Heat Transfer: Conduction

Heat conduction in a multilayer pipe wall with convective resistances

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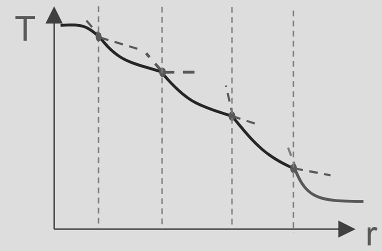
Learning goals

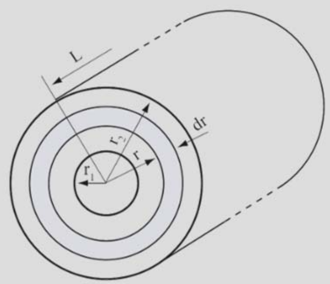
Temperature profile in multilayer pipe wall with convection:

- How does the surface area change in a multilayer pipe wall?
- How is the temperature profile in a multilayer pipe wall?



Heat flow in a multilayer pipe wall with convection:

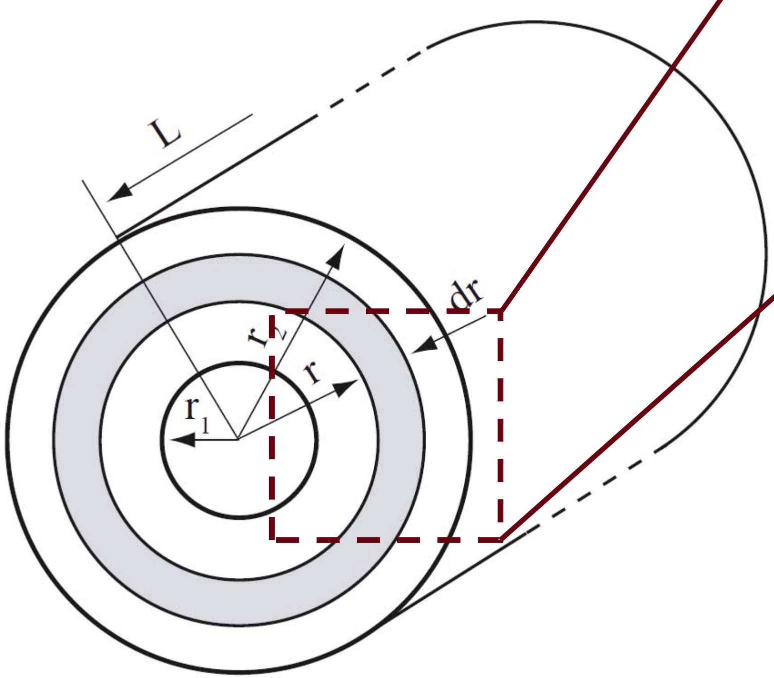
- How is calculated the total thermal resistance in a multilayer pipe wall?
- How is calculated the heat flow in a multilayer pipe wall?

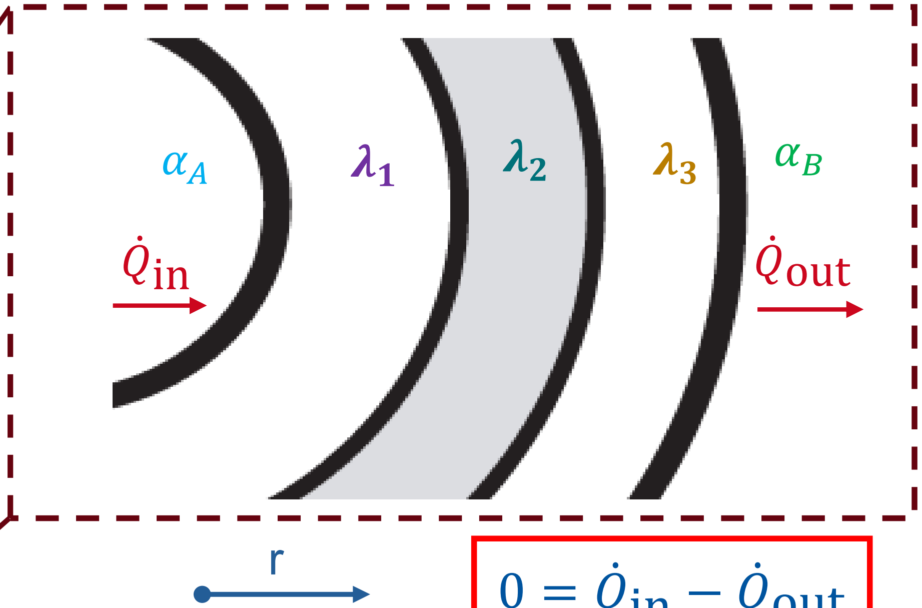


Multilayer pipewall with convection

Assumption:

- Steady state
- One-dimensional
- Constant material properties





$$\boxed{0 = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}}$$

Warning: $\boxed{\dot{q}''_{\text{in}} \neq \dot{q}''_{\text{out}}}$

With increasing radius from inside to outside the surface becomes larger!

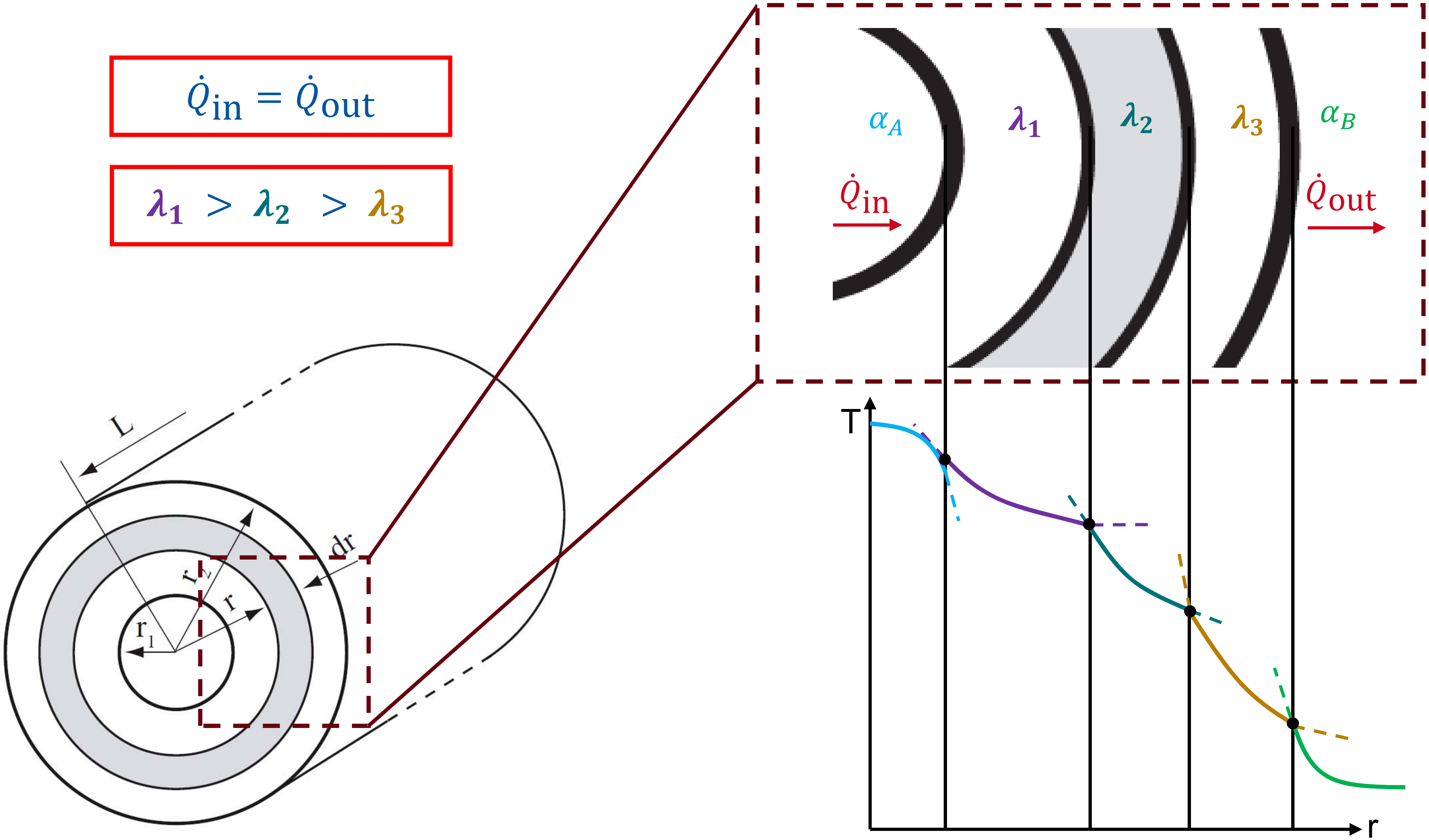
$$\dot{Q} = \dot{q}'' \cdot A$$

as: $A_{\text{out}} > A_{\text{in}} \Rightarrow \dot{q}''_{\text{out}} < \dot{q}''_{\text{in}}$

Temperature profile in a multilayer pipe wall with convection

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}}$$

$$\lambda_1 > \lambda_2 > \lambda_3$$

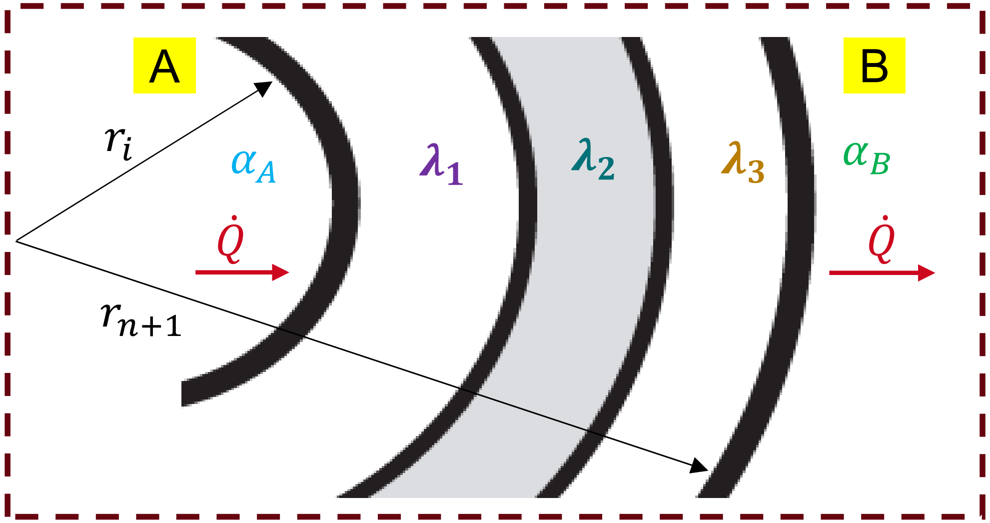


Heat flow in a multilayer pipe wall with convection

Convective resistance:

$$R_{\text{conv.},A} = \frac{1}{\alpha_A \cdot A_A} \qquad A_A = 2\pi r_i \cdot L$$

$$R_{\text{conv.},B} = \frac{1}{\alpha_B \cdot A_B} \qquad A_B = 2\pi r_{n+1} \cdot L$$



Heat flow in a multilayer pipe wall with convection

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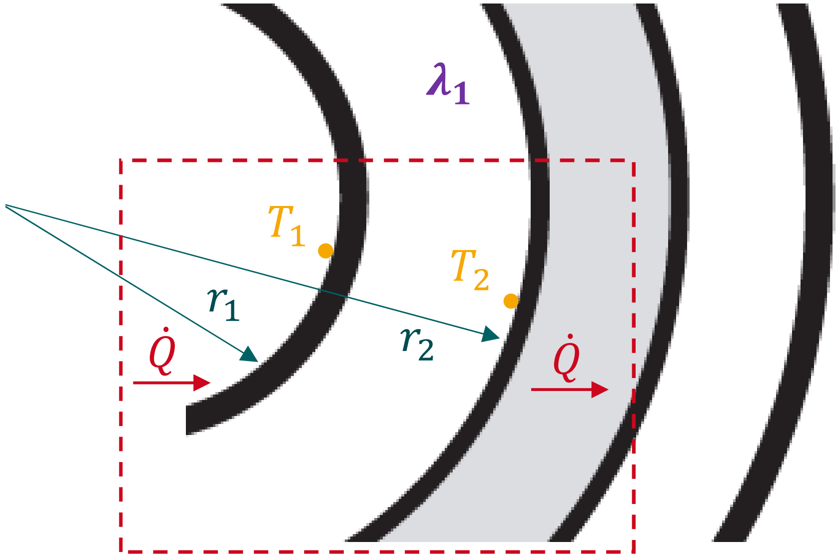

Review:

Heat conduction in pipe wall:

$$\dot{Q}_r = -\lambda_1 \cdot 2 \cdot \pi \cdot L \cdot \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}}$$

$$\dot{\mathrm{Q}} = \frac{\text{Temperature difference}}{\boxed{\text{Thermal resistance}}}$$

Heat flow in a multilayer pipe wall with convection

Convective resistance:

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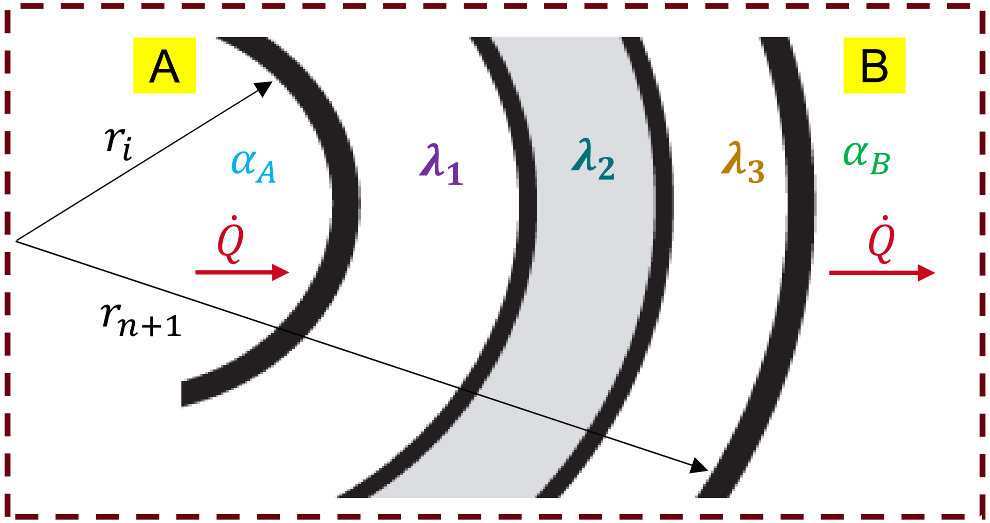
Resistance due to heat conduction:

$$R_c = \frac{1}{\lambda_i} \cdot \frac{1}{2\pi L} \cdot \ln \frac{r_{i+1}}{r_i}$$

$$\sum_i R_{c,i} = \frac{1}{2\pi L} \cdot \sum_{i=1}^{n} \frac{1}{\lambda_i} \cdot \ln \frac{r_{i+1}}{r_i}$$

$$\dot{Q} = \frac{T_A - T_B}{R_{conv.,A} + \sum R_c + R_{conv.,B}}$$

$$\dot{Q} = k \cdot A^* \cdot (T_A - T_B)$$



$$k \cdot A^* = \frac{1}{\sum R} = \frac{1}{\frac{1}{\alpha_A \pi d_i L} + \frac{1}{2\pi L} \cdot \sum_{i=1}^{n} \ln \frac{d_{i+1}}{d_i} + \frac{1}{\alpha_B \pi d_{n+1} L}}$$

Overall heat resistance in series:

$$\frac{1}{k} = \frac{d^*}{\alpha_1 d_1} + \frac{d^*}{2} \sum_{i=1}^{n} \ln \frac{d_{i+1}}{d_i} + \frac{d^*}{\alpha_B d_{n+1}}$$

d^*: Reference diameter
A*: Reference area

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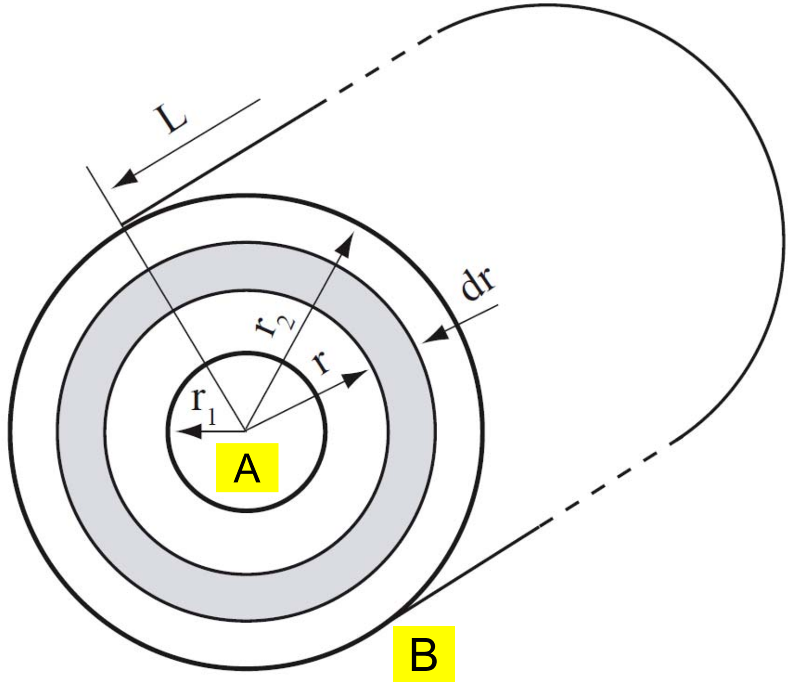

Surface area of the cylinder: $2\pi \, L \cdot r_i$

Equation in the book of formulary:

$$\dot{Q}_i = \frac{2\pi L}{\frac{1}{\alpha_A r_1} + \sum_{i=1}^{n} \frac{1}{\lambda_i} \ln\frac{r_{i+1}}{r_i} + \frac{1}{\alpha_B r_{n+1}}} (T_A - T_B)$$

Comprehension questions

How does the curved surface of a pipe affect the temperature gradient at constant heat flow and constant thermal conductivity?

What reference area and reference diameter must be considered when calculating the total heat transfer coefficient k for a pipe wall problem?

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