Heat Transfer: Conduction

Example: Clinical thermometer

Prof. Dr.-Ing. Reinhold Kneer

Prof. Dr.-Ing. Dr. rer. pol. Wilko Rohlfs

HEATQUIZ

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

Learning goals

Clinical Thermometer

- Example of the lumped capacity model
  → Procedure

[1]

[1] sciencing.com/different-parts-of-a-mercury-thermometer-12073649.html

How does a liquid thermometer work

[1]

The tip is heated by contact with the object to be measured.

Clinical Thermometer:

- Build with mercury as an indicator
  → no longer in use today

Principle:
linear volume expansion of the liquid through temperature rise

[1] sciencing.com/different-parts-of-a-mercury-thermometer-12073649.html

How does a liquid thermometer work

[1]

Given parameter:

$L = 30 \times 10^{-3}\,\mathrm{m}$ $\quad d = 4 \times 10^{-3}\,\mathrm{m}$

$T_0 = 20°\mathrm{C}$

$c_{Hg} = 140\ \mathrm{J/(kg \cdot K)}$

$\rho_{Hg} = 14 \times 10^3\ \mathrm{kg/m^3}$

$\lambda_{Hg} = 9\ \mathrm{W/(m \cdot K)}$

Problem discription:

- A child measures its body temperature at two different times:

$$t_1 = 40\ s \quad T_1 = 34\ °\mathrm{C} \qquad t_2 = 100\ s \quad T_2 = 39\ °\mathrm{C}$$

a) How high is the fever?

b) What is the heat transfer coefficient α?

c) How long does it take to determine the temperature with a precision of 0.1K?

d) Are the assumptions of the calculation valid?

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Calculation of the fever temperature

a) How high is the fever?

Still unknown:

- Parameter m
- „Fever temperature“ T_f

In principle calculable, requires iteration:

- simply start with good estimate: $T_f = 40°\mathrm{C}$

Lumped capacity model:

$$\Theta^* = \frac{T - T_0}{T_f - T_0} = 1 - e^{\underbrace{-\frac{\alpha A}{\rho c_p V}}_{\mathrm{m}} t}$$

$$1 - \frac{T - T_0}{T_f - T_0} = \frac{T_f - T}{T_f - T_0} = e^{-mt}$$

$$\ln \frac{T_f - T}{T_f - T_0} = -mt$$

$$\begin{cases} \ln \dfrac{T_f - T_1}{T_f - T_0} = -m t_1 \; (1) \\ \ln \dfrac{T_f - T_2}{T_f - T_0} = -m t_2 \; (2) \end{cases}$$

$$\frac{(1)}{(2)}: \frac{t_1}{t_2} = \frac{\ln \frac{T_f - T_1}{T_f - T_0}}{\ln \frac{T_f - T_2}{T_f - T_0}}$$

substitute 0.4   0.402 match

$$m = -\frac{1}{t_1} \ln \frac{T_f - T_1}{T_f - T_0} \approx 0.03$$

Calculation of the heat transfer coefficient

b) How large is α ?

Lumped capacity model:

$$m = \frac{\alpha A}{\rho c_p V} \approx 0.03$$

$$\text{with } A = \pi d L;\ V = \frac{\pi d^2 L}{4} \Rightarrow \frac{A}{V} = \frac{4}{d}$$

$$\alpha = \frac{m \rho c_p d}{4} = 58.8 \frac{\mathrm{W}}{\mathrm{m^2 K}}$$

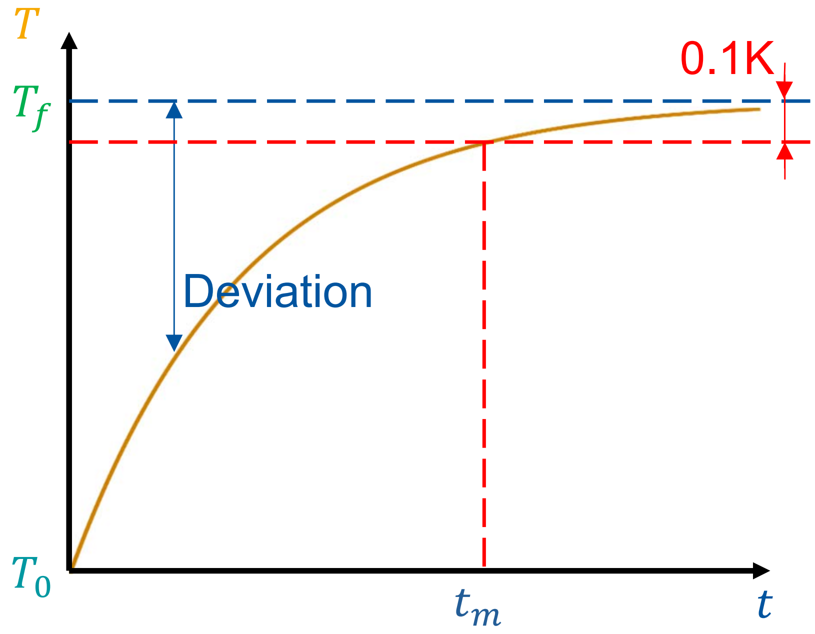
d) Are the assumptions of the lumped capacity model correct?

$$Bi \ll 1\ ?$$

$$Bi = \frac{\alpha \cdot d/2}{\lambda_{Hg}} = 0.013 \ll 1$$

Calculation of the measurement time

c) **How long must be measured to determine the temperature with an accuracy of 0.1K?**



Body with "high" thermal conductivity:

- Measuring time t_m for a 0.1K precision

$$t_m = -\frac{1}{m} \ln \frac{T_f - T}{T_f - T_0}$$

$$t_m = -\frac{1}{0.03} \ln \frac{0.1}{40-20} = 176.6\mathrm{s}$$

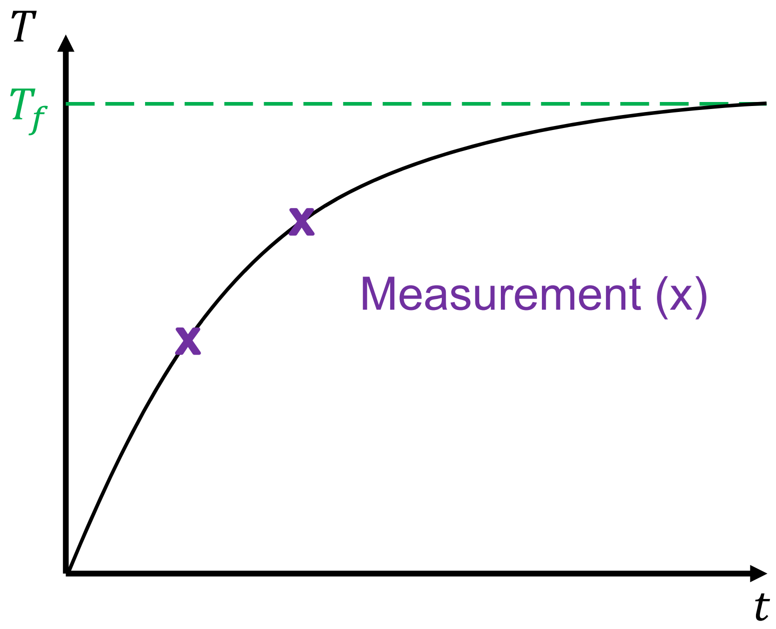
Comprehension questions

For safety reasons, mercury thermometers are no longer available in retail. Thermometers filled with alcohol are also hardly used anymore.
Why? What are the disadvantages of these measuring instruments?

The standard devices currently in use are digital thermometers.
How can the body temperature be determined with them?

I will present the solution after approx. 15 sec on the next slide (stop the video here if you want to think for yourself first).

Digital device:



is stored on a chip in the device

with unknown m and T_f $\rightarrow$
2 measurements and T_f can **be calculated** by evaluating the stored function