Mass Transfer: Diffusion

Derivation of the conservation equation of mass diffusion and analogy to heat transfer

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HEATQUIZ

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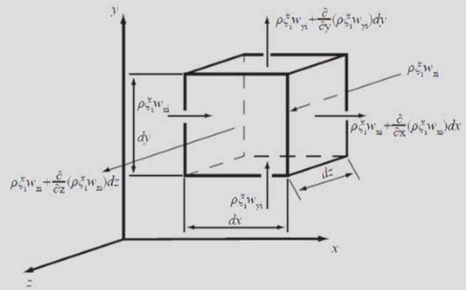
Learning goals

Conservation equations and analogy:

- Understanding of the necessary steps to develop the conservation equation
- Knowledge of the common features of heat, mass, and momentum transfer



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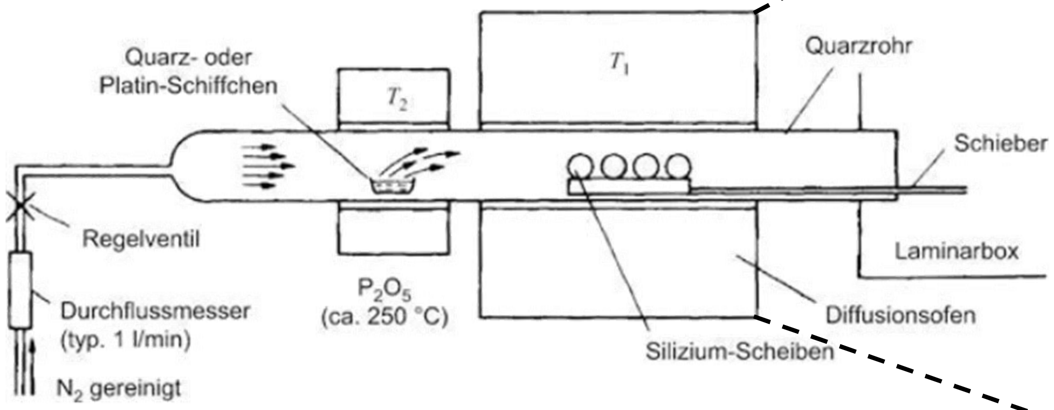
Practical example

Diffusion oven in the production of semiconductor elements and solar cells



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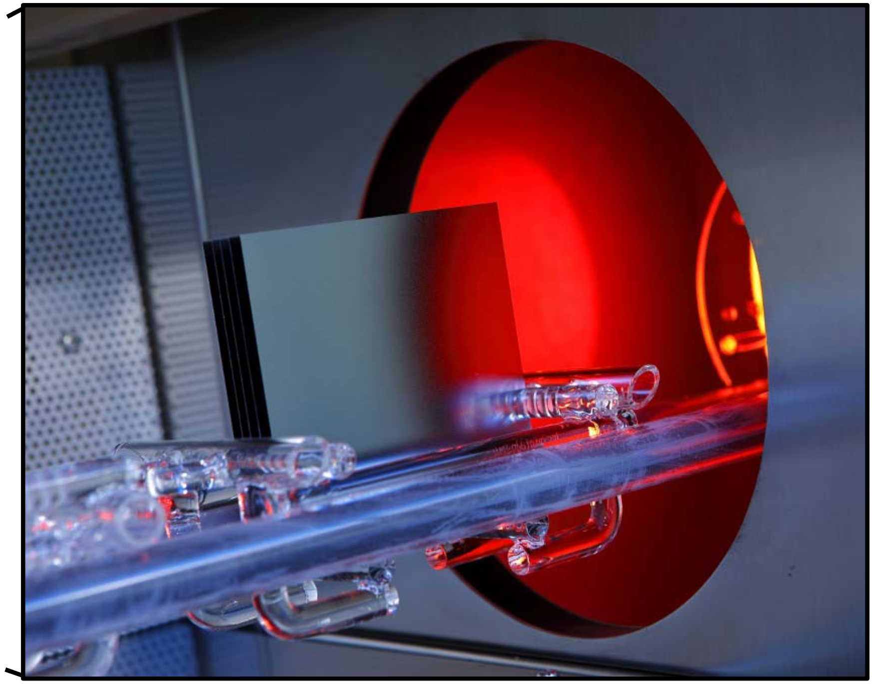

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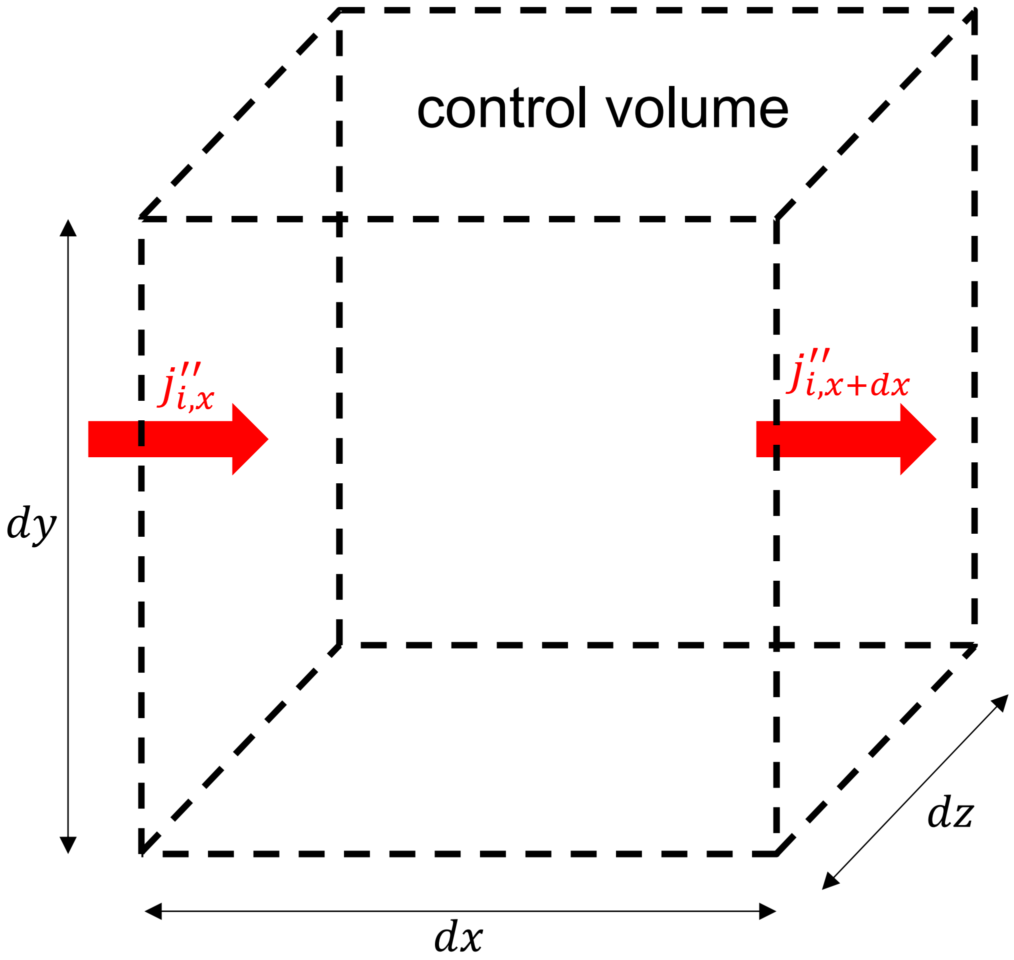
[1] Ruge und Mader, 1991
[2] https://isfh.de/wp-content/uploads/2017/01/IndustrielleSolarzellenBox.jpeg

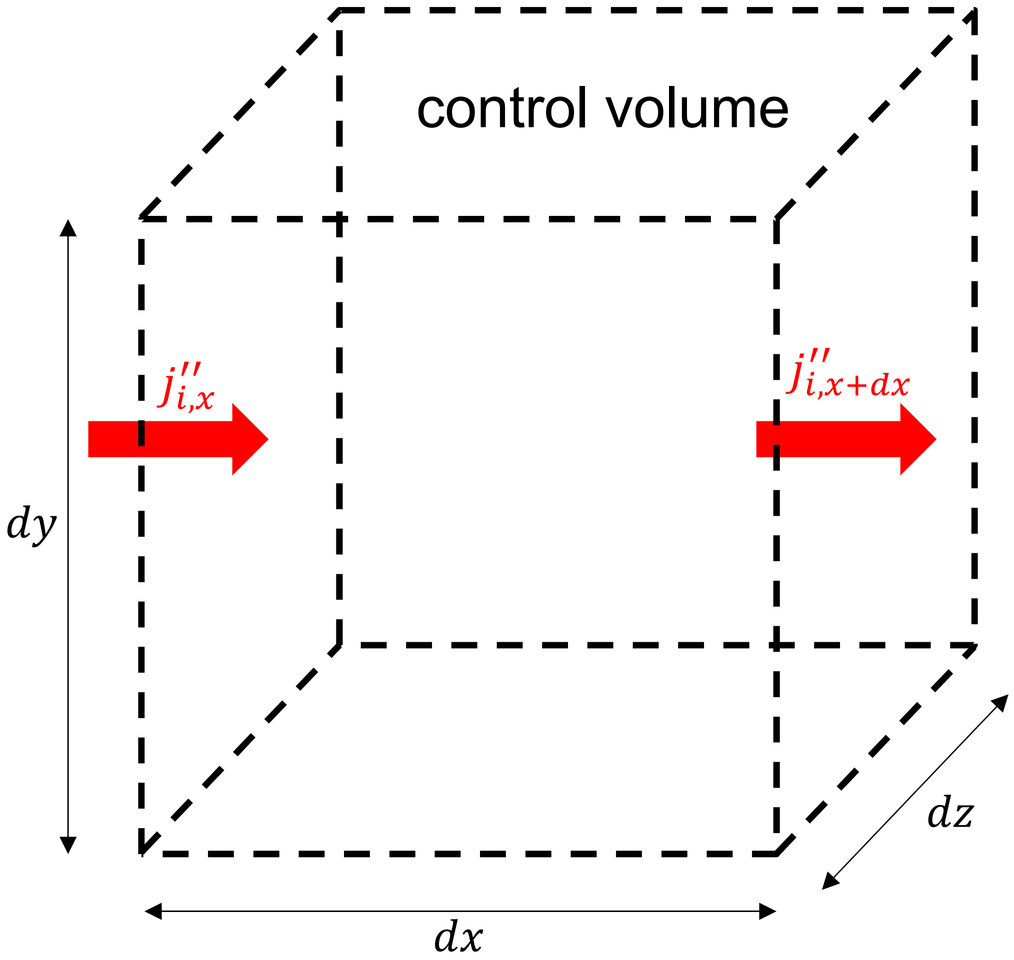
Derivation of the differential equation for diffusive mass transport (1D)



Procedure for concentration curves:

- determine control volume
- identify relevant flows
- set up balance
- develop differential equation
- solve differential equation

Derivation of the differential equation for diffusive mass transport (1D)



Balance (steady state):

$$0 = j''_{i,x} - j''_{i,x+dx}$$

Diffusion flux density:

$$j''_i = -D \cdot \frac{d\rho_i}{dx} = -\rho \cdot D \cdot \frac{d\xi_i}{dx}$$

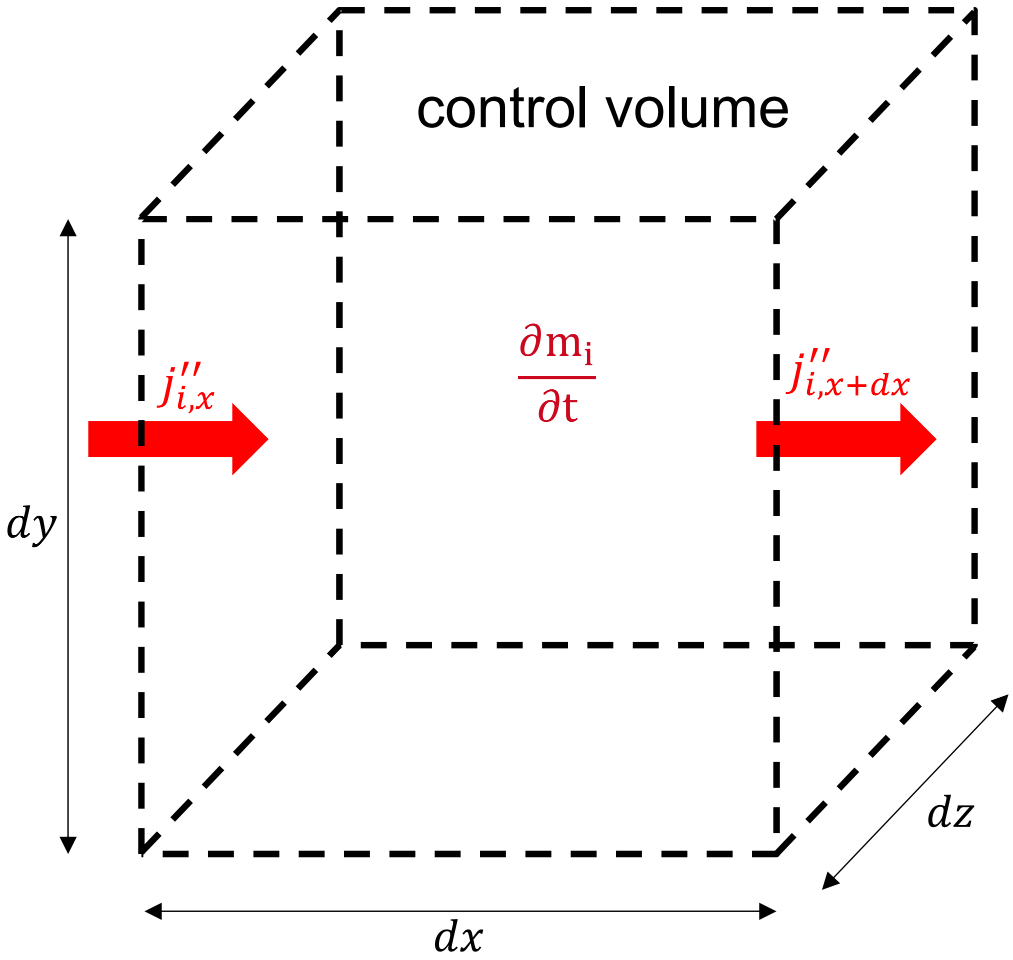
Taylor series expansion

Differential equation:

$$\frac{\partial j''_i}{\partial x} = \frac{\partial}{\partial x}\left(-D_{ij}\frac{\partial \rho_i}{\partial x}\right)$$

$$0 = \rho D_{ij}\frac{\partial^2 \xi_i}{\partial x^2}$$

Transient one-dimensional diffusion (1D)



Temporal change:

$$\frac{\partial m_i}{\partial t} = \frac{\partial \rho_i V}{\partial t} = \frac{\partial \rho_i dxdydz}{\partial t}$$

Balance

$$\frac{\partial m_i}{\partial t} = j''_{i,x} - j''_{i,x+dx}$$

Taylor series expansion

Differential equation:

$$\frac{\partial \rho_i}{\partial t} = -\frac{\partial j''_i}{\partial x}$$

$$\frac{\partial \rho_i}{\partial t} = \frac{\partial}{\partial x}\left(\rho D_{ij} \frac{\partial \xi_i}{\partial x}\right) = \rho D_{ij} \frac{\partial^2 \xi_i}{\partial x^2}$$

$$\frac{\partial \rho_i/\rho}{\partial t} = \frac{\partial \xi_i}{\partial t} = D_{ij} \frac{\partial^2 \xi_i}{\partial x^2}$$

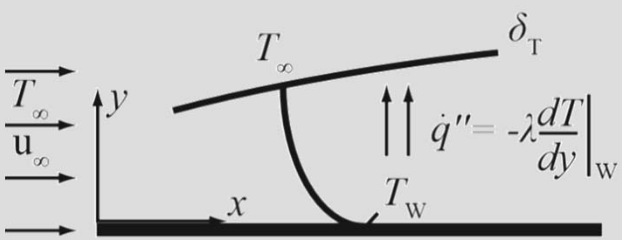
Analogy between heat, momentum, and mass transfer

Transient heat transfer:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}$$

with a in $\left[\frac{m^2}{s}\right]$

Thermal diffusivity



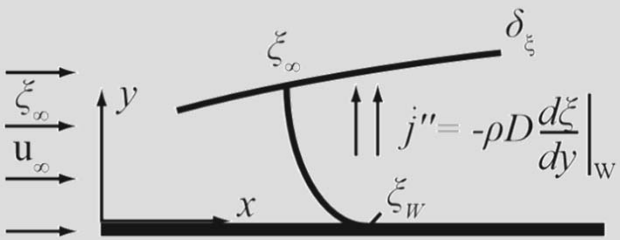
Heat Transfer: Fourier‘s Law

Transient mass transfer:

$$\frac{\partial \xi}{\partial t} = D\frac{\partial^2 \xi}{\partial x^2}$$

with D in $\left[\frac{m^2}{s}\right]$

Diffusion coefficient



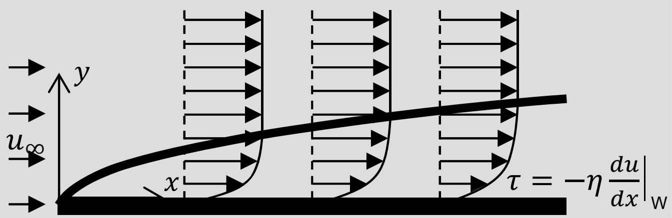
Mass Transfer: Fick‘s Law

Transient momentum transfer:

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial x^2}$$

with ν in $\left[\frac{m^2}{s}\right]$

Kinematic viscosity or momentum diffusivity



Momentum Transfer: Newton‘s Law

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Comprehension questions

What is the analogy of the diffusion coefficient in heat transfer and momentum transport?