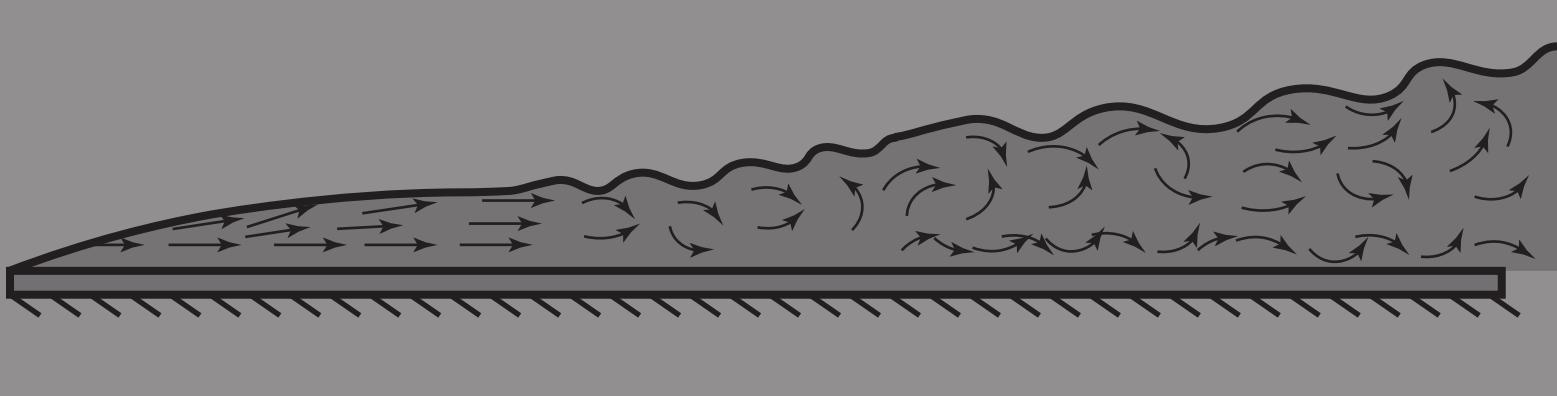




# HEATQUIZ

# HEAT TRANSFER

Book of Formularies



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## SECTION D

## Dimensionless numbers

### D.1 Fluid mechanics

**Definition****Reynolds number:**

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho u L}{\eta} = \frac{u L}{\nu} \quad (-), \quad (\text{D.1})$$

where  $L$  is the characteristic length.**Definition****Grashof number:**

$$\text{Gr} = \frac{\text{Bouyancy forces}}{\text{Viscous forces}} = \frac{\beta g \rho^2 (T_w - T_\infty) L^3}{\eta^2} = \frac{\beta g (T_w - T_\infty) L^3}{\nu^2} \quad (-), \quad (\text{D.2})$$

where  $L$  is the characteristic length.**Definition****Archimedes number:**

$$\text{Ar} = \frac{\text{Gr}}{\text{Re}^2} = \frac{\text{Bouyance forces}}{\text{Inertia forces}} = \frac{g_x \beta L (T_w - T_\infty)}{u_\infty^2} \quad (-), \quad (\text{D.3})$$

where  $L$  is the characteristic length.

## D.2 Heat transfer

**Definition**

**Biot number:**

$$Bi = \frac{\text{Conductive thermal resistance in body}}{\text{Convective thermal resistance at surface}} = \frac{\alpha L}{\lambda_s} \quad (-), \quad (\text{D.4})$$

where  $\lambda_s$  is the thermal conductivity of the solid and  $L = V/A$ .

**Definition**

**Fourier number:**

$$Fo = \frac{\text{Rate of diffusivity}}{\text{Rate of storage}} = \frac{\alpha t}{L^2} \quad (-), \quad (\text{D.5})$$

where  $L$  is the characteristic length.

**Definition**

**Nusselt number:**

$$Nu = \text{Dimensionless heat transfer coefficient} = \frac{\alpha L}{\lambda_f} \quad (-), \quad (\text{D.6})$$

where  $\lambda_f$  is the thermal conductivity of the fluid and  $L$  is the characteristic length.

**Definition**

**Prandtl number:**

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\eta}{\lambda/c_p} \quad (-). \quad (\text{D.7})$$

**Definition**

**Peclet number:**

$$Pe = Re Pr = \frac{\text{Rate of advection}}{\text{Rate of diffusion}} = \frac{\rho c_p u L}{\lambda} \quad (-), \quad (\text{D.8})$$

where  $L$  is the characteristic length.

## SECTION C

## Conduction

### C.1 Fundamentals

**Fundamental EQ** Fourier's law:

$$\dot{q}'' = -\lambda \frac{\partial T}{\partial x}. \quad (\text{C.1})$$

### C.2 Conservation equations

**Fundamental EQ** Equation of energy conservation for solids in Cartesian coordinates  $(x,y,z,t)$ :

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + \dot{\Phi}'''. \quad (\text{C.2})$$

**Fundamental EQ** Equation of energy conservation for solids in cylindrical coordinates  $(r,\theta,z,t)$ :

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( \lambda \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + \dot{\Phi}'''. \quad (\text{C.3})$$

**Fundamental EQ** Equation of energy conservation for solids in spherical coordinates  $(r,\theta,\phi,t)$ :

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \lambda \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \left( \lambda \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( \lambda \frac{\partial T}{\partial \phi} \right) + \dot{\Phi}'''. \quad (\text{C.4})$$

**Definition**

Thermal diffusivity:

$$a = \frac{\lambda}{\rho c} \left( \frac{m^2}{s} \right). \quad (\text{C.5})$$

### C.3 Multi-layer walls

**Fundamental EQ** Rate of heat transfer through a solid multi-layer wall without convection:

$$\dot{Q} = \frac{1}{\sum_{i=1}^n R_{\text{cond},i}} (T_1 - T_{n+1}). \quad (\text{C.6})$$

**Definition**Conductive resistance of a solid plane layer  $i$ :

$$R_{\text{cond},i} = \frac{\delta_i}{A \lambda_i} \left( \frac{\text{K}}{\text{W}} \right). \quad (\text{C.7})$$

**Definition**Conductive resistance of a solid cylindrical layer  $i$ :

$$R_{\text{cond},i} = \frac{1}{2\pi L \lambda_i} \ln \frac{r_{i+1}}{r_i} \left( \frac{\text{K}}{\text{W}} \right). \quad (\text{C.8})$$

**Fundamental EQ****Rate of heat transfer through a solid multi-layer wall with convection:**

$$\dot{Q} = \frac{1}{R_{\text{conv},A} + \sum_{i=1}^n R_{\text{cond},i} + R_{\text{conv},B}} (T_A - T_B). \quad (\text{C.9})$$

**Definition****Convective resistance of a solid plane layer  $j$ :**

$$R_{\text{conv},j} = \frac{1}{A_j \alpha_j} \left( \frac{\text{K}}{\text{W}} \right). \quad (\text{C.10})$$

**Definition****Convective resistance of a solid cylindrical layer  $j$ :**

$$R_{\text{conv},j} = \frac{1}{A_j \alpha_j} \left( \frac{\text{K}}{\text{W}} \right), \quad (\text{C.11})$$

with  $A_A = 2\pi r_1 L$  for convection on the inside and  $A_B = 2\pi r_{n+1} L$  for convection on the outside.

**Fundamental EQ****Rate of heat transfer through a solid multi-layer wall with convection:**

$$\dot{Q} = kA(T_A - T_B). \quad (\text{C.12})$$

**Definition****Overall heat transfer coefficient solid multi-layer plane wall system:**

$$k = \left( \frac{1}{\alpha_A} + \sum_{i=1}^n \frac{\delta_i}{\lambda_i} + \frac{1}{\alpha_B} \right)^{-1} \left( \frac{\text{W}}{\text{m}^2 \text{K}} \right). \quad (\text{C.13})$$

**Definition****Overall heat transfer coefficient solid multi-layer cylindrical wall system:**

$$k = \left( \frac{1}{\alpha_A} \frac{r}{r_1} + r \sum_{i=1}^n \frac{1}{\lambda_i} \ln \frac{r_{i+1}}{r_i} + \frac{1}{\alpha_B} \frac{r}{r_{n+1}} \right)^{-1} \left( \frac{\text{W}}{\text{m}^2 \text{K}} \right), \quad (\text{C.14})$$

where any reference radius  $r$  may be used.

#### C.4 Fins

**Definition****Fin parameter:**

$$m = \sqrt{\frac{\alpha U}{\lambda A_c}} \left( \frac{1}{\text{m}} \right), \quad (\text{C.15})$$

where for a rod fin  $m = \sqrt{\frac{4\alpha}{\lambda d}}$ , and for a plane fin  $m = \sqrt{\frac{2\alpha}{\lambda \delta}}$ .

**Fundamental EQ****Homogeneous fin equation:**

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \quad (\text{C.16})$$

where  $\theta = T(x) - T_a$ .

Fundamental EQ

**General solution of the fin equation:**

$$\theta = A e^{mx} + B e^{-mx} = A^* \sinh(mx) + B^* \cosh(mx). \quad (\text{C.17})$$

Fundamental EQ

**Temperature profile of a fin with base temperature  $T_b$  and an adiabatic tip:**

$$\theta = \theta_b \frac{\cosh[m(L-x)]}{\cosh(mL)}. \quad (\text{C.18})$$

Fundamental EQ

**Rate of heat transfer for a fin with base temperature  $T_b$  and an adiabatic tip:**

$$\dot{Q} = \lambda A_c m \theta_b \tanh(mL). \quad (\text{C.19})$$

Definition

**Fin efficiency:**

$$\eta_F = \frac{\text{transferred heat}}{\text{maximum transferable heat}} = \frac{\dot{Q}}{\dot{Q}_{\max}} \quad (-). \quad (\text{C.20})$$

## C.5 Unsteady heat conduction

Fundamental EQ

**Lumped capacity model:**

$$\theta^* = 1 - \exp\left(-\frac{\alpha}{\rho c V} t\right), \quad (\text{C.21})$$

which is valid for  $\text{Bi} \ll 1$ , and where  $\theta^* = \frac{T-T_0}{T_a-T_0}$ .

Fundamental EQ

**Temperature profile semi-infinite plate with negligible thermal surface resistance:**

$$\theta^* = 1 - \operatorname{erf}\left(\frac{1}{\sqrt{4\text{Fo}}}\right), \quad (\text{C.22})$$

where  $\theta^* = \frac{T-T_0}{T_a-T_0}$ , and  $\text{Fo} = \frac{at}{x^2}$ .

Fundamental EQ

**Penetration depth semi-infinite plate with negligible thermal surface resistance:**

$$\delta(t) = 3.6\sqrt{at}, \quad (\text{C.23})$$

for  $\text{Bi} \gg 1$ .

Fundamental EQ

**Heat flux semi-infinite plate with negligible thermal surface resistance:**

$$\dot{q}'' \Big|_{x=0} = \frac{\lambda}{\sqrt{\pi at}} (T_a - T_0), \quad (\text{C.24})$$

for  $\text{Bi} \gg 1$ .

Fundamental EQ

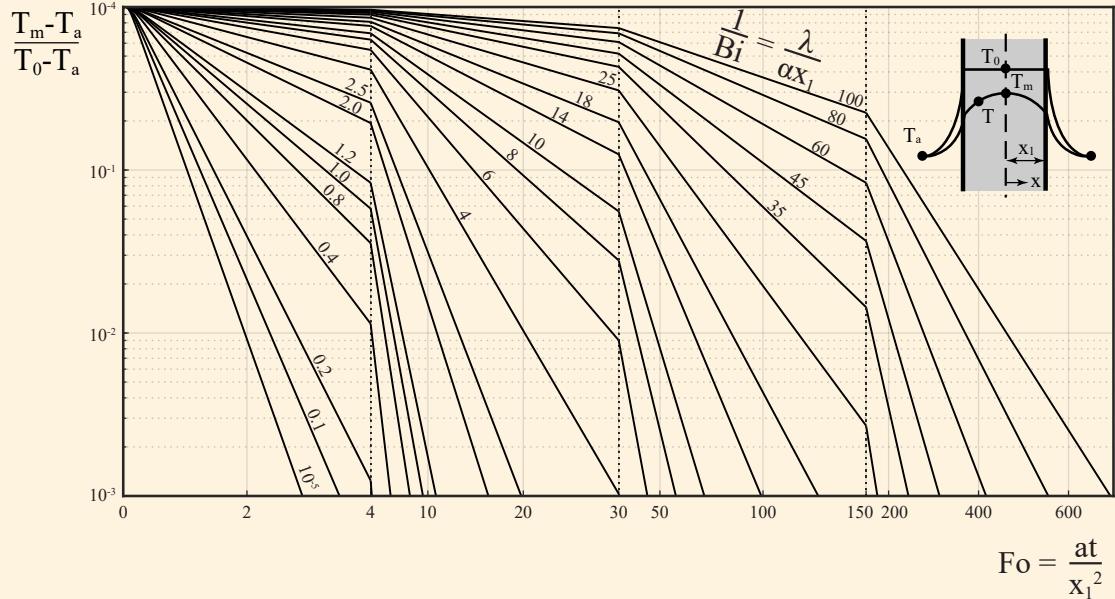
**Temperature profile semi-infinite plate with non-negligible thermal surface resistance:**

$$\theta^* = 1 - \operatorname{erf} \left( \frac{1}{\sqrt{4Fo}} \right) - [\exp(Bi + Fo Bi^2)] \left[ 1 - \operatorname{erf} \left( \frac{1}{\sqrt{4Fo}} + \sqrt{Fo} \cdot Bi \right) \right] \quad (\text{C.25})$$

for  $Bi \gg 1$ , where  $\theta^* = \frac{T-T_0}{T_a-T_0}$ ,  $Fo = \frac{\alpha t}{x^2}$ , and  $Bi = \frac{\alpha x}{\lambda}$ .

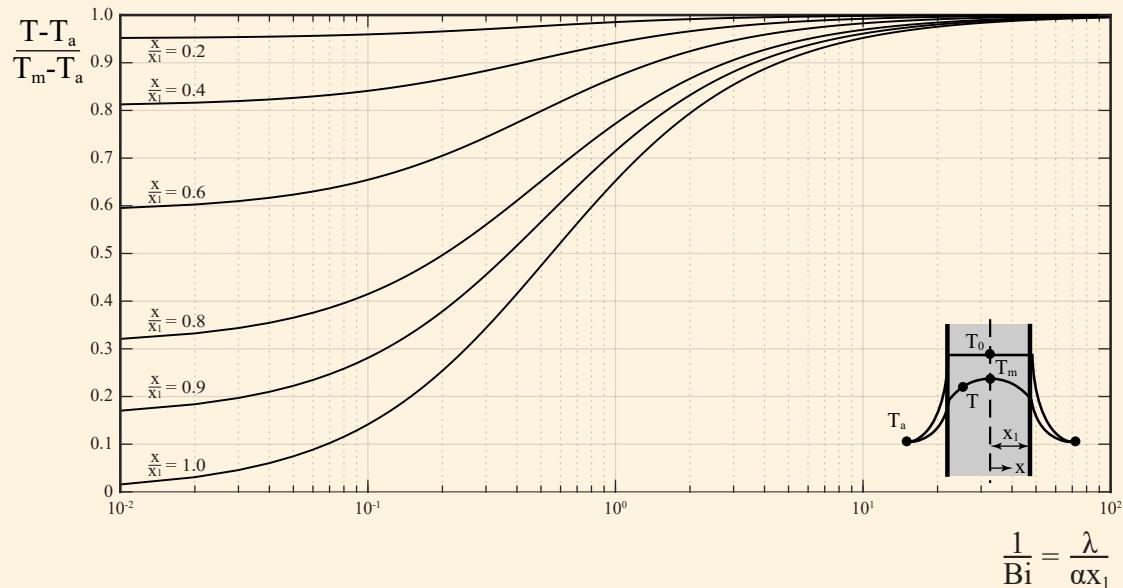
Fundamental EQ

**Temperature profile mid-plane of a plate with thickness  $2x_1$ :**



Fundamental EQ

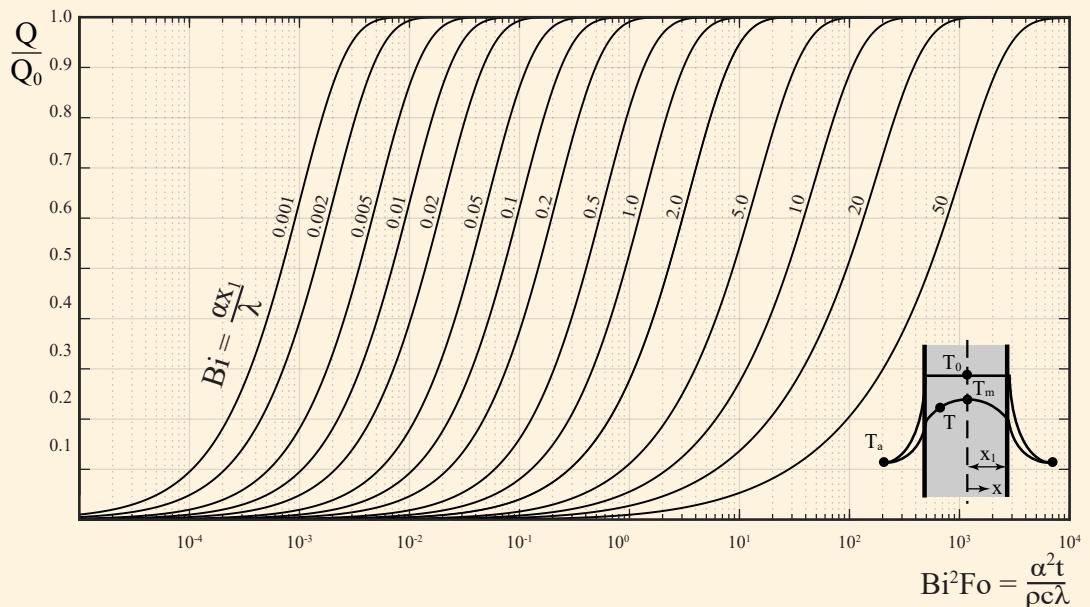
**Temperature distribution in a plate with thickness  $2x_1$ :**



which is for  $Fo > 0.2$ .

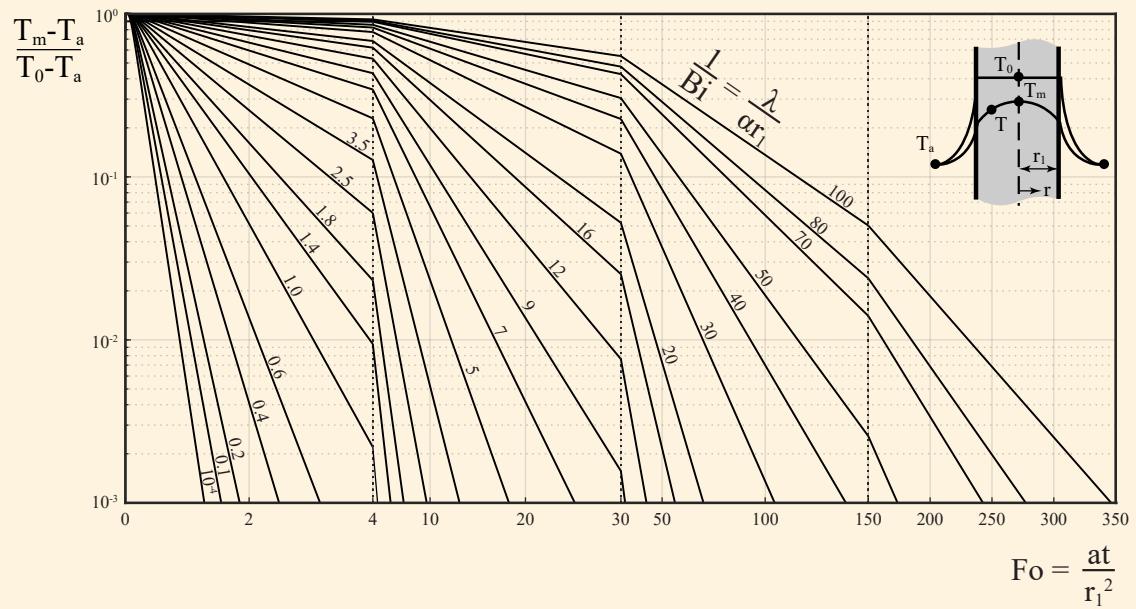
Fundamental EQ

Heat loss of a plate:

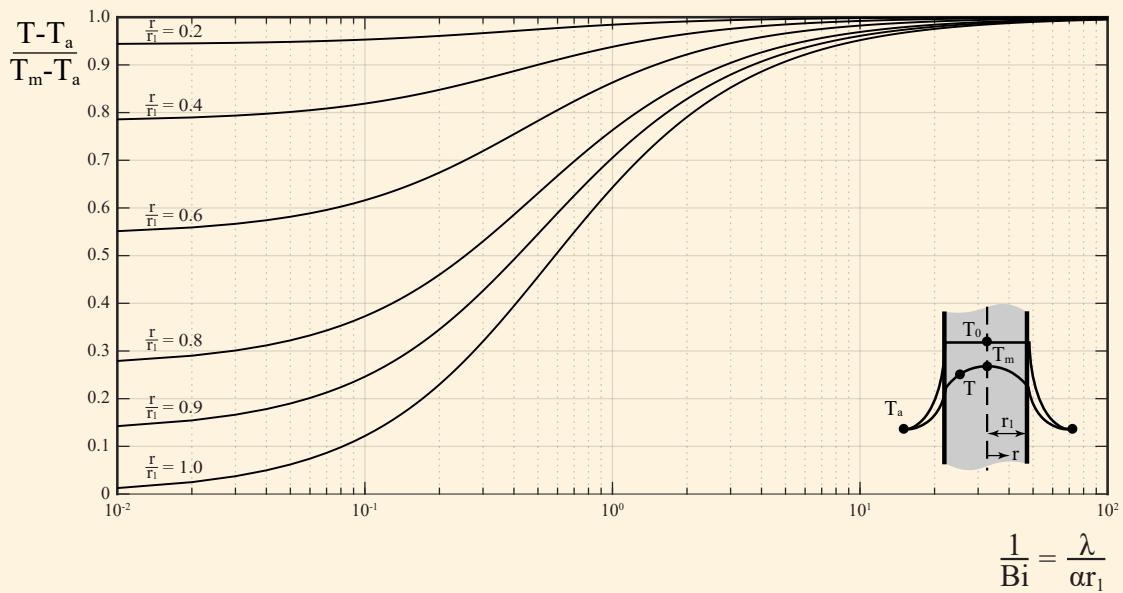


where  $Q = mc(T(t) - T_a)$  and  $Q_0 = mc(T_0 - T_a)$ .

Fundamental EQ

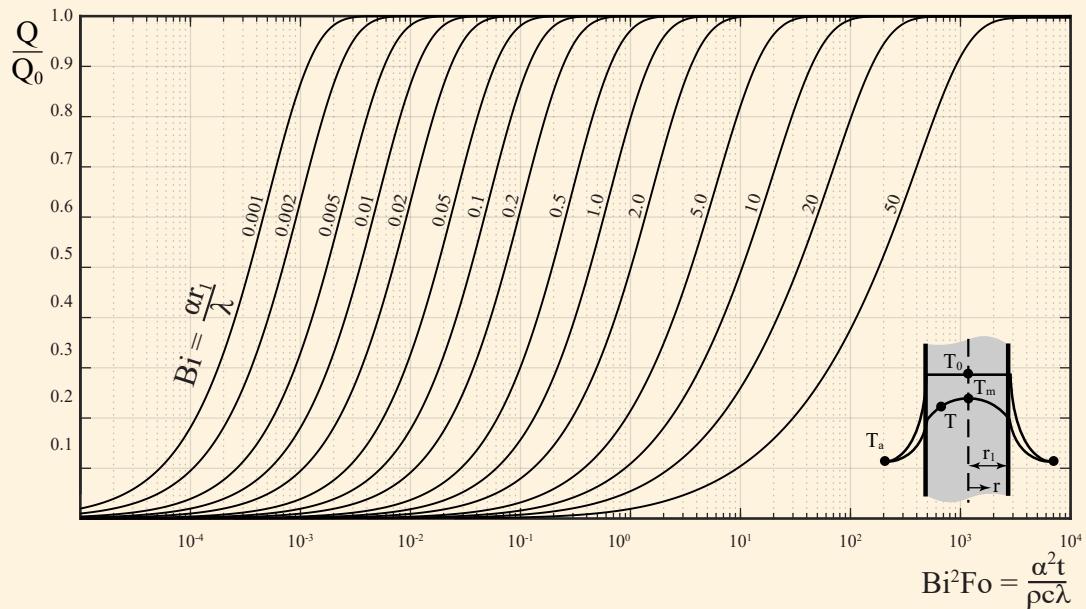
Temperature along the axis of a cylinder with radius  $r_1$ :

Fundamental EQ

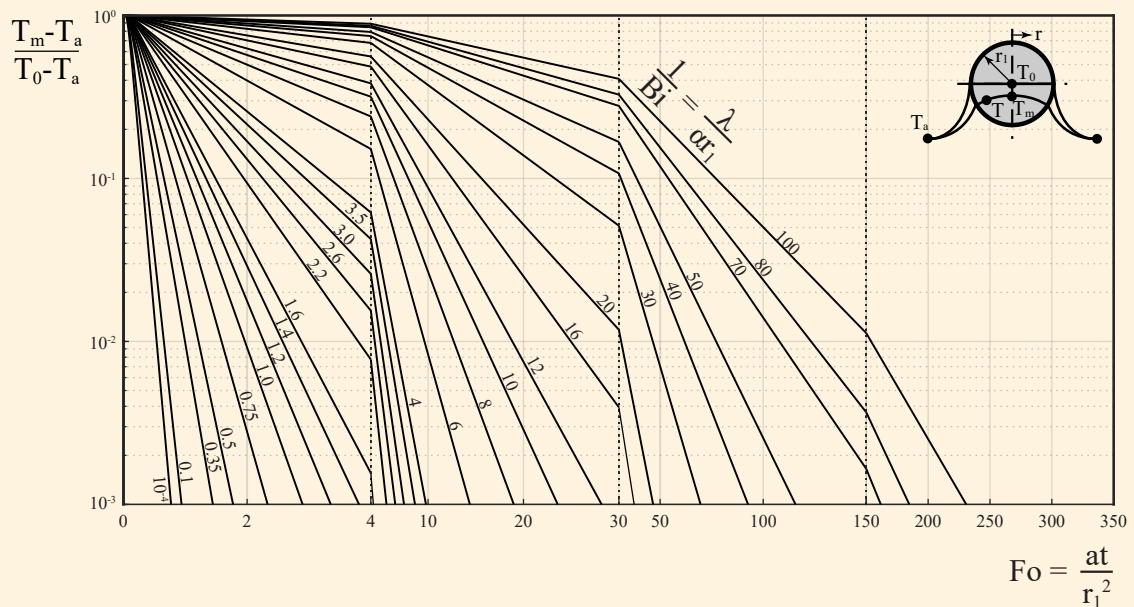
Temperature distribution in a cylinder with radius  $r_1$ :which is for  $Fo > 0.2$ .

Fundamental EQ

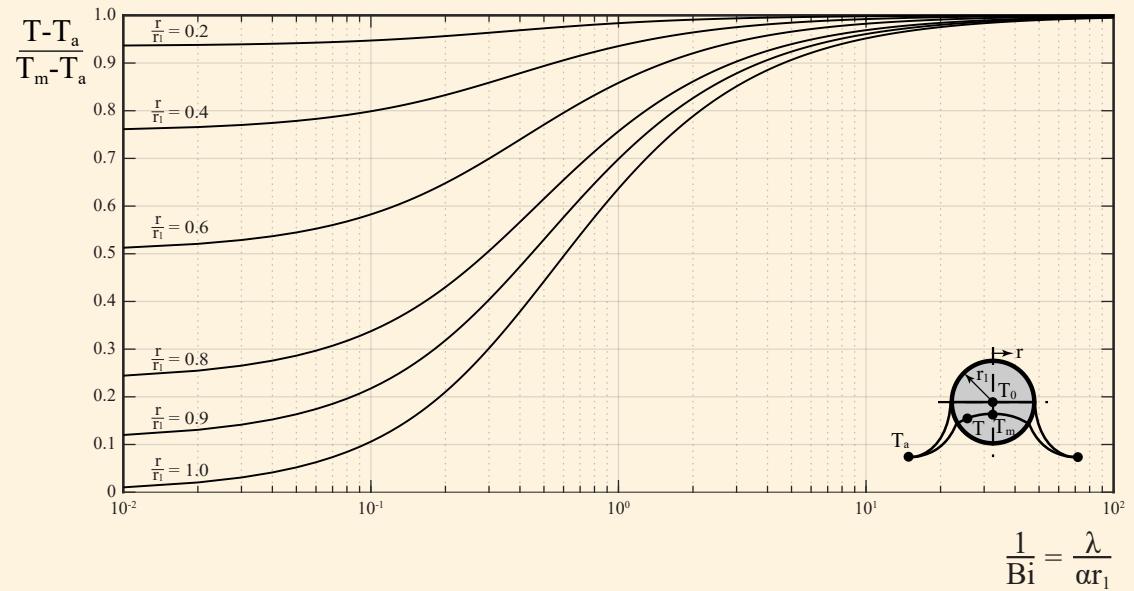
Heat loss of a cylinder:

where  $Q = mc(T(t) - T_a)$  and  $Q_0 = mc(T_0 - T_a)$ .

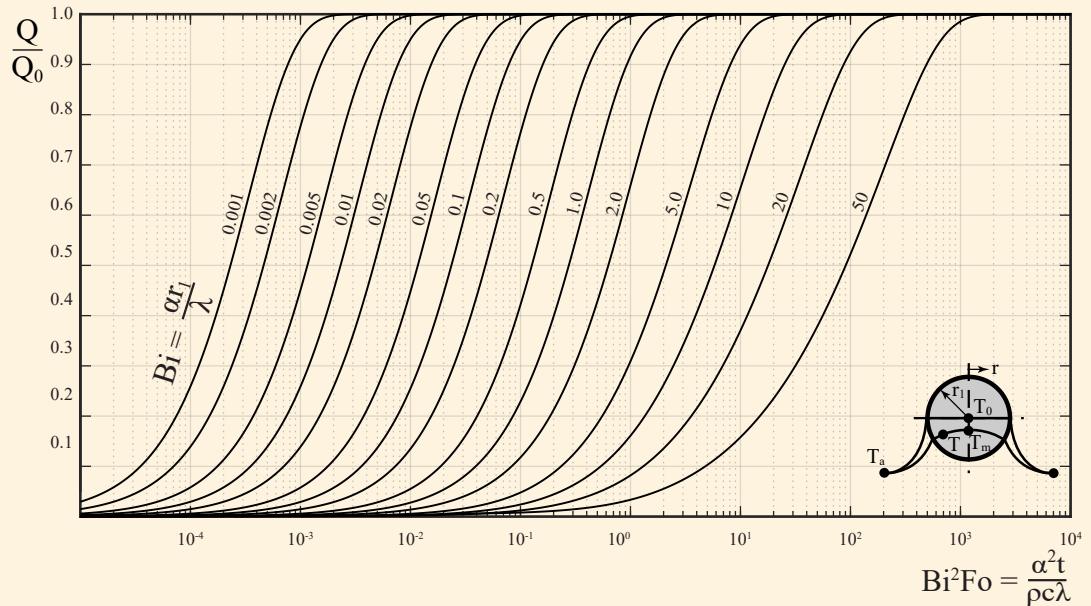
Fundamental EQ

Temperature in the centre of a sphere with radius  $r_1$ :

Fundamental EQ

Temperature distribution in a sphere with radius  $r_1$ :which is for  $Fo > 0.2$ .

Fundamental EQ

**Heat loss of a sphere:**

where  $Q = mc(T(t) - T_a)$  and  $Q_0 = mc(T_0 - T_a)$ .

## SECTION K

# Convection

## K.1 Fundamentals

Fundamental EQ

**Newton's law of cooling:**

$$\dot{Q} = \alpha A (T_W - T_A). \quad (\text{K.1})$$

Definition

**Convective heat transfer coefficient:**

$$\alpha = \frac{-\left(\lambda_f \frac{\partial T_f}{\partial y}\right)_W}{T_W - T_A} = \frac{-\left(\lambda_s \frac{\partial T_s}{\partial y}\right)_W}{T_W - T_A} \left( \frac{W}{m^2 K} \right). \quad (\text{K.2})$$

Definition

**Volumetric expansion coefficient:**

$$\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p = \frac{\rho_\infty - \rho}{\rho(T - T_\infty)} \left( \frac{1}{K} \right). \quad (\text{K.3})$$

Definition

**Volumetric expansion coefficient for ideal gases:**

$$\beta = \frac{1}{T} \left( \frac{1}{K} \right). \quad (\text{K.4})$$

## K.2 Conservation equations

Fundamental EQ

**Equation of continuity:**

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0. \quad (\text{K.5})$$

Fundamental EQ

**Equations of momentum:**

$$\rho \left( \vec{u} \cdot \nabla \right) \vec{u} = -\nabla p + \eta \nabla^2 \vec{u} + \rho \vec{g}. \quad (\text{K.6})$$

Fundamental EQ

**Equation of energy conservation:**

$$\rho \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{\lambda}{c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{\Phi}''. \quad (\text{K.7})$$

### K.3 Boundary layer equations laminar flow

Fundamental EQ

**Velocity boundary layer thickness for a linear velocity profile:**

$$\frac{\delta_u}{x} \approx \sqrt{\frac{12}{\text{Re}_x}}, \quad (\text{K.8})$$

for  $\text{Re}_x < 2 \cdot 10^5$ .

Fundamental EQ

**Velocity boundary layer thickness for a linear velocity profile:**

$$\frac{\delta_T}{x} \approx \left( \frac{\lambda}{\eta c_p} \right)^{1/3} \sqrt{\frac{12\eta}{\rho u_\infty x}} = \frac{1}{\text{Pr}^{1/3}} \sqrt{\frac{12}{\text{Re}_x}}, \quad (\text{K.9})$$

for  $\text{Re}_x < 2 \cdot 10^5$ , and  $0.6 < \text{Pr} < 10$ .

Fundamental EQ

**Velocity boundary layer thickness for flow over a flat plate:**

$$\frac{\delta_u}{x} = \frac{4.91}{\sqrt{\text{Re}_x}}, \quad (\text{K.10})$$

for  $\text{Re}_x < 2 \cdot 10^5$ .

Fundamental EQ

**Thermal boundary layer thickness for flow over a flat plate:**

$$\frac{\delta_T}{x} = \frac{4.91}{\sqrt{\text{Re}_x} \text{Pr}^{1/3}}, \quad (\text{K.11})$$

for  $\text{Re}_x < 2 \cdot 10^5$ , and  $0.6 < \text{Pr} < 10$ .

#### K.4 Internal forced convection

Fundamental EQ

**Heat transfer rate in ducts:**

$$\dot{Q} = \overline{\alpha} A \Delta T_m, \quad (\text{K.12})$$

where  $\Delta T_m$  is a representative temperature difference between the temperature at the wall  $T_w$  and the energetically averaged caloric mean temperature of the fluid  $T_{fl}$ .

Definition

**LMTD for flow along ducts with isothermal surface:**

$$\Delta T_m = \Delta T_{ln} = \frac{\Delta T_{in} - \Delta T_{out}}{\ln \frac{\Delta T_{in}}{\Delta T_{out}}} \text{ (K),} \quad (\text{K.13})$$

where  $\Delta T_{in} = T_{in} - T_w$ , and  $\Delta T_{out} = T_{out} - T_w$ .

Definition

**Mean temperature difference for flow along ducts with a constant impressed heat flux:**

$$\Delta T_m = (T_w - T_{fl})_m \text{ (K).} \quad (\text{K.14})$$

Definition

**Average fluid velocity:**

$$u_m = \frac{\int \rho u dA_c}{\int \rho dA_c} \left( \frac{\text{m}}{\text{s}} \right). \quad (\text{K.15})$$

Definition

**Caloric mean temperature:**

$$T_m = \frac{\int \rho u c_p T dA_c}{\int \rho u c_p dA_c} \text{ (K).} \quad (\text{K.16})$$

Fundamental EQ

**Laminar hydrodynamic entry length:**

$$L_h \approx 0.05 \text{ Re}_d d. \quad (\text{K.17})$$

Fundamental EQ

**Laminar thermodynamic entry length:**

$$L_{th} \approx 0.05 \text{ Re}_d \text{Pr} d. \quad (\text{K.18})$$

Fundamental EQ

**Turbulent hydrodynamic entry length:**

$$L_h \approx 10 d. \quad (\text{K.19})$$

Fundamental EQ

**Turbulent thermodynamic entry length:**

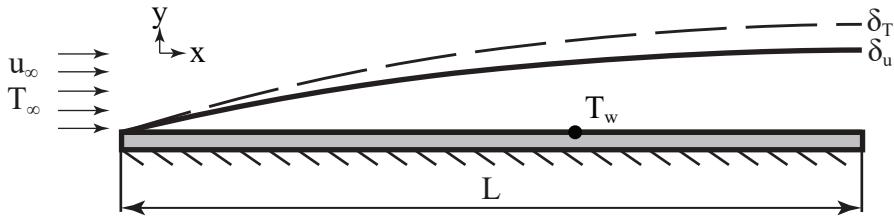
$$L_{th} \approx 10 d. \quad (\text{K.20})$$

## SECTION H

**Heat transfer correlations****H.1 External forced convection****Definition****Fluid property temperature external forced convection:**

$$T_{\text{prop}} = \frac{T_w + T_\infty}{2} \text{ (K)}, \quad (\text{H.1})$$

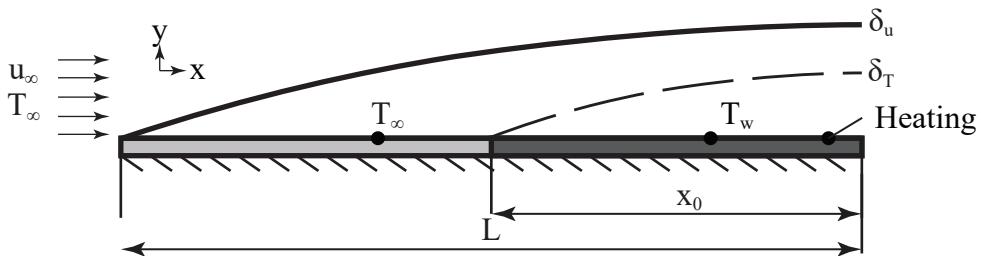
if not stated otherwise.

**HTC Local Nusselt number for forced laminar flow over a flat plate with isothermal surface:**

$$\text{Nu}_x = 0.332 \text{Re}_x^{\frac{1}{2}} \text{Pr}^{\frac{1}{3}}, \quad (\text{HTC.1})$$

for  $\text{Re}_x < 2 \cdot 10^5$ ,  $0.6 < \text{Pr} < 10$ , and where  $\text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5$ .**HTC Average Nusselt number for forced laminar flow over a flat plate with isothermal surface:**

$$\overline{\text{Nu}}_L = 0.664 \text{Re}_L^{\frac{1}{2}} \text{Pr}^{\frac{1}{3}}, \quad (\text{HTC.2})$$

for  $\text{Re}_L < 2 \cdot 10^5$ ,  $0.6 < \text{Pr} < 10$ , and where  $\text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5$ .**HTC Local Nusselt number for forced laminar flow over a flat plate with isothermal surface and first hydrodynamic inflow:**

$$\text{Nu}_x = 0.332 \text{Re}_x^{\frac{1}{2}} \text{Pr}^{\frac{1}{3}} \left[ 1 - \left( \frac{x_0}{x} \right)^{\frac{3}{4}} \right]^{-\frac{1}{3}}, \quad (\text{HTC.3})$$

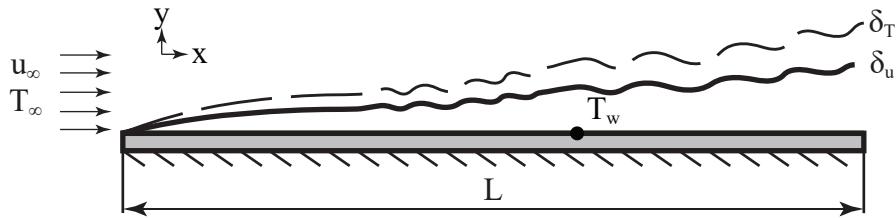
for  $\text{Re}_x < 2 \cdot 10^5$ ,  $0.6 < \text{Pr} < 10$ , and where  $\text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5$ .

HTC

Average Nusselt number for forced laminar flow over a flat plate with isothermal surface and first hydrodynamic inflow:

$$\overline{\text{Nu}}_L = \frac{L}{L - x_0} \frac{1}{\lambda} \int_{x_0}^L \alpha(x) dx = 0.664 \text{Re}_L^{1/2} \text{Pr}^{1/3} \frac{\left[1 - \left(\frac{x_0}{L}\right)^4\right]^{2/3}}{\left[1 - \frac{x_0}{L}\right]}, \quad (\text{HTC.4})$$

for  $\text{Re}_L < 2 \cdot 10^5$ ,  $0.6 < \text{Pr} < 10$ , and where  $\text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5$ .



HTC

Local Nusselt number for forced turbulent flow over a flat plate with isothermal surface:

$$\text{Nu}_x = 0.0296 \text{Re}_x^{0.8} \text{Pr}^{0.43}, \quad (\text{HTC.5})$$

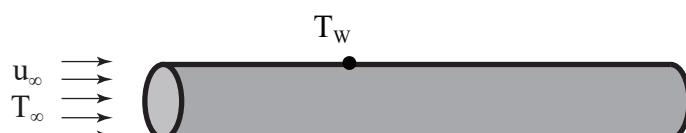
for  $5 \cdot 10^5 < \text{Re}_x < 10^7$ , where  $\text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5$ .

HTC

Average Nusselt number for forced turbulent flow over a flat plate with isothermal surface:

$$\overline{\text{Nu}}_L \approx 0.036 \text{Pr}^{0.43} \left( \text{Re}_L^{0.8} - 9400 \right), \quad (\text{HTC.6})$$

for  $5 \cdot 10^5 < \text{Re}_L < 10^7$ , where  $\text{Re}_{x,\text{crit}} \approx 2 \cdot 10^5$ .

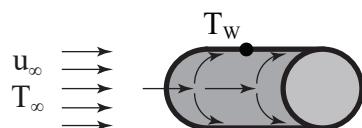


Criterion

Criterion for approximating parallel flow along a cylinder's longitudinal axis as flow over a flat plate:

$$d \gg \delta.$$

If so, [HTC.1](#) to [HTC.6](#) are applicable.



**HTC**

Average Nusselt number for forced flow perpendicular to the longitudinal axis of a circular cylinder with isothermal surface:

$$\overline{Nu}_d = C \text{Re}_d^m \text{Pr}^{1/3}, \quad (\text{HTC.7})$$

for  $\text{Pr} > 0.7$ , where:

$\text{Re}_d$	$C$	$m$
0.4 - 4	0.989	0.330
4 - 40	0.911	0.385
40 - 4000	0.683	0.466
4000 - 40,000	0.193	0.618
40,000 - 400,000	0.0266	0.805

**HTC**

Average Nusselt number for forced flow perpendicular to the longitudinal axis of a circular cylinder with isothermal surface:

$$\overline{Nu}_d = \left( 0.4 \text{Re}_d^{1/2} + 0.06 \text{Re}_d^{2/3} \right) \text{Pr}^{0.4} \left( \frac{\eta_\infty}{\eta_w} \right)^{1/4}, \quad (\text{HTC.8})$$

for  $1.0 < \text{Re} \cdot 10^5, 0.67 < \text{Pr} < 300, 0.25 < \frac{\eta_\infty}{\eta_w} < 5.2$ , and where  $T_{\text{prop}} = T_\infty$ .

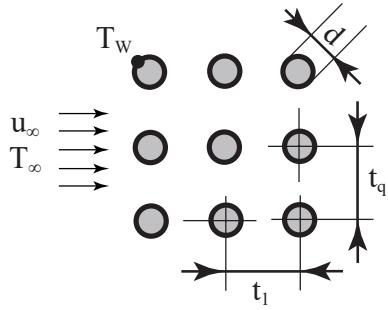
**HTC**

Average Nusselt number for forced flow perpendicular to the longitudinal axis of a non-circular cylinder with isothermal surface:

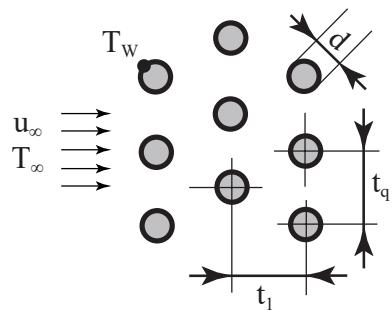
$$\overline{Nu}_d = C \text{Re}_d^m \text{Pr}^{1/3}, \quad (\text{HTC.9})$$

where:

Cross-section	$\text{Re}_d$	$C$	$m$	Medium
	5000-100,000	0.246	0.588	Gas
	5000-100,000	0.102	0.675	Gas
	5000-19,500 19,500-100,000	0.160 0.0385	0.638 0.782	Gas
	5000-100,000	0.153	0.638	Gas
	4000-15,000	0.228	0.731	Gas
	2500-15,000	0.248	0.612	Gas



(a) In-line arrangement



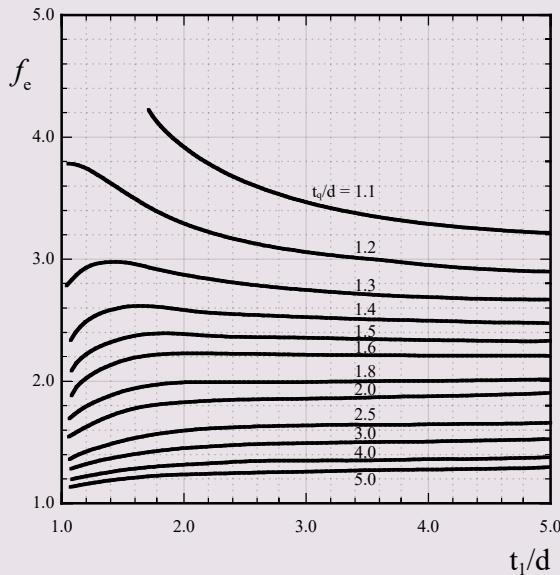
(b) Staggered arrangement

**HTC** Average Nusselt number for forced flow perpendicular to the longitudinal axis of a bundle of smooth tubes:

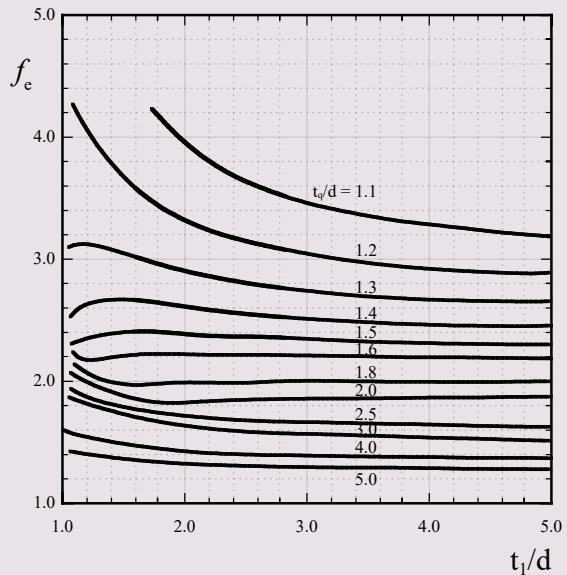
$$\overline{Nu}_d = 0.287 \text{ Re}_d^{0.6} \text{ Pr}^{0.36} \cdot f_e, \quad (\text{HTC.10})$$

where  $T_{\text{prop}} = \frac{T_{\text{out}}+T_{\text{in}}}{2}$ .

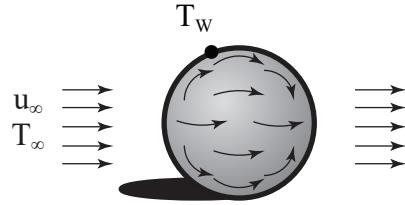
The tube arrangement factor  $f_e$  depends on the relative longitudinal distance/diameter ratio  $t_l/d$ , the relative transverse distance/diameter ratio  $t_q/d$  of the tubes, as well as the tube arrangement. This factor can be retrieved from:



(a) In-line arrangement



(b) Staggered arrangement.



**HTC** Average Nusselt number for forced flow around a sphere with isothermal surface:

$$\overline{Nu}_d = 2 + \left( 0.4 \text{Re}_d^{1/2} + 0.06 \text{Re}_d^{2/3} \right) \text{Pr}^{0.4} \left( \frac{\eta_\infty}{\eta_w} \right)^{1/4}, \quad (\text{HTC.11})$$

for  $3.5 < \text{Re}_d < 7.6 \cdot 10^4$ ,  $0.7 < \text{Pr} < 380$ ,  $1.0 < \frac{\eta_\infty}{\eta_w} < 3.2$ , and where  $T_{\text{prop}} = T_\infty$ .

## H.2 Internal forced convection

**Definition**

Fluid property temperature internal forced convection:

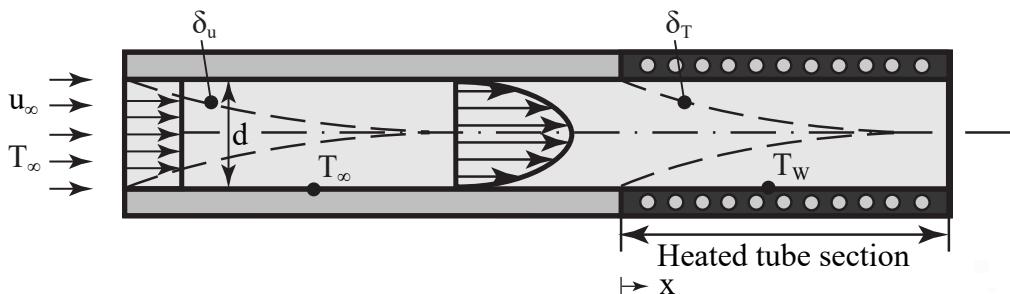
$$T_{\text{prop}} = \frac{T_{\text{out}} + T_{\text{in}}}{2} \text{ (K)}, \quad (\text{H.2})$$

if not stated otherwise.

**Definition**

Characteristic length of pipes:

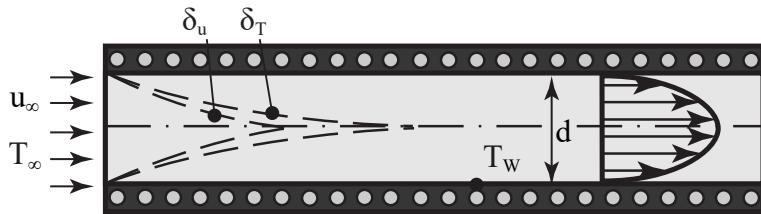
$$d_h = 4 \frac{\text{cross-section area}}{\text{wetted perimeter}} = 4 \frac{A_c}{U} \text{ (m)}. \quad (\text{H.3})$$



**HTC** Average Nusselt number for forced laminar flow within a circular pipe being hydrodynamic developed at the start of the isothermal heated or cooled section:

$$\overline{Nu}_d = \left( 3.66 + \frac{0.19 \left( \text{Re}_d \text{Pr} \frac{d}{L} \right)^{0.8}}{1 + 0.117 \left( \text{Re}_d \text{Pr} \frac{d}{L} \right)^{0.467}} \right) \left( \frac{\eta}{\eta_w} \right)^{0.14}, \quad (\text{HTC.12})$$

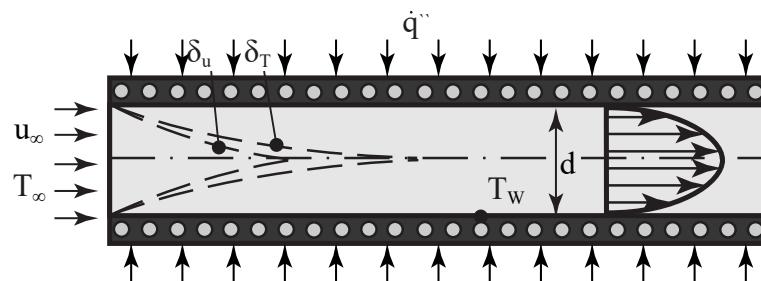
for  $\text{Re}_d < 2300$ , where  $\text{Re}_{d,\text{crit}} \approx 2300$ .



**HTC** Average Nusselt number for forced laminar flow within an isothermal circular pipe with simultaneous hydrodynamic and thermal start:

$$\overline{Nu}_d = \left( 3.66 + \frac{0.0677 (\text{Re}_d Pr \frac{d}{L})^{1.33}}{1 + 0.1 Pr (\text{Re}_d \frac{d}{L})^{0.83}} \right) \left( \frac{\eta}{\eta_W} \right)^{0.14}, \quad (\text{HTC.13a})$$

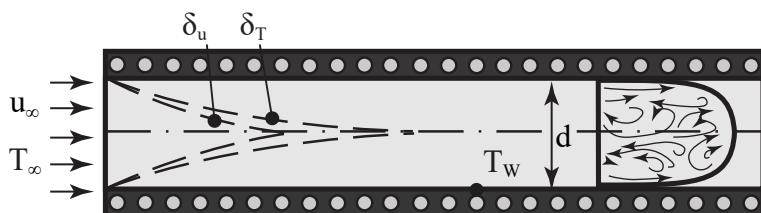
for  $\text{Re}_d < 2300$ , where  $\text{Re}_{d,\text{crit}} \approx 2300$ .



**HTC** Average Nusselt number for forced laminar flow within a circular pipe being fully developed with impressed heat flow:

$$\overline{Nu}_d = 4.36, \quad (\text{HTC.13b})$$

for  $\text{Re}_d < 2300$  and  $L \gg L_{\text{th}}$ , where  $\text{Re}_{d,\text{crit}} \approx 2300$ .



**HTC** Average Nusselt number for forced turbulent flow within a pipe with simultaneous hydrodynamic and thermal start:

$$\overline{Nu}_d = 0.0235 (\text{Re}_d^{0.8} - 230) (1.8 \text{Pr}^{0.3} - 0.8) \left( 1 + \left( \frac{d}{L} \right)^{\frac{2}{3}} \right) \left( \frac{\eta}{\eta_W} \right)^{0.14}, \quad (\text{HTC.14})$$

for  $\text{Re}_d > 2300$ ,  $0.6 < \text{Pr} < 500$  and  $\frac{L}{d} > 1$ , where  $\text{Re}_{d,\text{crit}} \approx 2300$ .

**HTC** Average Nusselt number for forced turbulent flow within a pipe being fully developed:

$$\overline{\text{Nu}}_d = 0.027 \text{Re}_d^{0.8} \text{Pr}^{\frac{1}{3}} \left( \frac{\eta}{\eta_w} \right)^{0.14}, \quad (\text{HTC.15})$$

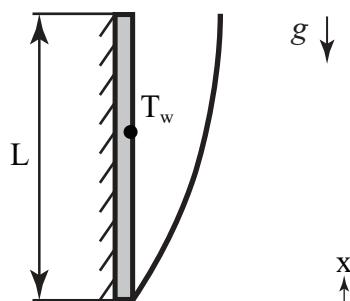
for  $3000 < \text{Re}_d < 10^5$  and  $\frac{L}{d} > 40$ , where  $\text{Re}_{d,\text{crit}} \approx 2300$ .

### H.3 External natural convection

**Definition** Fluid property temperature external natural convection:

$$T_{\text{prop}} = \frac{T_w + T_\infty}{2} \text{ (K)}, \quad (\text{H.4})$$

if not stated otherwise.



**HTC** Local Nusselt number for natural laminar flow along a vertical plate with isothermal surface:

$$\text{Nu}_x = 0.508 \left( \frac{\text{Pr}}{0.952 + \text{Pr}} \right)^{\frac{1}{4}} (\text{Gr}_x \text{Pr})^{\frac{1}{4}}, \quad (\text{HTC.16})$$

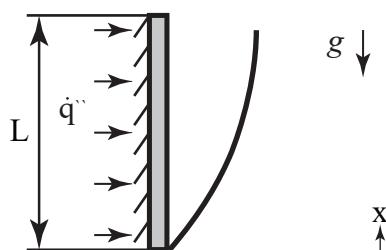
for  $\text{Gr}_x \cdot \text{Pr} < 4 \cdot 10^9$ .

**HTC** Average Nusselt number for natural laminar flow along a vertical plate with isothermal surface:

$$\overline{\text{Nu}}_L = C (\text{Gr}_L \text{Pr})^{\frac{1}{4}}, \quad (\text{HTC.17})$$

for  $\text{Gr}_L \cdot \text{Pr} < 4 \cdot 10^9$ , and:

Pr	0.003	0.01	0.03	0.72	1	2	10	100	1000	$\infty$
C	0.182	0.242	0.305	0.516	0.535	0.568	0.620	0.653	0.665	0.670



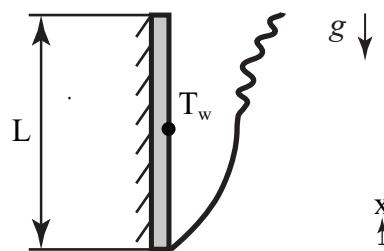
**Definition** Modified Grashof number for natural flow along a vertical plate with an impressed heat flux:

$$\text{Gr}_x^* = \text{Gr}_x \text{Nu}_x = \frac{\rho^2 g \beta \dot{q}_w'' x^4}{\lambda \eta^2} \quad (-) \quad (\text{H.5})$$

**HTC** Local Nusselt number for natural laminar flow along a vertical plate with an impressed heat flux:

$$\text{Nu}_x = 0.60 (\text{Gr}_x^* \text{Pr})^{\frac{1}{5}}, \quad (\text{HTC.18})$$

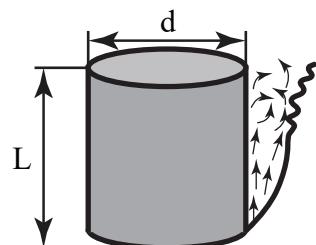
for  $10^5 < \text{Gr}_x^* < 10^{11}$ .



**HTC** Average Nusselt number for natural turbulent flow along a vertical plate with isothermal surface:

$$\overline{\text{Nu}}_L = 0.13 (\text{Gr}_L \text{Pr})^{\frac{1}{3}}, \quad (\text{HTC.19})$$

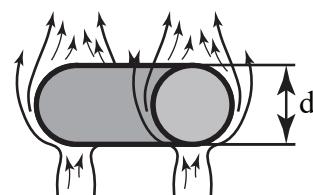
for  $10^9 < \text{Gr}_L \cdot \text{Pr} < 10^{12}$



**Criterion** Criterion for approximating a vertical cylinder as a vertical plate:

$$\frac{d}{L} > 35 \cdot \text{Gr}_L^{-\frac{1}{4}}.$$

If so, [HTC.16](#) to [HTC.19](#) are applicable.



**HTC** Average Nusselt number for natural laminar flow around a horizontal cylinder with isothermal surface:

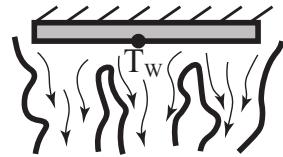
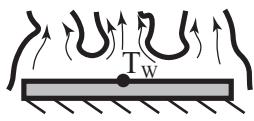
$$\overline{Nu}_d = 0.53 (\text{Gr}_d \text{Pr})^{\frac{1}{4}}, \quad (\text{HTC.20})$$

for  $10^4 < \text{Gr}_d \text{Pr} < 10^9$ .

**HTC** Average Nusselt number for natural turbulent flow around a horizontal cylinder with isothermal surface:

$$\overline{Nu}_d = 0.13 (\text{Gr}_d \text{Pr})^{\frac{1}{3}}, \quad (\text{HTC.21})$$

for  $10^9 < \text{Gr}_d \text{Pr} < 10^{12}$ .



**Definition** Characteristic length horizontal plates:

$$L = \frac{\text{Surface area}}{\text{Perimeter}} = \frac{A}{U} \text{ (m).} \quad (\text{H.6})$$

**HTC** Average Nusselt number for natural laminar flow over a horizontal plate with a heated upper or cooled lower:

Isothermal surface:

$$\overline{Nu}_L = 0.54 (\text{Gr}_L \text{Pr})^{\frac{1}{4}}, \quad (\text{HTC.22a})$$

for  $2 \cdot 10^4 < \text{Gr}_L \text{Pr} < 8 \cdot 10^6$ .

Impressed heat flow:

$$\overline{Nu}_L = 0.13 (\text{Gr}_L \text{Pr})^{\frac{1}{3}}, \quad (\text{HTC.22b})$$

for  $\text{Gr}_L \text{Pr} < 2 \cdot 10^8$ .

**HTC** Average Nusselt number for natural turbulent flow over a horizontal plate with a heated upper or cooled lower:

Isothermal surface:

$$\overline{Nu}_L = 0.15 (\text{Gr}_L \text{Pr})^{\frac{1}{3}}, \quad (\text{HTC.23a})$$

for  $8 \cdot 10^6 < \text{Gr}_L \text{Pr} < 10^{11}$ .

Impressed heat flow:

$$\overline{Nu}_L = 0.16 (\text{Gr}_L \text{Pr})^{\frac{1}{3}}, \quad (\text{HTC.23b})$$

for  $2 \cdot 10^8 < \text{Gr}_L \text{Pr} < 10^{11}$ .



**HTC** Average Nusselt number for natural laminar flow over a horizontal plate with a cooled upper or heated lower:

Isothermal surface:

$$\overline{Nu}_L = 0.27 (\text{Gr}_L \text{Pr})^{\frac{1}{4}}, \quad (\text{HTC.24a})$$

for  $10^5 < \text{Gr}_L \text{Pr} < 10^{10}$ .

Impressed heat flow:

$$\overline{Nu}_L = 0.58 (\text{Gr}_L \text{Pr})^{\frac{1}{5}}, \quad (\text{HTC.24b})$$

for  $10^6 < \text{Gr}_L \text{Pr} < 10^{11}$ .

#### H.4 Internal natural convection

**Definition** Fluid property temperature internal natural convection:

$$T_{\text{prop}} = \frac{T_1 + T_2}{2} \text{ (K)}, \quad (\text{H.7})$$

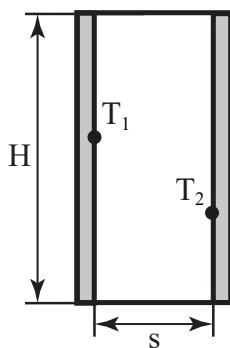
if not stated otherwise.

**Definition** Characteristic length enclosures:

$$L = s \text{ (m)}. \quad (\text{H.8})$$

**Definition** Grashof number enclosures:

$$\text{Gr}_s = \frac{\beta g \rho^2 (T_{\max} - T_{\min}) s^3}{\eta^2} \text{ (-)}. \quad (\text{H.9})$$



**HTC** Average Nusselt number for natural laminar flow within a vertical enclosure with isothermal surfaces:

Negligible flow:

$$\overline{\text{Nu}}_s = 1, \quad (\text{HTC.25a})$$

for  $\text{Gr}_s < 2 \cdot 10^3$ , and  $3.1 < H/s < 42.2$ .

Laminar flow:

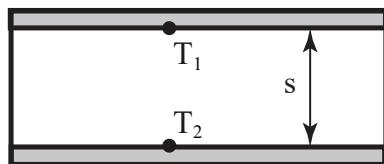
$$\overline{\text{Nu}}_s = 0.20 \left( \frac{H}{s} \right)^{-\frac{1}{9}} (\text{Gr}_s \text{Pr})^{\frac{1}{4}}, \quad (\text{HTC.25b})$$

for  $2 \cdot 10^3 < \text{Gr}_s < 2 \cdot 10^4$ , and  $3.1 < H/s < 42.2$ .

**HTC** Average Nusselt number for natural turbulent flow within a vertical enclosure with isothermal surfaces:

$$\overline{\text{Nu}}_s = 0.071 \left( \frac{H}{s} \right)^{-\frac{1}{9}} (\text{Gr}_s \text{Pr})^{\frac{1}{3}}, \quad (\text{HTC.26})$$

for  $2 \cdot 10^5 < \text{Gr}_s < 10^7$ , and  $3.1 < H/s < 42.2$ .



**HTC** Average Nusselt number for natural laminar flow within a horizontal enclosure with isothermal surfaces:

Negligible flow:

$$\text{Nu}_s = 1, \quad (\text{HTC.27a})$$

for  $\text{Gr}_s < 2 \cdot 10^3$ , or  $T_1 > T_2$ .

Laminar flow:

$$\overline{\text{Nu}}_s = 0.21 (\text{Gr}_s \text{Pr})^{\frac{1}{4}}, \quad (\text{HTC.27b})$$

for  $10^4 < \text{Gr}_s < 3.2 \cdot 10^5$ .

**HTC** Average Nusselt number for natural turbulent flow within a horizontal enclosure with isothermal surfaces:

$$\overline{\text{Nu}}_s = 0.075 (\text{Gr}_s \text{Pr})^{\frac{1}{3}}, \quad (\text{HTC.28})$$

for  $3.2 \cdot 10^5 < \text{Gr}_s < 10^7$ .

## SECTION R

# Radiation

## R.1 Fundamentals

**Definition** **Wavelength in a medium:**  $\lambda = \frac{c}{\nu}$  (m). (R.1)

**Fundamental EQ** **Photon energy:**  $E = h \nu = \frac{hc}{\lambda}$ , where Planck's constant  $h = 6.626 \cdot 10^{-34}$  J · s. (R.2)

**Definition** **Wavenumber:**  $\eta = \frac{1}{\lambda} \left( \frac{1}{\text{m}} \right)$ . (R.3)

**Fundamental EQ** **Planck's distribution law:**  $\dot{q}_{b\lambda}'' = \frac{c_1 \lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1} \left( \frac{\text{W}}{\text{m}^2 \text{m}} \right)$ , where b refers to being a black body, and  $\lambda$  to being wavelength-specific. Besides,  $c_1 = 3.741 \cdot 10^{-16}$  W m<sup>2</sup> and  $c_2 = 1.439 \cdot 10^{-2}$  mK. (R.4)

**Fundamental EQ** **Wien's law of displacement:**  $\lambda_{\max} = \frac{2898 \mu\text{mK}}{T}$ . (R.5)

**Fundamental EQ** **Stefan-Boltzmann law:**  $\dot{q}_b'' = \int_0^{\infty} \dot{q}_{b\lambda}'' d\lambda = \sigma T^4$ , where the Stefan-Boltzmann constant  $\sigma = 5.67 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$ . (R.6)

**Fundamental EQ** **Black body radiation in the spectral range between 0 and  $\lambda$ :**  $\dot{q}_{b,0 \rightarrow \lambda}'' = F_{0 \rightarrow \lambda T} \cdot \sigma T^4$ , where: 

$\lambda T$	( $\mu\text{mK}$ )	1000	1250	1500	1750	2000	2500
$F_{0 \rightarrow \lambda T}$	(-)	0.00031	0.00308	0.01283	0.03363	0.06663	0.16115
$\lambda T$	( $\mu\text{mK}$ )	3000	3500	4000	5000	6000	8000
$F_{0 \rightarrow \lambda T}$	(-)	0.27322	0.38250	0.48085	0.63315	0.73715	0.85556

(R.7)

Fundamental EQ

**Black body radiation in the spectral range between  $\lambda_1$  and  $\lambda_2$ :**

$$\dot{q}_{b,\lambda_1 \rightarrow \lambda_2}'' = (F_{0 \rightarrow \lambda_2 T} - F_{0 \rightarrow \lambda_1 T}) \cdot \sigma T^4. \quad (\text{R.8})$$

Definition

**Total reflectivity:**

$$\rho = \frac{\text{Total radiation reflected}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}_{\lambda\rho}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda 0}'' d\lambda} = \frac{\int_0^\infty \rho_\lambda \dot{q}_{\lambda 0}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda 0}'' d\lambda} (-). \quad (\text{R.9})$$

Definition

**Total absorptivity:**

$$\alpha = \frac{\text{Total radiation absorbed}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}_{\lambda\alpha}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda 0}'' d\lambda} = \frac{\int_0^\infty \alpha_\lambda \dot{q}_{\lambda 0}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda 0}'' d\lambda} (-). \quad (\text{R.10})$$

Definition

**Total transmissivity:**

$$\tau = \frac{\text{Total radiation transmitted}}{\text{Total incident radiation}} = \frac{\int_0^\infty \dot{q}_{\lambda\tau}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda 0}'' d\lambda} = \frac{\int_0^\infty \tau_\lambda \dot{q}_{\lambda 0}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda 0}'' d\lambda} (-). \quad (\text{R.11})$$

Definition

**Total emissivity:**

$$\varepsilon = \frac{\text{Total radiation emitted}}{\text{Total blackbody radiation}} = \frac{\int_0^\infty \dot{q}_{\lambda\varepsilon}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda b}'' d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda \dot{q}_{\lambda b}'' d\lambda}{\int_0^\infty \dot{q}_{\lambda b}'' d\lambda} (-). \quad (\text{R.12})$$

Fundamental EQ

**Relation between  $\rho_\lambda$ ,  $\alpha_\lambda$ , and  $\tau_\lambda$  for a real body:**

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1. \quad (\text{R.13})$$

Fundamental EQ

**Relation between  $\rho$ ,  $\alpha$ , and  $\tau$  for bodies:**

$$\rho + \alpha + \tau = 1. \quad (\text{R.14})$$

Fundamental EQ

**Kirchoff's law of thermal radiation:**

$$\alpha_\lambda = \varepsilon_\lambda. \quad (\text{R.15})$$

**Fundamental EQ**

**Kirchoff's law of thermal radiation for total absorptivity and emissivity:**

$$\alpha = \varepsilon, \quad (\text{R.16})$$

if  $T_{\text{rad}} = T_{\text{body}}$  or the surfaces of the body are grey.

## R.2 View factors

**Definition**

**View factor:**

$$\Phi_{ij} = \frac{\text{Radiation leaving body } i \text{ intercepted by body } j}{\text{Radiation leaving body } i} = \frac{\dot{Q}_{ij}}{\dot{Q}_i} \quad (-). \quad (\text{R.17})$$

**Definition**

**Radiosity or radiation density:**

$$L = \frac{\text{Power of radiator}}{\text{Projected area} \times \text{Solid angle}} = \frac{\dot{q}''}{\Omega} \left( \frac{\text{W}}{\text{m}^2} \right). \quad (\text{R.18})$$

**Fundamental EQ**

**Reciprocity rule:**

$$A_i \Phi_{ij} = A_j \Phi_{ji}. \quad (\text{R.19})$$

**Fundamental EQ**

**Summation rule:**

$$\sum_{j=1}^n \Phi_{ij} = \Phi_{i1} + \Phi_{i2} + \Phi_{i3} + \dots + \Phi_{in} = 1. \quad (\text{R.20})$$

**Fundamental EQ**

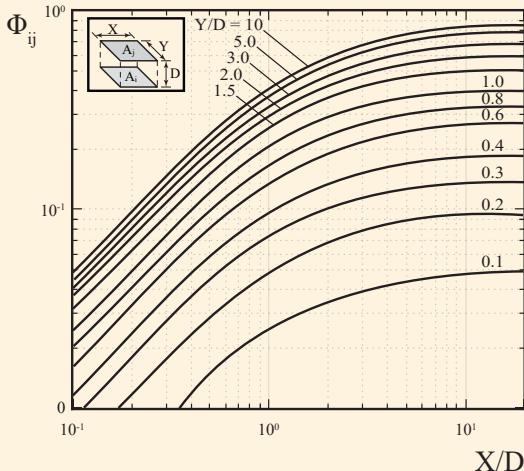
**Symmetry rule:**

$$\Phi_{ij} = \Phi_{ik}, \quad (\text{R.21})$$

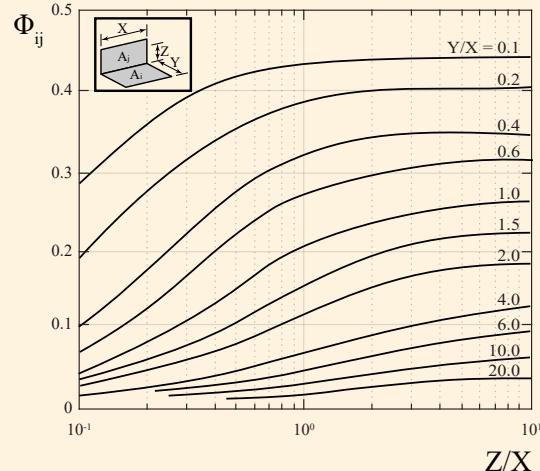
if two or more surfaces display symmetry about a third surface, they will have identical view factors from that surface.

**Fundamental EQ**

**View factor diagrams:**



(a) Radiation transfer between parallel, rectangular plates.



(b) Radiation transfer between perpendicular, rectangular plates.

### R.3 Radiative transport

Definition

Surface brightness of surface  $i$ :

$$\dot{Q} = \dot{Q}_{i,\rho} + \dot{Q}_{i,\tau} + \dot{Q}_{i,\varepsilon} \text{ (W).} \quad (\text{R.22})$$

Fundamental EQ Inner energy balance closed system without heat generation:

$$\frac{\partial U}{\partial t} = \alpha \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\varepsilon}. \quad (\text{R.23})$$

Fundamental EQ Outer energy balance closed system without heat generation:

$$\frac{\partial U}{\partial t} = \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}}. \quad (\text{R.24})$$

Fundamental EQ Net radiative heat flow from body  $i$  to  $n$  bodies:

$$\dot{Q}_{i,\text{net}} = \dot{Q}_i - \sum_{j=1}^n \dot{Q}_{j \rightarrow i}. \quad (\text{R.25})$$

Fundamental EQ Radiative heat exchange between bodies  $i$  and  $j$ :

$$\dot{Q}_{i \leftrightarrow j} = \dot{Q}_{i \rightarrow j} - \dot{Q}_{j \rightarrow i}. \quad (\text{R.26})$$

Fundamental EQ Radiative heat exchange between two black bodies  $i$  and  $j$ :

$$\dot{Q}_{i \leftrightarrow j} = A_i \Phi_{ij} \sigma (T_i^4 - T_j^4) = A_j \Phi_{ji} \sigma (T_i^4 - T_j^4). \quad (\text{R.27})$$

Fundamental EQ Radiative heat exchange between two grey plates  $i$  and  $j$ :

$$\dot{q}_{i \leftrightarrow j}'' = \frac{1}{\frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_j} - 1} \sigma (T_i^4 - T_j^4), \quad (\text{R.28})$$

when the plates are plane, parallel, and infinitely long.

Fundamental EQ Radiative heat exchange between two grey bodies  $i$  and  $j$ :

$$\dot{Q}_{i \leftrightarrow j} = \frac{A_i}{\frac{1}{\varepsilon_i} + \frac{A_i}{A_j} \left( \frac{1}{\varepsilon_j} - 1 \right)} \sigma (T_i^4 - T_j^4), \quad (\text{R.29})$$

when body  $j$  encloses body  $i$  ( $A_j > A_i$ ) and body  $i$  does not see itself ( $\Phi_{ii} = 0$ ).

## SECTION P

**Properties of materials****P.1 Thermophysical - metals**

Metals at 20°C.

	$\rho$ $10^3 \text{ kg/m}^3$	$c$ $\text{kJ/kg K}$	$\lambda$ $\text{W/m K}$	$a$ $10^{-6} \text{ m}^2/\text{s}$
Aluminum	2.70	0.888	237	98.80
Lead	11.34	0.129	35	23.90
Chromium	6.92	0.440	91	29.90
Iron	7.86	0.452	81	22.80
Gold	19.26	0.129	316	127.20
Copper	8.93	0.382	399	117.00
Magnesium	1.74	1.020	156	87.90
Manganese	7.42	0.473	21	6.00
Molybdenum	10.20	0.251	138	53.90
Sodium	9.71	1.220	133	11.20
Nickel	8.85	0.448	91	23.00
Platinum	21.37	0.133	71	25.00
Silver	10.50	0.235	427	173.00
Titanium	4.50	0.522	22	9.40
Wolfram	19.00	0.134	173	67.90
Zinc	7.10	0.387	121	44.00
Tin, white	7.29	0.225	67	40.80
Bronze	8.80	0.377	62	18.70
Cast iron	7.80	0.540	42...50	10...12
Carbon steel (<0.4% C)	7.85	0.465	42...50	12...15
Cr-Ni-steel (X12CrNi 18.8)	7.80	0.500	15	3.80

**P.2 Thermophysical - non-metal solids**

Non-metal solids at 20°C.

	$\rho$ $10^3 \text{ kg/m}^3$	$c$ $\text{kJ/kg K}$	$\lambda$ $\text{W/m K}$	$a$ $10^{-6} \text{ m}^2/\text{s}$
Acryl glass	1.18	1.44	0.184	0.108
Asphalt	2.12	0.92	0.7	0.36
Concrete	2.1	0.88	1	0.54
Ice (water 0 C)	0.917	2.04	2.25	1.203
Soil coarse gravel	2.04	1.84	0.52	0.14
Sand, dry	1.65	0.8	0.27	0.2
Sand, wet	1.75	1	0.58	0.33
Clay	1.45	0.88	1.28	1
Glass.				
window	2.48	0.7	0.87	0.5
mirror	2.7	0.8	0.76	0.35
quarz	2.21	0.73	1.4	0.87
Glass wool	1.2	0.66	0.046	0.58
Gypsum	1	1.09	0.51	0.47
Granite	2.75	0.89	2.9	1.18
Cork	0.19	1.88	0.041	0.115
Marble	2.6	0.8	2.8	1.35
Mortar	1.9	0.8	0.93	0.61
Paper	0.7	1.2	0.12	0.14
Polyethylene	0.92	2.3	0.35	0.17
Polytetrafluorethylene	2.2	1.04	0.23	0.1
PVC	1.38	0.96	0.15	0.11
Porcelain (95 C)	2.4	1.08	1.03	0.4
Hard coal	1.35	1.26	0.26	0.15
Fir wood (radial)	0.415	2.72	0.14	0.12
Plaster	1.69	0.8	0.79	0.58
Bricks	1.6...1.8	0.84	0.38...0.52	0.28...0.34

### P.3 Thermophysical - liquids

Liquids at 1 bar.

	<i>T</i>	<i>ρ</i>	<i>c</i>	<i>λ</i>	<i>ν</i>	<i>a</i>	Pr
	°C	10 <sup>3</sup> kg/m <sup>3</sup>	kJ/kg K	W/m K	10 <sup>-6</sup> m <sup>2</sup> /s	10 <sup>-6</sup> m <sup>2</sup> /s	1
Nitrogen	-190	0.861	1.988	0.161	0.321	0.0939	3.42
Water	0	0.9998	4.218	0.561	1.793	0.133	13.48
	20	0.9982	4.181	0.598	1.004	0.1434	7.001
	40	0.9922	4.177	0.631	0.658	0.1521	4.3280
	60	0.9832	4.184	0.654	0.475	0.1591	2.983
	80	0.9718	4.197	0.67	0.365	0.1643	2.221
	99.63	0.9586	4.216	0.679	0.295	0.168	1.757
Aqueous non-organic solution							
21% NaCl	-10	1.187	3.312	0.528	4.02	0.136	29.5
Benzene	20	0.879	1.738	0.154	0.74	0.101	7.33
Methanol	20	0.792	2.495	0.22	0.737	0.111	6.57
Fuel oil	20	0.819	2	0.116	1.82	0.0709	25.7
	100	0.766	2.38	0.104	0.711	0.0572	12.4
Mercury	20	13.55	0.139	9.3	0.115	4.9	0.023

### P.4 Thermophysical - gases

Gases at 1 bar.

	<i>T</i>	<i>ρ</i>	<i>c</i>	<i>λ</i>	<i>ν</i>	<i>a</i>	Pr
	°C	kg/m <sup>3</sup>	kJ/kg K	10 <sup>-3</sup> W/m K	10 <sup>-6</sup> m <sup>2</sup> /s	10 <sup>-6</sup> m <sup>2</sup> /s	1
Air	-200	5.106	1.186	6.886	0.979	1.137	0.8606
	-100	2.019	1.011	16.2	5.829	7.851	0.7423
	0	1.275	1.006	24.18	13.52	18.83	0.7179
	20	1.188	1.007	25.69	15.35	21.47	0.7148
	40	1.112	1.007	27.16	17.26	24.24	0.7122
	80	0.9859	1.01	30.01	21.35	30.14	0.7083
	100	0.9329	1.012	31.39	23.51	33.26	0.707
	200	0.7356	1.026	37.95	35.47	50.3	0.7051
	400	0.517	1.069	49.96	64.51	90.38	0.7137
	600	0.3986	1.116	61.14	99.63	137.5	0.7247
	800	0.3243	1.155	71.54	140.2	191	0.7342
	1000	0.2734	1.185	80.77	185.9	249.2	0.7458
Steam	100	0.5896	2.042	25.08	20.81	20.83	0.999
	200	0.4604	1.975	33.28	35.14	36.6	0.96
	400	0.3223	2.07	54.76	75.86	82.07	0.9243
	600	0.2483	2.203	79.89	131.4	146.1	0.8993
	800	0.2019	2.343	107.3	199.9	226.8	0.8816
	1000	0.1702	2.478	163.3	280	323.2	0.8665
Hydrogen	0	0.0886	14.24	176	95	139	0.68
	50	0.0748	14.36	202	126	188	0.67
	100	0.0649	14.44	229	159	244	0.65
Carbon dioxide	0	1.95	0.829	14.3	7.1	8.86	0.8
	50	1.648	0.875	17.8	9.8	12.3	0.8
	100	1.428	0.925	21.3	12.4	16.1	0.8
Helium	27	0.1625	5.193	155.7	122.6	184.5	0.655

## P.5 Emissivity - solids

Emissivity of various solids (Total emissivity  $\epsilon$ , Emissivity in normal direction of the surface  $\epsilon_n$ ).

Surface	$T$ K	$\epsilon_n$	$\epsilon$	Surface	$T$ K	$\epsilon_n$	$\epsilon$
<b>Metals</b>							
Aluminum, plain	443	0.039	0.049	Zinc, highly polished	500		0.045
... polished	373	0.095		Iron plate, galvanized	600		0.055
... heavily oxidized	366	0.2		... plain	301	0.228	
	777	0.31		... grey oxidized	297	0.276	
Aluminum oxide	550	0.63		Tin, non oxidized	298		0.043
	1100	0.26			373	0.05	
	1089	0.052		<b>Non-Metals</b>			
Chromium, polished	423	423	423	Asbestos, paper	296	0.96	
				... Papier	311	0.93	
Gold, highly polished	500	0.018		Concrete, rough	273 – 366		0.94
	900	900		Roofing felt	294	0.91	
Copper, polished	293	0.03		Gips	293	0.8 – 0.9	
... struck	293	0.037		Glas	293	0.94	
... black oxidized	293	0.78		Quartz (7mm thick)	555	0.93	
... oxidized	403	0.76			1111	0.47	
Inconel, rolled	1089		0.69	Rubber	293	0.92	
... sandblasted	1089		0.79	Wood			
Iron and steel,				Oak, planed	273 – 366		0.9
... highly polished	450	0.052		Beech	343	0.94	0.91
... polished	700	0.144		Ceramics			
	1300	0.377		White $\text{Al}_2\text{O}_3$	366		0.9
... sanded	293	0.242		Carbon			
Cast iron, polished	473	0.21		... not oxidized	298		0.81
Cast steel, polished	1044	0.52			773		0.79
	1311	0.56		... Fibers	533		0.95
Iron sheet				... Graphite	373		0.76
... heavy rusty	292	0.685		Corundum, rough	353	0.85	0.84
... rolled	294	0.657		Coating, colors:			
Cast iron.				Oil paint black	366		0.92
... oxidized at 866 K	472	0.64		... green	366		0.95
	872	0.78		... red	366		0.97
Steel,				... white	373		0.94
... oxidized at 866 K	472	0.79		Coating, white	373	0.925	
	872	0.79		... flat black	353		0.97
Brass, not oxidized	298	0.035		Bakelite coating	353	0.935	
	373	0.035		Mennig color	373	0.93	
... oxidized	473	0.61		Radiator (acc. to VDI-74)	373	0.925	
	873	0.59		Enamel, white on iron	292	0.897	
Nickel, not oxidized	298	0.045		Marble			
	373	0.06		light grey, polished	273 – 366		0.9
	873	0.478		Paper	273		0.92
... oxidized	473	0.37			366		0.94
Platinum	422	0.022		Porcelain, white	295		0.924
	1089	0.123		Clay, glassy	298		0.9
Mercury,				... flat	298		0.93
... not oxidized	298	0.1		Water	273	0.95	
	373	0.12			373	0.96	
Silver, polished	311	0.022		Ice, smooth with water	273	0.966	0.92
	644	0.031		... rough surface	273	0.985	
Wolfram	298		0.024	Bricks, red	273 – 366		0.93
	1273		0.15				
	1773		0.23				

## SECTION M

**Mathematical function****M.1 Error functions****Definition****Error function**

$$\operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_{\xi=0}^{\xi=\eta} \exp(-\xi^2) d\xi, \quad (\text{M.1})$$

which has the characteristic  $\operatorname{erf}(\infty) = 1$     $\operatorname{erf}(-\eta) = -\operatorname{erf}(\eta)$     $\frac{d}{d\eta} [\operatorname{erf}(\eta)] = \frac{2}{\sqrt{\pi}} \exp(-\eta^2)$ .

**Definition****Complementary error function**

$$\operatorname{erfc}(\eta) = 1 - \operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_{\xi=\eta}^{\xi=\infty} \exp(-\xi^2) d\xi. \quad (\text{M.2})$$

Evaluation of the error function.

$\eta$	$\operatorname{erf}(\eta)$	$\operatorname{erfc}(\eta)$	$2/\sqrt{\pi} \exp(-\eta^2)$
0	0	1	1.128
0.05	0.056	0.944	1.126
0.1	0.112	0.888	1.117
0.15	0.168	0.832	1.103
0.2	0.223	0.777	1.084
0.25	0.276	0.724	1.060
0.3	0.329	0.671	1.031
0.35	0.379	0.621	0.998
0.4	0.428	0.572	0.962
0.45	0.475	0.525	0.922
0.5	0.520	0.480	0.879
0.55	0.563	0.437	0.834
0.6	0.604	0.396	0.787
0.65	0.642	0.378	0.740
0.7	0.678	0.322	0.691
0.75	0.711	0.289	0.643
0.8	0.742	0.258	0.595
0.85	0.771	0.229	0.548
0.9	0.797	0.203	0.502
0.95	0.821	0.179	0.458
1	0.843	0.157	0.415
1.1	0.880	0.120	0.337
1.2	0.910	0.090	0.267
1.3	0.934	0.066	0.208
1.4	0.952	0.048	0.159
1.5	0.966	0.034	0.119
1.6	0.976	0.024	0.087
1.7	0.984	0.016	0.063
1.8	0.989	0.011	0.044
1.9	0.993	0.007	0.030
2	0.995	0.005	0.021