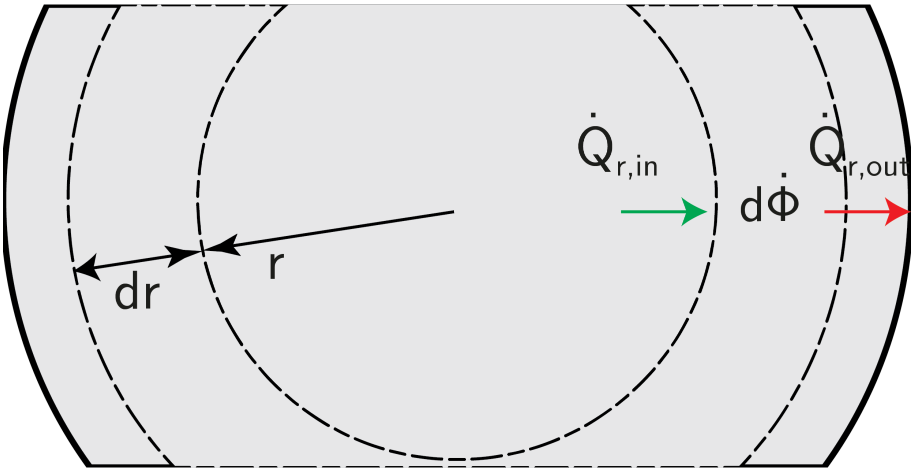
EB - Cond. - IE 13

Develop an energy balance to calculate the temperature profile inside the cylinder and give the boundary conditions. The cylinder is losing heat to the environment. Assume one-dimensional steady-state heat with a source.



Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} + d\dot{\Phi} = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda \cdot 2 \cdot \pi \cdot r \cdot L \cdot \frac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} = -\lambda \cdot 2 \cdot \pi \cdot r \cdot L \cdot \frac{\partial T}{\partial r} + \frac{\partial \dot{Q}_{r,in}}{\partial r} \cdot dr$$

$$d\dot{\Phi} = \dot{\Phi}''' \cdot 2 \cdot \pi \cdot r \cdot L \cdot dr$$

The heat entering the system is transferred from the centre of the cylinder by conductive heat transfer. Heat is generated because of the source. $\dot{Q}_{\mathrm{r,out}}$ can be approximated by use of the Taylor series expansion.

Boundary Conditions:

$$\frac{\partial T(r=0)}{\partial r} = 0$$

$$T(r = r_1) = T_1$$

The first boundary condition describes that the temperature gradient in the center equals zero. This is because of symmetry. The second one describes that the temperature at the surface equals T_1.