Heat Transfer: Convection

Forced Convection in Internal Flow and the LMTD

Prof. Dr.-Ing. Reinhold Kneer

Prof. Dr.-Ing. Dr. rer. pol. Wilko Rohlfs

Prof. dr.ir. C.H. Venner (Kees)

Learning goals

Forced convection in internal flows:

- Knowledge of meaning of the **logarithmic mean temperature difference (LMTD)**
- Ability to apply and calculate the LMTD

Change of mean temperature in pipe flow with constant temperature b.c.

How to determine axial temperature profile in the pipe and outlet temperature?

Development of energy balance:

- Develop local energy balance for the temperature profile

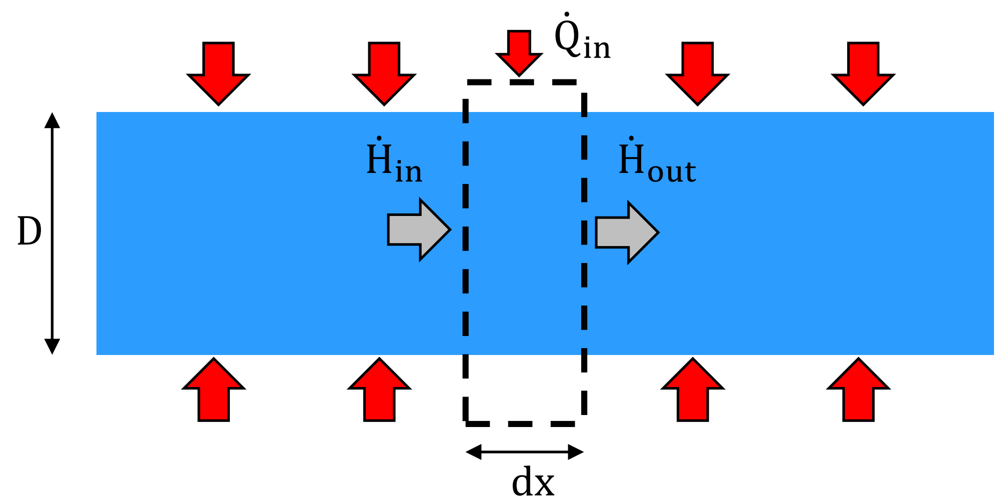
→ **Energy balance**: $0 = \dot{H}_{\mathrm{in}} - \dot{H}_{\mathrm{out}} + \dot{Q}_{\mathrm{in}}$

$$\dot{\mathrm{H}}_{\mathrm{in}} = \dot{m} c_p T_{\mathrm{m}}(x)$$

$$\dot{\mathrm{H}}_{\mathrm{out}} = \dot{m} c_p T_{\mathrm{m}}(x + dx)$$

$$\dot{Q}_{\mathrm{in}} = \alpha A (T_{\mathrm{w}} - T_{\mathrm{m}}) = \alpha \pi D dx (T_{\mathrm{w}} - T_{\mathrm{m}}(x))$$

Inhomogeneous differential equation



Differential equation:

$$0 = -\dot{m} c_p \frac{\partial T_{\mathrm{m}}(x)}{\partial x} + \alpha \pi D (T_{\mathrm{w}} - T_{\mathrm{m}}(x))$$

$$\frac{\partial T_{\mathrm{m}}(x)}{\partial x} = \frac{\alpha \pi D (T_{\mathrm{w}} - T_{\mathrm{m}}(x))}{\dot{m} c_p}$$

→ depends on $T_{\mathrm{m}}(x)$ → Inhomogeneous differential equation

Change of mean temperature in pipe flow with constant temperature b.c.

Homogenization of partial differential equation:

$$\frac{\partial T_\mathrm{m}(x)}{\partial x} = \frac{\alpha \pi D (T_\mathrm{w} - T_\mathrm{m}(x))}{\dot{m} c_p}$$

$$\Theta = T_\mathrm{m}(x) - T_\mathrm{w}$$

Solution for Θ:

$$\frac{\partial \Theta}{\partial x} = \frac{\partial T_\mathrm{m}(x)}{\partial x} = -\frac{\Theta \alpha \pi D}{\dot{m} c_p}$$

$$\frac{\partial \Theta}{\Theta} = -\frac{\alpha \pi D}{\dot{m} c_p} \partial x$$

Integration: $\ln(\Theta) = -\frac{\alpha \pi D}{\dot{m} c_p} x + C$

$$\Theta = e^{-\frac{\alpha \pi D}{\dot{m} c_p} x + C} = C^* \cdot e^{-\frac{\alpha \pi D}{\dot{m} c_p} x}$$

Solution for the temperature T:

- Back transformation:

$$T_\mathrm{m}(x) = C^* \cdot e^{-\frac{\alpha \pi D}{\dot{m} c_p} x} + T_\mathrm{w}$$

- Temperature profile:

$$\boxed{T_\mathrm{m}(x) = (T_\mathrm{in} - T_\mathrm{w}) e^{-\frac{\alpha \pi D}{\dot{m} c_p} x} + T_\mathrm{w}}$$

- Boundary condition

$$T_\mathrm{m}(0) = T_\mathrm{in}$$

Logarithmic mean temperature difference

Outlet temperature:

$$T_{\text{out}} = (T_{\text{in}} - T_{\text{w}}) e^{-\frac{\alpha \pi D}{\dot{m} c_p} L} + T_{\text{w}}$$

$$\frac{T_{\text{out}} - T_{\text{w}}}{T_{\text{in}} - T_{\text{w}}} = e^{-\frac{\alpha \pi D}{\dot{m} c_p} L} \rightarrow \ln\left(\frac{T_{\text{out}} - T_{\text{w}}}{T_{\text{in}} - T_{\text{w}}}\right) = -\frac{\alpha A}{\dot{m} c_p}$$

$$\frac{1}{\ln\left(\frac{T_{\text{out}} - T_{\text{w}}}{T_{\text{in}} - T_{\text{w}}}\right)} = -\frac{\dot{m} c_p}{\alpha A}$$

Total heat flux from fluid to wall:

$$\dot{\text{Q}}_{\text{tot}} = \alpha A \Delta T = \dot{\text{H}}_{\text{in}} - \dot{\text{H}}_{\text{out}}$$

What is the driving potential that describes the heat flux $\dot{\text{Q}}_{\text{tot}}$ adequately?

$$\dot{\text{H}}_{\text{out}} = \dot{m} c_p T_{\text{out}} \qquad \dot{\text{H}}_{\text{in}} = \dot{m} c_p T_{\text{in}}$$

$$\rightarrow \Delta T = \frac{\dot{m} c_p}{\alpha A} (T_{\text{in}} - T_{\text{out}})$$

UNIVERSITY OF TWENTE.
Lehrstuhl für Wärme- und Stoffübertragung
RWTH AACHEN UNIVERSITY

Logarithmic mean temperature difference

Outlet temperature:

$$T_{\mathrm{out}} = (T_{\mathrm{in}} - T_{\mathrm{w}})e^{-\frac{\alpha\pi D}{\dot{m}c_p}L} + T_{\mathrm{w}}$$

$$\frac{T_{\mathrm{out}} - T_{\mathrm{w}}}{T_{\mathrm{in}} - T_{\mathrm{w}}} = e^{-\frac{\alpha\pi D}{\dot{m}c_p}L} \rightarrow \ln\left(\frac{T_{\mathrm{out}} - T_{\mathrm{w}}}{T_{\mathrm{in}} - T_{\mathrm{w}}}\right) = -\frac{\alpha A}{\dot{m}c_p}$$

$$\frac{1}{\ln\left(\frac{T_{\mathrm{out}} - T_{\mathrm{w}}}{T_{\mathrm{in}} - T_{\mathrm{w}}}\right)} = -\frac{\dot{m}c_p}{\alpha A}$$

Total heat flux from fluid to wall:

$$\dot{Q}_{\mathrm{tot}} = \alpha A \Delta T = \dot{H}_{\mathrm{in}} - \dot{H}_{\mathrm{out}}$$

What is the driving potential that describes the heat flux $\dot{Q}_{\mathrm{tot}}$ adequately?

$$\dot{H}_{\mathrm{out}} = \dot{m}c_p T_{\mathrm{out}} \qquad \dot{H}_{\mathrm{in}} = \dot{m}c_p T_{\mathrm{in}}$$

$$\rightarrow \Delta T = \frac{\dot{m}c_p}{\alpha A}(T_{\mathrm{in}} - T_{\mathrm{out}})$$

Solution:

$$\frac{1}{\ln\left(\frac{T_{\mathrm{in}} - T_{\mathrm{w}}}{T_{\mathrm{out}} - T_{\mathrm{w}}}\right)} = \frac{\dot{m}c_p}{\alpha A}$$

$$\frac{\Delta T}{T_{\mathrm{in}} - T_{\mathrm{out}}} = \frac{\dot{m}c_p}{\alpha A}$$

- **L**ogarithmic **m**ean **t**emperature **d**ifference (LMTD):

$$\Delta T = \frac{(T_{\mathrm{in}} - T_{\mathrm{out}})}{\ln\left(\frac{T_{\mathrm{in}} - T_{\mathrm{w}}}{T_{\mathrm{out}} - T_{\mathrm{w}}}\right)} = \frac{(\Delta T_{\mathrm{in}} - \Delta T_{\mathrm{out}})}{\ln\left(\frac{\Delta T_{\mathrm{in}}}{\Delta T_{\mathrm{out}}}\right)} = \Delta T_{\mathrm{ln}}$$

What is the purpose of the LMTD?

$$\dot{Q}_{\mathrm{tot}} = \alpha A \frac{(\Delta T_{\mathrm{in}} - \Delta T_{\mathrm{out}})}{\ln\left(\frac{\Delta T_{\mathrm{in}}}{\Delta T_{\mathrm{out}}}\right)}$$

Logarithmic mean temperature difference

Outlet temperature:

- **L**ogarithmic **m**ean **t**emperature **d**ifference (LMTD):

$$\dot{Q}_{\mathrm{tot}} = \alpha A \frac{(\Delta T_{\mathrm{in}} - \Delta T_{\mathrm{out}})}{\ln\left(\frac{\Delta T_{\mathrm{in}}}{\Delta T_{\mathrm{out}}}\right)}$$

- Applicable to calculate the heat flux transferred from fluid to wall in a heat exchenger if :

  $\rightarrow$ the heat transfer coefficient α is constant

  $\rightarrow$ the specific heat of the fluid is constant (non-temperature dependent)

  $\rightarrow$ the wall temperature is constant

  $\rightarrow$ the mean fluid temperature changes spatially

  $\rightarrow$ the problem is stationary

Often this equation has to be used iteratively because the outlet temperature as well as the heat flux are unknown.

What is the meaning of the logarithmic mean temperature difference, and when do we need to apply this?