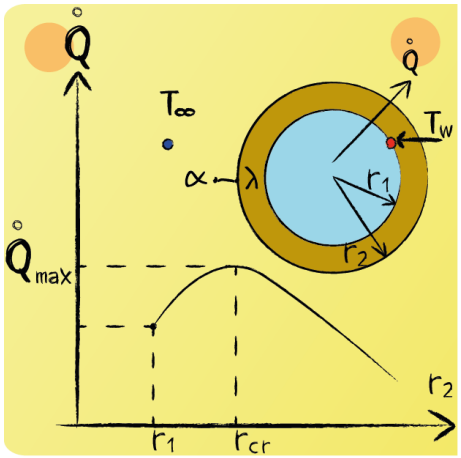
Lecture 8 Question 6

For a very thin wall pipe wall, an effect of increasing the wall thickness is an increase of the rate of heat transfer. Give an expression for the critical radius.



Between T_w and T_∞ we have 1 solid layer and convection is taking place at the outside.

$$\dot{Q} = \frac{\Delta T}{R_\mathrm{total}} = \frac{2\cdot\pi\cdot L\cdot(T_\mathrm{w}-T_\infty)}{\frac{1}{\lambda}\cdot\ln\frac{r_2}{r_1}+\frac{1}{\alpha\cdot r_2}} \Rightarrow R_\mathrm{total} = \frac{\frac{1}{\lambda}\cdot\ln\frac{r_2}{r_1}+\frac{1}{\alpha\cdot r_2}}{2\cdot\pi\cdot L}$$

In the figure it can be seen that for $r = r_\mathrm{cr}$ a result is $\frac{d\dot{Q}}{dr_2} = 0$ and thus $\frac{dR_\mathrm{total}}{dr_2} = 0$

$$\frac{d}{dr_2}\left(\frac{\frac{1}{\lambda}\cdot\ln\frac{r_2}{r_1}+\frac{1}{\alpha\cdot r_2}}{2\cdot\pi\cdot L}\right)\Bigg|_{r_2=r_\mathrm{cr}} = 0$$

$$\frac{d}{dr_2}\left(\frac{1}{\lambda}\cdot(\ln r_2 - \ln r_1)+\frac{1}{\alpha\cdot r_2}\right)\Bigg|_{r_2=r_\mathrm{cr}} = 0$$

$$\left(\frac{1}{\lambda\cdot r_2}-\frac{1}{\alpha\cdot r_2^2}\right)\Bigg|_{r_2=r_\mathrm{cr}} = 0$$

$$r_\mathrm{cr} = \frac{\lambda}{\alpha}$$