Heat Transfer: Conduction

Heat conduction in a multilayer plane wall with convection

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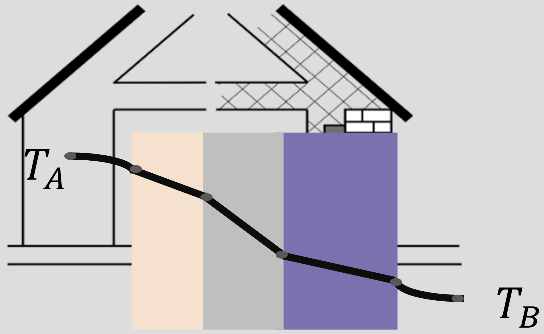
Learning goals

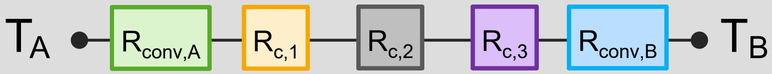
Temperature profile in multilayer wall with convection:

- What is the temperature profile in a multilayer plane wall considering convection resistances?



Thermal resistance in a multilayer wall with convection:

- What is the total resistance in a multilayer plane wall with convection?
- How to calculate the heat flow in a multilayer plane wall with convection?



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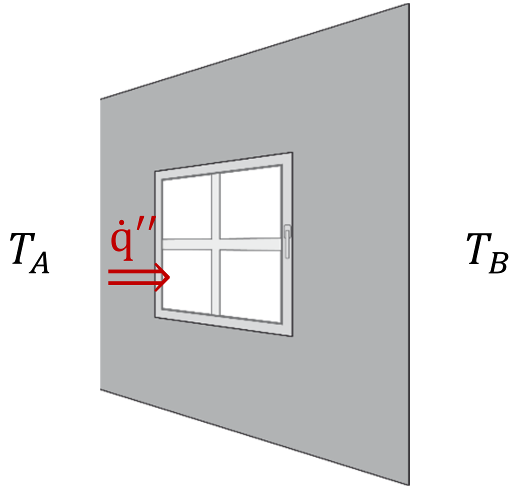
Comparison of heat loss with and without convective thermal resistance

Heat loss without convective resistance:

$$R_c = \frac{\delta_G}{\lambda_G} = \frac{4 \cdot 10^{-3} m}{0.8 \frac{W}{mK}} = 0.005 \frac{K}{W}$$

$$\dot{q}'' = -\frac{T_B - T_A}{W_L} = -\frac{-20°C - 20°C}{0.005} = 8{,}000 \frac{W}{m^2}$$



Heat loss with convection:

$$\dot{q}'' = ? \frac{W}{m^2}$$

Values assumed:

Indoor temperature:	T_A = 20°C
Outdoor temperature:	T_B = -20°C
Glass thickness:	δ_G = 4 mm
Thermal conductivity of glass:	λ_G = 0.8 W / mK
Heat transfer coefficient:	α_A, α_B = ? W / m²K

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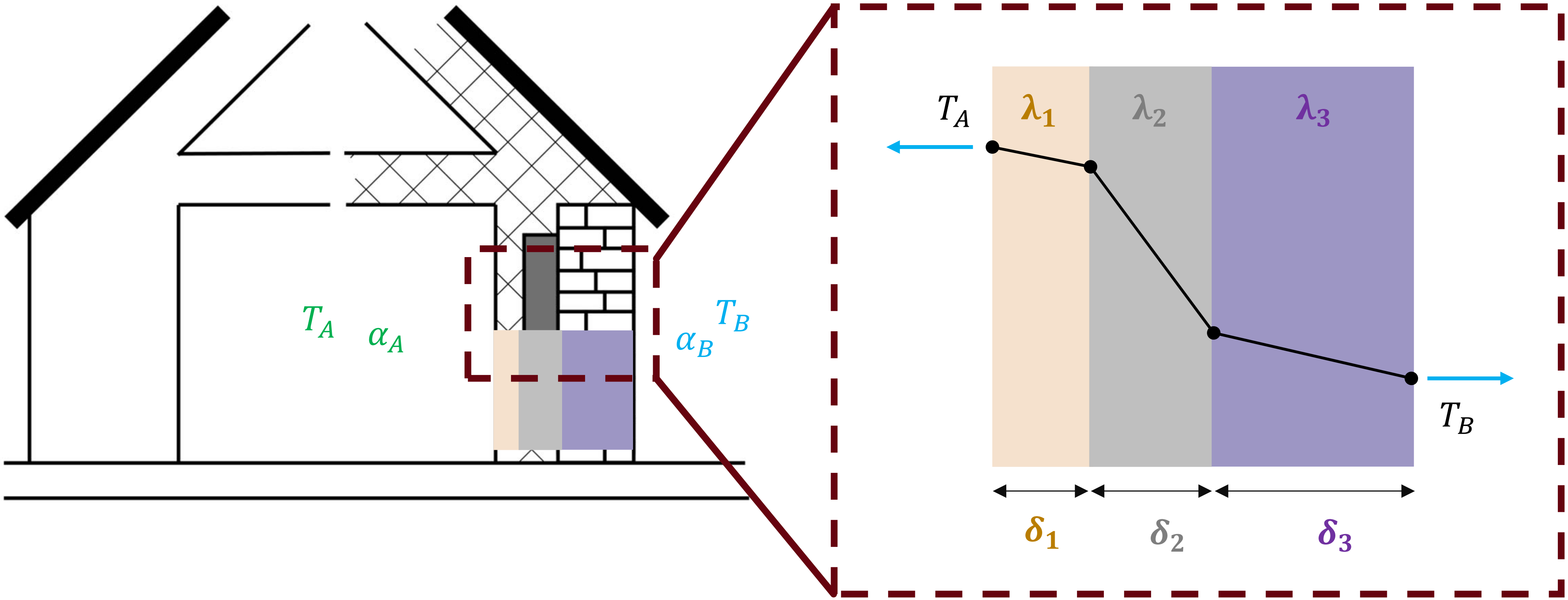
Multilayer wall with convection
Example: House wall



Temperature profile in multilayer wall with convection

Assumptions:

- Steady state
- One-dimensional
- Constant material properties
- Constant cross section area

Values assumed:

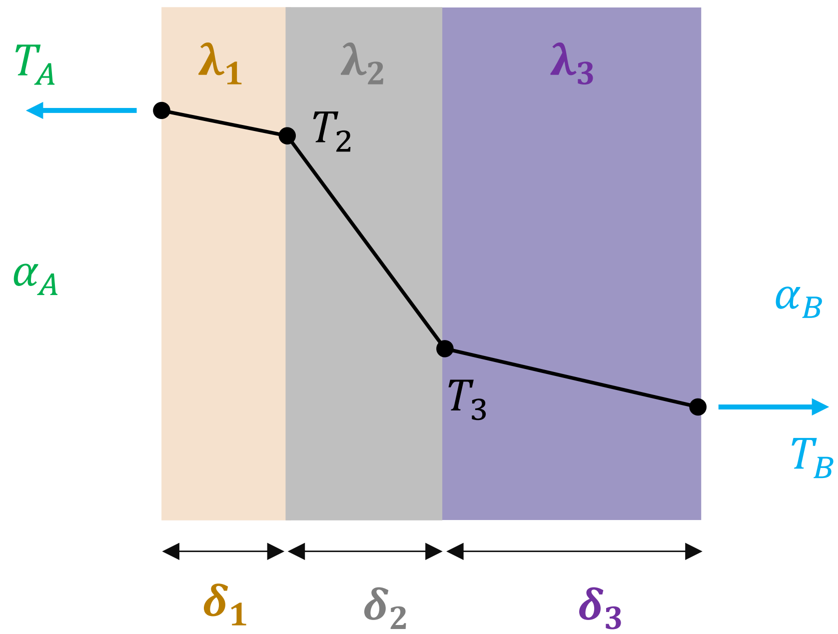
$\boldsymbol{\lambda_i}$: Thermal conductivity in each layer

$$\boxed{\boldsymbol{\lambda_2 = \lambda_{iso} \ll \lambda_1, \lambda_3 \quad \left[\frac{W}{m\,K}\right]}}$$

$\boldsymbol{\alpha_A}$: Heat Transfer coefficient indoor

$\boldsymbol{\alpha_B}$: Heat Transfer coefficient outdoor

$$\boxed{\mathrm{\alpha_A, \alpha_B} : \quad \left[\frac{\mathrm{W}}{\mathrm{m^2 K}}\right]}$$



Temperature profile in multilayer wall with convection

Assumptions:

- Steady state
- One-dimensional
- Constant material properties
- Constant cross section area

Equations:

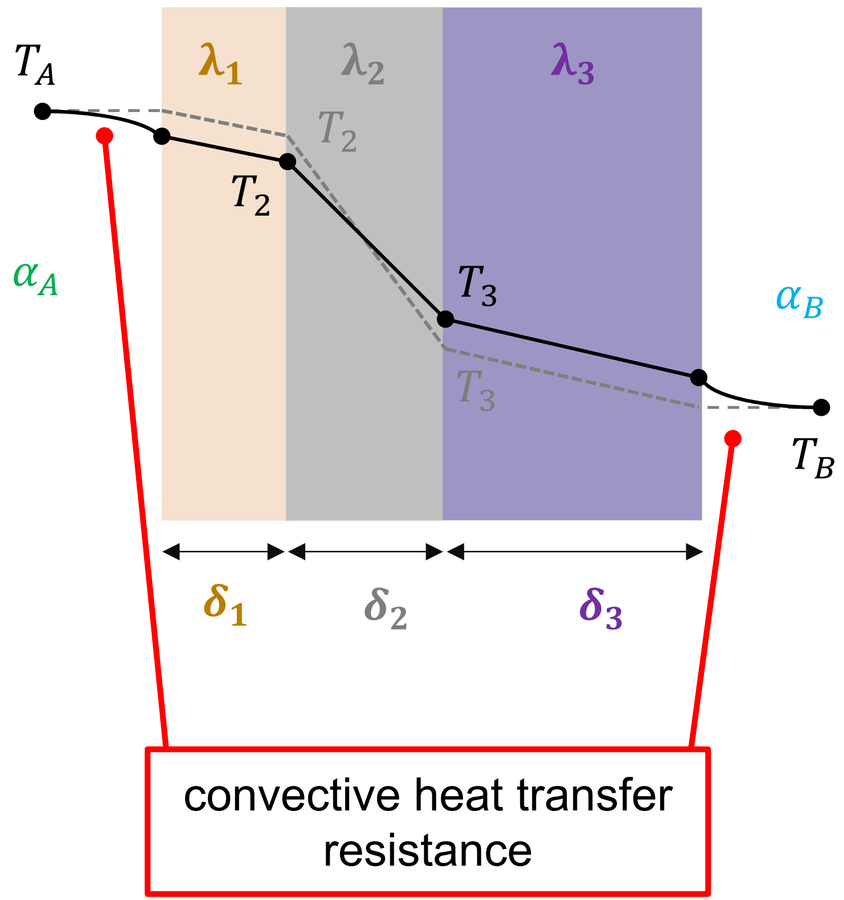
λ_i: Thermal conductivity in each layer

$$\boxed{\lambda_2 = \lambda_{iso} \ll \lambda_1, \lambda_3 \qquad \left[\frac{\mathrm{W}}{\mathrm{m\,K}}\right]}$$

α_A: Heat Transfer coefficient indoor

α_B: Heat Transfer coefficient outdoor

$$\boxed{\alpha_A, \alpha_B : \qquad \left[\frac{\mathrm{W}}{\mathrm{m^2 K}}\right]}$$



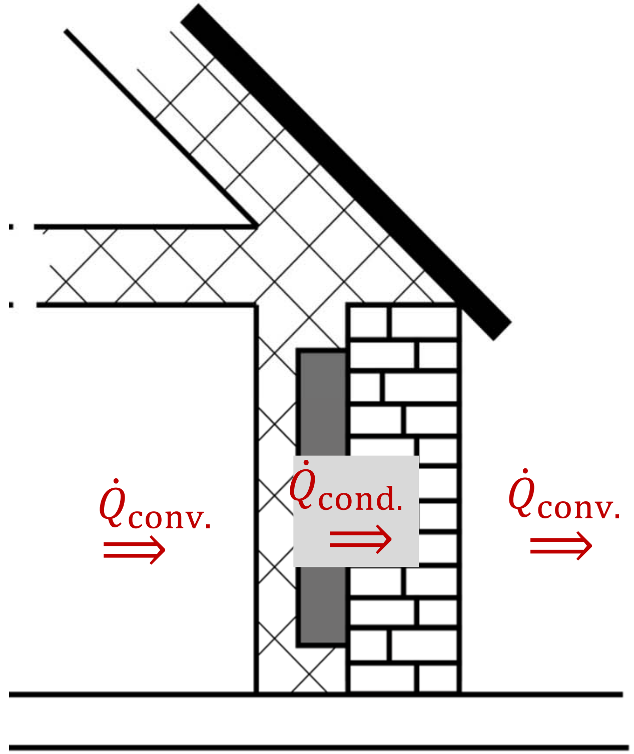
Temperature profile in multilayer wall with convection

Assumptions:

- Steady state
- One-dimensional
- Constant material properties
- Constant cross section area

Equations:

$$\boxed{\dot{Q}_{\text{conv., indoor}} = \dot{Q}_{\text{cond., wall}} = \dot{Q}_{\text{conv., outdoor}}}$$



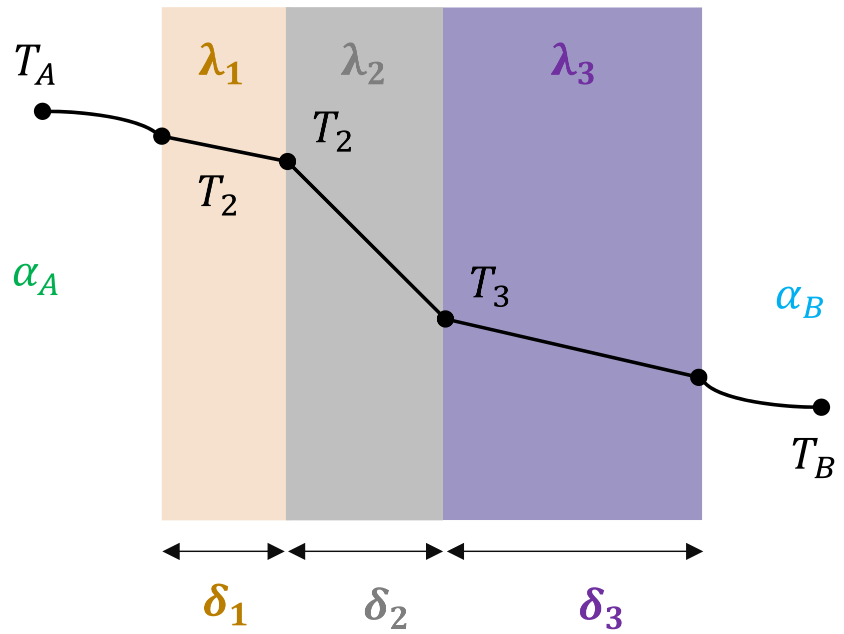
Thermal resistance in a multilayer plane wall with convection

Convection on the surface:

- Inner side: $\dot{Q}_{\mathrm{conv.}A} = \dfrac{T_A - T_1}{\dfrac{1}{\alpha_A A}}$
- Outer side: $\dot{Q}_{\mathrm{conv.}B} = \dfrac{T_4 - T_B}{\dfrac{1}{\alpha_B A}}$

Heat conduction in the wall:

- First wall layer: $\dot{Q}_{c_1} = \dfrac{T_1 - T_2}{\dfrac{\delta_1}{\lambda_1 A}}$
- Second wall layer: $\dot{Q}_{c_2} = \dfrac{T_2 - T_3}{\dfrac{\delta_2}{\lambda_2 A}}$
- Third wall layer: $\dot{Q}_{c_3} = \dfrac{T_3 - T_4}{\dfrac{\delta_3}{\lambda_3 A}}$



$$\dot{Q}_{\mathrm{conv.}A} = \dot{Q}_{c_1} = \dot{Q}_{c_2} = \dot{Q}_{c_3} = \dot{Q}_{\mathrm{conv.}B}$$

$$\dot{\mathrm{Q}} = \frac{\text{Temperature difference}}{\boxed{\text{Thermal resistance}}}$$

Thermal resistance in a multilayer plane wall with convection

Heat transfer resistances:

$$R_{conv.A} = \frac{1}{\alpha_A\, A}$$

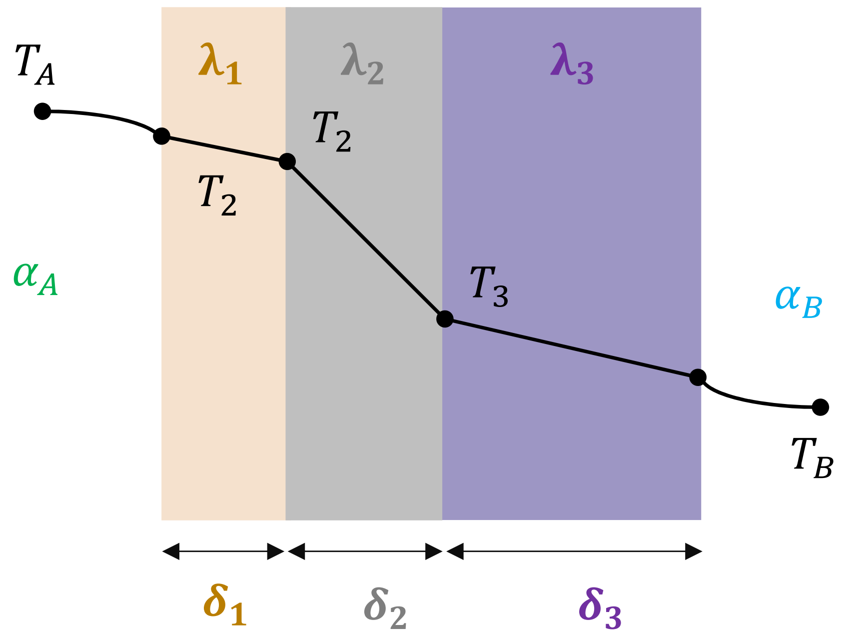
$$R_{conv.B} = \frac{1}{\alpha_B\, A}$$

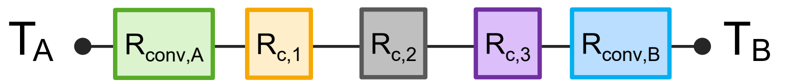
Heat conduction resistances:

$$R_{c,1} = \frac{\delta_1}{\lambda_1\, A}$$

$$R_{c,2} = \frac{\delta_2}{\lambda_2\, A}$$

$$R_{c,3} = \frac{\delta_3}{\lambda_3\, A}$$





$$\dot{Q} = \frac{T_A - T_B}{R_{conv.A} + R_{c,1} + R_{c,2} + R_{c,3} + R_{conv.B}}$$

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Heat flow in a multilayer plane wall with convection

$$\dot{Q} = \frac{T_A - T_B}{R_{conv.A} + R_{c,1} + R_{c,2} + R_{c,3} + R_{conv.B}}$$

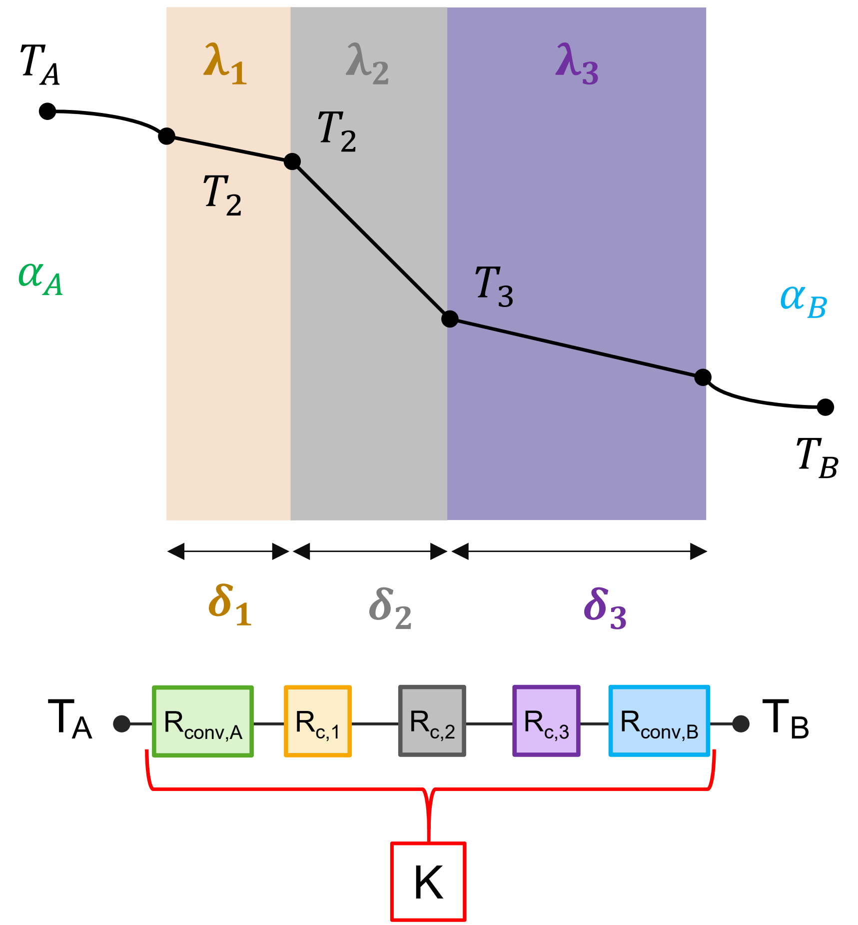
Heat flow in a multilayer plane wall :

$$\dot{Q}_i = \frac{1}{\frac{1}{\alpha_A} + \boxed{\sum_{i=1}^{n} \frac{\delta_i}{\lambda_i}} + \frac{1}{\alpha_B}} (T_A - T_B)$$

Heat transfer coefficient, *k*:

$$k = \frac{1}{\sum R} = \frac{1}{\frac{1}{\alpha_A} + \sum_{i=1}^{n} \frac{\delta_i}{\lambda_i} + \frac{1}{\alpha_B}} \qquad \left[\frac{W}{m^2 K}\right]$$

$$\boxed{\dot{Q}_i = k\, A\, (T_A - T_B)}$$



Comparison of heat loss with and without convective thermal resistance

Heat loss without convective resistance:

$$R_c = \frac{\delta_G}{\lambda_G} = \frac{4 \cdot 10^{-3} m}{0.8 \frac{W}{mK}} = 0.005 \frac{K}{W}$$

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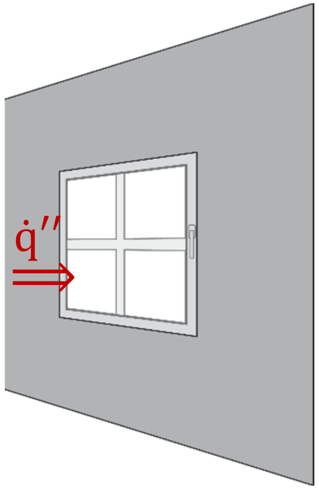

Heat loss including convective resistance:

$$\dot{q}'' = ? \frac{W}{m^2}$$

Values assumed:

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Comparison of heat loss with and without convective thermal resistance

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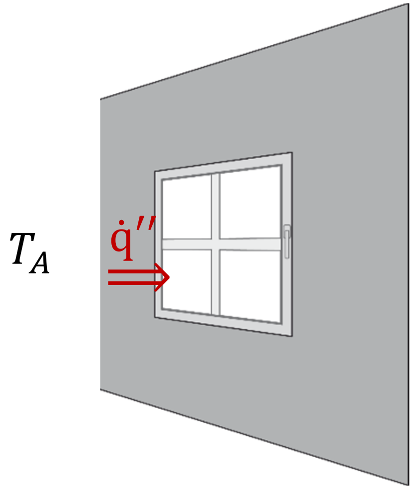

Heat loss including convective resistance:

$$R_{tot} = R_{conv.,A} + R_c + R_{conv.,B} = \frac{1}{\alpha_A} + \frac{\delta_g}{\lambda_g} + \frac{1}{\alpha_B}$$

$$= \frac{1}{10 \frac{W}{m^2K}} + \frac{4 \cdot 10^{-3} m}{0.8 \frac{W}{mK}} + \frac{1}{10 \frac{W}{m^2K}} = 0.205 \frac{K}{W}$$

$$\dot{q}'' = \frac{T_A - T_B}{W_{tot.}} = -\frac{-20°C - 20°C}{0.205 \frac{K}{W}} \approx 200 \frac{W}{m^2}$$

Values assumed:

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Heat transfer coefficient:	α_A, α_B = 10 W / m²K

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What influence does the additional consideration of convection have on the total heat transfer?