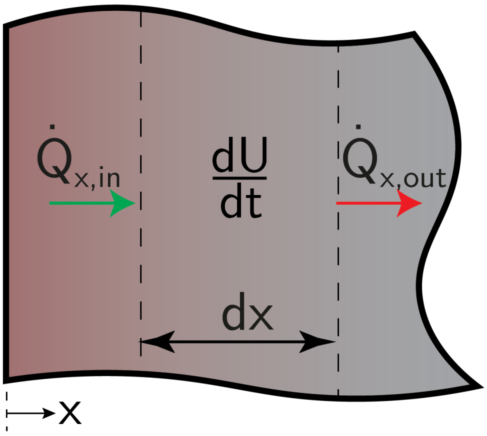
Boundary Conditions - Cond. - IE 17

A very thick wall $\mathrm{Fo} \ll 1$, initially at a homogeneous temperature T_0, is heated up at the left-hand side. Specify the boundary and initial conditions to solve the governing energy balance used for calculating the temperature profile inside the body:



Given the differential equation:

$$\rho c_\mathrm{p} \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2}$$

In order to solve the differential equation, two boundary conditions and one initial condition are required. This can be seen from the fact that the variable T has been differentiated twice with respect to x and once with respect to t.

Initial condition:

$$T(t=0) = T_0$$

The initial condition should apply for the entire domain for $t = 0$. The given condition describes that initially the body was at a homogeneous temperature T_0.

Boundary conditions:

$$\frac{\partial T}{\partial x}|_{x=0} = -\frac{\dot{q}''}{\lambda}$$

$$\frac{\partial T}{\partial x}|_{x\to\infty} = 0$$

The first boundary condition yields from an energy balance at the interface, which states that the imposed heat flux is conducted through the body: $\dot{Q} = \dot{q}''A = -\lambda A \frac{\partial T}{\partial x}|_{x=0}$

The second boundary condition yield from the fact that we are dealing with a semi-infinite body, which has the property that no heat is conducted anymore when $x \to \infty$, which can be written as: $\dot{Q} = -\lambda A \frac{\partial T}{\partial x}|_{x=0} = 0$