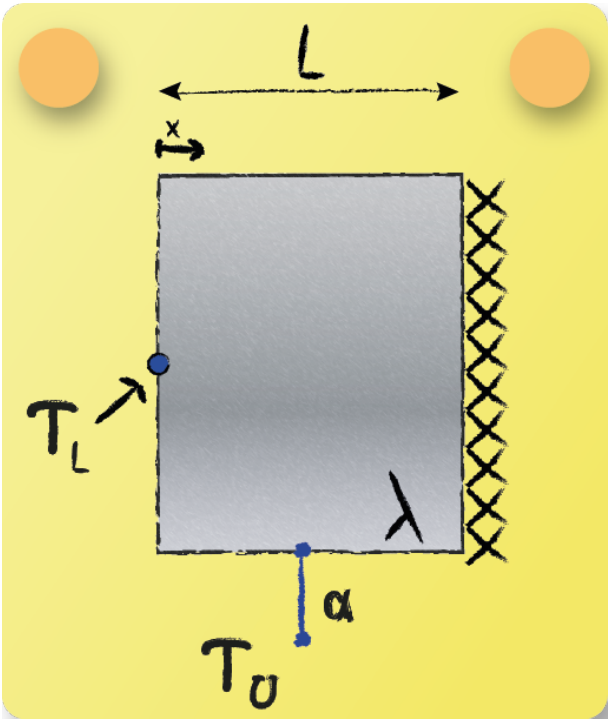
Energy Balance: Task 7



Choose the differential equation that describes the steady temperature profile in x-direction for $\mathrm{Bi} \ll 1$.

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The condition $\mathrm{Bi} \ll 1$ states that thermal resistance of convective heat transfer is orders of magnitude greater than conductive thermal resistance. This yields to the assumption that in vertical direction temperature distribution is treated as uniform within the solid and only depending on x. For an energy balance of an infinitesimal slice in x-direction one obtains equation:

$$0 = -A\lambda\frac{\partial T}{\partial x} + A\lambda(\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2}dx) - \alpha U(T - T_\mathrm{u})dx$$

Canceling out recurring terms and replacing the temperature difference $T - T_\mathrm{u} = \Theta$ one obtains the differential equation for a fin:

$$0 = \frac{\partial^2 \Theta}{\partial x^2} - \frac{\alpha U}{\lambda A_\mathrm{q}}\Theta$$