Heat Transfer

Application of Dimensional Analysis

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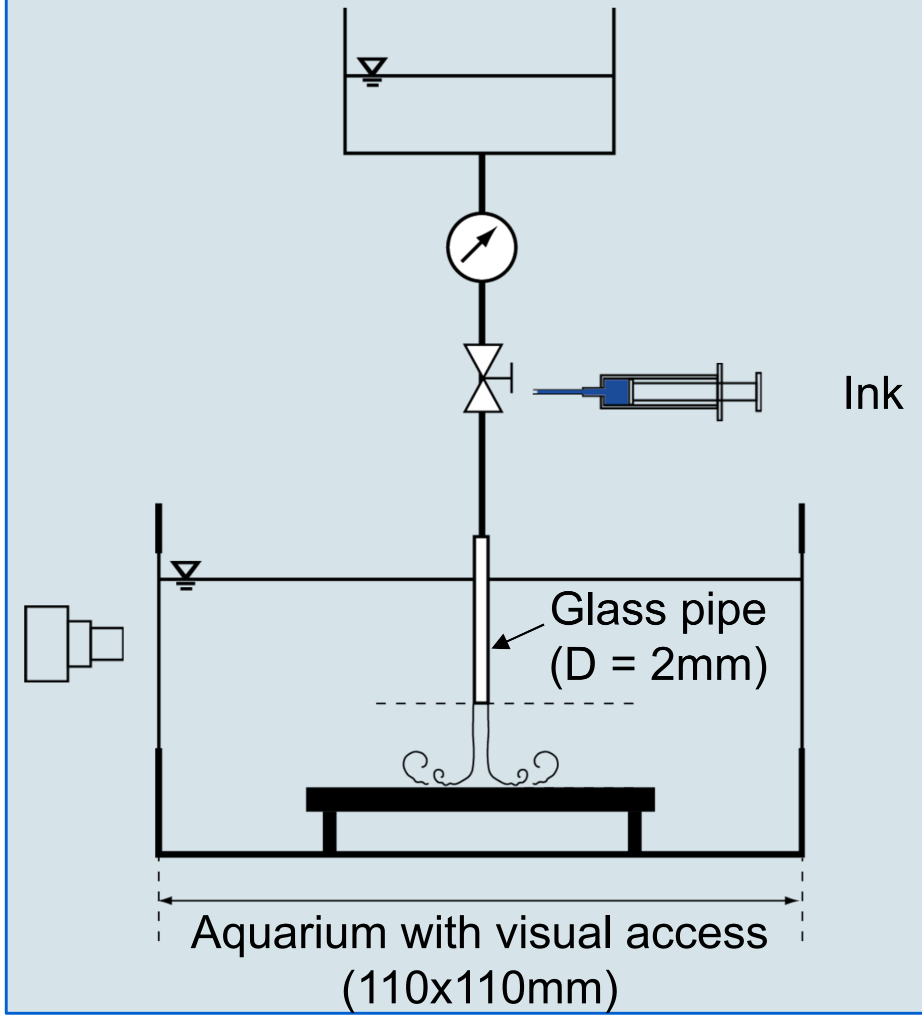
Learning Goals

- Dimensional Analysis in Heat and Mass Transfer
  - Basic understanding of Dimensional Analysis.
  - Understand the physical meanings of relevant dimensionless numbers that can describe a convection problem.
  - Ability to distinguish different convective heat transfer problems in terms of flow and boundary conditions.

$$Nu = Nu(Re, Gr, Pr)$$

Dimensional Analysis

Impact Jet: Experiment



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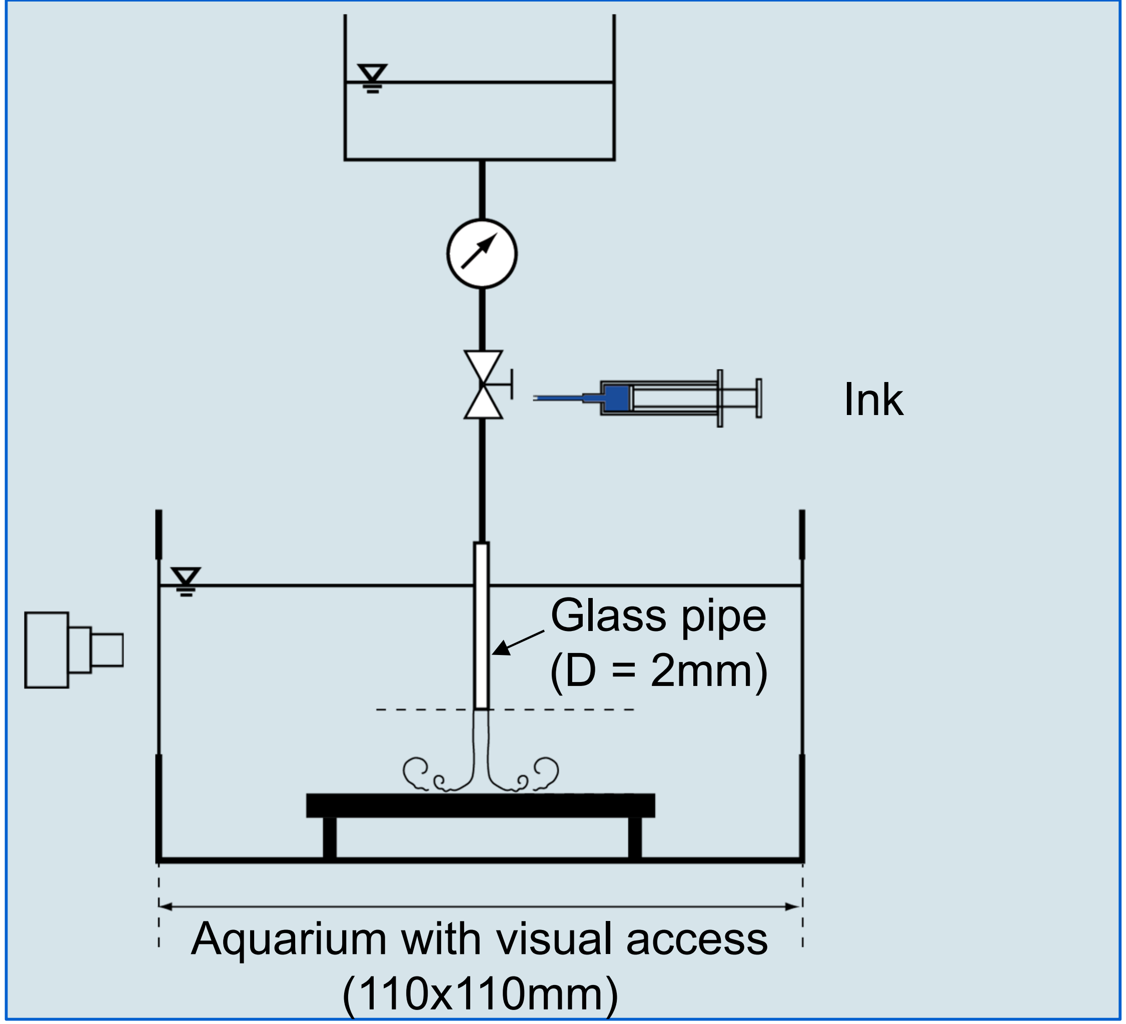
Dimensional Analysis

Impact Jet: Experiment



Dimensional Analysis

Which physical quantities are decisive?

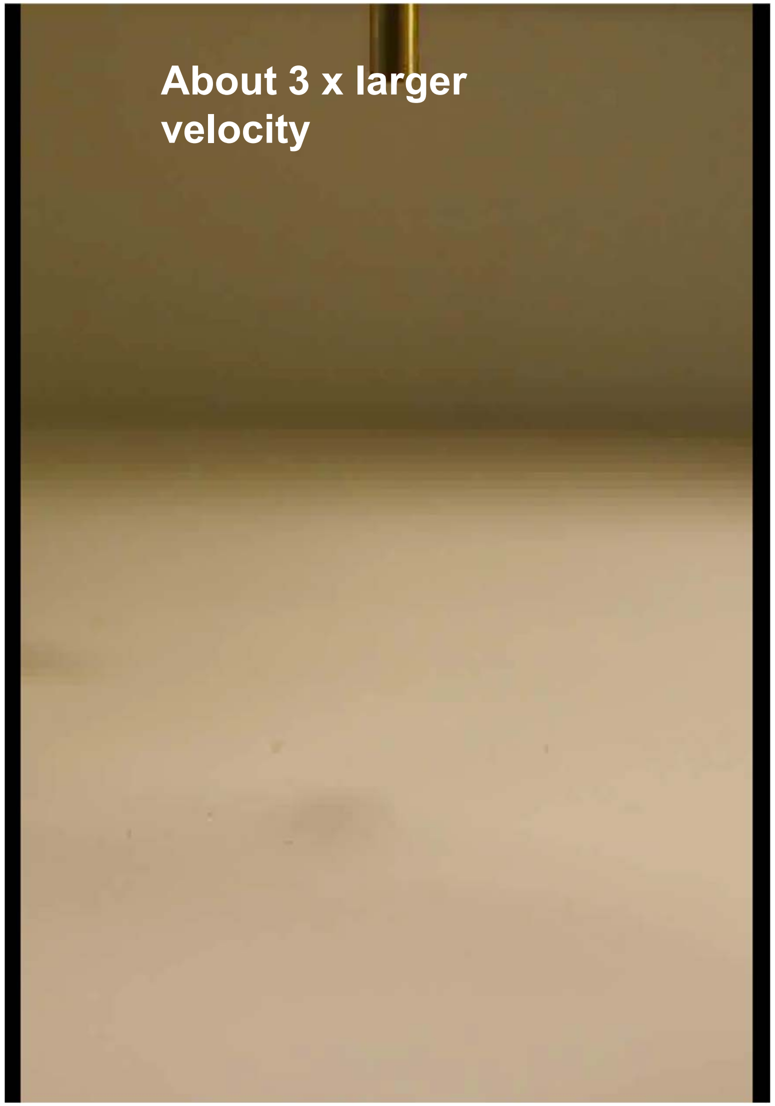
Substance properties:
- Viscosity
- Density

Flow Conditions:
- Velocity

Geometry:
- Nozzle diameter
- Distance of impact plate

Are experiments with oil and water comparable?

When all ratios between „forces“ (terms in the equations) are identical: yes



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Which forces play a role: Conservation Equations considerations

Continuity equation

Mass Flows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation

Momentum Flows | Pressure | Shear stresses

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Which forces play a role: Conservation Equations considerations

Continuity equation

Mass Flows

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

De-scaling

$$x^* = \frac{x}{L},\ y^* = \frac{y}{L}, u^* = \frac{u}{u_\infty}, v^* = \frac{v}{u_\infty}, p^* = \frac{p}{\rho {u_\infty}^2}$$

Momentum equation

Momentum Flows | Pressure | Shear stresses

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

The Reynolds number is **the** relevant Dimensionless number

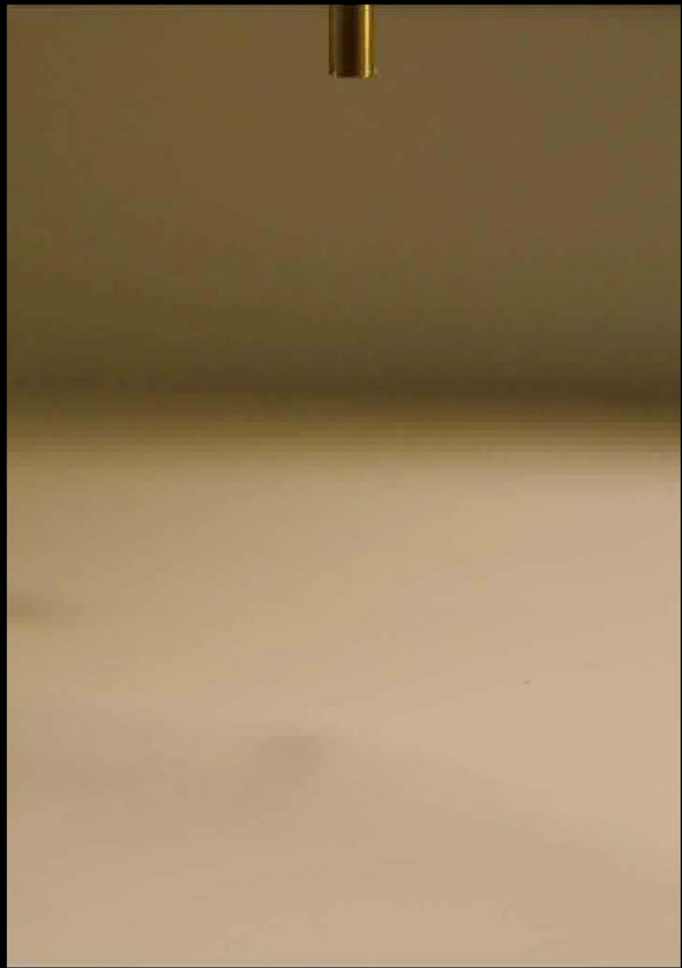
Crucial to this problem is the ratio between viscous forces and inertial forces

Attention: Often further effects come into play due to the boundary conditions

Re = 2500

Re = 1600

Re = 800



Examples of convective heat transport configurations

External Flow

Internal Flow

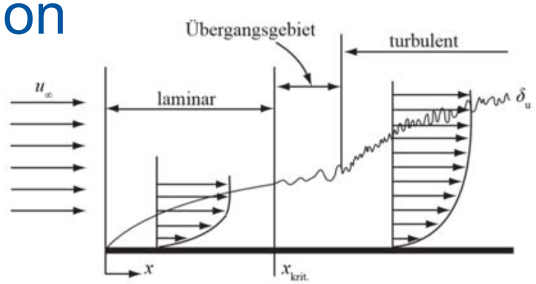
Plates

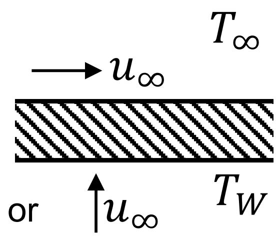
Cylinder (axial / radial)

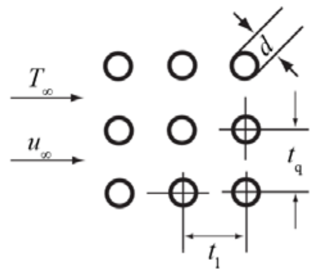
Smooth pipe bundle etc. …

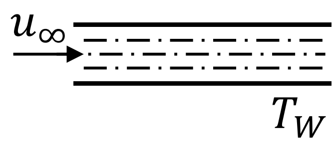
Pipe

Forced Convection

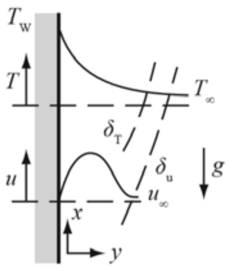


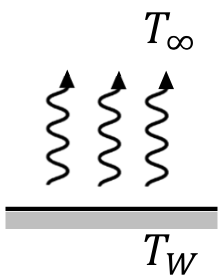


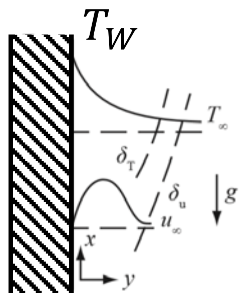


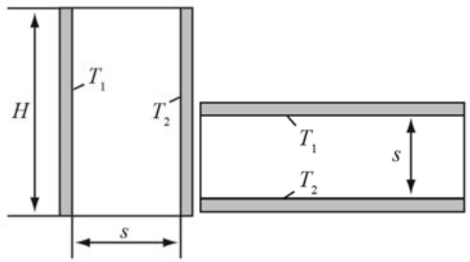


Natural Convection









Examples of convective heat transport configurations

External Flow

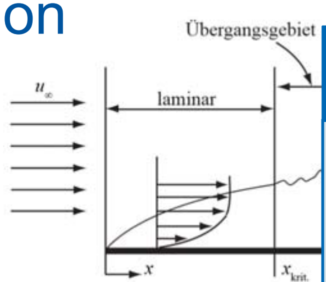
Internal Flow

Plates
(Boundary Layers, partly already learned)

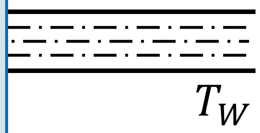
Cylinder (axial / radial)

Smooth pipe bundle etc. …

Forced Convection



Pipe



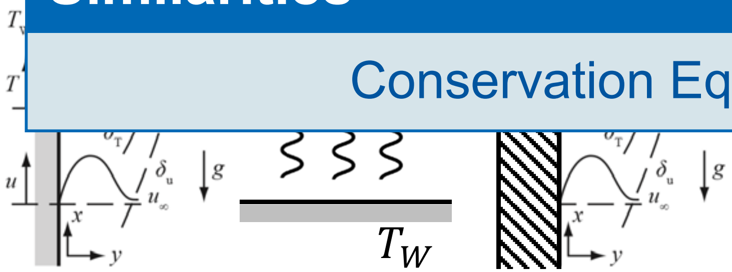
Distinctions

- Geometry (plates, cylinder etc.)
- Flow (laminar, turbulent)
- Boundary Conditions (dynamic, thermal)

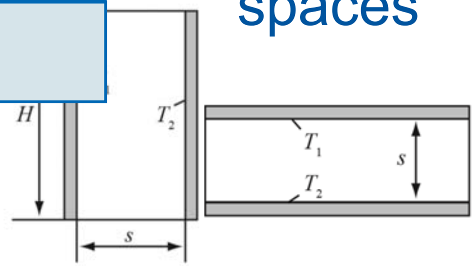
Similarities

Conservation Equations

Natural Convection



Enclosed spaces



Review: Forced Convection

Continuity equation

Mass Flows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation

Momentum Flows | Pressure | Shear stresses

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Energy equation

Enthalpy Flows | Heat Conduction

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

Review: Forced Convection

De-scaling

$$x^* = \frac{x}{L},\ y^* = \frac{y}{L},\ u^* = \frac{u}{u_\infty},\ v^* = \frac{v}{u_\infty},\ p^* = \frac{p}{\rho {u_\infty}^2},\ \Theta^* = \frac{T - T_\infty}{T_W - T_\infty}$$

Continuity equation

Mass Flows

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Momentum equation

Momentum Flows | Pressure | Shear stresses

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

Energy equation

Enthalpy Flows | Heat Conduction

$$u^* \frac{\partial \Theta^*}{\partial x^*} + v^* \frac{\partial \Theta^*}{\partial y^*} = \frac{1}{\underbrace{RePr}_{Pe}}\left(\frac{\partial^2 \Theta^*}{\partial x^{*2}} + \frac{\partial^2 \Theta^*}{\partial y^{*2}}\right)$$

$$Nu = Nu(Re, Pr)$$

Review: Natural Convection

Continuity equation

Mass Flows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation

Momentum Flows | Pressure | Shear stresses | Gravity

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \beta g(T - T_\infty)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Energy equation

Enthalpy Flows | Heat Conduction

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

Review: Natural Convection

De-scaling

$$x^* = \frac{x}{L},\; y^* = \frac{y}{L},\; u^* = \frac{u}{u_\infty},\; v^* = \frac{v}{u_\infty},\; p^* = \frac{p}{\rho {u_\infty}^2},\; \Theta^* = \frac{T - T_\infty}{T_W - T_\infty}$$

Continuity equation

Mass Flows

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Momentum equation

Momentum Flows | Pressure | Shear stresses | Gravity

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) + \underbrace{Gr \cdot \left(\frac{1}{Re}\right)^2}_{Ar} \Theta^*$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

Energy equation

Enthalpy Flows | Heat Conduction

$$u^* \frac{\partial \Theta^*}{\partial x^*} + v^* \frac{\partial \Theta^*}{\partial y^*} = \frac{1}{\underbrace{RePr}_{Pe}}\left(\frac{\partial^2 \Theta^*}{\partial x^{*2}} + \frac{\partial^2 \Theta^*}{\partial y^{*2}}\right)$$

$$Nu = Nu(Re, Pr, Gr)$$

Summary: Dimensionless numbers

General form of the heat transfer coefficient α

$$Nu \equiv \frac{\alpha L}{\lambda} = \text{Dimensionless heat transfer coefficient}$$
$$= C \cdot Re^m \cdot Pr^n \cdot Gr^p$$

with

$$Re \equiv \frac{\rho u_\infty L}{\eta} = \frac{\text{Inertial Forces}}{\text{Viscosity Forces}}$$

$$Pr \equiv \frac{\eta c_p}{\lambda} = \frac{\nu}{a} = \frac{\text{Diffusive Momentum transport}}{\text{Diffusive Heat transport}}$$

$$Gr \equiv \frac{\beta g \rho^2 (T_W - T_\infty) L^3}{\eta^2} = \frac{\text{Buoyancy Forces}}{\text{Viscosity Forces}}$$

$$Pe \equiv Re \cdot Pr = \frac{\text{Advective Heat Flow}}{\text{Diffusive Heat Flow}}$$

$$Ar \equiv \frac{Gr}{Re^2} = \frac{\text{Buoyancy Forces}}{\text{Friction Forces}}$$

Comprehension Questions

What does the Dimensional Analysis say and what must be taken into account so that the solutions of two different problems are identical?

Which Dimensionless numbers are essential for the empirically found heat transfer laws?