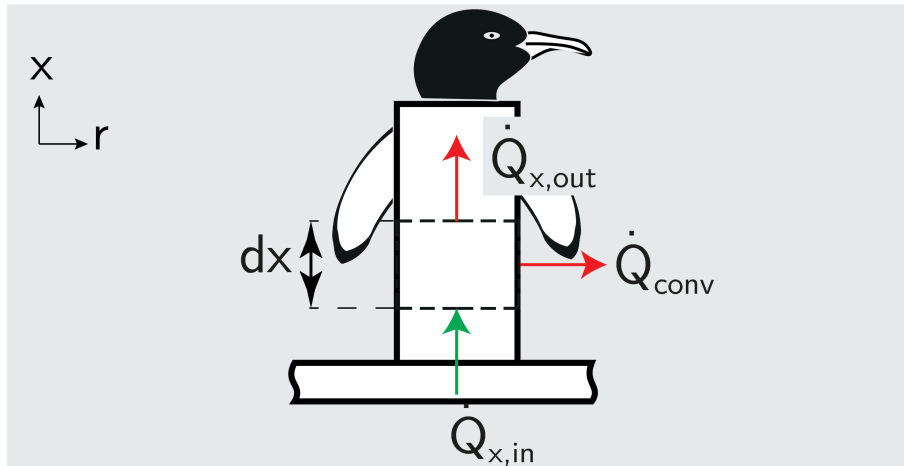# EB - Cond. - IE 11

Derive a homogeneous differential equation to describe the axial temperature distribution for the body of a penguin. Assume one-dimensional, steady-state heat transfer in the x-direction with no sources/sinks.

Homogenization:

$$\Theta = T(x) - T_\infty$$

$$m^2 = \frac{2 \cdot \alpha}{\lambda \cdot R}$$

Energy balance:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} - \dot{Q}_{conv}(x) = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \frac{\partial T}{\partial x} \;\rightarrow\; \dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \frac{\partial \Theta}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda \cdot \pi R^2 \cdot \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx \;\rightarrow\; \dot{Q}_{x,out} = \dot{Q}_{x,in} - \lambda \cdot \pi R^2 \cdot \frac{\partial^2 \Theta}{\partial x^2} dx$$

$$\dot{Q}_{conv} = \alpha \cdot 2\pi R dx \cdot (T(x) - T_\infty) \rightarrow \dot{Q}_{conv} = \alpha \cdot 2\pi R dx \cdot \Theta$$

Substituting and rewriting:

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} - \dot{Q}_{conv}(x) = 0$$

$$-\lambda \cdot \pi R^2 \cdot \frac{\partial \Theta}{\partial x} + \lambda \cdot \pi R^2 \cdot \frac{\partial \Theta}{\partial x} + \lambda \cdot \pi R^2 \cdot \frac{\partial^2 \Theta}{\partial x^2} dx - \alpha \cdot 2\pi R dx \cdot \Theta = 0$$

$$\Rightarrow \frac{\partial^2 \Theta}{\partial x^2} - m^2 \Theta = 0$$