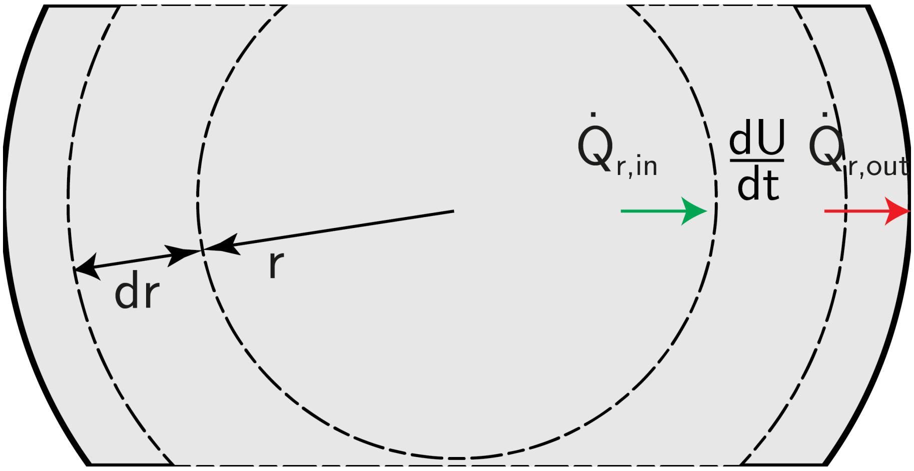
EB - Cond. - IE 21

A cylinder, initially at a homogeneous temperature T_0, is heated by convection at the sides. Give the energy balance to derive the heat conduction equation. Assume one-dimensional transient conditions in the radial direction at constant atmospheric pressure.



Energy balance:

$$\frac{\partial U}{\partial t} = \dot{Q}_{r,in} - \dot{Q}_{r,out}$$

For unsteady heat transfer, the internal energy will change over time and equals the sum of the in- and outgoing heat fluxes.

Change of internal energy over time:

$$\frac{\partial U}{\partial t} = \rho \cdot c_p \cdot dV \cdot \frac{\partial T}{\partial t} = \rho \cdot c_p \cdot 2\pi \cdot r \cdot L \cdot \frac{\partial T}{\partial t}$$

The internal energy of a constant volume can be described as $U = m \cdot c_p \cdot T$. Where $dV = (\pi(r+dr)^2 - \pi r^2)\, c\dot{L} \approx 2\pi r L$. L is the length of the cylinder in the axial direction.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda \cdot 2\pi \cdot r \cdot L \cdot \frac{\partial T}{\partial r}$$

$$\dot{Q}_{x,out} = \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

The ingoing flux can be described by use of Fourier's equation. The outgoing flux can be approximated by the use of the Taylor series expansion.

Conditions

Initial condition:

$$T(t=0) = T_0$$

Boundaries:

$$\frac{\partial T}{\partial r}|_{r=0} = 0$$

$$-\lambda \cdot 2\pi \cdot r \cdot L \cdot \frac{\partial T}{\partial r}|_{r=r_1} = \alpha \cdot 2\pi \cdot r \cdot L(T_\infty - T(r=r_1))$$