Mass Transfer: Diffusion

Example:
Evaporation of a droplet - Stefan flow

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Fuel in the combustion engine and droplets (aerosols) while speaking

COVID-19

- Main infection pathway:
  ⇨ aerosol droplets,
  exhalation, speaking, sneezing

[1]

[2]

[1] DISI Motor Animation (Source: BOSCH)
[2] https://www.zm-online.de/news/gesellschaft/aerosole-bleiben-beim-sprechen-durchschnittlich-acht-minuten-in-der-luft/

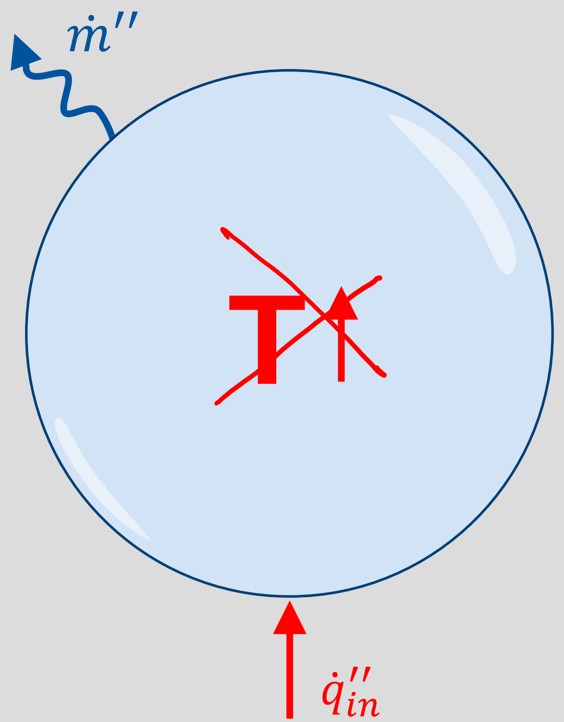
Procedure

Investigation of:

- Balance at the droplet
- Equilibrium temperature during evaporation of a droplet
- Mass flow of the evaporated fuel $\dot{m}''$
- Duration of complete evaporation of a droplet

Balance at the droplet

What happens when a drop evaporates?



- Initially, the droplet is colder than the environment
  - → Surface temperature is low
  - → Diffusion in the environment is low because the driving potential is low

$$\dot{m}'' \sim (\xi_{Surface} - \xi_{\infty})$$

- After a certain time, an equilibrium is reached between:
  - → amount of heat provided
  - → amount of heat required for evaporation

- Then the droplet temperature does not change anymore
  - → This is the case considered here

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Balance at the droplet

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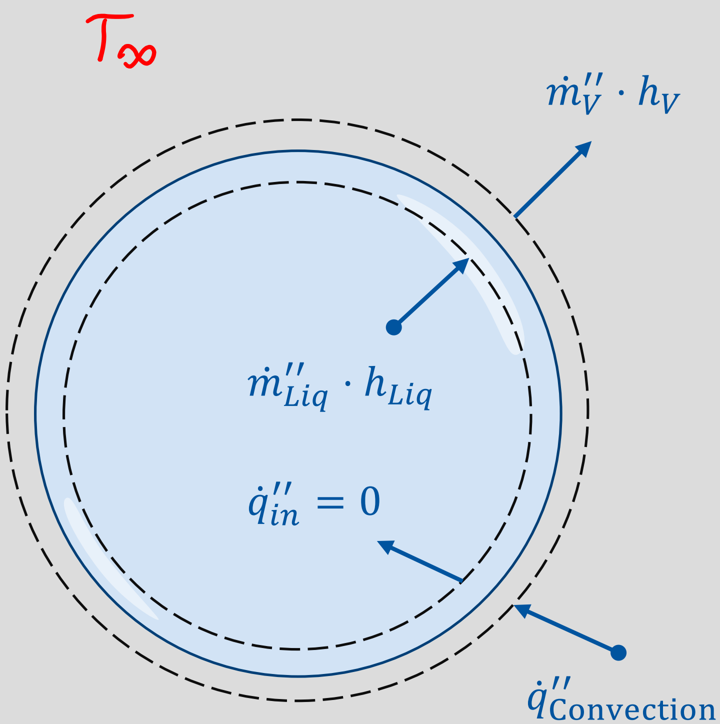


- Balance

$$\dot{m}''_V \cdot h_V - \dot{m}''_{Liq} \cdot h_{Liq} = \dot{q}''_{\mathrm{Convection}}$$

$$\dot{m}''_V = \dot{m}''_{Liq} = \dot{m}''$$

$$\dot{m}'' \cdot (h_V - h_{Liq}) = \dot{q}''_{\mathrm{Convection}}$$

$$\Delta h_v = h_V - h_{Liq}$$

$$\dot{q}''_{\mathrm{Convection}} = \alpha \left(T_\infty - T_{Liq,\,Surface} \right)$$

$$\boxed{\dot{m}'' \cdot \Delta h_v = \alpha \left(T_\infty - T_{Liq,\,Surface} \right)}$$

Equilibrium temperature and evaporation amount $\dot{m}''$

Calculate droplet surface temperature:

Formula seems straightforward (at the first glance)
⇨ this does not work

$$\dot{m}'' \cdot \Delta h_v = \alpha \left(T_\infty - T_{Liq,\,Surface} \right)$$

$$\Rightarrow \cancel{T_{Liq,Surface} = T_\infty - \frac{\dot{m}'' \cdot \Delta h_v}{\alpha}}$$

$$\Delta h_v = f\left(T_{Liq,Surface}\right)$$

$$\dot{m}'' = ?$$

How large is $\dot{m}''$?

$$\dot{m}'' = \boxed{g} \cdot \frac{\xi_{\text{Vapor},\,Surface} - \xi_{\text{Vapor},\infty}}{\boxed{1 - \xi_{\text{Vapor},\,Surface}}}$$

$$\xi_{\text{Vapor},\,Surface} = f\left(T_{Liq,Surface}\right)$$

Stefan Factor

Lewis Law: $g = \frac{\boxed{\alpha}}{c_p}$

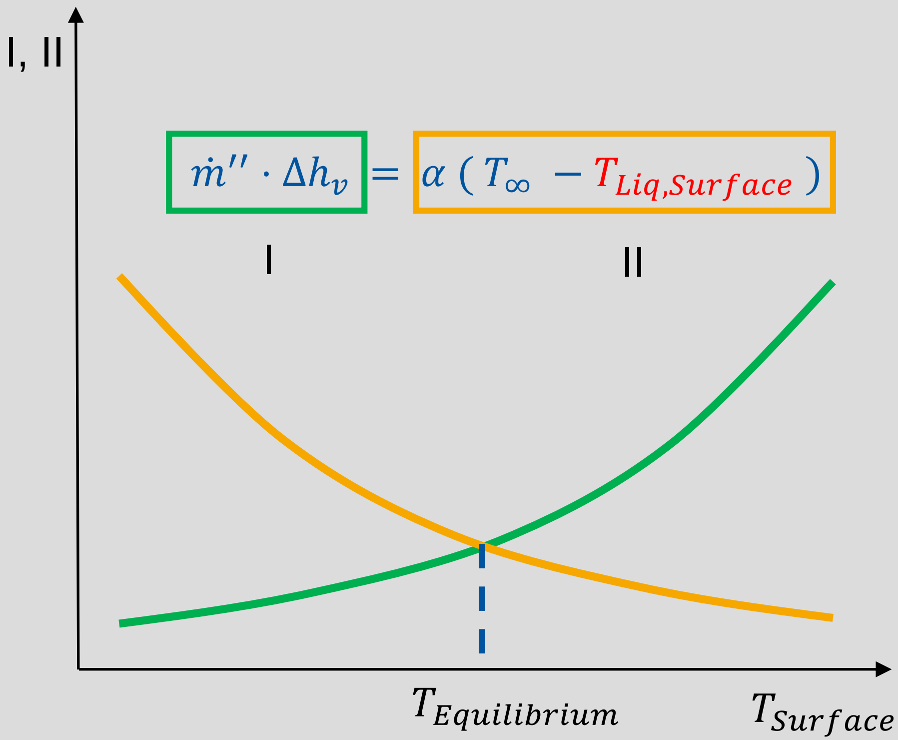
using Nusselt correlation

Equilibrium temperature and evaporation amount $\dot{m}''$

Calculate droplet surface temperature:

Temperature can only be determined **iteratively**, since both enthalpy and $\xi_{Vapor,Surface}$ are temperature dependent

$$\boxed{\dot{m}'' \cdot \Delta h_v} = \boxed{\alpha \left(T_\infty - T_{Liq,Surface} \right)}$$



How large is $\dot{m}''$?

$$\dot{m}'' = \boxed{g} \cdot \frac{\xi_{Vapor,Surface} - \xi_{Vapor,\infty}}{\boxed{1 - \xi_{Vapor,Surface}}}$$

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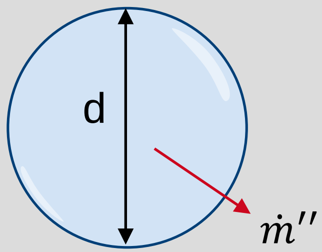
Evaporation time

After what time has the droplet completely evaporated?



$$\frac{dm}{dt} = -\dot{m}'' \cdot A \;\Rightarrow\; \boxed{\rho \frac{dV}{dt}} = -g \cdot \boxed{\frac{\xi_{V,S} - \xi_{V,\infty}}{1-\xi_{V,S}}}^{\,B} \cdot A \;\Rightarrow\; \rho \frac{d(\frac{4}{3}\pi r^3)}{dt} = -g \cdot B \cdot 4\pi r^2$$

$$\frac{\cancel{4}}{3} \cdot \cancel{\pi} \cdot \rho_L \frac{dr^3}{dt} = -g \cdot B \cdot \cancel{4} \cdot \cancel{\pi} \cdot r^2$$

Chain rule: $\frac{dr^3}{dt} = 3r^2 \frac{dr}{dt}$

$$\frac{1}{\cancel{3}} \cdot \rho_L \cdot \cancel{3} \cdot \cancel{r^2} \frac{dr}{dt} = -g \cdot B \cdot \cancel{r^2}$$

$$\hookrightarrow \frac{\rho_L}{g} dr = -B\, dt$$

g estimated with $\mathrm{Sh} = 2$ (resting droplets):

$$g = \frac{Sh \cdot \rho_V \cdot D}{d} = \frac{2 \cdot \rho_V \cdot D}{d} = \frac{\rho_V \cdot D}{r}$$

$$\int_{r_0}^{r} \frac{\rho_L \cdot r}{\rho_V \cdot D} dr = \int_{o}^{t} -B\, dt$$

$$\Rightarrow \frac{1}{2}(r^2 - r_0^2) = -\frac{\rho_V}{\rho_L} B \cdot D \cdot t$$

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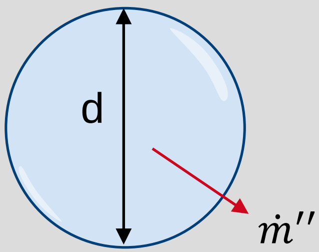

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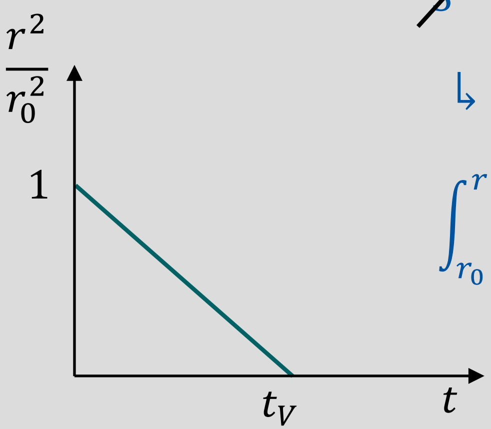

$$\Rightarrow \frac{1}{2}(r^2 - r_0^2) = -\frac{\rho_V}{\rho_L} B \cdot D \cdot t \;\Rightarrow\; \frac{r^2}{r_0^2} = 1 - \frac{2 \cdot B \cdot D \cdot t}{r_0^2} \cdot \frac{\rho_V}{\rho_L}$$

Evaporation time

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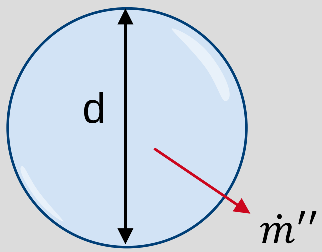

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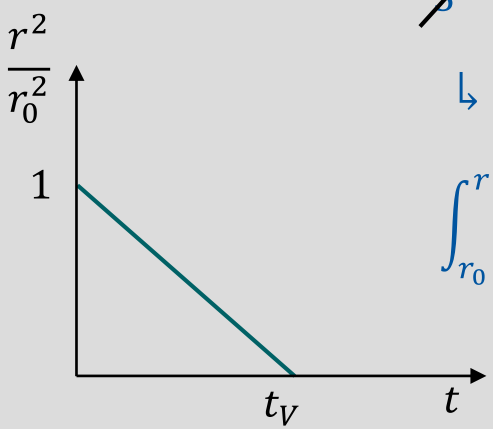
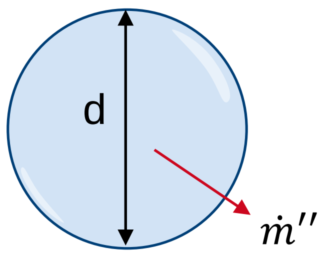
$$\int_{r_0}^{r} \frac{\rho_L \cdot r}{\rho_V \cdot D} dr = \int_o^t -B\, dt$$

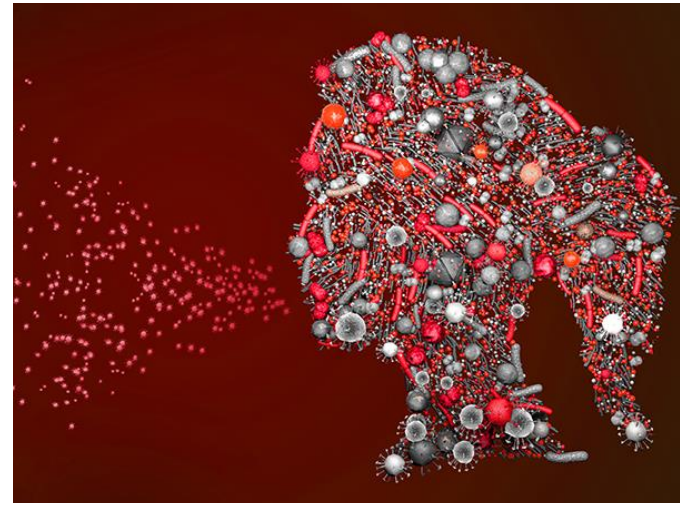


$$\Rightarrow \frac{1}{2}(r^2 - r_0^2) = -\frac{\rho_V}{\rho_L} B \cdot D \cdot t \;\Rightarrow\; t_V \text{ at } r = 0: \boxed{t_V = \frac{1}{2} \cdot \frac{r_0^2 \cdot \rho_L}{B \cdot D \cdot \rho_V}}$$

Evaporation time: aerosol droplets



$$t_V = \frac{1}{2} \cdot \frac{r_0^2 \cdot \rho_L}{\boldsymbol{B} \cdot D \cdot \rho_V}$$



Numerical values: (taken from example of video 8)

$$\boldsymbol{B} = \frac{\xi_{H_2O,s} - \xi_{H_2O,\infty}}{1 - \xi_{H_2O,s}} = 0.0102; \quad D = 2 \cdot 10^{-5} \frac{m^2}{s}$$

$$\rho_L = 1000 \left[\frac{kg}{m^3}\right]$$

$$\rho_V = \frac{p_V}{(R/M) \cdot T} = 0{,}01722 \frac{kg}{m^3}$$

$$p_V = 2330 \frac{N}{m^2}$$

Cox-Antoine relation of table 8, Appendix of lecture skript

$$R/M = 461.4 \frac{J}{kg\ K} \qquad T = 293{,}15\ K$$

	Initial diameter (= 2 r_0)		
	1 µm	10 µm	100 µm
t_V [sec]	**0,0355**	**3,55**	**355,7**

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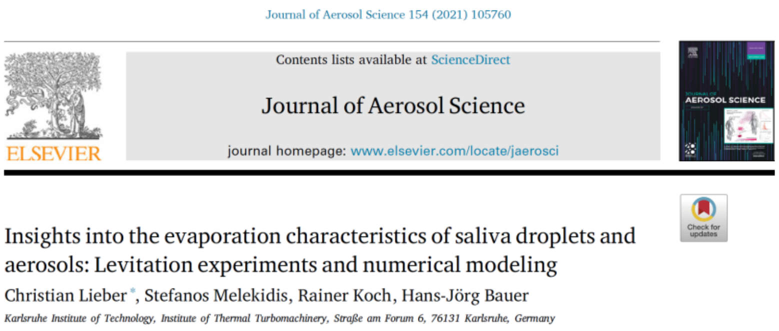
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Insights into the evaporation characteristics of saliva droplets and aerosols: Levitation experiments and numerical modeling

Christian Lieber*, Stefanos Melekidis, Rainer Koch, Hans-Jörg Bauer

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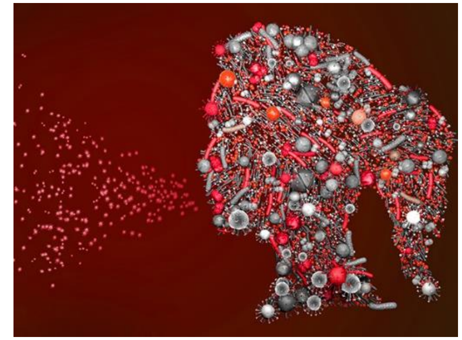

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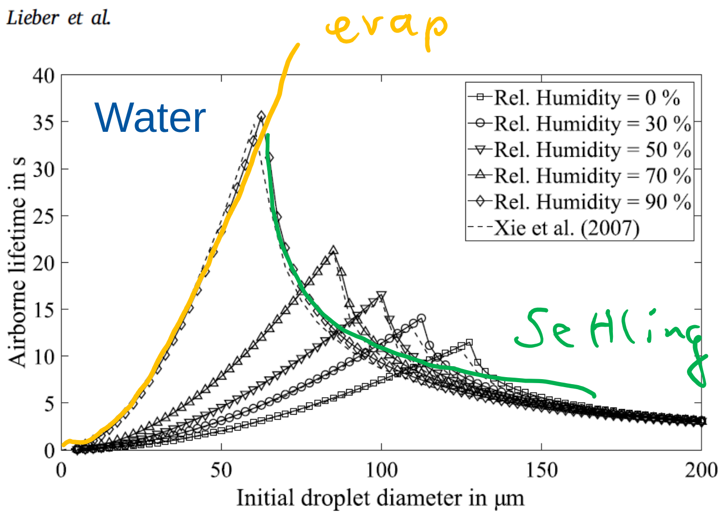

(a)

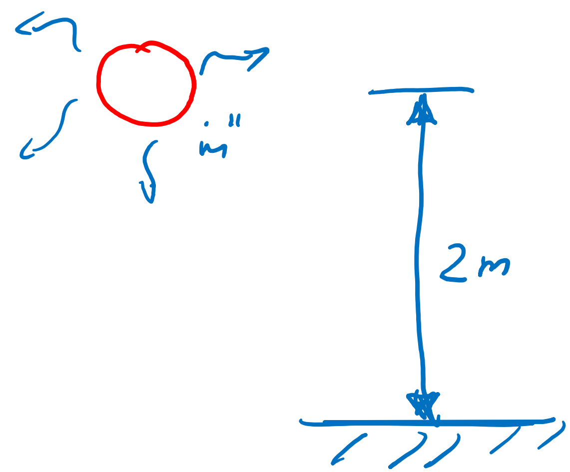

Fig. 12. Results of recalculating the evaporation-falling curve by Wells (1934) for (a) water droplets, and (b) saliva droplets using the ratio between equilibrium and initial diameter as determined in the present study.

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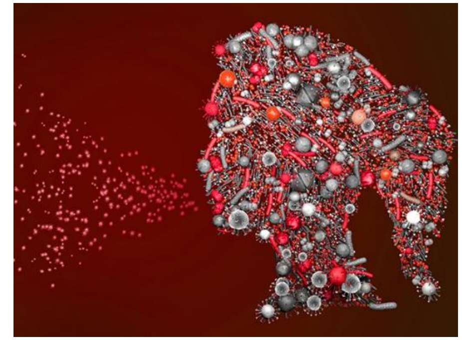

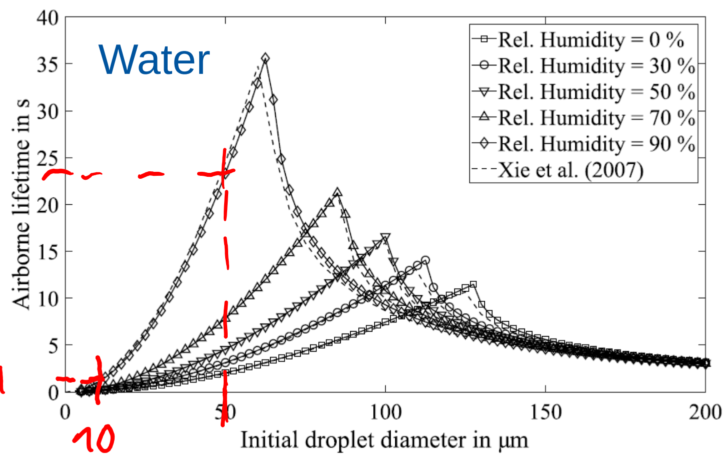

(a)

Fig. 12. Results of recalculating the evaporation-falling curve by Wells (1934) for (a) water droplets, and (b) saliva droplets using the ratio between equilibrium and initial diameter as determined in the present study.

	Initial diameter (= 2 r_0)		
	1 µm	10 µm	100 µm
t_V [sec]	**0,0355**	**3,55**	**355,7**

/4 ~ 80 sec

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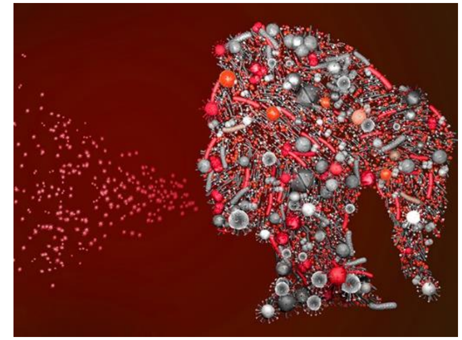
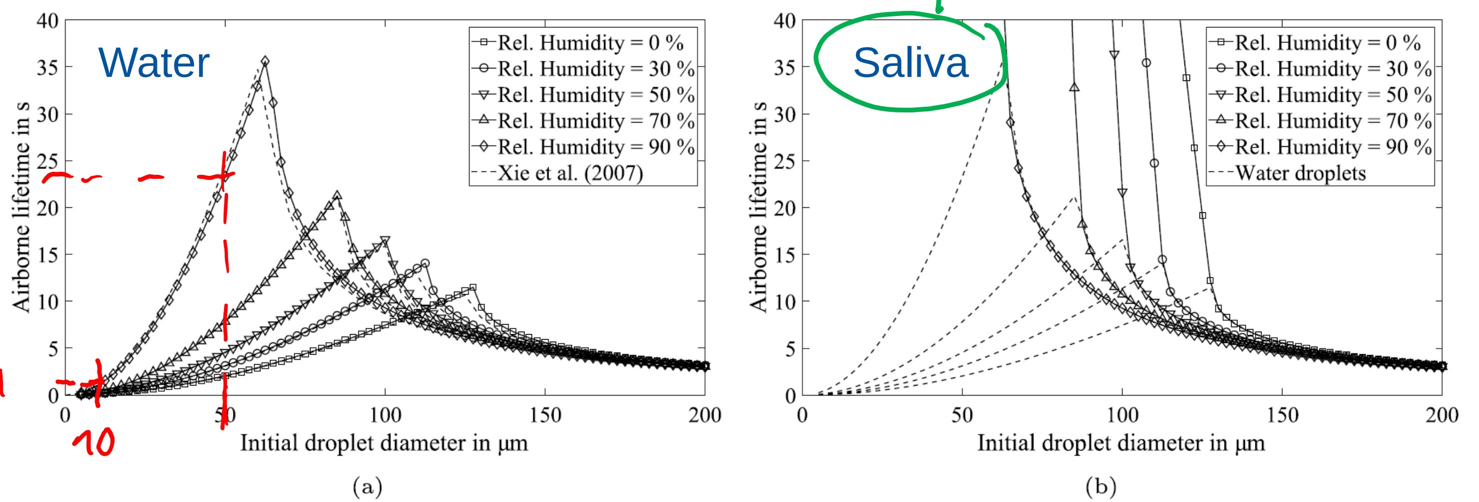


Liquid in our mouth



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Comprehension questions

Why is the determination of the surface temperature only possible iteratively?

What are the considerations behind the estimation of the evaporation time of a droplet?

Why is the evaporation time of an exhaled droplet relatively large?