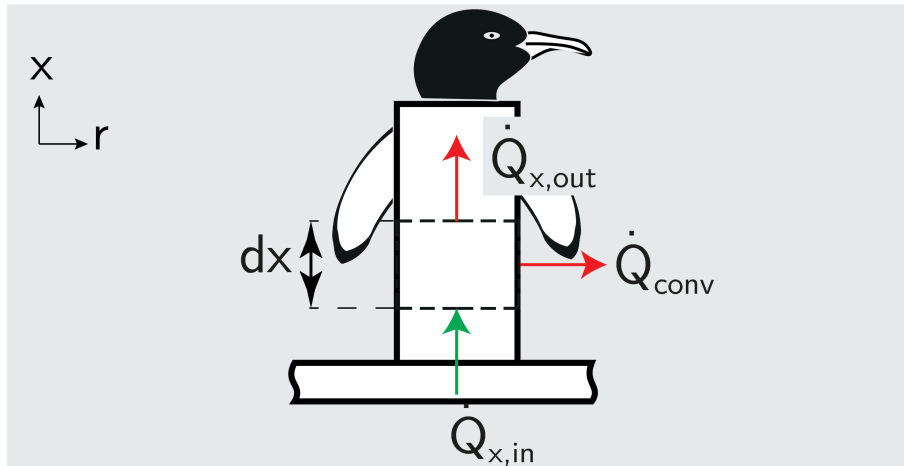# Temperature Function - Cond. - IE 11

A penguin's body has a circular cross-section. One-dimensional, steady-state heat transfer in the direction from base to tip can be assumed. Provide the temperature function derived from the heat conduction equation

Given the differential equation and boundary conditions:

$$0 = \frac{\partial^2 \Theta}{\partial x^2} - m^2 \Theta$$

$$\Theta(x = 0) = \Theta_B \qquad\qquad \Theta(x = L) = \Theta_H$$

Solving the equation:

The standard solution for the given differential equation is:

$$\Theta(x) = A \cosh(mx) + B \sinh(mx)$$

Substituting the first boundary condition:

$$\Theta(0) = A = \Theta_B$$

Substituting the second boundary condition:

$$\Theta(L) = \Theta_B \cosh(mL) + B \sinh(mL) = \Theta_H$$

$$\Rightarrow B = \frac{\Theta_H}{\sinh(mL)} - \frac{\Theta_B}{\tanh(mL)}$$

Which yields the solution:

$$\Theta(x) = \Theta_B \left(\cosh(mx) - \frac{\sinh(mx)}{\tanh(mL)} \right) + \Theta_H \frac{\sinh(mx)}{\sinh(mL)}$$