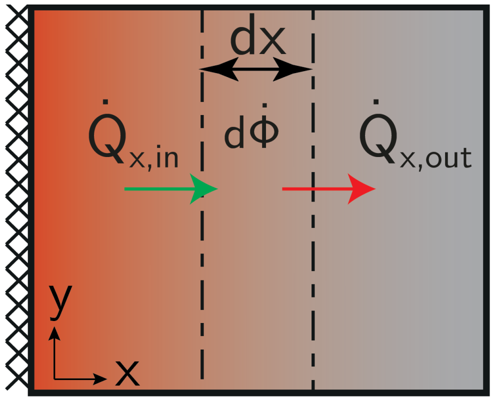
EB - Cond. - IE 15

Set up the energy balance for a one-dimensional steady-state heat transfer through the wall, which is adiabatic on the left-hand side with the cross-sectional area A. There is a source $\dot{\Phi}'''$ in the wall.



Energy Balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} + d\dot{\Phi} = 0$$

Heat Fluxes:

$$\dot{Q}_{x,in} = -\lambda A \tfrac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = \dot{Q}_{x,in} + \tfrac{\partial \dot{Q}_{x,in}}{\partial x} dx = -\lambda A \tfrac{\partial T}{\partial x} + \tfrac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

$$d\dot{\Phi} = \dot{\Phi}''' \cdot dV = \dot{\Phi}''' A dx$$

The in and outgoing fluxes should equal each other and are characterized by conductive heat transfer. The use of the Taylor series expansion can approximate the outgoing flux.

Substituting and rewriting:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} + d\dot{\Phi} = 0$$

$$-\lambda A \frac{\partial T}{\partial x} + \lambda A \frac{\partial T}{\partial x} - \frac{\partial}{\partial x}\left(-\lambda A \frac{\partial T}{\partial x}\right) dx + \dot{\Phi}''' A dx = 0 \tag{4}$$

$$\Rightarrow \lambda \frac{\partial^2 T}{\partial x^2} + \dot{\Phi}''' = 0 \tag{5}$$