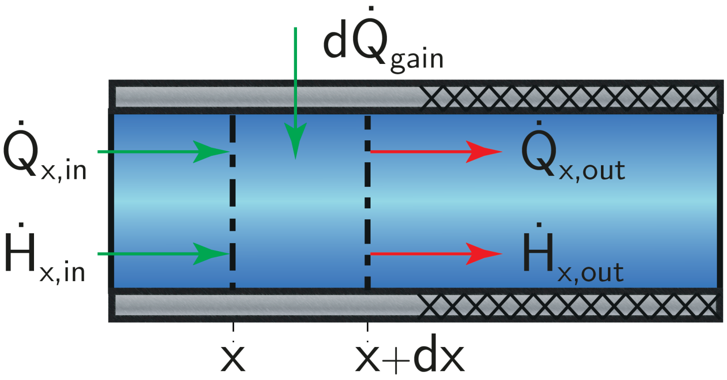
# EB - Conv. - IE 4

**Through a very long pipe with diameter $D$ flows a fluid. The first half of the pipe is being heated with a constant rate $\dot{q}''$. The second half of the pipe is fully adiabatic.**

**Derive the differential equation for the temperature profile in the flow direction in the first segment of the pipe, while not neglecting the diffusive heat transport in the direction of the flow.**




**Energy balance:**

$$0 = \dot{Q}_{\text{x,in}} + \dot{Q}_{\text{x,out}} + \dot{H}_{\text{x,in}} - \dot{H}_{\text{x,out}} + d\dot{Q}_{\text{gain}}$$

**Definition of fluxes:**

$$\dot{Q}_{\text{x,in}} = -\lambda \cdot \tfrac{\pi \cdot D^2}{4} \cdot \tfrac{\partial T}{\partial x}$$

$$\dot{Q}_{\text{x,out}} = \dot{Q}_{\text{x,in}} + \tfrac{\partial \dot{Q}_{\text{x,in}}}{\partial x} \cdot dx$$

$$\dot{H}_{\text{x,in}} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{\text{x,out}} = \dot{H}_{\text{x,in}} + \tfrac{\partial \dot{H}_{\text{x,in}}}{\partial x} \cdot dx$$

$$d\dot{Q}_{\text{gain}} = \dot{q}'' \cdot \pi \cdot D \cdot dx$$

Where:

$$\dot{m} = u \cdot \tfrac{\pi \cdot D^2}{4} \cdot \rho$$

**Substituting and rewriting:**

$$0 = \lambda \cdot \frac{\pi \cdot D^2}{4} \cdot \frac{\partial^2 T}{\partial x^2} \cdot dx - u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho \cdot c \cdot \frac{\partial T}{\partial x} \cdot dx + \dot{q}'' \cdot \pi \cdot D \cdot dx$$

$$0 = \frac{\lambda \pi D^2}{4} \frac{\partial^2 T}{\partial x^2} - \frac{u \rho c \pi D^2}{4} \frac{\partial T}{\partial x} + \dot{q}'' \pi D$$