Heat Transfer: Radiation

Black Body Radiation

Prof. Dr.-Ing. Reinhold Kneer

Prof. Dr.-Ing. Dr. rer. pol. Wilko Rohlfs

HEATQUIZ

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

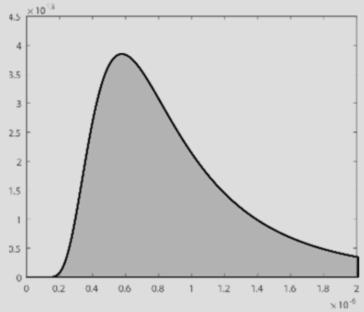
Learning goals

Radiation Properties:

- Understanding of the Wave-Quantum Duality
- Black Body:
  Description of the spectral radiation intensity according to Planck

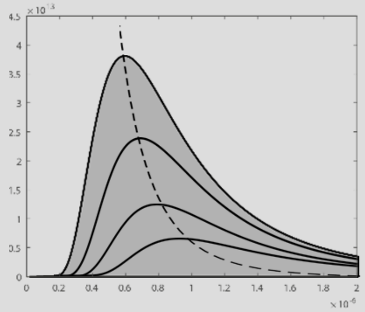
Stefan-Boltzmann Law:

- Solution approach for integration of the Planck‘s Distribution Law
- Use of Stefan-Boltzmann Law



Wien‘s Law of Displacement:

- Relationship from temperature and position of maximum spectral radiation intensity



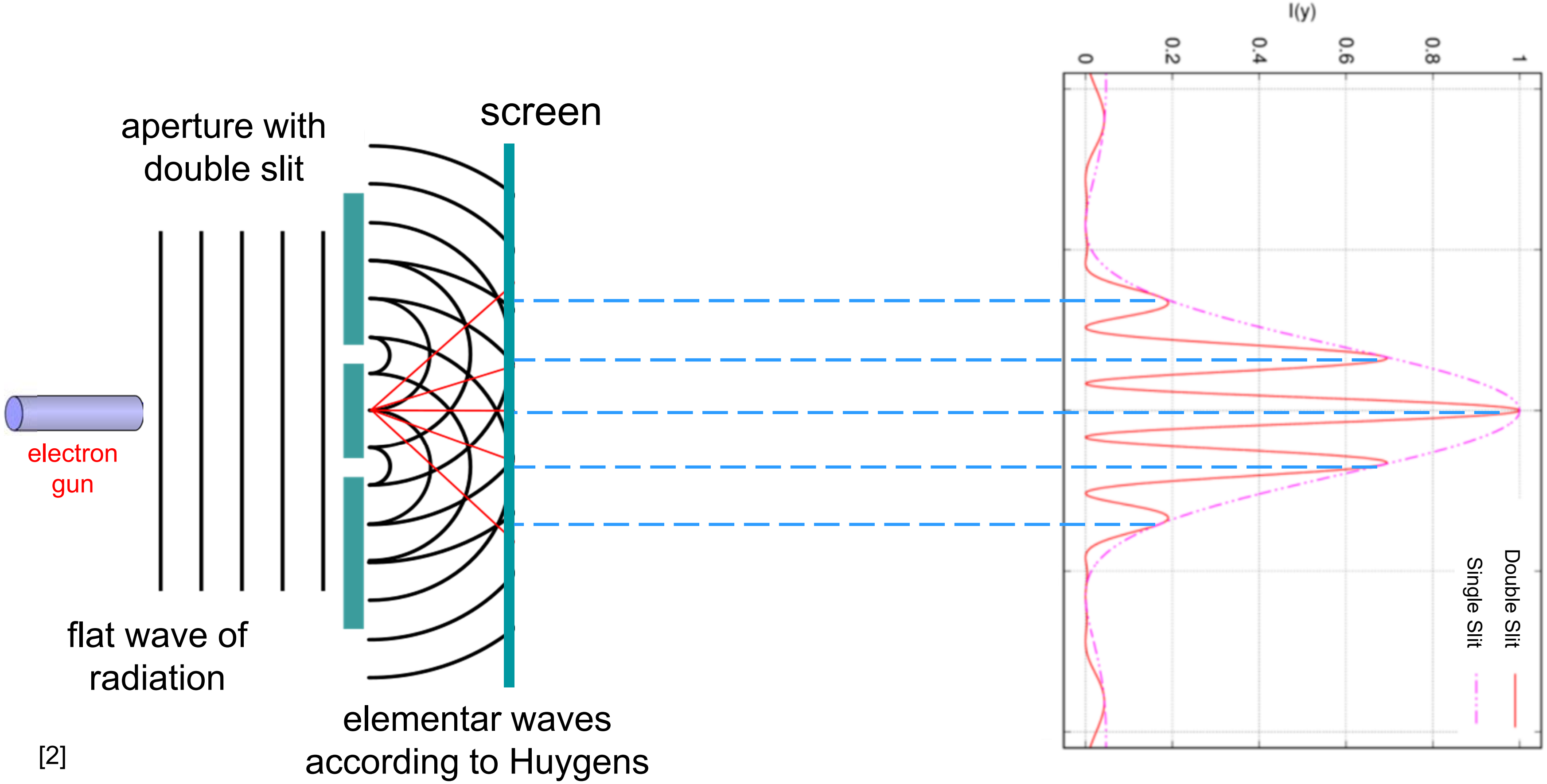
[1] Max Planck

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

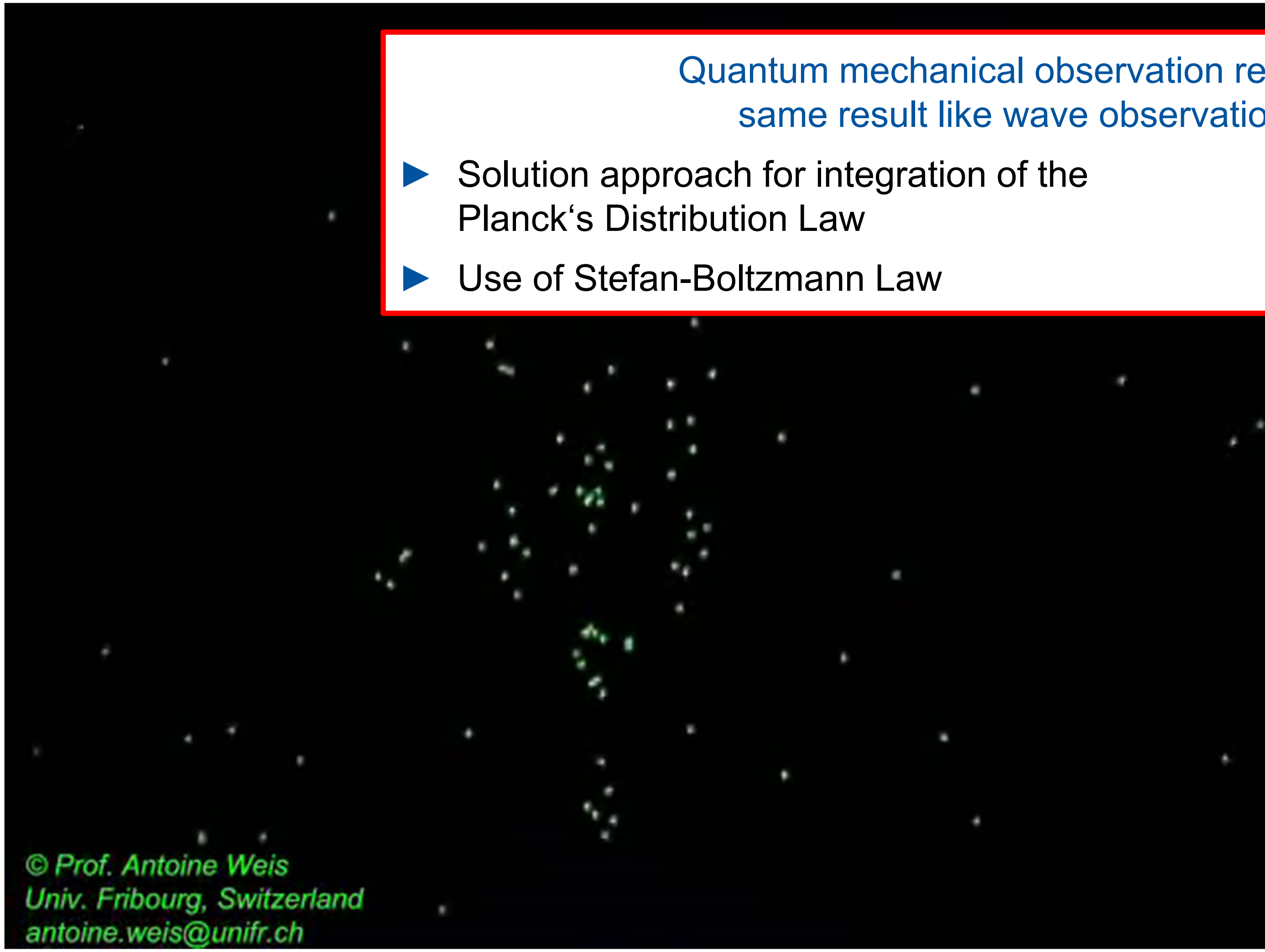
Description of Radiation (Wave-Quantum Dualty)



[2] [3]

[1] An illustration of the 'Double-slit experiment' in physics. Johannes Kalliauer
[2] http://mondbrand.de/Doppelspaltexperiment.htm
[3] wwwex.physik.uni-ulm.de / Interferenz- und Beugungsmuster

Double-slit Experiment



Quantum mechanical observation returns same result like wave observation

- Solution approach for integration of the Planck's Distribution Law
- Use of Stefan-Boltzmann Law

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

Quantum mechanics description of Radiation

Speed o

Energy

Visible li

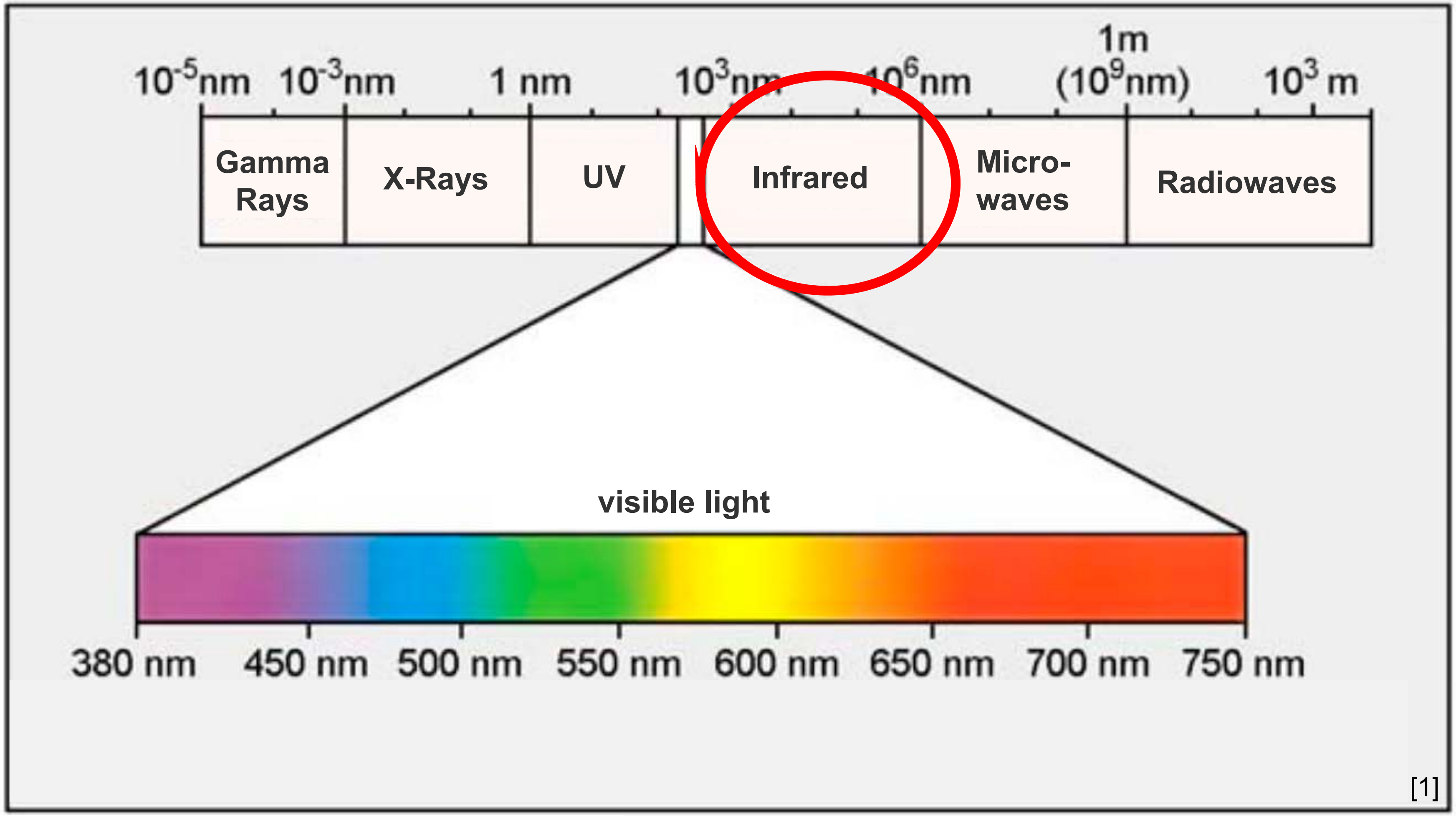
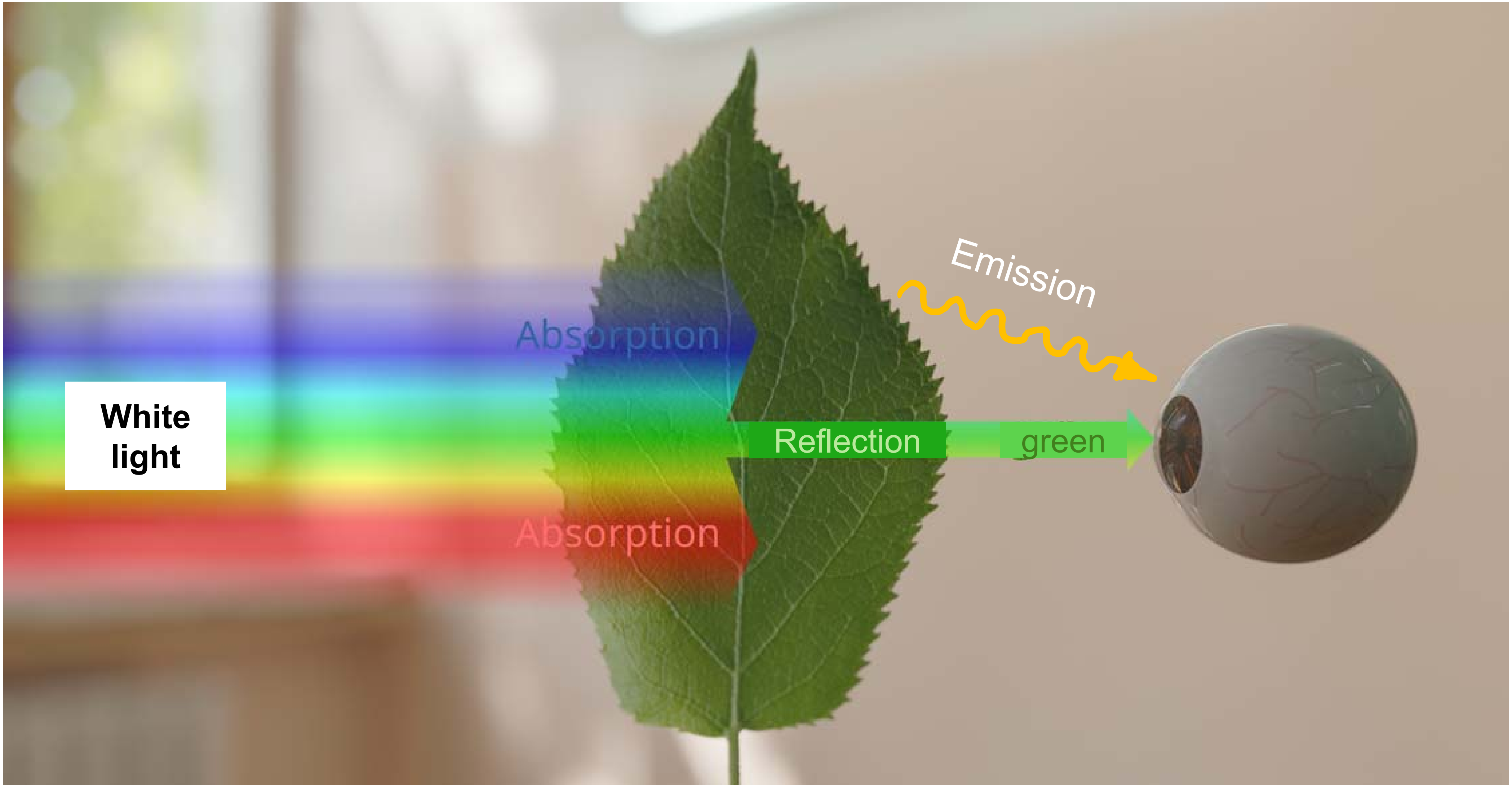


Image: © Karin Kiefer

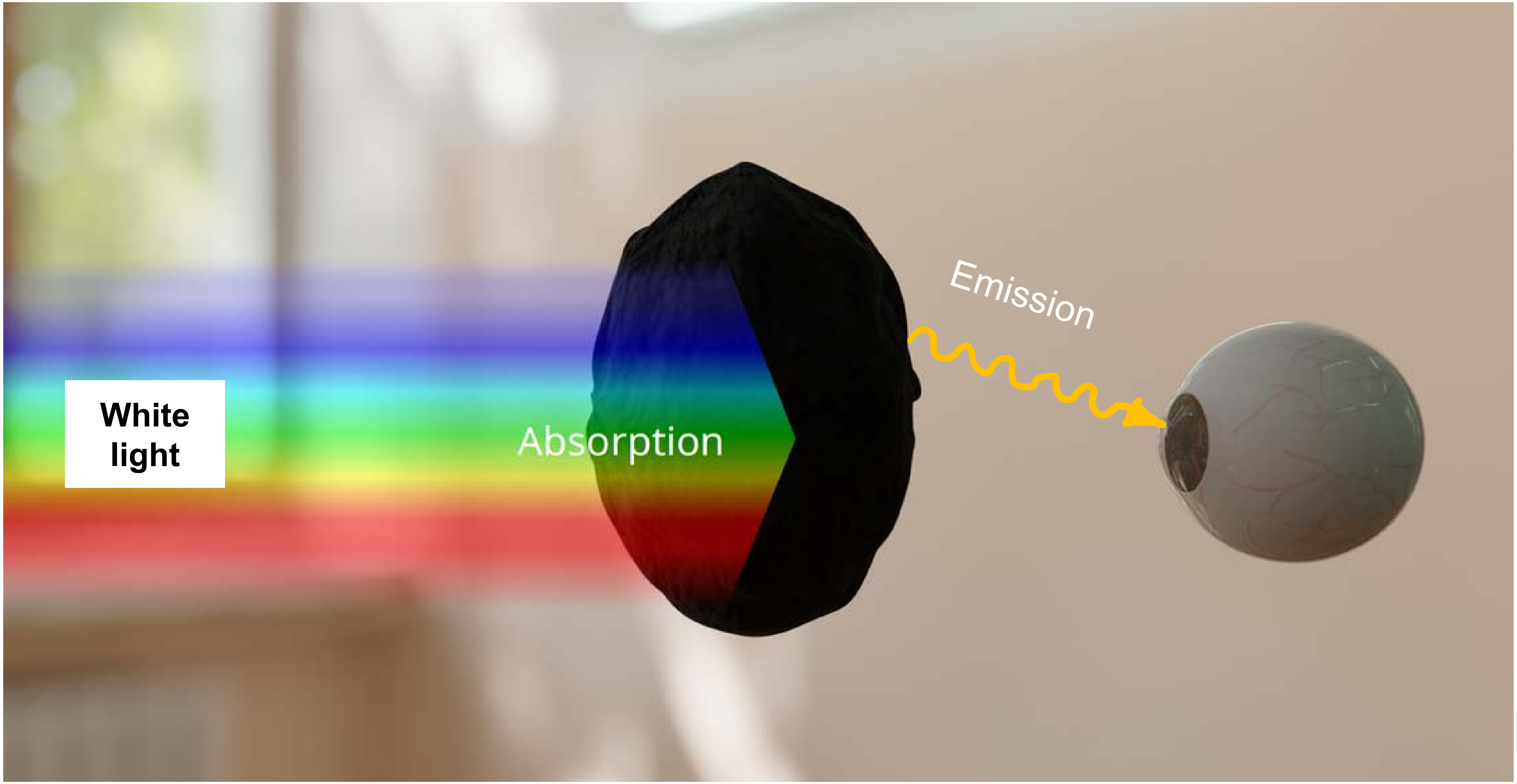
[1] Energy Wavelength

Our perception of Objects



[1] tec-science.com

Special Object: „Black Body“



[1] tec-science.com

Planck‘s Distribution Law

Explanation:

- Radiation depends on the temperature of the body
- Model representation „Black Body“ (ideally thermal radiation source):
  - All incident radiation is absorbed
  - Radiation is emitted in whole wavelength range
  - A black body emits a maximum at a given temperature

Max Planck (from quantum theory) ➡Distribution of the radiation intensity of a black body as function of wavelength

Planck‘s distribution law:

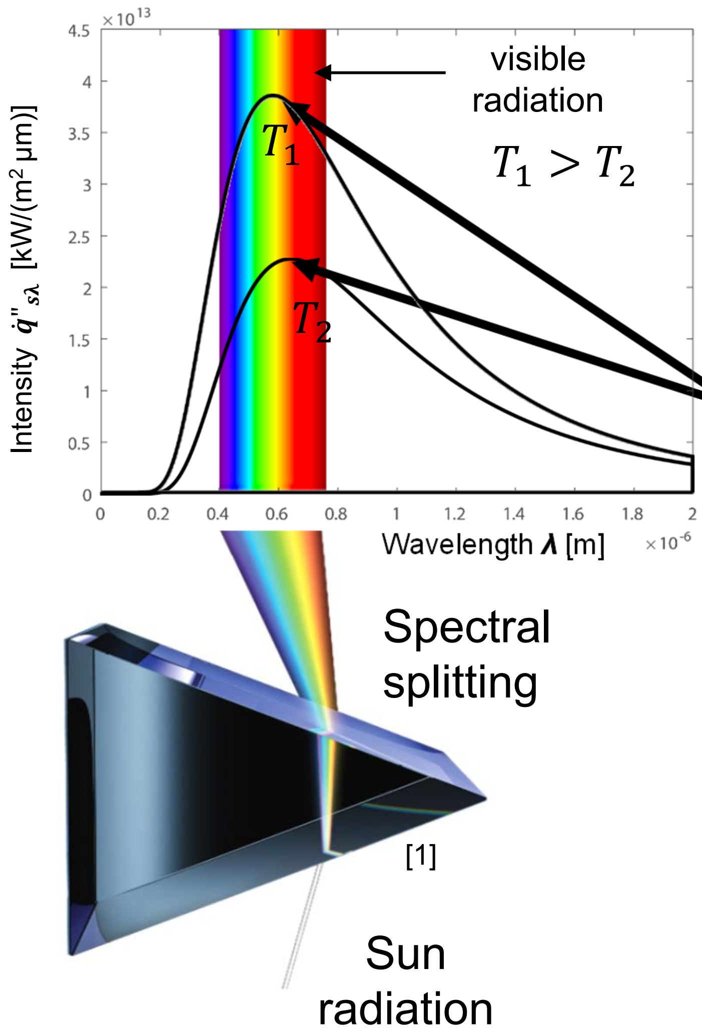
$$\dot{q}''_{s\lambda} = \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1} \left[\frac{W}{m^2 m}\right]$$

Constants:

$$c_1 = 3{,}741\ 10^{-16}\ [Wm^2]$$

$$c_2 = 1{,}439\ 10^{-2}\ [mK]$$

Planck's Spectral Intensity Distribution

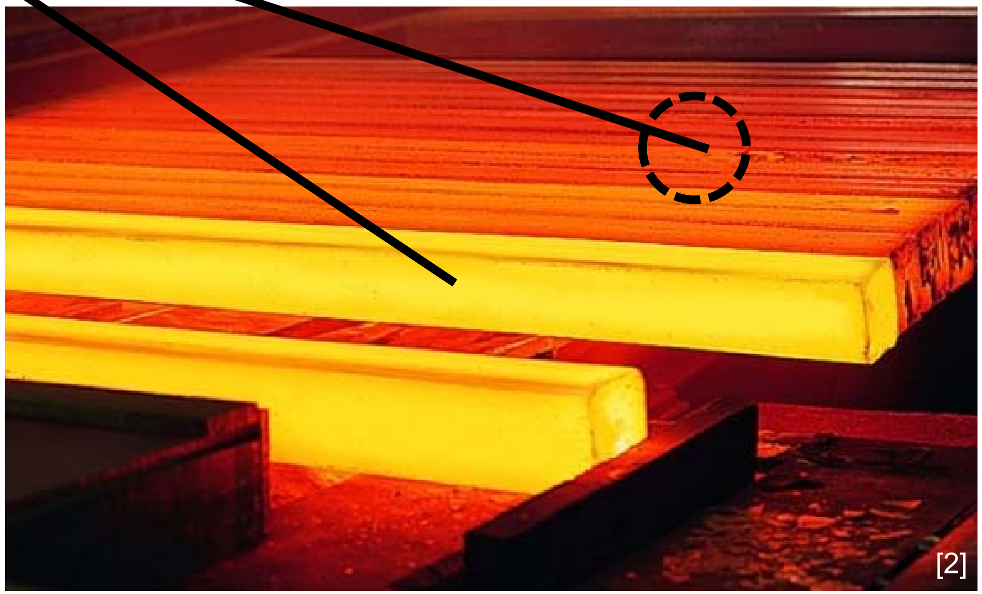


Planck's distribution law:

Time derivation Area derivation

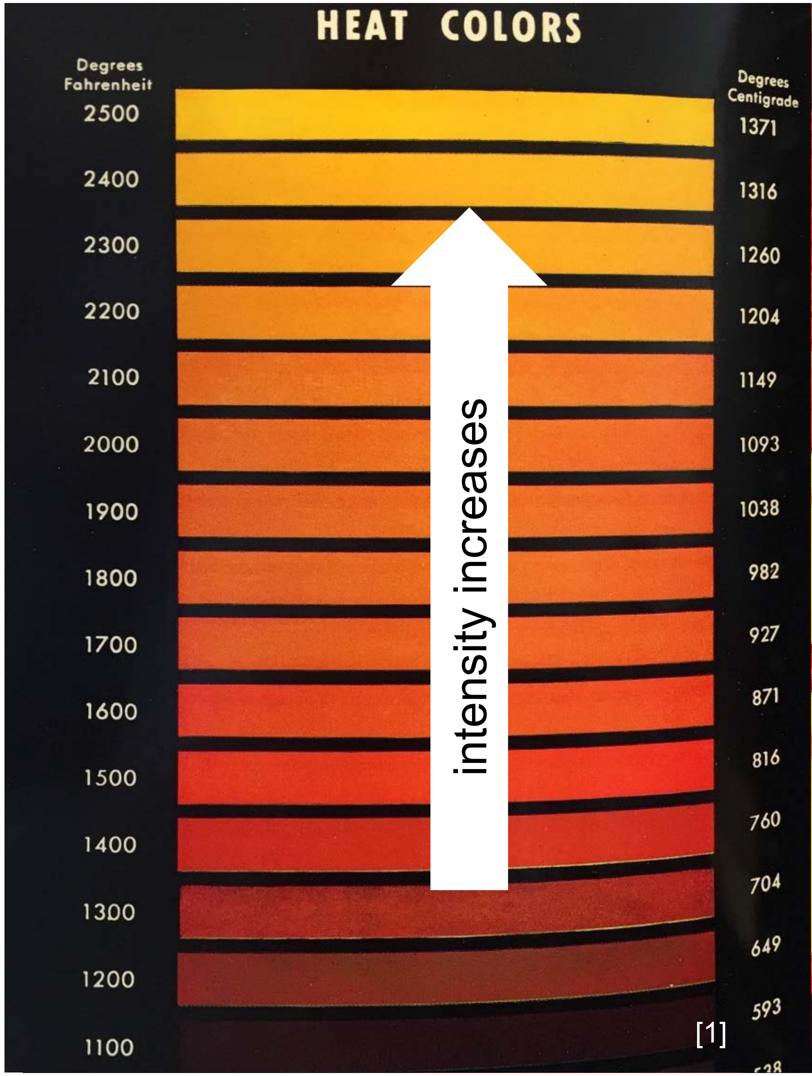
$$\dot{q}''_{s\lambda} = \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1} \left[\frac{W}{m^2\, m}\right]$$

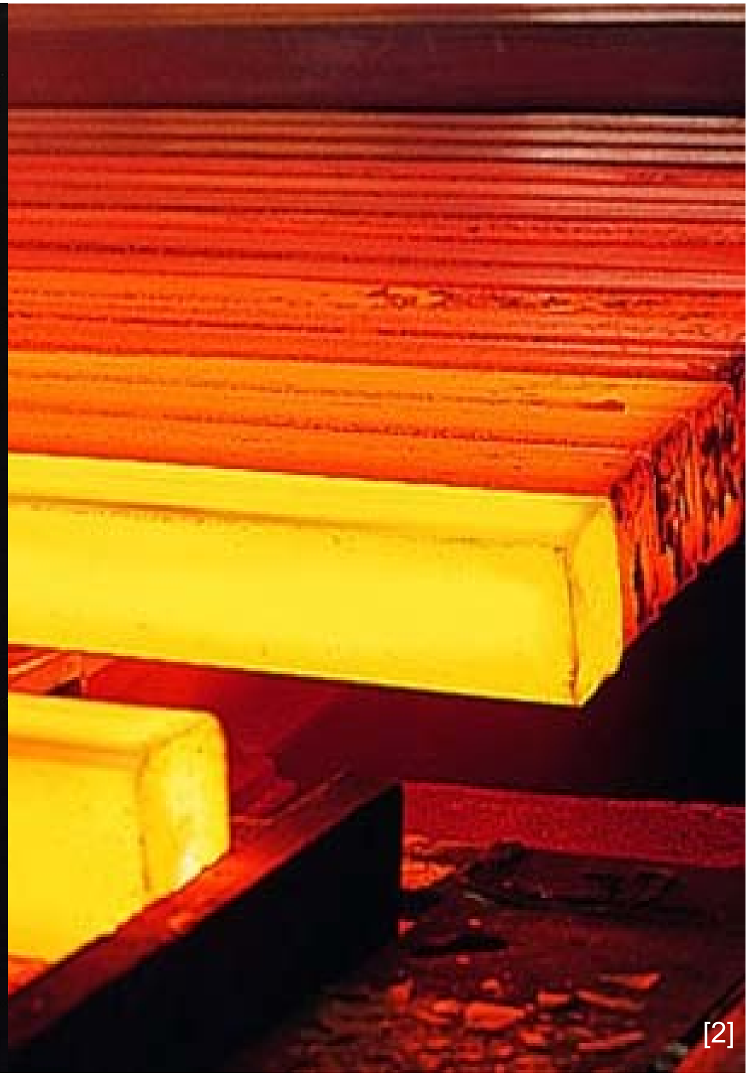
Black body Wavelength specific



[1] weisse-licht-scheint-durch-das-prisma
[2] http://www.steadfastfze.com

Temperature colors: Metal processing





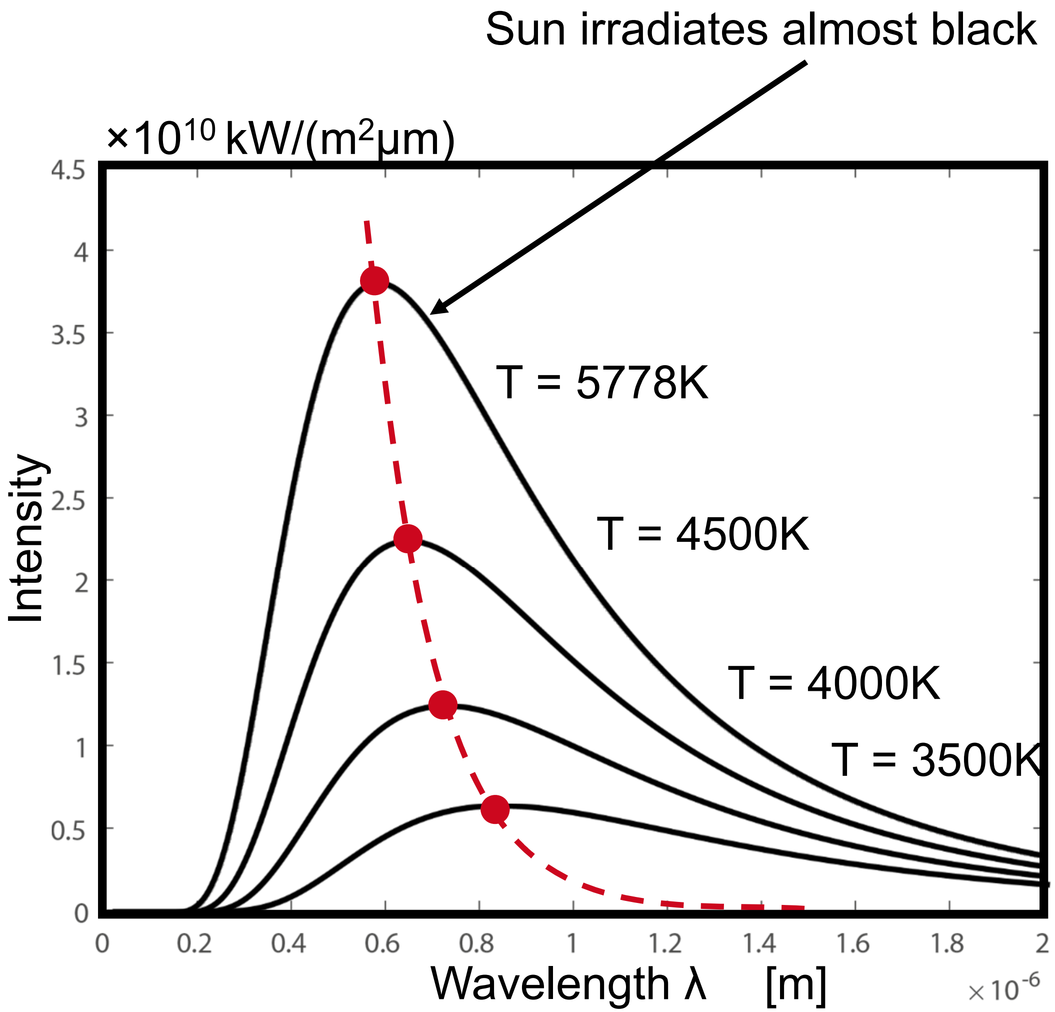
[1] http://www.steadfastfze.com
[2] http://www.blksmth.com

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

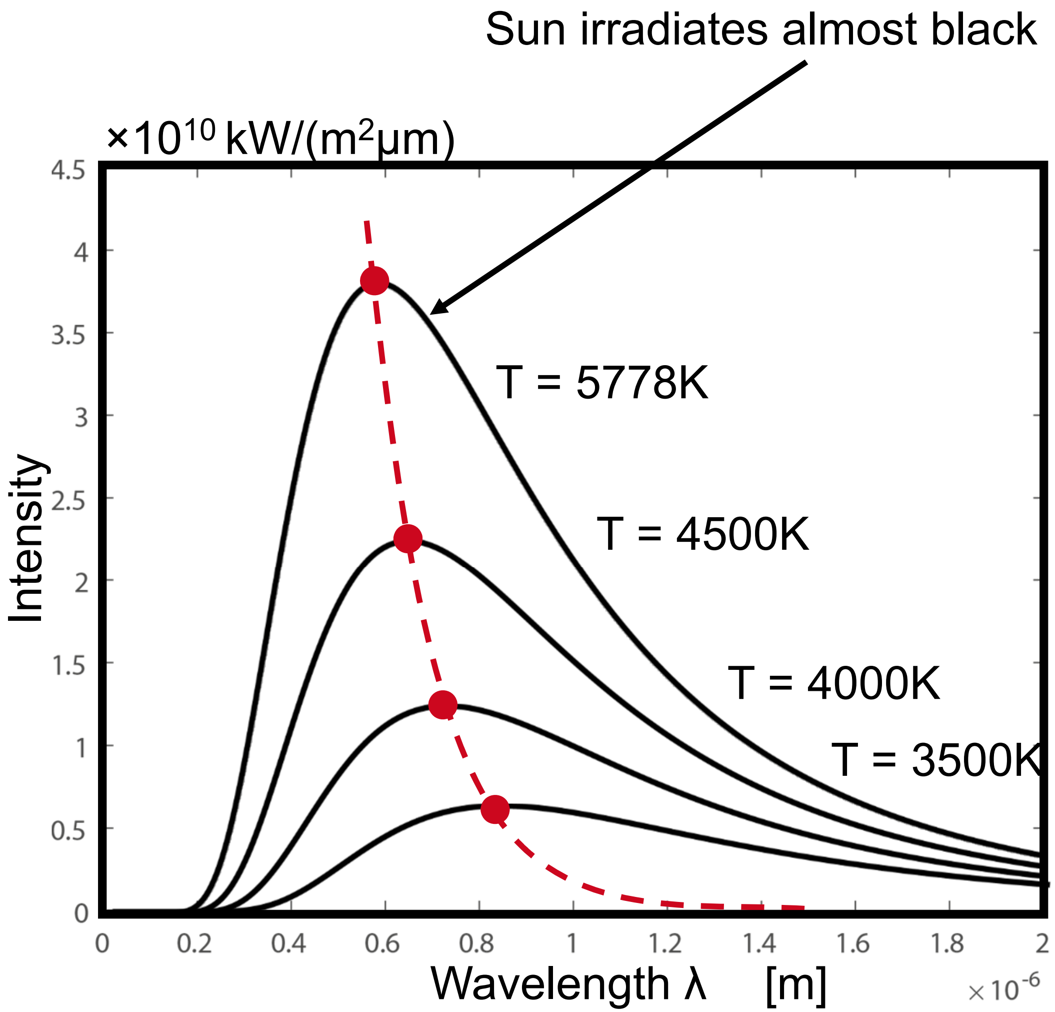
Wien‘s Law of Displacement



Planck‘s distribution law:

$$\dot{q}''_{s\lambda} = \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1} \left[\frac{W}{m^2 m}\right]$$

Wien‘s Law of Displacement



Wien‘s law of displacement:

$$\lambda_{\max} T = 2898\ [\mu m\ K]$$

- Describes the position of the maximum of the spectral emissions

Sun temperature:

$$T = \frac{2898}{0.5} [K] \approx 5800\ [K]$$

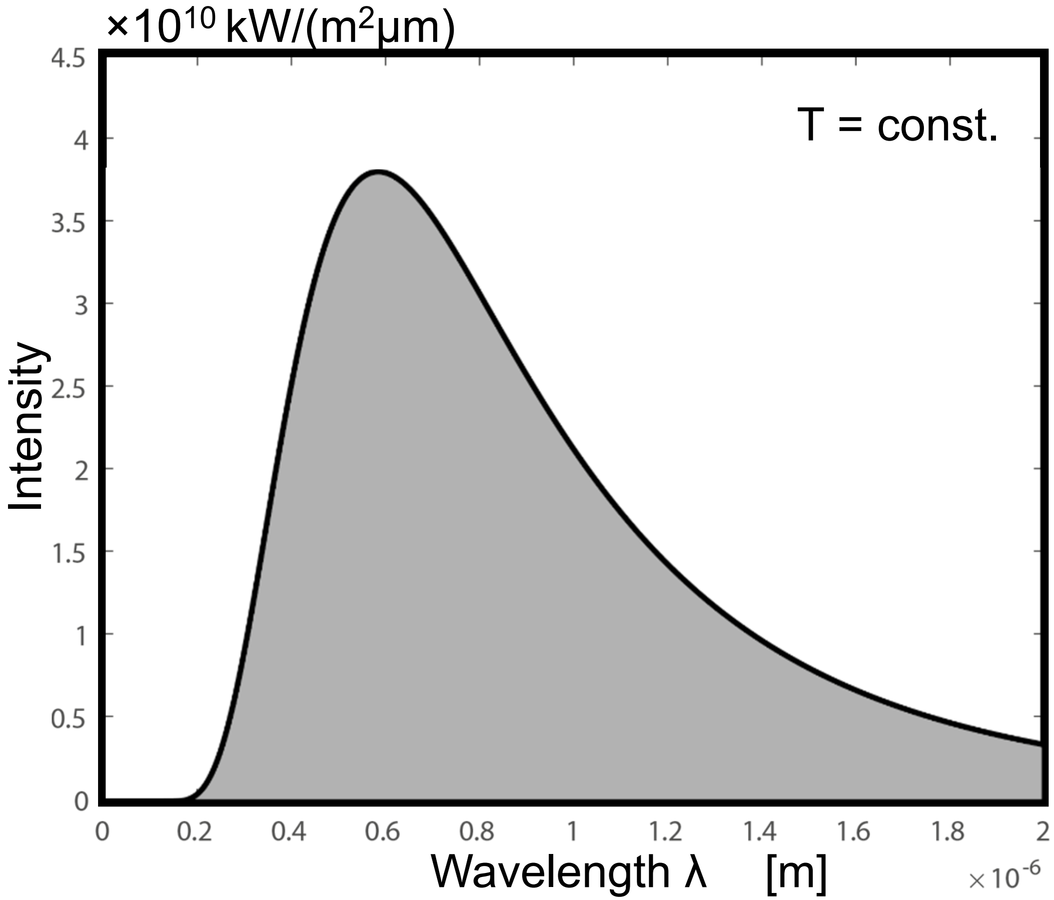
$$\boxed{\lambda \approx 0.4 - 0.7\ [\mu m]}$$

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

Stefan-Boltzmann Law



Integration of Planck distribution:

$$\dot{q}''_s = \int_0^\infty \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1} \, \mathrm{d}\lambda$$

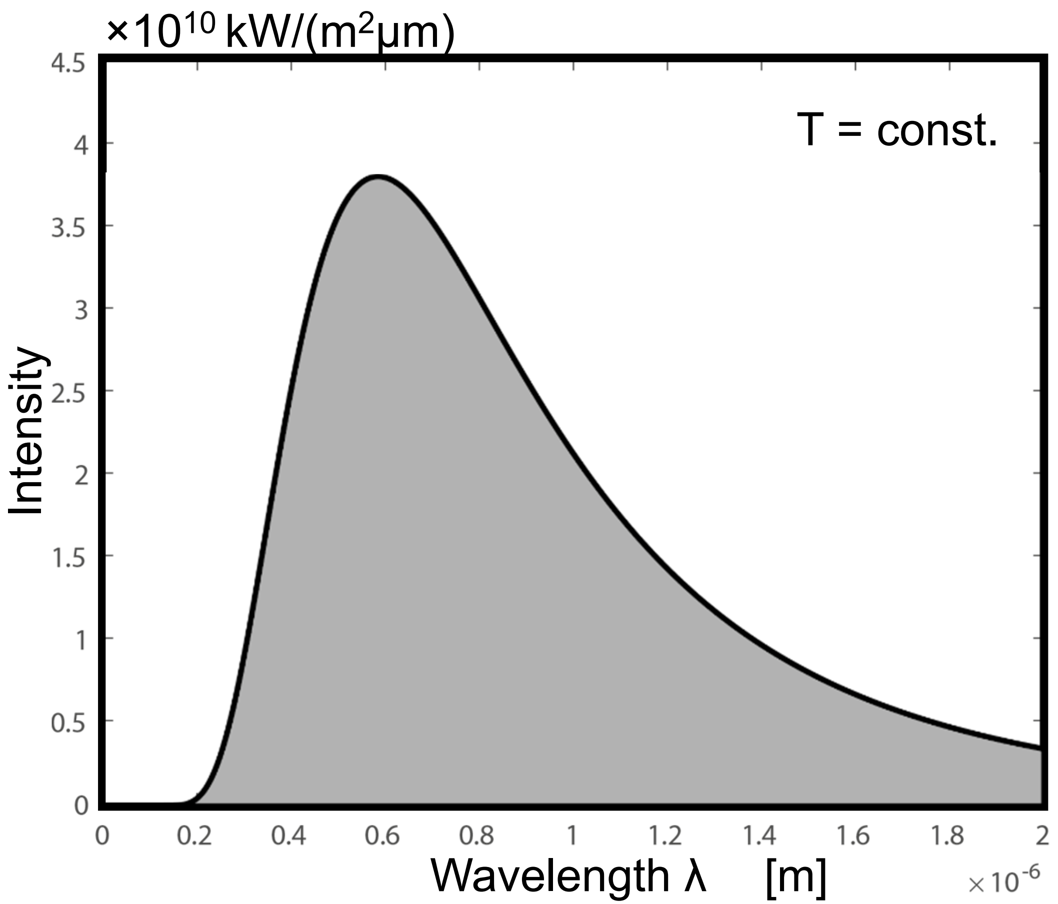
integration

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

Stefan-Boltzmann Law



Integration of Planck distribution:

$$\dot{q}''_s = \int_0^\infty \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1} \, d\lambda$$

Stefan-Boltzmann law:

$$\dot{q}''_s = \sigma T^4 \qquad \sigma = 5.67 \times 10^{-8} \left[\frac{W}{m^2 K^4}\right]$$

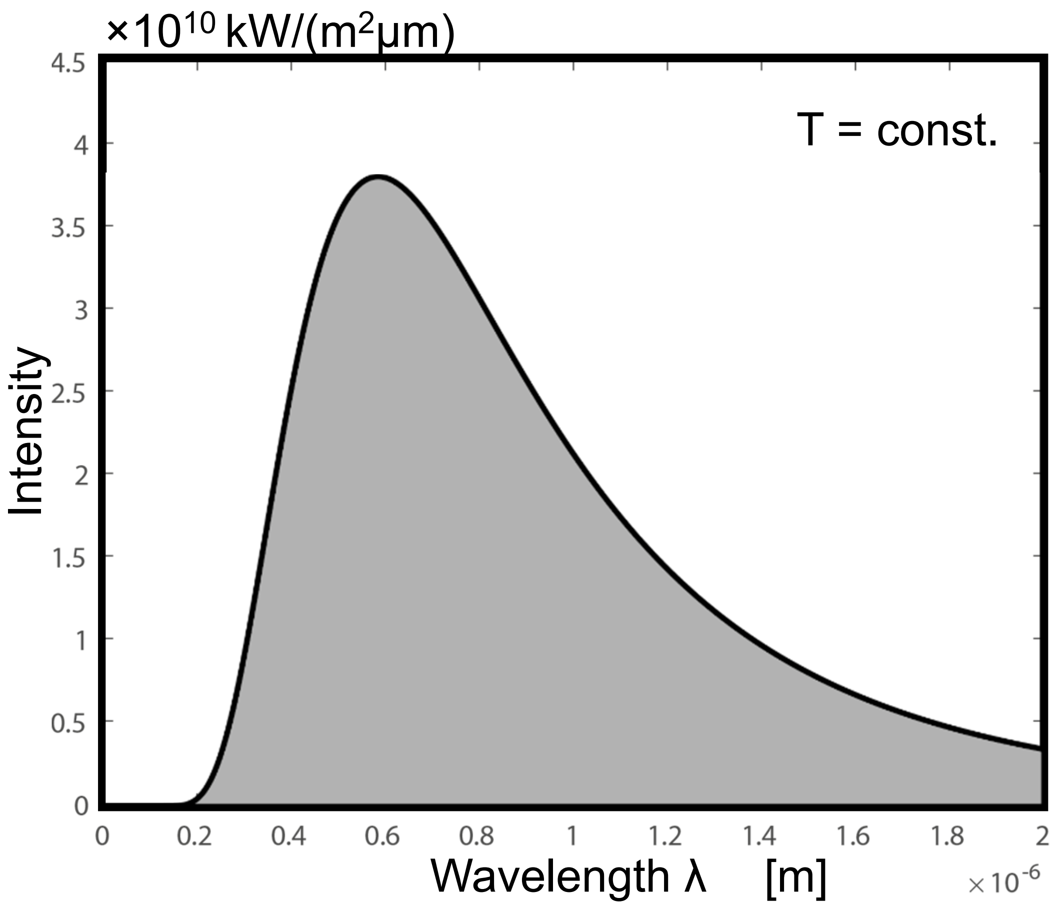
- Total radiation heat flux
  (of a black body, at a given temperature)

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

Stefan-Boltzmann Law



Integration of Planck distribution:

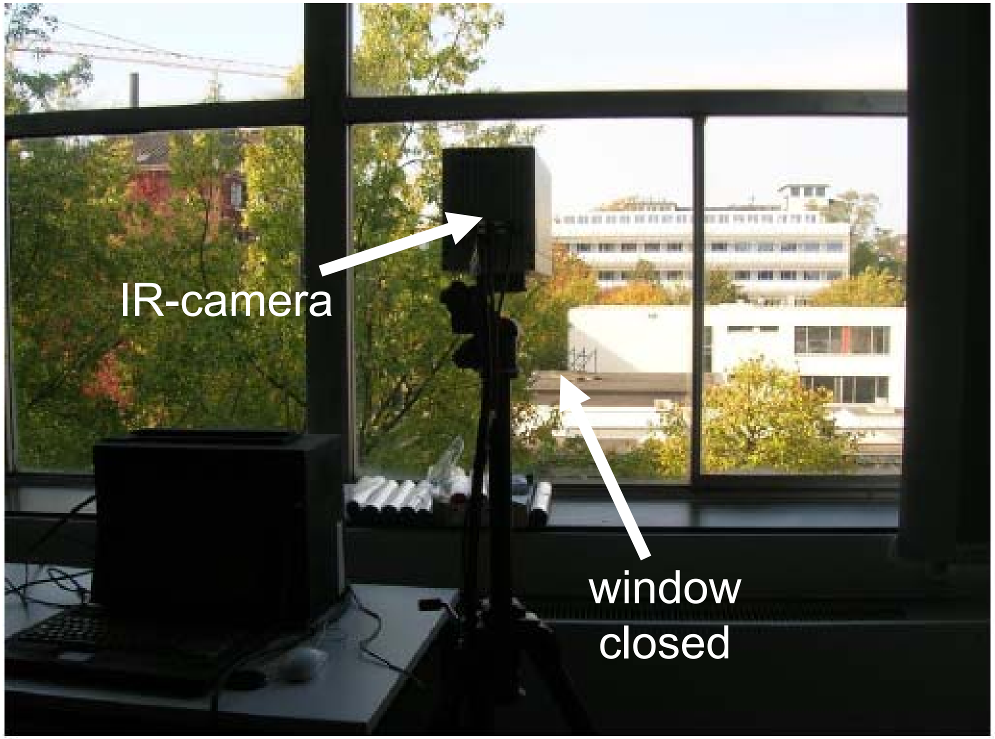
$$\dot{q}''_s = \int_0^\infty \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1}\, \mathrm{d}\lambda$$

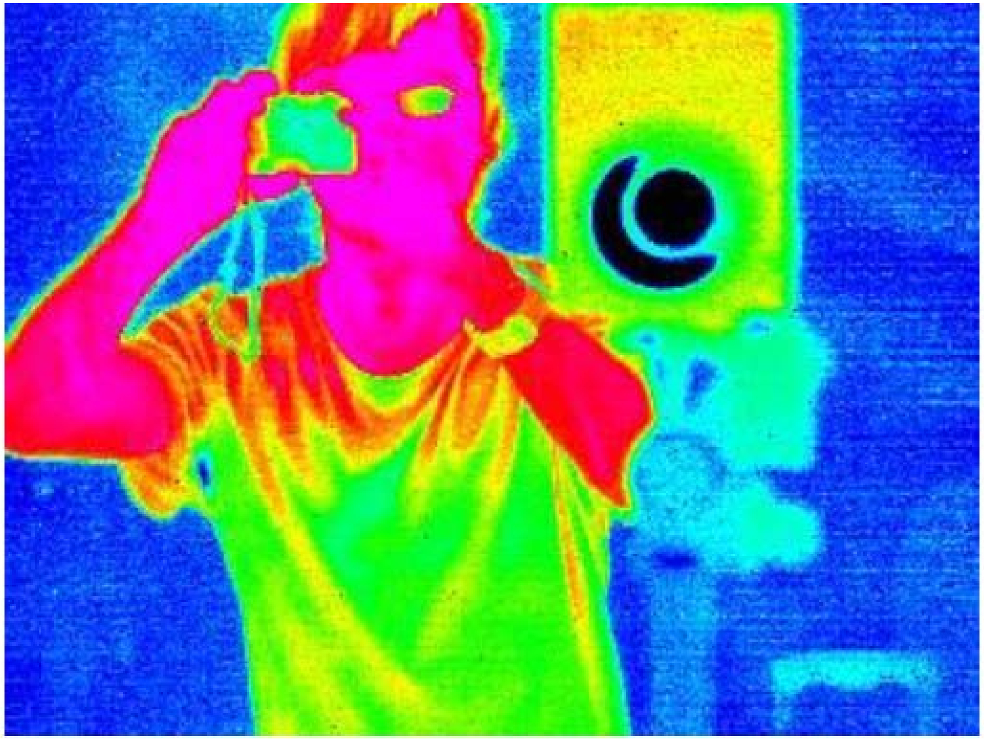
Stefan-Boltzmann law:

$$\dot{q}''_s = \sigma T^4 \qquad \sigma = 5{,}67 \times 10^{-8} \left[\frac{W}{m^2 K^4}\right]$$

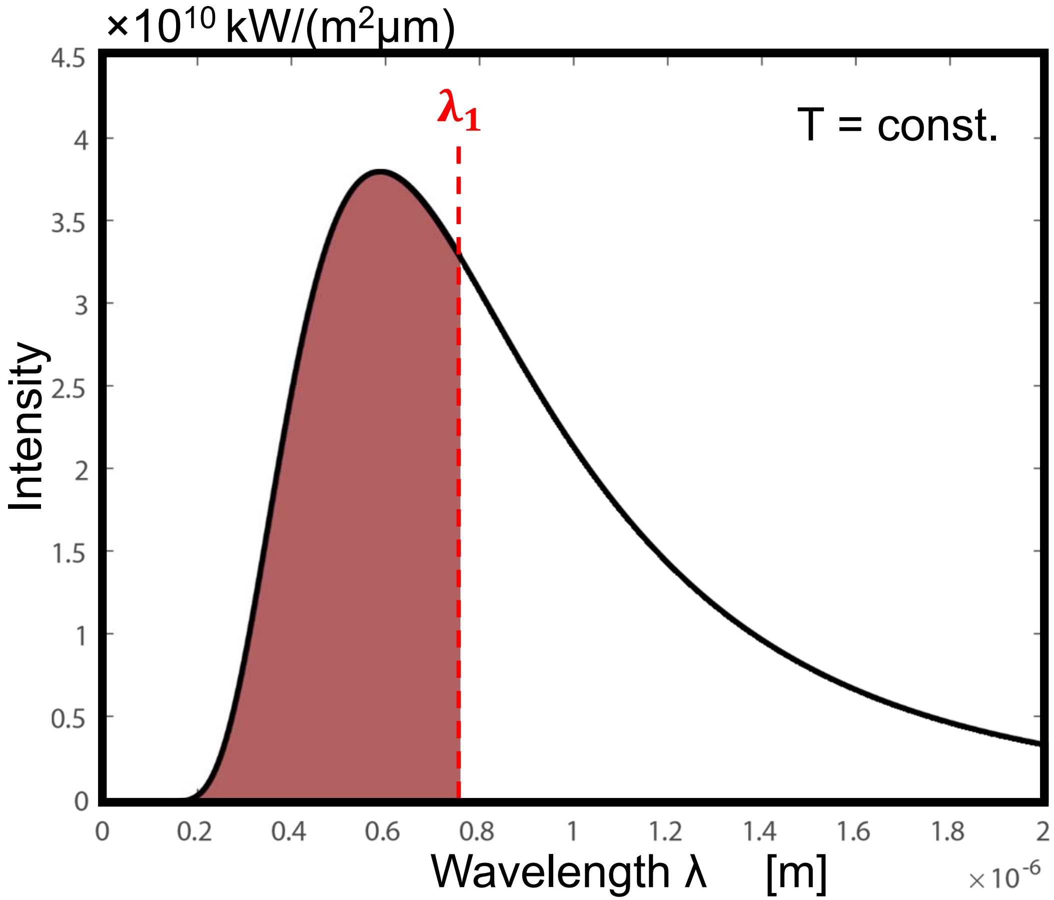
- Stefan-Boltzmann constant: **σ**

Wavelength dependence of radiation properties





Radiation in specific wavelength ranges



Black body radiation in a specific wavelength range:

$$\dot{q}''_{s,0-\lambda\_1} = \int_0^{\lambda_1} \dot{q}''_{s\lambda} \cdot d\lambda$$

$$\dot{q}''_{s,0-\lambda\_1} = \int_0^{\lambda_1} \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1} d\lambda$$

Relative fraction of radiation in relation to the total radiation:

$$F(\lambda) = \frac{1}{\sigma T^4} \int_0^{\lambda_1} \dot{q}''_{s\lambda} \cdot d\lambda$$

Radiation in specific wavelength ranges

HMT formulary

λT in µm K	1000,0	1250,0	1500,0	1750,0	2000,0	2500,0
$F(\lambda)$	0,00031	0,00308	0,01283	0,03363	0,06663	0,16115
λT in µm K	3000,0	3500,0	4000,0	5000,0	6000,0	8000,0
$F(\lambda)$	0,27322	0,38250	0,48085	0,63315	0,73715	0,85556

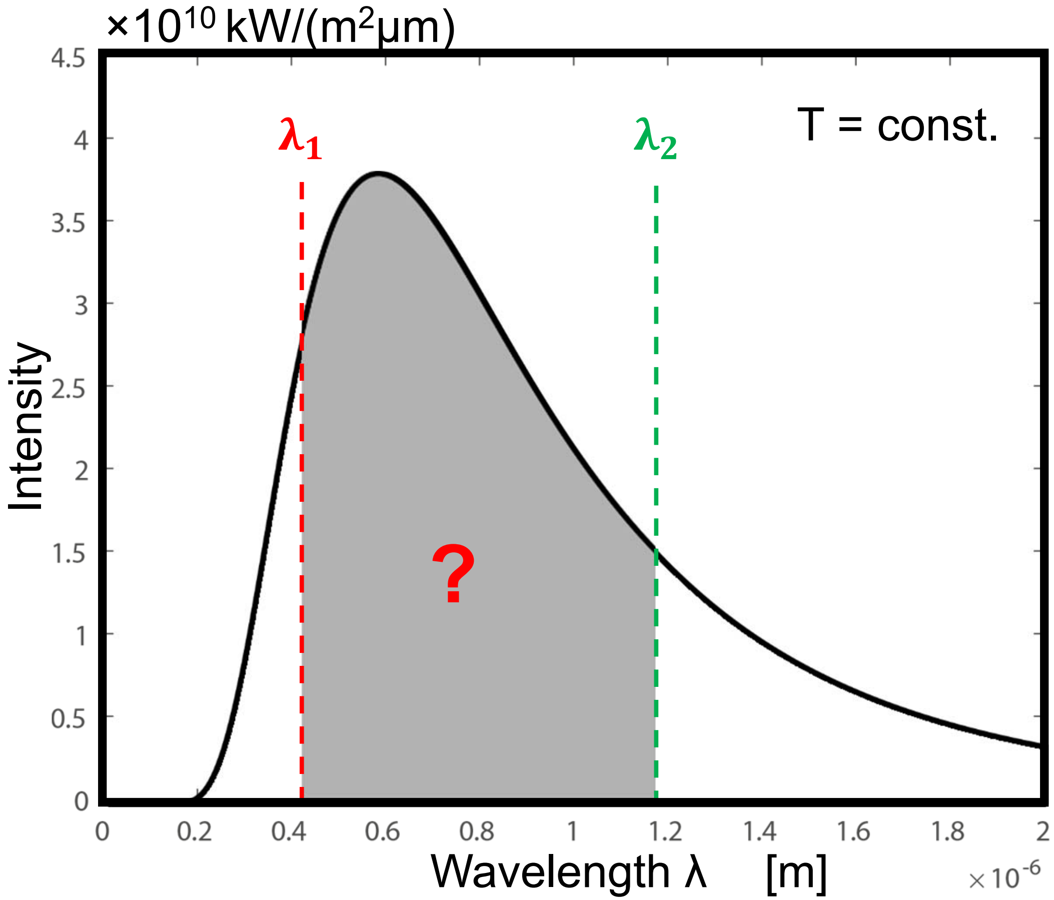
Distribution of black body radiation: $F(\lambda) = \int_0^\lambda \dot{q}''_{\lambda\mathrm{b}} d\lambda \,/\, \sigma T^4$

Upper integration limit
(multiplication of wavelength and temperature)

Relative fraction of radiation in relation to the total radiation:

$$F(\lambda) = \frac{1}{\sigma T^4} \int_0^{\lambda_1} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

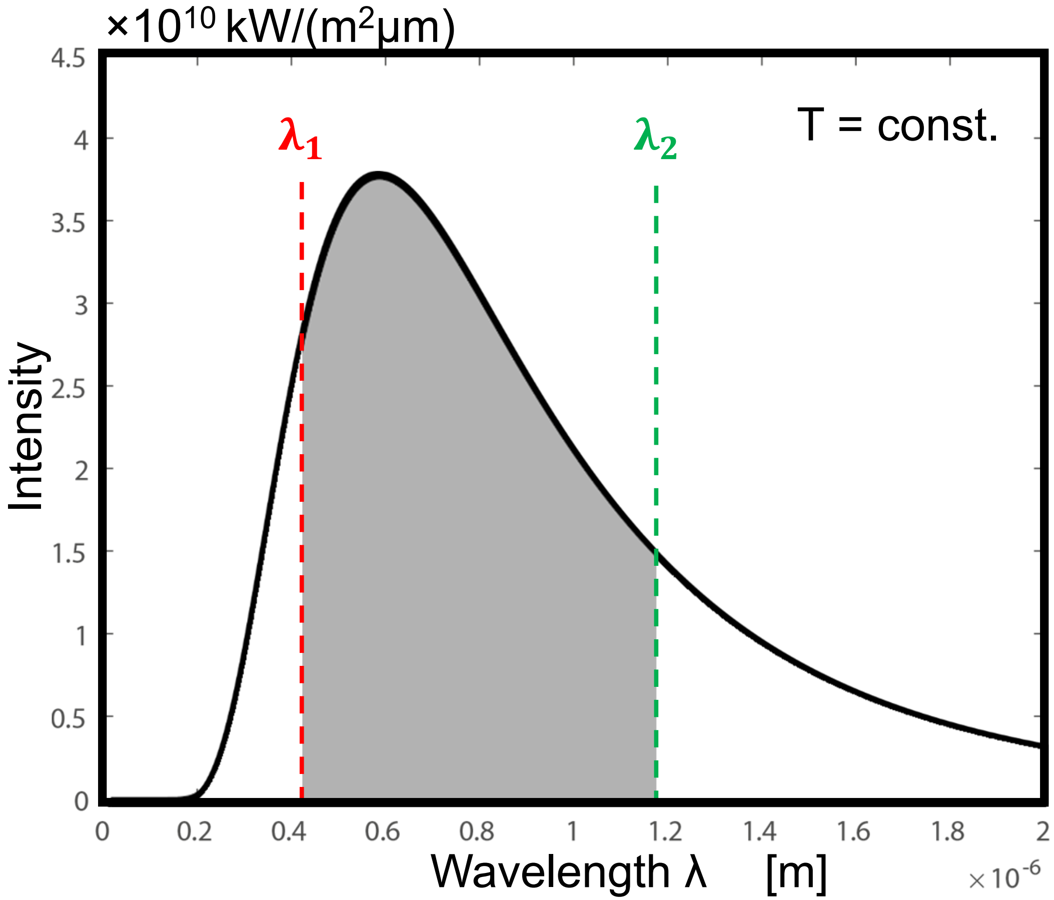
Stefan-Boltzmann Law



Relative fraction of radiation in relation to the total radiation:

$$F(\lambda_1 \rightarrow \lambda_2) = \frac{1}{\sigma T^4} \int_{\lambda_1}^{\lambda_2} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

Radiation in specific wavelength ranges



Relative fraction of radiation in relation to the total radiation:

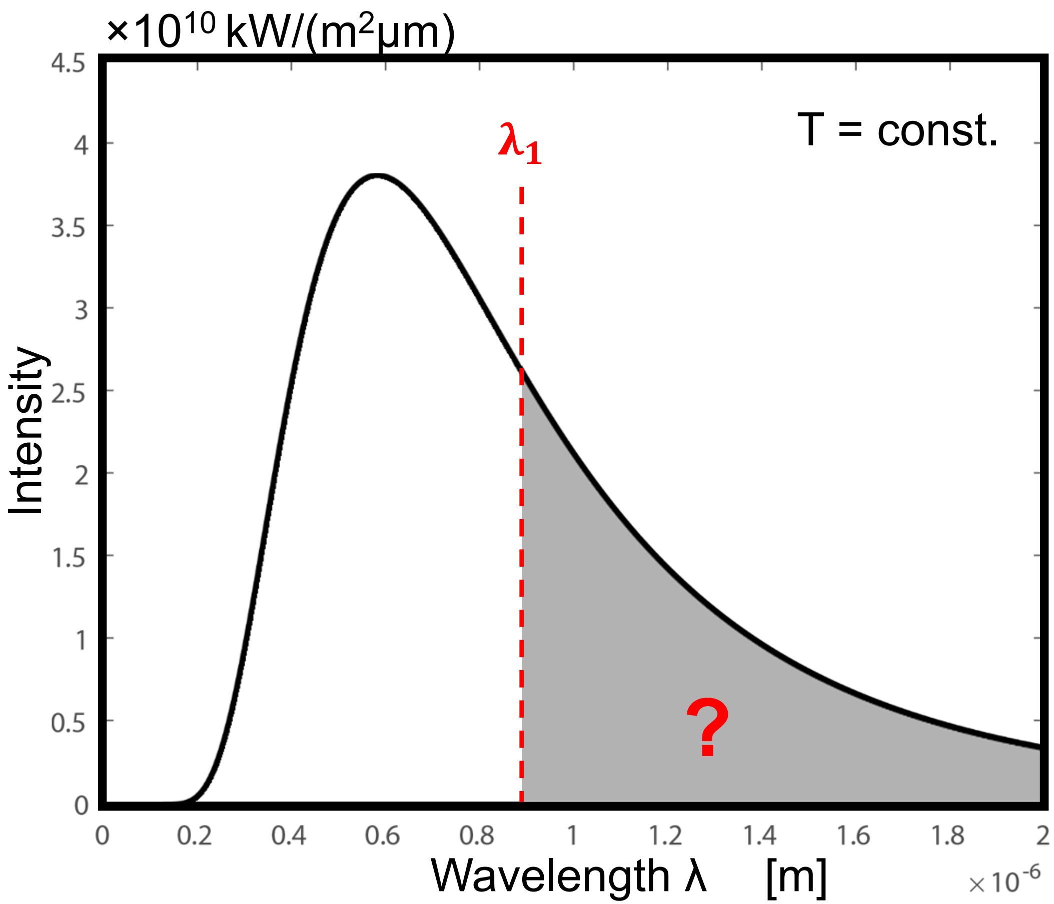
$$F(\lambda_1 \rightarrow \lambda_2) = \frac{1}{\sigma T^4} \int_{\lambda_1}^{\lambda_2} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

$$F(0 \rightarrow \lambda_2) = \frac{1}{\sigma T^4} \int_{0}^{\lambda_2} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

$$F(0 \rightarrow \lambda_1) = \frac{1}{\sigma T^4} \int_{0}^{\lambda_1} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

$$\underbrace{F(\lambda_1 \rightarrow \lambda_2) = F(0 \rightarrow \lambda_2) - F(0 \rightarrow \lambda_1)}_{\boxed{\text{Principle of superposition}}}$$

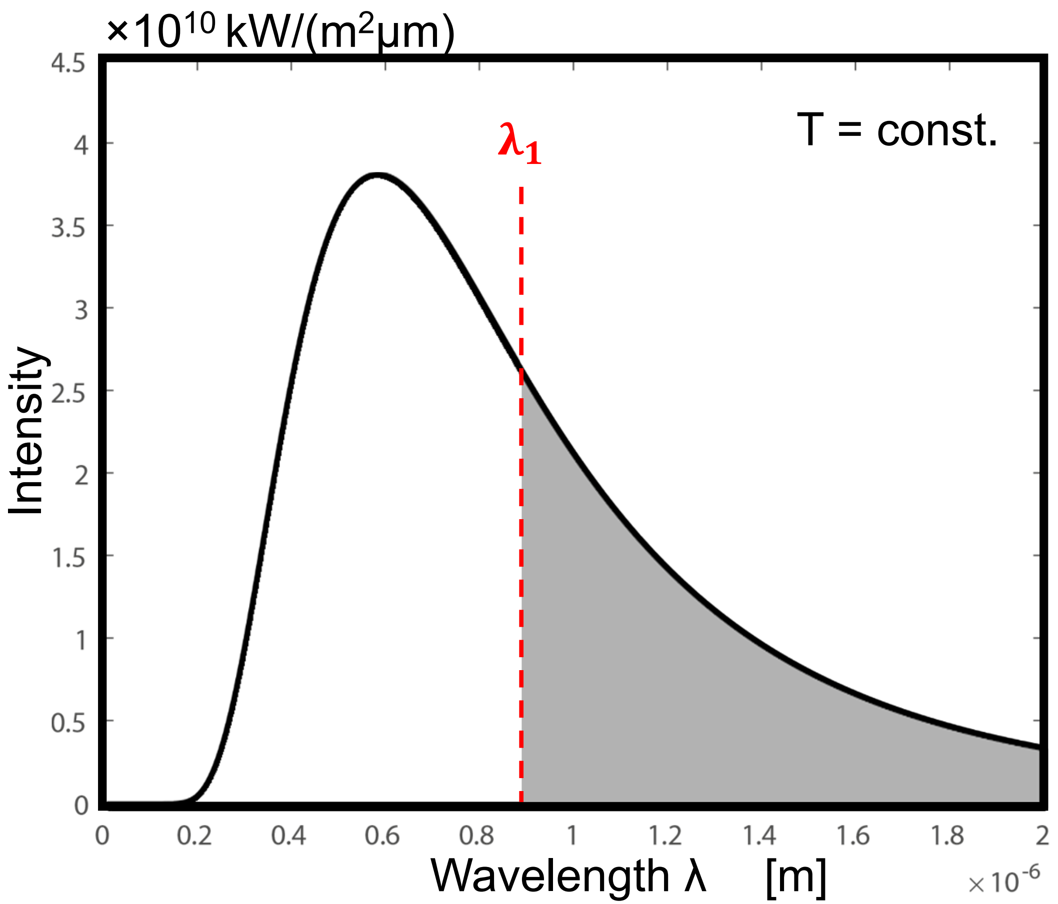
Radiation in specific wavelength ranges



Relative fraction of radiation in relation to the total radiation:

$$F(\lambda) = \frac{1}{\sigma T^4} \int_{\lambda_1}^{\infty} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

Radiation in specific wavelength ranges



Relative fraction of radiation in relation to the total radiation:

$$F(\lambda) = \frac{1}{\sigma T^4} \int_{\lambda_1}^{\infty} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

$$F(0 \to \infty) = \frac{1}{\sigma T^4} \int_{0}^{\infty} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

$$F(0 \to \lambda_1) = \frac{1}{\sigma T^4} \int_{0}^{\lambda_1} \dot{q}''_{s\lambda} \cdot d\,\lambda$$

$$F(\lambda_1 \to \infty) = F(0 \to \infty) - F(0 \to \lambda_1)$$

Comprehension questions

What is a “Black Body”?

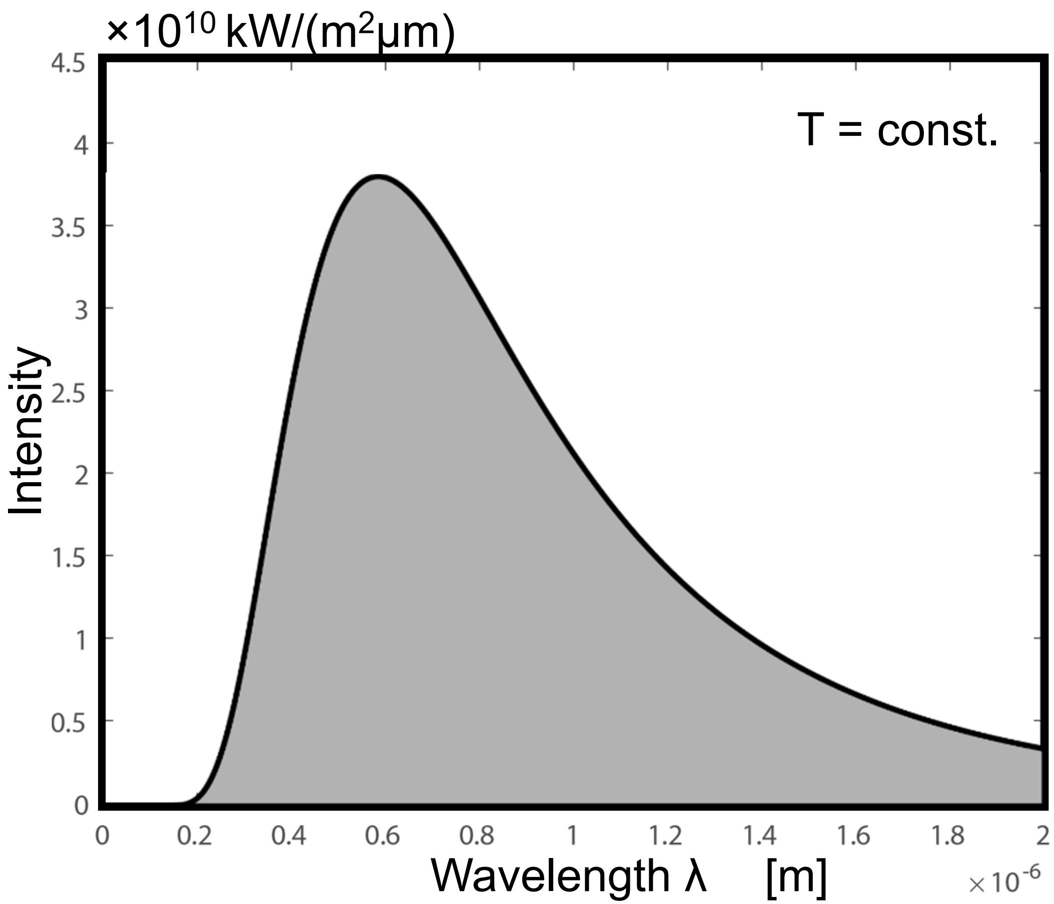
Which assumptions are valid for the calculation of „Black Bodies“?

Which law can be used to determine the wavelength at the intensity maximum of a “Black Body”?

Which approach was used to determine the Stefan-Boltzmann constant?

How can the radiation intensity in a certain wavelength range $\lambda_1 - \lambda_2$ be calculated?

Stefan-Boltzmann Law



going back *(click)*

Integration of Planck Distribution:

$$\dot{q}''_s = \int_0^\infty \frac{c_1 \lambda^{-5}}{exp[c_2/(\lambda T)] - 1}\, \mathrm{d}\lambda$$

Substitution: $x = \dfrac{\lambda\, T}{c_2} \quad \rightarrow \quad \mathrm{d}\lambda = \dfrac{c_2}{T}\, \mathrm{d}x$

insert: $\dot{q}''_s = c_2 \displaystyle\int_0^\infty \frac{c_1 T^4}{{c_2}^5\, x^5\, \left(exp\left[\frac{1}{x}\right] - 1\right)}\, dx$

$$\dot{q}''_s = \frac{c_1}{{c_2}^4} T^4 \underbrace{\int_0^\infty \frac{dx}{x^5 \left(exp\left[\frac{1}{x}\right] - 1\right)}}_{\boxed{= \frac{\pi^4}{15}}}$$

$$\boxed{\dot{q}''_s = \sigma T^4 \quad \rightarrow \quad \sigma = \frac{c_1\ \pi^4}{{c_2}^4\ 15}}$$

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY