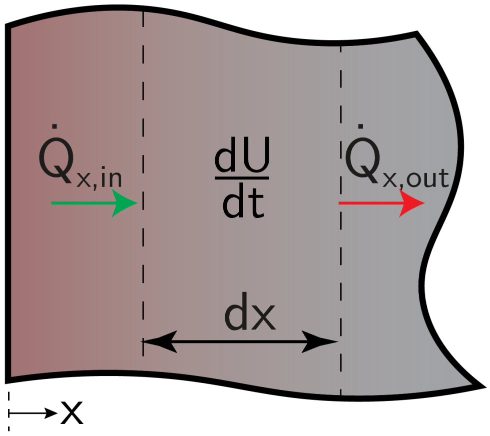
EB - Cond. - IE 18

A very thick wall Fo $<< 1$, initially at a homogeneous temperature T_0, is heated up at the left-hand side. Give the energy balance to derive the heat conduction equation. Assume one-dimensional transient conditions in x-direction at constant atmospheric pressure.



Energy balance:

$$\frac{\partial U}{\partial t} = \dot{Q}_{x,in} - \dot{Q}_{x,out}$$

For unsteady heat transfer the internal energy will change over time and equals the sum of the in- and outgoing heat fluxes.

Change of internal energy over time:

$$\frac{\partial U}{\partial t} = \rho \cdot c_p \cdot dx \cdot A \cdot \frac{\partial T}{\partial t}$$

The internal energy of a constant volume can be described as: $U = m \cdot c_p \cdot T$.

Heat fluxes:

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

The ingoing flux can be described by use of Fourier's equation. The outgoing flux can be approximated by use of the Taylor series expansion.

Conditions

Initial condition:

$$T(t=0) = T_0$$

Boundaries:

$$-\lambda A \frac{\partial T}{\partial x}|_{x=0} = \alpha A \left(T_\infty - T(x=0)\right)$$

$$-\lambda A \frac{\partial T}{\partial x}|_{x\to\infty} = 0$$