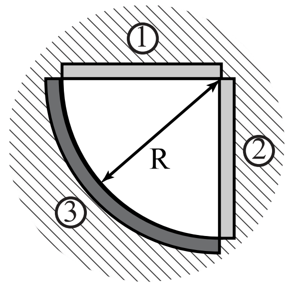
Section I

Radiation solutions

Exercise I.1: (Infinite pipe segment ⋆⋆)

Consider an infinite long pipe segment as in the figure.



Tasks:

a) Specify the view factors Φ_{12}, Φ_{31} and Φ_{33} as a function of Φ_{13}.

b) Determine Φ_{13}.

Solution I.1: (Infinite pipe segment ⋆⋆)

Task a)

When determining the view factors, recall the general rules that apply to view factors. First the summation rule:

$$\sum_{j=1}^{n} \Phi_{ij} = \Phi_{i1} + \Phi_{i2} + \Phi_{i3} + ... + \Phi_{in} = 1,$$

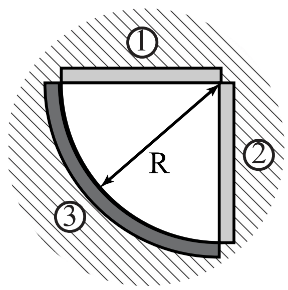
second, the reciprocity rule:

$$A_i\Phi_{ij} = A_j\Phi_{ji},$$

and third the symmetry rule:

$$\Phi_{ij} = \Phi_{ik},$$

which applies if two or more surfaces display symmetry about a third surface, they have identical view factors from that surface.



From the figure, some information regarding the view factors can already be found. Surfaces 1 and 2 are flat plates, and therefore, cannot see themselves:

$$\Phi_{11} = 0 \ \ (-)\,, \tag{I.1.1}$$

and:

$$\Phi_{22} = 0 \ \ (-)\,. \tag{I.1.2}$$

With this given, Φ_{12} is expressed in terms of Φ_{13} by using the summation rule:

$$\cancelto{0}{\Phi_{11}} + \Phi_{12} + \Phi_{13} = 1 \tag{I.1.3}$$

$$\Rightarrow \boldsymbol{\Phi_{12} = 1 - \Phi_{13}}.$$

Furthermore, Φ_{31} is expressed in terms of Φ_{13} by using the reciprocity rule, where L is the infinite length of the pipe segment:

$$\Phi_{31}A_3 = \Phi_{13}A_1 \tag{I.1.4}$$

$$\Rightarrow \boldsymbol{\Phi_{31} = \Phi_{13} \cdot \frac{RL}{\frac{2\pi}{4} \cdot RL} = \frac{2}{\pi} \cdot \Phi_{13}}. \tag{I.1.5}$$

The figure shows that surfaces 1 and 2 display symmetry about surface 3. Therefore Φ_{31} and Φ_{32} are equal to each other:

$$\Phi_{32} = \frac{2}{\pi} \cdot \Phi_{13}. \tag{I.1.6}$$

Lastly, with Φ_{31} and Φ_{32} known, the summation rule is used to determine Φ_{33}

$$\Phi_{31} + \Phi_{32} + \Phi_{33} = 1 \tag{I.1.7}$$
$$\Rightarrow \boldsymbol{\Phi_{33} = 1 - \frac{4}{\pi} \cdot \Phi_{13}}.$$

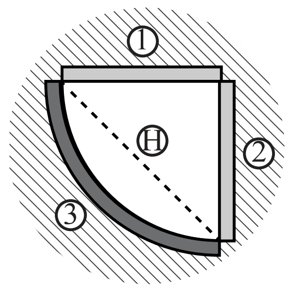
Conclusion

Which thus yields view factors Φ_{12}, Φ_{31}, and Φ_{33} as a function of Φ_{13}:

$$\Phi_{12} = 1 - \Phi_{13}, \qquad \Phi_{31} = \frac{2}{\pi} \cdot \Phi_{13}, \qquad \text{and} \qquad \Phi_{33} = 1 - \frac{4}{\pi} \cdot \Phi_{13}.$$

Task b)

To determine Φ_{13} the following auxiliary plane H is introduced.



The figure shows symmetry about plane H, and thus:

$$\Phi_{H1} = \frac{1}{2} \ (-). \tag{I.1.8}$$

The reciprocity rule is used to find the numerical value for Φ_{1H}:

$$\Phi_{1H} A_1 = \Phi_{H1} A_H \tag{I.1.9}$$
$$\Rightarrow \Phi_{1H} = \Phi_{H1} \frac{\sqrt{2} RL}{RL} = \frac{\sqrt{2}}{2} \ (-).$$

The figure shows that surface 3 only sees the auxiliary plane H, and therefore:

$$\Phi_{H3} = 1 \ (-). \tag{I.1.10}$$

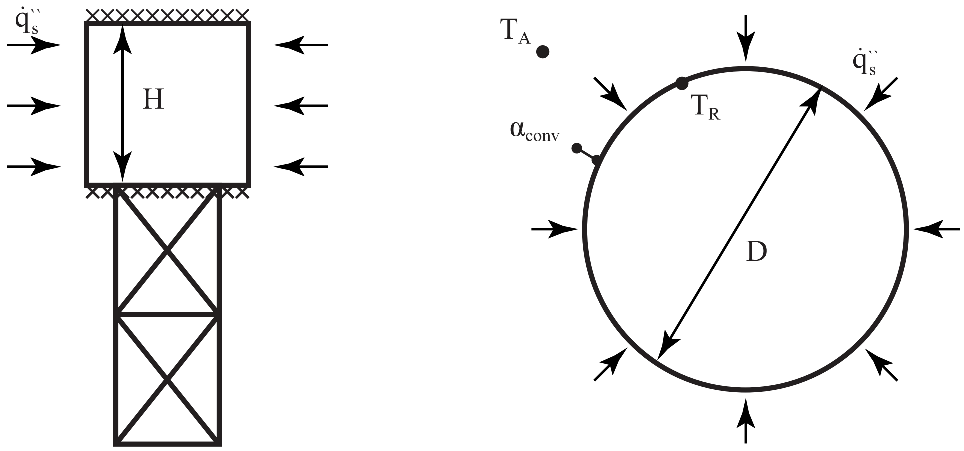
Conclusion

Thus, all radiation emitted from body 1 and transferred to surface H is directed towards body 3:

$$\Phi_{13} = \Phi_{1H} = \frac{\sqrt{2}}{2} \ (-). \tag{I.1.11}$$

Exercise I.2: (Solar power tower ⋆)

Solar radiation is uniformly and radially redirected toward a central cylindrical receiver in a solar tower plant by a surrounding mirror field (radiation density $\dot{q}''_{\mathrm{S}}$). Consequently, the surface of the receiver is heated to a temperature of T_{R}, and the thermal power output of the plant is $\dot{Q}_{\mathrm{th}}$.



(a) Side view (b) Top view

Given parameters:

- Receiver height: H
- Receiver outer diameter: D
- Receiver surface temperature: T_{R}
- Receiver emissivity of the surface: ϵ
- Heat transfer coefficient: α_{conv}
- Ambient temperature: T_{A}

Hints:

- Heat losses in the interior of the receiver as well as at its ends can be neglected.
- Radiation from the ambient can be neglected.
- The receiver can be considered as a grey body.

Tasks:

a) From a balance around the receiver, determine the mean radiation density $\dot{q}''_{\mathrm{S}}$ as a function of the thermal load $\dot{Q}_{\mathrm{th}}$.

Solution I.2: (Solar power tower ⋆)

Task a)

To determine the mean radiation density $\dot{q}''_S$ as a function of the thermal load $\dot{Q}_{th}$, an energy balance around the receiver must be established.

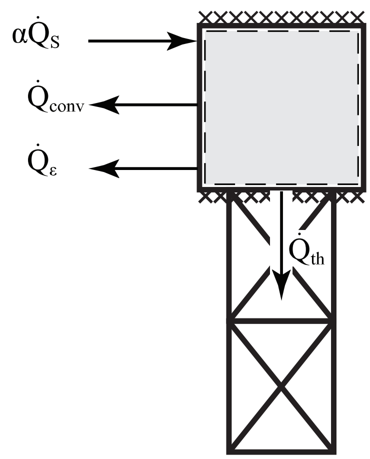
Since the values are unknown, the most straightforward approach to setting an inner energy balance:

$$\underbrace{\frac{\partial U}{\partial t}}_{\substack{\text{Temporal change}\\ \text{of inner energy}}} = \underbrace{\alpha \sum \dot{Q}_{\text{in,rad}}}_{\substack{\text{Absorbed}\\ \text{radiation}}} - \underbrace{\sum \dot{Q}_{\epsilon}}_{\substack{\text{Emission}\\ \text{radiation}}} + \underbrace{\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}}}_{\substack{\text{External}\\ \text{transport}}},$$

where $\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}}$ describes the transport that is not due to radiative heat, e.g. convection and thermal power generation.

1 Setting up the balance:

The sun is radiating on the solar power tower with a heat flux density $\dot{q}''_S$, but at the same time the receiver is losing heat due to emitting radiation as a grey body and convection. The remaining thermal load is transferred inside the solar power tower where the energy is converted into electrical power.



The inner energy balance reads:

$$0 = \underbrace{\alpha \dot{Q}_S}_{\substack{\text{Absorbed}\\ \text{radiation}}} - \underbrace{\dot{Q}_{\epsilon}}_{\substack{\text{Emission}\\ \text{radiation}}} - \underbrace{\dot{Q}_{\text{conv}}}_{\substack{\text{Convective}\\ \text{losses}}} - \underbrace{\dot{Q}_{\text{th}}}_{\substack{\text{Thermal}\\ \text{load}}}. \tag{I.2.1}$$

2 Defining the elements within the balance:

Kirchoff's law states for grey bodies that:

$$\alpha = \epsilon. \tag{I.2.2}$$

The solar radiation absorbed by the receiver can be expressed as:

$$\begin{aligned} \alpha \dot{Q}_{\mathrm{S}} &= \epsilon \dot{q}_{\mathrm{S}}'' A_{\mathrm{S}} \\ &= \epsilon \dot{q}_{\mathrm{S}}'' \pi D H. \end{aligned} \tag{I.2.3}$$

The rate of heat loss due to convection is determined from Newton's law of cooling:

$$\begin{aligned} \dot{Q}_{\mathrm{conv}} &= \alpha_{\mathrm{conv}} A_{\mathrm{s}} \left(T_{\mathrm{R}} - T_{\mathrm{A}} \right) \\ &= \alpha_{\mathrm{conv}} \pi D H \left(T_{\mathrm{R}} - T_{\mathrm{A}} \right). \end{aligned} \tag{I.2.4}$$

The emitted radiation by the solar receiver is found from:

$$\begin{aligned} \dot{Q}_{\epsilon} &= \epsilon \sigma A_{\mathrm{s}} T_{\mathrm{R}}^4 \\ &= \epsilon \sigma \pi D H T_{\mathrm{R}}^4. \end{aligned} \tag{I.2.5}$$

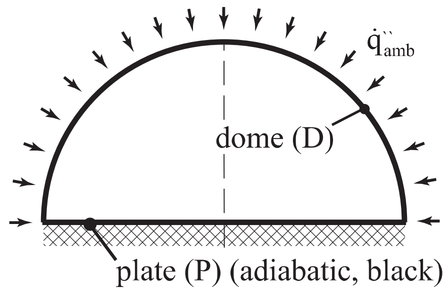
Conclusion

3 **Inserting and rearranging:**

$$\dot{q}_{\mathrm{S}}'' = \frac{\alpha_{\mathrm{conv}}}{\epsilon} \left(T_{\mathrm{R}} - T_{\mathrm{A}} \right) + \sigma T_{\mathrm{R}}^4 + \frac{\dot{Q}_{\mathrm{th}}}{\epsilon \pi D H}. \tag{I.2.6}$$

Exercise I.3: (Hemispherical dome ⋆⋆)

A thin circular plate (P) is covered by a hemispherical, transparent, grey dome (D). A radiative heat flux from the ambient $\dot{q}''_{\text{amb}}$ is uniformly falling on the dome.



Given parameters:

- Temperature of the dome: T_{D}
- Surfaces of the plate and dome: $A_{\text{P}},\ A_{\text{D}}$
- Radiative heat flux: $\dot{q}''_{\text{amb}}$
- View factor: Φ_{DP}
- Absorptivity of the plate: $\alpha_{\text{P}} = 1$
- Reflectivity of the dome: $\rho_{\text{D}} = 0$
- Transmissivity of the dome: τ_{D}
- Emissivity of the dome: ϵ_{D}

Hints:

- Conduction and convection are to be neglected.
- All surfaces are radiating diffusely.

Tasks:

a) Derive an expression for the temperature of the plate T_{P}.

Solution I.3: (Hemispherical dome ⋆⋆)

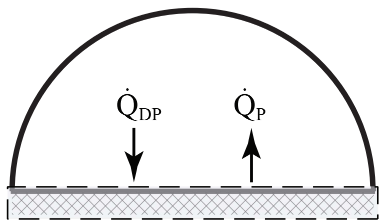
Task a)

The temperature of the plate T_P is determined by setting up an energy balance around the plate. Both an inner and outer energy balance offer solution routes that result in an identical expression.

Outer energy balance:

1 Setting up the balance:

Partially the dome is radiating its surface brightness on the plate. Besides, the plate emits its surface brightness as well.



Therefore the steady-state outer balance reads:

$$0 = \underbrace{\dot{Q}_\mathrm{DP}}_{\substack{\text{S.B. dome}\\ \text{on plate}}} - \underbrace{\dot{Q}_\mathrm{P}}_{\substack{\text{S.B.}\\ \text{plate}}}. \tag{I.3.1}$$

2 Defining the elements within the balance:

The plate acts as a black body radiator and therefore the plate's surface brightness is expressed as:

$$\dot{Q}_\mathrm{P} = \sigma A_\mathrm{P} T_\mathrm{P}^4. \tag{I.3.2}$$

The radiative transport from the dome to the plate is found from the respective view factor and the surface brightness of the dome:

$$\dot{Q}_\mathrm{DP} = \Phi_\mathrm{DP}\dot{Q}_\mathrm{D}. \tag{I.3.3}$$

The dome emits radiation as a grey body, but the dome also transmits some of the ambient radiation. Therefore the surface brightness of the dome's inner surface is expressed as:

$$\dot{Q}_\mathrm{D} = \dot{Q}_{\mathrm{D},\epsilon} + \cancelto{\rho_\mathrm{D} = 0}{\dot{Q}_{\mathrm{D},\rho}} + \dot{Q}_{\mathrm{D},\tau}, \tag{I.3.4}$$

where the emission term is defined as:

$$\dot{Q}_{\mathrm{D},\epsilon} = \epsilon_\mathrm{D}\sigma A_\mathrm{D} T_\mathrm{D}^4, \tag{I.3.5}$$

and the transmission term:

$$\dot{Q}_{\mathrm{D},\tau} = \tau_\mathrm{D}\dot{q}''_\mathrm{amb} A_\mathrm{D}. \tag{I.3.6}$$

Conclusion

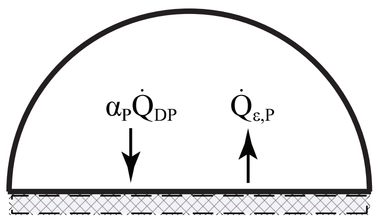
3 Inserting and rearranging:

$$T_\mathrm{P} = \sqrt[4]{\Phi_\mathrm{DP}\epsilon_\mathrm{D}\frac{A_\mathrm{D}}{A_\mathrm{P}}T_\mathrm{D}^4 + \Phi_\mathrm{DP}\tau_\mathrm{D}\dot{q}''_\mathrm{amb}\frac{A_\mathrm{D}}{\sigma A_\mathrm{P}}} \tag{I.3.7}$$

Inner energy balance:

1 Setting up the balance:

Partially the dome is radiating its surface brightness on the plate. Besides, the plate emits its surface brightness as well.



Therefore the steady-state inner balance reads:

$$0 = \underbrace{\alpha_\mathrm{P}\dot{Q}_\mathrm{DP}}_{\substack{\text{Absorbed S.B.}\\ \text{dome by plate}}} - \underbrace{\dot{Q}_{\epsilon,\mathrm{P}}}_{\substack{\text{Emission}\\ \text{plate}}}. \tag{I.3.8}$$

2 Defining the elements within the balance:

The radiation emitted by the plate is described as the radiation emitted by a black body:

$$\dot{Q}_{\epsilon,\mathrm{P}} = \sigma A_\mathrm{P} T_\mathrm{P}^4.$$

First, it is given that the absorptivity of the plate equals 1:

$$\alpha_\mathrm{P} = 1 \;\; (-) \tag{I.3.9}$$

The radiative transport from the dome to the plate yields from the respective view factor and the surface brightness of the dome:

$$\dot{Q}_\mathrm{DP} = \Phi_\mathrm{DP}\dot{Q}_\mathrm{D}. \tag{I.3.10}$$

The dome emits radiation as a grey body, but it also transmits some of the ambient radiation. Therefore the surface brightness of the dome's inner surface is expressed as:

$$\dot{Q}_\mathrm{D} = \dot{Q}_{\mathrm{D},\epsilon} + \cancelto{\rho_\mathrm{D} = 0}{\dot{Q}_{\mathrm{D},\rho}} + \dot{Q}_{\mathrm{D},\tau}, \tag{I.3.11}$$

where the emission term is defined as:

$$\dot{Q}_{\mathrm{D},\epsilon} = \epsilon_\mathrm{D}\sigma A_\mathrm{D} T_\mathrm{D}^4, \tag{I.3.12}$$

and the transmission term:

$$\dot{Q}_{\mathrm{D},\tau} = \tau_\mathrm{D}\dot{q}''_\mathrm{amb} A_\mathrm{D}. \tag{I.3.13}$$

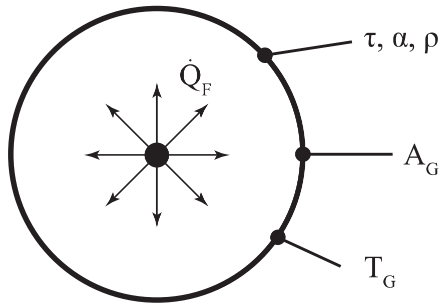
Conclusion

3 Inserting and rearranging:

$$T_{\mathrm{P}} = \sqrt[4]{\Phi_{\mathrm{DP}}\epsilon_{\mathrm{D}}\frac{A_{\mathrm{D}}}{A_{\mathrm{P}}}T_{\mathrm{D}}^{4} + \Phi_{\mathrm{DP}}\tau_{\mathrm{D}}\dot{q}''_{\mathrm{amb}}\frac{A_{\mathrm{D}}}{\sigma A_{\mathrm{P}}}}. \tag{I.3.14}$$

Exercise I.4: (Light bulb ⋆⋆)

The filament of a light bulb emits diffuse radiation $\dot{Q}_{\mathrm{F}}$. The glass of the bulb is thin, spherical, and acts as a gray body. The surface of the filament is small in comparison to the glass body and the problem is steady in time.



Given parameters:

- Power consumption of the filament: $\dot{Q}_{\mathrm{F}}$
- Glass properties: τ, α, ρ
- Surface of the glass sphere: A_{G}

Hints:

- The surface of the filament in comparison to the glass body is small.

Tasks:

a) Provide the energy balance in terms of given variables for determining the glass temperature T_{G}, while neglecting radiation from the environment.

Solution I.4: (Light bulb ★★)

Task a)

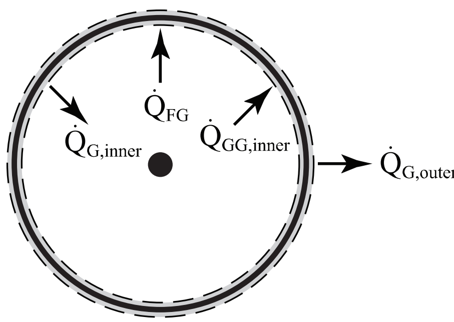
A possible way of deriving the glass temperature T_G is by setting up the outer or inner energy balance for the glass bulb.

Outer energy balance:

1 **Setting up the balance:**

Inside the glass bulb, both the filament and the inner surface of the glass bulb emit their surface brightness onto the glass. Some of this radiation is reflected by the glass bulb. Finally, the inner surface emits radiation as a grey body radiator.

On the exterior, a portion of the radiated surface brightness from the inner surface of the glass bulb and the filament is transmitted. Additionally, the outer surface of the glass bulb emits radiation as a grey body radiator.



With this given, the outer energy balance for the glass bulb is:

$$0 = \underbrace{\dot{Q}_{FG}}_{\text{S.B. filament on glass}} + \underbrace{\dot{Q}_{GG,inner}}_{\text{S.B. inner glass on glass}} - \underbrace{\dot{Q}_{G,inner}}_{\text{S.B. inner glass}} - \underbrace{\dot{Q}_{G,outer}}_{\text{S.B. outer glass}} . \tag{I.4.1}$$

2 **Defining the elements within the balance:**

The radiative transport from the filament to the glass bulb yields from the respective view factor and the surface brightness of the filament:

$$\dot{Q}_{FG} = \Phi_{FG}\dot{Q}_F . \tag{I.4.2}$$

Since the surface of the filament in comparison to the glass body is small, the view factor of the glass of the filament is negligible, thus:

$$\Phi_{FG} = 1 \ (-) , \tag{I.4.3}$$

and

$$\Phi_{GG,inner} = 1 \ (-) . \tag{I.4.4}$$

The surface brightness by the inner surface of the glass bulb on the glass bulb itself is written as:

$$\dot{Q}_{GG,inner} = \Phi_{GG,inner}\dot{Q}_{G,inner} . \tag{I.4.5}$$

The surface brightness of the inside of the glass bulb is:

$$\dot{Q}_{\mathrm{G,inner}} = \dot{Q}_{\epsilon,\mathrm{G,inner}} + \dot{Q}_{\rho,\mathrm{G,inner}} + \dot{Q}_{\tau,\mathrm{G,inner}}. \tag{I.4.6}$$

The emission term is stated as:

$$\dot{Q}_{\epsilon,\mathrm{G,inner}} = \epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}}. \tag{I.4.7}$$

The reflection term is derived from the reflected surface brightness of the filament and the inner surface of the glass bulb:

$$\dot{Q}_{\rho,\mathrm{G,inner}} = \rho\left(\dot{Q}_{\mathrm{FG}} + \dot{Q}_{\mathrm{G,inner}}\right). \tag{I.4.8}$$

Lastly, since no radiation from the outside is transmitted, the transmission term of the inner surface is equal to zero:

$$\dot{Q}_{\tau,\mathrm{G,inner}} = 0. \tag{I.4.9}$$

Plugging in equations (I.4.7) - (I.4.9) into the definition of the surface brightness given in equation (I.4.6):

$$\dot{Q}_{\mathrm{G,inner}} = \frac{\epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho}. \tag{I.4.10}$$

Furthermore, the surface brightness of the outer surface of the glass bulb is described as:

$$\dot{Q}_{\mathrm{G,outer}} = \dot{Q}_{\epsilon,\mathrm{G,outer}} + \dot{Q}_{\rho,\mathrm{G,outer}} + \dot{Q}_{\tau,\mathrm{G,outer}}, \tag{I.4.11}$$

where the emission term is described as:

$$\dot{Q}_{\epsilon,\mathrm{G,outer}} = \epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}}, \tag{I.4.12}$$

since no radiation from the ambient is received from the ambient, the outer surface does not reflect any radiation:

$$\dot{Q}_{\rho,\mathrm{G,outer}} = 0, \tag{I.4.13}$$

and the transmitted radiation arises from fractions of the surface brightness of the filament and the inner surface of the glass that passes through the outer surface:

$$\dot{Q}_{\tau,\mathrm{G,outer}} = \tau\left(\dot{Q}_{\mathrm{FG}} + \dot{Q}_{\mathrm{GG,inner}}\right). \tag{I.4.14}$$

Conclusion

3 Inserting and rearranging:

$$0 = \dot{Q}_{\mathrm{F}} - \epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}} - \tau\left(\dot{Q}_{\mathrm{F}} + \frac{\epsilon\sigma T_{\mathrm{G}}^{4} A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho}\right), \tag{I.4.15}$$

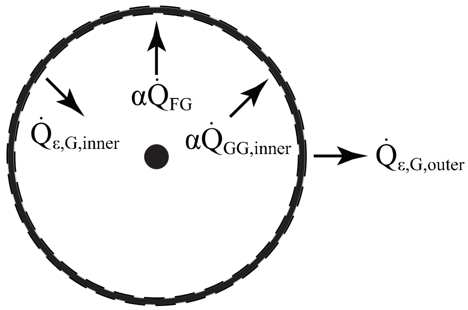
where T_{G} is the only unknown parameter and thus can be determined from this energy balance.

Inner energy balance:

1 Setting up the balance:

Inside the glass bulb, both the filament and the inner surface of the glass bulb emit radiation onto the glass that is partially absorbed.

On the exterior, also radiation is emitted by the outer surface of the glass.



The inner energy balance reads:

$$0 = \underbrace{\alpha\dot{Q}_{\mathrm{FG}}}_{\substack{\text{absorbed S.B.}\\ \text{filament on glass}}} + \underbrace{\alpha\dot{Q}_{\mathrm{G,inner}}}_{\substack{\text{absorbed S.B. inner}\\ \text{glass on glass}}} - \underbrace{\dot{Q}_{\epsilon,\mathrm{G,inner}}}_{\substack{\text{Emission}\\ \text{inner glass}}} - \underbrace{\dot{Q}_{\epsilon,\mathrm{G,outer}}}_{\substack{\text{Emission}\\ \text{outer glass}}}. \tag{I.4.16}$$

2 Defining the elements within the balance:

The radiative transport from the filament to the glass bulb is found from the respective view factor and the surface brightness of the filament:

$$\dot{Q}_{\mathrm{FG}} = \Phi_{\mathrm{FG}}\dot{Q}_{\mathrm{F}}. \tag{I.4.17}$$

Since the surface of the filament in comparison to the glass body is small, the view factor of the glass on the filament is negligible, thus:

$$\Phi_{\mathrm{FG}} = 1 \ \ (-)\,, \tag{I.4.18}$$

and

$$\Phi_{\mathrm{GG,inner}} = 1 \ \ (-)\,. \tag{I.4.19}$$

The emitted radiation by the in- and outside of the glass bulb is defined as:

$$\dot{Q}_{\epsilon,\mathrm{G,inner}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}}, \tag{I.4.20}$$

and:

$$\dot{Q}_{\epsilon,\mathrm{G,outer}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}}. \tag{I.4.21}$$

Kichoff's law states that:

$$\epsilon = \alpha. \tag{I.4.22}$$

The surface brightness of the inside of the glass bulb is given as:

$$\dot{Q}_{\mathrm{G,inner}} = \dot{Q}_{\epsilon,\mathrm{G,inner}} + \dot{Q}_{\rho,\mathrm{G,inner}} + \dot{Q}_{\tau,\mathrm{G,inner}}. \tag{I.4.23}$$

The emission term is written as:

$$\dot{Q}_{\epsilon,\mathrm{G,inner}} = \epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}}. \tag{I.4.24}$$

The reflection term yields from the reflected surface brightness of the filament and the inner surface of the glass bulb:

$$\dot{Q}_{\rho,\mathrm{G,inner}} = \rho\left(\dot{Q}_{\mathrm{FG}} + \dot{Q}_{\mathrm{G,inner}}\right). \tag{I.4.25}$$

Lastly, since no radiation from the outside is transmitted, the transmission term of the inner surface is equal to zero:

$$\dot{Q}_{\tau,\mathrm{G,inner}} = 0. \tag{I.4.26}$$

Plugging in equations (I.4.24) - (I.4.26) into the definition of the surface brightness given in equation (I.4.23):

$$\dot{Q}_{\mathrm{G,inner}} = \frac{\epsilon\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \rho\dot{Q}_{\mathrm{F}}}{1-\rho}. \tag{I.4.27}$$

Conclusion

3 **Inserting and rearranging:**

$$\epsilon\dot{Q}_{\mathrm{F}} + \frac{\epsilon^2\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} + \epsilon\rho\dot{Q}_{\mathrm{F}}}{1-\rho} - 2\sigma T_{\mathrm{G}}^4 A_{\mathrm{G}} = 0, \tag{I.4.28}$$

where T_{G} is the only unknown parameter and thus can be determined from this energy balance.