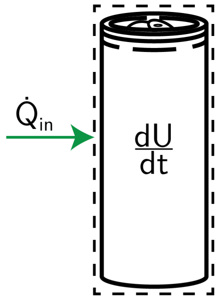
Heating of a Can

Derive the energy balance describing the temperature change of the can over time.



1) Setting up an energy balance:

The standard energy balance describing the change in energy of a control volume over time can be expressed as:

$$\frac{dU}{dt} = \sum \dot{Q}_{\mathrm{in}} - \sum \dot{Q}_{\mathrm{out}}$$

Which for the given scenario will be:

$$\frac{dU}{dt} = \dot{Q}_{\mathrm{in}}$$

2) Defining the fluxes:

The change of internal energy over the course of our control volume can be expressed as:

$$\begin{aligned}\frac{dU}{dt} &= \frac{d}{dt}(mcT)\\ &= \frac{1}{4}\pi\rho D^2 Lc\frac{dT}{dt}\end{aligned}$$

And the incoming rate of heat transfer can be described by use of Newton's law of cooling, where the temperatures are switched due to the fact that it uses the convention of cooling:

$$\begin{aligned}\dot{Q}_{\mathrm{in}} &= \alpha A_{\mathrm{s}}\left(T(t) - T_{\mathrm{A}}\right)\\ &= \alpha\pi DL\left(T(t) - T_{\mathrm{A}}\right)\end{aligned}$$

3) Inserting and rearranging:

Inserting the found fluxes into the energy balance yields into the balance describing the temperature change of the can over the course of time:

$$\frac{1}{4}\pi\rho D^2 Lc\frac{dT}{dt} = \alpha\pi DL\left(T(t) - T_{\mathrm{A}}\right)$$

$$\boxed{\rightarrow \frac{dT}{dt} = \frac{4\alpha}{\rho Dc}\left(T(t) - T_{\mathrm{A}}\right)}$$