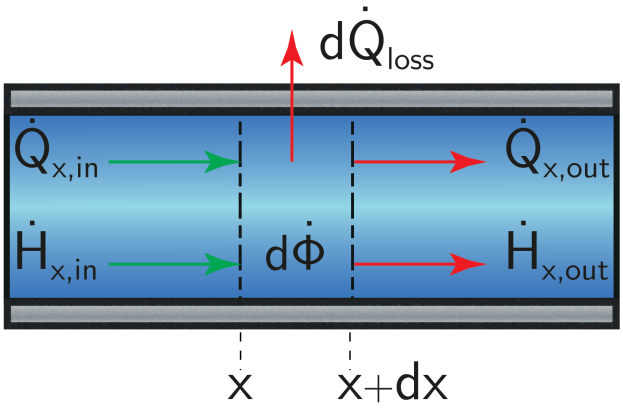
Boundary Conditions - Conv. - IE 1

Through a very long pipe with diameter D flows a heat-generating fluid (homogeneous and constant source strength $\dot{\Phi}''' > 0$). In addition, the pipe has a uniform, constant wall temperature T_w

Give the correct conditions to solve the given differential equation for deriving the temperature profile in the flow direction.



Given the differential equation:

$$0 = \frac{\lambda \pi D^2}{4}\frac{\partial^2 T}{\partial x^2} - \frac{u\rho c \pi D^2}{4}\frac{\partial T}{\partial x} - \alpha \pi D(T(x) - T_\mathrm{w}) + \frac{\pi D^2}{4}\dot{\Phi}'''$$

In order to solve the differential equation, two boundary conditions are required. This can be seen from the fact that the variable T has been differentiated twice with respect to x.

Boundary conditions:

$$T(x = 0) = T_1$$

$$T(x = L) = T_2$$

The first boundary condition describes that the temperature of the fluid equals T_1 at the entrance of the pipe and the second one states that the temperature of the fluid equals T_2 at the exit of the pipe, as can be seen from the figure.