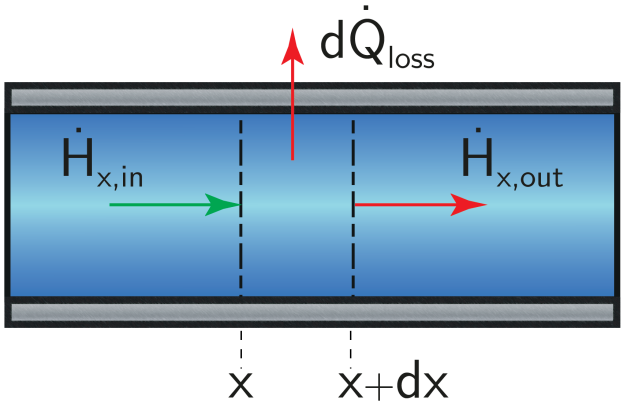
EB - Conv. - IE 3

Through a very long pipe with diameter D flows a fluid. In addition, the pipe has a uniform, constant wall temperature T_w.

Derive the differential equation for the temperature profile in the flow direction, while neglecting the diffusive heat transport in the direction of the flow. Assume steady-state conditions.



Energy balance:

$$0 = \dot{H}_{x,in} - \dot{H}_{x,out} - d\dot{Q}_\mathrm{loss}$$

Definition of fluxes:

$$\dot{H}_{x,in} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{x,out} = \dot{H}_{x,in} + \frac{\partial \dot{H}_{x,in}}{\partial x} \cdot dx$$

$$d\dot{Q}_\mathrm{loss} = \alpha \cdot \pi \cdot D \cdot dx \cdot (T - T_\mathrm{w})$$

Where:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

Substituting and rewriting:

$$0 = -u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho \cdot c \cdot \frac{\partial T}{\partial x} - \alpha \cdot \pi \cdot D \cdot (T - T_\mathrm{w})$$

$$\Rightarrow 0 = -u\rho cD\frac{\partial T}{\partial x} - 4\alpha\,(T - T_\mathrm{w})$$