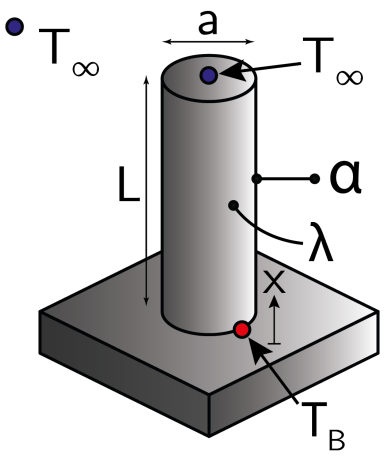
Fins - Flux 3

Calculate the rate of heat transfer for a fin with its head temperature equal to the ambient.



Given the fin temperature profile:

$$\Theta(x) = \Theta_{\mathrm{B}} \left(\cosh(mx) - \frac{1}{\tanh(mL)} \sinh(mx) \right)$$

Where $\Theta(x) = T(x) - T_\infty$ and $m = \sqrt{\frac{4\alpha}{\lambda a}}$.

The rate of heat transfer through the fin can be expressed as:

$$\dot{Q} = -\lambda A_{\mathrm{c}} \frac{\partial T}{\partial x}|_{x=0} = -\lambda \frac{\pi a^2}{4} \frac{\partial \Theta}{\partial x}|_{x=0}$$

Differentiation of $\Theta(x)$ with respect to x yields:

$$\frac{\partial \Theta}{\partial x} = \Theta_{\mathrm{B}} m \left(\sinh(mx) - \frac{1}{\tanh(mL)} \cosh(mx) \right)$$

There for at $x = 0$ gives:

$$\frac{\partial \Theta}{\partial x}|_{x=0} = -\frac{\Theta_{\mathrm{B}} m}{\tanh(mL)}$$

Substitution into the expression for the rate of heat transfer yields:

$$\dot{Q} = \lambda \frac{\pi a^2}{4} \frac{\Theta_{\mathrm{B}} m}{\tanh(mL)}$$