Heat Transfer

Introduction to convection and the conservation equations

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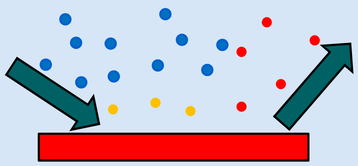
Learning Goals

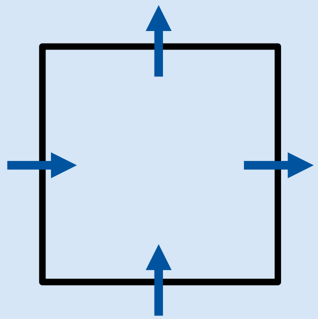
• Classification

- ➢ Understanding Convection and the distinction from Advection
- ➢ Convection as the interaction of heat Conduction and Advection
- ➢ Classification of convection problems



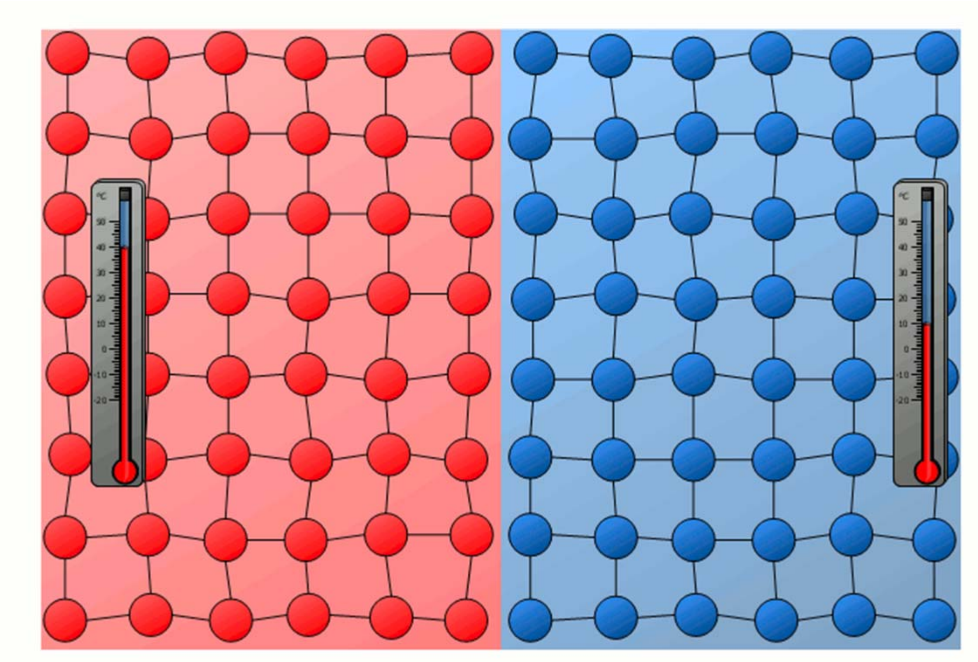
• Conservation Equation

- ➢ Derive the conservation equations for mass, momentum and energy
- ➢ Understand the similarity between momentum and energy transport



How is the heat transferred?

Heat Conduction (conduction/diffusion)



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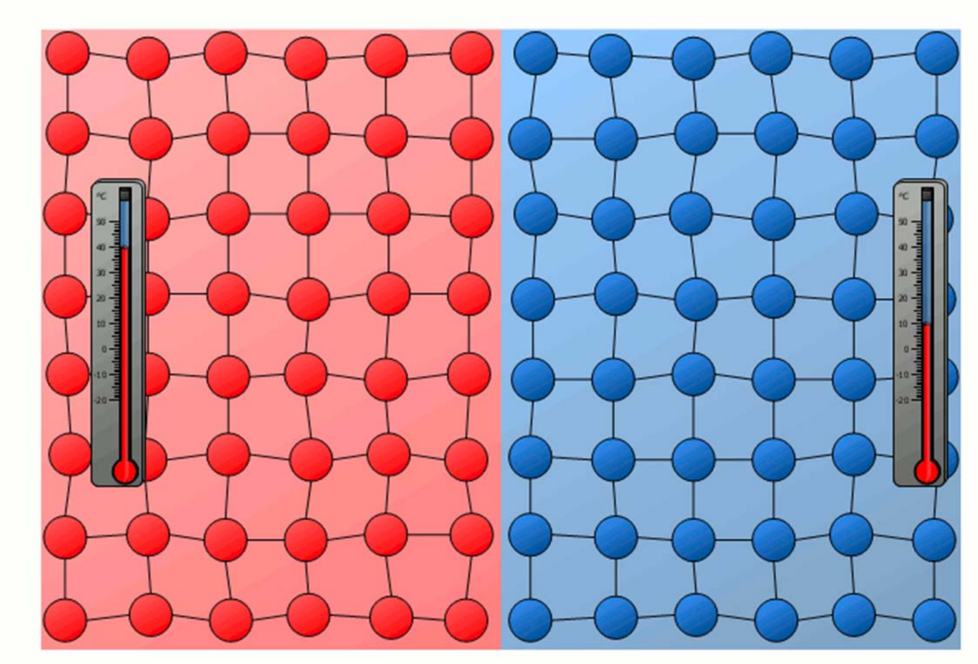

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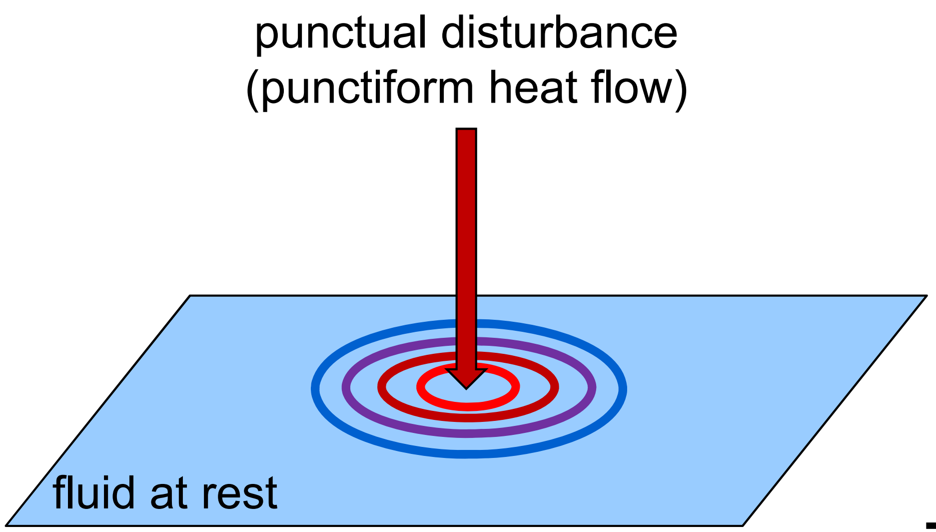
Convection

How is the heat transferred?

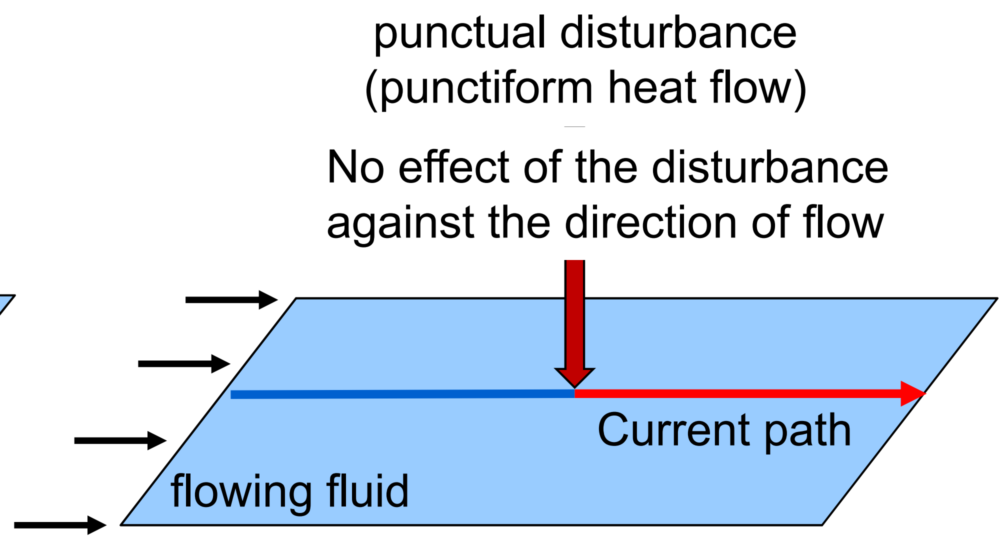
Heat Conduction (conduction/diffusion)



Heat flow in radial directional along the gradients

Fourier Law $\dot{q}'' = -\lambda \nabla T$

Advection



Heat is transported by fluid movement along a current path

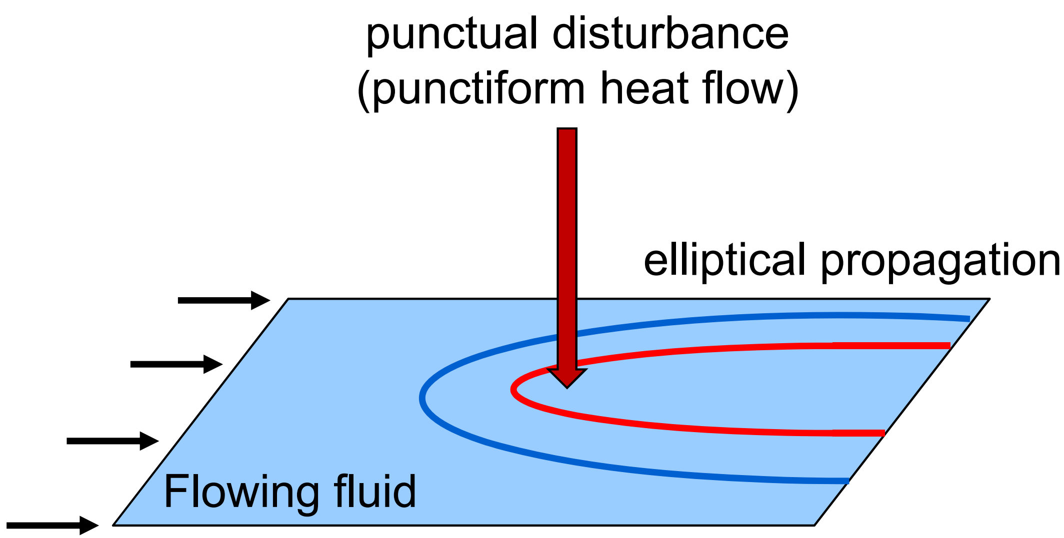
Enthalpy flow density $\dot{h}'' = \rho u c_p T$

How is the heat transferred?

Heat Conduction (conduction/diffusion) + Advection

=

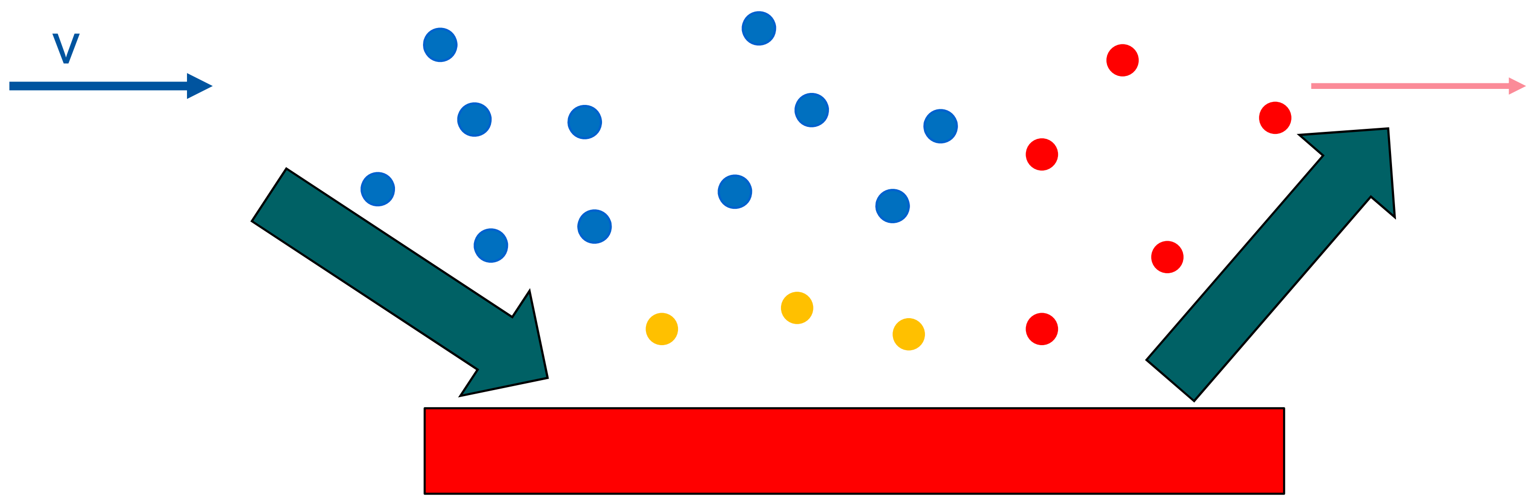
Convective Heat Transfer (convection)



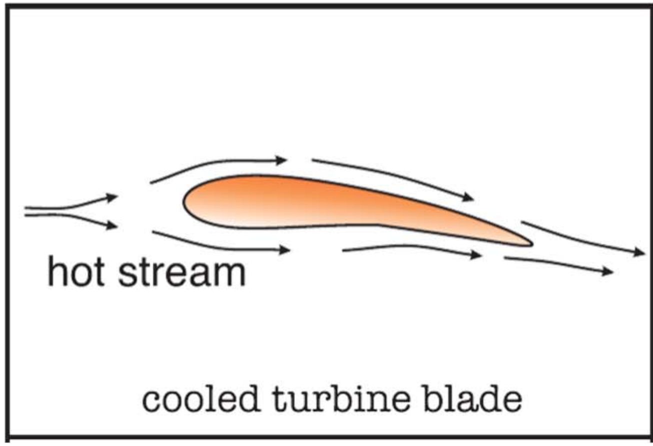
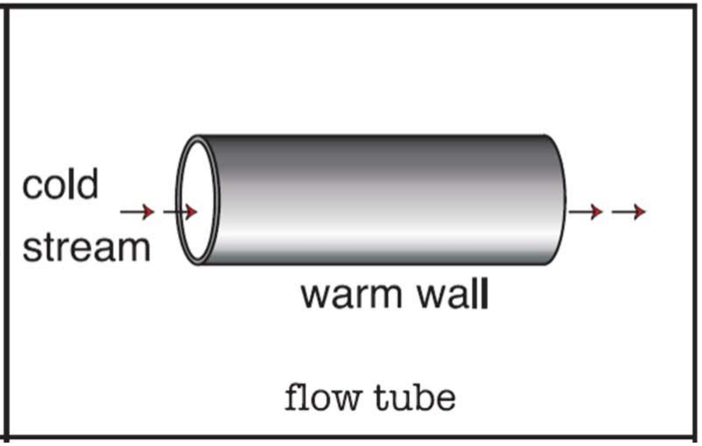
Transport along the current (flow) paths: **Convection (and Conduction)**
Transport perpendicular to the current paths: **<u>only</u> Conduction**

Mechanism of convective heat transfer

What is the difference in comparison to pure heat conduction?



Classifications according to flow condition

	External	Internal
Forced Convection • Driven by externally generated movement of the fluid/object		

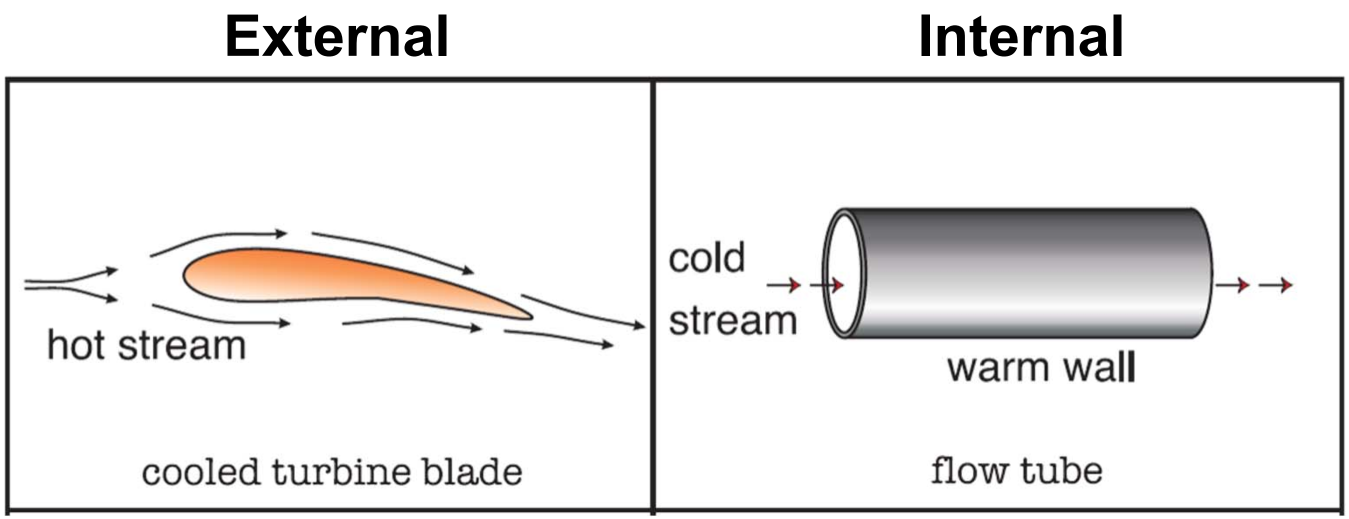
Classifications according to flow condition

Forced Convection

- Driven by externally generated movement of the fluid/object

Free Convection

- Inherently driven due to heat transfer (density differences)



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Classifications according to flow condition

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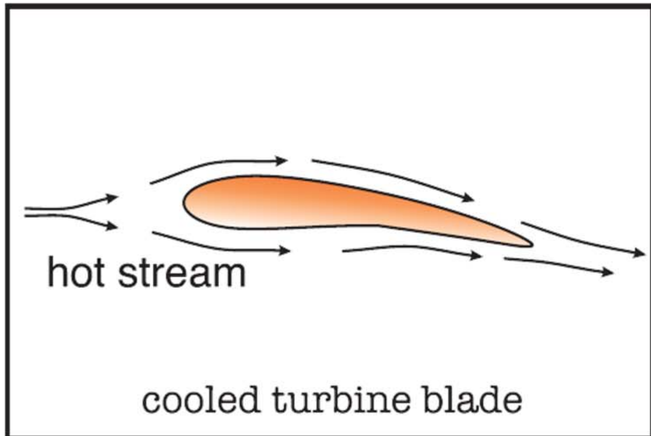
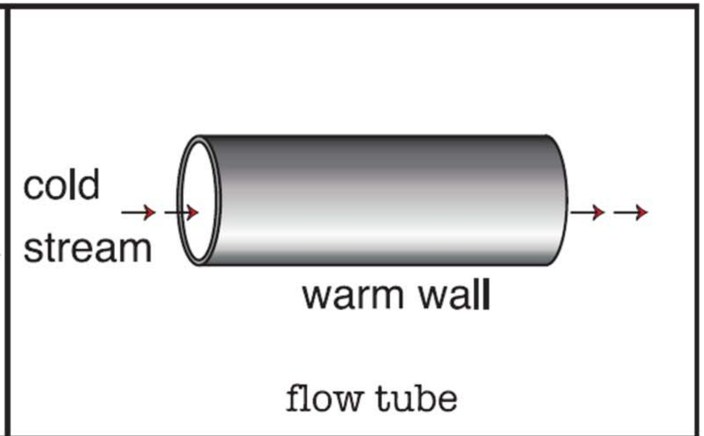
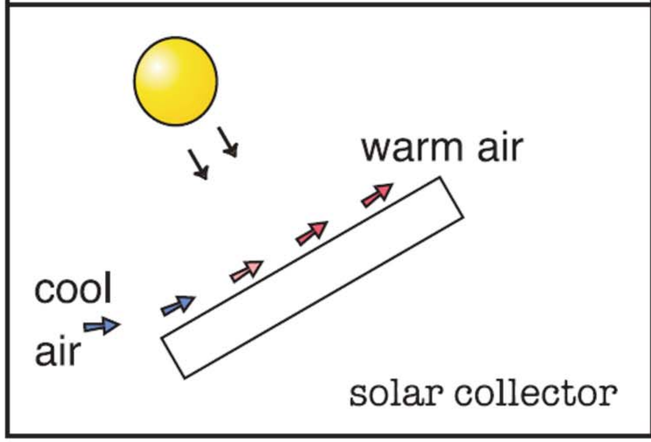
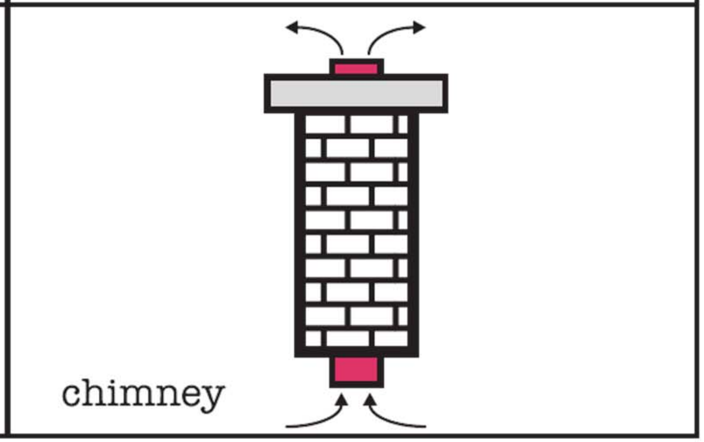
Forced Convection • Driven by externally generated movement of the fluid/object		
Free Convection • Inherently driven due to heat transfer (density differences)		

Empirical description by the heat transfer coefficient

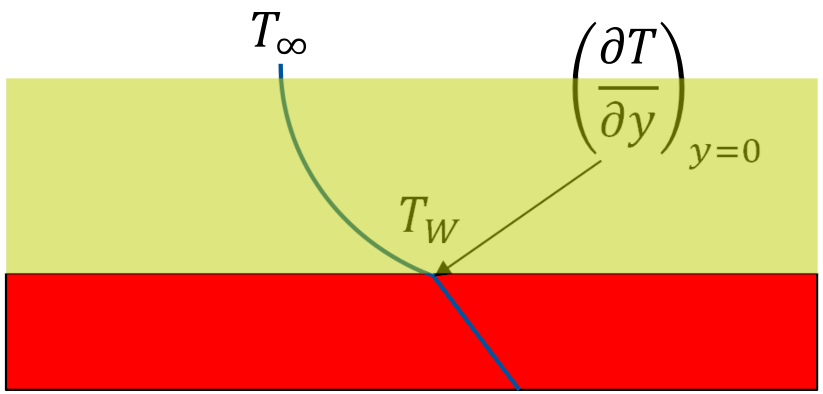
$$\dot{Q} = \alpha\, A\, (T_W - T_\infty)$$

Fourier's Heat Conduction Law

$$\dot{Q} = -A\lambda_f \left(\frac{\partial T}{\partial y}\right)_{y=0,f}$$

The heat transfer coefficient α describes the approximately linear relationship between the amount of heat transferred and the temperature gradient. α is a SYSTEM parameter, not a material property !

$$\alpha \equiv \frac{-\lambda_f \left(\frac{\partial T}{\partial y}\right)_{y=0,f}}{(T_W - T_\infty)}$$



Nusselt number

- Dimensionless heat transfer coefficient with the reference length L

$$Nu = \frac{\alpha L}{\lambda} = L\,\frac{-\left(\frac{\partial T}{\partial y}\right)_{y=0,f}}{(T_W - T_\infty)}$$

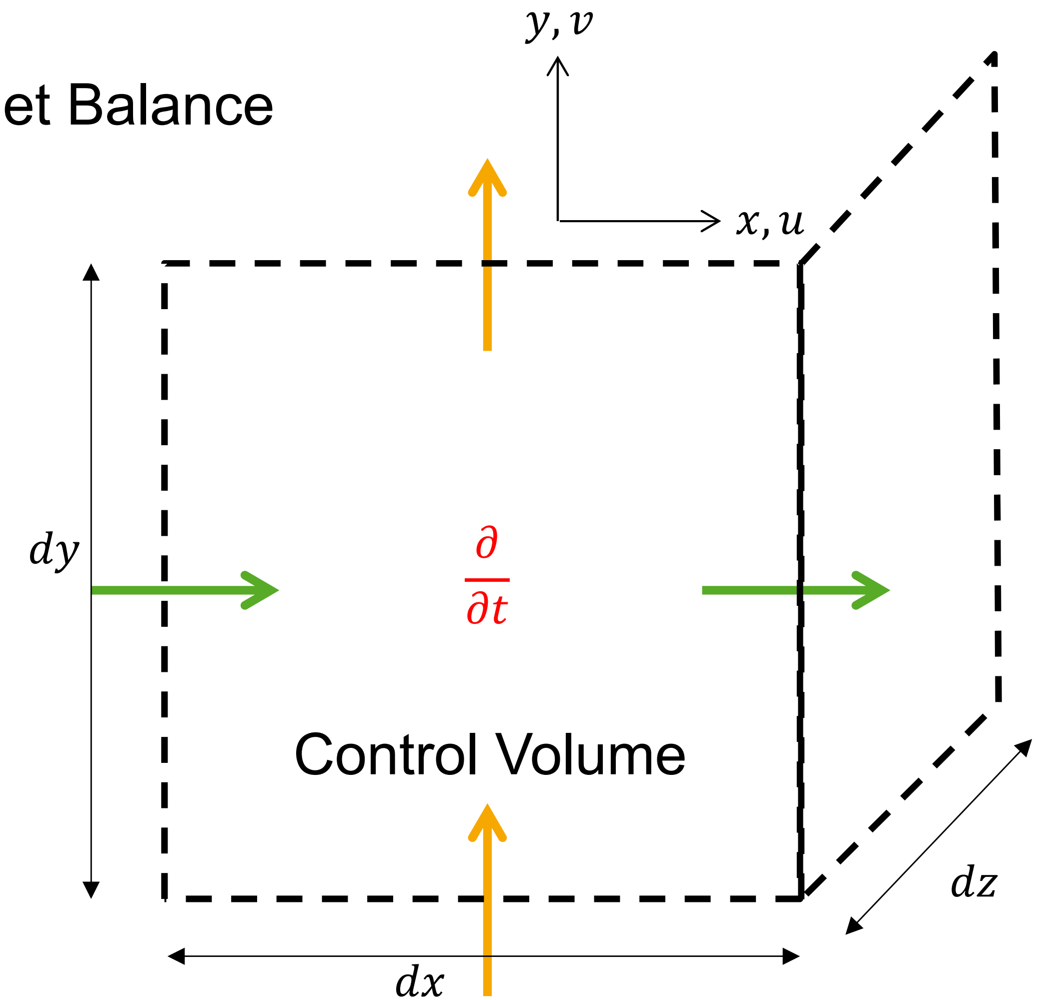
Boundary Layer

- Near-wall layer with significant gradient of Velocity and Temperature
- What happens here? → Conservation Equation

Conservation Equation

For Mass $\dot{m}$, Momentum $\dot{I}$, Energy h, $\dot{q}''$.

Set Balance



General Balance

Temporal change of a quantity inside the control volume

=

Net transport of the quantity across the boundaries of the control volume

+

External forces (for momentum equation)

+

Work output of the external forces (for energy equation)

Continuity Equation

Set Balance

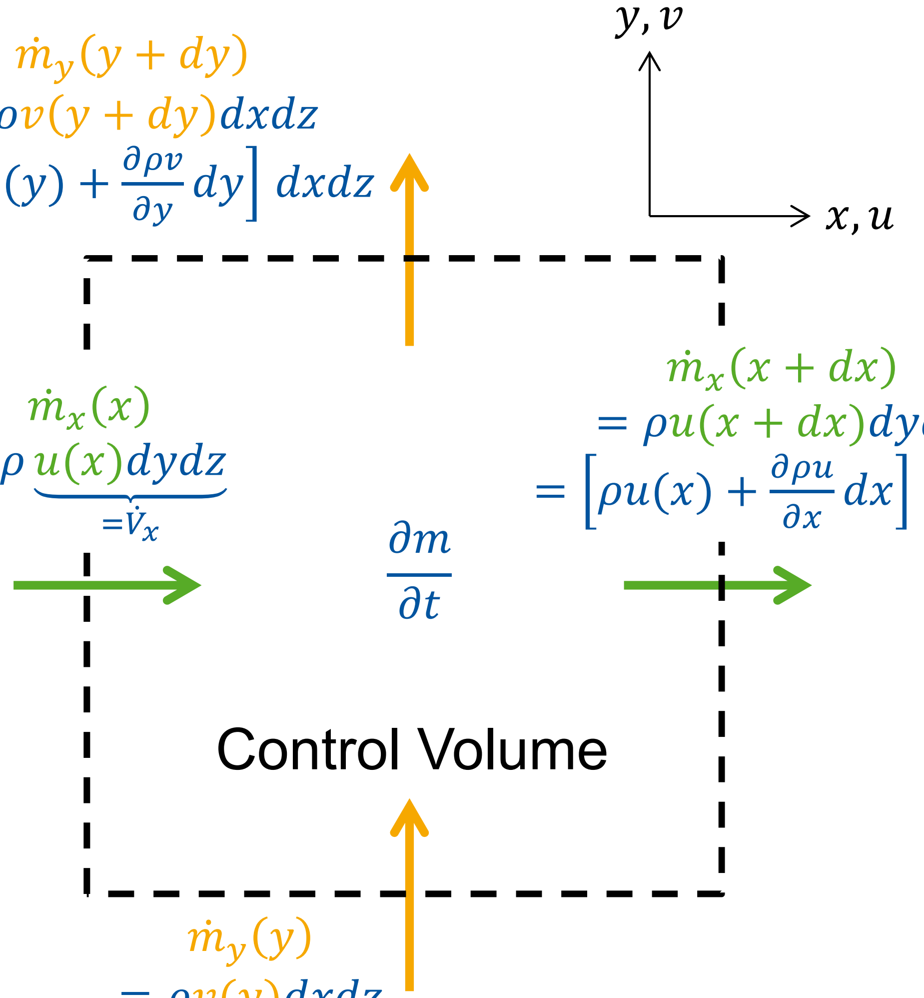
$$\dot{m}_y(y+dy) = \rho v(y+dy)dxdz = \left[\rho v(y) + \frac{\partial \rho v}{\partial y}dy\right] dxdz$$

$$\dot{m}_x(x) = \rho \underbrace{u(x)dydz}_{=\dot{V}_x}$$

$$\frac{\partial m}{\partial t}$$

$$\dot{m}_x(x+dx) = \rho u(x+dx)dydz = \left[\rho u(x) + \frac{\partial \rho u}{\partial x}dx\right] dydz$$

$$\dot{m}_y(y) = \rho v(y)dxdz$$



Mass Flows

$$\frac{\partial m}{\partial t} = \dot{m}_x(x) - \dot{m}_x(x+dx) + \dot{m}_y(y) - \dot{m}_y(y+dy)$$

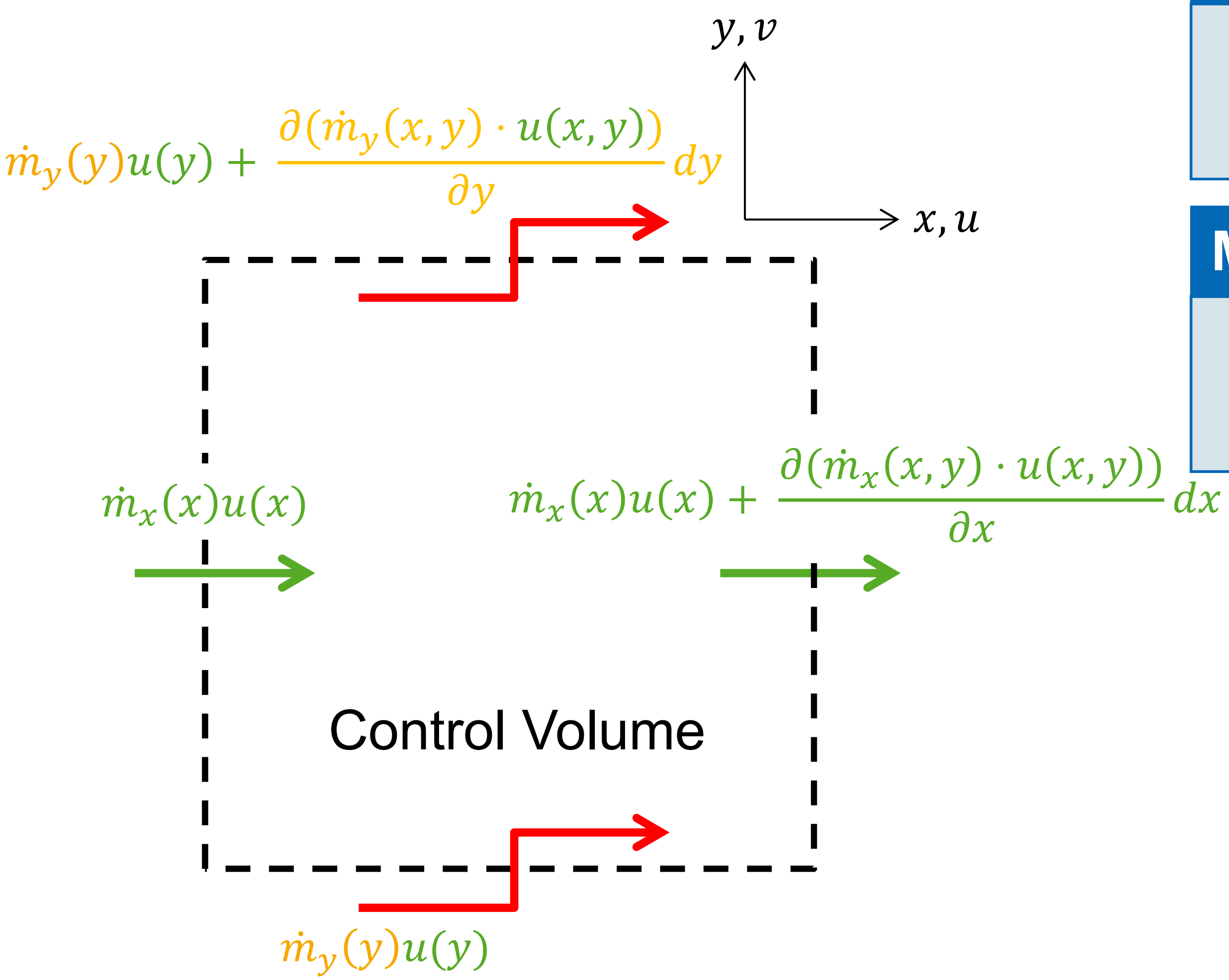
$$\frac{\partial \rho}{\partial t}\cancel{dV} = -\frac{\partial \rho u}{\partial x}\cancel{dxdydz} - \frac{\partial \rho v}{\partial y}\cancel{dxdydz}$$

incompressible $\rho = const.$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum Equation: x-direction

Set Balance



Temporal Change

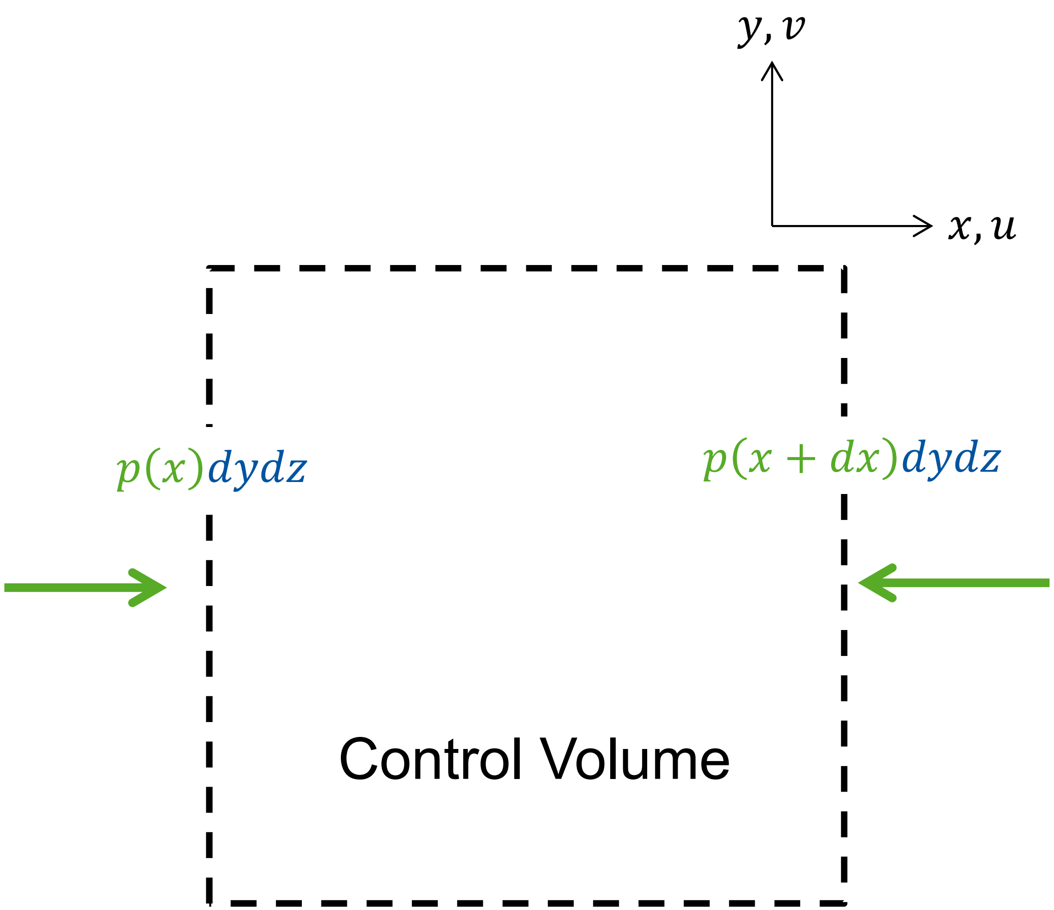
Steady state $\frac{\partial I_x}{\partial t} dV = 0$

Momentum Flow

$$-\left(\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y}\right) dxdydz$$

Momentum Equation: x-direction

Set Balance



Temporal Change

Steady state $\frac{\partial I_x}{\partial t} dV = 0$

Momentum Flow

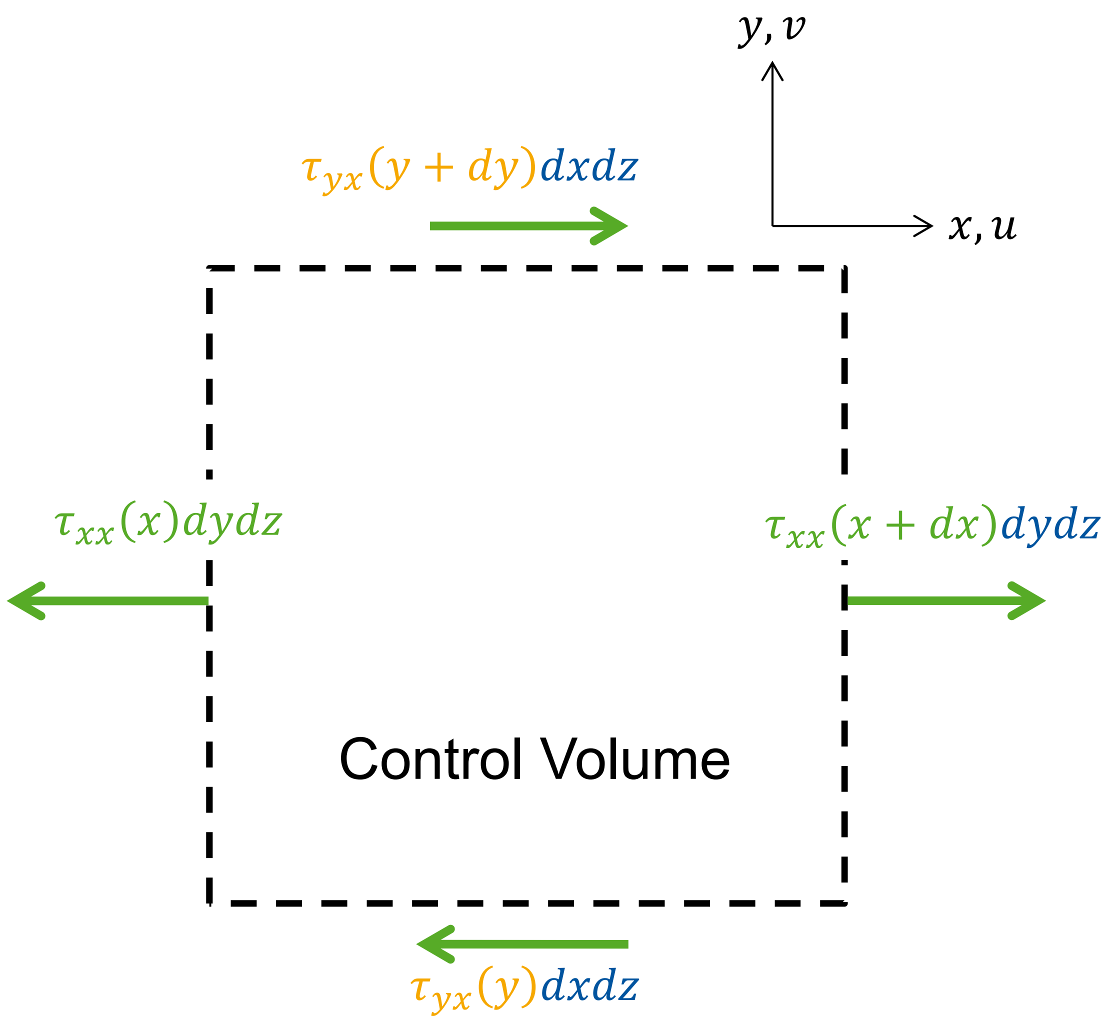
$$-\left(\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y}\right) dxdydz$$

External Forces acting ON the volume

Pressure Change $-\frac{\partial p}{\partial x} dxdydz$

Momentum Equation: x-direction

Set Balance



Temporal Change

Steady state $\quad \frac{\partial I_x}{\partial t} dV = 0$

Momentum Flow

$$-\left(\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y}\right) dxdydz$$

External Forces acting ON the volume

Pressure Change $\quad -\frac{\partial p}{\partial x} dxdydz$

Shear Stress

(if incompressible)

$$\eta \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) dxdydz$$

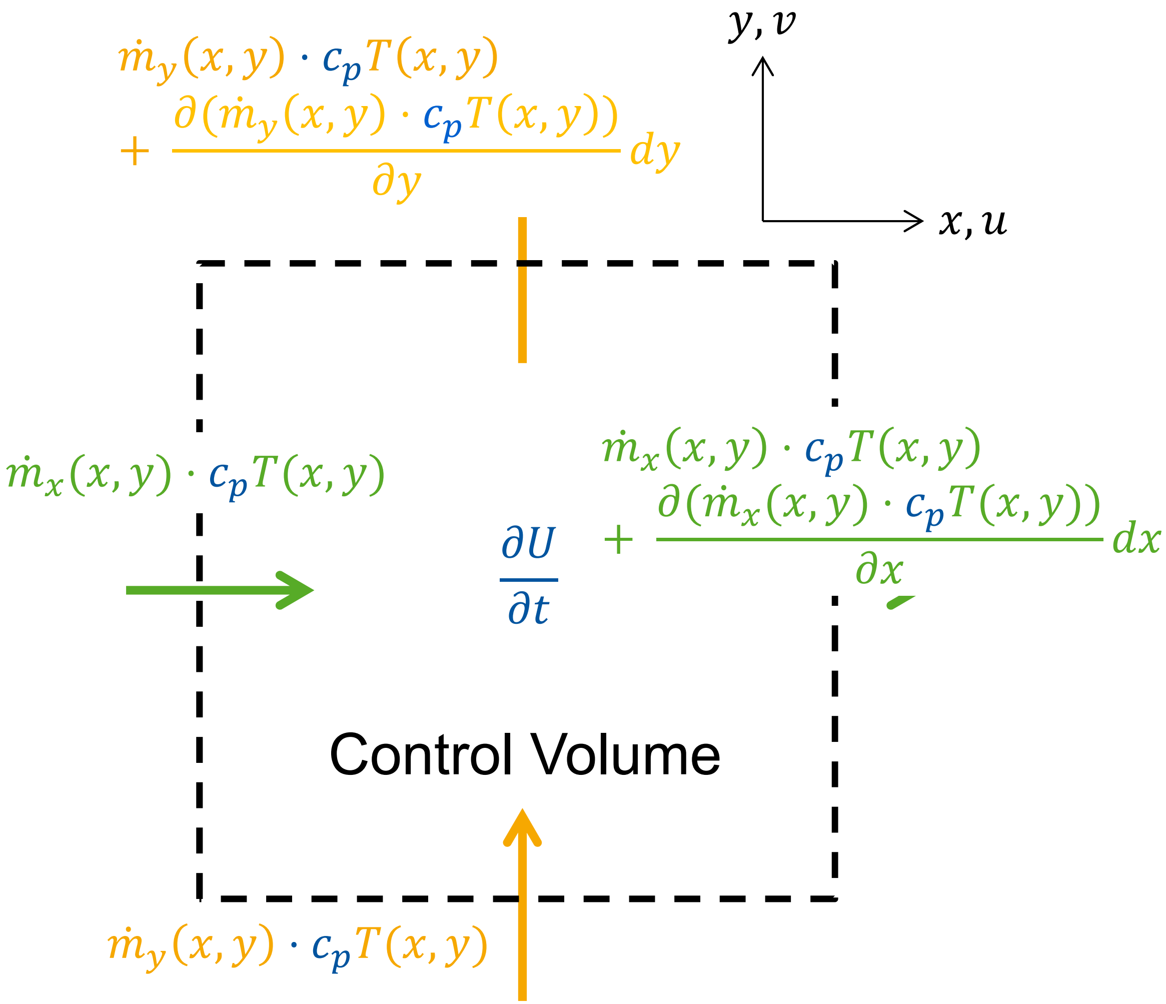
Momentum Equation (steady state, incompressible)

	Momentum Flows	Pressure	Shear Stress
x-direction	$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} =$	$- \frac{\partial p}{\partial x}$	$+ \eta \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$
y-direction	$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} =$	$- \frac{\partial p}{\partial y}$	$+ \eta \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$
z-direction	$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} =$	$- \frac{\partial p}{\partial z}$	$+ \eta \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$

+ Volume forces (e.g. Gravitation)

Energy Conservation: Enthalpy Flows

Set Balance



Temporal Change

$$\frac{\partial U}{\partial t} = \rho c_p \frac{\partial T}{\partial t} dV \quad \text{(steady state } \frac{\partial U}{\partial t} = 0\text{)}$$

Enthalpy Flows

$$-\left(\rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial y}\right) dxdydz$$

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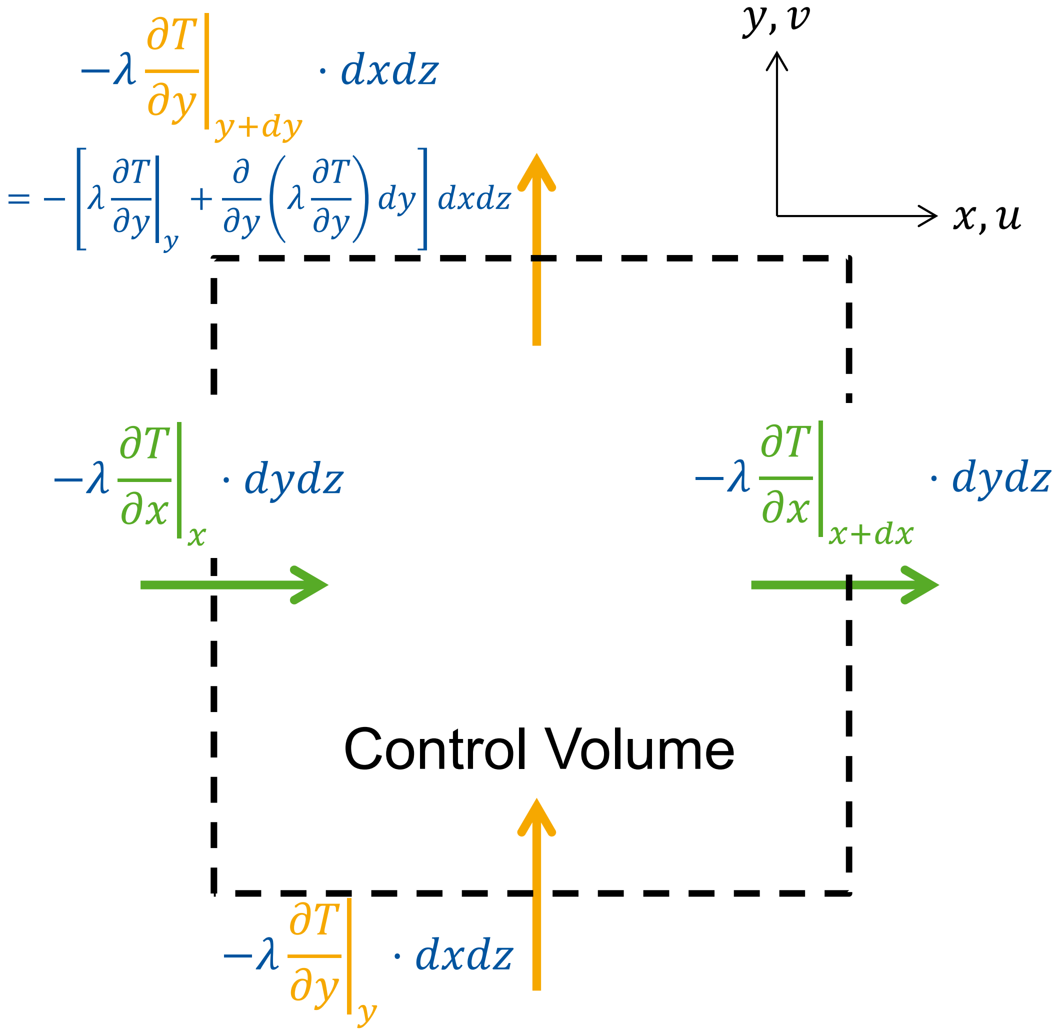
Energy Conservation: Heat conduction / diffusion

Set Balance



Temporal Change

$$\frac{\partial U}{\partial t} = \rho c_p \frac{\partial T}{\partial t} dV \quad \text{(stationär } \frac{\partial U}{\partial t} = 0\text{)}$$

Enthalpy Flows

$$-\left(\rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial y}\right) dxdydz$$

Heat Conduction

(if λ homogeneous)

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) dxdydz$$

Energy Conservation (steady state, incompressible, λ homogeneous)

Enthalpy Flows / Heat Conduction

$$\cancel{\rho} u \cancel{c}_p \frac{\partial T}{\partial x} + \cancel{\rho} v \cancel{c}_p \frac{\partial T}{\partial y} + \cancel{\rho} w \cancel{c}_p \frac{\partial T}{\partial z} = \cancel{\lambda}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

+ Work against pressure, shear stresses, volume forces

$$a = \frac{\lambda}{\rho c_p}$$

Compared to Conservation of Momentum

Impulse Flows / Pressure / Shear Stresses

$$\cancel{\rho} u \frac{\partial u}{\partial x} + \cancel{\rho} v \frac{\partial u}{\partial y} + \cancel{\rho} w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \cancel{\eta}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

+ Volume forces (e.g. Gravitation)

$$\frac{1}{\rho} \qquad \upsilon = \frac{\eta}{\rho}$$

Similarity between Momentum and Energy transport

Momentum Flows | Pressure | Shear Stresses

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{\nu}{Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

Enthalpy Flows (advective transport) | Heat Conduction

Prandtl number

$$Pr=\frac{\nu}{a}=\frac{\text{Diffusive Momentum transport}}{\text{Diffusive Heat transport}}$$

Comprehension questions

What is meant by a heat transfer coefficient and what does it describe?

Why does the Fourier's law of heat conduction also apply on the fluid side in the immediate vicinity of the wall?

What does the dimensionless Nusselt number mean?

What is the difference between natural and forced convection?