Heat Transfer: Conduction

Conduction in a multilayer plane wall

Prof. Dr.-Ing. Reinhold Kneer

Prof. Dr.-Ing. Dr. rer. pol. Wilko Rohlfs

HEATQUIZ

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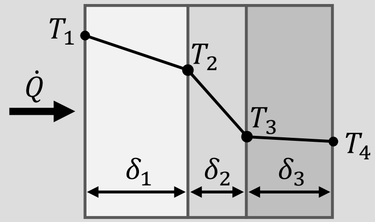
Learning goals

Temperature profile of a multilayer wall:

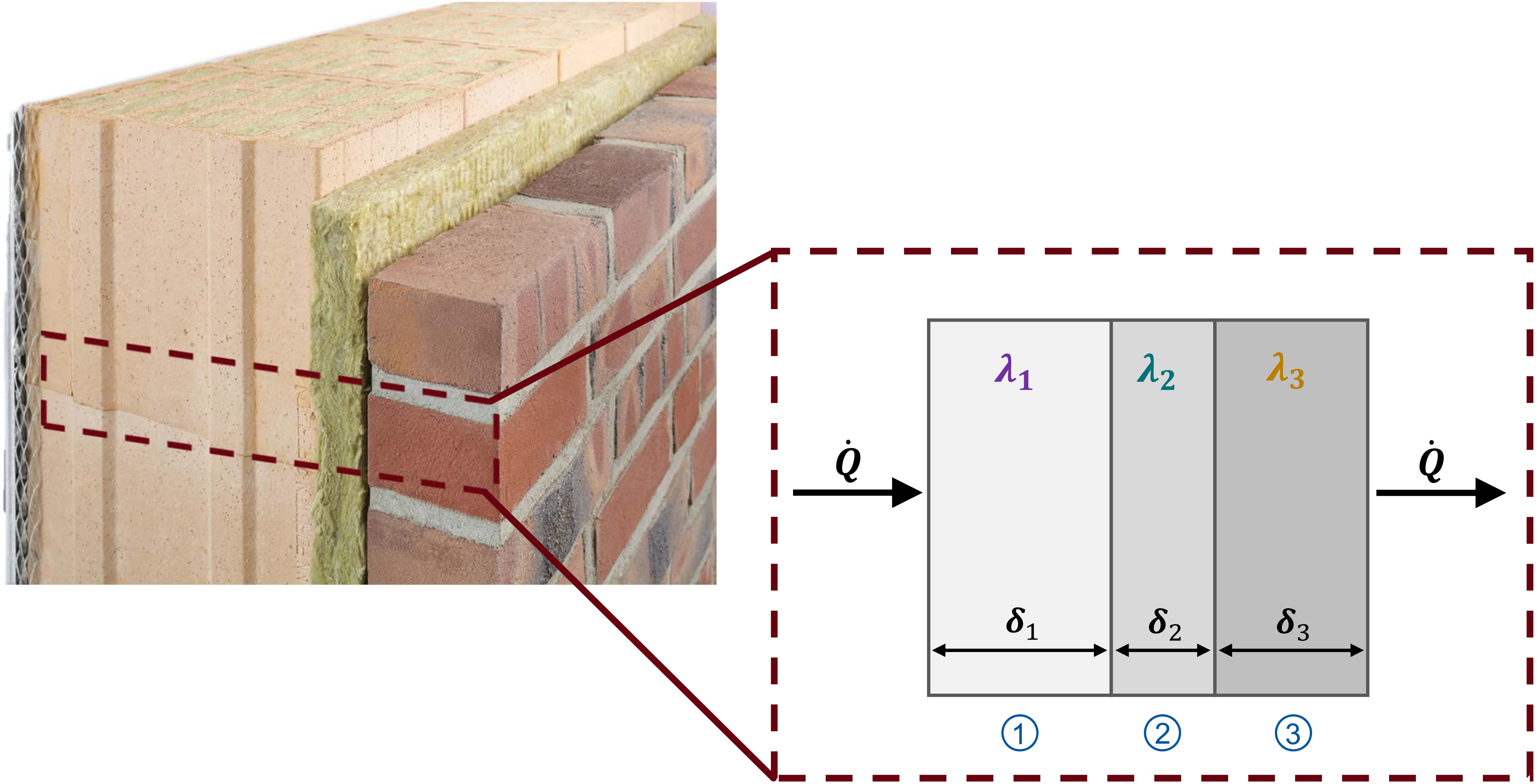
- Consideration of temperature profile of a multilayer wall under steady state conditions



Thermal resistances in a multilayer wall:

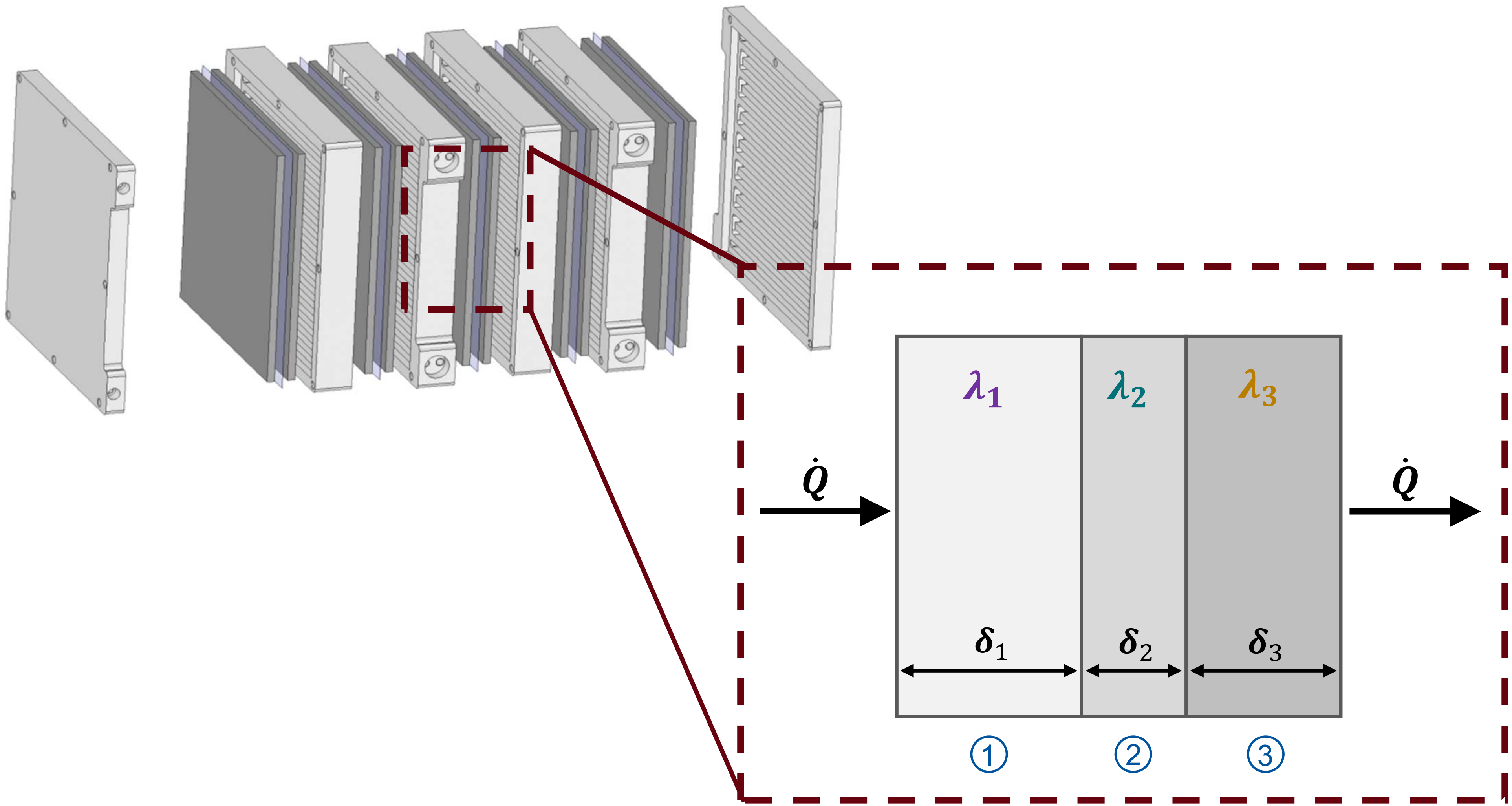
- Combining the thermal resistors connected in series to define the total resistance

Multilayer wall – Example: Brick wall



https://www.baunetzwissen.de

Multilayer wall – Example: Battery stack



https://www.crtech.com/applications/flow-battery

Assumptions and conditions

Conditions:

- Steady state
- One-dimensional heat transport
- Constant material properties
- Constant cross section area

Why do the discontinuities occur?

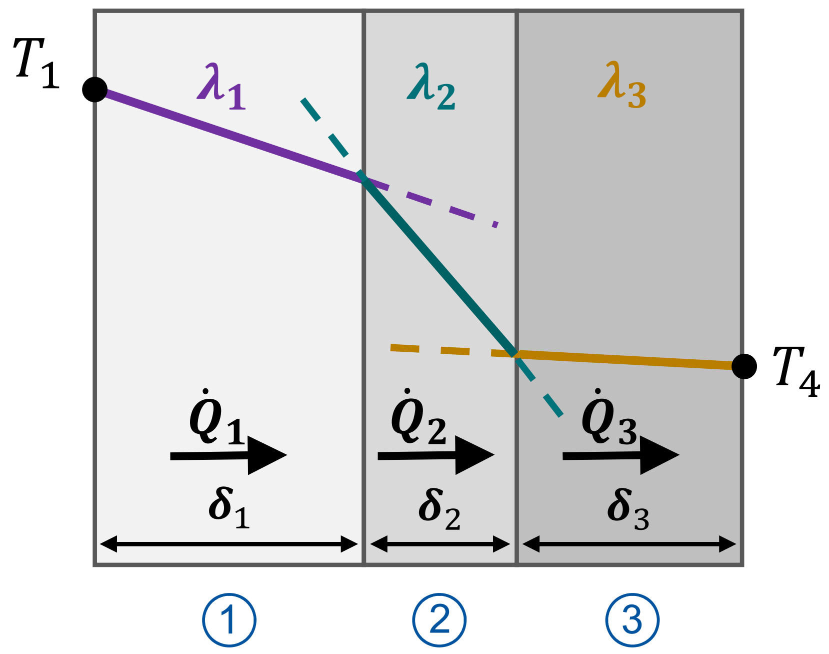
$$\dot{Q} = -\lambda \cdot A \cdot \frac{dT}{dx}$$

- The heat flux $\dot{Q}$ is constant throughout the entire layer: $\dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3$

Example

1. Sand-lime brick
2. Insulation
3. Brick

$\lambda_3 \gg \lambda_1 > \lambda_2$



Assumptions and fundamentals

Conditions:

- Steady state
- One-dimensional heat transport
- Constant material properties
- Constant cross section area

Why do the discontinuities occur?

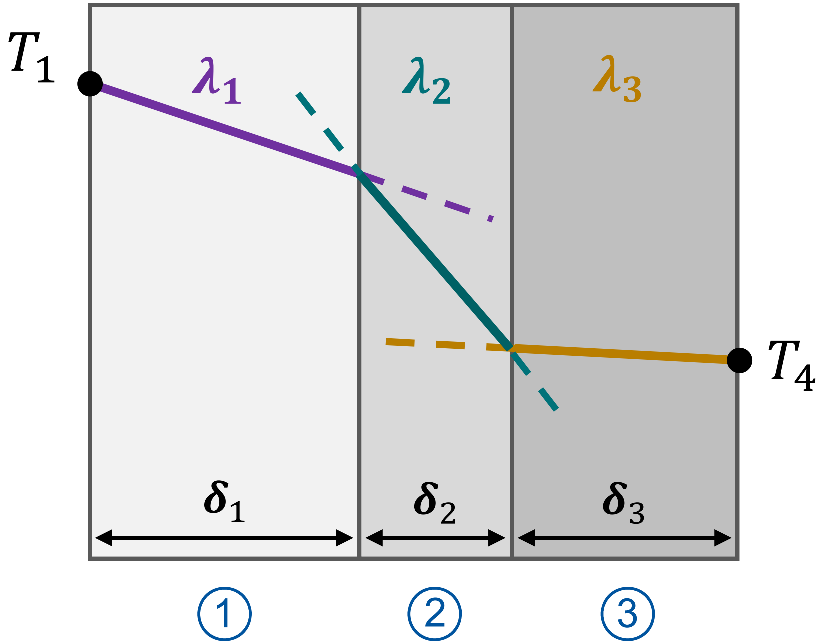
$$\lambda \uparrow \quad \Rightarrow \quad \frac{dT}{dx} \downarrow$$

$$\lambda \downarrow \quad \Rightarrow \quad \frac{dT}{dx} \uparrow$$

Example

① Sand-lime brick
② Insulation
③ Brick

$$\lambda_3 \gg \lambda_1 > \lambda_2$$



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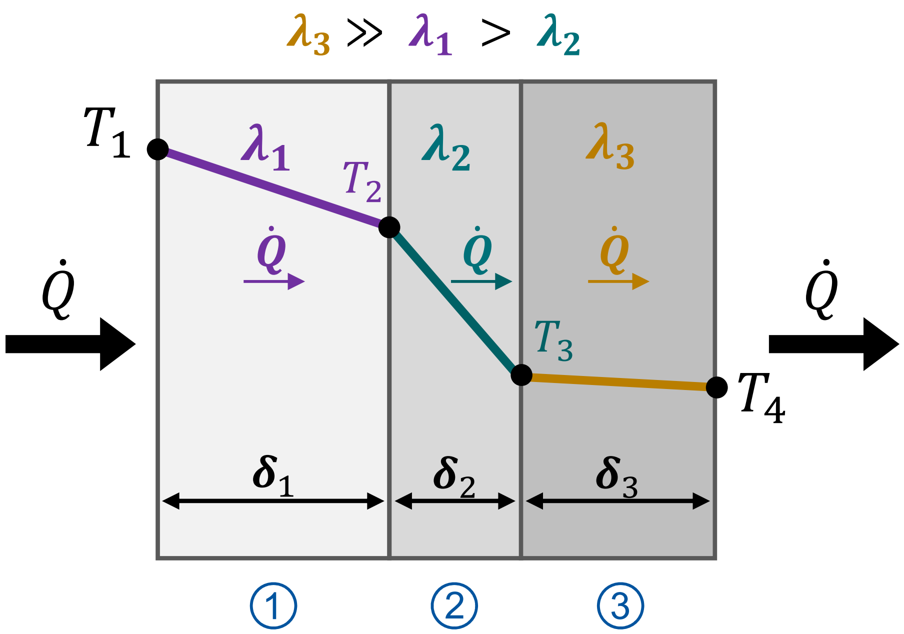
Heat flow in a multilayer wall

Question:

How can the heat flow be determined if only the temperatures T_1 and T_4 are known?



1 Heat flow layer 1

$$\dot{Q} = \lambda_1 \cdot \frac{A}{\delta_1}(T_1 - T_2)$$

2 Heat flow layer 2

$$\dot{Q} = \lambda_2 \cdot \frac{A}{\delta_2}(T_2 - T_3)$$

3 Heat flow layer 3

$$\dot{Q} = \lambda_3 \cdot \frac{A}{\delta_3}(T_3 - T_4)$$

For steady state without sources and sinks:

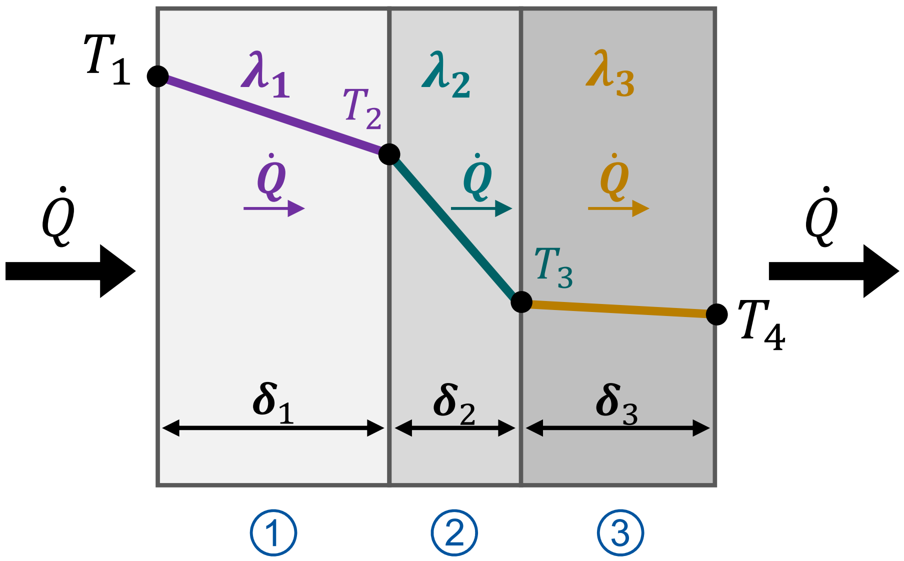
$$\dot{Q} = \dot{Q} = \dot{Q}$$

Heat flow in a multilayer wall

Question:

How can the heat flow be determined if only the temperatures T_1 and T_4 are known?

$\lambda_3 \gg \lambda_1 > \lambda_2$



Rearrange the equations 2 and 3 for the unknown temperatures T_2 and T_3:

- From equation 2 $\quad T_2 = \dfrac{\dot{Q}}{\lambda_2 \cdot \dfrac{A}{\delta_2}} + T_3$
- From equation 3 $\quad T_3 = \dfrac{\dot{Q}}{\lambda_3 \cdot \dfrac{A}{\delta_3}} + T_4$
- To eliminate T_3, insert 3 in 2:

4 $$T_2 = \frac{\dot{Q}}{\lambda_2 \cdot \dfrac{A}{\delta_2}} + \frac{\dot{Q}}{\lambda_3 \cdot \dfrac{A}{\delta_3}} + T_4$$

- To eliminate T_2, insert 4 in 1:

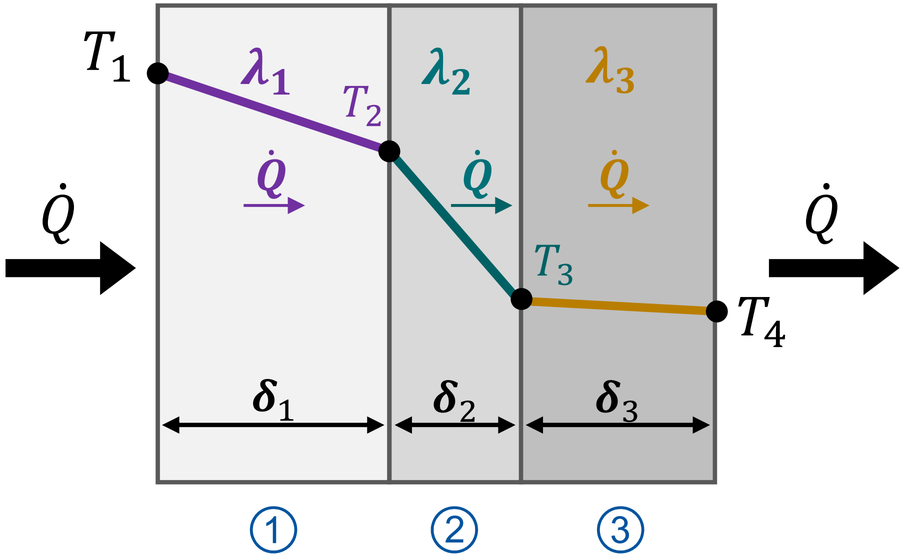
5 $$\dot{Q} = \lambda_1 \cdot \frac{A}{\delta_1}\left[T_1 - \frac{\dot{Q}}{\lambda_2 \dfrac{A}{\delta_2}} - \frac{\dot{Q}}{\lambda_3 \dfrac{A}{\delta_3}} - T_4\right]$$

Heat flow in a multilayer wall

Question:

How can the heat flow be determined if only the temperatures T_1 and T_4 are known?

$\lambda_3 \gg \lambda_1 > \lambda_2$



Rearrange the equations 2 and 3 for the unknown temperatures T_2 and T_3:

5 $$\dot{Q} = \lambda_1 \cdot \frac{A}{\delta_1}\left[T_1 - \frac{\dot{Q}}{\lambda_2 \frac{A}{\delta_2}} - \frac{\dot{Q}}{\lambda_3 \frac{A}{\delta_3}} - T_4\right]$$

- In 5 - exclude $\dot{Q}$ from bracket

$$\dot{Q}\left[\frac{1}{\lambda_1 \cdot \frac{A}{\delta_1}} + \frac{1}{\lambda_2 \frac{A}{\delta_2}} + \frac{1}{\lambda_3 \frac{A}{\delta_3}}\right] = [T_1 - T_4]$$

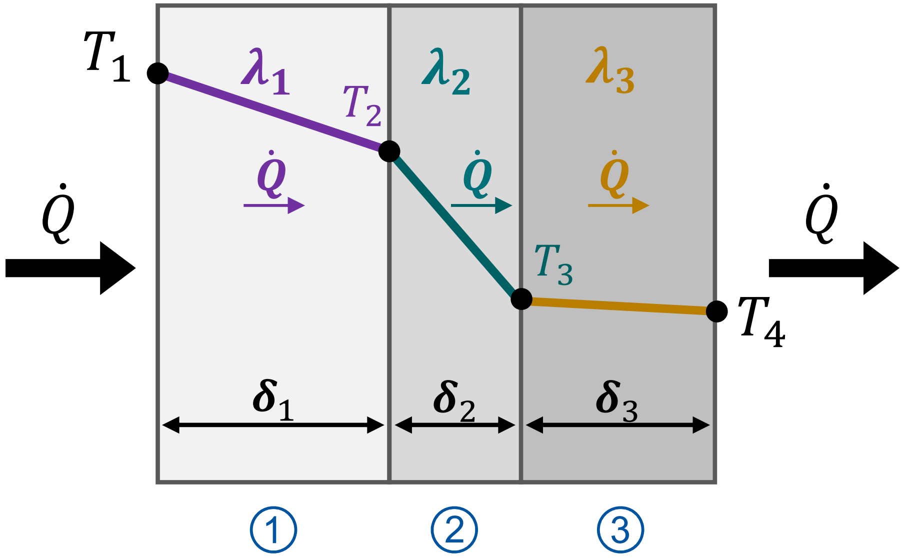
$$\dot{Q} = \frac{A}{\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3}}(T_1 - T_4)$$

Heat flow in a multilayer wall

Question:

How can the heat flow be determined if only the temperatures T_1 and T_4 are known?

$$\lambda_3 \gg \lambda_1 > \lambda_2$$



$$\boxed{\text{Flow} = \frac{\text{Driving potential}}{\text{Resistance}}}$$

Rearrange the equations 2 and 3 for the unknown temperatures T_2 and T_3:

5 $$\dot{Q} = \lambda_1 \cdot \frac{A}{\delta_1}\left[T_1 - \frac{\dot{Q}}{\lambda_2 \frac{A}{\delta_2}} - \frac{\dot{Q}}{\lambda_3 \frac{A}{\delta_3}} - T_4\right]$$

- In 5 - exclude $\dot{Q}$ from bracket

$$\dot{Q}\left[\frac{1}{\lambda_1 \cdot \frac{A}{\delta_1}} + \frac{1}{\lambda_2 \frac{A}{\delta_2}} + \frac{1}{\lambda_3 \frac{A}{\delta_3}}\right] = [T_1 - T_4]$$

$$\boxed{\dot{Q}} = \boxed{\frac{A}{\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3}}}\boxed{(T_1 - T_4)}$$

flow

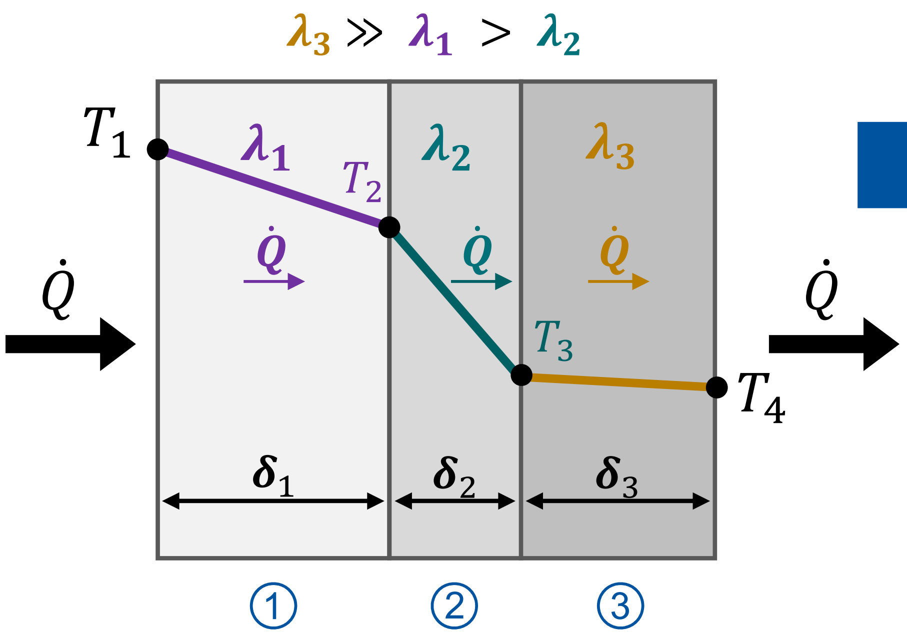
resistance (reciprocal)

driving potential

Thermal resistance: Definition and application

$$\text{Flow} = \frac{\text{Driving potential}}{\text{Resistance}}$$

$$\dot{Q} = \frac{1}{R_{c,tot}} (T_1 - T_{n+1})$$



Equivalent circuit diagram of thermal resistors in series:

$$R_{c,tot} = \sum_{i=1}^{n} R_{c_i} = \sum_{i=1}^{n} \frac{\delta_i}{A_i\, \lambda_i}$$

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Analogy to electrical engineering

Heat flow can be considered as an analogy to electric current!

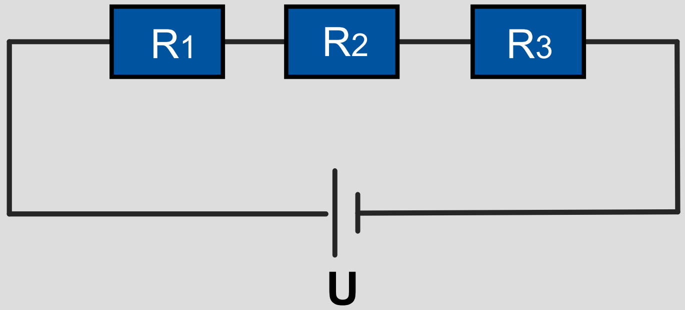
Heat flux equals current flow: $\dot{Q} \equiv I$

$$\text{Flow(Flux)} = \frac{\text{Driving potential}}{\text{Resistance}}$$

Electrical circuit, resistances in series:

$$I = \frac{U}{R_{tot}}$$

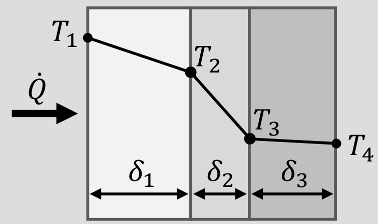
with: $\mathrm{R}_{\mathrm{tot}} = \sum_{i=1}^{n} R_i$



Heat Transfer in multilayer wall:

$$R_{\mathrm{c,tot}} = \sum_{i=1}^{n} R_{\mathrm{c}_i} = \sum_{i=1}^{n} \frac{\delta_i}{A_i\,\lambda_i}$$

$$\boxed{\dot{Q} = \frac{1}{R_{\mathrm{c,tot}}}\left(T_1 - T_{n+1}\right)}$$



Comprehension questions

What is the course of the temperature profile in a flat wall without heat sources and sinks in steady state?

Under what conditions can it be assumed that the heat flow remains constant in all layers?

How is the thermal resistance of a plane wall defined? How can the thermal resistance be calculated for a wall of n layers?

$$\dot{Q} = \frac{1}{R_{\mathrm{c,tot}}}\left(T_1 - T_{n+1}\right) \qquad R_{\mathrm{c,tot}} = \sum_{i=1}^{n} R_{\mathrm{c}_i} = \sum_{i=1}^{n} \frac{\delta_i}{A_i\,\lambda_i}$$