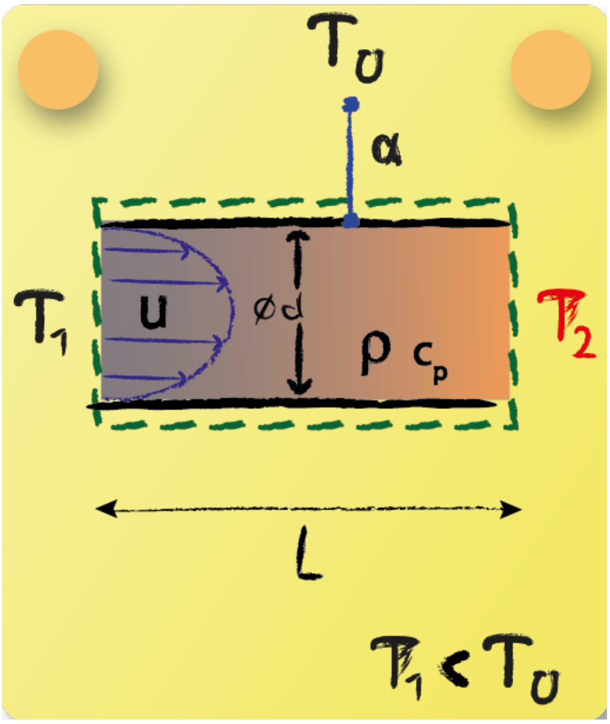
Energy Balance: Task 13



Derive an equation to determine T_2

The differential equation that is to be solved to determine T_2 emerges from the energy balance of an infinitesimal slice of pipe in x-direction:

$$0 = \frac{\partial T}{\partial x} + \frac{4\alpha}{\rho u d c_{\mathrm{p}}}[T - T_{\mathrm{u}}]$$

To solve the equation the common procedure for first order linear inhomogeneous differential equations is performed. With boundary condition $T(x=0) = T_1$ the temperature profile is given as:

$$T(x) = T_{\mathrm{u}} + [T_1 - T_u]e^{\frac{-4\alpha}{\rho u d c_{\mathrm{p}}}x}$$

1 and T_2 consequently as:

$$T_2 = T(x=L) = T_{\mathrm{u}} + [T_1 - T_u]e^{\frac{-4\alpha}{\rho u d c_{\mathrm{p}}}L}$$

However the collection of formulas already contains an expression for heat flux from wall to fluid via the logarithmic temperature difference that results from the differential equation.

$$\dot{Q} = \alpha A \Delta T_{\mathrm{ln}} = \alpha A \frac{T_1 - T_2}{\ln(\frac{T_1 - T_u}{T_2 - T_u})}$$

The energy balance than writes as:

$$0 = u\frac{\pi d^2}{4}\rho c_{\mathrm{p}}(T_1 - T_2) + \alpha \pi d L \frac{T_1 - T_2}{\ln(\frac{T_1 - T_u}{T_2 - T_u})}$$