

Heat Transfer: Conduction

Biot number

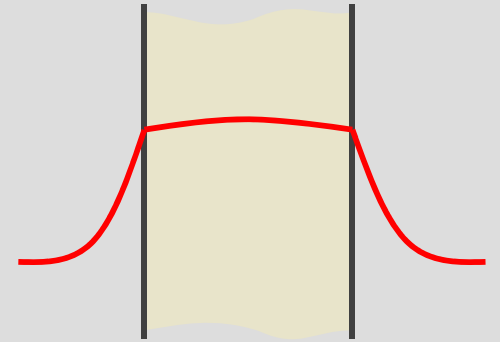
Prof. Dr.-Ing. Reinhold Kneer

Prof. Dr.-Ing. Dr. rer. pol. Wilko Rohlfs

Learning goals

Dimensionless Number: Biot number

- ▶ Characterization of the dominant thermal resistances by using the relevant dimensionless number.
- ▶ Simplify complex multidimensional heat conduction problems based on the problem-defining thermal resistances.



Relevance of thermal resistances

Questions:

- ▶ Which thermal resistances are relevant?
- ▶ Can certain resistances be neglected?

Temperature distribution inside a cylinder:

- ▶ Heat is supplied to the cylinder from one side (here by a flame).



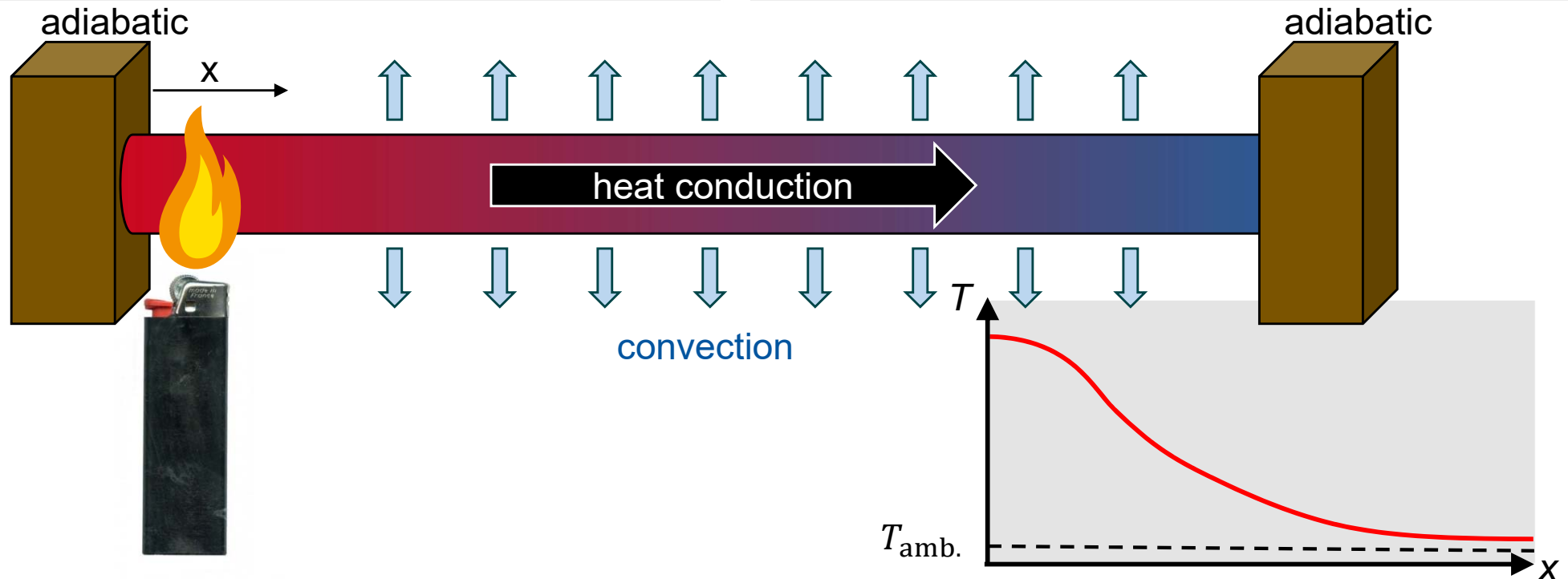
Steady state temperature distribution in a cylinder

Questions:

- ▶ Which thermal resistances are relevant?
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Temperature distribution inside a cylinder:

- ▶ Heat is supplied to the cylinder from one side (here by a flame).
- ▶ Heat is transferred by conduction inside the cylinder and by convection across the shell surface.



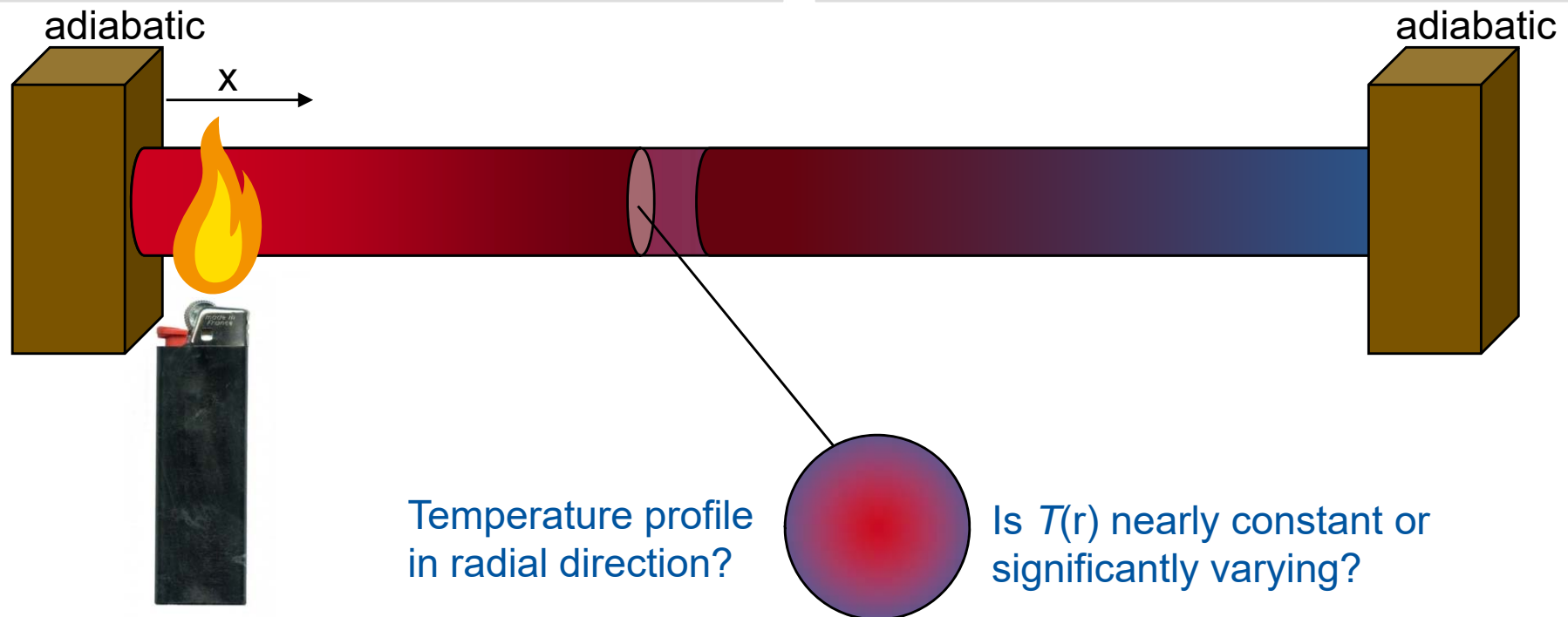
Radial thermal resistance

Questions:

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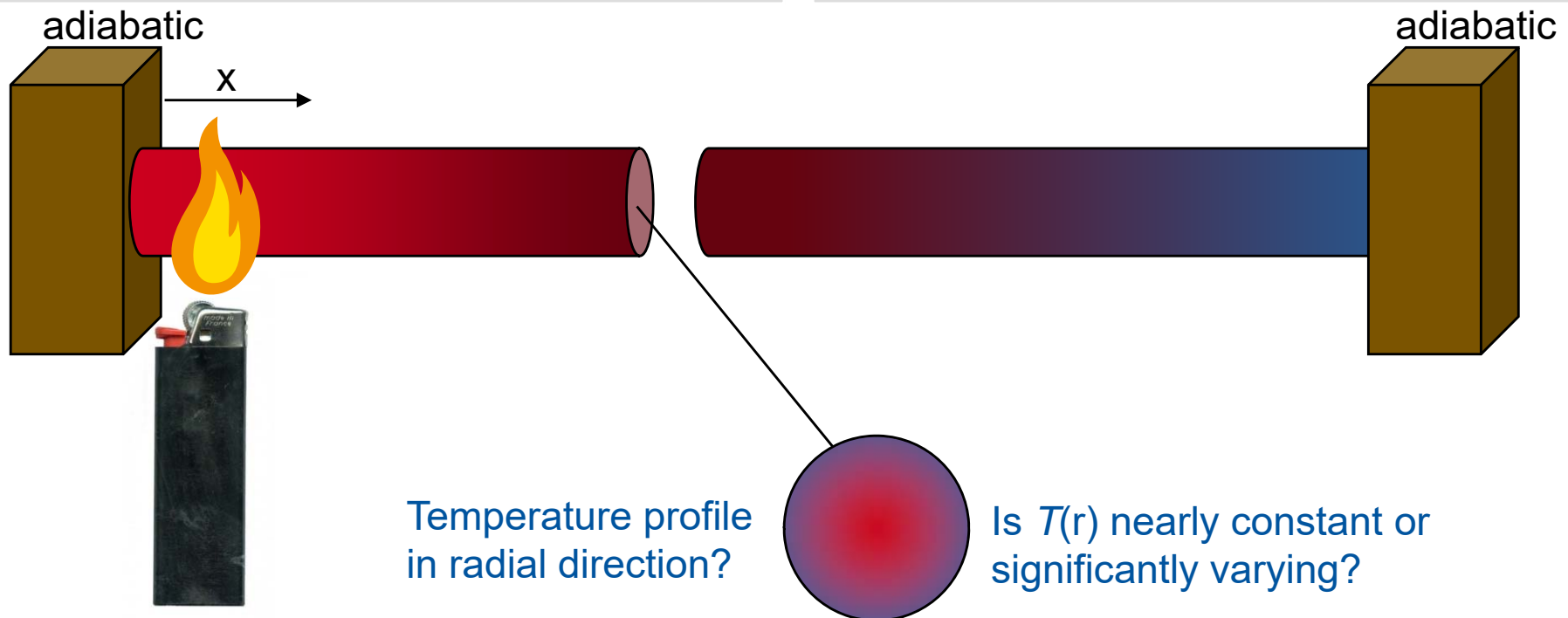
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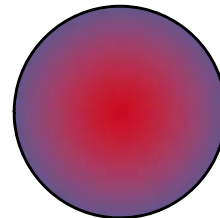
Radial thermal resistance

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Temperature distribution inside a cylinder:

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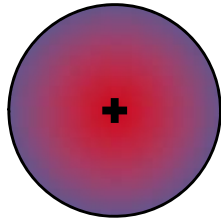


Radial thermal resistance

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Temperature distribution inside a cylinder:



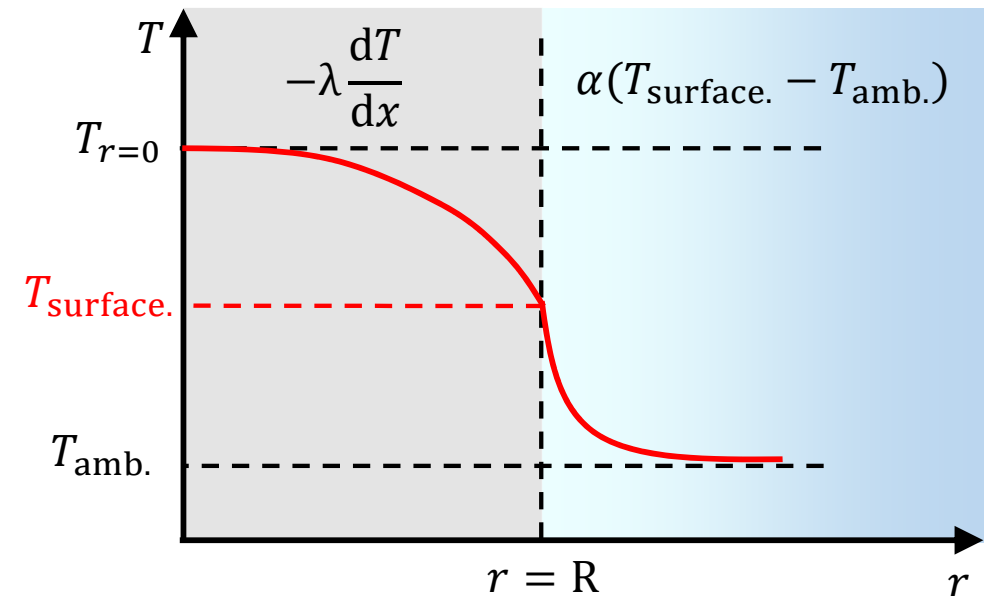
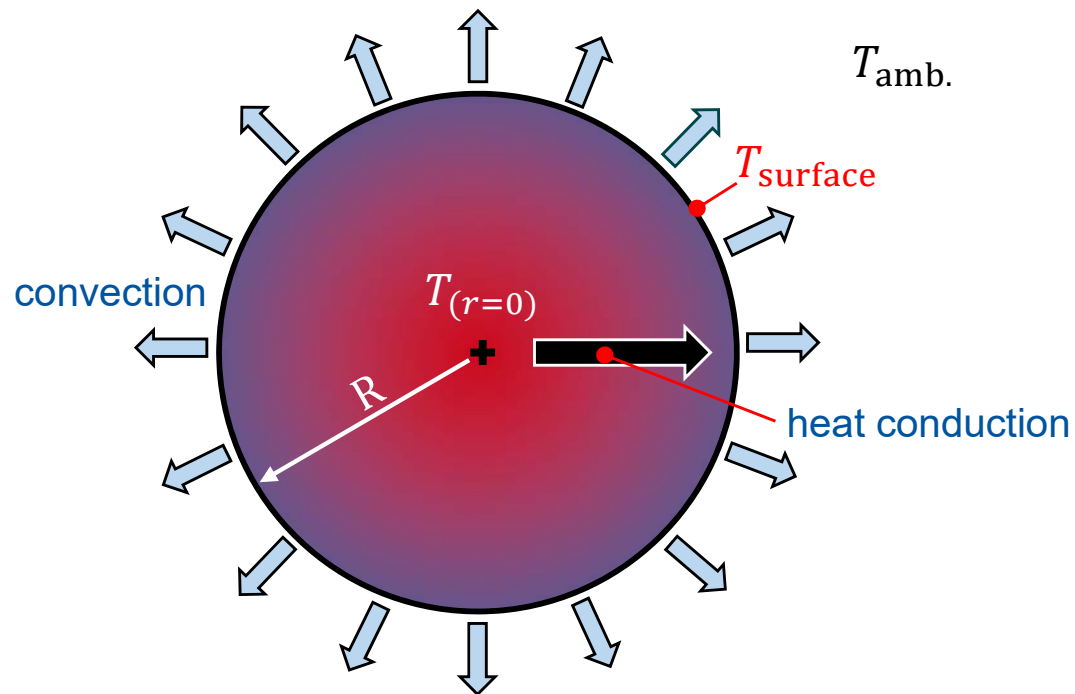
Radial temperature profile

Questions:

- ▶ Which thermal resistances are relevant?
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Temperature distribution inside a cylinder:

- ▶ Heat flows in both axial and radial directions (2D view inside the cylinder)



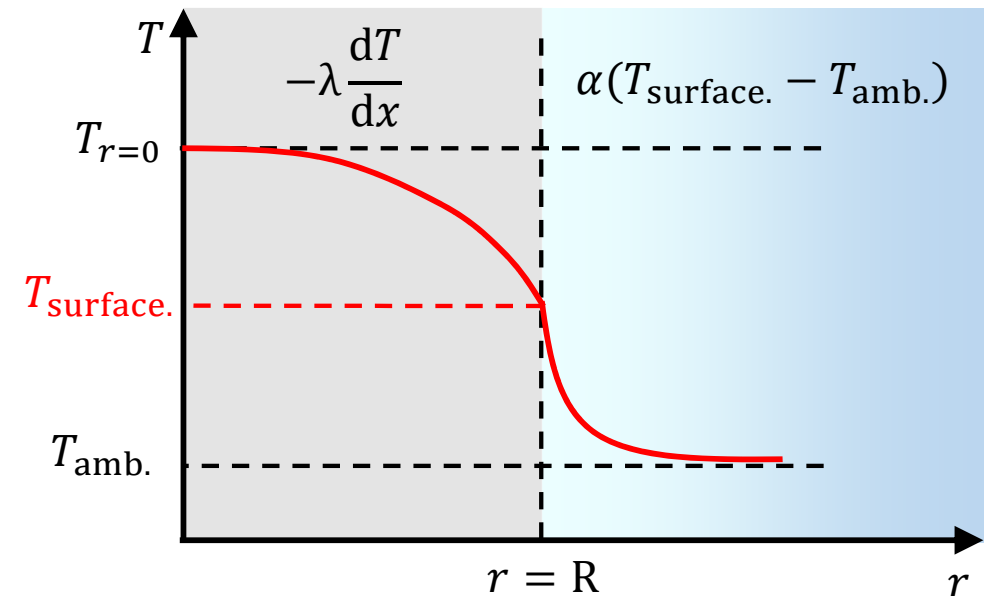
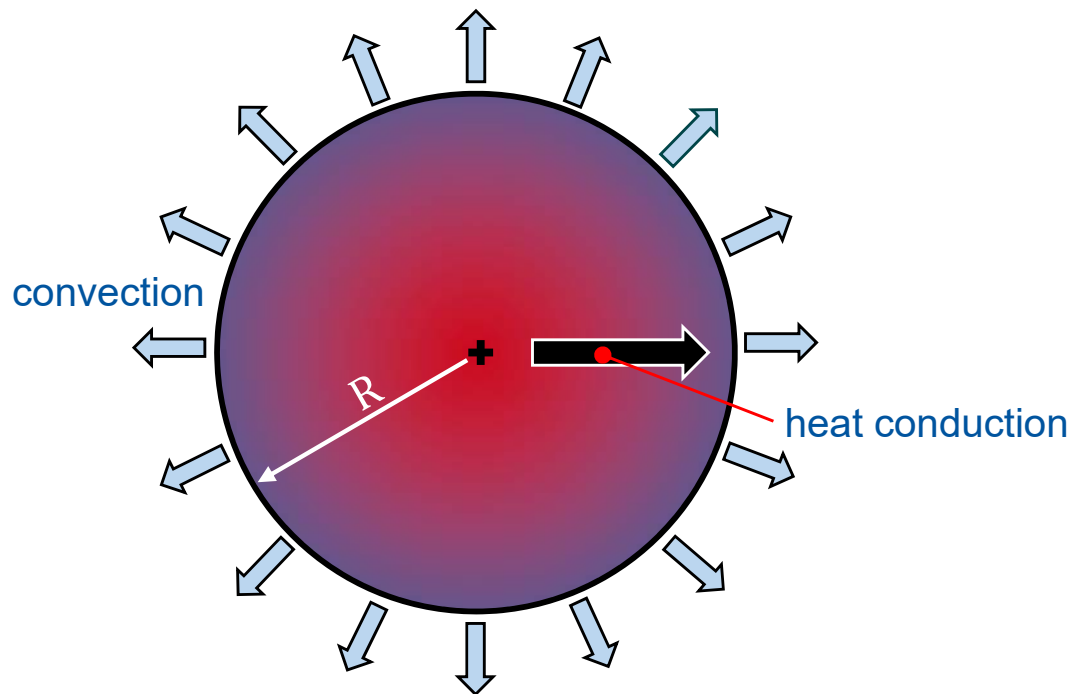
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Change of temperature profile with increase of thermal conductivity

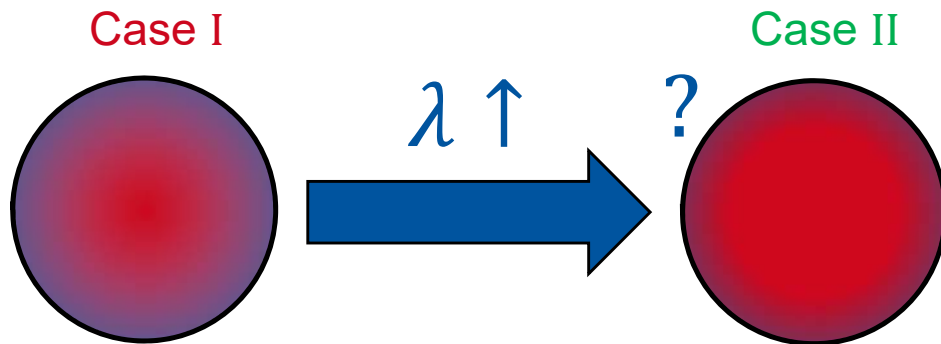
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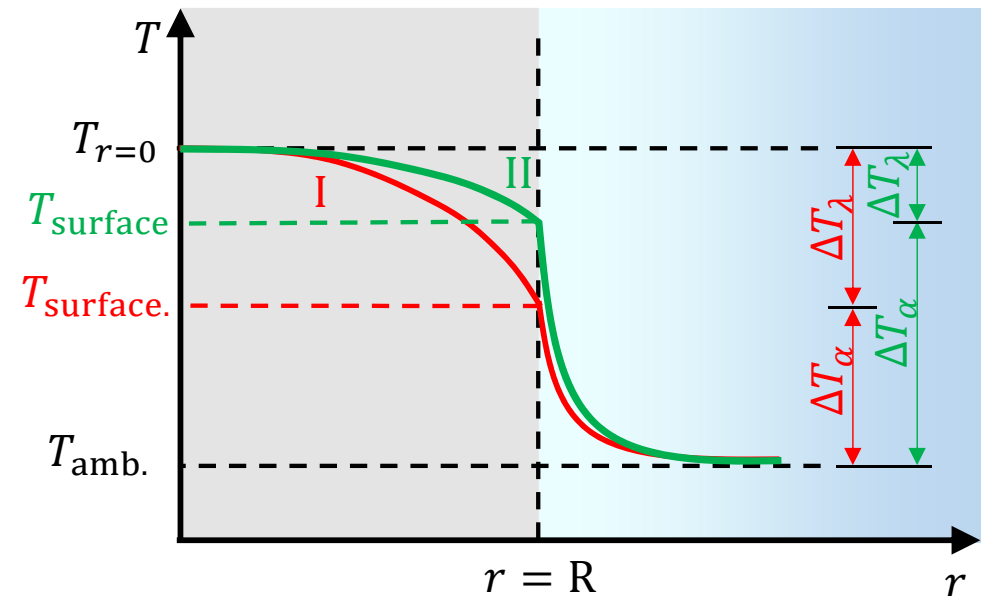
The greater the thermal conductivity becomes, the lower the thermal resistance becomes due to conduction.



Case II:

- ▶ Thermal resistance lower than resistance due to convection:

$$\Delta T_{\lambda} < \Delta T_{\alpha}$$



Change of temperature profile with reduction of the diameter

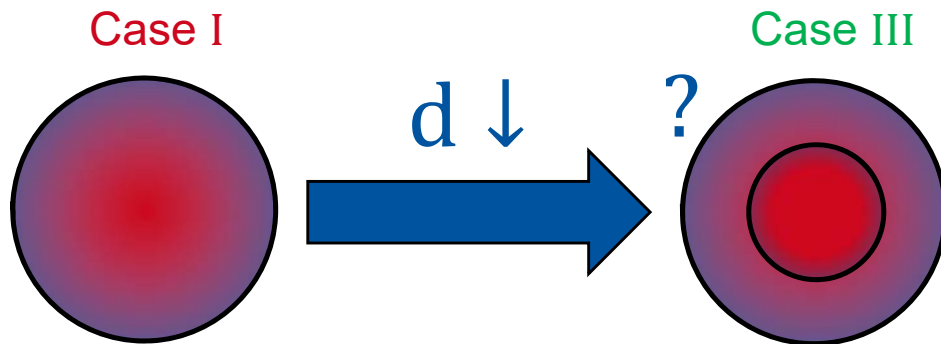
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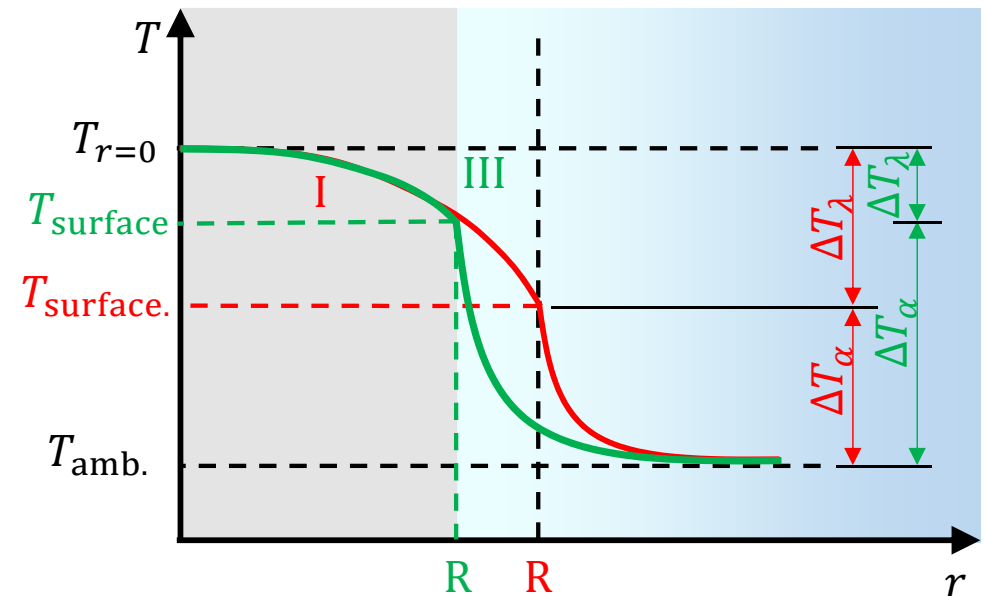
The smaller the diameter becomes, the lower the thermal resistance becomes due to conduction in the radial direction.



Case II:

- ▶ Thermal resistance lower than resistance due to convection:

$$\Delta T_{\lambda} < \Delta T_{\alpha}$$



Change of temperature profile with reduction of the diameter

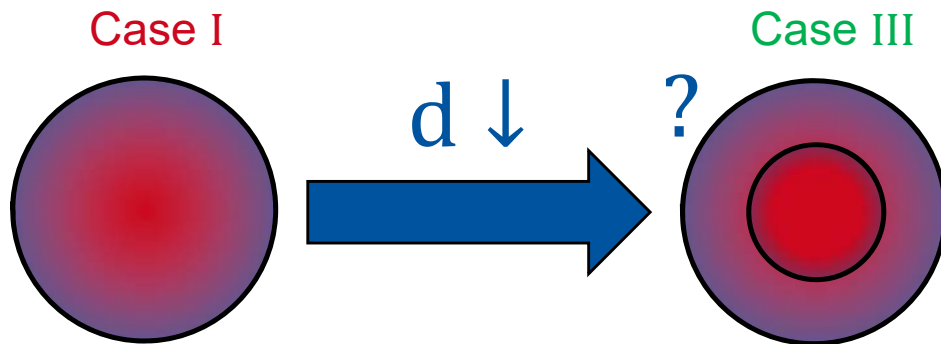
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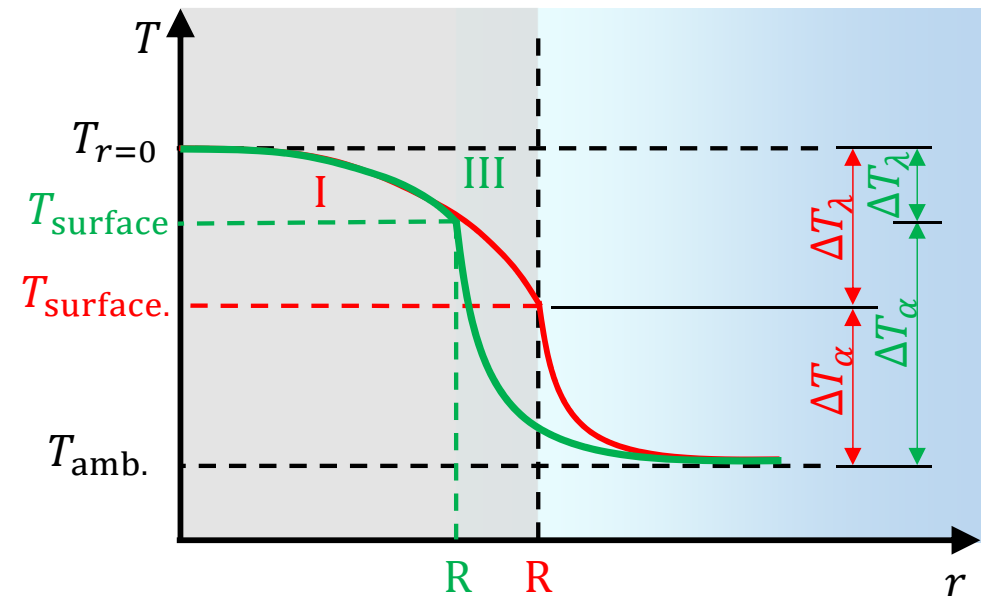
The smaller the diameter becomes, the lower the thermal resistance becomes due to conduction in the radial direction.



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- ▶ Thermal resistance lower than resistance due to convection:

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Change of temperature profile with reduction of convection

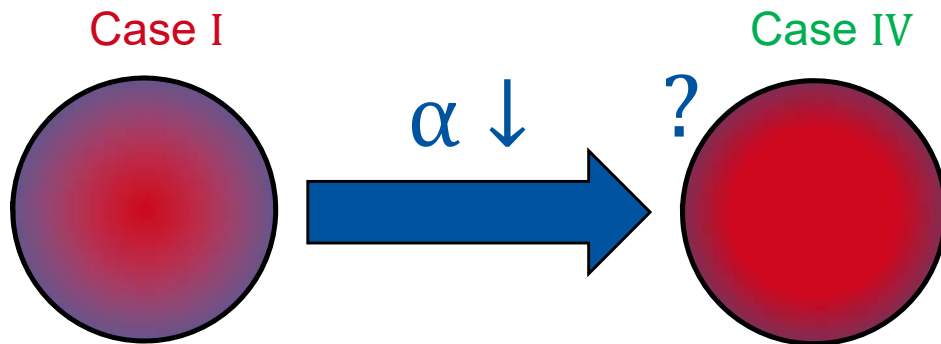
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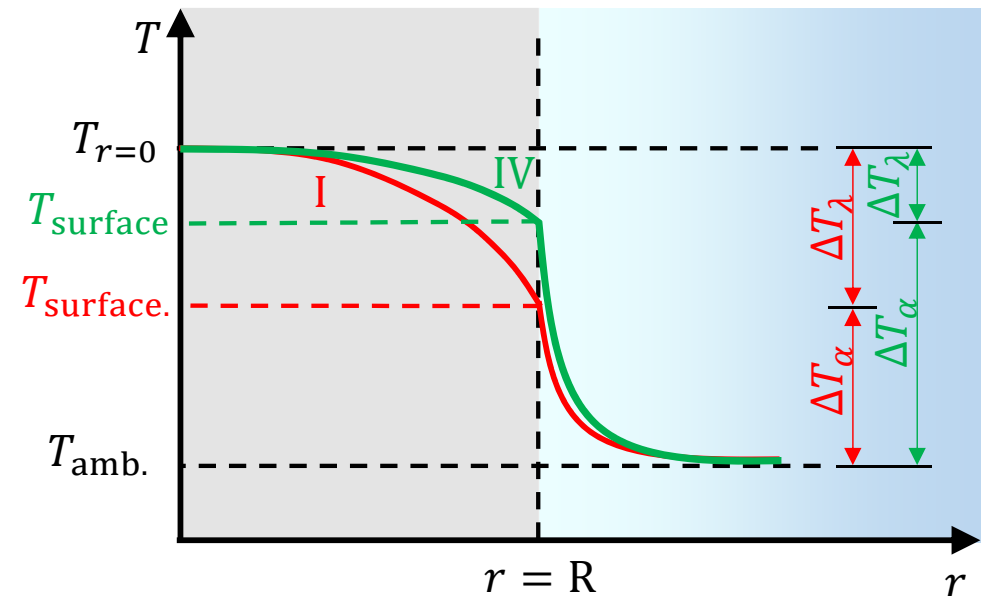
The lower the convection, the more it becomes the limiting factor in heat transfer.



Case II:

- ▶ Thermal resistance lower than resistance due to convection:

$$\Delta T_{\lambda} < \Delta T_{\alpha}$$



Change of temperature profile with reduction of convection

Questions:

- ▶ Which thermal resistances are relevant?
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Temperature distribution inside a cylinder:

- ▶ Heat flows in both axial and radial directions (2D view inside the cylinder)

The lower the convection, the more it becomes the limiting factor in heat transfer.

$$\text{Temperature profile} = f(\alpha, d, \lambda)$$

Dimensionless number: Biot number

Biot number:

To characterize the problem dominant resistance, the **resistances of heat conduction inside the body and convection outside the body** are brought into relation.

Dimensionless parameter: **Biot number**.

$$Bi = \frac{R_{\lambda}}{R_{\alpha}} = \frac{\frac{L}{\lambda}}{\frac{1}{\alpha}}$$

$$Bi = \frac{\alpha L}{\lambda}$$

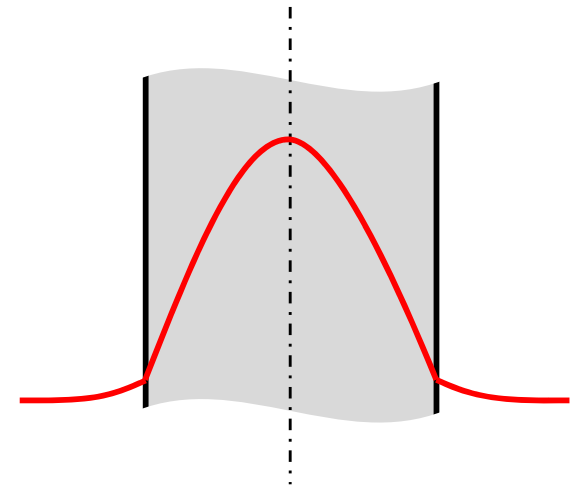
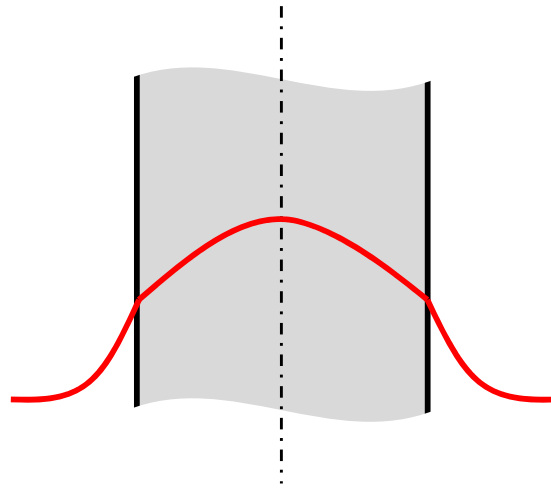
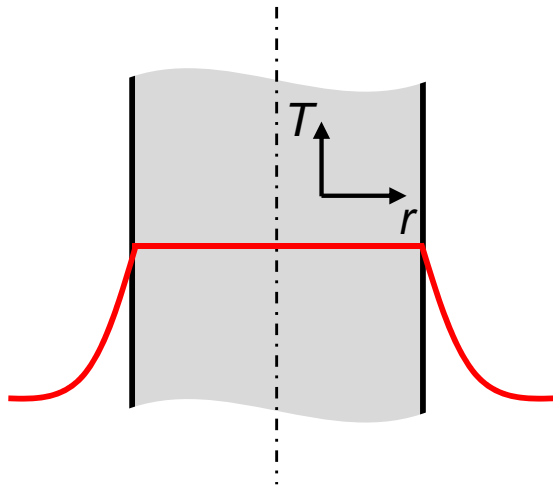
$$Bi = \frac{\text{Thermal resistance inside the body}}{\text{Convective thermal resistance at the surface}}$$

Explannation:

L:	Characteristic length determining thermal resistance [m]
λ :	Thermal conductivity $\left[\frac{\text{W}}{\text{m K}}\right]$
α :	Heat transfer coefficient $\left[\frac{\text{W}}{\text{K}}\right]$

Regime classification by Biot number:

The Biot number: $Bi = \frac{\alpha L}{\lambda}$



$Bi \ll 1 :$

- ▶ Homogeneous temperature in the body
- ▶ R_λ negligible
- ▶ Small bodies or bodies with high thermal conductivity

$Bi \approx 1 :$

- ▶ Similar contributions of heat conduction and convection
- ▶ $R_\lambda \approx R_\alpha$

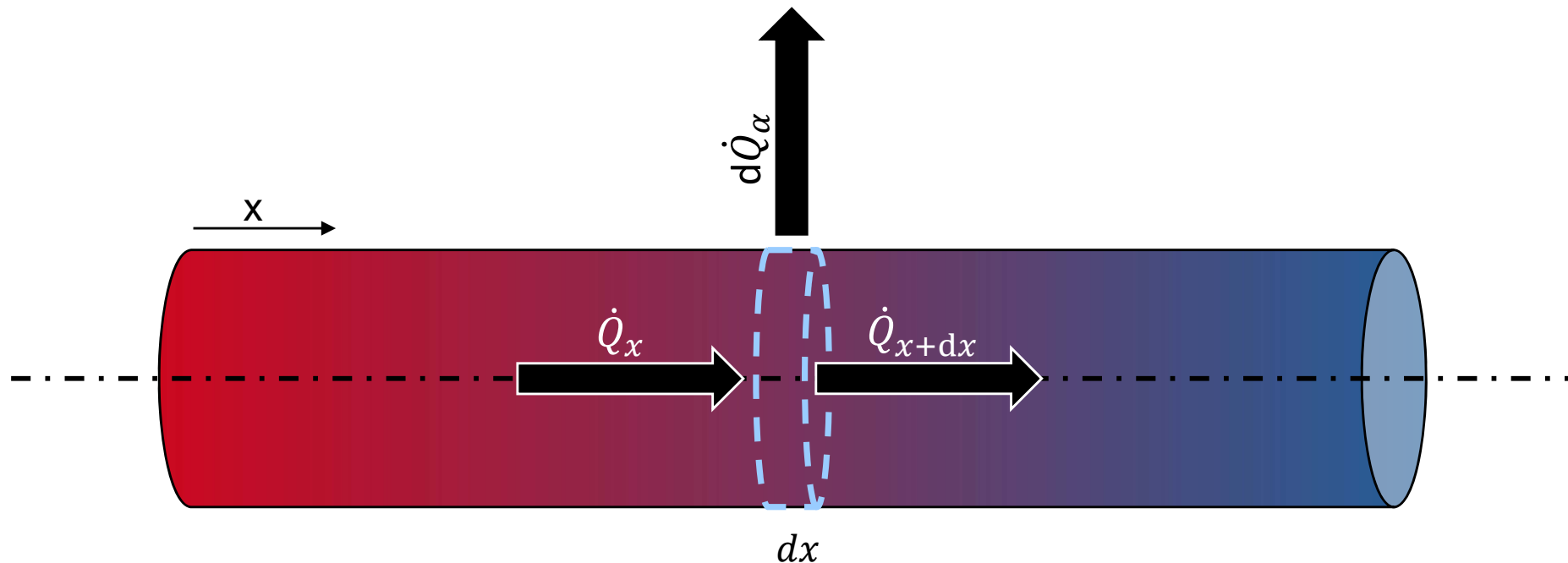
$Bi \gg 1 :$

- ▶ Higher thermal resistance
- ▶ $R_\lambda \gg R_\alpha$
- ▶ Frequent in bodies with low thermal conductivity

Bi \ll 1: One-dimensional approach

Bi \ll 1 :

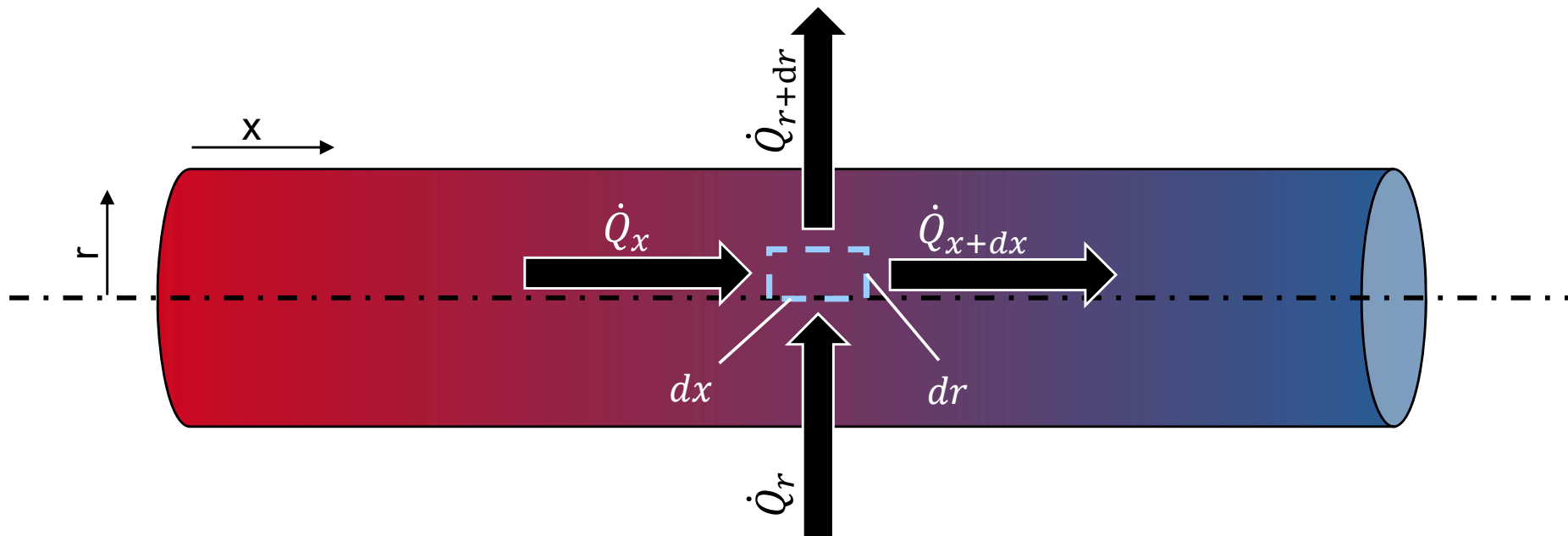
Bi \ll 1 thus leads to a one-dimensional approach within the cylinder



$Bi \approx 1$ or $Bi \gg 1$: Two/Three-dimensional approach

$Bi \approx 1$ and $Bi \gg 1$:

$Bi \approx 1$ and $Bi \gg 1$ leads to a two-dimensional approach inside the cylinder



Comprehension questions

What information does the Biot number provide?

Which assumptions may be made for $Bi \ll 1$?

For a classical fin problem, is the Biot number high or low?