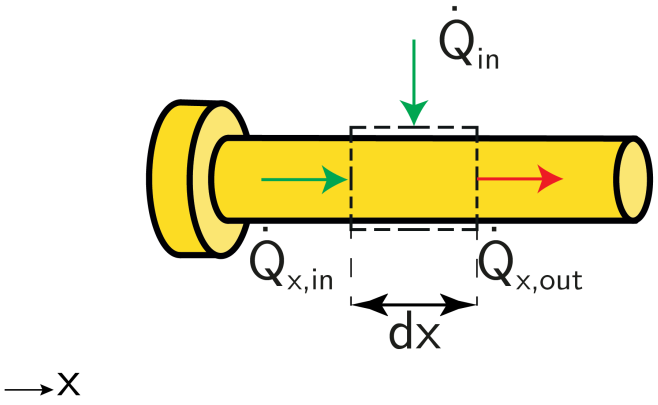
EB - Cond. - IE 23

Specify the energy balance to derive the temperature distribution. Assume one-dimensional, steady-state heat transfer in the x-direction with a heat flux imposed at a constant rate $\dot{q}''$.



Energy balance:

$$0 = \dot{Q}_{\mathrm{x,in}} - \dot{Q}_{\mathrm{x,out}} + \dot{Q}_{\mathrm{in}}$$

Since the heat transfer is characterized as a steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{\mathrm{x,in}} = -\lambda A_{\mathrm{c}} \frac{\partial T}{\partial x}$$

$$\dot{Q}_{\mathrm{x,out}} = -\lambda A_{\mathrm{c}} \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{\mathrm{x,in}}}{\partial x} \cdot dx$$

$$\dot{Q}_{\mathrm{in}} = \dot{q}'' \pi \cdot d \cdot dx$$

Inserting and rewriting:

$$0 = \lambda \frac{\pi d^2}{4} \frac{\partial^2 T}{\partial x^2} + \dot{q}'' \pi d$$