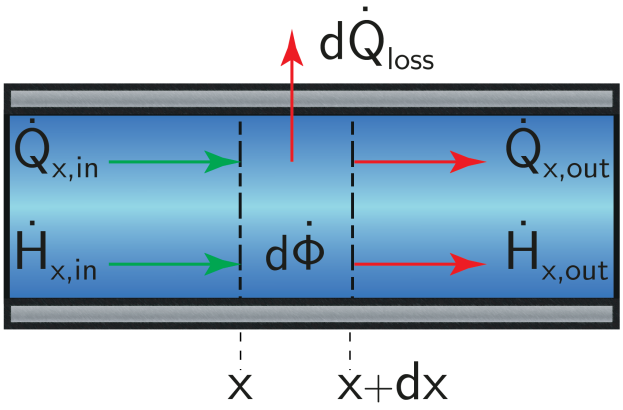
EB - Conv. - IE 1

Through a very long pipe with diameter D flows a heat generating fluid (homogeneous and constant source strength $\dot{\Phi}''' > 0$). In addition, the pipe has a uniform, constant wall temperature T_w.

Derive the differential equations for the temperature profile in the flow direction, not neglecting the diffusive heat transport in the direction of the flow.



Energy balance:

$$\dot{Q}_{x,in} + \dot{Q}_{x,out} + \dot{H}_{x,in} - \dot{H}_{x,out} - d\dot{Q}_\mathrm{loss} + d\dot{\Phi} = 0$$

Energy fluxes:

$$\dot{Q}_{x,in} = -\lambda \cdot \tfrac{\pi \cdot D^2}{4} \cdot \tfrac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = \dot{Q}_{x,in} + \tfrac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx$$

$$\dot{H}_{x,in} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{x,out} = \dot{H}_{x,in} + \tfrac{\partial \dot{H}_{x,in}}{\partial x} \cdot dx$$

$$d\dot{Q}_\mathrm{loss} = \alpha \cdot \pi \cdot D \cdot dx \cdot (T - T_\mathrm{w})$$

$$d\dot{\Phi} = \dot{\Phi}''' \cdot \tfrac{\pi \cdot D^2}{4} \cdot dx$$

Mass flow rate:

$$\dot{m} = u \cdot \tfrac{\pi \cdot D^2}{4} \cdot \rho$$