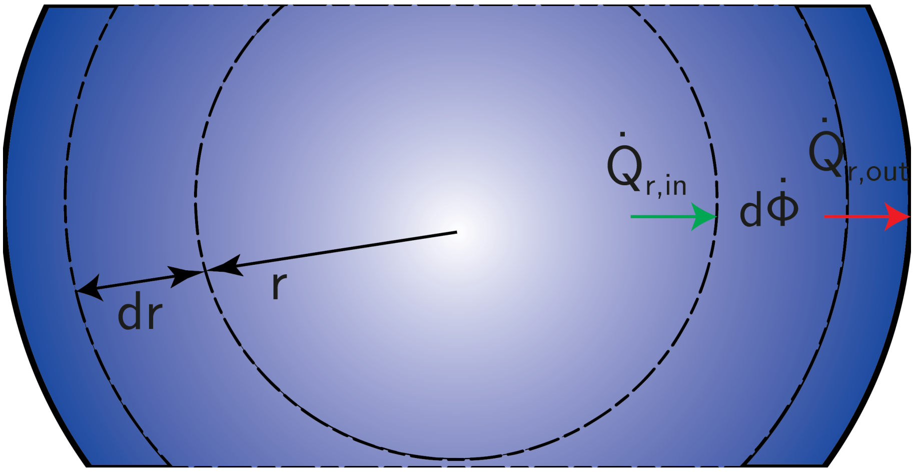
EB - Cond. - IE 16

Develop an energy balance to calculate the temperature profile inside the sphere and give the boundary conditions. The sphere is losing heat to the environment. Assume one-dimensional steady-state heat with a source.



Energy Balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} + d\dot{\Phi} = 0$$

Heat Fluxes:

$$\dot{Q}_{r,in} = -\lambda A(r)\frac{\partial T}{\partial r} = -\lambda 4\pi r^2 \frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} = \dot{Q}_{r,in} + \frac{\partial \dot{Q}_{r,in}}{\partial r}dr = -\lambda 4\pi r^2 \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}(-\lambda 4\pi r^2 \frac{\partial T}{\partial r})dr$$

$$d\dot{\Phi} = dV \cdot \dot{\Phi}''' = 4 \cdot \pi \cdot r^2 dr \cdot \dot{\Phi}'''$$

The ingoing flux can be described by use of Fourier's law and the outgoing flux can be approximated by use of the Taylor series expansion. Note that for the energy generation, the infinitesimal volume can be approximated by removing higher order terms for dr. This will have a neglectable influence on the volume.

$$dV = \frac{4}{3} \cdot \pi \cdot (r + dr)^3 - \frac{4}{3} \cdot \pi \cdot r^3 = \frac{4}{3} \cdot \pi \cdot \left(3r^2 dr + 3r dr^2 + \mathrm{dr}^3\right) \approx 4 \cdot \pi \cdot r^2 dr$$

Boundary conditions:

$$\frac{\partial T(r=0)}{\partial r} = 0$$

$$T(r = r_1) = T_1$$

The first boundary condition describes that the temperature gradient in the center equals zero. This is because of symmetry. The second one describes that the temperature on the surface equals T_1.