Heat Transfer: Conduction

Fin efficiency

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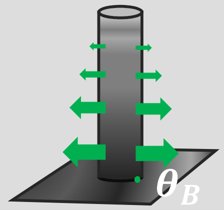
Learning goals

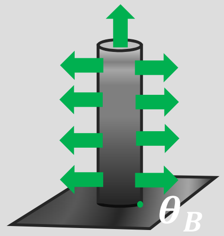
Fin design:

- Fin material
- Geometry



Fin efficiency:

- Derivation
- Interpretation



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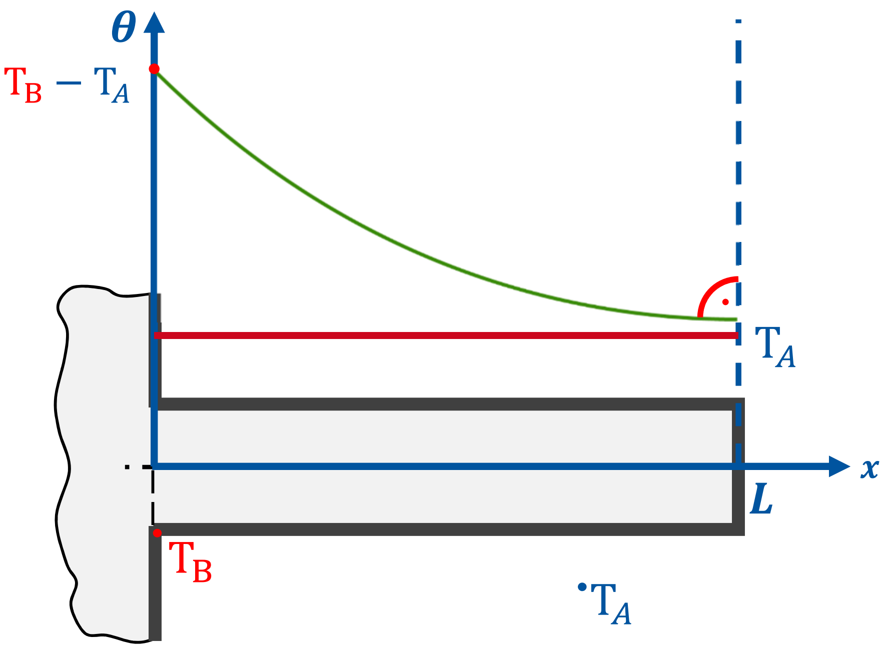
Review: Temperature profile in a fin

Boundary conditions:

At $x = 0$: $\theta(x) = \theta_B$

At $x = L$: $\dot{Q}_{head} = 0 \implies \left.\frac{d\theta}{dx}\right|_{x=L} = 0$

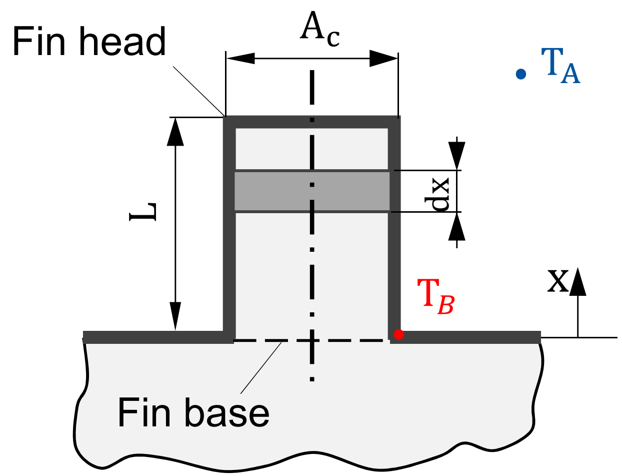


Fin temperature profile with given boundary conditions:

$$\theta(x) = \theta_B \cdot \left(\frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL} + e^{-mL}}\right)$$

or

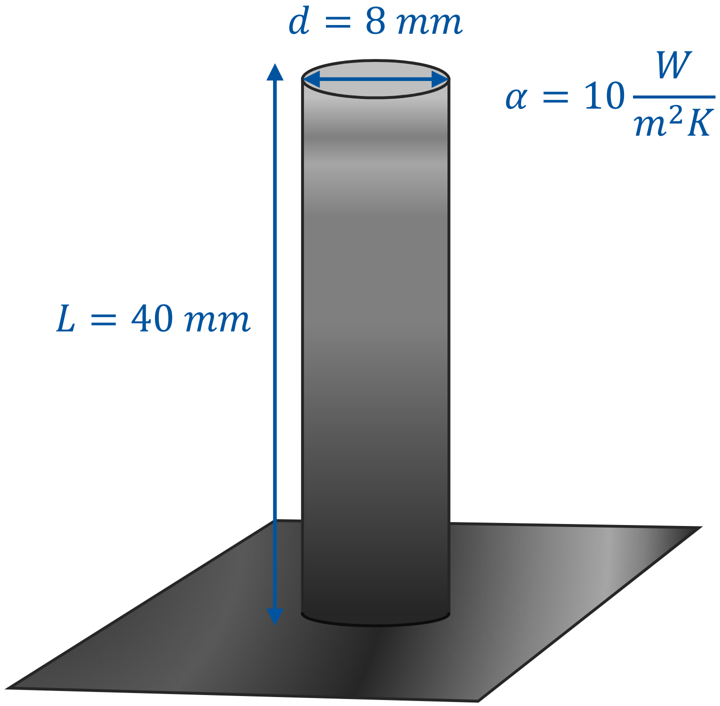
$$\theta(x) = \theta_B \cdot \left(\frac{\cosh(m(L-x))}{\cosh(mL)}\right)$$



Temperature profile: Comparison for different fin materials

Materials:

Copper:	$\lambda_{\mathrm{Cu}} = 385 \frac{W}{m\ K}$
Chrome-Nickel-Steel:	$\lambda_{\mathrm{Cr-Ni}} = 16 \frac{W}{m\ K}$
Glass:	$\lambda_{\mathrm{Glass}} = 0.8 \frac{W}{m\ K}$



Cylindrical pin fin ➡ $m^2 = \frac{4\,\alpha}{\lambda\, d}$

Temperature profile: Comparison for different fin materials

$$\theta(x) = \theta_B \cdot \left(\frac{e^{m\,(L-x)} + e^{-m\,(L-x)}}{e^{m\,L} + e^{-m\,L}}\right)$$

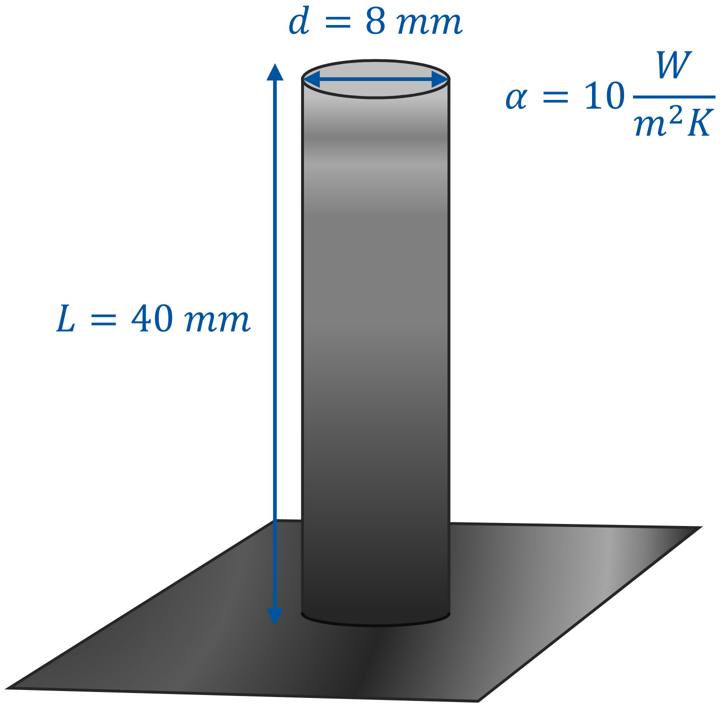
$$\theta(x) = \theta_B \cdot \left(\frac{\cosh(m(L-x))}{\cosh(mL)}\right)$$

Temperatures at the head and base:

At $x = 0$: $\theta(x) = \theta_B$

At $x = L$: $\frac{\theta(x=L)}{\theta_B} = \frac{e^0 + e^0}{e^{m\,L} + e^{-m\,L}} = \frac{2}{e^{m\,L} + e^{-m\,L}}$

$$m^2 = \frac{4\,\alpha}{\lambda\,d}$$



Temperature profile: Comparison for different fin materials

$$\frac{\theta(L)}{\theta_B} = \frac{2}{e^{m\,L} + e^{-m\,L}}$$

$$m^2 = \frac{4\,\alpha}{\lambda\,d} \quad \left[\frac{1}{m}\right]$$

Copper:

$$\lambda_{Cu} = 385 \frac{W}{m\,K}$$

$$m_{Cu}^2 = 13\,m^{-1}$$

$$(m \cdot L)_{Cu} = 0.144$$

$$\left(\frac{\theta(L)}{\theta_B}\right)_{Cu} = 0.9887$$

Chrome-Nickel-Steel:

$$\lambda_{Cr-Ni} = 16 \frac{W}{m\,K}$$

$$m_{Cr-Ni}^2 = 312.5\,m^{-1}$$

$$(m \cdot L)_{Cr-Ni} = 0.707$$

$$\left(\frac{\theta(L)}{\theta_B}\right)_{Cr-Ni} = 0.7939$$

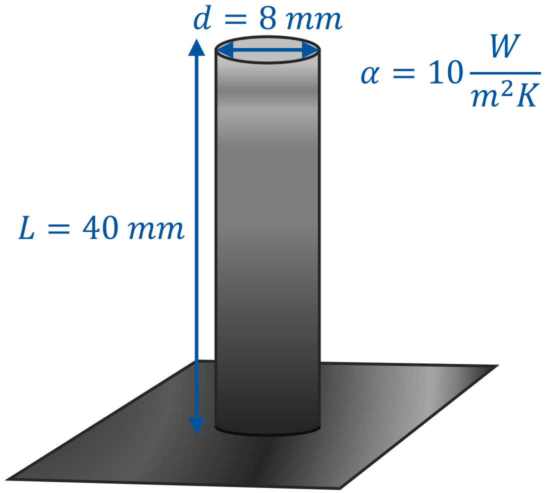
Glass:

$$\lambda_{Glass} = 0.8 \frac{W}{m\,K}$$

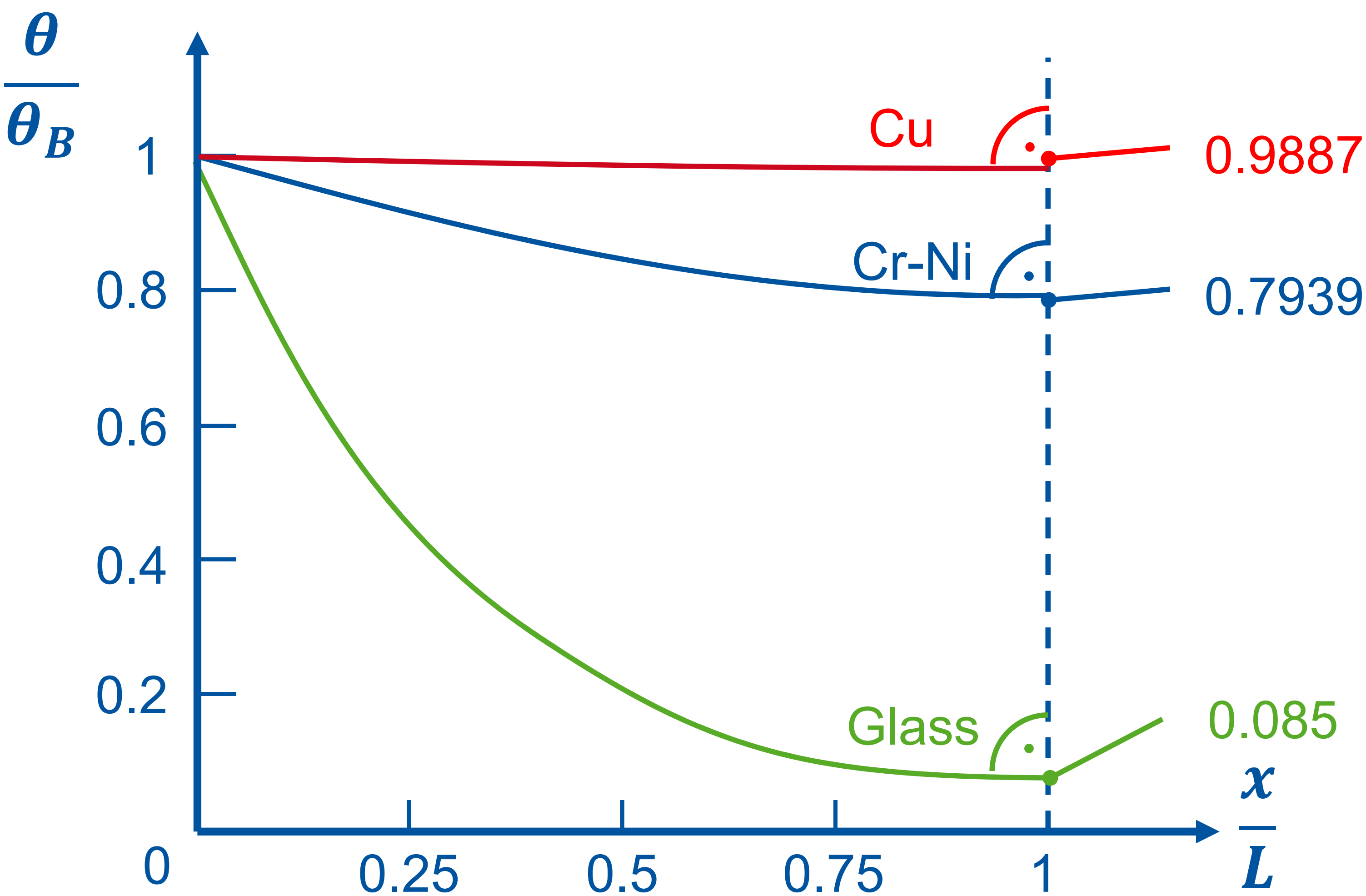
$$m_{Glass}^2 = 6250\,m^{-1}$$

$$(m \cdot L)_{Glass} = 3.16$$

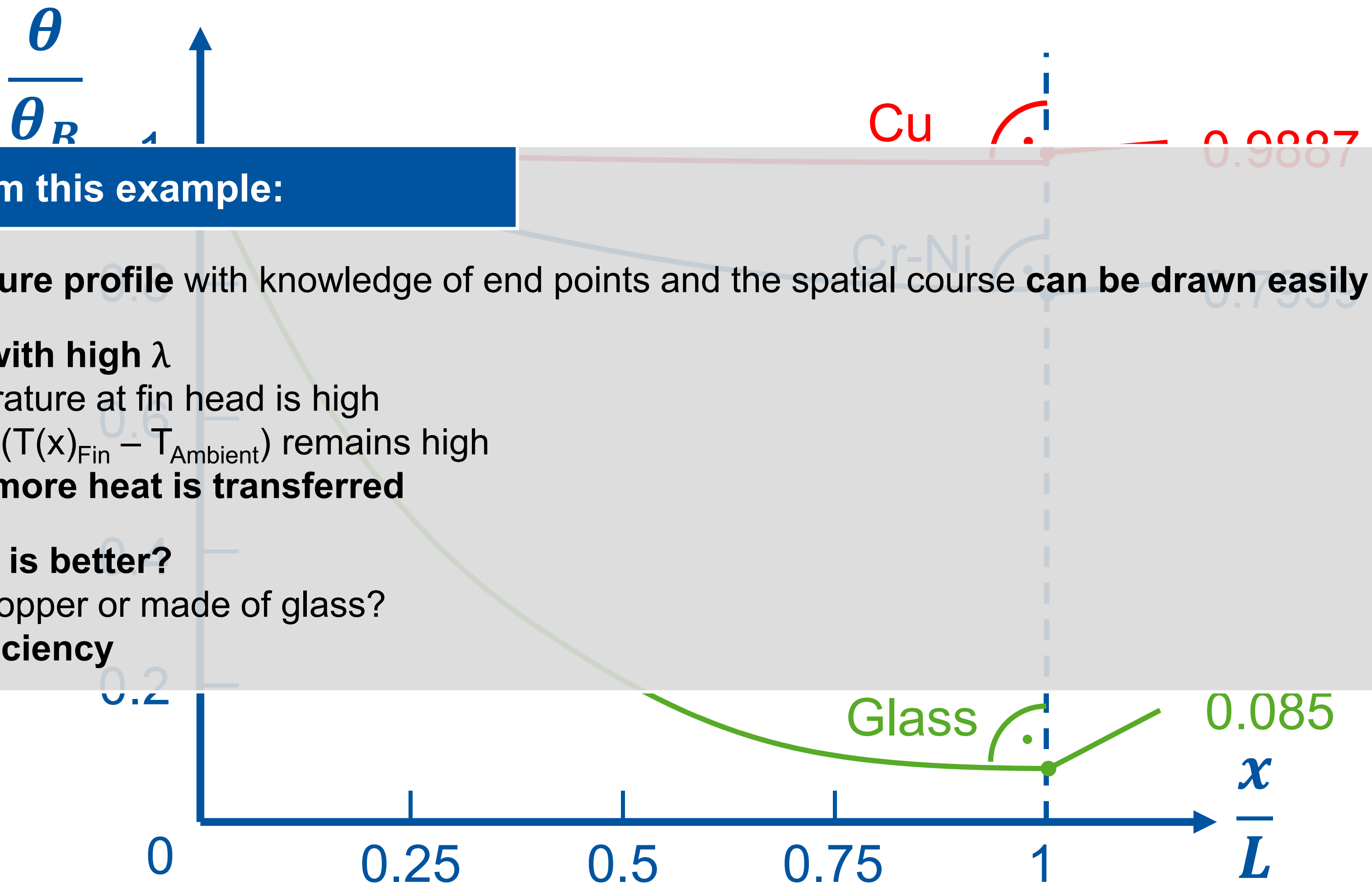
$$\left(\frac{\theta(L)}{\theta_B}\right)_{Glass} = 0.085$$



Dimensionless temperature profile for the three different materials



Dimensionless temperature profile for the three different materials



Findings from this example:

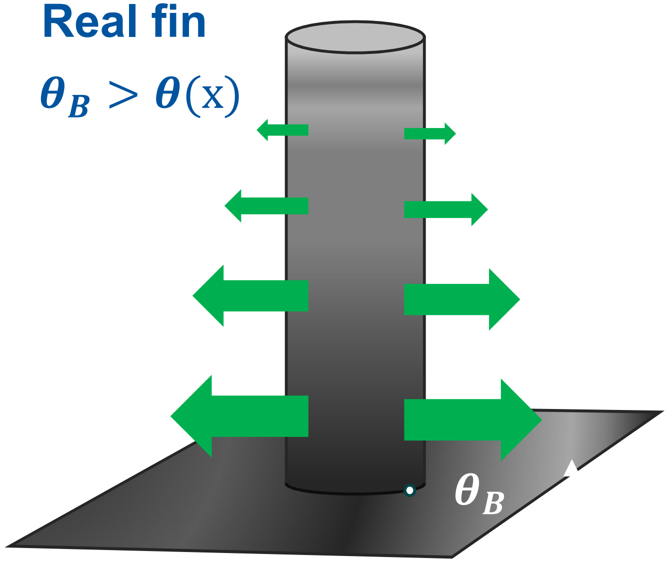
1. **Temperature profile** with knowledge of end points and the spatial course **can be drawn easily**
2. **Material with high λ**
   → Temperature at fin head is high
   → ΔT ($T(x)_{Fin} - T_{Ambient}$) remains high
   → **more heat is transferred**
3. **Which fin is better?**
   Made of copper or made of glass?
   → **Fin efficiency**

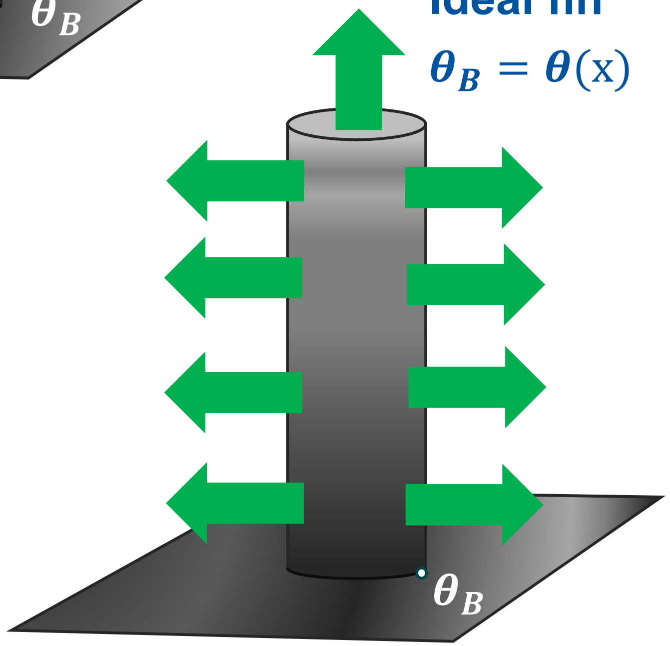
Fin efficiency

$$\eta_R = \frac{\text{Heat transferred from a fin}}{\text{Maximum amount of transferable heat}}$$

Maximum transferable heat flow is achieved when the temperature remains equal to the base temperature along the entire length of the fin.

$$\boldsymbol{T = T_{\text{base}}}$$
$$\boldsymbol{\neq f(x)}$$





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Calculate total heat transferred by fin surface

$$\eta_R = \frac{\boxed{\text{Heat transferred from a fin}}}{\text{Maximum amount of transferable heat}}$$

Heat flow transferred from the fin:

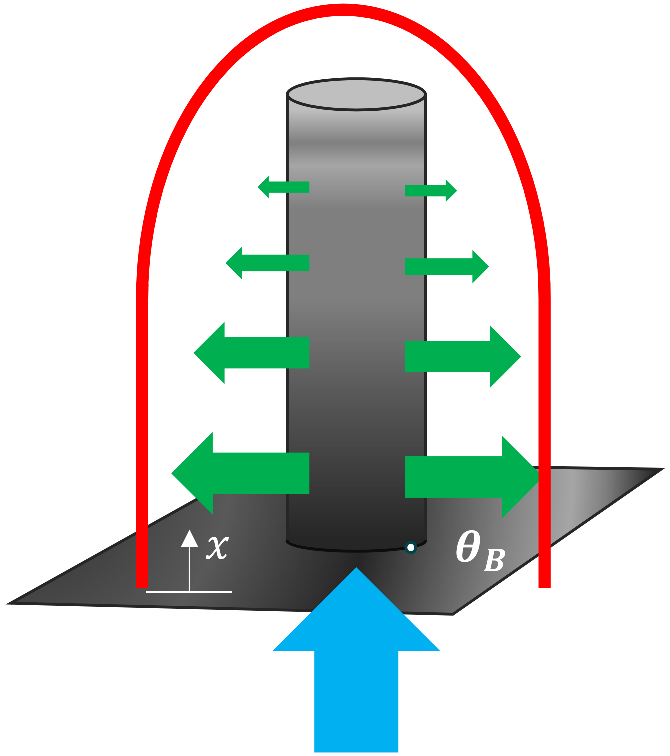
Transferred (convective) heat flow on the shell surface:

$$\dot{Q}_{Conv} = \int_0^L \dot{q}_{conv.}\, dx = \pi d \cdot \int_0^L \alpha \cdot \underbrace{(T(x) - T_A)}_{\theta(x)} dx$$

Alternatively:

Heat flow by conduction at the base:

$$\dot{Q}_{cond.,base} = -\lambda \cdot A_c \cdot \left.\frac{dT}{dx}\right|_{x=0} = -\lambda \cdot A_c \cdot \left.\frac{d\theta}{dx}\right|_{x=0}$$



$$\dot{Q}_{conv} = \dot{Q}_{cond.,base}$$

Calculate total heat transferred by fin surface

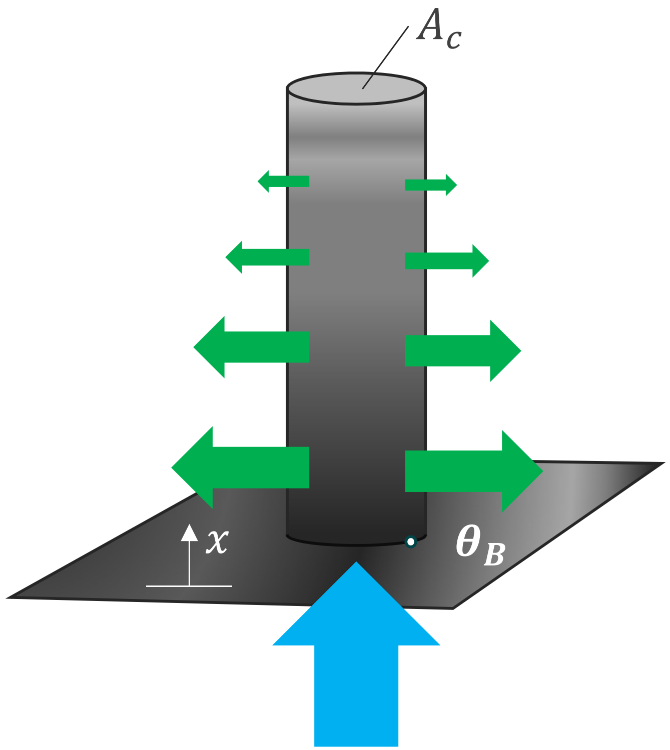
$$\theta(x) = \theta_B \cdot \left(\frac{e^{m\,(L-x)} + e^{-m\,(L-x)}}{e^{m\,L} + e^{-m\,L}} \right)$$

Heat flow transferred from the fin:

$$\dot{Q}_{cond.,base} = -\lambda \cdot A_c \cdot \left.\frac{dT}{dx}\right|_{x=0} = -\lambda \cdot A_c \cdot \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = -m \cdot \theta_B \cdot \left(\frac{e^{m\,L} - e^{-m\,L}}{e^{m\,L} + e^{-m\,L}} \right) = -m \cdot \theta_B \cdot \tanh(mL)$$

$$\boxed{\dot{Q}_{cond.,base} = \lambda \cdot A_c \cdot m \cdot \theta_B \cdot \tanh(mL)}$$



$$\dot{Q}_{conv} = \dot{Q}_{cond.,base}$$

Calculate maximum amout of transferable heat

$$\boxed{\eta_R = \frac{\text{Heat transferred from a fin}}{\text{Maximum amount of transferable heat}}}$$

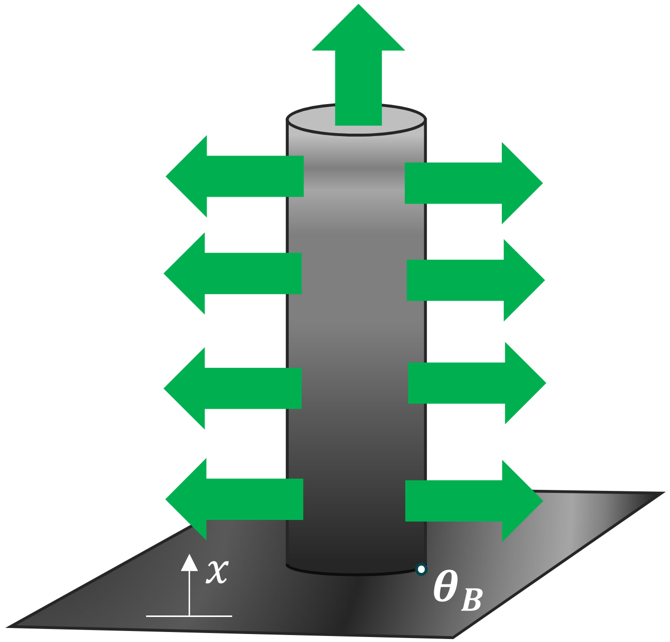
Fin efficiency:

$$\dot{Q}_{max} = \pi \cdot d \cdot \alpha \underbrace{\int_0^L \theta_B dx}_{\theta_B \cdot L} = \pi \cdot d \cdot \alpha \cdot \theta_B \cdot L$$

$$\boxed{m^2 = \frac{4\,\alpha}{\lambda\, d}}$$

$$\frac{1}{m^2} \cdot m$$

$$\eta_R = \frac{\dot{Q}_{cond,base}}{\dot{Q}_{max}} = \frac{\lambda \cdot A_c \cdot m \cdot \cancel{\theta_B} \cdot \tanh(mL)}{\pi \cdot d \cdot \cancel{\alpha} \cdot \cancel{\theta_B} \cdot L} \Rightarrow \boxed{\eta_R = \frac{\tanh(mL)}{mL}}$$

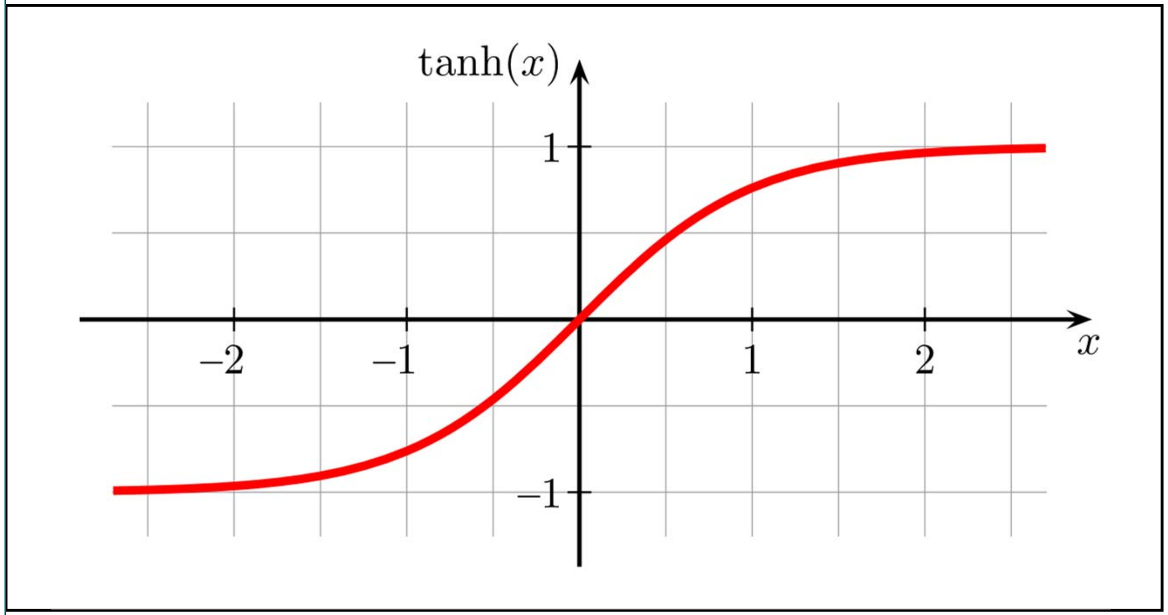


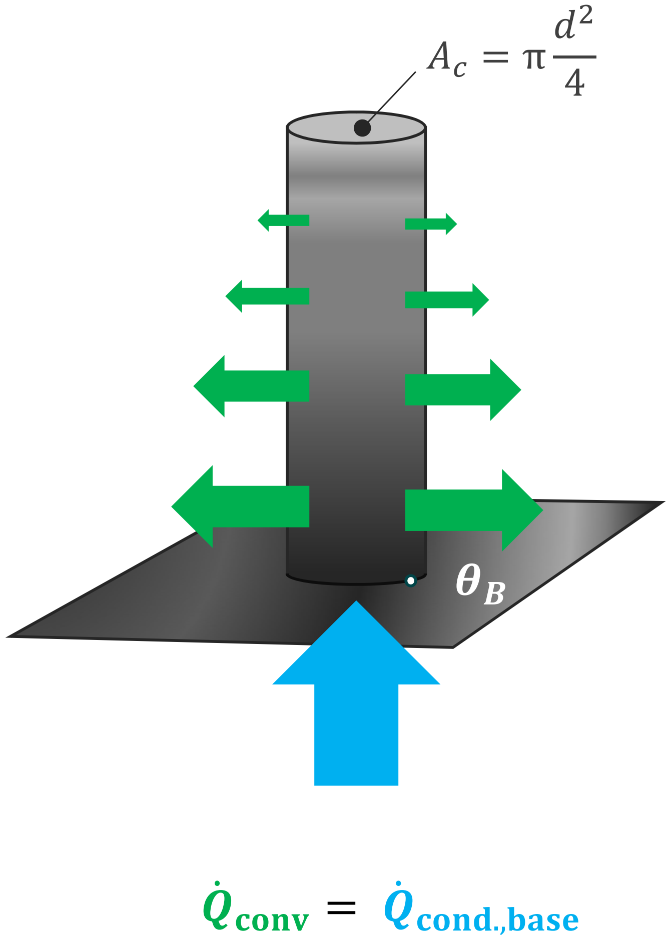
$$\dot{Q}_{conv} = \dot{Q}_{cond.,base}$$

Course of the fin efficiency

$$\eta_R = \frac{\text{Heat transferred from a fin}}{\text{Maximum amount of transferable heat}}$$

$$\eta_R = \frac{\tanh(mL)}{mL}$$

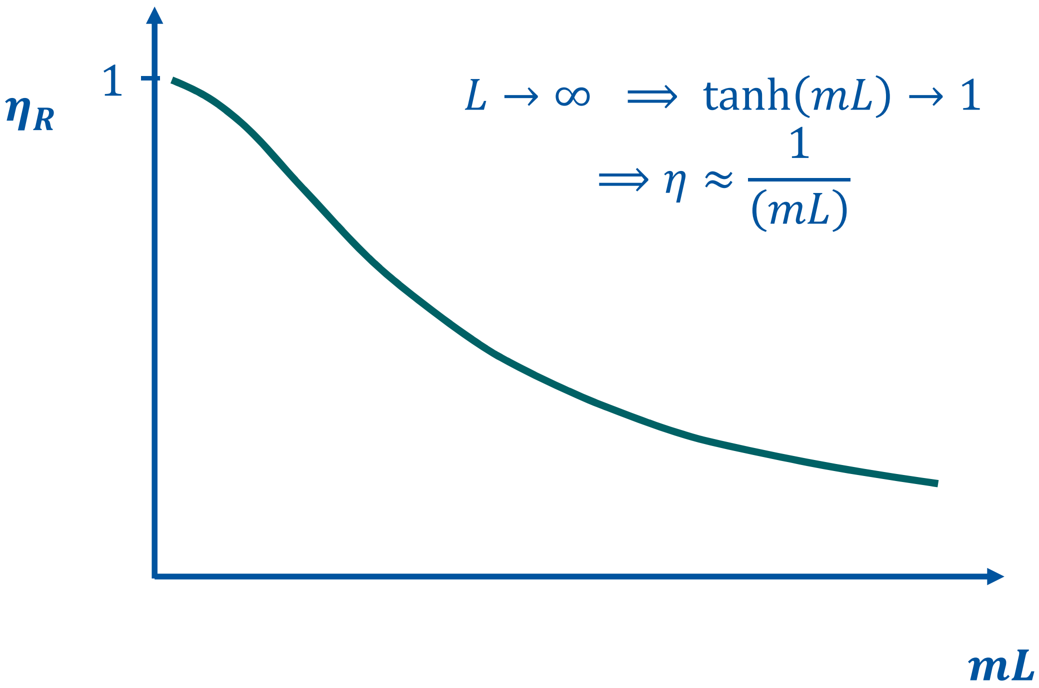




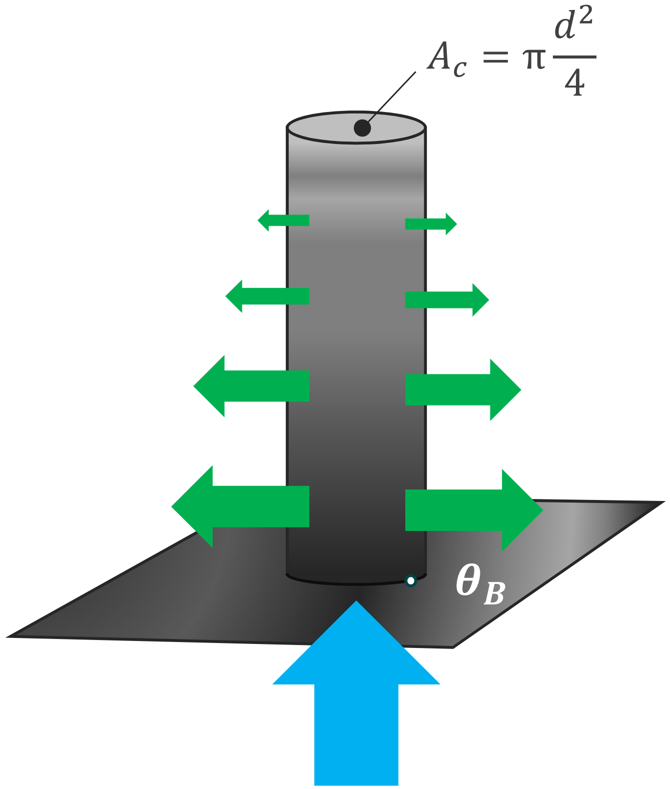
Course of the fin efficiency

$$\eta_R = \frac{\text{Heat transferred from a fin}}{\text{Maximum amount of transferable heat}}$$

$$\eta_R = \frac{\tanh(mL)}{mL}$$



$$L \to \infty \implies \tanh(mL) \to 1$$
$$\implies \eta \approx \frac{1}{(mL)}$$



$$\dot{Q}_{\text{conv}} = \dot{Q}_{\text{cond.,base}}$$

Fin efficiency: Recapitulation of fin parameter

Fin parameter m:

$$m = \left(\frac{\alpha\, U}{\lambda\, A_c}\right)^{1/2}$$

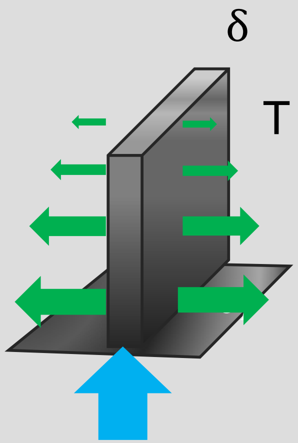
$$\eta_R = \frac{\tanh(mL)}{mL}$$

Plain fin:

$$m = \left(\frac{2\,(\delta + T)\,\alpha}{\lambda\,\delta\,t}\right)^{1/2}$$

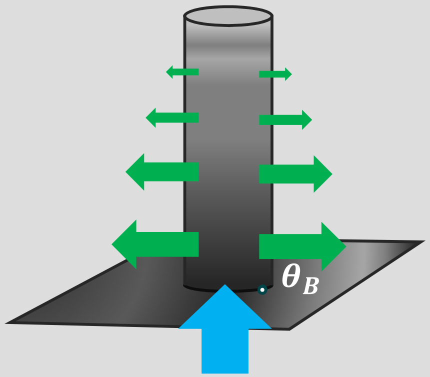
$$= \left(\frac{2\,\alpha}{\lambda}\left(\frac{1}{\delta} + \frac{1}{T}\right)\right)^{1/2}$$

with $\frac{1}{\delta} \gg \frac{1}{T}$ ⇨ $m \approx \left(\frac{2\,\alpha}{\lambda\delta}\right)^{1/2}$



Pin fin:

$$m = \left(\frac{\alpha\,\pi d}{\lambda\,\pi\frac{d^2}{4}}\right)^{\frac{1}{2}} = \left(\frac{4\,\alpha}{\lambda\,d}\right)^{1/2}$$

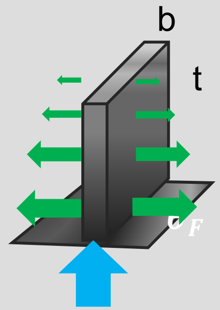


Fin efficiency: Physical dependencies

$$\eta_R = \frac{\tanh(mL)}{mL}$$

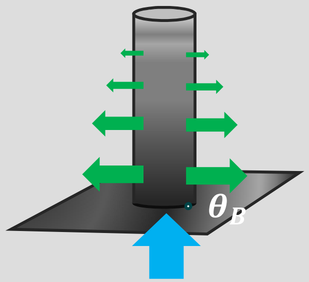
Plain fin:

$$m = \left(\frac{2\,\alpha}{\lambda\,\delta}\right)^{\frac{1}{2}}$$



Pin fin:

$$m = \left(\frac{4\,\alpha}{\lambda\,d}\right)^{\frac{1}{2}}$$



Physical dependencies:

- Material:

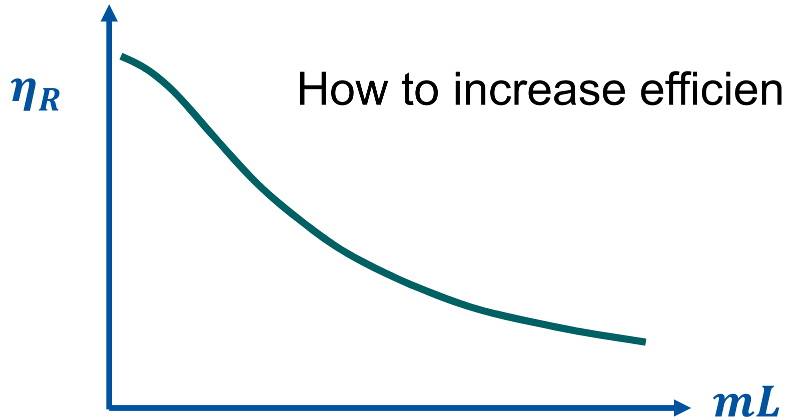
$$\lambda \uparrow \quad \Longrightarrow \quad m^2 \downarrow \quad \Longrightarrow \quad \eta \uparrow$$

- Fin geometry (L = $const.$) :

$$d \uparrow \quad \Longrightarrow \quad m^2 \downarrow \quad \Longrightarrow \quad \eta \uparrow$$

- Convective heat transfer:

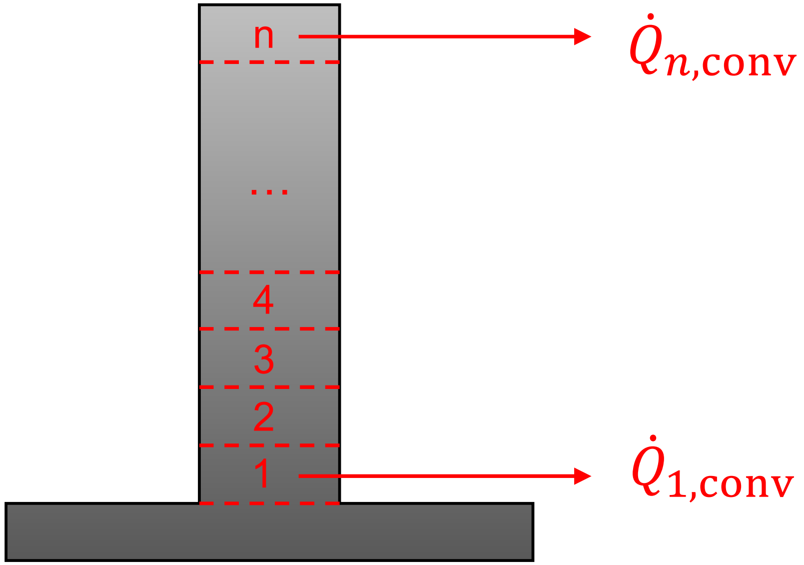
$$\alpha \downarrow \quad \Longrightarrow \quad m^2 \downarrow \quad \Longrightarrow \quad \eta \uparrow$$



How to increase efficiency?

Fin efficiency: Limitations

Fin efficiency only indicates how well single sections of a fin are used!



for $\eta \uparrow \Rightarrow \dot{Q}_{1,\mathrm{conv}} \approx \dot{Q}_{n,\mathrm{conv}}$

η_R does not provide information about the heat flow transferred → although it decreases when a fin is extended → $\dot{Q}_{\mathrm{transferred}}$ increases

Comprehension questions

What relationship does the fin efficiency describe?

What is the assumption for the theoretical maximum transferable heat of a fin?

How can the fin efficiency be increased?