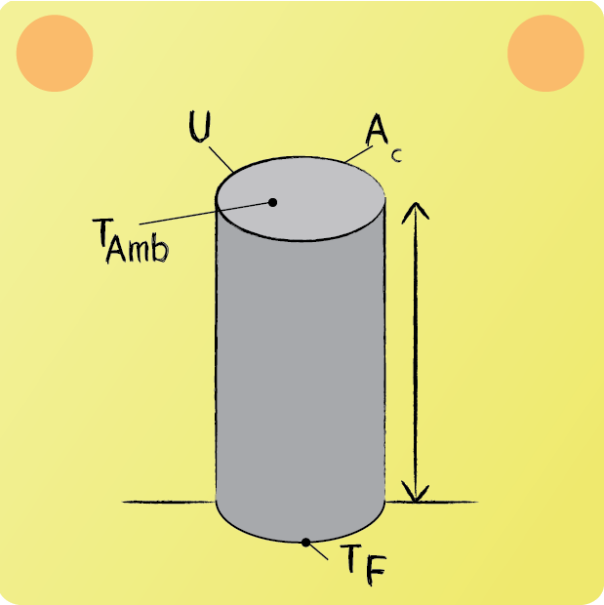
Exam Preparation Conduction 07



A rod-fin, with a temperature equal to the ambient temperature T_{Amb} at the end of the fin is given. What is the correct expression of the total heat flux carried away from the fin

The following equation for the fin temperature can be taken from the formulary:

$$\Theta(x) = A \cdot cosh(m \cdot x) + B \cdot sinh(m \cdot x)$$

with

$$\Theta(x) = T(x) - T_{\text{amb}}$$

and

$$m^2 = \frac{\alpha \cdot U}{\lambda \cdot A_{\text{Q}}}$$

The constants A and B should be determined using the boundary conditions.
The temperature at the fin base is T_{b}:

$$\Theta(x = 0) = A = T_{\text{b}} - T_{\text{amb}} = \Theta_{\text{b}}$$

The temperature at the fin head is T_{amb}:

$$\Theta(x = L) = A \cdot cosh(m \cdot L) + B \cdot sinh(m \cdot L) = T_{\text{amb}} - T_{\text{amb}} = 0$$

$$\Rightarrow B = -A \cdot \frac{1}{tanh(m \cdot L)}$$

Plugging in the constants yields the equation for the fin temperature:

$$\Theta(x) = \Theta_{\text{b}} \cdot \left(cosh(m \cdot x) - \frac{sinh(m \cdot x)}{tanh(m \cdot L)} \right)$$

Dissipated heat through the fin:

$$\dot{Q} = -\lambda \cdot A_{\text{C}} \cdot \frac{d\Theta}{dx}|_{x=0}$$

with

$$\frac{d\Theta}{dx}|_{x=0} = \Theta_{\text{b}} \cdot \left(m \cdot sinh(m \cdot 0) - \frac{m \cdot cosh(m \cdot 0)}{tanh(m \cdot L)} \right) = -\frac{\Theta_{\text{b}} \cdot m}{tanh(m \cdot L)}$$