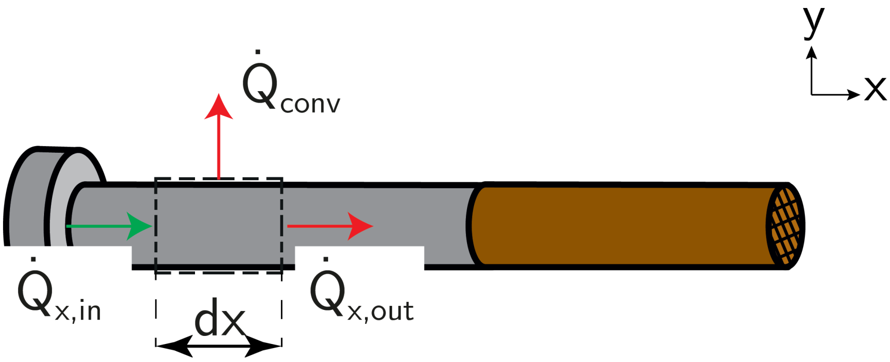
# EB - Cond. - IE 12

**Derive a homogeneous differential equation to describe the axial temperature distribution for the left fin. Assume one-dimensional, steady-state heat transfer in the x-direction with no sources/sinks.**




**Energy balance:**

$$\dot{Q}_{x,in} - \dot{Q}_{x,out} - \dot{Q}_{conv}(x) = 0$$

Since the heat transfer is characterized as steady-state, the sum of the in- and outgoing heat fluxes for the control volume should equal zero.

**Homogenization:**

$$\Theta \ = \ T(x) \ - \ T_{\infty}$$

$$m^2 = \ \tfrac{2 \cdot \alpha}{\lambda \cdot R}$$

**Heat fluxes:**

$$\dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \tfrac{\partial T}{\partial x} \ \rightarrow \dot{Q}_{x,in} = -\lambda \cdot \pi R^2 \cdot \tfrac{\partial \Theta}{\partial x}$$

$$\dot{Q}_{x,out} \ = \ -\lambda \cdot \pi R^2 \cdot \tfrac{\partial T}{\partial x} + \tfrac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx \ \rightarrow \dot{Q}_{x,out} \ = \dot{Q}_{x,in} - \lambda \cdot \pi R^2 \cdot \tfrac{\partial^2 \Theta}{\partial x^2} dx$$

$$\dot{Q}_{conv} \ = \ \alpha \cdot 2\pi R dx \cdot (T(x) - T_{\infty}) \rightarrow \dot{Q}_{conv} \ = \alpha \cdot 2\pi R dx \cdot \Theta$$

**Boundary Conditions:**

$$\Theta(x=0) \ = T_B - \ T_{\infty}$$

$$\tfrac{\partial \Theta}{\partial x}\,(x = L) = -\tfrac{\dot{q}'' 2L}{\lambda R}$$

The first boundary condition results from the fact that $T(x=0) = T_{\mathrm{B}}$ and the second one from the fact that $Q_{\mathrm{cond}}(x=L) = \dot{q}'' 2\pi RL$ .

$$\frac{\partial^2 \Theta}{\partial x^2} \ - m^2 \Theta \ = 0$$