Mass Transfer: Evaporation

Short calculation example:
Evaporation on a liquid surface - Stefan Flow

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How much water evaporates over Lake Constance (Bodensee) in summer?

Considerations:

- How can the problem be described?

- Does mass transport limit the problem?
  $\longrightarrow$ Mass flow = transport coefficient * driving potential

- Does heat transport limit the problem?
  $\longrightarrow$ Heat flow = heat transfer coefficient * driving potential
  $\longrightarrow$ Mass flow = heat flow / evaporation enthalpy

Question

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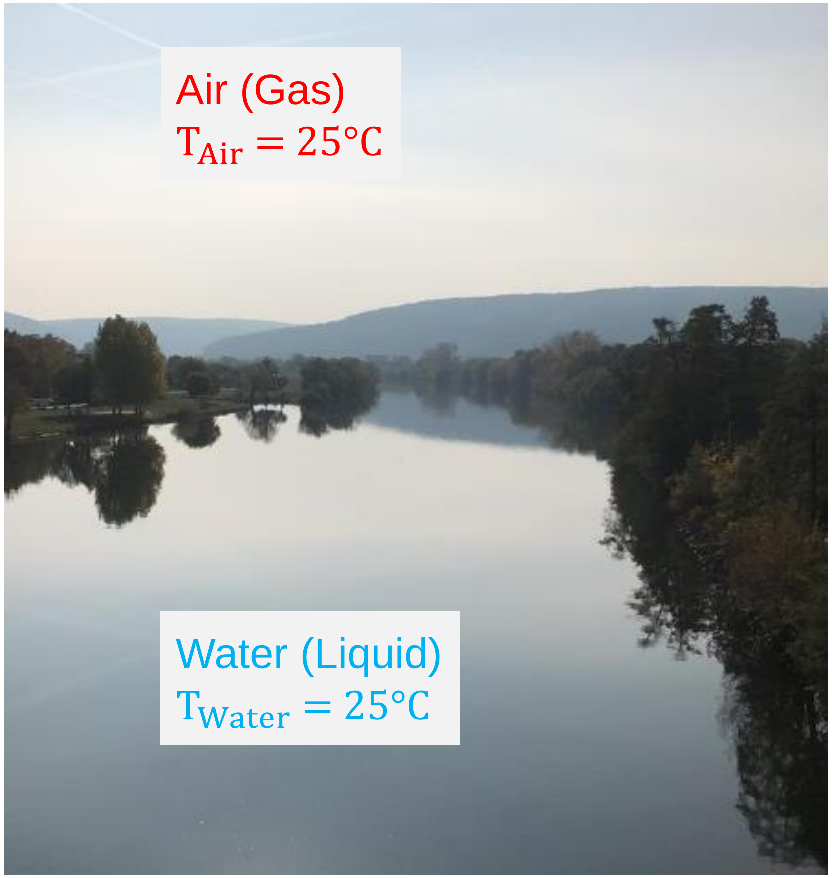

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Assumption:
Air temperature and water temperature are given and remain constant, the water temperature does not drop due to the evaporating mass flow

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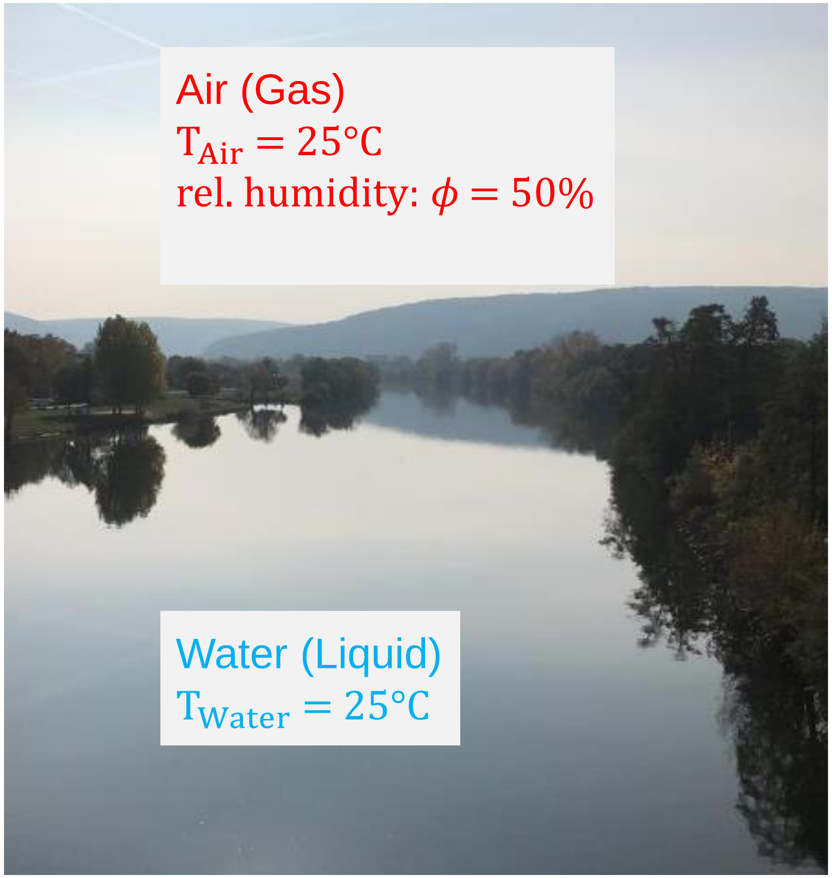

Considerations:

- How to determine the driving potential?

  Assumption:
  The surface of the water is impermeable to air, therefore the **Stefan Flow** must be taken into account in the driving potential

$$\dot{m}''_w \propto \frac{\xi_{H_2O,s} - \xi_{H_2O,\infty}}{1 - \xi_{H_2O,s}}$$

- How can the mass fractions be determined?
  - The water mass fraction in the air depends on the saturation
  - At the water surface, the air is fully saturated with water (ϕ = 1)
  - The temperature of the air at the water surface is equal to the water temperature
  - The air humidity in the environment must be assumed

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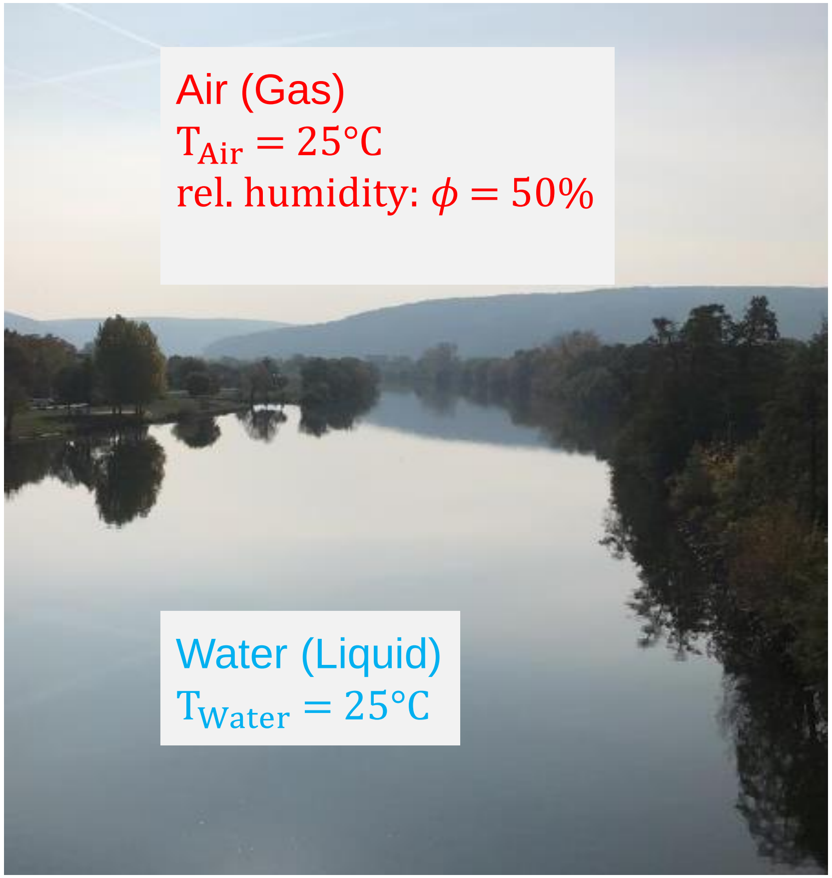

Considerations:

- How to determine the mass transfer coefficient?

$$\dot{m}''_w \propto g$$

- How can the mass fractions be determined?

Assumption:
Lewis' Law is applicable, the mass transfer coefficient g can be determined by the heat transfer coefficient α

Prerequisite:
Thermal Diffusion ≈ Mass Diffusion, i.e. $Pr \approx Sc$

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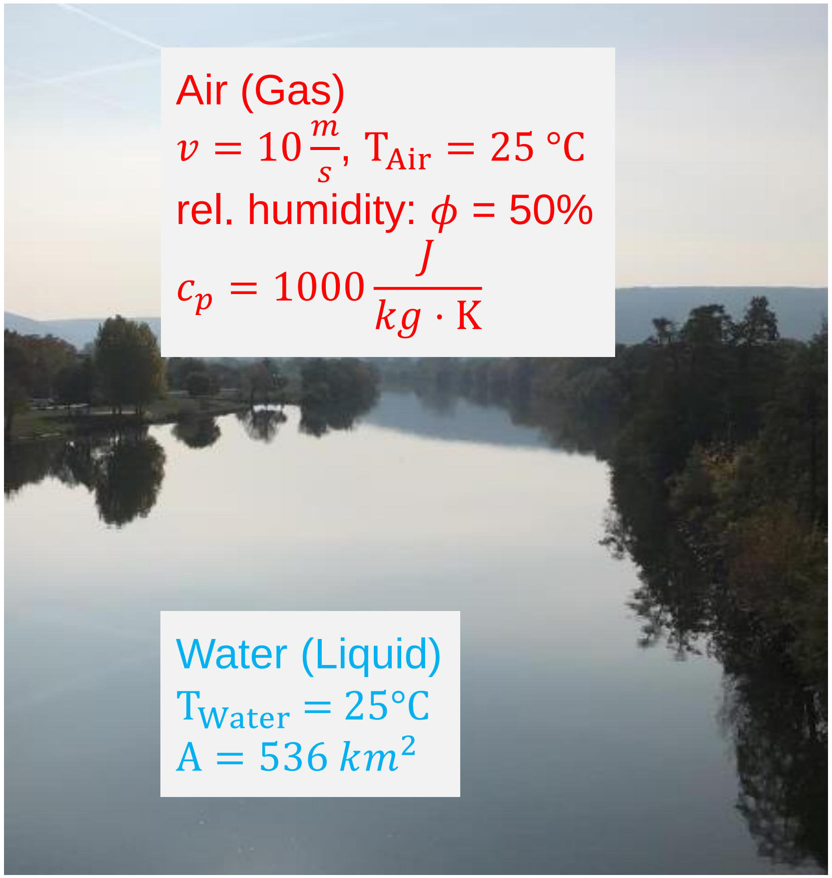

Evaporation of the liquid on the water surface:

- Diffusion of Water and Air
- Water surface as phase interface at $x = 0$
- Phase interface permeable for Water vapor and impermeable for Air

  → Semi-permeable phase interface or one-sided diffusion

Calculation Procedure:

1. Determination of the heat transfer coefficient $\bar{\alpha}$
2. Derivation of an equation for the mass fraction $\xi_{H_2O,i}$
3. Mass fraction of water vapor at the surface $\xi_{H_2O,s}$
4. Mass fraction of water vapor in air $\xi_{H_2O,\infty}$
5. Lewis Law and mass transfer coefficient g
6. Mass of evaporated water at the surface $\dot{m}_w''$
7. Required heat flow $\dot{Q}$

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Determination of the heat transfer coefficient $\bar{\alpha}$

Tabelle 4: Gases at 1 bar

	T	ρ	c	λ	ν	a	Pr
	°C	kg/m^3	$kJ/kg\,K$	$10^{-3}\ W/m\,K$	$10^{-6}\ m^2/s$	$10^{-6}\ m^2/s$	1
Air	-200	5,106	1,186	6,886	0,979	1,137	0,8606
	-100	2,019	1,011	16,2	5,829	7,851	0,7423
	0	1,275	1,006	24,18	13,52	18,83	0,7179
	20	1,188	1,007	25,69	15,35	21,47	0,7148
	40	1,112	1,007	27,16	17,26	24,24	0,7122
	80	0,9859	1,01	30,01	21,35	30,14	0,7083
	100	0,9329	1,012	31,39	23,51	33,26	0,707
	200	0,7356	1,026	37,95	35,47	50,3	0,7051
	400	0,517	1,069	49,96	64,51	90,38	0,7137
	600	0,3986	1,116	61,14	99,63	137,5	0,7247
	800	0,3243	1,155	71,54	140,2	191	0,7342
	1000	0,2734	1,185	80,77	185,9	249,2	0,7458
Steam	100	0,5896	2,042	25,08	20,81	20,83	0,999

Derivation of an equation for the mass fraction $\xi_{H_2O,i}$

Values of the HTC $\bar{\alpha}$ at the water surface?:

$$\xi_{H_2O,i} = \frac{m_{H_2O}}{m_{tot}} = \frac{m_{H_2O}}{m_{Air} + m_{H_2O}}$$

Gas Law:

$$m_{Air} = \frac{p_{Air} \cdot V}{R_{Air} \cdot T_{Air}} \qquad m_{H_2O} = \frac{p_{H_2O} \cdot V}{R_{H_2O} \cdot T_{H_2O}}$$

$$= \frac{\frac{p_{H_2O} \cdot V}{R_{H_2O} \cdot T_{H_2O}}}{\frac{p_{Air} \cdot V}{R_{Air} \cdot T_{Air}} + \frac{p_{H_2O} \cdot V}{R_{H_2O} \cdot T_{H_2O}}} = \frac{\frac{p_{H_2O}}{R_{H_2O}}}{\frac{p_{Air}}{R_{Air}} + \frac{p_{H_2O}}{R_{H_2O}}} = \frac{p_{H_2O} \cdot M_{H_2O}}{\underbrace{p_{Air}}_{(p_{tot} - p_{H_2O}) = (1 - p_{Saturation})} \cdot M_{Air} + p_{H_2O} \cdot M_{H_2O}}$$

$$(\, T_{Air} = T_{H_2O} \,)$$

$$\boxed{\xi_{H_2O,i} = \frac{1}{\frac{1 - p_{H_2O}}{p_{H_2O}} \cdot \frac{M_{Air}}{M_{H_2O}} + 1}}$$

Derivation of an equation for the mass fraction $\xi_{H_2O,i}$

Appendix

Table 8: Water vapor of pure substances: Antoine Equation $\boxed{\log p^* = A - \frac{B}{T+C}}$; $(p^* \text{ in mbar, T in } °C)$

Substance	Temperature Range [°C]	A -	B -	C -
Aceton	-13-55	7,24208	1210,595	229,664
Aceton	57-205	7,75624	1566,690	273,419
Ethanol	20-93	8,23714	1592,864	226,184
Benzol	8-80	7,00481	1196,760	219,161
i-Butanol	72-107	7,32625	1157,000	168,270
Chloroform	-10-60	7,07959	1170,966	226,232
n-Heptan	-3-127	7,01880	1264,370	216,640
Methanol	15-84	8,20591	1582,271	239,726
Methanol	25-56	7,89373	1408,360	223,600
i-Octan	24-100	6,92798	1252,590	220,119
Propan		6,95467	813,200	248,000
Sauerstoff		7,11577	370,757	273,200
Stickstoff		6,99100	308,365	273,200
Water	1-100	8,19625	1730,630	233,426

Lewis Law and mass transfer coefficient g

Derivation Lewis Law for all gases:

$$\left(\frac{Sh}{Nu}\right) = \left(\frac{Sc}{Pr}\right)^{n}$$

$$\frac{g}{\alpha / c_p} = \left(\frac{Sc}{Pr}\right)^{n-1}$$

- For gases, the **Prandtl number** and the **Schmidt number** are almost equal

$$\frac{g}{\alpha / c_p} = 1 \quad \text{or} \quad g = \frac{\alpha}{c_p}$$

Calculation of the mass transfer coefficient g:

$$\alpha = 3.89 \, \frac{W}{m^2 K}$$ (from Part 1)

$$c_p = 1000 \, \frac{J}{kg \cdot \mathrm{K}}$$ (from A.S.)

$$g = \frac{\alpha}{c_p} = \frac{3.89 \, \frac{W}{m^2 K}}{1000 \, \frac{J}{kg \cdot \mathrm{K}}} = 3.89 \cdot 10^{-3} \, \frac{kg \cdot s}{m^2}$$

Mass of evaporating water $\dot{m}_w$ or $\dot{m}''_w$ and Heat Flow $\dot{Q}$

Evaporating mass flow per area $\dot{m}''_w$:

$$\dot{m}''_w = g \cdot \frac{\xi_{H_2O,s} - \xi_{H_2O,\infty}}{1 - \xi_{H_2O,s}} = \frac{\alpha}{c_p} \cdot \frac{\xi_{H_2O,s} - \xi_{H_2O,\infty}}{1 - \xi_{H_2O,s}} = \frac{3.89 \frac{W}{m^2 \cdot \mathrm{K}}}{1000 \frac{J}{kg \cdot \mathrm{K}}} \cdot \frac{(20 - 10) \cdot 10^{-3}}{(1 - 20 \cdot 10^{-3})} = 3.97 \cdot 10^{-5} \frac{kg}{m^2 \cdot s}$$

Evaporating mass flow over the entire lake surface $\dot{m}_W$:

$$\dot{m}_W = \dot{m}''_w \cdot A = 3.97 \cdot 10^{-5} \frac{kg}{m^2 \cdot s} \cdot 536 \cdot 10^6\, m^2 = 2.13 \cdot 10^4 \frac{kg}{s}$$

What heat flow $\dot{Q}$ is required for this?:

$$\dot{Q} = \dot{m}_W \cdot \Delta h_v = 2.13 \cdot 10^4 \frac{kg}{s} \cdot 2500 \frac{kJ}{kg} = 5.32 \cdot 10^7\ \mathrm{kW}$$

Comprehension questions

How are mass fractions calculated?

Under what conditions does the Lewis‘ law apply?

How is the mass transfer coefficient calculated using Lewis‘ law?

How is the mass of evaporating water determined?