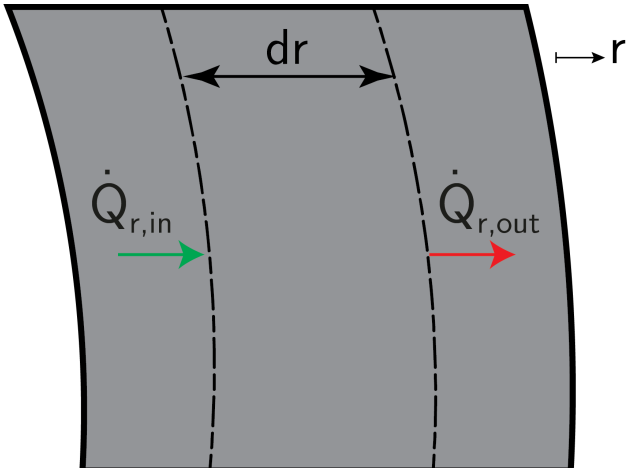
Boundary Conditions - Cond. - IE 9

Hot water flows through a long pipe of length L. The water temperature and external surface temperature of the pipe are constant and equal to T_∞ and T_1 respectively. Define the boundary conditions needed for solving the given radial heat conduction equation for the pipe wall



Given the differential equation:

$$0 = \frac{\partial}{\partial r}\left(\lambda 2\pi r L \frac{\partial T}{\partial r}\right)$$

In order to solve the differential equation, two boundary conditions are required. This can be seen from the fact that the variable T has been differentiated twice with respect to r.

Boundary conditions:

$$\frac{\partial T(r=r_1)}{\partial r} = -\frac{\alpha}{\lambda}(T_\infty - T(r=r_1))$$

$$T(r=r_2) = T_1$$

The first boundary condition results from the fact that $\dot{Q}_{r=r_1} = -\lambda A(r)\frac{\partial\, T(r=r_1)}{\partial\, r} = \alpha\ A(r)(T_\infty - T(r=r_1))$, the second one describes that the temperature at the surface equals T_1.