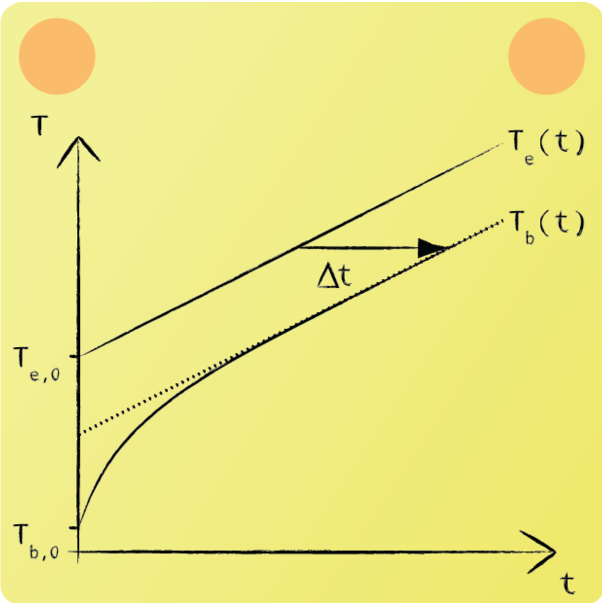
Exam Preparation Conduction 08

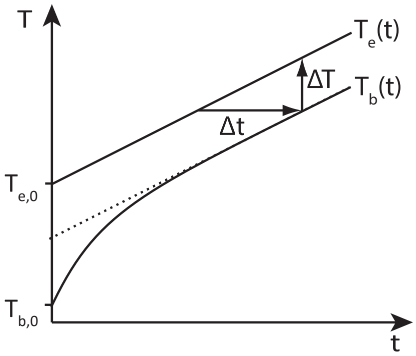


A body with a temperature of T_{b} is located within an environment with the lineary rising temperature T_{e} and heats up accordingly the given profile. The body temperature is uniform within the body and the environment and its temperature is not affected by the body. As $t \to \infty$, the temperature of the body follows that of the environment with a constant time delay Δt. Determine Δt.

Regarding the body the following equation for the temperature profile over time can be stated:

$$mc_p\frac{dT}{dt} = \alpha A \Delta T$$

The term on the left-hand side describes the transient behavior of the heating-up process, the right term describes the heat transfer into the body.



For $t \to \infty$ the time derivative can be stated as $\frac{dT}{dt} = \frac{\Delta T}{\Delta t}$, since the temperature of the body increases linearly. An analysis of the differential equation shows the linearity of the body's temperature profile for $t \to \infty$. The constant ΔT or Δt can also be determined analytically, which yields the body temperature. The fact that ΔT can't be null can be explained with an easy deliberation. If ΔT has to be null, either the change in environment temperature or one of the other parameters have to be null (m, c_p) or infinite (α, A), otherwise the equation can't be satisfied. Since due to definition the temperature change is a nonzero number and the other parameters can't be infinite or null (non-physical), ΔT can't be null.Plugging in $\frac{dT}{dt} = \frac{\Delta T}{\Delta t}$ yields:

$$mc_p\frac{\Delta T}{\Delta t} = \alpha A \frac{dT}{dt}\Delta t$$

After cancelling ΔT the equation can be solved for Δt:

$$\Delta t = \frac{mc_p}{\alpha A}$$