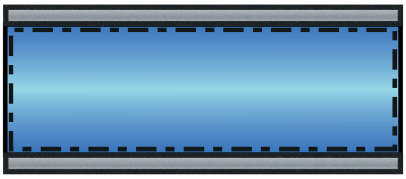
Control Volume - Conv. - Body 1

Water flows through a pipe with an average velocity u, inlet temperature T_1 and a constant wall temperature T_{w}. Assume steady-state conditions.
Pick the suitable control volume for determining the outlet's mean temperature T_2.



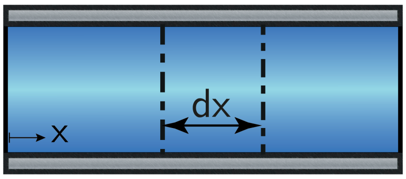
Defining the domain:

The outlet temperature needs to be determined. As we are dealing with steady-state conditions, the outlet temperature can be determined from a global energy balance having the in- and outlet at the boundaries of the domain. Therefore a global energy balance around the entire body is suitable for determining the outlet's mean temperature T_2.

Control Volume - Conv. - I.E. 1

Through a very long pipe with diameter D flows a heat-generating fluid (homogeneous and constant source strength $\dot{\Phi}''' > 0$). In addition, the pipe has a uniform, constant wall temperature $T_\mathrm{w} < T(x)$.

Pick the correct control volume for setting up the energy balance to calculate the temperature profile of the fluid in the flow direction. If needed, also define the coordinate system.



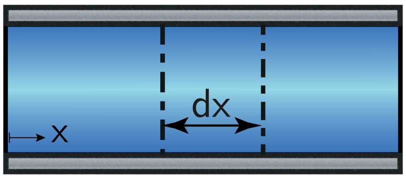
Defining the domain:

The temperature profile in the flow direction needs to be determined. The temperature profile can be derived from setting up the energy balance of an infinitesimal element in the flow direction and defining its boundary and/or initial conditions. Therefore an energy balance around an infinitesimal element with the flow-direction being parallel to the x-direction is suitable.

Control Volume - Conv. - I.E. 2

A fluid flows through a long cylindrical tube. A constant heat flux density $\dot{q}''$ is imposed on the fluid.

Pick the correct control volume for setting up the energy balance to calculate the temperature profile of the fluid in the flow direction. If needed, also define the coordinate system.



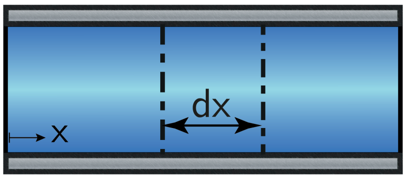
Defining the domain:

The temperature profile in the flow direction needs to be determined. The temperature profile can be derived from setting up the energy balance of an infinitesimal element in the flow direction and defining its boundary and/or initial conditions. Therefore an energy balance around an infinitesimal element with the flow direction being parallel to the x-direction is suitable.

Control Volume - Conv. - I.E. 3

Through a very long pipe with diameter D flows a fluid. In addition, the pipe has a uniform, constant wall temperature $T_\mathrm{w} < T(x)$.

Pick the correct control volume for setting up the energy balance to calculate the temperature profile of the fluid in the flow direction. If needed, also define the coordinate system.



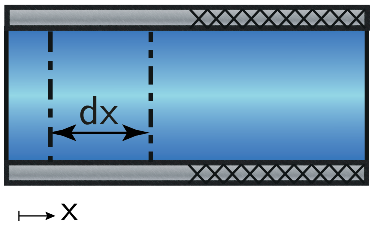
Defining the domain:

The temperature profile in the flow direction needs to be determined. The temperature profile can be derived from setting up the energy balance of an infinitesimal element in the flow direction and defining its boundary and/or initial conditions. Therefore an energy balance around an infinitesimal element with the flow-direction being parallel to the x-direction is suitable.

Control Volume - Conv. - I.E. 4

Through a very long pipe with diameter D flows a fluid. The first half of the pipe is being heated with a constant rate $\dot{q}''$. The second half of the pipe is fully adiabatic.

Pick the correct control volume for setting up the energy balance to calculate the temperature profile of the fluid in the flow direction in the first segment of the pipe. If needed, also define the coordinate system.



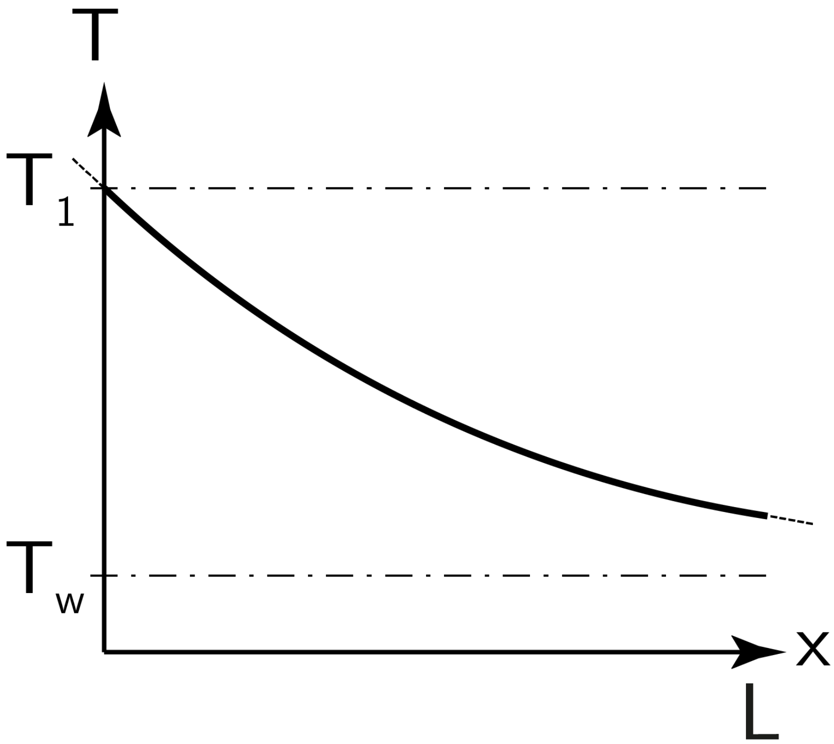
Defining the domain:

The temperature profile in the flow direction needs to be determined for the segment of the pipe. The temperature profile can be derived from setting up the energy balance of an infinitesimal element in the flow direction in the first segment of the pipe and defining its boundary and/or initial conditions. Therefore an energy balance around the drawn infinitesimal element with the flow direction being parallel to the x-direction is suitable.

Temperature Profile - Conv. - IE 3

Through a very long pipe with a diameter D flows a fluid. In addition, the pipe has a uniform, constant wall temperature T_w.

Sketch the profile of the average fluid temperature $T(x)$ in the flow direction.



The fluid enters the pipe with a temperature T_1.

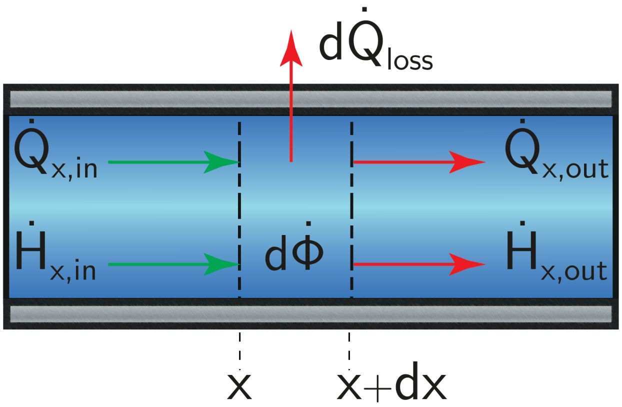
When moving in the direction of the flow, the difference between fluid and wall temperature becomes smaller, and so does the rate of heat loss. Therefore the slope of the profile decreases when moving in the direction of the flow.

This will continue until the temperature approaches T_w with a horizontal slope, but in the given example this does not happen as the pipe is not long enough. Therefore this is the only suitable solution.

nBC - Conv. - IE 1

Through a very long pipe with diameter D flows a heat-generating fluid (homogeneous and constant source strength $\dot{\Phi}''' > 0$). In addition, the pipe has a uniform, constant wall temperature T_w

How many conditions are required to solve its differential equation and find an expression for the temperature profile in the flow direction?



Given the differential equation:

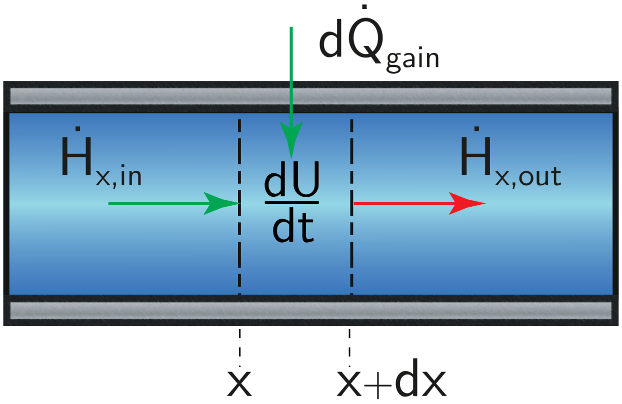
$$0 = \frac{\lambda \pi D^2}{4} \frac{\partial^2 T}{\partial x^2} - \frac{u \rho c \pi D^2}{4} \frac{\partial T}{\partial x} - \alpha \pi D (T(x) - T_\mathrm{w}) + \frac{\pi D^2}{4} \dot{\Phi}'''$$

In order to solve the differential equation, two boundary conditions are required. This can be seen from the fact that the variable T has been differentiated twice with respect to x.

nBC - Conv. - IE 2

A fluid flows through a long cylindrical tube. A constant heat flux density $\dot{q}''$ is imposed on the fluid.

How many conditions are required to solve its differential equation and find an expression for the temperature profile in the flow direction?



Given the differential equation:

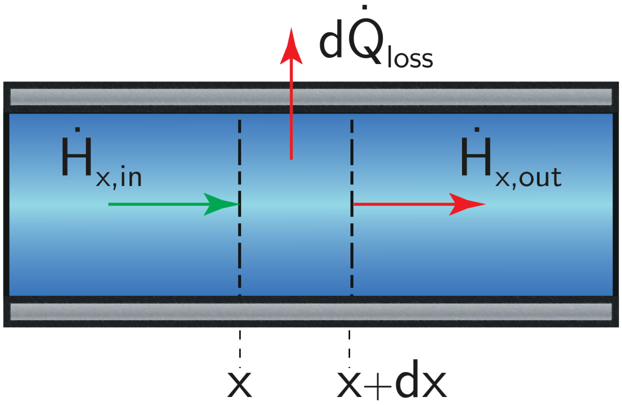
$$\frac{\rho c \pi D^2}{4}\frac{\partial T}{\partial t} = -\frac{u \rho c \pi D^2}{4}\frac{\partial T}{\partial x} + \dot{q}''\pi D$$

In order to solve the differential equation, one boundary condition and one initial condition are required. This can be seen from the fact that the variable T has been differentiated once with respect to x and once with respect to t.

nBC - Conv. - IE 3

Through a very long pipe with diameter D flows a fluid. In addition, the pipe has a uniform, constant wall temperature T_{w}.

How many conditions are required to solve its differential equation and find an expression for the temperature profile in the flow direction?



Given the differential equation:

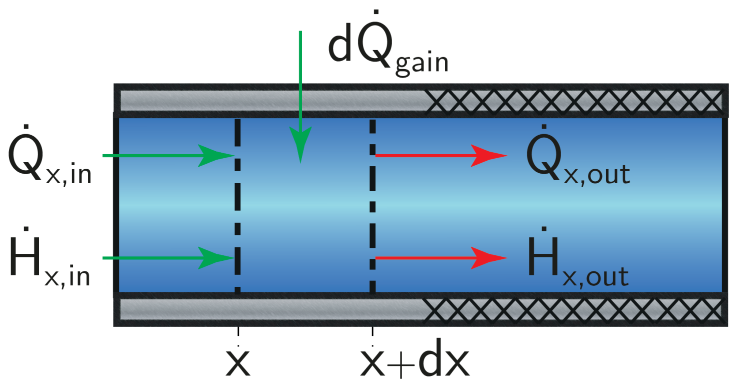
$$0 = \lambda \frac{\partial^2 T}{\partial x^2}$$

In order to solve the differential equation, one boundary condition is required. This can be seen from the fact that the variable T has been differentiated once with respect to x.

nBC - Conv. - IE 4

Through a very long pipe with diameter D flows a fluid. The first half of the pipe is being heated with a constant rate $\dot{q}''$. The second half of the pipe is fully adiabatic.

How many conditions are required to solve its differential equation and find an expression for the temperature profile in the flow direction?



Given the differential equation:

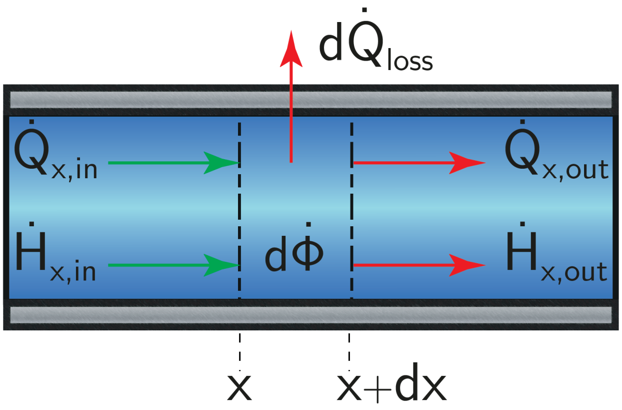
$$0 = \frac{\lambda \pi D^2}{4} \frac{\partial^2 T}{\partial x^2} - \frac{u \rho c \pi D^2}{4} \frac{\partial T}{\partial x} + \dot{q}'' \pi D$$

In order to solve the differential equation, two boundary conditions are required, where one of them should be a coupling condition for both segments. This can be seen from the fact that the variable T has been differentiated twice with respect to x.

Boundary Conditions - Conv. - IE 1

Through a very long pipe with diameter D flows a heat-generating fluid (homogeneous and constant source strength $\dot{\Phi}''' > 0$). In addition, the pipe has a uniform, constant wall temperature T_w

Give the correct conditions to solve the given differential equation for deriving the temperature profile in the flow direction.



Given the differential equation:

$$0 = \frac{\lambda \pi D^2}{4} \frac{\partial^2 T}{\partial x^2} - \frac{u \rho c \pi D^2}{4} \frac{\partial T}{\partial x} - \alpha \pi D (T(x) - T_\mathrm{w}) + \frac{\pi D^2}{4} \dot{\Phi}'''$$

In order to solve the differential equation, two boundary conditions are required. This can be seen from the fact that the variable T has been differentiated twice with respect to x.

Boundary conditions:

$$T(x = 0) = T_1$$

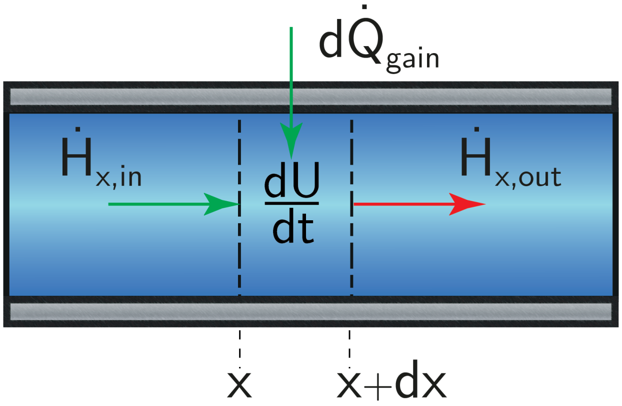
$$T(x = L) = T_2$$

The first boundary condition describes that the temperature of the fluid equals T_1 at the entrance of the pipe and the second one states that the temperature of the fluid equals T_2 at the exit of the pipe, as can be seen from the figure.

Boundary Conditions - Conv. - IE 2

A fluid flows through a long cylindrical tube. A constant heat flux density $\dot{q}''$ is imposed on the fluid. Initially, before heating, the pipe is at a uniform temperature T_1. During the process, the fluid always enters the pipe at a temperature of T_1.

Give the correct conditions to solve the given differential equation for deriving the temperature profile in the flow direction.



Given the differential equation:

$$\frac{\rho c \pi D^2}{4}\frac{\partial T}{\partial t} = -\frac{u \rho c \pi D^2}{4}\frac{\partial T}{\partial x} + \dot{q}''\pi D$$

In order to solve the differential equation, one boundary condition and one initial condition are required. This can be seen from the fact that the variable T has been differentiated once with respect to x and once with respect to t.

Boundary and initial conditions:

$$T(x=0) \;=\; T_1$$

The boundary condition above describes that the temperature of the fluid equals T_1 at the entrance of the pipe, as can be seen from the figure.

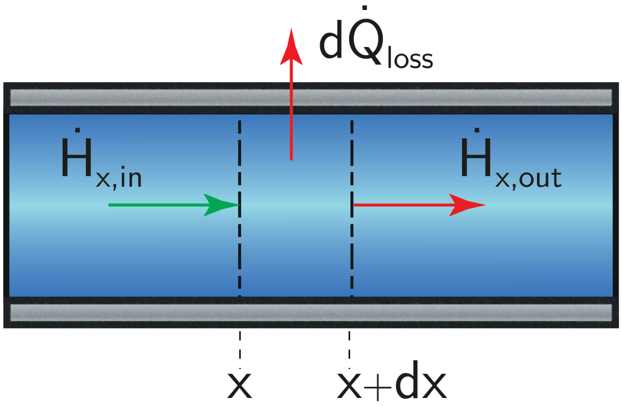
$$T(t=0) \;=\; T_1$$

The initial condition above describes that the temperature of the fluid is uniform for the entire domain T_1 at $t = 0$.

Boundary Conditions - Conv. - IE 3

Through a very long pipe with diameter D flows a fluid. In addition, the pipe has a uniform, constant wall temperature T_w.

Give the correct boundary conditions to solve the given differential equation for deriving the temperature profile in the flow direction.



Given the differential equation:

$$0 = \lambda \frac{\partial^2 T}{\partial x^2}$$

In order to solve the differential equation, one boundary condition is required. This can be seen from the fact that the variable T has been differentiated once with respect to x.

Boundary conditions:

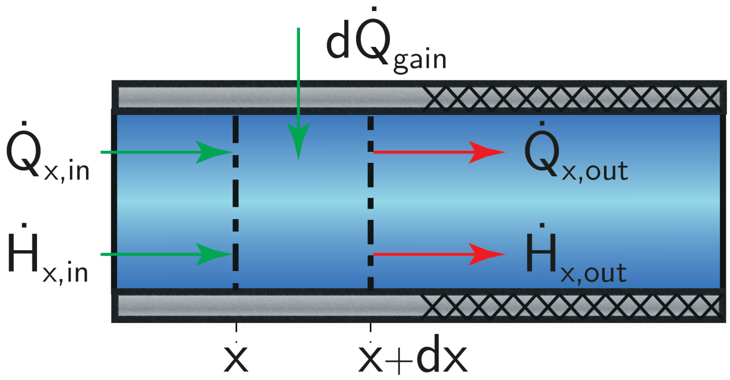
$$T(x=0) \;=\; T_1$$

The boundary condition above describes that the temperature of the fluid equals T_1 at the entrance of the pipe, as can be seen from the figure.

Boundary Conditions - Conv. - IE 4

Through a very long pipe with diameter D flows a fluid. The first half of the pipe is being heated with a constant rate $\dot{q}''$. The second half of the pipe is fully adiabatic.

Give the correct boundary and coupling conditions to solve the given differential equation for deriving the temperature profile in the flow direction of the first segment of the pipe $T_\mathrm{I}(x)$.



Given the differential equation:

$$0 = \frac{\lambda\pi D^2}{4}\frac{\partial^2 T}{\partial x^2} - \frac{u\rho c\pi D^2}{4}\frac{\partial T}{\partial x} + \dot{q}''\pi D$$

In order to solve the differential equation, two boundary conditions are required. This can be seen from the fact that the variable T has been differentiated twice with respect to x.

Boundary conditions:

$$T_\mathrm{I}(x=0) \ = \ T_1$$

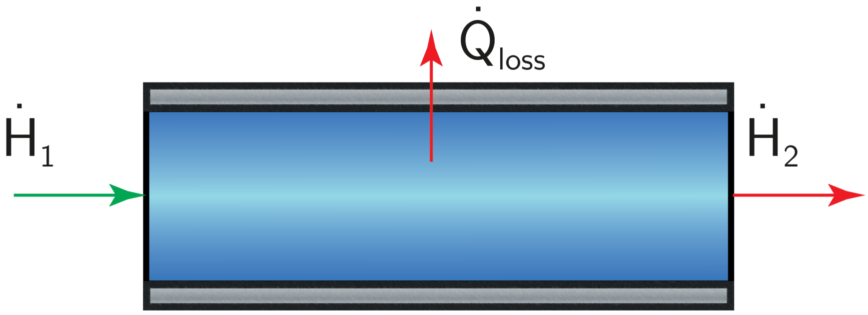
The boundary condition above describes that the temperature of the fluid equals T_1 at the entrance of the pipe, as can be seen from the figure.

$$T_\mathrm{I}(x=L/2) \ = \ T_\mathrm{II}(x=L/2)$$

The boundary condition above describes that the temperature of the fluid at $x = L/2$ should be equal for both segments.

EB - Conv. - Body 1

Water flows through a pipe an average velocity u and inlet temperature T_1. Provide the energy balance to determine the water temperature T_2.
Hint: $T_w < T_2 < T_1$



Energy balance:

$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

Definition of fluxes:

Enthalpies entering and leaving:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

Convective heat losses:

$$\dot{Q}_{\text{loss}} = \alpha \cdot \pi \cdot D \cdot L \cdot \Delta T$$

Logarithmic mean temperature difference:

$$\Delta T = \frac{\dot{m} \cdot c}{\alpha \cdot \pi \cdot D \cdot L} (T_1 - T_2) = \frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

Mass flow rate:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

Substituting and rewriting:

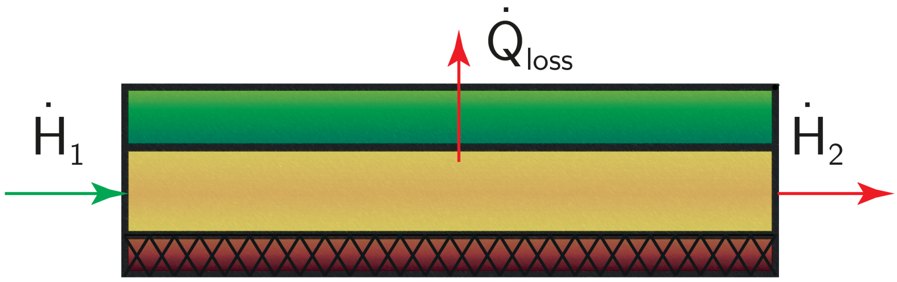
$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

$$u \frac{\pi D^2}{4} \rho c T_1 - u \frac{\pi D^2}{4} \rho c T_2 - \alpha \pi D L \frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

$$\Rightarrow 0 = \frac{u \rho c \pi D^2}{4} (T_1 - T_2) - \alpha \pi D L \frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

EB - Conv. - Body 2

A project consists to heat up the grass layer (width W, length L) by pumping warm water through a porous membrane. Derive an energy balance to calculate the exit temperature T_2.
Hint:
The overall heat transfer coefficient k fulfils a similar function as the convection heat transfer coefficient α.



Energy balance:

$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

Energy fluxes:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

$$\dot{Q}_{\text{loss}} = k \cdot W \cdot L \cdot \Delta T$$

Logarithmic mean temperature difference:

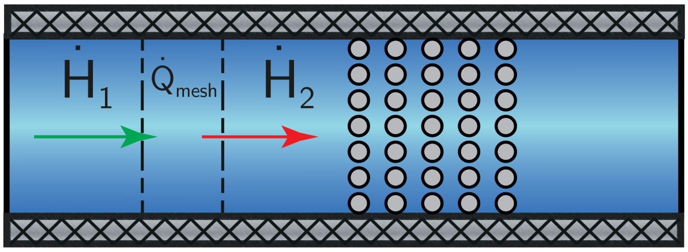
$$\Delta T = \frac{\dot{m} \cdot c}{k \cdot W \cdot L} (T_1 - T_2) = \frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

Mass flow rate:

$$\dot{m} = u \cdot H \cdot W \cdot \rho$$

EB - Conv. - Body 3

Water flows through a rectangular duct (height H, width W) with a velocity u and inlet temperature T_1. The water is heated with a wire mesh (heating power per surface $\dot{q}''$), located at the beginning of the duct. Behind the mesh a heat exchanger in a pipe bundle configuration is mounted perpendicularly to the flow. Provide the energy balance to determine the water temperature T_2 directly after the wire mesh.



Energy balance:

$$\dot{H}_1 - \dot{H}_2 + \dot{Q}_{\text{mesh}} = 0$$

Energy fluxes:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

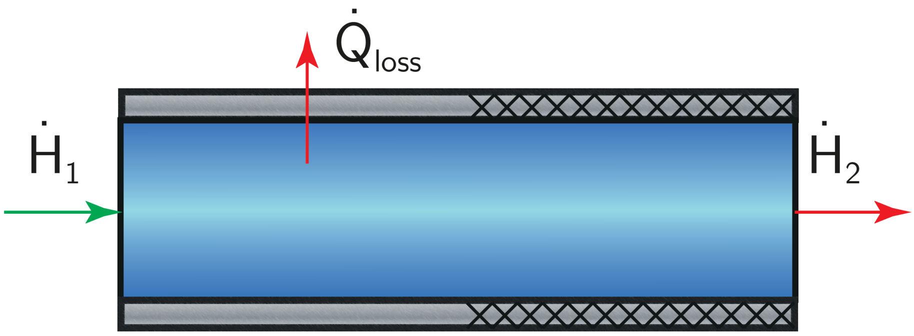
$$\dot{Q}_{\text{mesh}} = H \cdot W \cdot \dot{q}''$$

Mass flow rate:

$$\dot{m} = u \cdot H \cdot W \cdot \rho$$

EB - Conv. - Body 4

Water flows through a pipe an average velocity u and inlet temperature T_1. Provide the energy balance to determine the water temperature T_2.
Hint: $T_w < T_2 < T_1$



Energy balance:

$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

Energy fluxes:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

$$\dot{Q}_{\text{loss}} = \tfrac{1}{2} \cdot \alpha \cdot \pi D \cdot L \cdot \Delta T$$

Logarithmic mean temperature difference:

$$\Delta T = \tfrac{2 \cdot \dot{m} \cdot c}{\alpha \cdot \pi D \cdot L} \left(T_1 - T_2\right) = \tfrac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

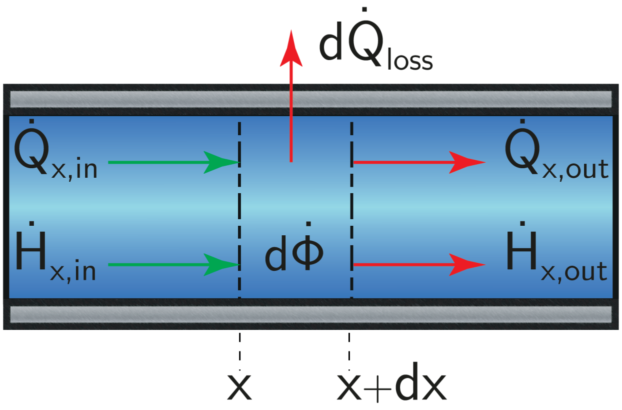
Mass flow rate:

$$\dot{m} = u \cdot \tfrac{\pi \cdot D^2}{4} \cdot \rho$$

EB - Conv. - IE 1

Through a very long pipe with diameter D flows a heat generating fluid (homogeneous and constant source strength $\dot{\Phi}''' > 0$). In addition, the pipe has a uniform, constant wall temperature T_{w}.

Derive the differential equations for the temperature profile in the flow direction, not neglecting the diffusive heat transport in the direction of the flow.



Energy balance:

$$\dot{Q}_{x,in} + \dot{Q}_{x,out} + \dot{H}_{x,in} - \dot{H}_{x,out} - d\dot{Q}_{\mathrm{loss}} + d\dot{\Phi} = 0$$

Energy fluxes:

$$\dot{Q}_{x,in} = -\lambda \cdot \frac{\pi \cdot D^2}{4} \cdot \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx$$

$$\dot{H}_{x,in} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{x,out} = \dot{H}_{x,in} + \frac{\partial \dot{H}_{x,in}}{\partial x} \cdot dx$$

$$d\dot{Q}_{\mathrm{loss}} = \alpha \cdot \pi \cdot D \cdot dx \cdot (T - T_{\mathrm{w}})$$

$$d\dot{\Phi} = \dot{\Phi}''' \cdot \frac{\pi \cdot D^2}{4} \cdot dx$$

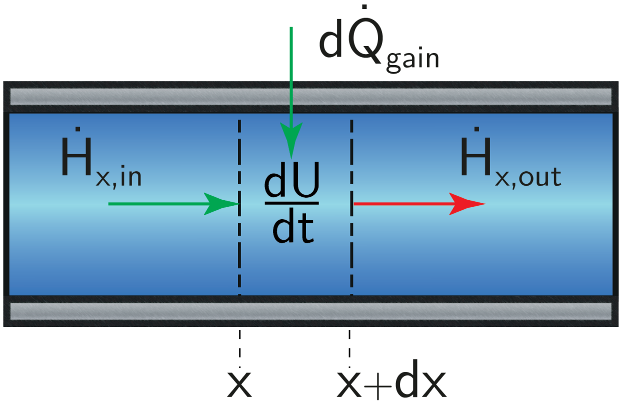
Mass flow rate:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

EB - Conv. - IE 2

A fluid flows through a long cylindrical tube. A constant heat flux density $\dot{q}''$ is imposed on the fluid.

Derive the transient differential energy balance for the averaged temperature in the fluid, using a stationary coordinate system in the x–direction. Axial heat conduction is negligible in this case.



Energy balance:

$$\dot{H}_{x,in} - \dot{H}_{x,out} + d\dot{Q}_{\text{gain}} = \frac{\partial U}{\partial t}$$

Energy fluxes:

$$\dot{H}_{x,in} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{x,out} = \dot{H}_{x,in} + \frac{\partial \dot{H}_{x,in}}{\partial x} \cdot dx$$

$$d\dot{Q}_{\text{gain}} = \dot{q}'' \cdot \pi \cdot D \cdot dx$$

$$\frac{\partial U}{\partial t} = \frac{\pi \cdot D^2}{4} \cdot dx \cdot \rho \cdot c \cdot \frac{\partial T}{\partial t}$$

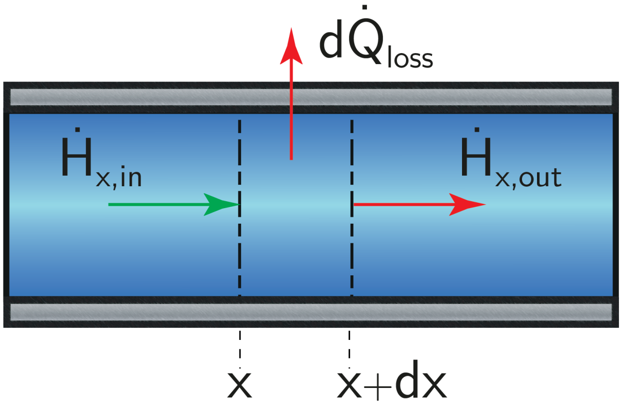
Mass flow rate:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

EB - Conv. - IE 3

Through a very long pipe with diameter D flows a fluid. In addition, the pipe has a uniform, constant wall temperature T_{w}.

Derive the differential equation for the temperature profile in the flow direction, while neglecting the diffusive heat transport in the direction of the flow. Assume steady-state conditions.



Energy balance:

$$0 = \dot{H}_{x,in} - \dot{H}_{x,out} - d\dot{Q}_{\mathrm{loss}}$$

Definition of fluxes:

$$\dot{H}_{x,in} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{x,out} = \dot{H}_{x,in} + \frac{\partial \dot{H}_{x,in}}{\partial x} \cdot dx$$

$$d\dot{Q}_{\mathrm{loss}} = \alpha \cdot \pi \cdot D \cdot dx \cdot (T - T_{\mathrm{w}})$$

Where:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

Substituting and rewriting:

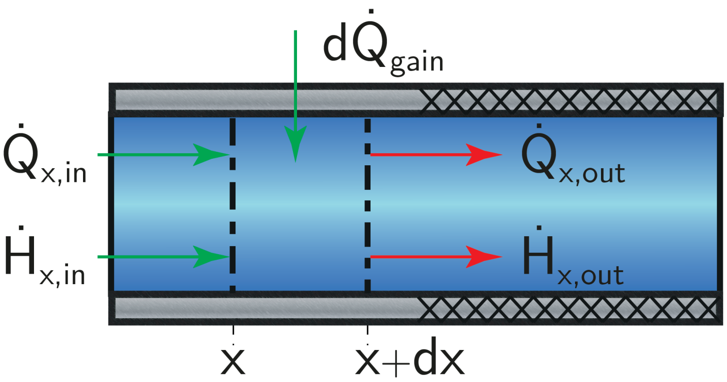
$$0 = -u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho \cdot c \cdot \frac{\partial T}{\partial x} - \alpha \cdot \pi \cdot D \cdot (T - T_{\mathrm{w}})$$

$$\Rightarrow 0 = -u\rho c D \frac{\partial T}{\partial x} - 4\alpha \, (T - T_{\mathrm{w}})$$

EB - Conv. - IE 4

Through a very long pipe with diameter D flows a fluid. The first half of the pipe is being heated with a constant rate $\dot{q}''$. The second half of the pipe is fully adiabatic.

Derive the differential equation for the temperature profile in the flow direction in the first segment of the pipe, while not neglecting the diffusive heat transport in the direction of the flow.



Energy balance:

$$0 = \dot{Q}_{\text{x,in}} + \dot{Q}_{\text{x,out}} + \dot{H}_{\text{x,in}} - \dot{H}_{\text{x,out}} + d\dot{Q}_{\text{gain}}$$

Definition of fluxes:

$$\dot{Q}_{\text{x,in}} = -\lambda \cdot \tfrac{\pi \cdot D^2}{4} \cdot \tfrac{\partial T}{\partial x}$$

$$\dot{Q}_{\text{x,out}} = \dot{Q}_{\text{x,in}} + \tfrac{\partial \dot{Q}_{\text{x,in}}}{\partial x} \cdot dx$$

$$\dot{H}_{\text{x,in}} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{\text{x,out}} = \dot{H}_{\text{x,in}} + \tfrac{\partial \dot{H}_{\text{x,in}}}{\partial x} \cdot dx$$

$$d\dot{Q}_{\text{gain}} = \dot{q}'' \cdot \pi \cdot D \cdot dx$$

Where:

$$\dot{m} = u \cdot \tfrac{\pi \cdot D^2}{4} \cdot \rho$$

Substituting and rewriting:

$$0 = \lambda \cdot \frac{\pi \cdot D^2}{4} \cdot \frac{\partial^2 T}{\partial x^2} \cdot dx - u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho \cdot c \cdot \frac{\partial T}{\partial x} \cdot dx + \dot{q}'' \cdot \pi \cdot D \cdot dx$$

$$0 = \frac{\lambda \pi D^2}{4} \frac{\partial^2 T}{\partial x^2} - \frac{u \rho c \pi D^2}{4} \frac{\partial T}{\partial x} + \dot{q}'' \pi D$$