Heat Transfer: Conduction

Dimensionless Numbers and Heisler Diagrams

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Learning goals

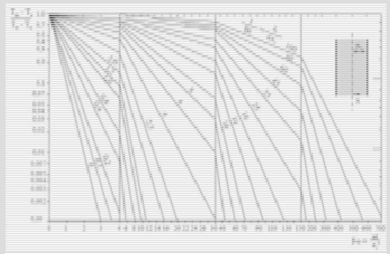
Dimensionless numbers

- Importance of dimensionless numbers, especially Fourier and Biot numbers for transient heat transfer.

$$\Theta^* = \Theta^*(x^*, y^*, z^*, t^*, Fo, Bi)$$

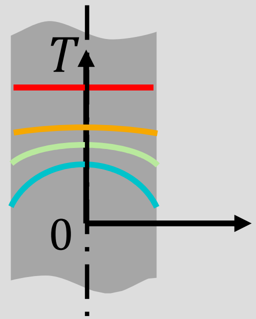
Heisler diagrams

- Understanding of the Heisler diagrams for the determination of the body core temperature, the local temperature profile and the heat flow.

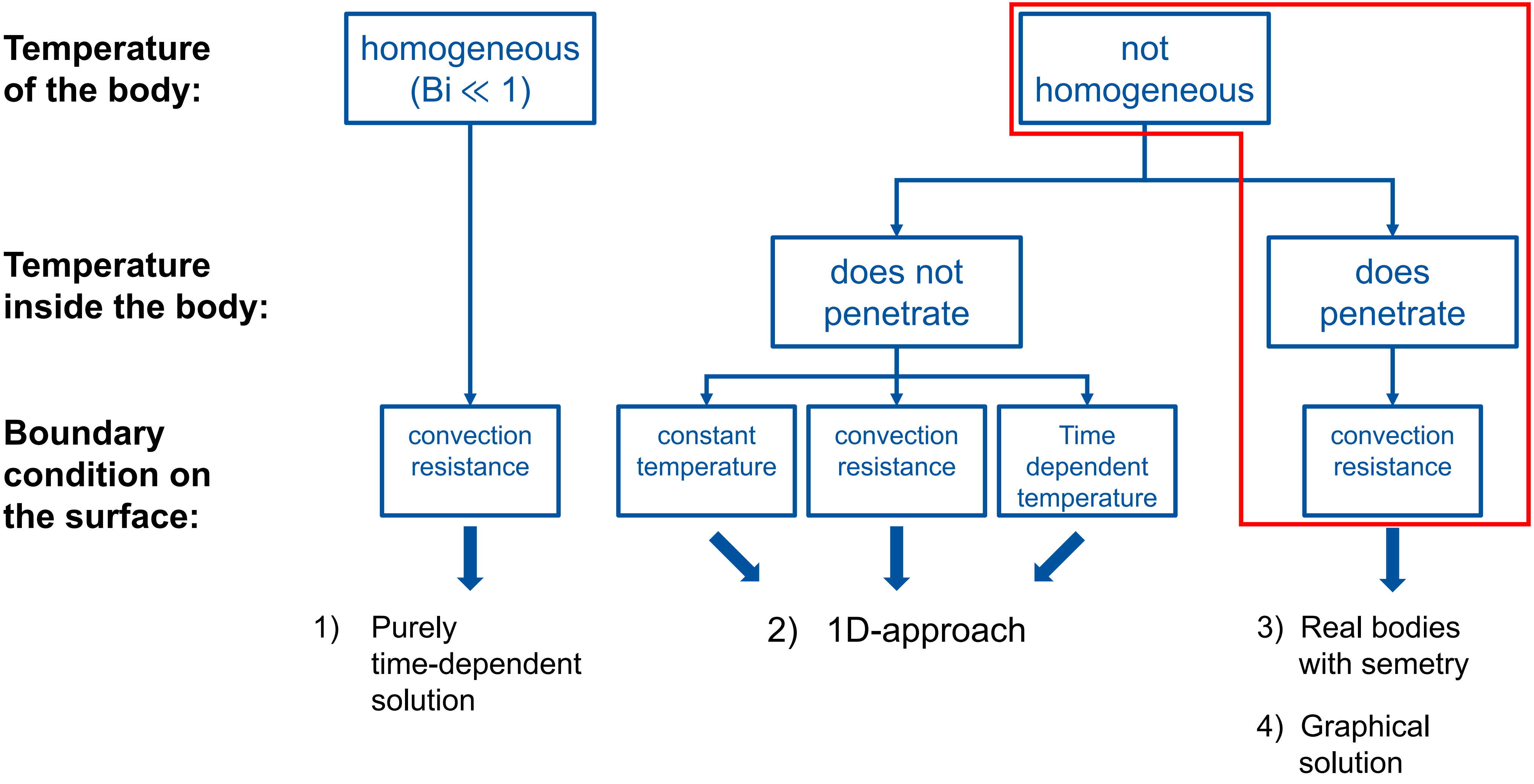


Example: Quenching of a steel plate

- Application of the Heisler diagrams.



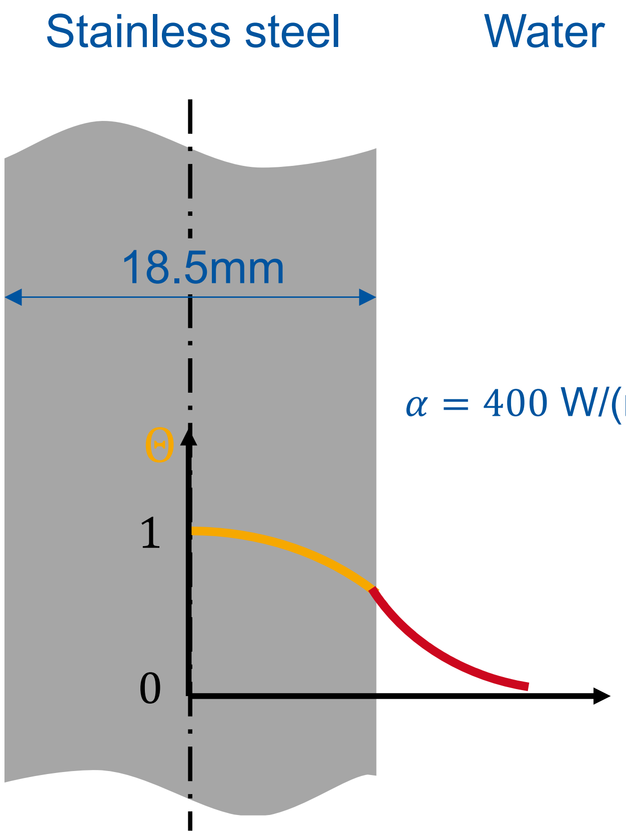
How to simplify the problem?



Dimensionless Form

Can the temperature distribution in two different cooled or heated systems look similar?

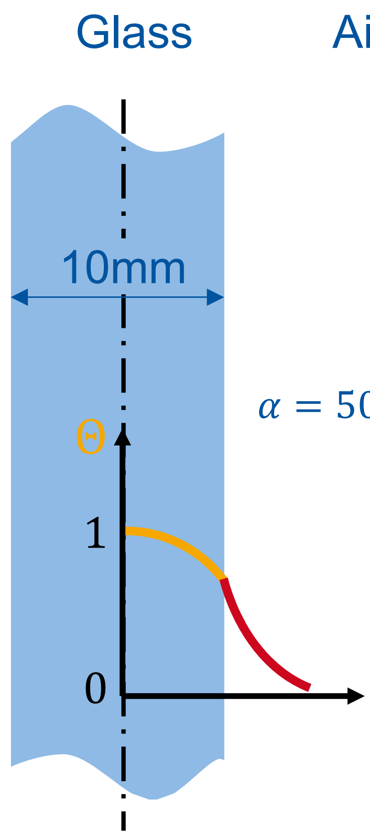
System A



$\lambda = 15$ W/(m·K)
$\rho = 7900$ kg/m³
$c_p = 500$ J/(kg·K)
$\downarrow$
$a = 3.8 \times 10^{-6}$ m²/s

$\alpha = 400$ W/(m²·K)

System B



$\lambda = 1$ W/(m·K)
$\rho = 2500$ kg/m³
$c_p = 850$ J/(kg·K)
$\downarrow$
$a = 4.7 \times 10^{-7}$ m²/s

$\alpha = 50$ W/(m²·K)

Dimensionless form

Transient heat conduction

3-D Conservation Equation without advection and source

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right)$$

Dimensionless equation

$$\frac{\partial \Theta^*}{\partial t^*} = Fo\left(\frac{\partial^2 \Theta^*}{\partial x^{*2}} + \frac{\partial^2 \Theta^*}{\partial y^{*2}} + \frac{\partial^2 \Theta^*}{\partial z^{*2}}\right)$$

Solution

$$T = T\left(x, y, z, t, \rho, c_p, \lambda, \underbrace{\text{Initial}}_{T_0} - \text{and } \underbrace{\text{Boundary Conditions}}_{\alpha, T_A}\right)$$

Dimensionless solution

$$\Theta^* = \Theta^*(x^*, y^*, z^*, t^*, Fo, Bi)$$

Example: Quenching of a steel plate

$$x^* = \frac{x}{\delta_x} \qquad y^* = \frac{y}{\delta_y} \qquad z^* = \frac{z}{\delta_z} \qquad t^* = \frac{t}{\tau} \qquad \Theta^* = \frac{T - T_A}{T_0 - T_A}$$

$$Bi = \frac{\alpha \delta}{\lambda} \qquad Fo = \frac{a\tau}{\delta^2} = \frac{\lambda}{\rho c_p}\frac{\tau}{\delta^2}$$

Time evolution of the core body temperature

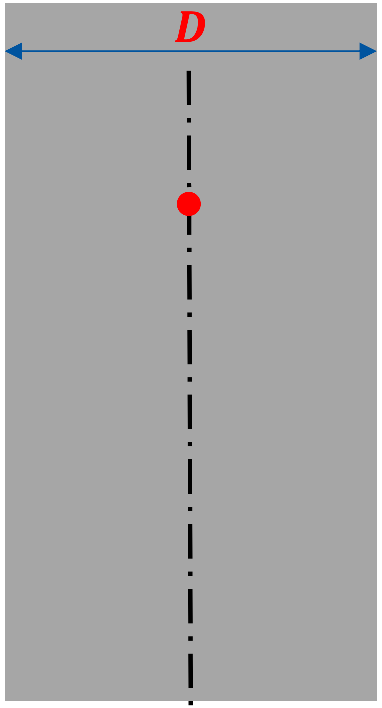
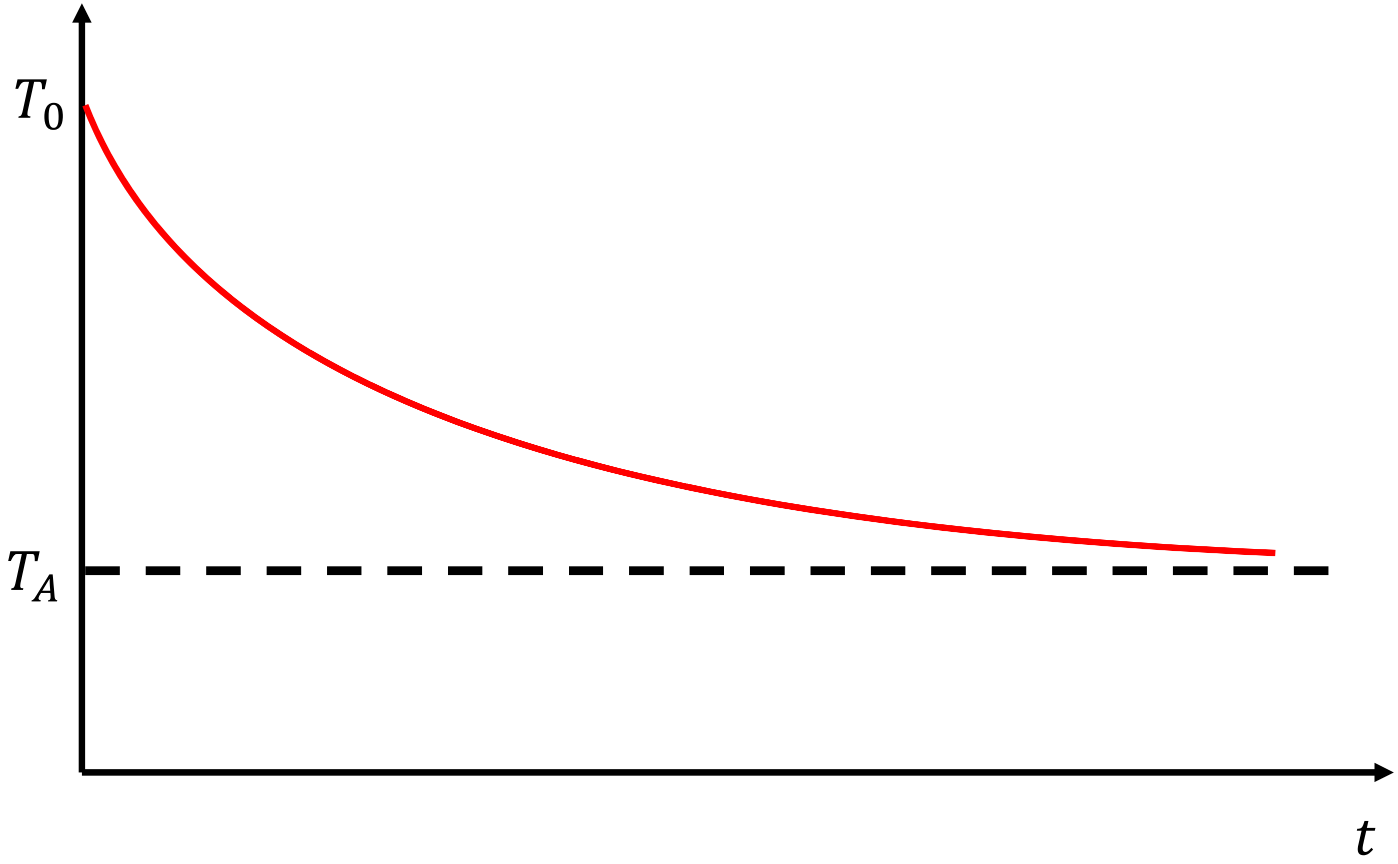


Plate
(infinite expansion)

Relevant dependencies: $Bi = \frac{\alpha\delta}{\lambda}$ $Fo = \frac{a\tau}{\delta^2} = \frac{\lambda}{\rho c_p}\frac{\tau}{\delta^2}$



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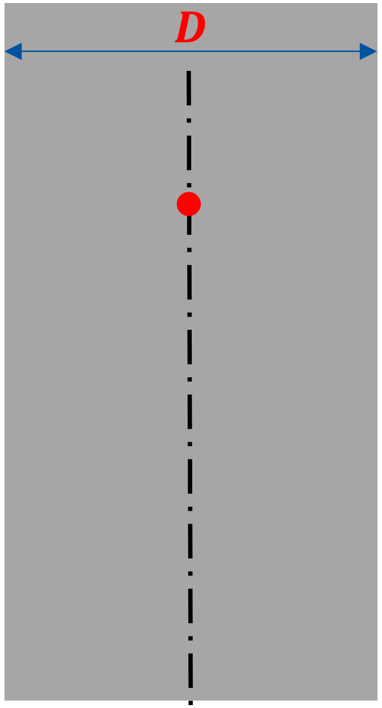

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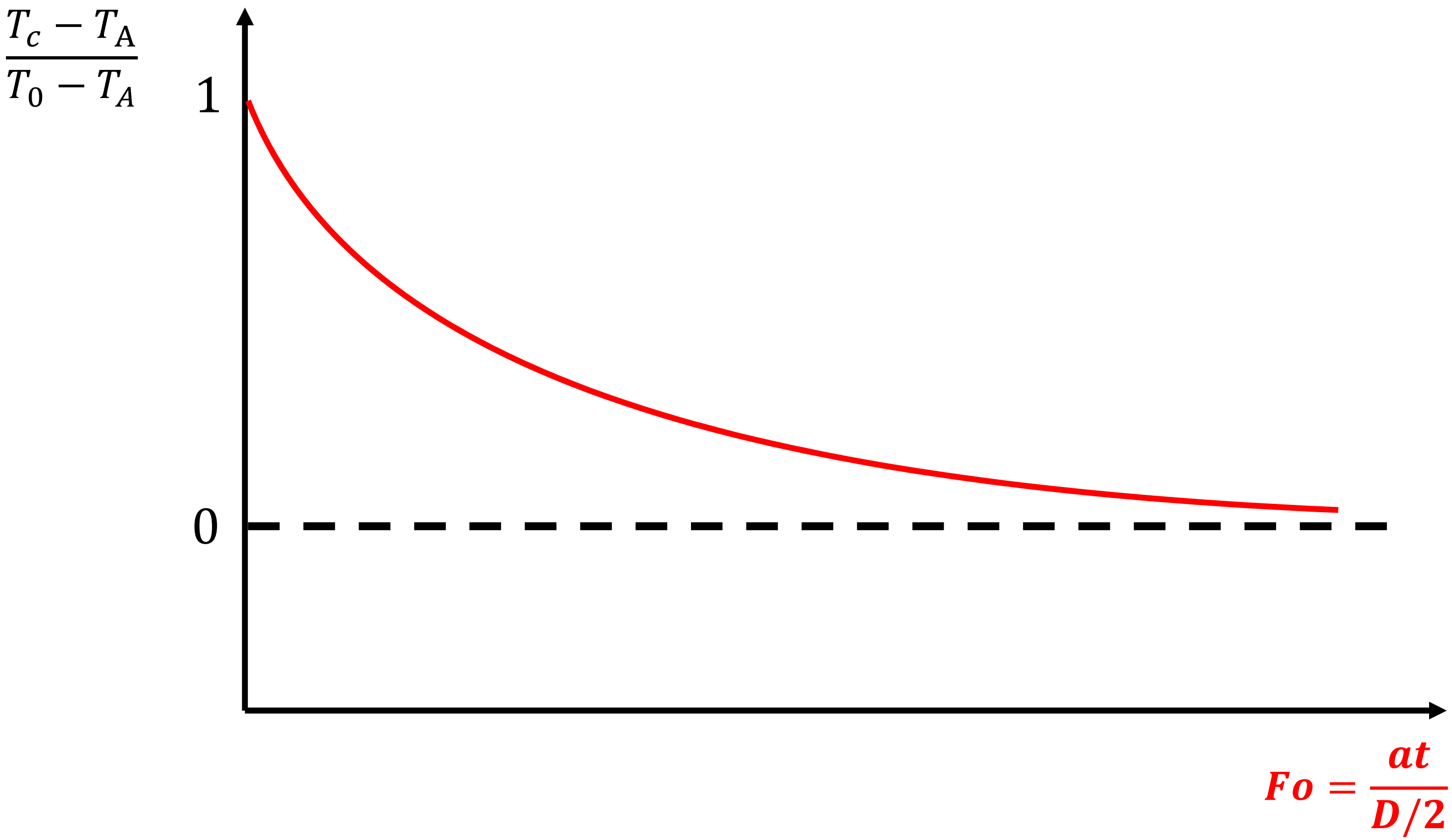

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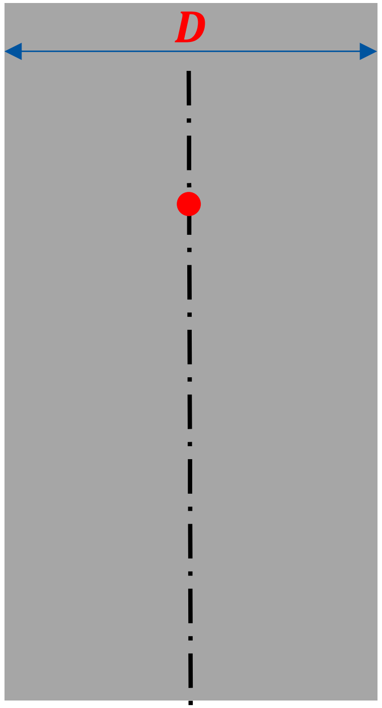
Time evolution of the core body temperature



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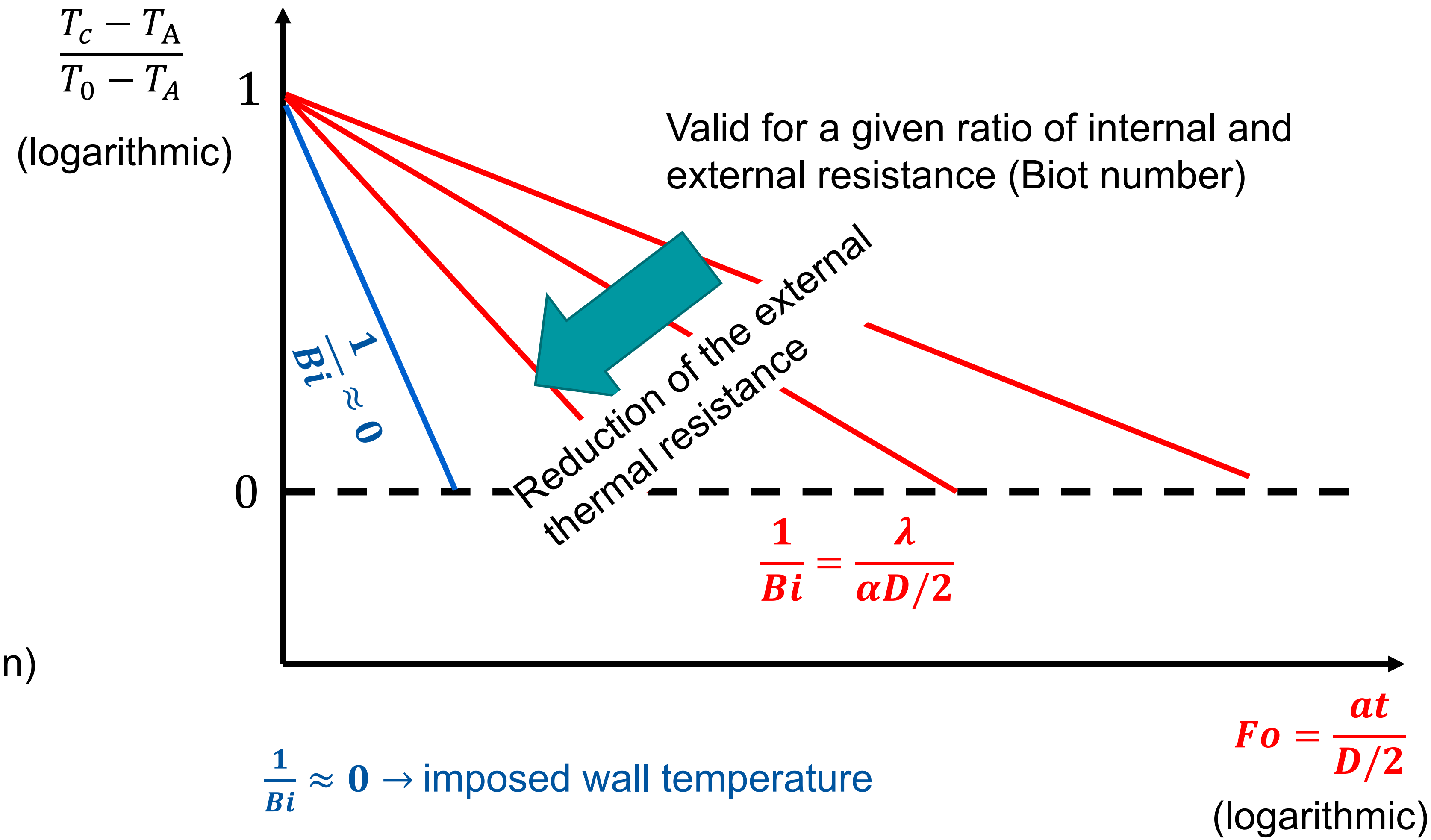

$\frac{1}{Bi} \approx 0$ → imposed wall temperature

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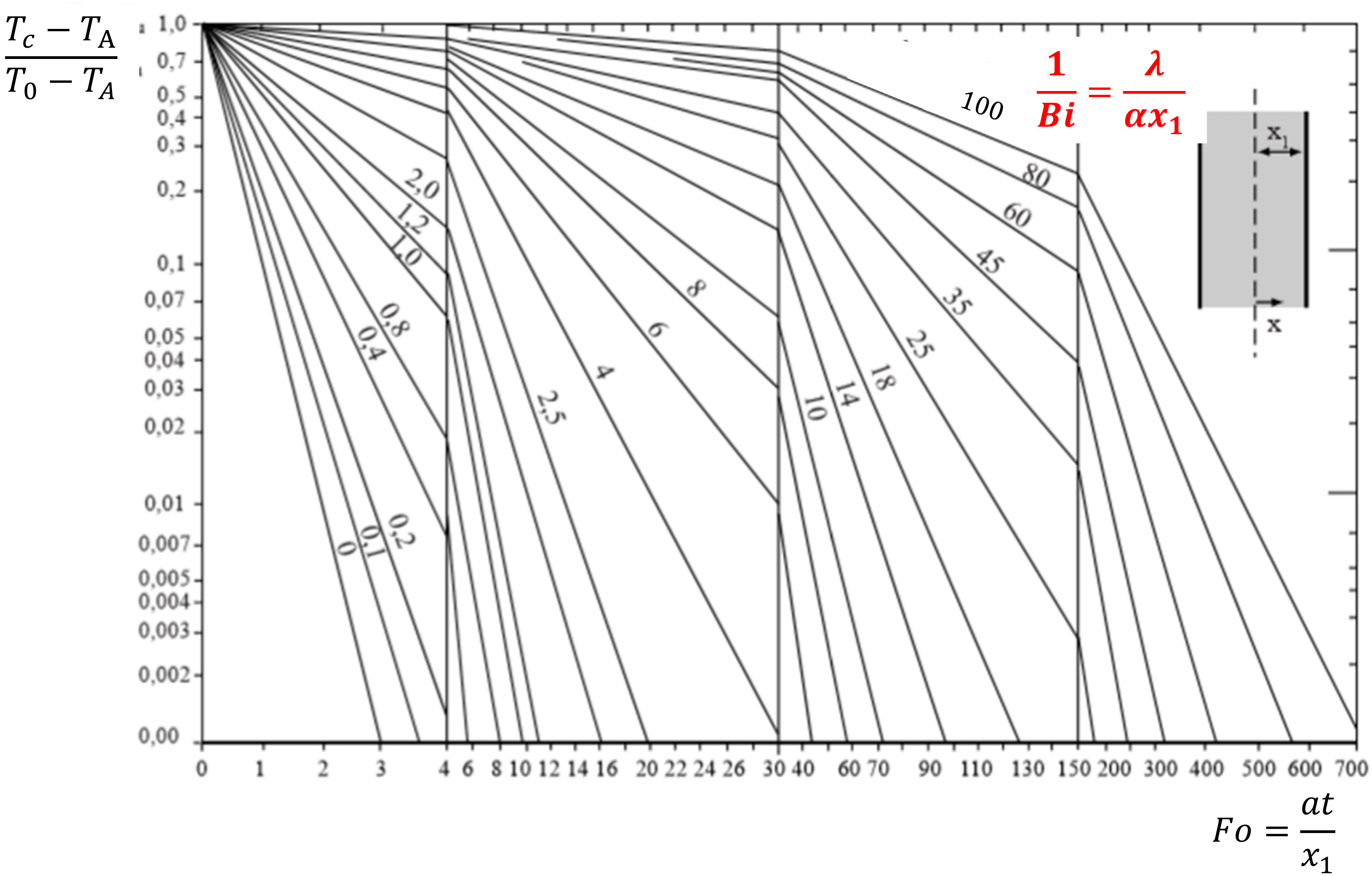
Heisler Diagram: Time evolution of the core body temperature



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Local distribution of body temperature

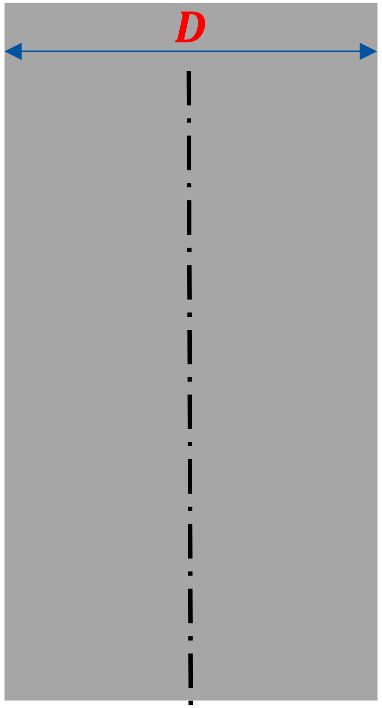
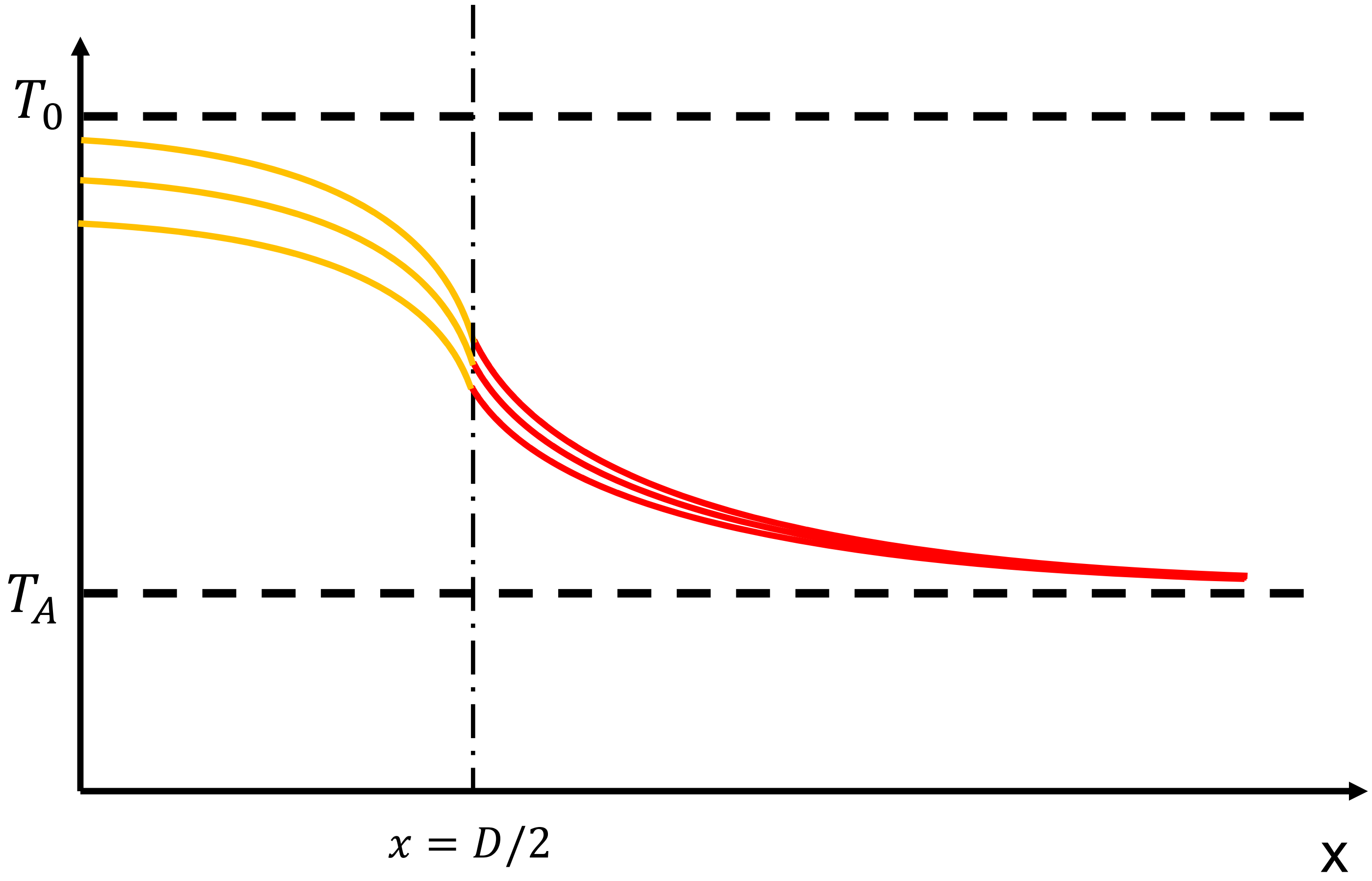


Plate
(infinite expansion)



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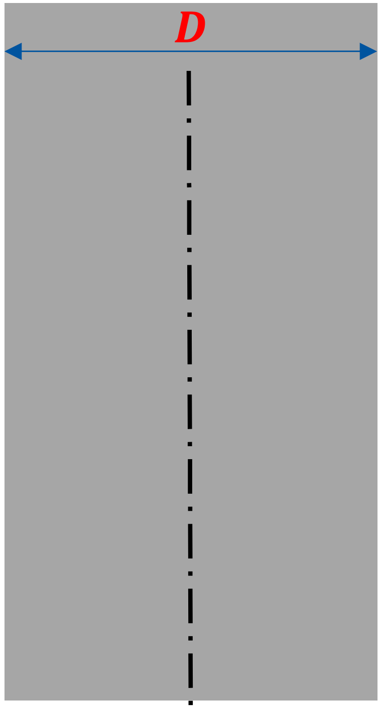
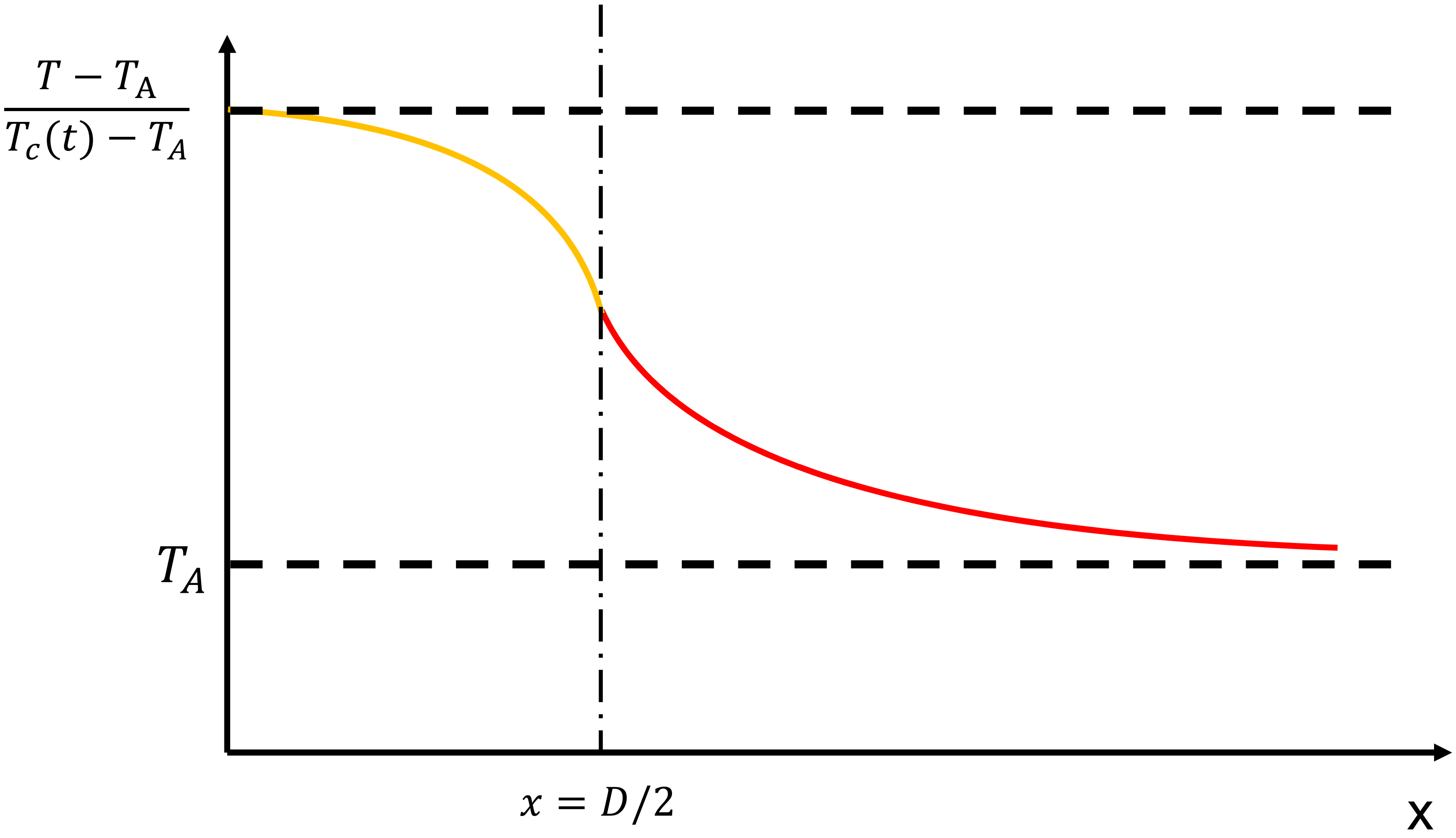
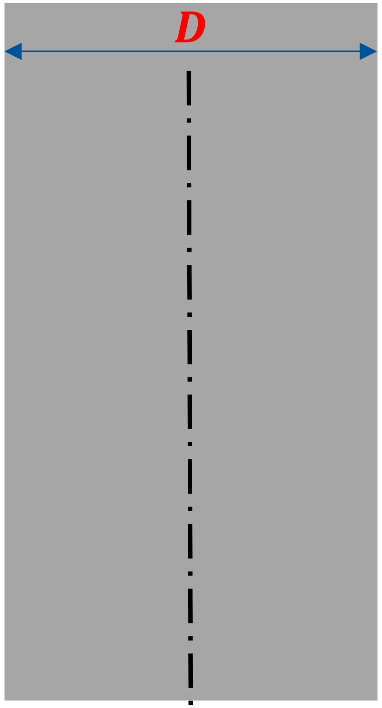


Plate
(infinite expansion)



Local distribution of body temperature



Plate
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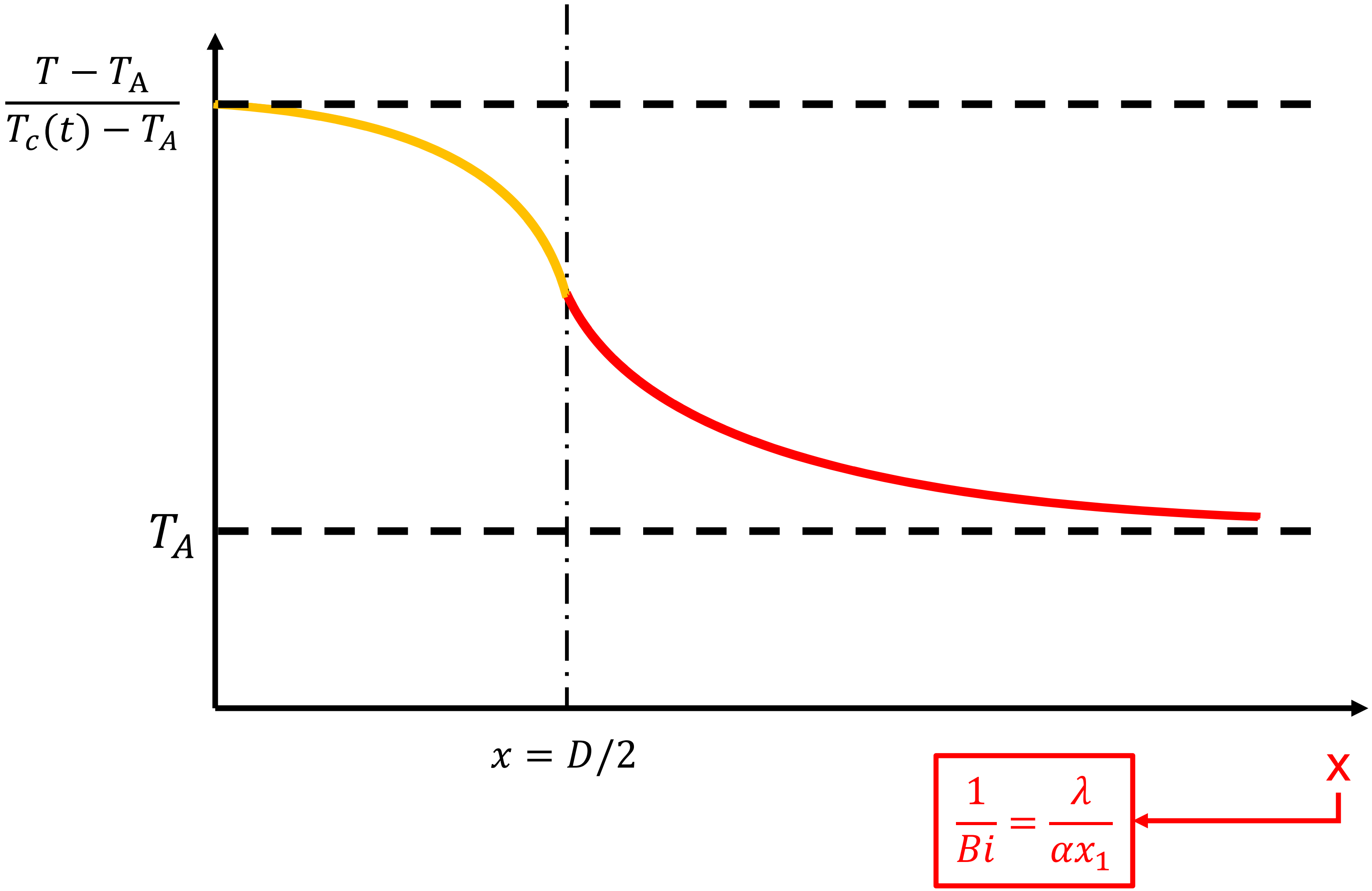
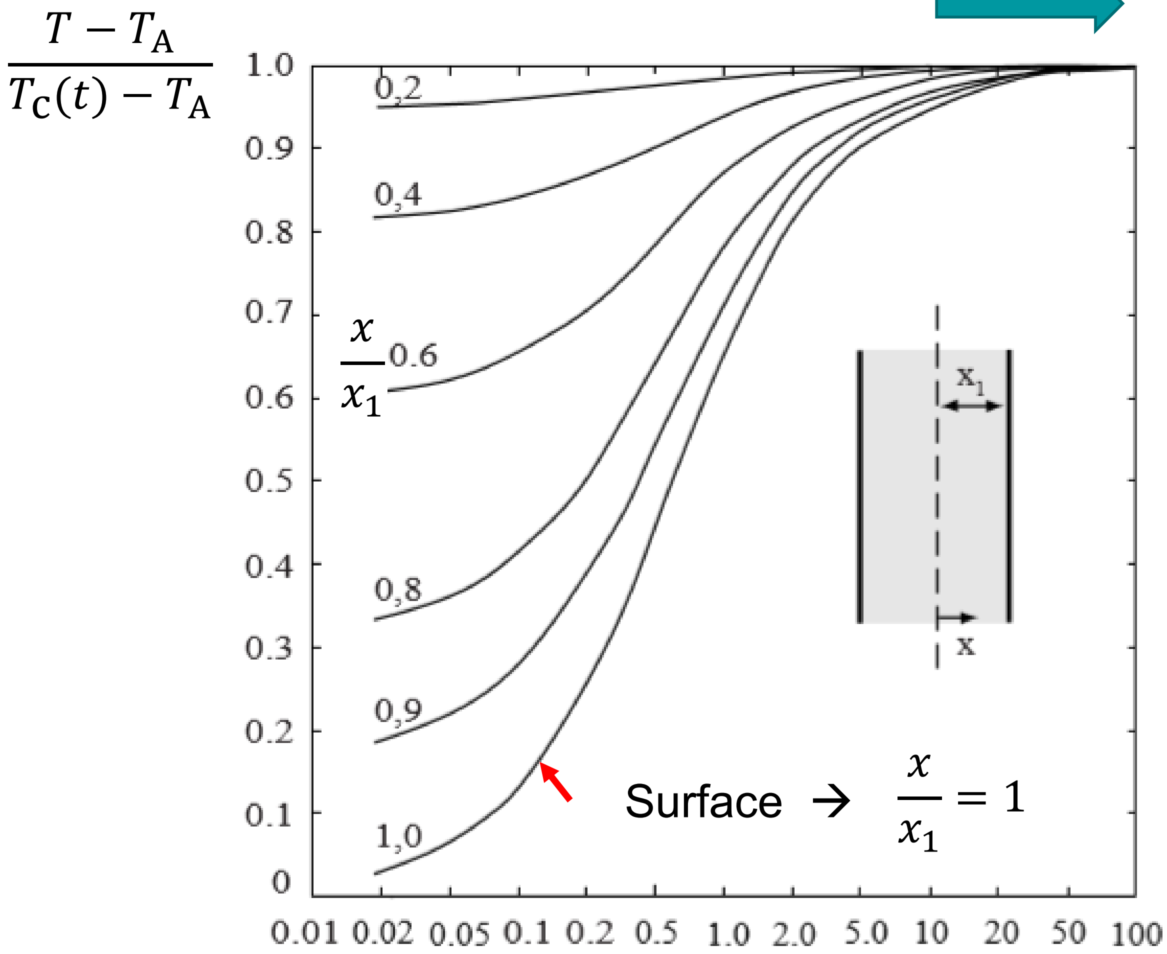


Heisler diagram: Local temperature distribution

Low Bi → homogeneous temperature

Body center: → $\frac{x}{x_1} = 0$



Surface → $\frac{x}{x_1} = 1$

$\frac{1}{Bi} = \frac{\lambda}{\alpha x_1}$ (logarithmic)

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Time evolution of the emitted heat

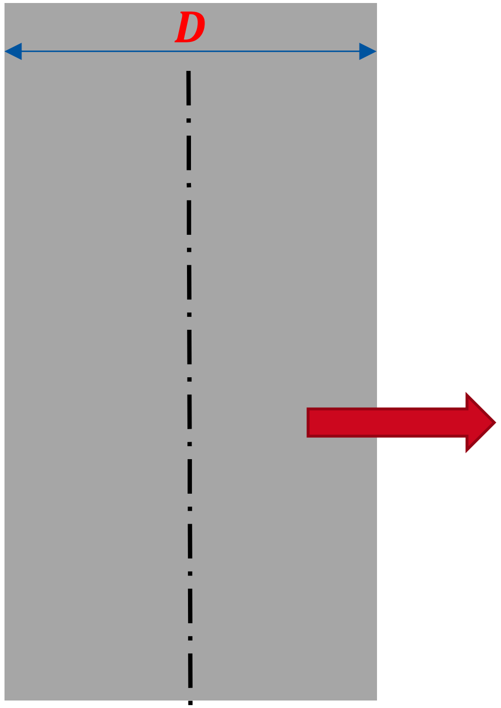
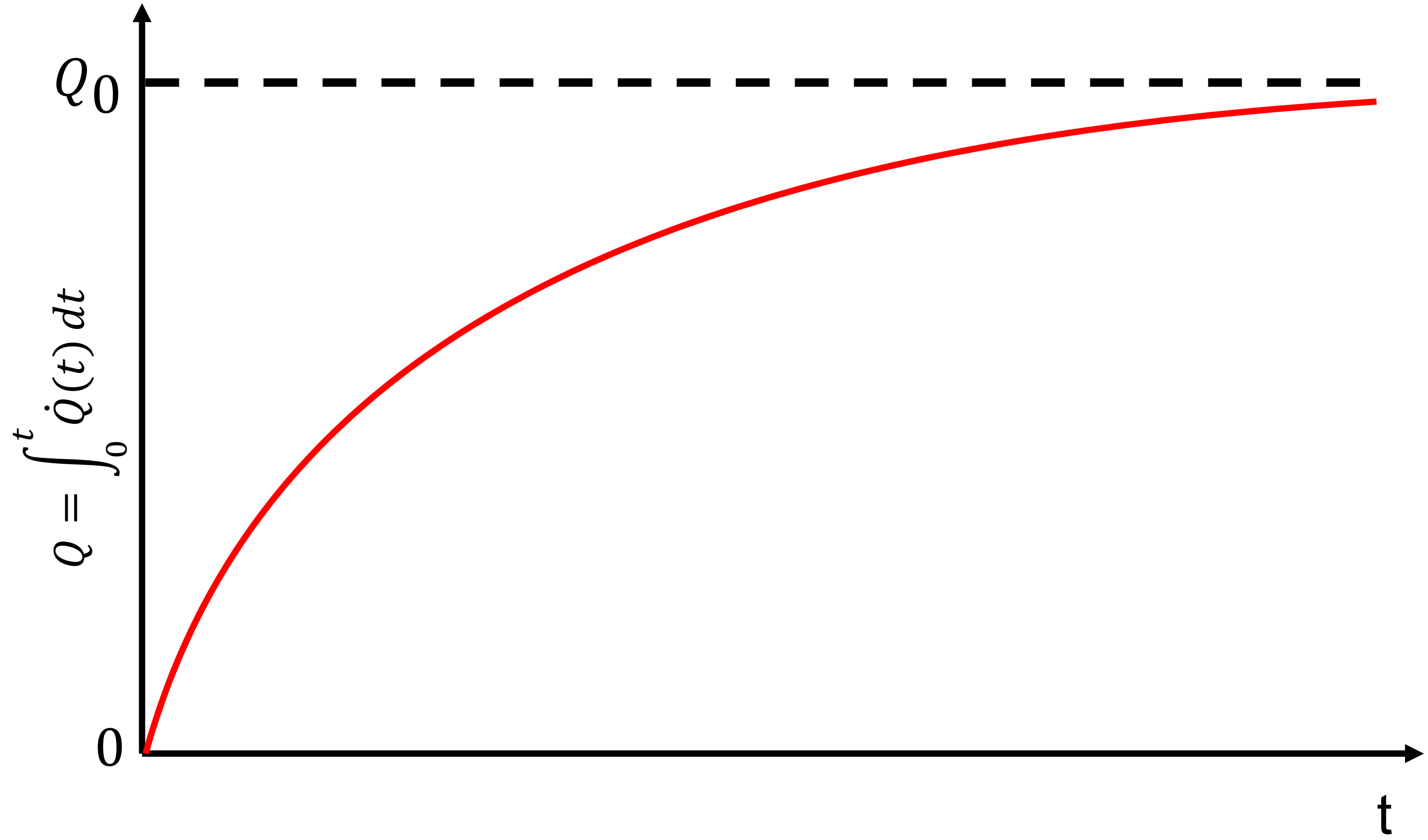
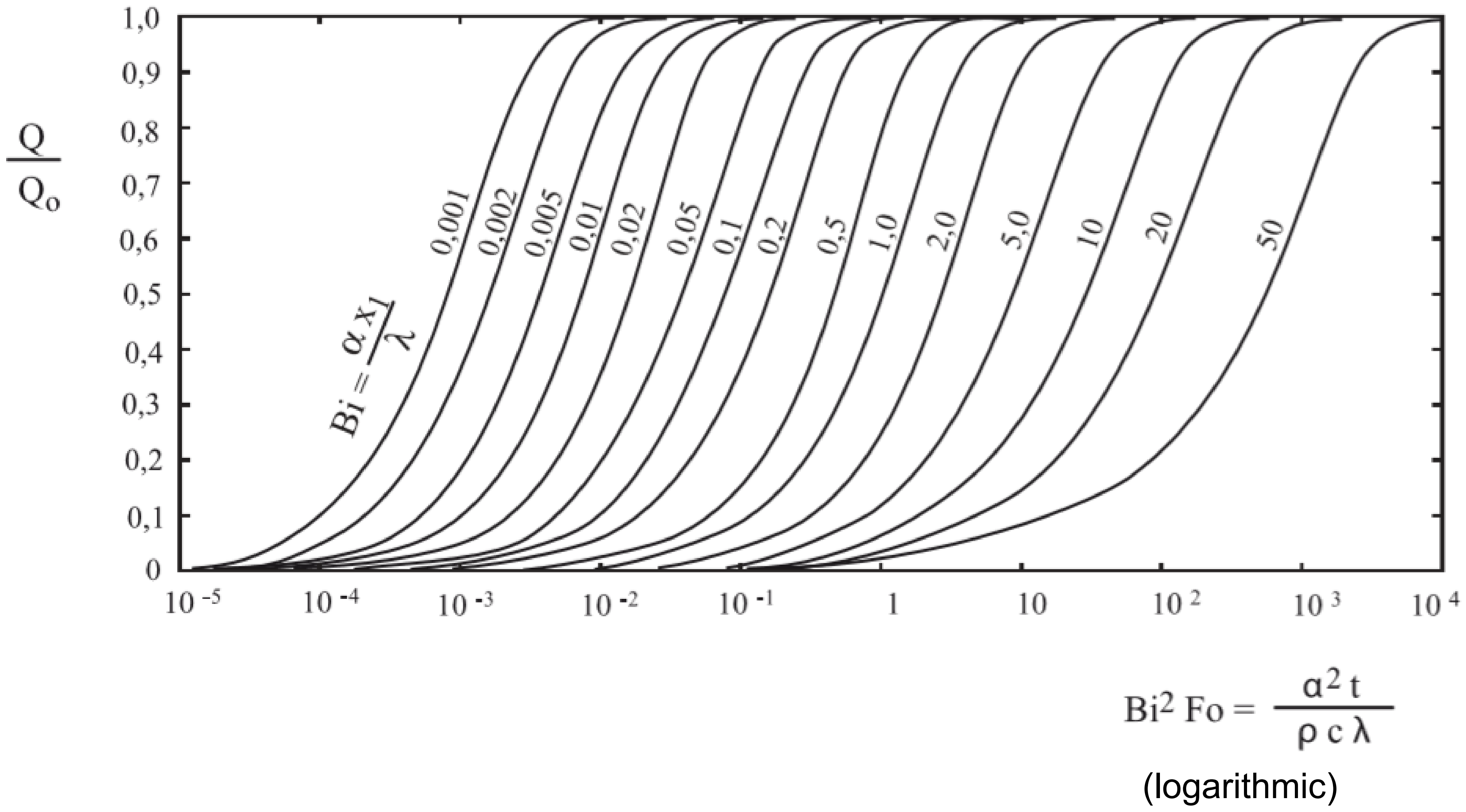


Plate
(infinite expansion)

Total heat stored in the object: $Q_0 = \mathrm{mc_p}(\mathrm{T_0} - \mathrm{T}_A)$



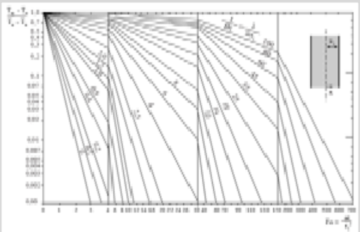
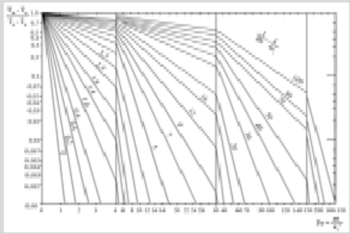
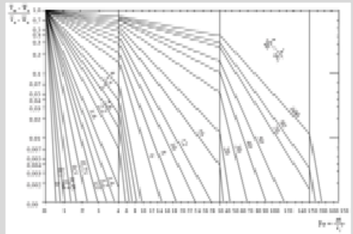
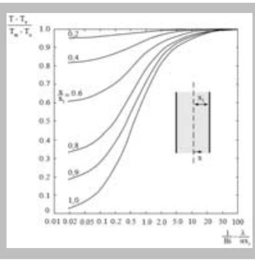
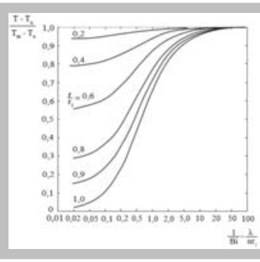
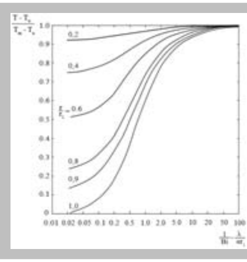
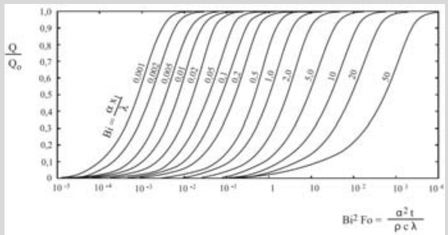
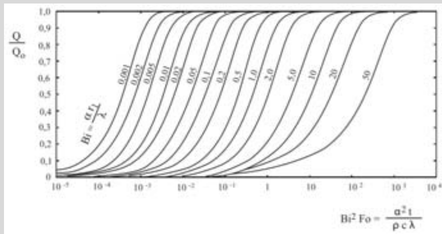
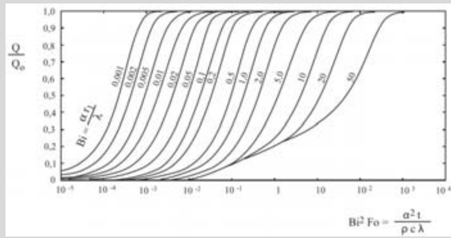
Heisler diagram: Time evolution of the emitted heat



Heisler Diagrams of different symmetrical bodies

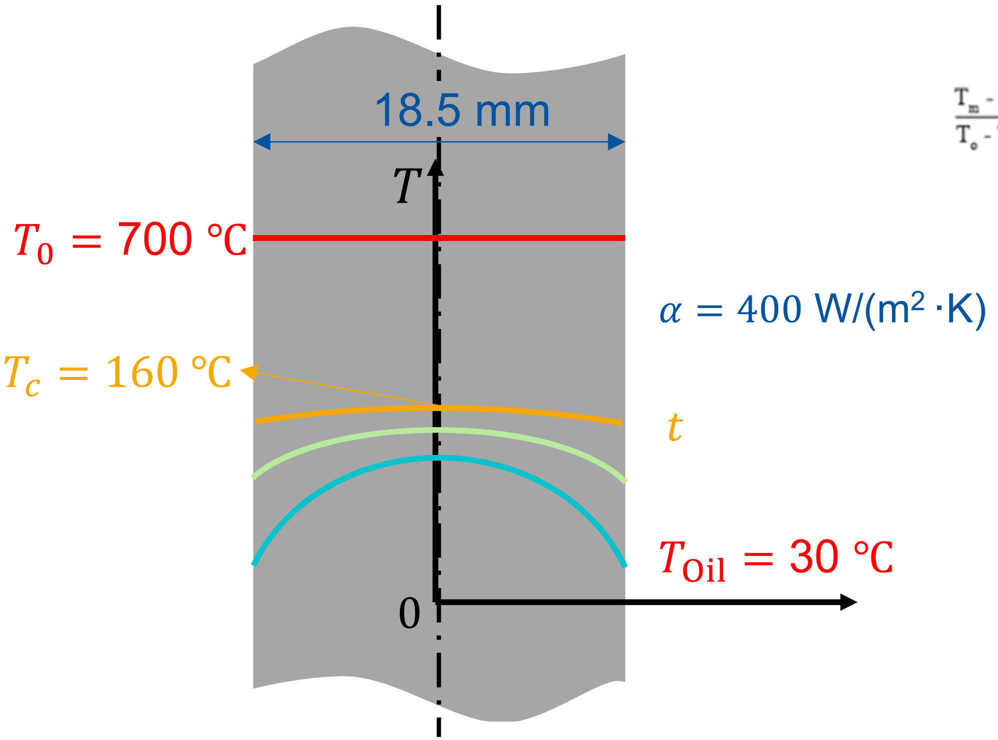
Dimensionless solution

$$\Theta^* = \Theta^*(x^*, y^*, z^*, t^*, Fo, Bi)$$

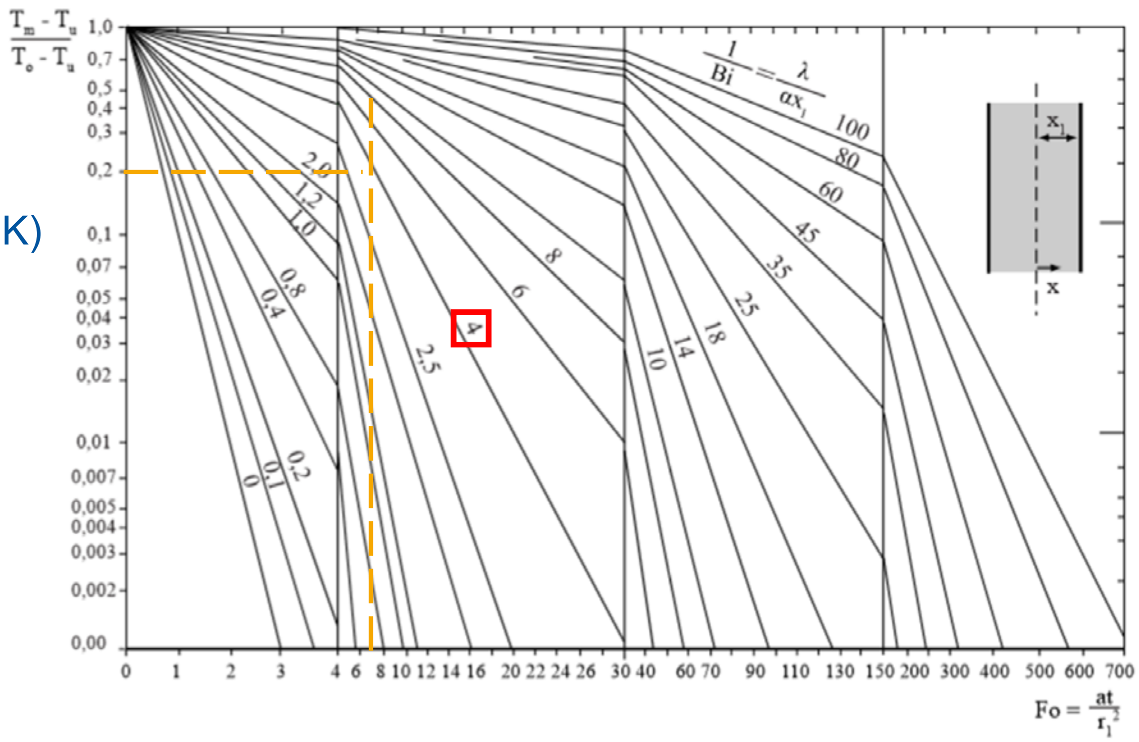
Geometry	Plate	Cylinder	Sphere
Temperature in the center of the object			
Temperature distribution			
Heat flux fraction			

Example: Quenching of a steel plate

Stainless steel　　　Oil



a) After what time has the center temperature T_c cooled down to 160 °C ?

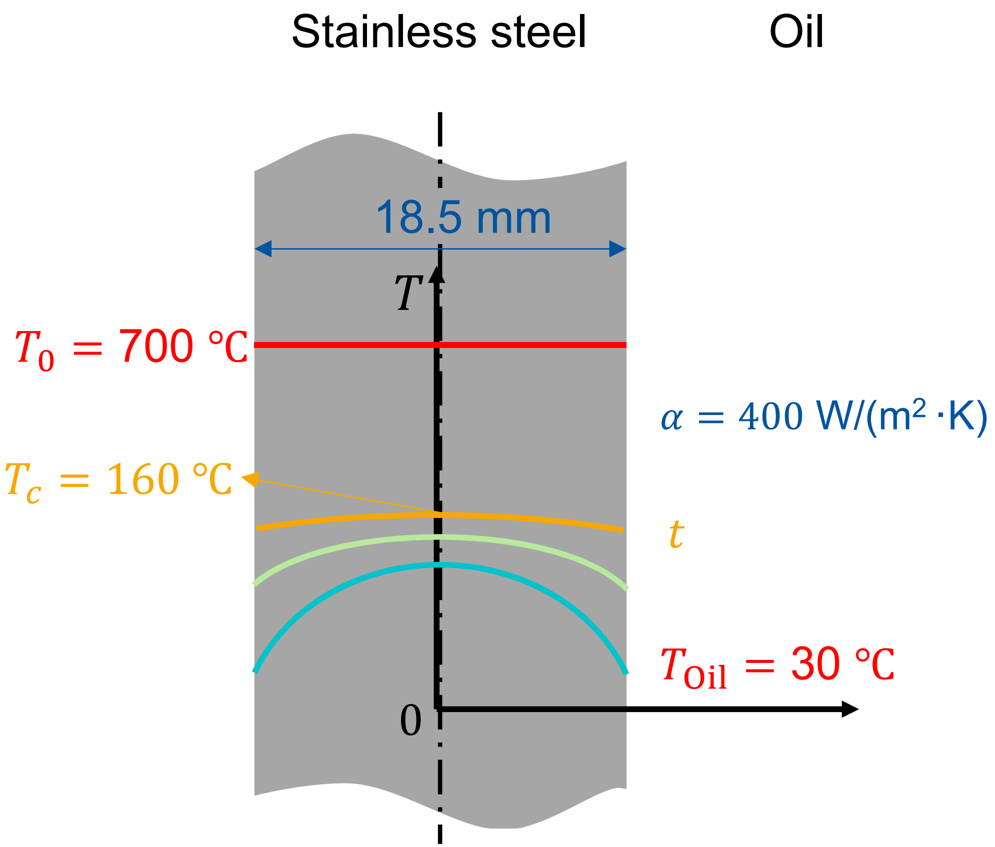


$\lambda = 15$ W/(m·K)
$\rho = 7900$ kg/m³
$c_p = 500$ J/(kg·K)
↓
$a = 3.8 \times 10^{-6}$ m²/s

$$\frac{1}{Bi} = \frac{\lambda}{\alpha x_1} = 4.05$$
$$x_1 = D/2$$

$$\frac{T_c - T_{\mathrm{Oil}}}{T_0 - T_{\mathrm{Oil}}} = 0.19 \Rightarrow Fo = \frac{at}{x_1^2} = 7 \Rightarrow t = 158\mathrm{s}$$

Example: Quenching of a steel plate



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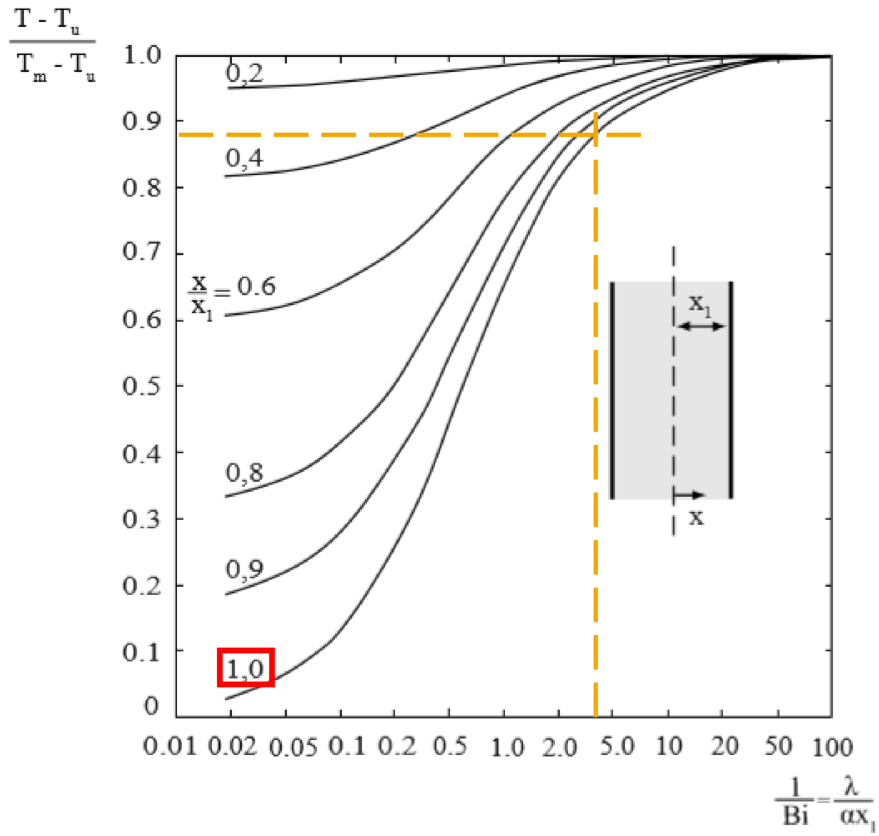
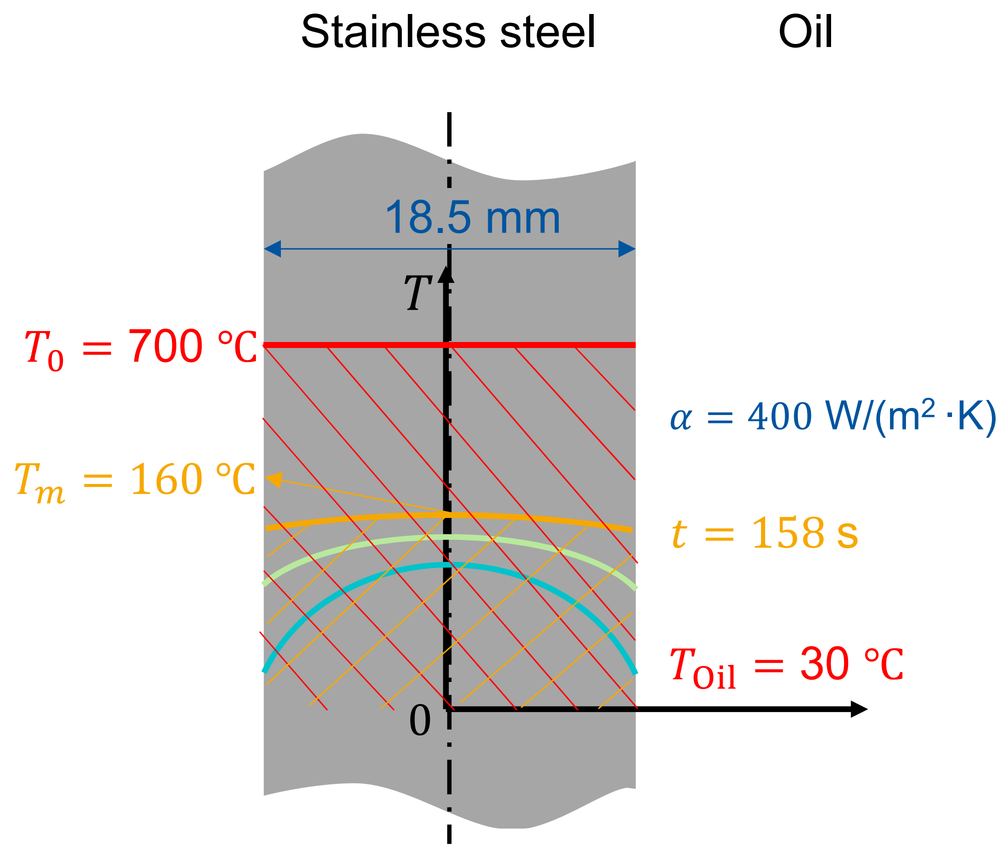
$$\frac{1}{Bi} = \frac{\lambda}{\alpha x_1} = 4.05$$
$$x_1 = D/2$$

b) What is the value of the temperature T at the plate surface after $t = 158\text{s}$?



$$\frac{1}{Bi} = \frac{\lambda}{\alpha x_1} \approx 4 \Rightarrow \frac{T - T_{\text{Oil}}}{T_c - T_{\text{Oil}}} = 0.88 \Rightarrow T = 144\text{°C}$$

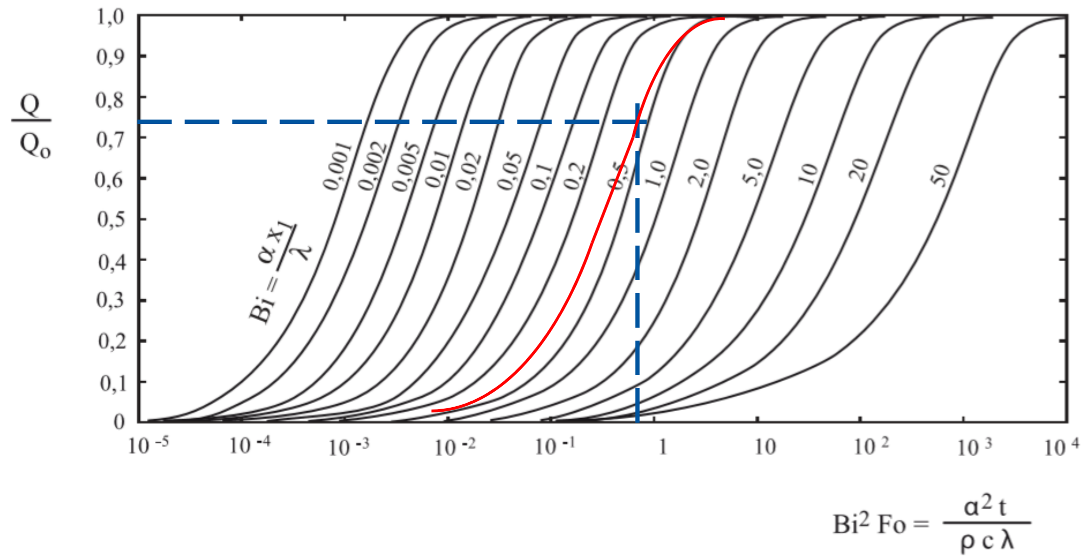
Example: Quenching of a steel plate



c) How much heat has the plate dissipated after $t = 158$ s ?

total heat Q_0 remaining heat Q_t

dissipated Heat $Q = Q_0 - Q_t$



$\lambda = 15$ W/(m·K)
$\rho = 7900$ kg/m³
$c_p = 500$ J/(kg·K)
↓
$a = 3.8 \times 10^{-6}$ m²/s

$Bi = 0.25$
$Fo = 7$

$$Bi^2 Fo = \frac{\alpha^2 t}{\rho c_p \lambda} = 0.43 \;\Rightarrow\; \frac{Q}{Q_0} = 0.74 \;\Rightarrow\; \frac{Q}{m} = 247.9 \frac{\text{kJ}}{\text{kg}}$$

with $Q_0 = m c_p (T_0 - T_{\text{Oil}})$

Comprehension questions

Which two dimensionless numbers are used to describe a transient heat transfer problem of a body with additional external thermal resistance?

Which tool allows the determination of the temperature profile or the amount of heat transferred for extended plates, long cylinders or spheres?