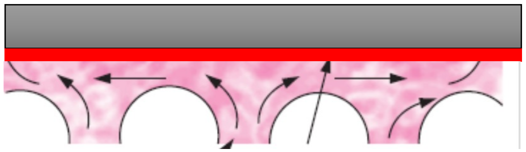
$$\overline{\mathrm{Nu_L}} = 0{,}15\,(\mathrm{Gr_L Pr})^{\frac{1}{3}} \qquad \text{(HTC.23a)}$$

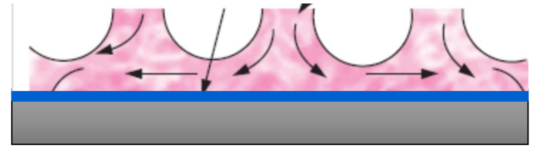
(Note: the relationships presented below are derived from measurements on square plates. They are approximately valid for rectangular and circular surfaces, as long as one mean side length or $(0{,}9d)$, respectively is taken as the characteristic length.)

★ Convection current visualisation: Cengel et al. “Heat and Mass Transfer”

after Holman (1976)

Horizontal plates in laminar and turbulent flow with isothermal surface (cooled upper side or heated lower side)





Temperature given

Isothermal surface, cooled upper plate side or heated lower side

- **laminar** range $10^5 < \mathrm{Gr_L Pr} < 10^{11}$

$$\overline{\mathrm{Nu_L}} = 0{,}27\,(\mathrm{Gr_L Pr})^{\frac{1}{4}} \qquad \text{(HTC.24a)}$$

after Holman (1976)

Horizontal plates in laminar and turbulent flow with constant heat flow and heated upper side or cooled lower side of plate

Heat flow given

- **laminar** range $\mathrm{Gr_L Pr} < 2 \cdot 10^8$

$$\overline{\mathrm{Nu_L}} = 0{,}13\,(\mathrm{Gr_L Pr})^{\frac{1}{3}} \qquad \text{(HTC.22b)}$$

- **turbulent** range $2 \cdot 10^8 < \mathrm{Gr_L Pr} < 10^{11}$

$$\overline{\mathrm{Nu_L}} = 0{,}16\,(\mathrm{Gr_L Pr})^{\frac{1}{3}} \qquad \text{(HTC.23b)}$$

after Holman (1976)

Horizontal plates in laminar and turbulent flow with constant heat flow and cooled plate top or heated underside

Heat flow given

- **laminar** range $10^6 < \mathrm{Gr_L Pr} < 10^{11}$

$$\overline{\mathrm{Nu_L}} = 0{,}58\,(\mathrm{Gr_L Pr})^{\frac{1}{5}} \qquad \text{(HTC.24b)}$$

after Holman (1976)

Comprehension Questions

Which dimensionless numbers must be taken into account when applying the heat transfer laws?

What is the driving potential in natural convection?

Which are the two different cases for horizontal plates and how do they differ from vertical plates?