Heat Transfer

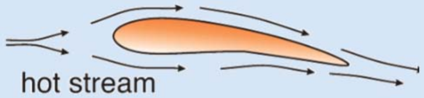
Heat transfer laws for the forced convection in External Flow

Prof. Dr.-Ing. Reinhold Kneer
Dr.-Ing. Dr. rer. pol. Wilko Rohlfs
Prof. dr. ir. Kees Venner

UNIVERSITY OF TWENTE.
Lehrstuhl für Wärme- und Stoffübertragung
RWTH AACHEN UNIVERSITY

Learning Goals

- Forced Convection in External Flow
  - Knowledge and understanding of the Dimensionless numbers
  - Overview of different application cases and associated correlations



cooled turbine blade

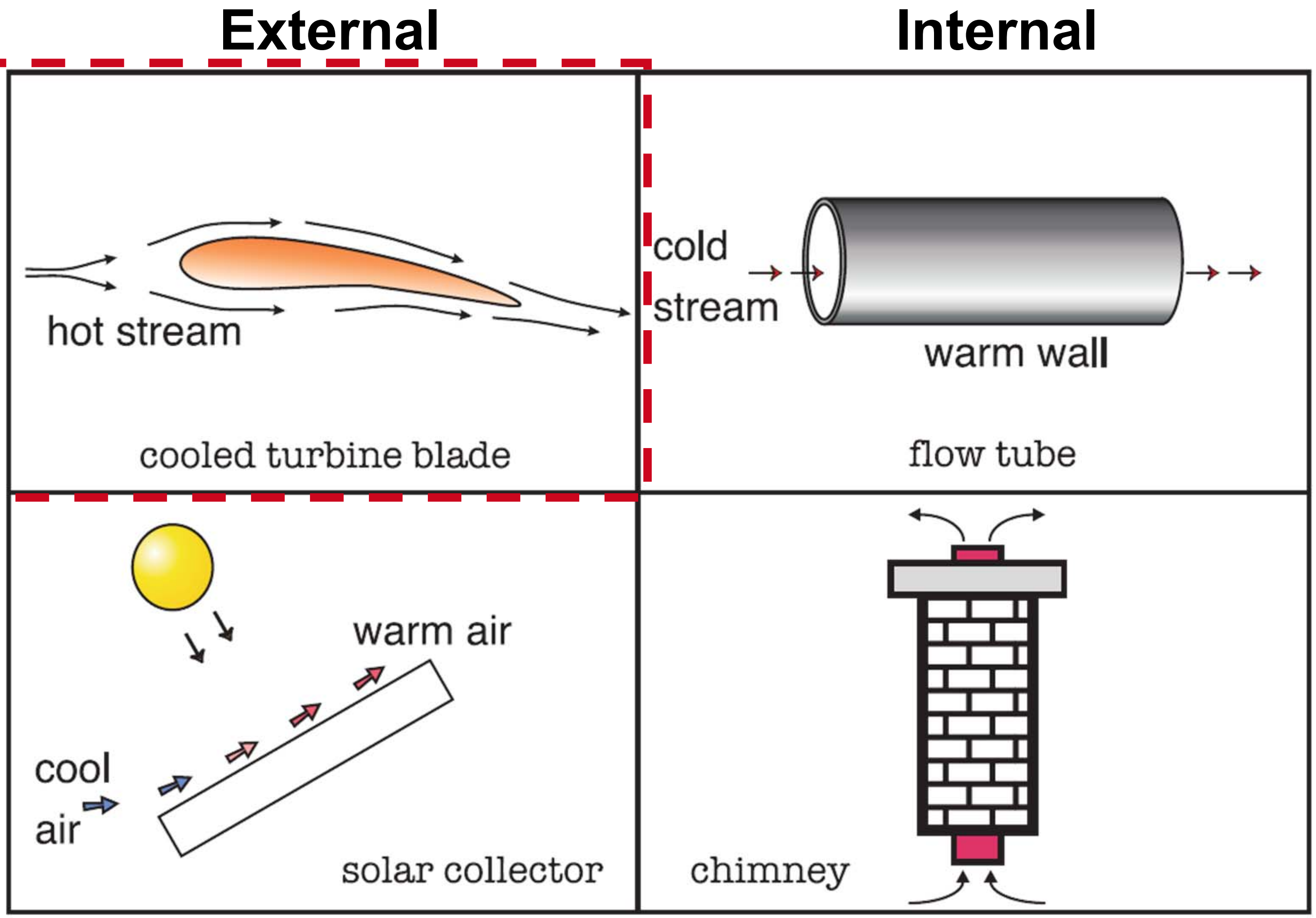
Classifications according to flow regime

Forced Convection

- Driven by externally generated movement of the fluid/object

Free Convection

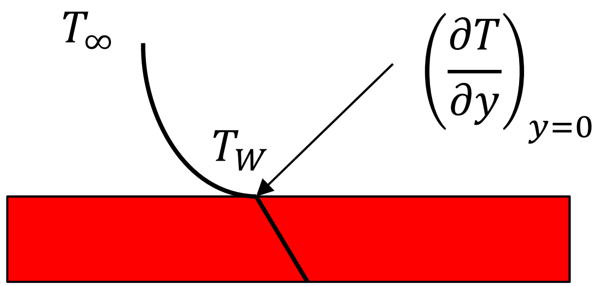
- Inherently driven due to heat transfer (density differences)



Heat flow and heat transfer coefficient for convection

$$\dot{Q} = \alpha\, A\, (T_W - T_\infty)$$

$$\alpha = \frac{-\lambda_f \left(\frac{\partial T}{\partial y}\right)_{y=0,f}}{(T_W - T_\infty)}$$



A = Convective transport area [m²]

ΔT = Temperature difference between wall temperature T_w and fluid temperature T_∞.

α = Heat transfer coefficient [W/m²K]

Alternative:

$$\alpha = \frac{-\lambda_W \left(\frac{\partial T}{\partial y}\right)_{y=0,W}}{(T_W - T_\infty)}$$

Important Dimensionless numbers

Nusselt number:	$Nu = \frac{\alpha L}{\lambda}$	The Nusselt number is the dimensionless heat transfer coefficient
Prandtl number:	$Pr = \frac{\eta c_p}{\lambda}$	The Prandtl number compares the momentum transport due to friction with the heat transport due to conduction/diffusion
Reynolds number:	$Re = \frac{\rho u_\infty L}{\eta}$	The Reynolds number gives the ratio of inertial forces to viscous forces
Grashof number:	$Gr = \frac{\rho^2 g\beta(T_W - T_{fl})L^3}{\eta^2}$	The Grashof number describes the relationship between the buoyancy forces of a fluid and the acting viscosity forces

Nusselt number function in Forced Convection

General Form

Forced Convection: $Nu = Nu(Re, Pr)$

Application criteria

- Geometry
- Flow regime
- Thermal boundary conditions

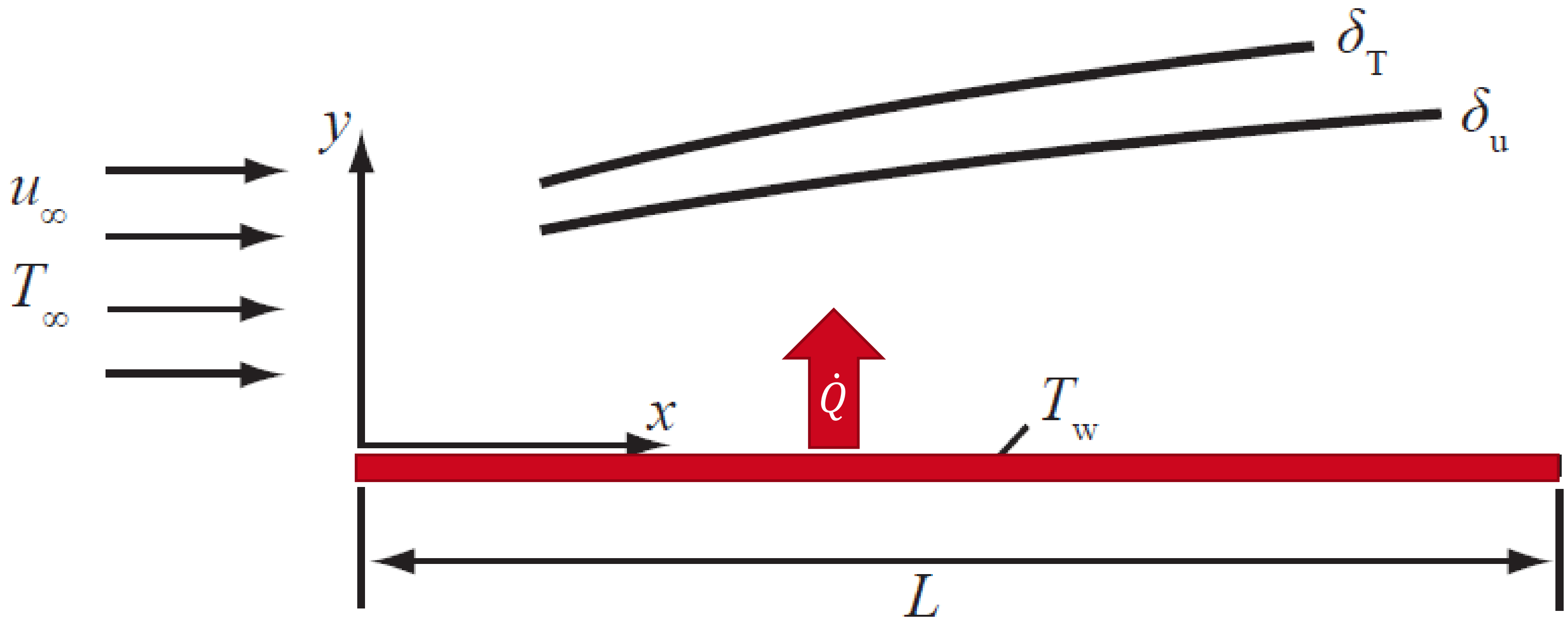
Substance Properties

For the calculation of the substance properties, use the average temperature:

$$T_{\mathrm{Prop}} = \frac{T_{\mathrm{W}} + T_{\infty}}{2}$$

Flat plate – laminar boundary layer flow

5.1.1.1 Flat plate - laminar boundary layer flow $\mathrm{Re}_{\mathrm{x,crit}} \approx 2 \cdot 10^5$



$$Nu_x = \frac{\alpha x}{\lambda}$$

The characteristic length x is perpendicular to the heat flow!

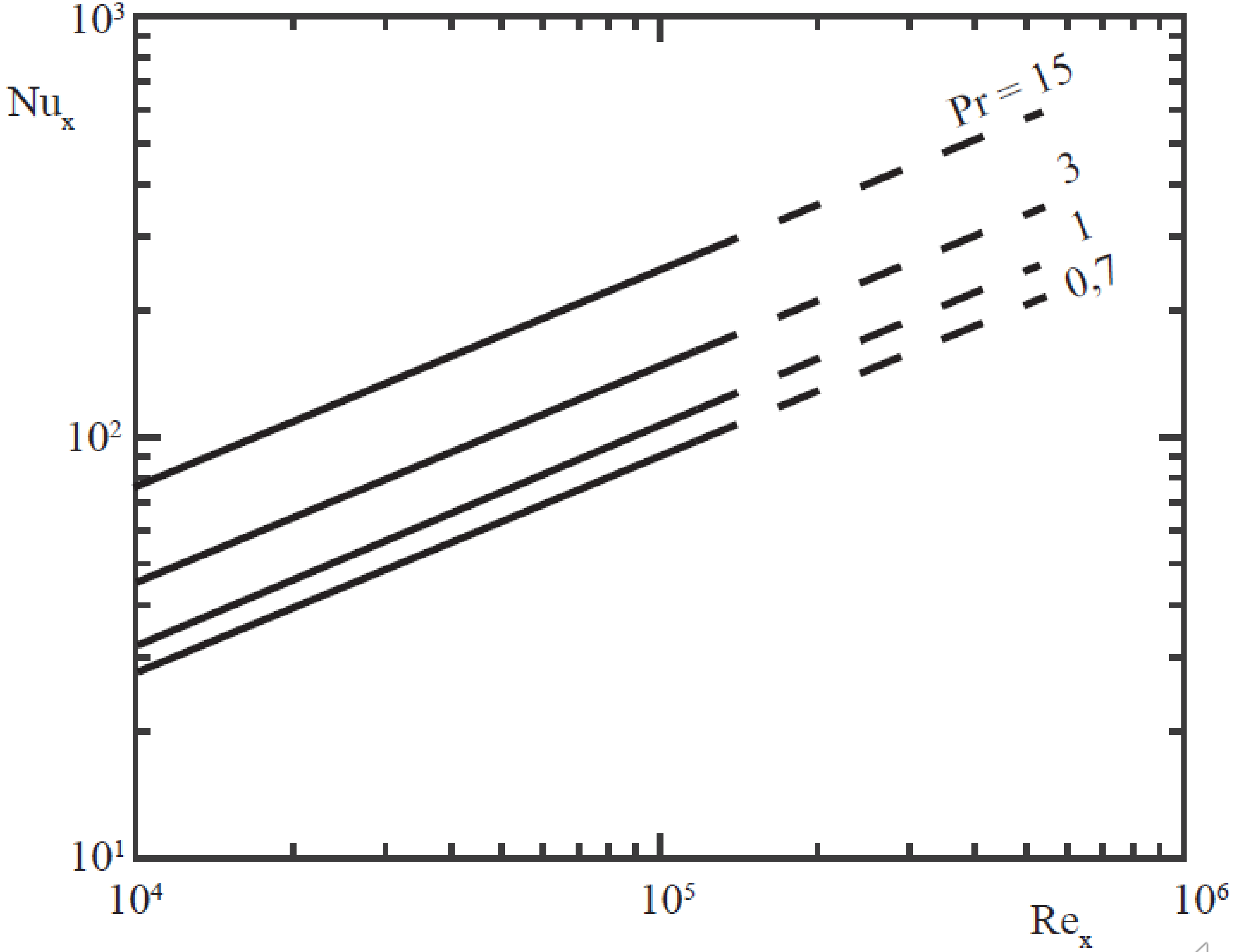
UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

Flat plate with laminar boundary layer, isothermal surface, simultaneous hydrodynamic and thermal inflow

$$Nu_x = \frac{\alpha x}{\lambda}$$



Flat plate with laminar boundary layer, isothermal surface, simultaneous hydrodynamic and thermal inflow

Nusselt law for the local heat transfer:

$$\mathrm{Nu_x} = 0{,}332\,\mathrm{Re_x}^{\frac{1}{2}}\,\mathrm{Pr}^{\frac{1}{3}} \tag{HTC.1}$$

$$\begin{aligned}&\text{for} \quad 0{,}6 < \mathrm{Pr} < 10\\ &\text{and} \quad \mathrm{Re_x} < \mathrm{Re_{x,crit}} \approx 2 \cdot 10^5\end{aligned}$$

$$Nu_x = \frac{\alpha x}{\lambda} = 0.332\left(\frac{\rho u x}{\eta}\right)^{1/2} \cdot Pr^{1/3}$$

$$\alpha = 0.332 \cdot \lambda \left(\frac{\rho u}{\eta x}\right)^{1/2} \cdot Pr^{1/3}$$

The heat transfer coefficient decreases in the x-direction in the same way as the thermal boundary layer increases

Flat plate with laminar boundary layer, isothermal surface, simultaneous hydrodynamic and thermal inflow

Nusselt law for the local heat transfer:

$$\mathrm{Nu_x} = 0{,}332 \mathrm{Re_x}^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{3}} \qquad \text{(HTC.1)}$$

$$\begin{aligned} &\text{for} \quad 0{,}6 < \mathrm{Pr} < 10 \\ &\text{and} \quad \mathrm{Re_x} < \mathrm{Re_{x,crit}} \approx 2 \cdot 10^5 \end{aligned}$$

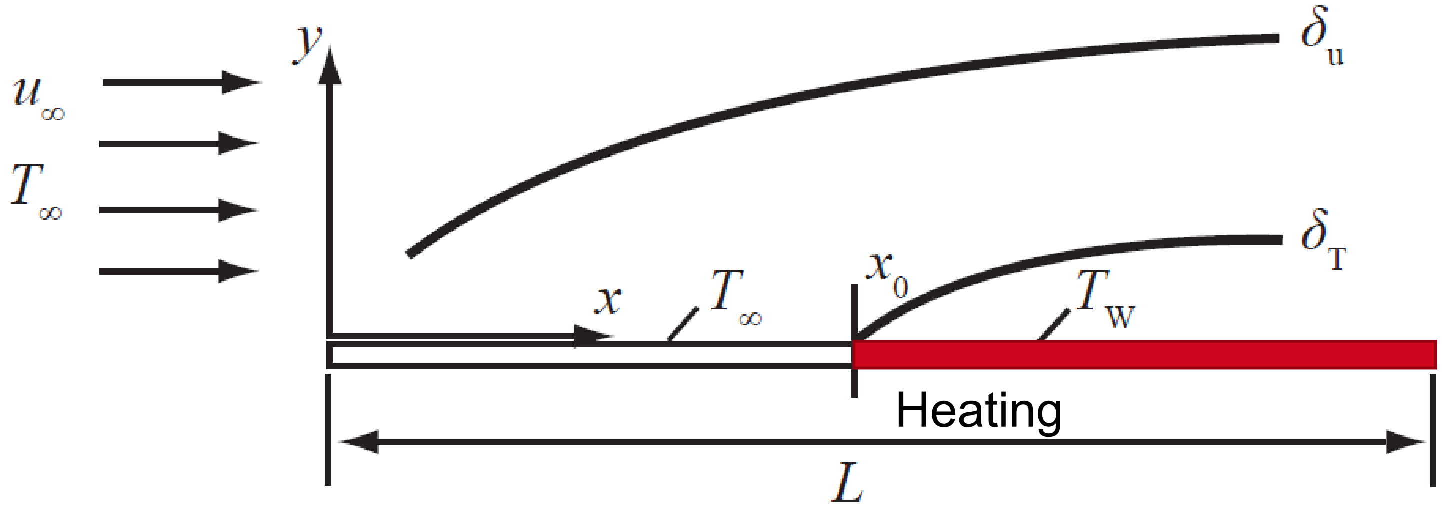
Nusselt law for the mean heat transfer:

$$\overline{\mathrm{Nu_L}} = \frac{\overline{\alpha} L}{\lambda} = \frac{1}{L} \int_0^L \alpha(x) dx = 2\ \mathrm{Nu_{x=L}}$$

$$\overline{\mathrm{Nu_L}} = 0{,}664\ \mathrm{Re_L}^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{3}} \qquad \text{(HTC.2)}$$

$$\begin{aligned} &\text{for} \quad 0{,}6 < \mathrm{Pr} < 10 \\ &\text{and} \quad \mathrm{Re_L} < \mathrm{Re_{L,crit}} \approx 2 \cdot 10^5 \end{aligned}$$

Flat plate with laminar boundary layer, isothermal surface, hydrodynamic inlet, heating or cooling starting at $x = x_0$

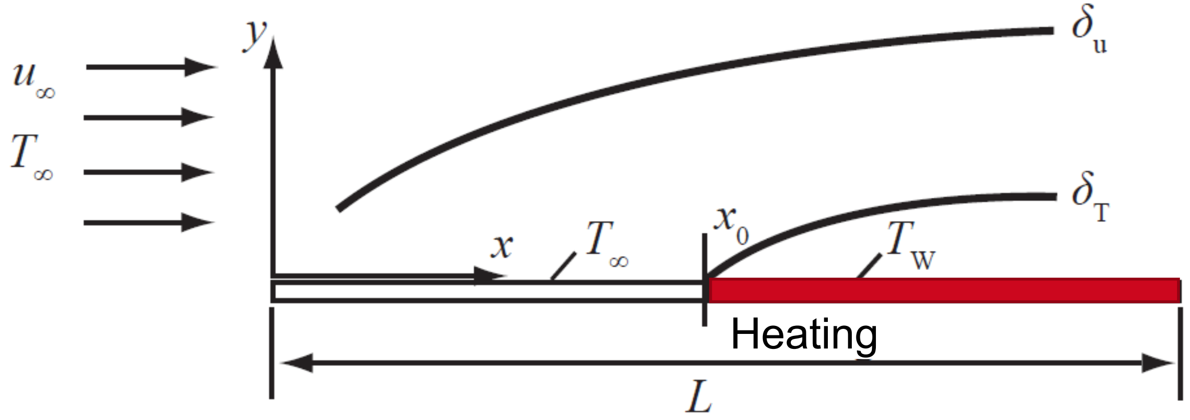


Nusselt law for the local heat transfer:

$$\mathrm{Nu_x} = 0{,}332\,\mathrm{Re_x}^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}\left[1 - \left(\frac{x_0}{x}\right)^{\frac{3}{4}}\right]^{-\frac{1}{3}} \qquad \text{(HTC.3)}$$

$$\begin{aligned} &\text{for} \quad 0{,}6 < \mathrm{Pr} < 10 \\ &\text{and} \quad \mathrm{Re_x} < \mathrm{Re_{x,crit}} \approx 2 \cdot 10^5 \end{aligned}$$

Flat plate with laminar boundary layer, isothermal surface, hydrodynamic inlet, heating or cooling starting at $x = x_0$



Nusselt law for the mean heat transfer:

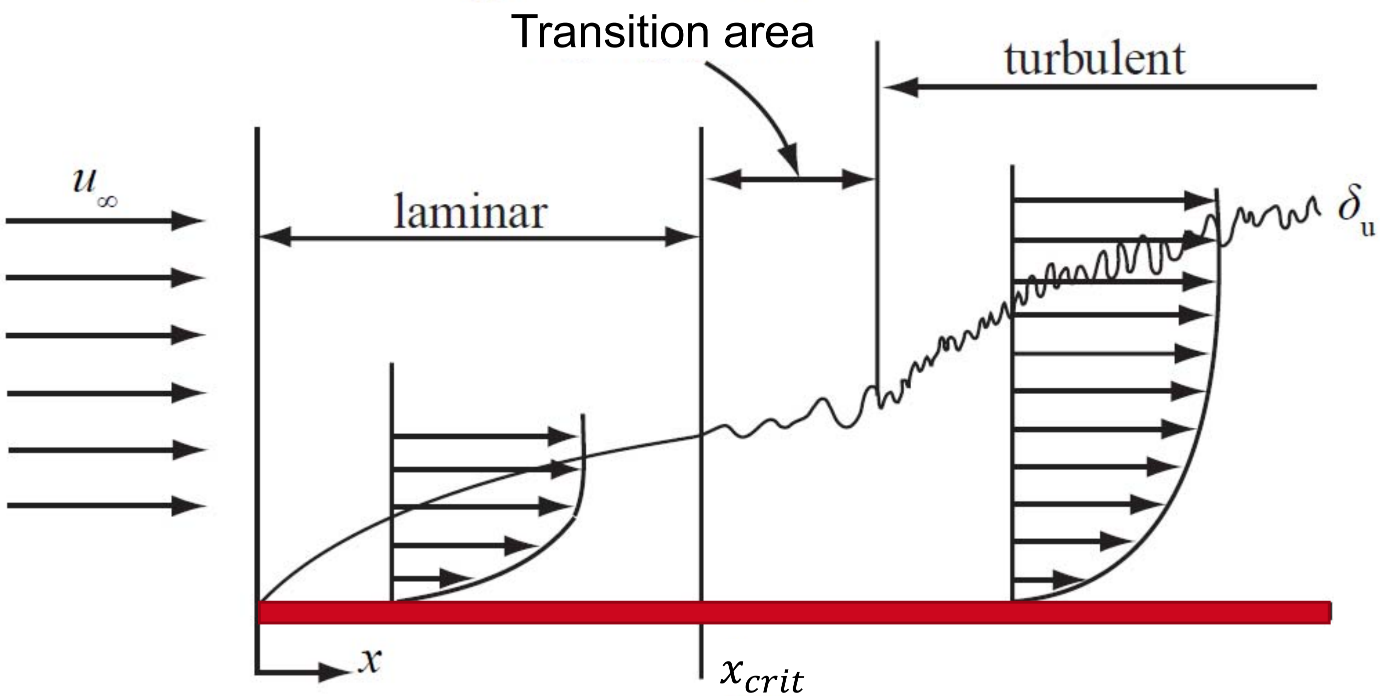
$$\mathrm{Nu_L} = \frac{L}{L - x_0} \frac{1}{\lambda} \int_{\mathrm{x_0}}^{\mathrm{L}} \alpha(x) dx$$

$$\mathrm{Nu_L} = 0{,}664\ \mathrm{Re_L}^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{3}} \frac{\left[1 - \left(\frac{x_0}{L}\right)^{\frac{3}{4}}\right]^{\frac{2}{3}}}{\left[1 - \frac{x_0}{L}\right]} \qquad \text{(HTC.4)}$$

for $\quad 0{,}6 < \mathrm{Pr} < 10$

and $\quad \mathrm{Re_x} < \mathrm{Re_{x,crit}} \approx 2 \cdot 10^5$

Flat plate with turbulent boundary layer and isothermal surface



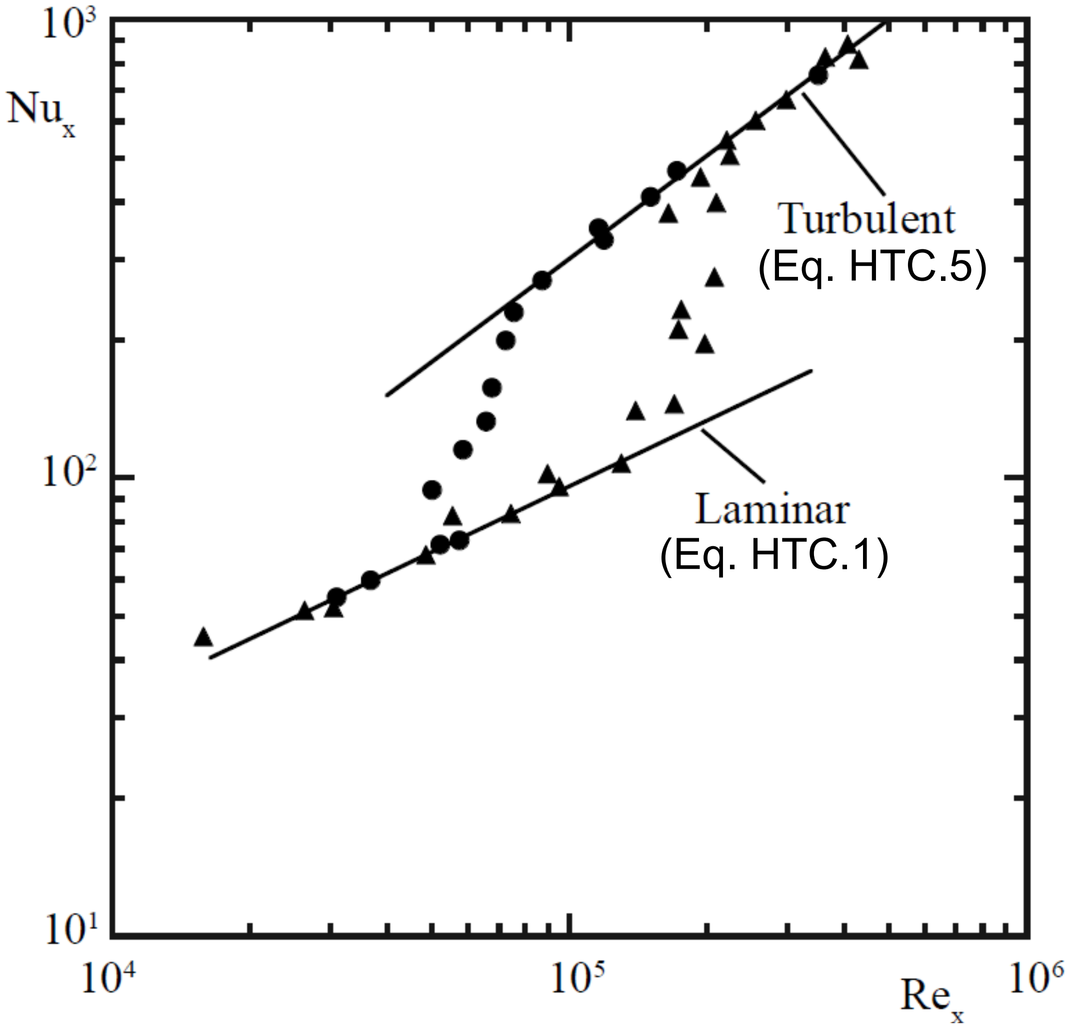
$$\mathrm{Re}_{\mathrm{x,crit}} \approx 2 \cdot 10^5$$

$$Re_{x,crit} = \frac{\rho u x_{crit.}}{\eta} = 2 \cdot 10^5$$

$$x_{crit.} = 2 \cdot 10^5 \frac{\eta}{\rho u}$$

The turbulent envelope shifts backwards with increasing viscosity and forwards with increasing flow velocity

Flat plate with turbulent boundary layer and isothermal surface



Influence of the average degree of turbulence of the incident flow on the laminar/turbulent transition (triangles and circles)

In both cases, the laminar or turbulent heat transfer is well represented by the two correlations (HTC.1 and HTC.5)

$$\mathrm{Nu_x} = 0{,}0296\ \mathrm{Re_x}^{0{,}8}\ \mathrm{Pr}^{0{,}43} \quad \text{for} \quad 5 \cdot 10^5 < \mathrm{Re}_x < 10^7 \qquad \text{(HTC.5)}$$

Average heat transfer for a flat plate with turbulent boundary layer and isothermal surface

Nusselt law for mean heat transfer:
(Note : The average heat transfer of a plate with length L can be calculated by integrating equations (WÜK.1) and (WÜK.5)

$$\overline{\mathrm{Nu_L}} = \frac{\overline{\alpha} L}{\lambda} = \frac{1}{\lambda}\left(\int_0^{\mathrm{x_{crit}}} \alpha(x)_{\mathrm{eq.\ (HTC.1)}}\, dx + \int_{\mathrm{x_{crit}}}^{L} \alpha(x)_{\mathrm{eq.\ (HTC.5)}}\, dx \right) \qquad (5.8)$$

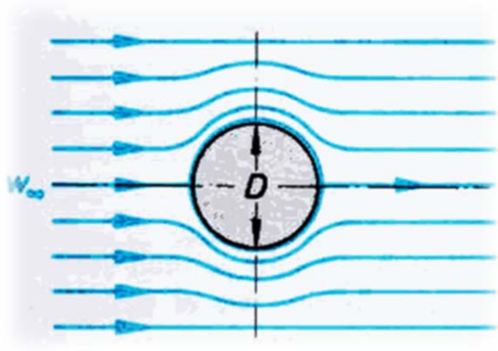
Hence, the result depends on the actual critical Reynolds number).

Nusselt law for mean heat transfer:

$$\overline{\mathrm{Nu_L}} \approx 0{,}036\ \mathrm{Pr}^{0{,}43} \left(\mathrm{Re_L}^{0{,}8} - 9400 \right) \qquad \text{(HTC.6)}$$

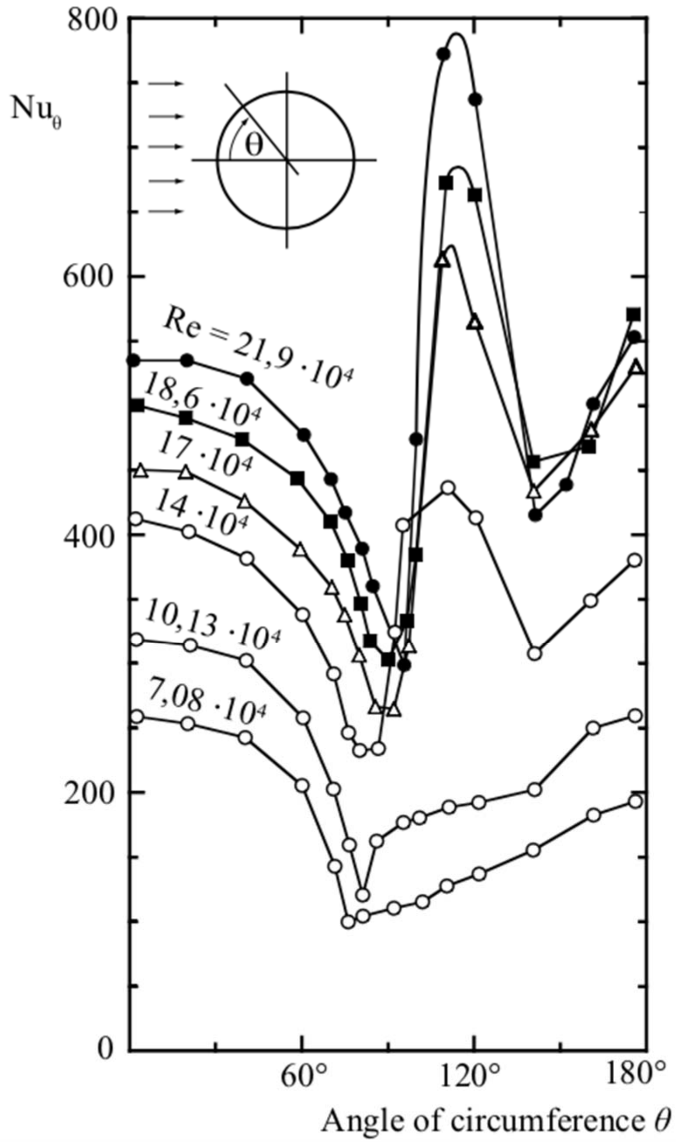
$$\text{for} \quad \mathrm{Re_{L,crit}} \approx 2 \cdot 10^5$$

Transverse flow around cylinder

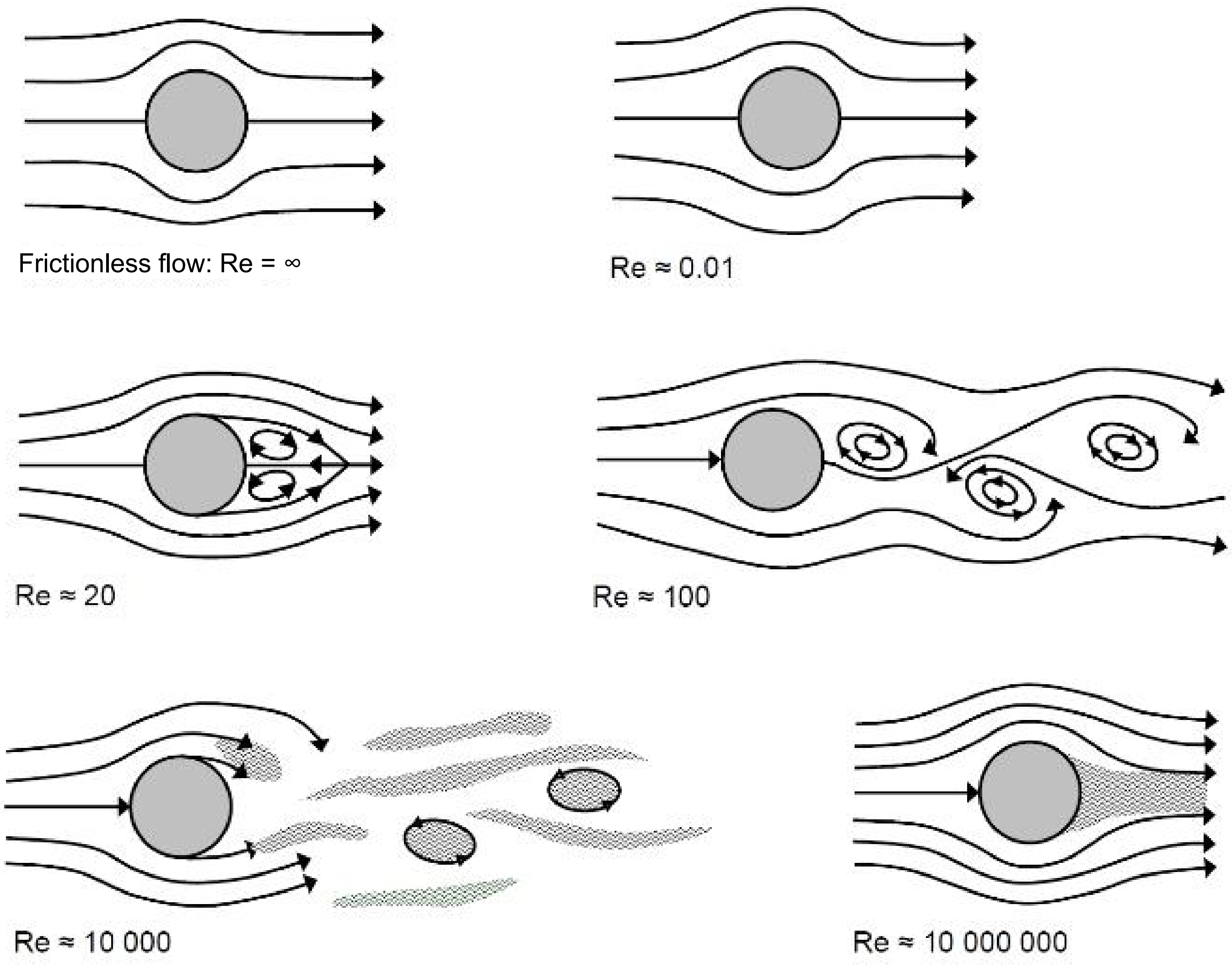


$$\overline{\mathrm{Nu_d}} = C\ \mathrm{Re_d}^m \mathrm{Pr}^{0,4}$$

$\mathrm{Re_d}$	C	m
0,4 - 4	0,989	0,330
4 - 40	0,911	0,385
40 - 4000	0,683	0,466
4000 - 40000	0,193	0,618
40000 - 400000	0,0266	0,805



Transverse flow around cylinder as a function of the Reynolds number



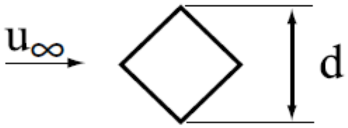
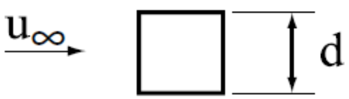
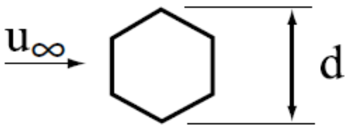
Transverse flow around non-circular cylinder

Mean heat transfer for non circular cylinders

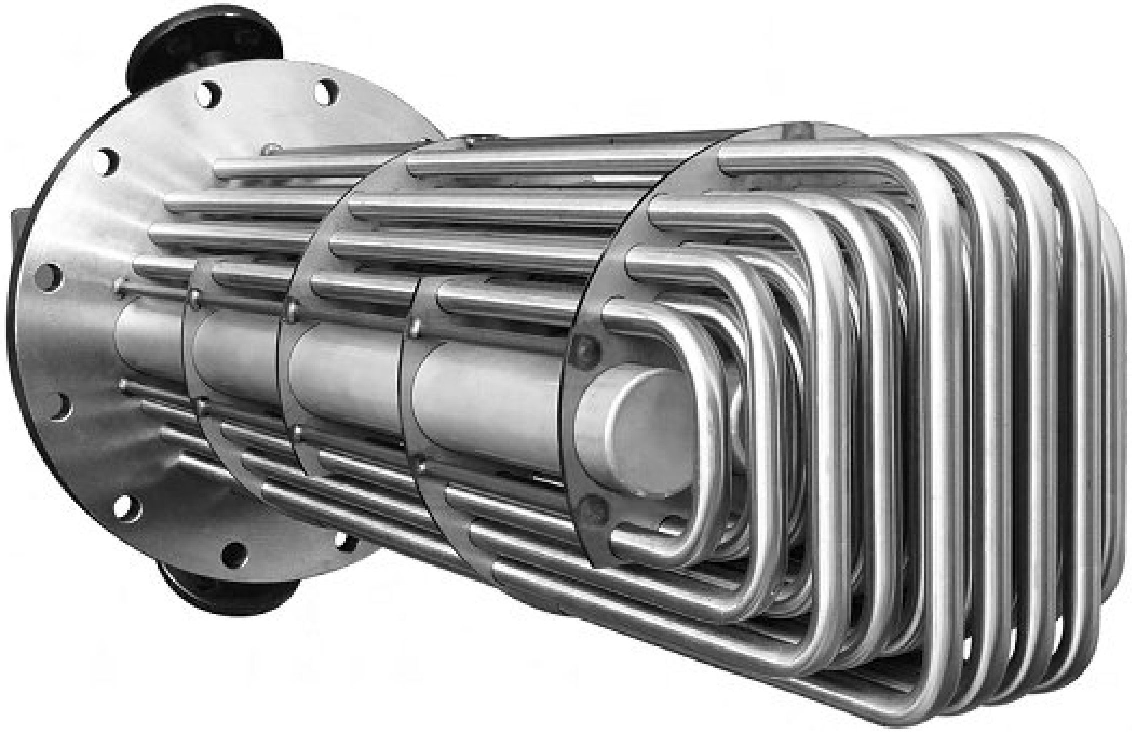
$$\overline{\mathrm{Nu_d}} = C\ \mathrm{Re_d}^m \mathrm{Pr}^{0,4}$$

Constants of equation (HTC.9) according to Jakob (1949)

Table 5.2: Constants C and m of equation (HTC.9)

Geometry	$\mathrm{Re_d}$	C	m
	$5 \cdot 10^3 - 10^5$	0,246	0,588
	$5 \cdot 10^3 - 10^5$	0,102	0,675
	$5 \cdot 10^3 - 1{,}94 \cdot 10^4$	0,160	0,638

Transverse flow around bundle of smooth tubes



https://www.heatsystems.de/produkt-details/rohrbuendel-waermetauscher.html

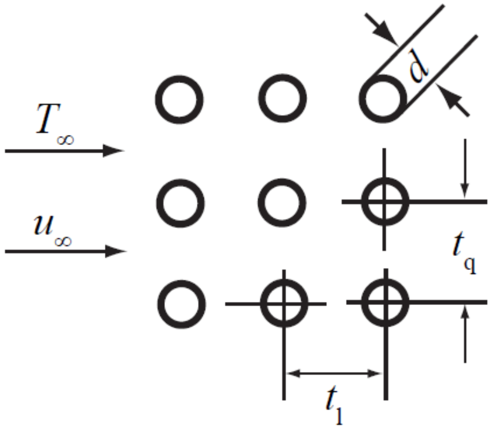
Transverse flow around bundle of smooth tubes

(Note: Since the temperature of the fluid in the tube bundle changes from row to row, a representative mean temperature difference must be used calculating the heat transfer.)

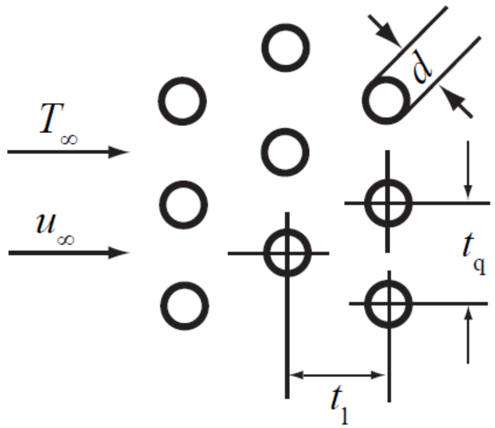
$$\dot{Q}_{\mathrm{W}} = \overline{\alpha} A \left(T_{\mathrm{W}} - T_{\mathrm{fl}}\right)_{\mathrm{m}} = \overline{\alpha} A \Delta T_{\mathrm{m}} \tag{5.9}$$

For many practical purposes, the logarithmic temperature difference is a good approximation

$$\Delta T_{\mathrm{m}} = \Delta T_{\mathrm{ln}} \equiv \frac{\Delta T_{\mathrm{inlet}} - \Delta T_{\mathrm{outlet}}}{\ln \frac{\Delta T_{\mathrm{inlet}}}{\Delta T_{\mathrm{outlet}}}}$$

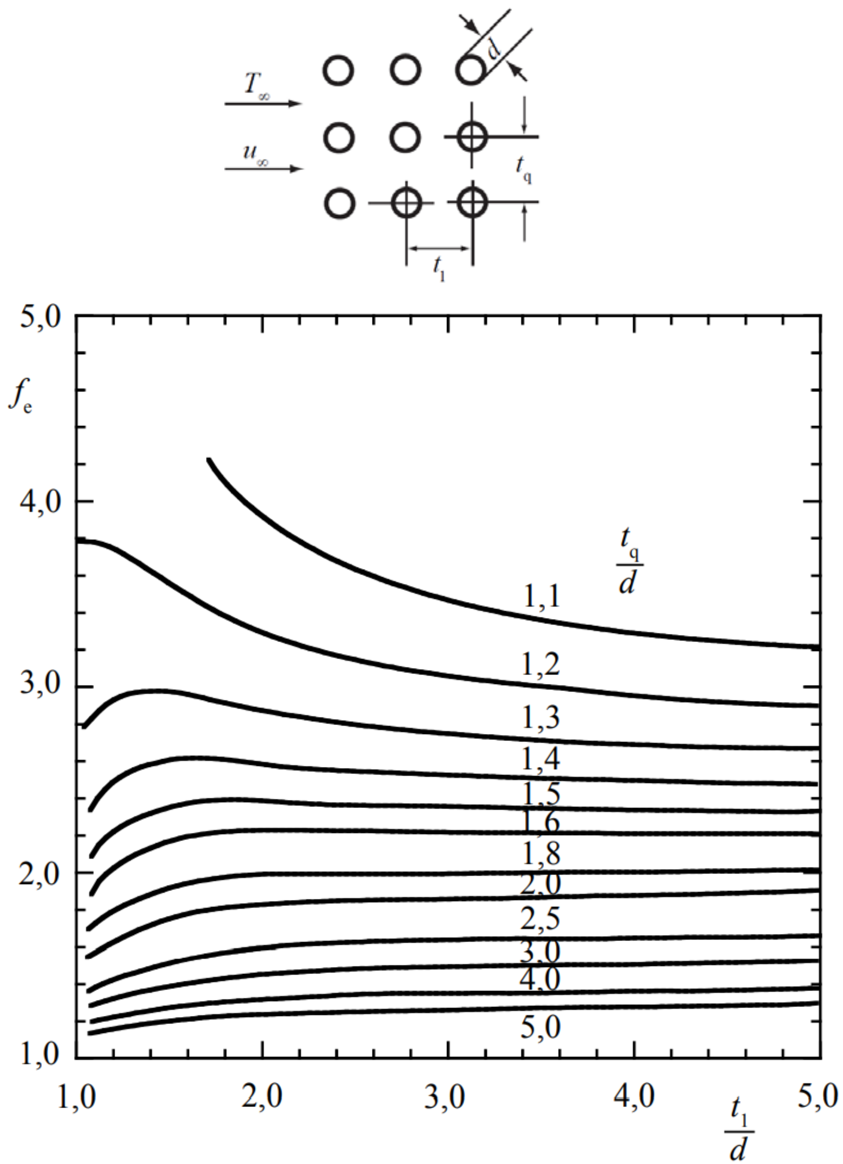


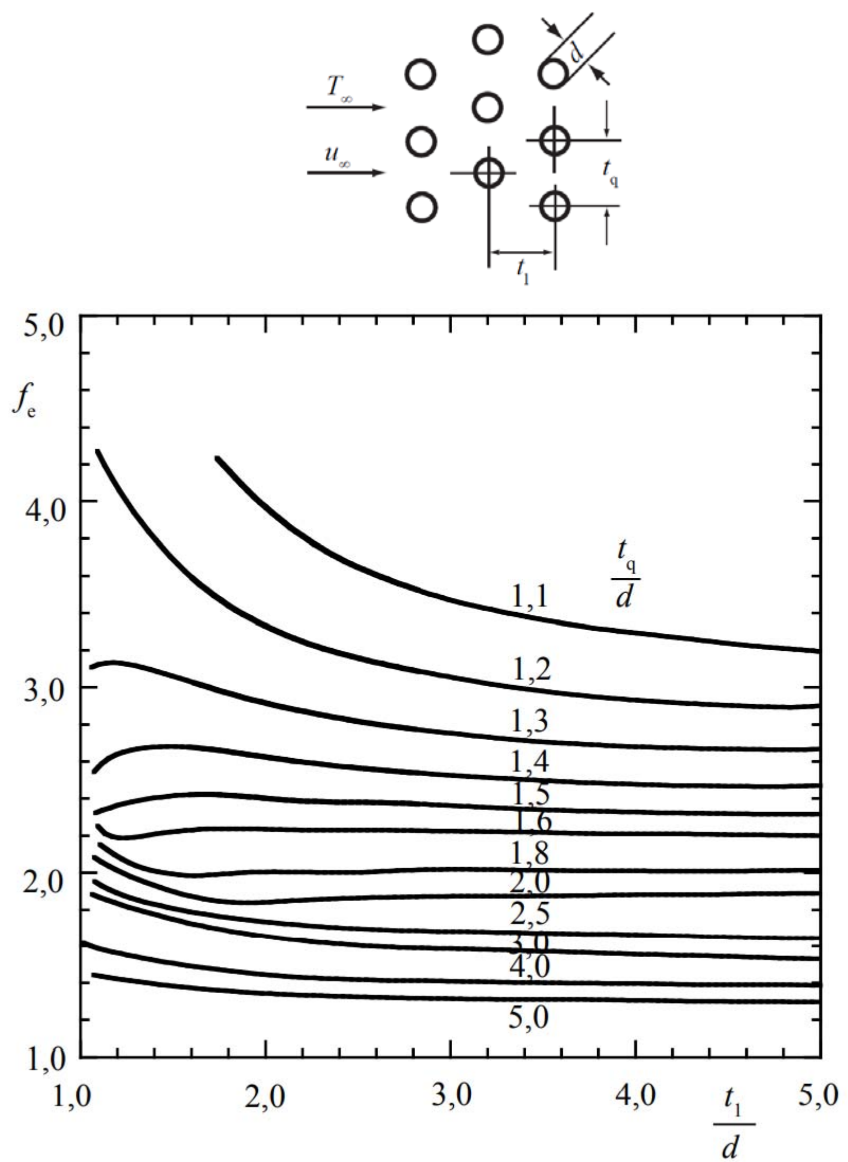
in-line arrangement



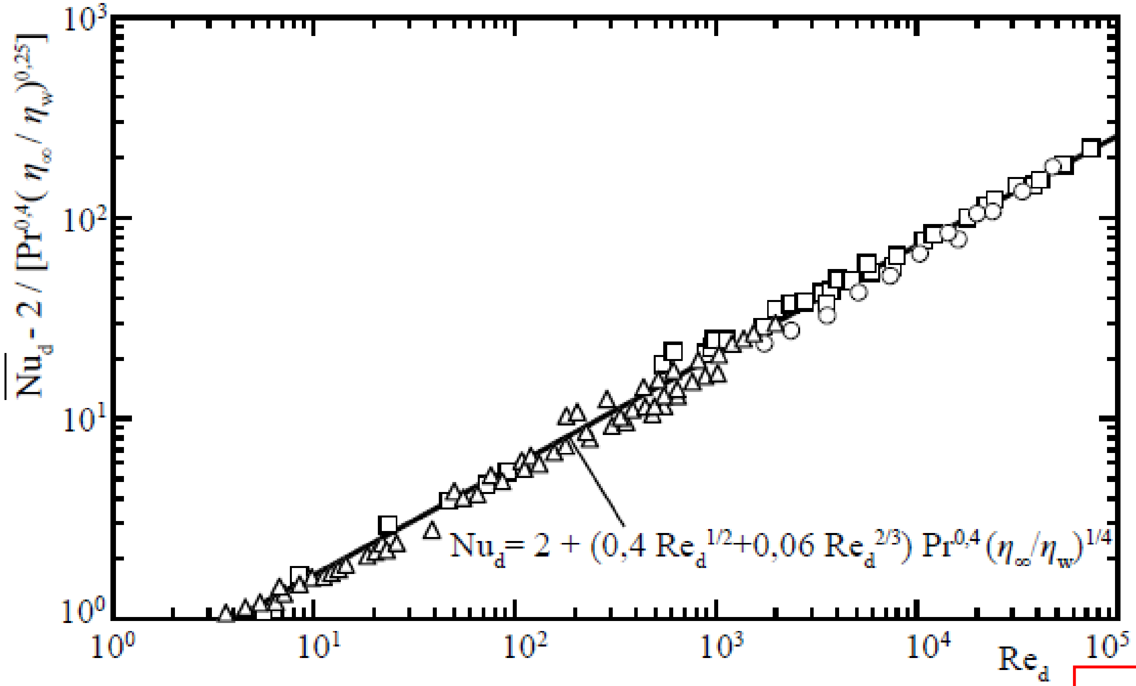
staggered arrangement

Transverse flow around bundle of smooth tubes





Flow around a sphere



- Whitaker correlation

$$\overline{Nu}_d = \boxed{2} + \left(0{,}4\, Re_d^{\frac{1}{2}} + 0{,}06\, Re_d^{\frac{2}{3}}\right) Pr^{0,4} \boxed{\left(\frac{\eta_\infty}{\eta_w}\right)^{\frac{1}{4}}} \qquad \text{(HTC.11)}$$

$$\text{for} \quad 0{,}7 < Pr < 380$$
$$\text{and} \quad 3{,}5 < Re_d < 8 \cdot 10^4$$

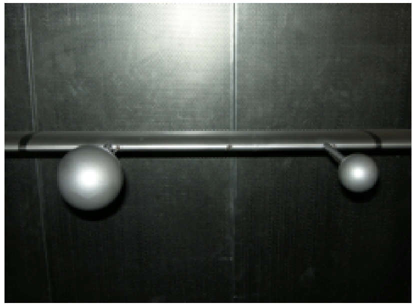
If $u = 0$ (sphere at rest) → $Nu_d = 2$

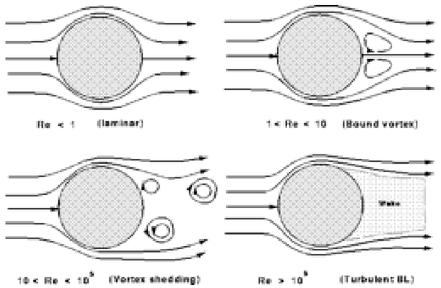
3 Whitaker, S. "Forced Convection Heat Transfer Correlations for Flow in Pipes, Past Flat Plates, Single Cylinders, Single Spheres, and for Flow in Packed Beds and Tube Bundles", A. I. Ch. E. Journal, Vol. 18 (1972), pp. 361-371

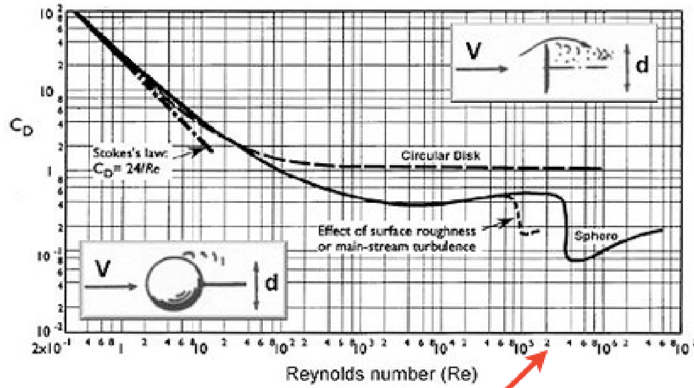
Flow around a sphere

Will, J., Venner, C.H., Kruyt, N.P., An experimental study of forced convective heat transfer from smooth solid spheres, J. of Heat and Mass Transfer, 109, 2017, 1059-1067

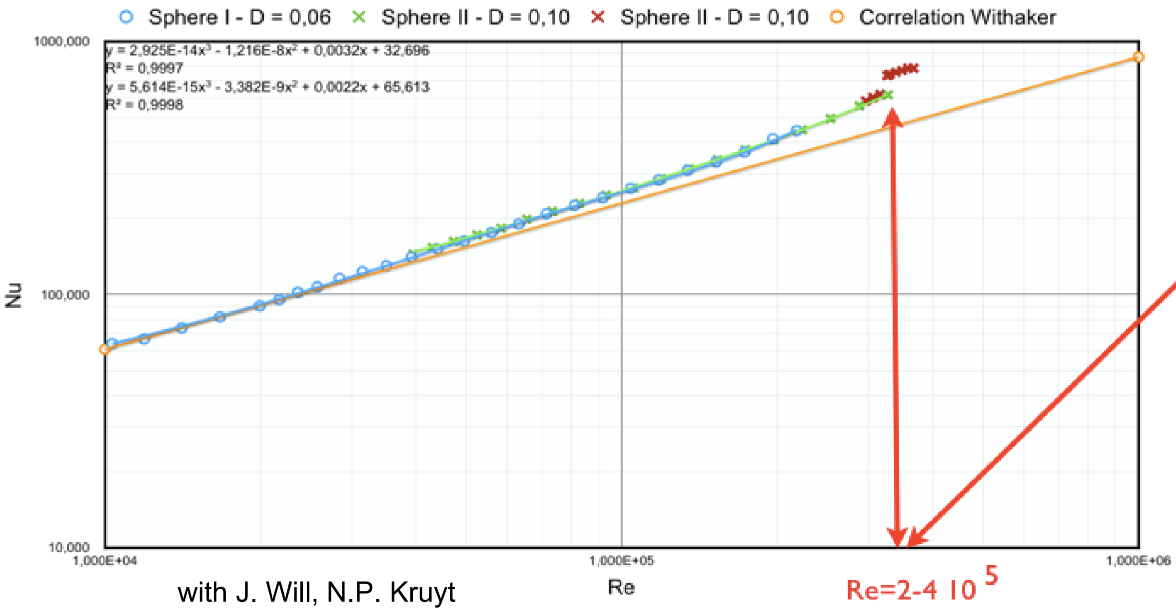
$$\alpha = \dot{Q}/(A\,(T_W - T_\infty))$$







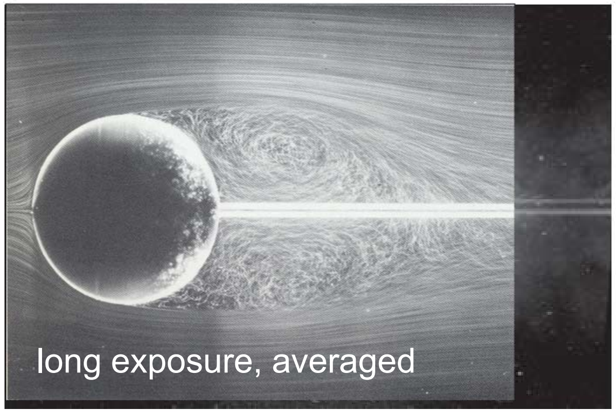
source: Dr J. Christy



with J. Will, N.P. Kruyt

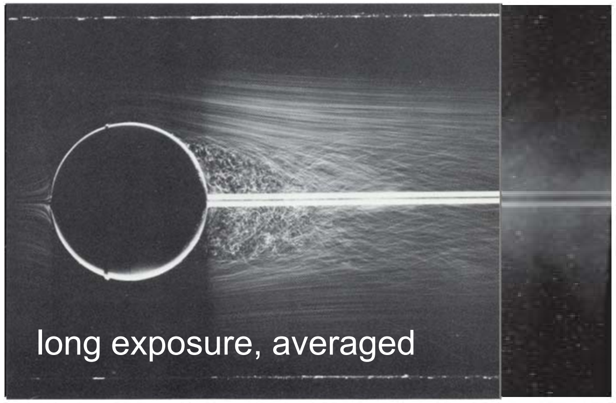
Re=2-4 10^5

Sphere in Flow: Visualization



(a)

Instantaneous flow past smooth sphere at R=15,000. Dye in water shows laminar boundary layer separating Before the equator and remaining laminar, and then Unstable and turbulent.



(b)

Instantaneous flow past a smooth sphere at Re=30,000 with a trip wire. Classical Experiment of Prandtl and Wieselsberger repeated. Boundary layer turning turbulent delays separation, reduces drag, and enhances heat transfer.

Source: ONERA, H. Werle

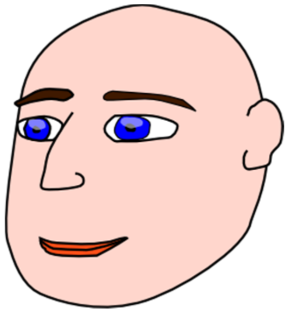
$$\overline{\mathrm{Nu_d}} = 2 + \left(0{,}4\ \mathrm{Re_d}^{\frac{1}{2}} + 0{,}06\ \mathrm{Re_d}^{\frac{2}{3}}\right) \mathrm{Pr}^{0{,}4} \left(\frac{\eta_\infty}{\eta_w}\right)^{\frac{1}{4}}$$

for $0{,}7 < \mathrm{Pr} < 380$
and $3{,}5 < \mathrm{Re_d} < 8 \cdot 10^4$

Example Application: Heat Transfer from Sphere

$T_\infty = -10\ [\ {}^\circ C]$

$U_\infty = 5\ \ [m/s]$



$T_W = 33\ \ {}^\circ C$

$D = 0.18\ [m]$

$A = \pi D^2 = 0.102\ [m^2]$

$Re = 62.230$

$Nu = 170.8$

$\alpha = Nu\lambda/D = 23.9\ [W/(m^2K)]$

$\dot{Q} = \alpha A(T_w - T_\infty) = 105\ [W]$

$\rho = 1.246\ [kg/m^3]$

$\lambda = 0.02476\ [W/(mK)]$

$\eta = 1.802\ 10^{-5}\ [Pa.s]$

$\eta_s = 1.872\ 10^{-5}\ [Pa.s]$

$Pr = 0.7323$

$T_W = 10\ \ {}^\circ C$

$Nu = 170.8$

$\alpha = Nu\lambda/D = 23.9\ [W/(m^2K)]$

$\dot{Q} = \alpha A(T_w - T_\infty) = 49\ [W]$

$U_\infty = 5\ \ [m/s]$

$T_\infty = -10\ [\ {}^\circ C]$

(where I assumed the air parameters to stay the same)

Comprehension Questions

Which dimensionless numbers have to be considered in forced convection?
How is the applicability of a correlation checked?

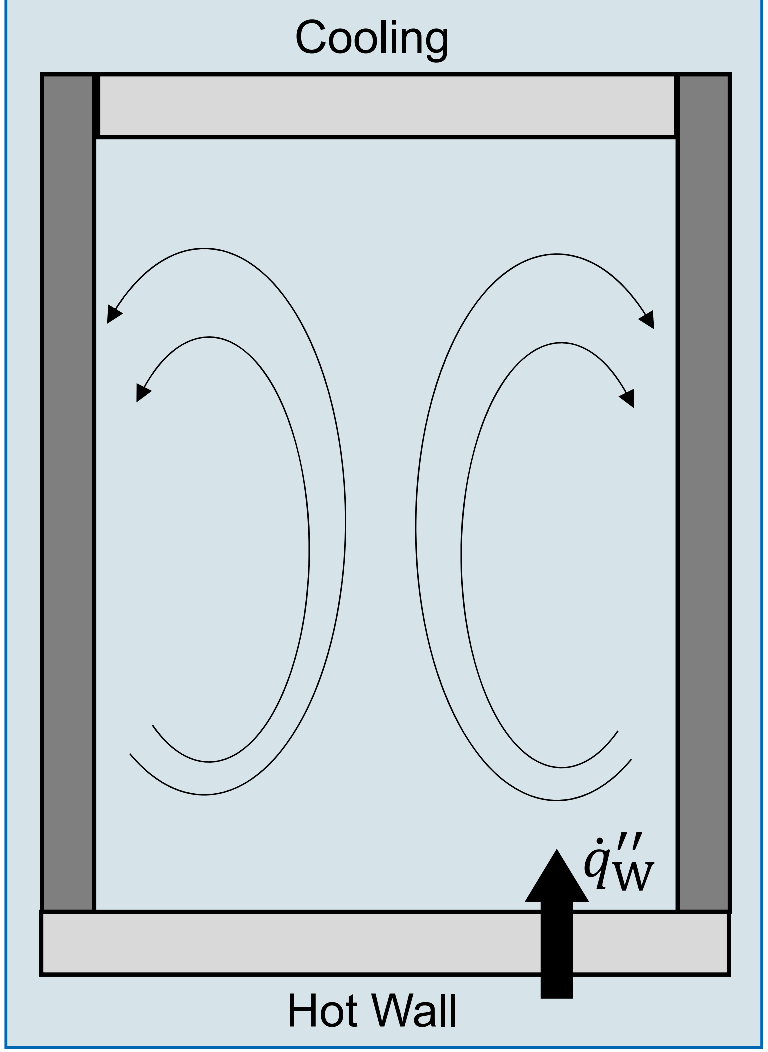
At what temperature are the material properties occurring in the dimensionless numbers to be determined?

What is the difference between local and average heat transfer in a flat plate with heating or cooling?

UNIVERSITY OF TWENTE.
Lehrstuhl für Wärme- und Stoffübertragung
RWTH AACHEN UNIVERSITY

Heat transfer with Natural and Forced Convection
Example: Cooling a hot surface

Natural Convection



Forced Convection