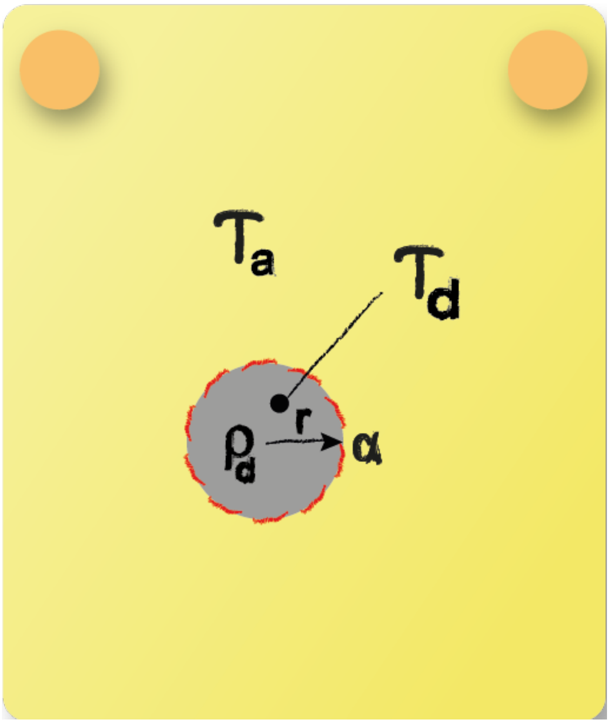
Energy Balance: Task 19



Choose an equation for the temporal derivative of the droplet's radius while it is evaporating in hot air.

To approach the problem it is suitable to set up an energy balance at the droplet's surface. There is an ingoing convective heat flux, that is the heat transfer coefficient times surface area and temperature difference as well as an outgoing energy flux consisting of the evaporated fluid mass times the specific latent heat:

$$0 = A\,\alpha\,(T_\mathrm{d} - T_\mathrm{a}) - \dot{m}\,\Delta h_\mathrm{F}$$

1 In order to obtain an expression for the temporal derivative of the droplet's radius the mass flux is expressed as the temporal change of the droplet's mass:

$$\dot{m} = -\frac{\partial m}{\partial t} = -\rho_\mathrm{d}\frac{\partial V}{\partial t} = -\rho_\mathrm{d}\frac{\partial V}{\partial r}\frac{\partial r}{\partial t}$$

$$= -\rho_\mathrm{d}\left(\tfrac{4}{3}\pi r^3\right)\frac{\partial}{\partial r}\frac{\partial r}{\partial t} = -\rho_\mathrm{d}\,4\pi r^2\,\frac{\partial r}{\partial t} = -\rho_\mathrm{d} A\frac{\partial r}{\partial t}$$

The surface area cancels out such that the final expression for the temporal derivative of the radius is given as:

$$\frac{\partial r}{\partial t} = \frac{\alpha}{\rho_\mathrm{d}\,\Delta h_\mathrm{V}}(T_\mathrm{d} - T_\mathrm{a})$$