Heat Transfer

Boundary Layer equations – Natural convection

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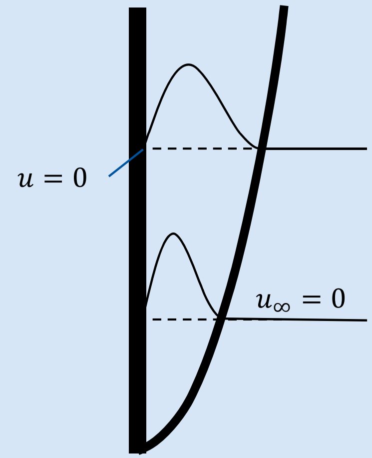
Learning Goals

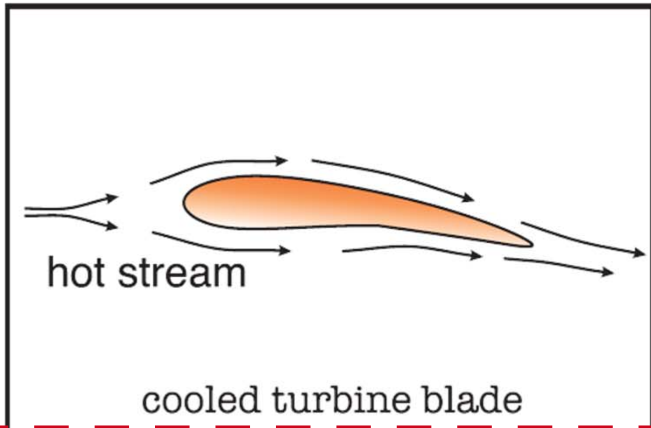
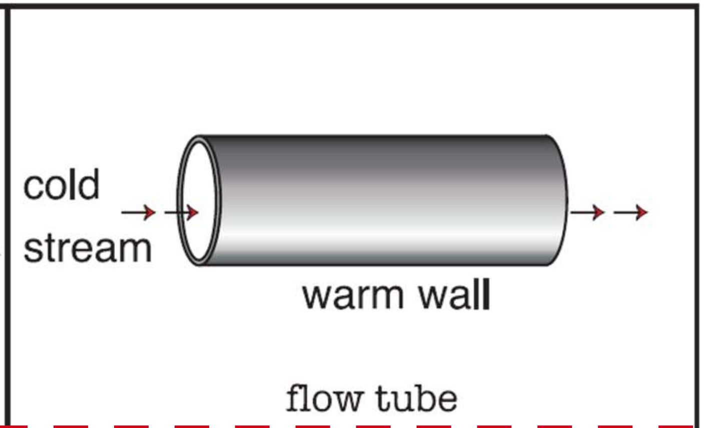
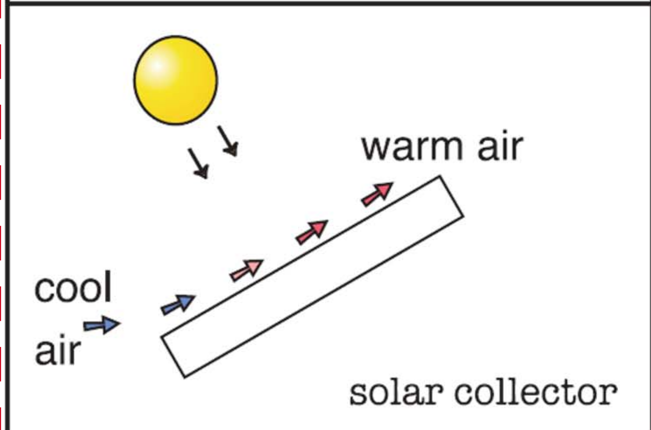
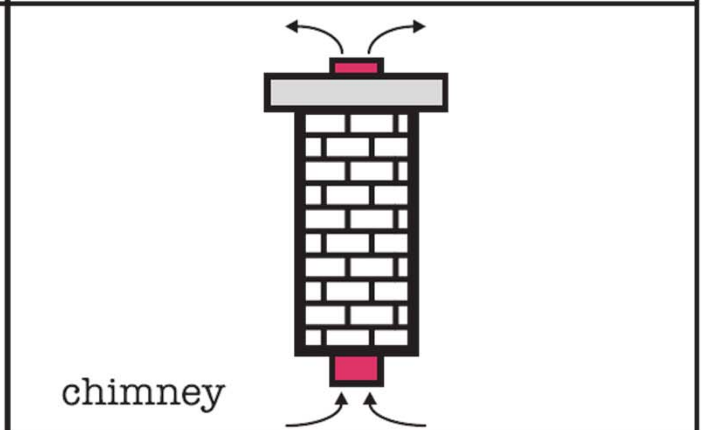
- Boundary Layer in Natural Convection
  - Understanding the Boundary Layer profile (Temperature and Velocity) on a flat plate with natural (free) convection
  - Derivation and meaning of the Grashof number
  - Knowledge of the differences between the Boundary Layer profiles for forced and free convection



(Note: "natural" or "free" convection, both terms are identical and can be used equivalently.)

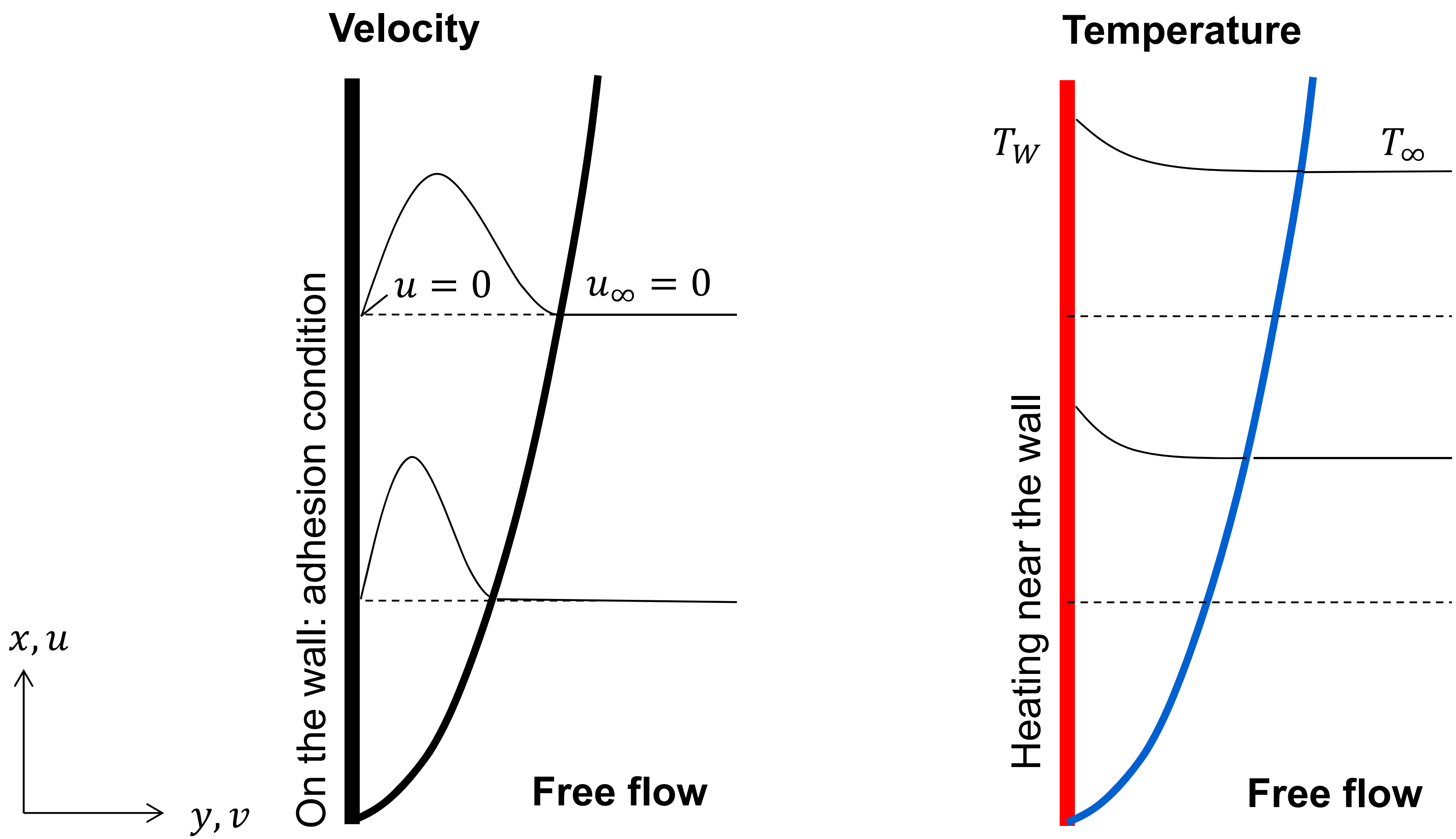
$u = 0$

Classification convection according to flow regime

	External	Internal
Forced Convection • Driven by externally generated movement of the fluid/object		
Free Convection • Inherently driven due to heat transfer (density differences)		

Free Convection

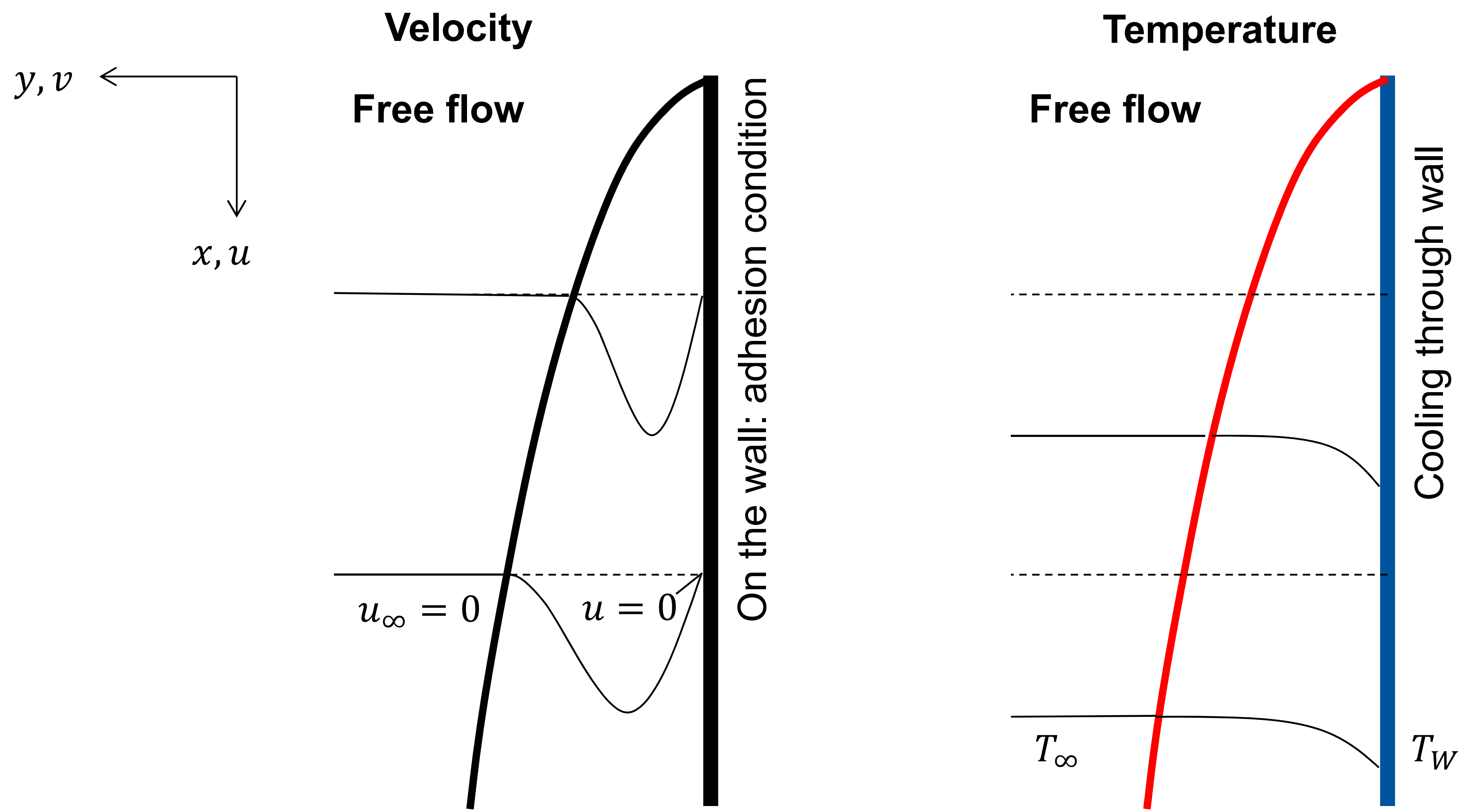
Free Convection: Boundary Layers



Heating near the wall leads to a decrease in density

Natural Convection = Upward flow

Free Convection: Initial Situation



Cooling near the wall leads to an increase in density

Natural Convection = Downward flow

Review: Conservation equations (2D, steady state, incompressible)

Continuity equation

Mass Flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\delta \ll L, \qquad u \gg v \rightarrow \frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$$

Momentum equation

Momentum Flow	Pressure	Shear stresses	Gravity
$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} =$		$\nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right)$	$+\frac{g}{\rho}(\rho_\infty - \rho)$
$\cancel{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}} =$	$-\frac{1}{\rho}\frac{\partial p}{\partial y}$	$\cancel{+\nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)}$	

No pressure variation accross boundary layer

$$\frac{\partial p}{\partial y} \ll \frac{\partial p}{\partial x} \approx 0$$

Energy equation

Enthalpy Flow	Heat Conduction
$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} =$	$\frac{\nu}{Pr}\left(\cancel{\frac{\partial^2 T}{\partial x^2}} + \frac{\partial^2 T}{\partial y^2}\right)$

Conservation equations (2D, steady state, incompressible)

Coefficient of volumetric expansion

$$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = \frac{\rho_\infty - \rho}{\rho(T - T_\infty)}$$

Continuity equation

Mass Flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation

Momentum Flow | Pressure | Shear stresses | Gravity

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right) + \beta g(T - T_\infty)$$

$$\cancel{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)}$$

negligible

Energy equation

Enthalpy Flow | Heat Conduction

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\left(\cancel{\frac{\partial^2 T}{\partial x^2}} + \frac{\partial^2 T}{\partial y^2}\right)$$

Coefficient of volumetric expansion of an ideal Gas

$$p \cdot V = nRT$$

Coefficient of volumetric expansion

$$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = \frac{\rho_\infty - \rho}{\rho(T - T_\infty)}$$

- n Amount of substance
- $R = 8{,}314 \frac{\mathrm{J}}{\mathrm{mol \cdot K}}$ Universal gas constant

$$\frac{V}{T} = \frac{nR}{p} = const. \rightarrow \left(\frac{\partial V}{\partial T}\right)_p = \frac{V}{T}$$

For ideal Gases

$$\beta = \frac{1}{T} \approx \frac{1}{T_{\mathrm{mean}}} = \frac{2}{T_W + T_\infty}$$

Conservation equations
(2D, steady state, incompressible, plane boundary layer)

Continuity equation

Mass Flow

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Scaling (dimensionless variables)

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, u^* = \frac{u}{u_\infty}, v^* = \frac{v}{u_\infty}, \Theta^* = \frac{T - T_\infty}{T_W - T_\infty}$$

Momentum equation

Momentum Flow | Shear stresses | Gravity

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} + \underbrace{\frac{\beta g L (T_W - T_\infty)}{{u_\infty}^2}}_{\frac{\beta g \rho^2 (T_W - T_\infty) L^3}{\eta^2} \cdot \left(\frac{\eta}{\rho u_\infty L}\right)^2} \Theta^*$$

$$\frac{\beta g \rho^2 (T_W - T_\infty) L^3}{\eta^2} \cdot \left(\frac{\eta}{\rho u_\infty L}\right)^2 \Rightarrow Gr \cdot \left(\frac{1}{Re}\right)^2$$

Energy equation

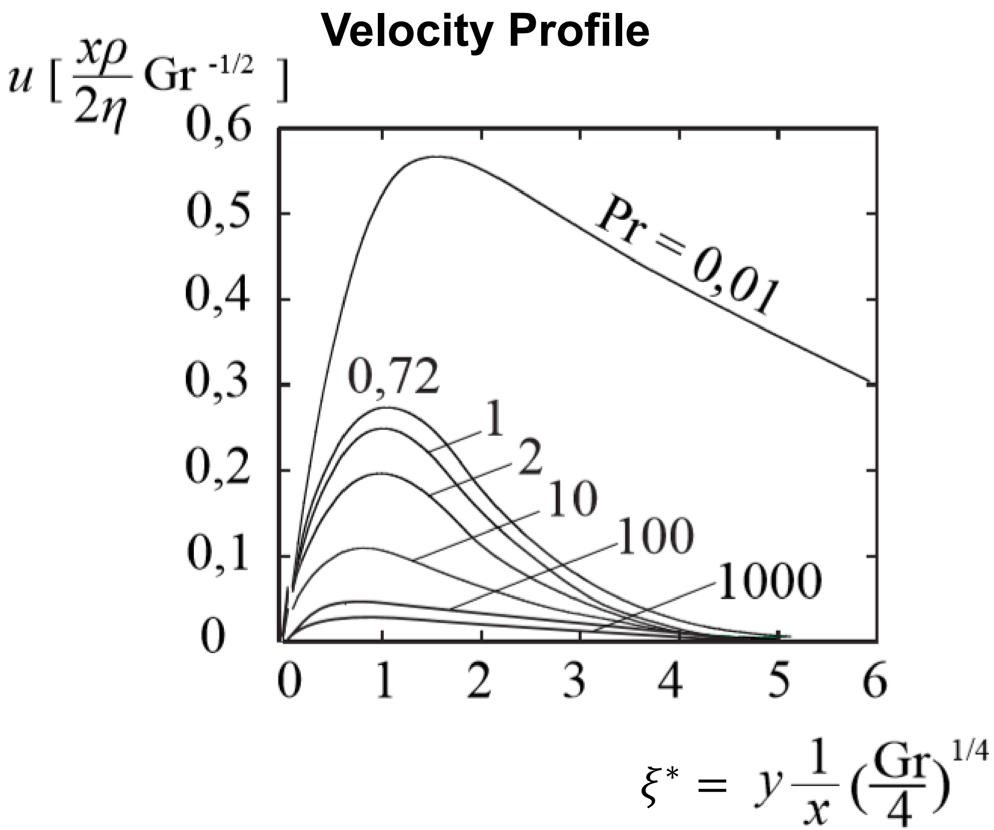
Enthalpy flow | Heat Conduction

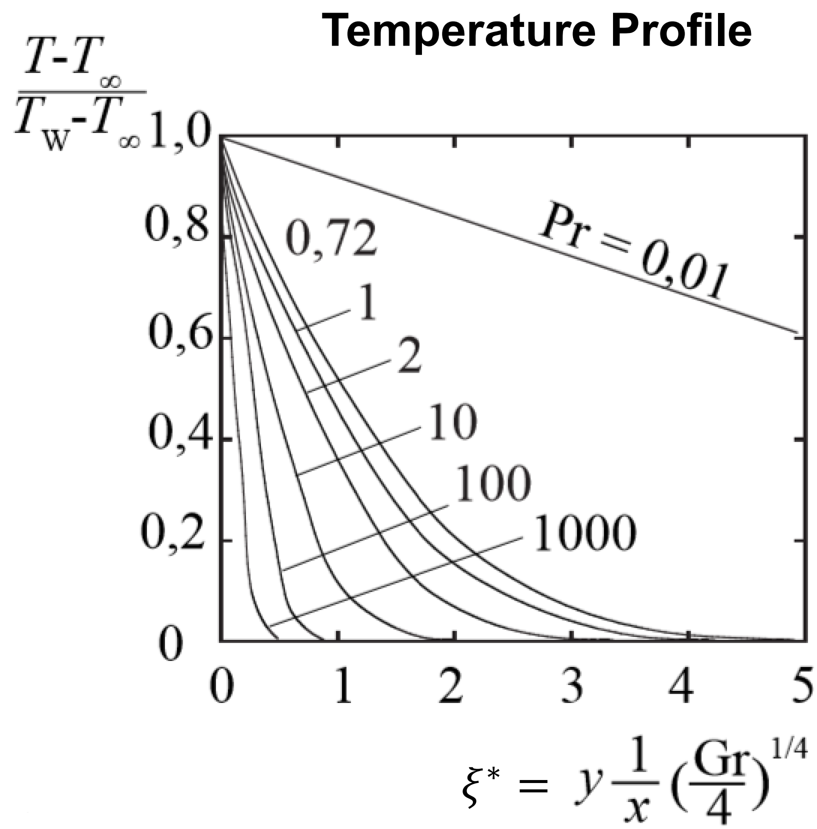
$$u^* \frac{\partial \Theta^*}{\partial x^*} + v^* \frac{\partial \Theta^*}{\partial y^*} = \frac{1}{\underbrace{RePr}_{Pe}} \frac{\partial^2 \Theta^*}{\partial y^{*2}}$$

Grashof number

$$Gr \equiv \frac{\beta g \rho^2 (T_W - T_\infty) L^3}{\eta^2} = \frac{\text{Buoyancy forces}}{\text{Viscosity forces}}$$

Exact Solutions





Dimensionless Heat transfer coefficient

$$Nu = \frac{\alpha x}{\lambda} = \left(\lambda \frac{\partial T}{\partial y}\bigg|_{y=0}\right)\frac{x}{\lambda} = \left(\frac{Gr}{4}\right)^{\frac{1}{4}} \frac{\partial \Theta^*}{\partial \xi^*}\bigg|_{\xi^*=0} = Nu(Gr, Pr)$$

Comparison between Forced and Free convection

Convection	Forced (laminar $Re_x < 2 \cdot 10^5$ isothermal $0{,}6 < Pr < 10$)	Free (laminar, isothermal $GrPr < 4 \cdot 10^9$)
Dependence	$Nu(Re, Pr)$	$Nu(Gr, Pr)$
Local	$Nu = 0{,}332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$	$Nu = 0{,}508 \left(\frac{Pr}{0{,}952 + Pr}\right)^{\frac{1}{4}} (Gr_x Pr)^{\frac{1}{4}}$
Average	$\overline{Nu} = \frac{\bar{\alpha} L}{\lambda} = \int_0^L \frac{Nu}{x} dx = 0{,}664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}}$	$\overline{Nu} = \frac{\bar{\alpha} L}{\lambda} = \int_0^L \frac{Nu}{x} dx = \underbrace{0{,}677 \left(\frac{Pr}{0{,}952 + Pr}\right)^{\frac{1}{4}}}_{C} (Gr_L Pr)^{\frac{1}{4}}$

Comprehension Questions

What is the driving potential of Natural Convection?

Why are buoyancy forces negligible in Forced Convection?