Mass Transfer: Diffusion

Example for analogy: Transient 1-D

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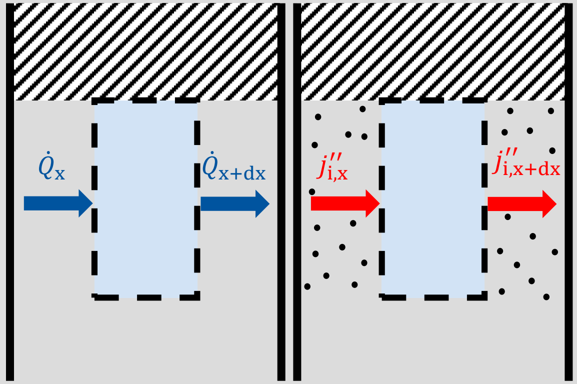
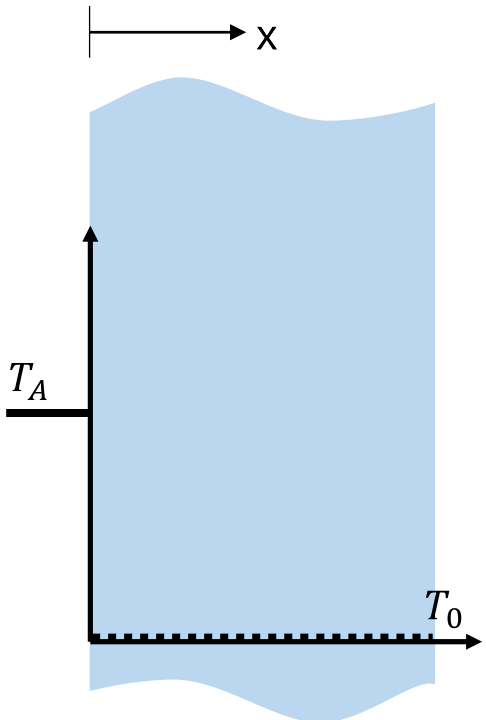
Learning goals

Example - Analogy transient heat conduction and diffusion:

- Review of the solution of the one-dimensional heat conduction problem
- Understand the steps to solve the one-dimensional diffusion problem
- Understand to apply Heat Conduction “knowledge” to Diffusion problems



Description of the 1-D transient heat conduction problem



Plate

T_0 = Initial temperature of the plate (---)
T_A = ambient temperature

Diff. Equation → Semi-infinite plate:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial x^2} = a \frac{\partial^2 T}{\partial x^2}$$

Substitution: $$\frac{d^2\theta}{d\eta^2} + 2\eta \frac{d\theta}{d\eta} = 0$$

Solution: $$\theta = 1 - \mathrm{erf}(\eta)$$

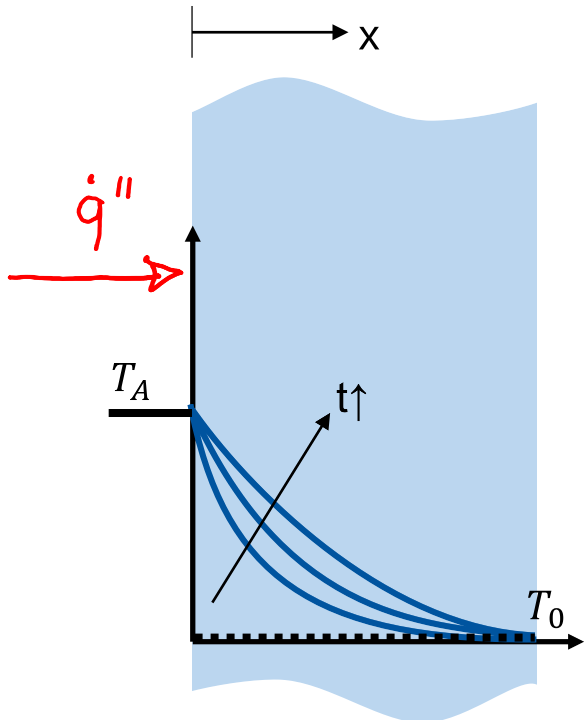
Initial and boundary conditions:

- $T(t = 0, x) = T_0$
- $T(t > 0, x = 0) = T_\mathrm{A}$
- $T(t > 0, x \to \infty) = T_0$

Temp. difference and transformation:

$$\theta = \frac{T - T_0}{T_A - T_0} \qquad \eta = \frac{x}{\sqrt{4at}}$$

Solution of the 1-D transient heat conduction problem



Plate

Heat flow at the wall:

$$\dot{q}''_{x=0} = -\lambda \left.\frac{\partial T}{\partial x}\right|_{x=0} = \frac{\lambda}{\sqrt{\pi a t}} (T_A - T_0)$$

$$\dot{q}''_{x=0} = \sqrt{\frac{\lambda \rho c_p}{\pi t}} (T_A - T_0)$$

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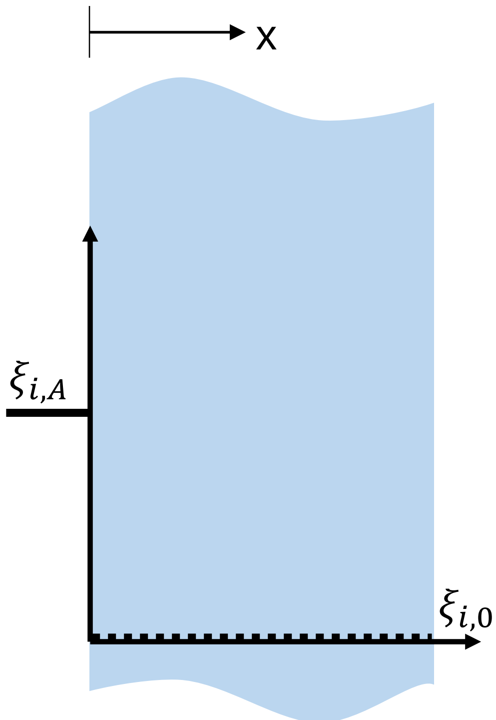
Solution of the 1-D transient diffusion problem



Plate

$\xi_{i,0}$ = Initial mass fraction of component i inside the plate
$\xi_{i,A}$ = ambient value of mass fraction of i

Diffusion Equation → Semi-infinite plate:

$$\frac{\partial \rho_i}{\partial t} = \rho D \frac{\partial^2 \xi_i}{\partial x^2} \Rightarrow \frac{\partial \xi_i}{\partial t} = D \frac{\partial^2 \xi_i}{\partial x^2} = a \frac{\partial^2 \xi_i}{\partial x^2}$$

Substitution: $\frac{d^2 \boldsymbol{\Xi}}{d\eta^2} + 2\eta \frac{d\boldsymbol{\Xi}}{d\eta} = 0$

Solution: $\boldsymbol{\Xi} = 1 - \mathrm{erf}(\eta)$

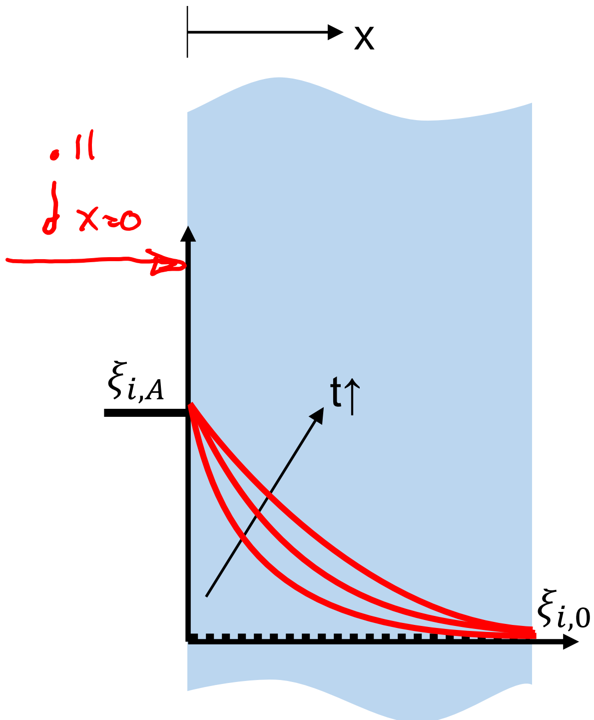
Initial and boundary conditions:

- $\xi_i(t = 0, x) = \xi_{i,0}$
- $\xi_i(t > 0, x = 0) = \xi_{i,A}$
- $\xi_i(t > 0, x \rightarrow \infty) = \xi_{i,0}$

Dimensionless concentration and transformation:

$$\boldsymbol{\Xi} = \frac{\xi_i - \xi_{i,0}}{\xi_{i,A} - \xi_{i,0}} \qquad \eta = \frac{x}{\sqrt{4Dt}}$$

Solution of the 1-D transient diffusion problem



Plate

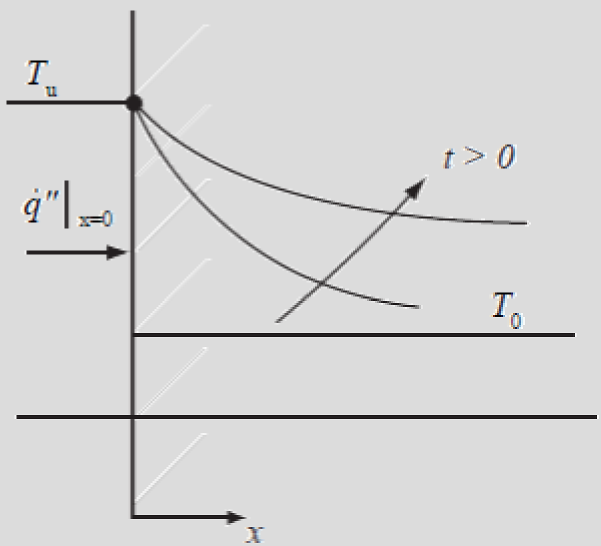
Diffusion flow at the surface:

$$j_i''\Big|_{x=0} = j_{i,x=0}'' = \rho\sqrt{\frac{D}{\pi t}}\left(\xi_{i,A} - \xi_{i,0}\right)$$

Comparison of thermal diffusion and mass diffusion

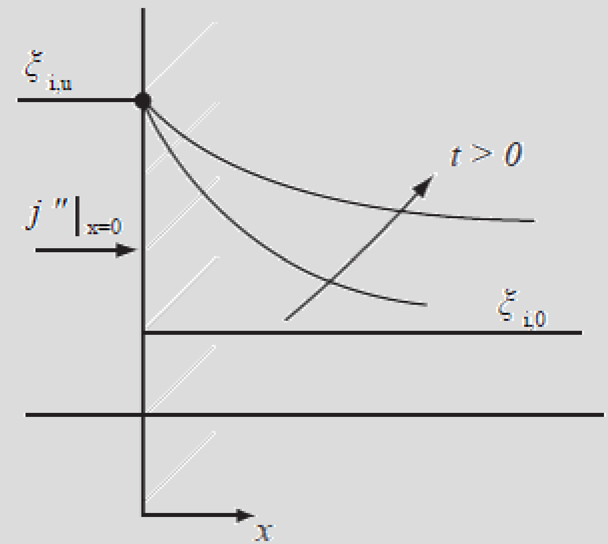
Thermal diffusion:

$$\dot{q}''_{x=0} = \sqrt{\frac{\lambda \rho c_p}{\pi t}}\,(T_A - T_0)$$



Mass diffusion:

$$j''_{i,x=0} = \rho \sqrt{\frac{D}{\pi t}}\left(\xi_{i,A} - \xi_{i,0}\right)$$



Comprehension questions

Which one is the "semi-infinite" bc? What does "semi-infinite" mean?
Can a piece of paper be regarded as being "semi-infinite?

Which initial and boundary conditions are chosen when solving the one-dimensional transient diffusion problem?

Assuming that temperature or mass fraction at the surface are identical to the free stream values: which value of α (heat transfer coefficient) or g (mass transfer coefficient) is defined by this assumption?