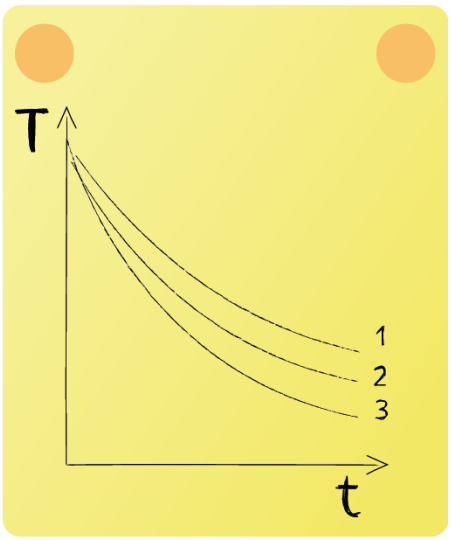
Lecture 6 Question 4

A sphere, a cube, and a thin circular plate, all made of the same material and having the same volume are initially heated up above room temperature. Afterward, they are left at room temperature. Assume all three to have a homogeneous temperature distribution. Which temperature profile do you expect to belong to which object? The effect of the difference in heat transfer coefficient is negligible.



When having a homogeneous temperature distribution, the change in body temperature is determined by the rate of heat loss. Heat is lost due to convection.

$$\dot{Q}_{conv} = \alpha \cdot A_s \cdot (T(t) - T_\infty)$$

Since they are under identical environmental conditions $\alpha_{sphere} = \alpha_{plate} = \alpha_{cube}$. For the surface Since the bodies are of the same volume, the relation of their surface areas is only depending on their shapes, yielding: $A_{plate} > A_{cube} > A_{sphere}$. Which is the order of fastest to slowest cool down.

$$V_{\text{cube}} = a^3 \Rightarrow a = \sqrt[3]{V}$$

$$A_{\text{cube}} = 6a^2 = 6V^{2/3}$$

$$V_{\text{sphere}} = \frac{4}{3}\pi a^3 \Rightarrow a = \sqrt[3]{\frac{3}{4\pi}V} \approx 4.8V^{2/3}$$

$$A_{\text{sphere}} = 4\pi a^2 = 4\pi \left(\frac{3}{4\pi}V\right)^{2/3}$$

$$V_{\text{thin plate}} = 2h\pi a^2 \rightarrow a = \sqrt{\frac{1}{2h\pi}V}$$

$$A_{\text{thin plate}} \approx \pi a^2 = \frac{1}{2h}V \approx 10^{\cdots}V$$

$A_{\text{thin plate}}$ becomes really big relative to A_{sphere} and A_{cube} as h is really small.