Heat Transfer

Turbulent Flow

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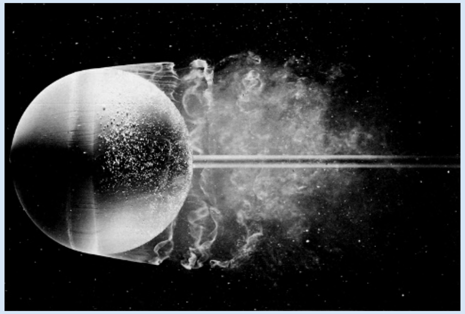
Learning Goals

- Turbulent Flow
  - ➢ Occurrence of turbulent flow
  - ➢ Understanding the macroscopic effect of turbulent fluctuations on mass and heat transport



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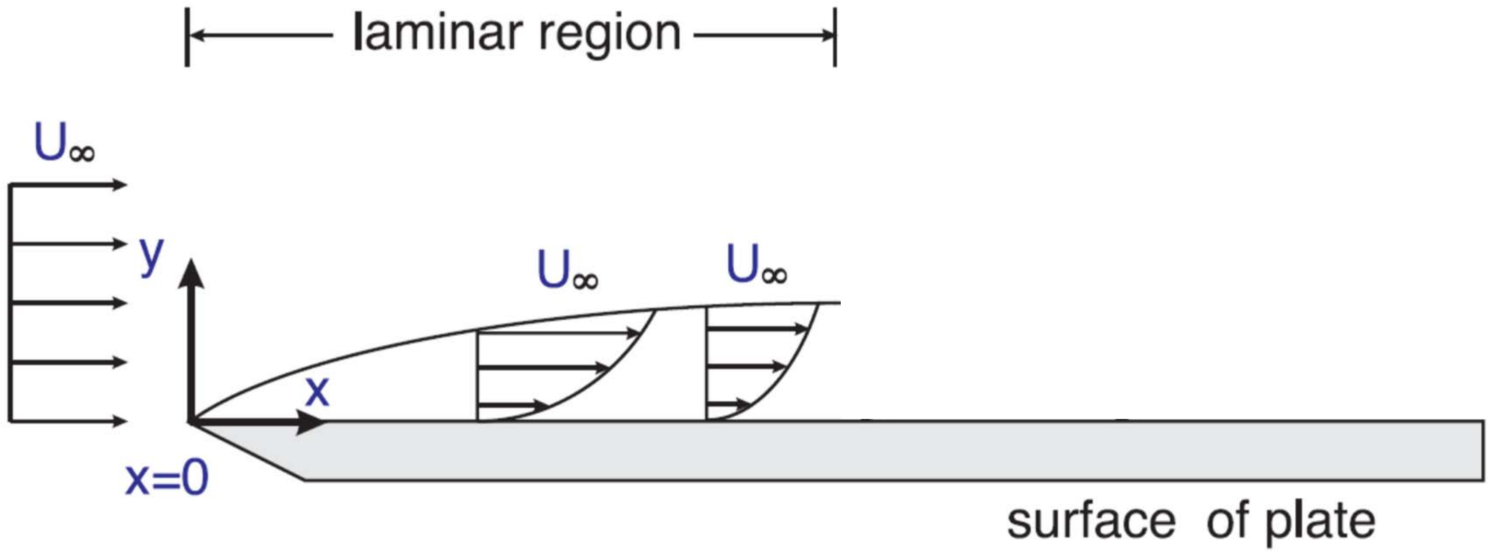
Turbulent Flows

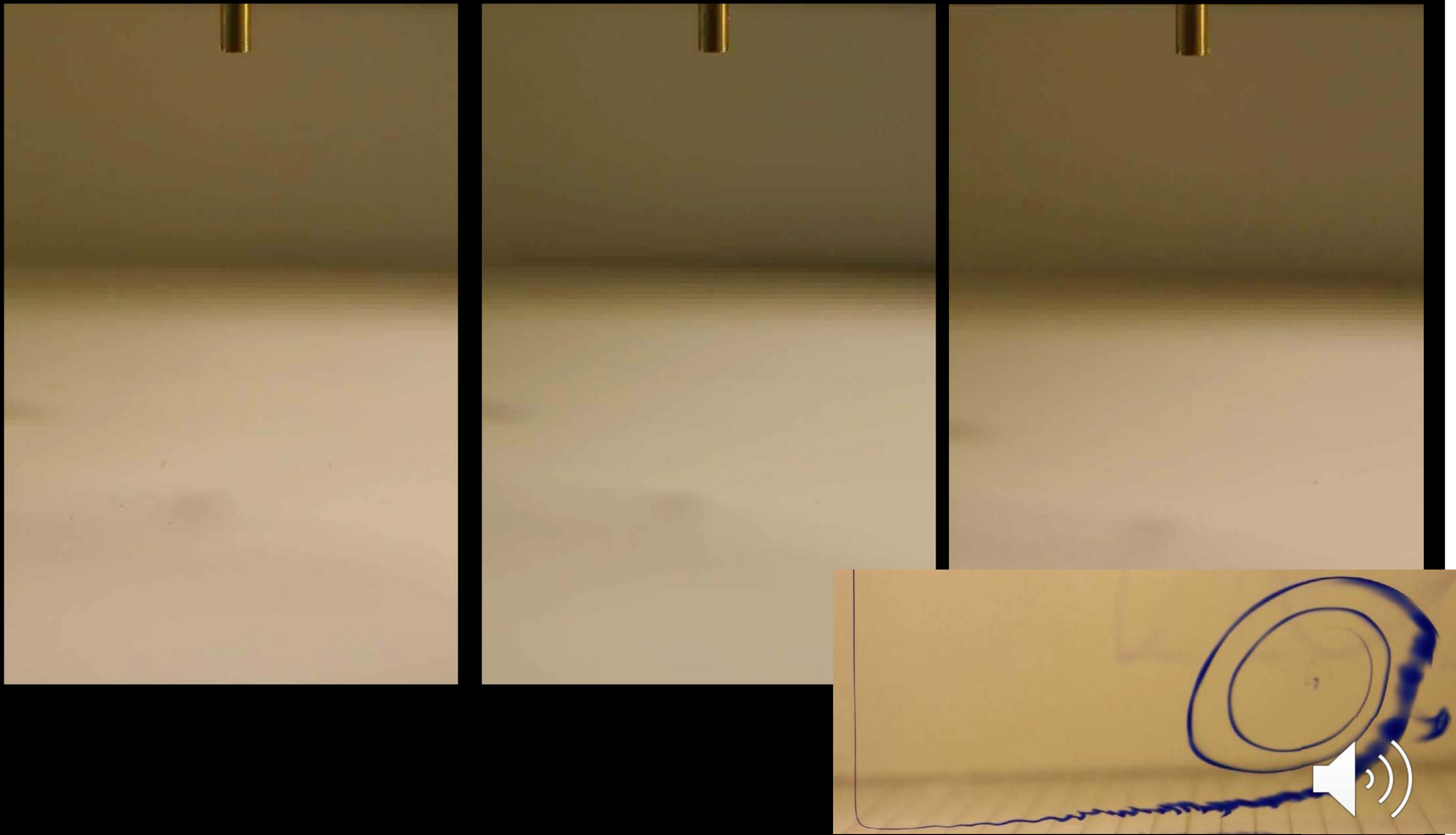
Is the flow along a plate always laminar?

$$Re = \frac{\rho u_\infty x}{\eta} = \frac{\text{Inertia Forces}}{\text{Friction Forces}} \Rightarrow x\uparrow\ Re\uparrow$$

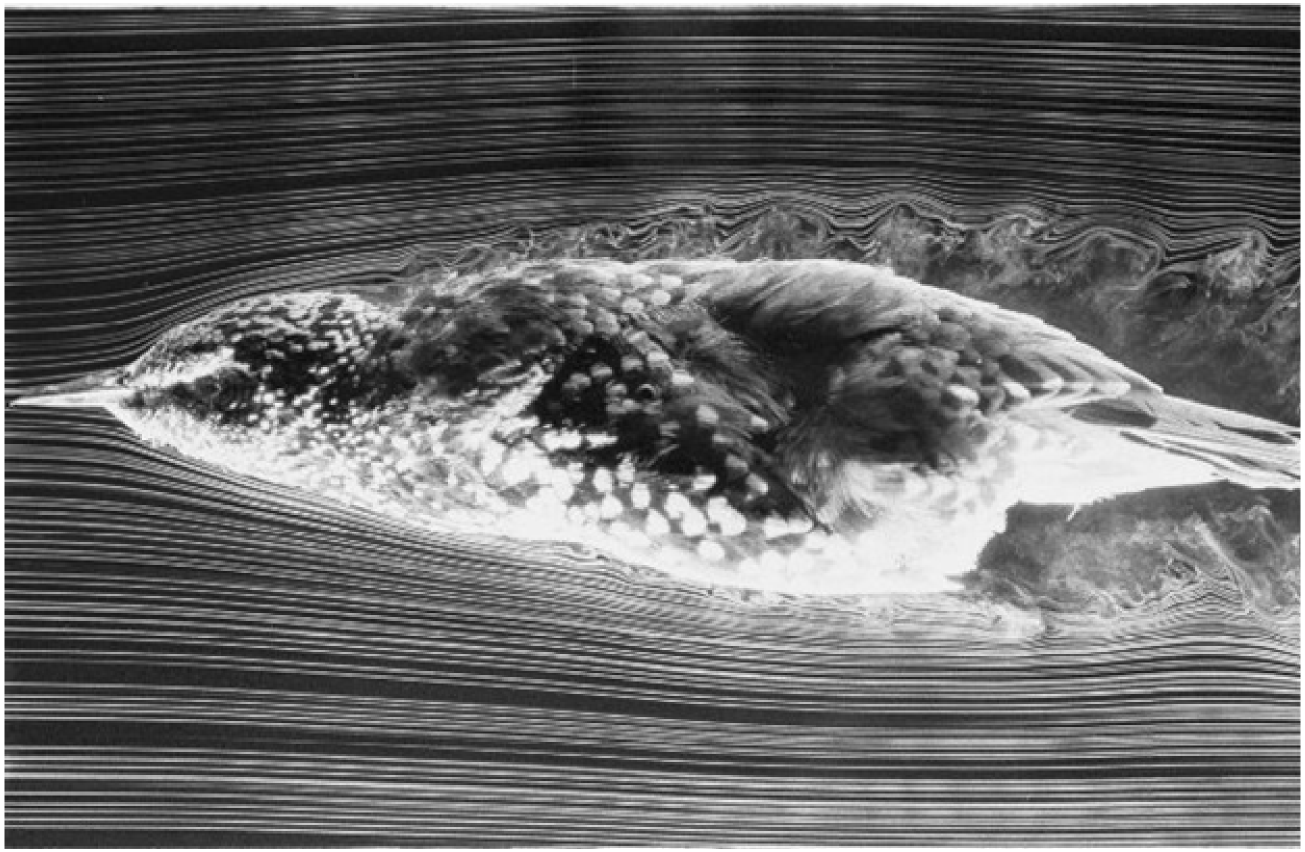
$\Rightarrow$ Inertial forces (Momentum Flow) can no longer be stabilized by viscous forces.



Re = 2500 Re = 1600 Re = 800

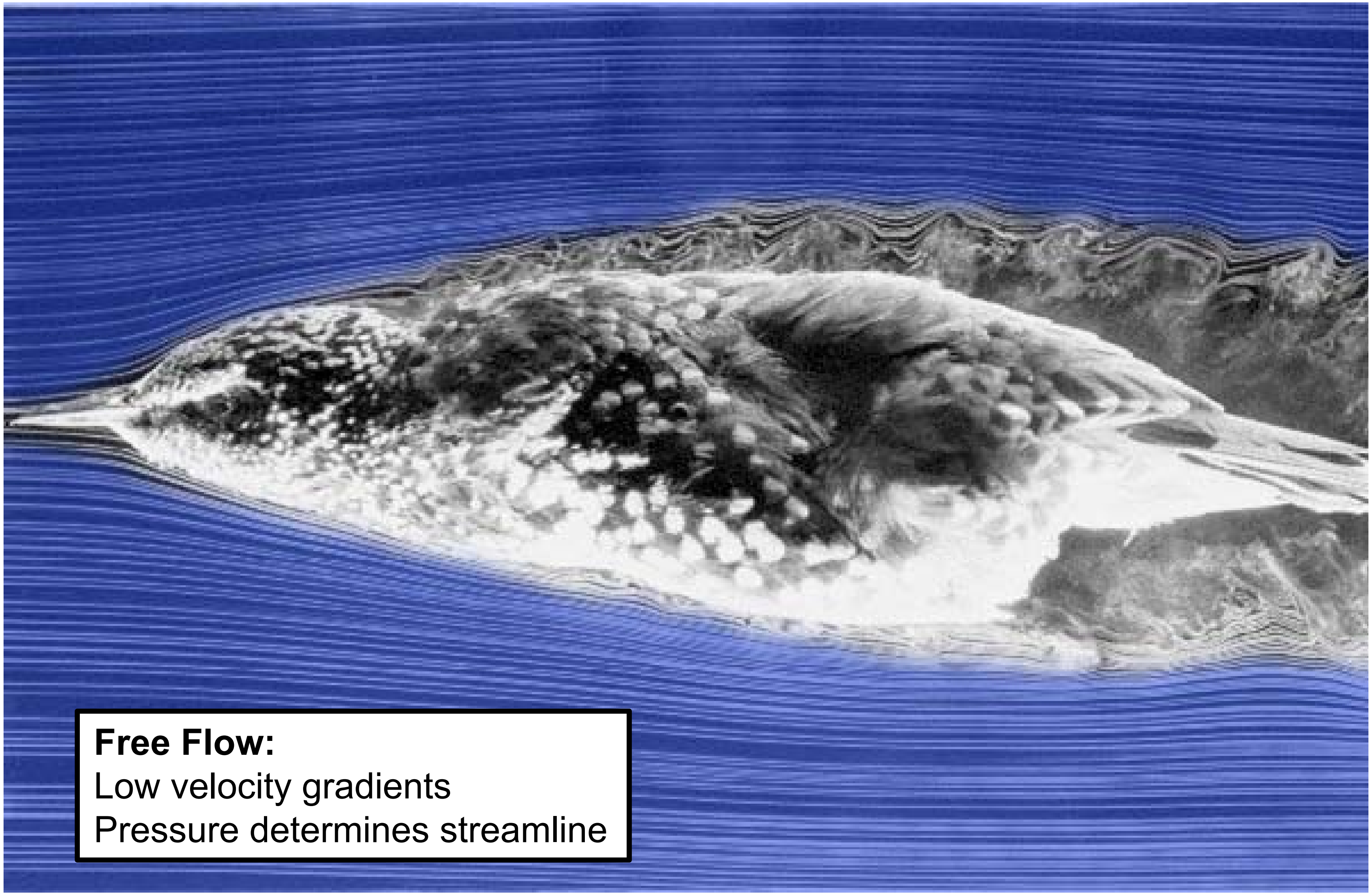


Turbulent Flows



Source: University of Iowa

Turbulent Flows



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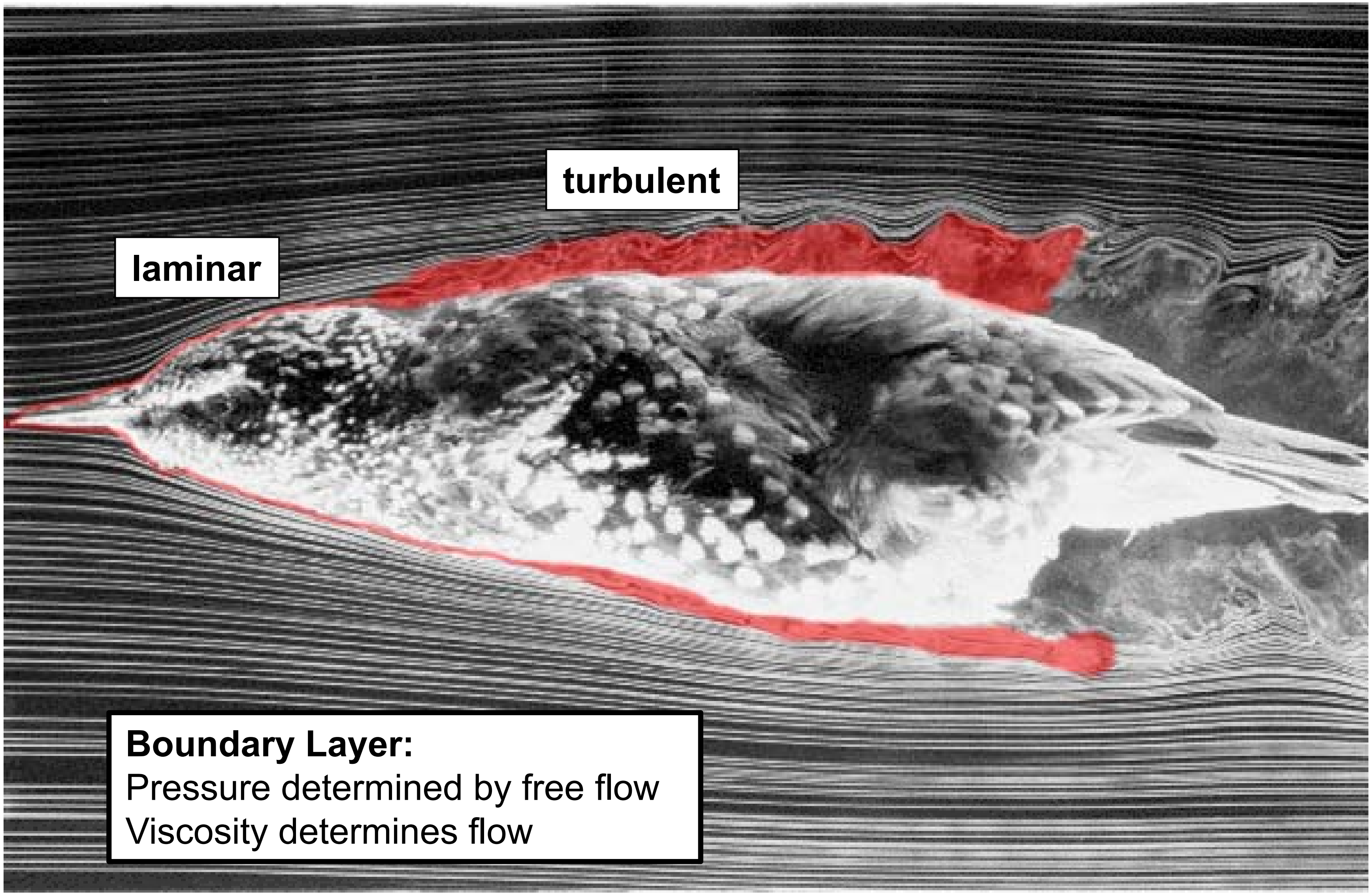
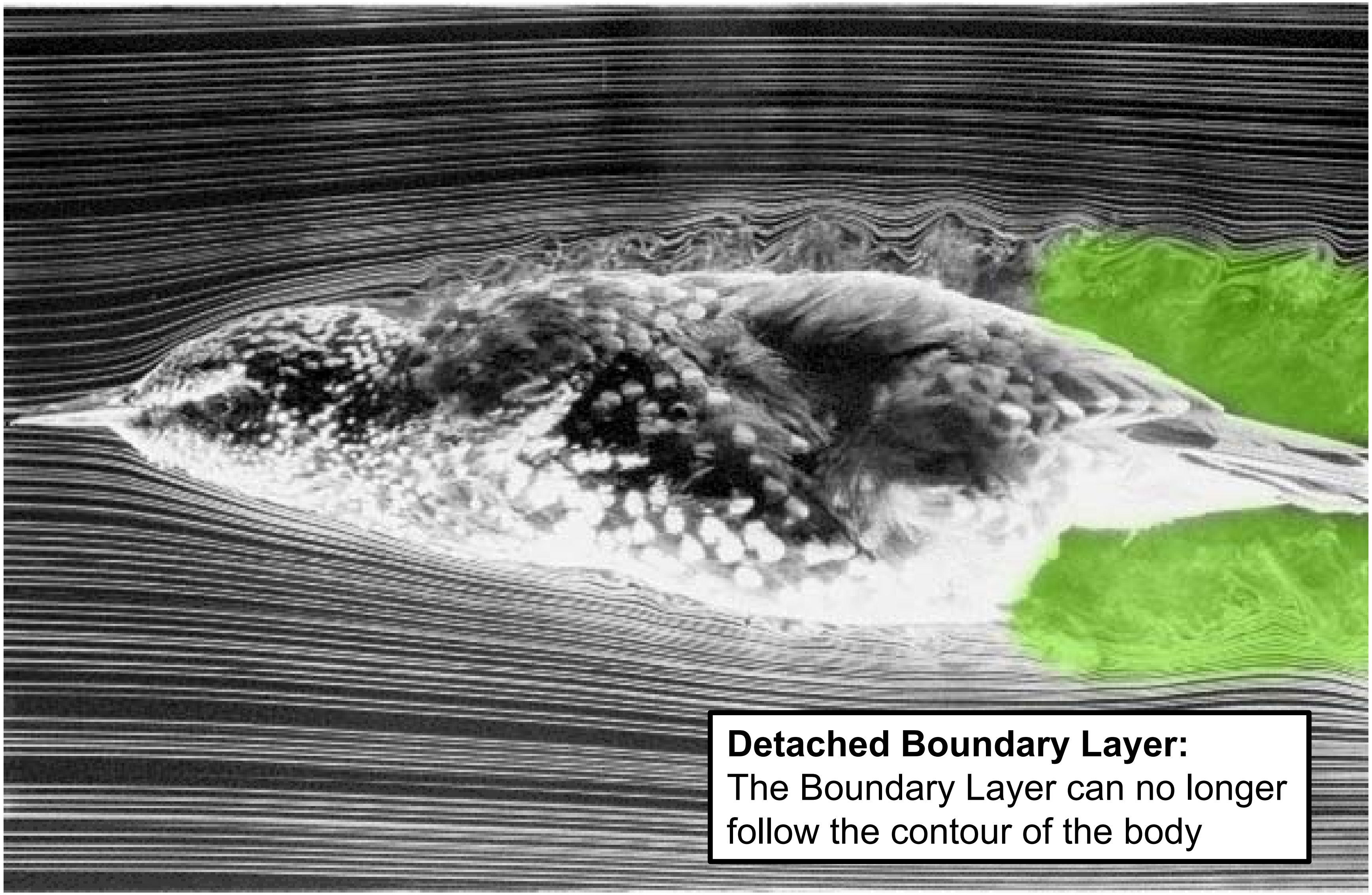
Turbulent Flows

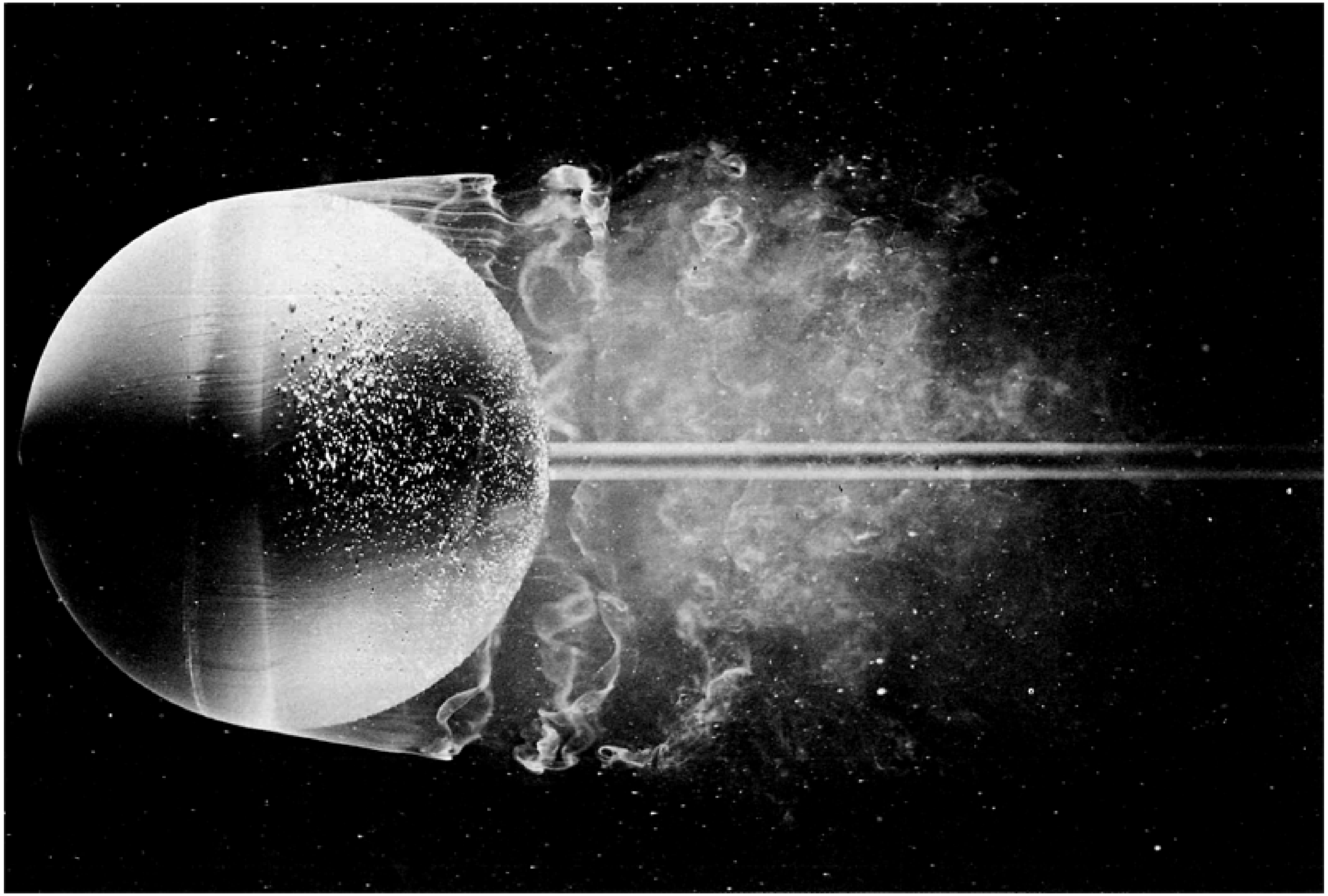


Turbulent Flows



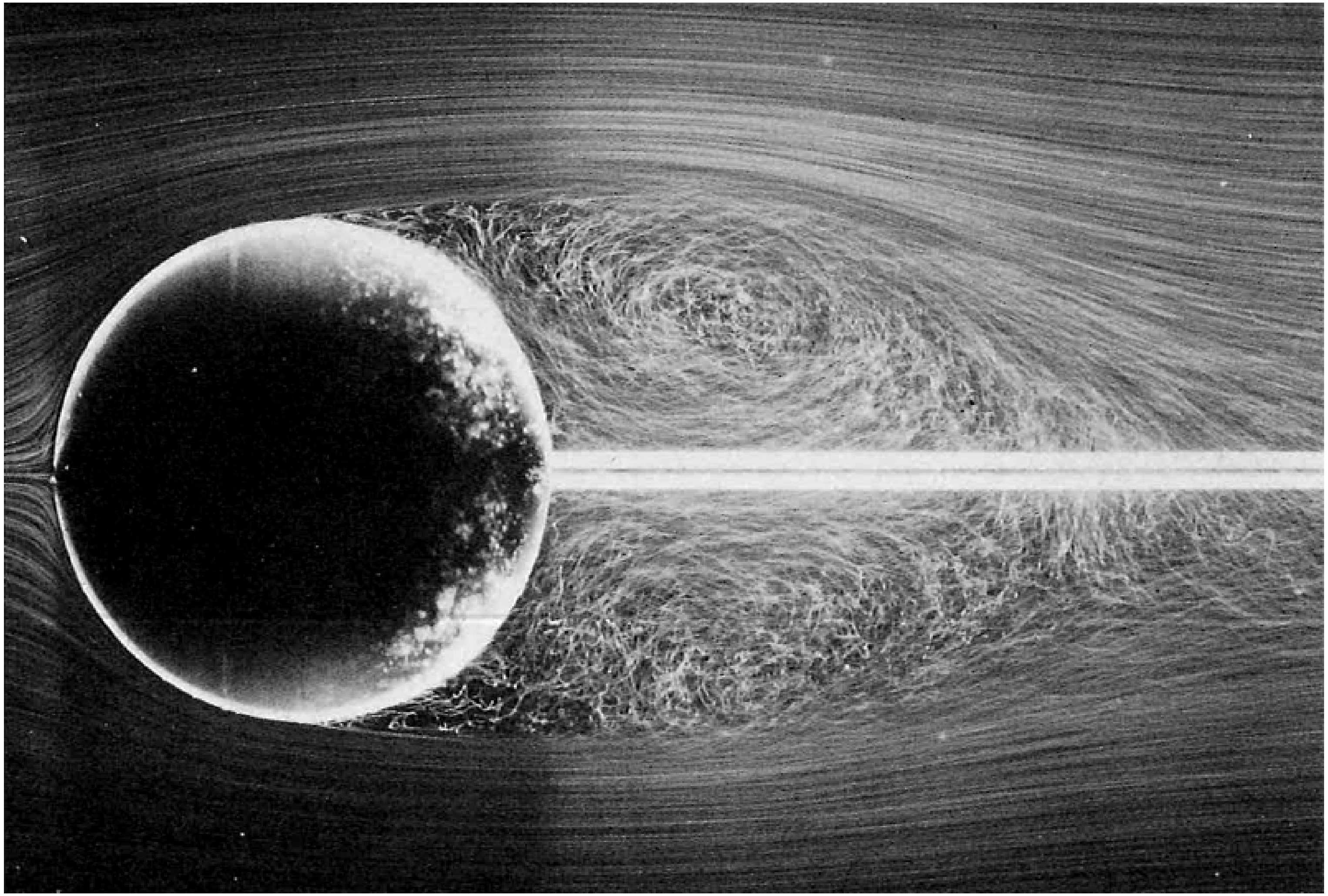
Detached Boundary Layer:
The Boundary Layer can no longer follow the contour of the body

Turbulent Flows



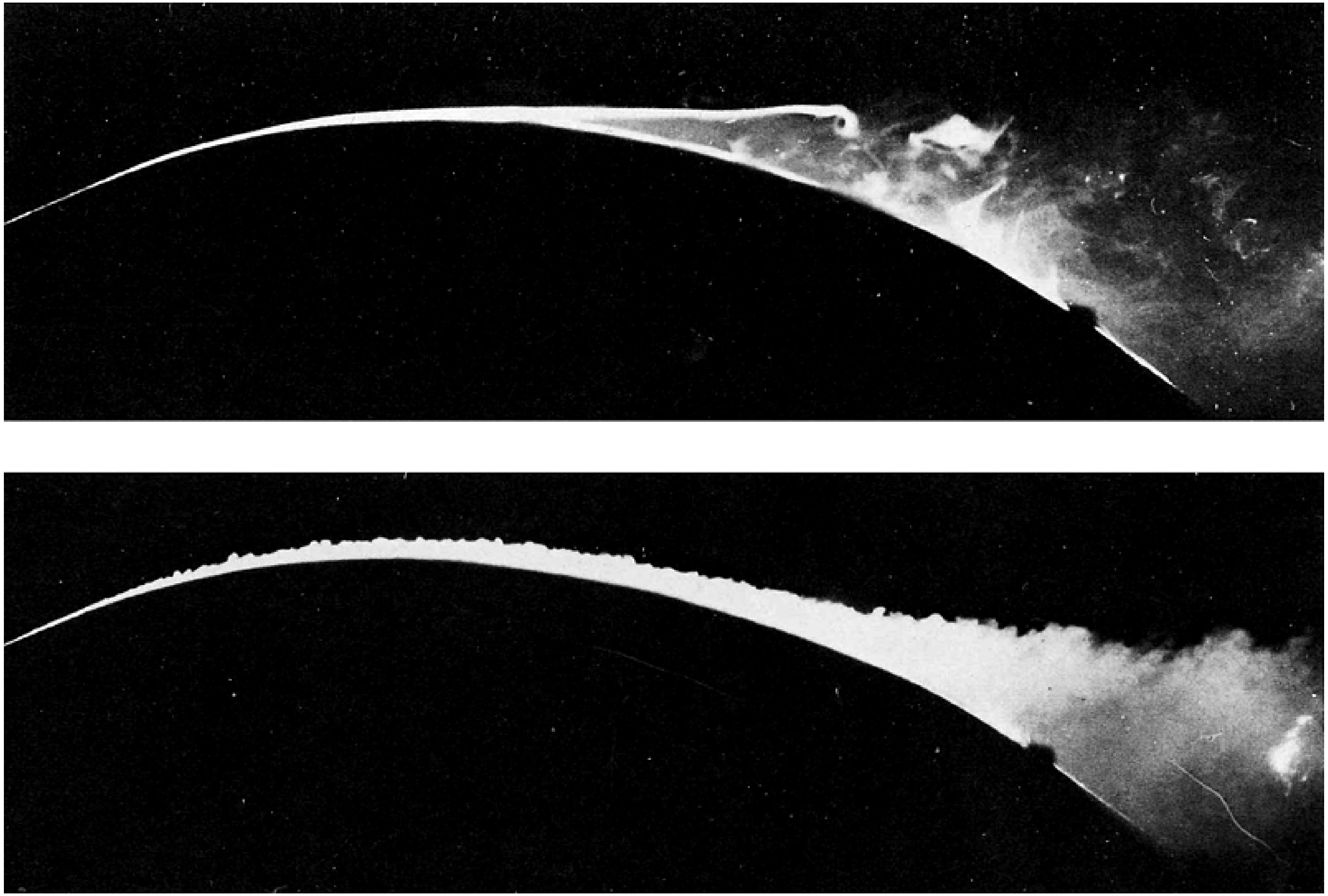
Source: Van Dyke, Handbook of Fluid Motion

Turbulent Flows



Source: Van Dyke, Handbook of Fluid Motion

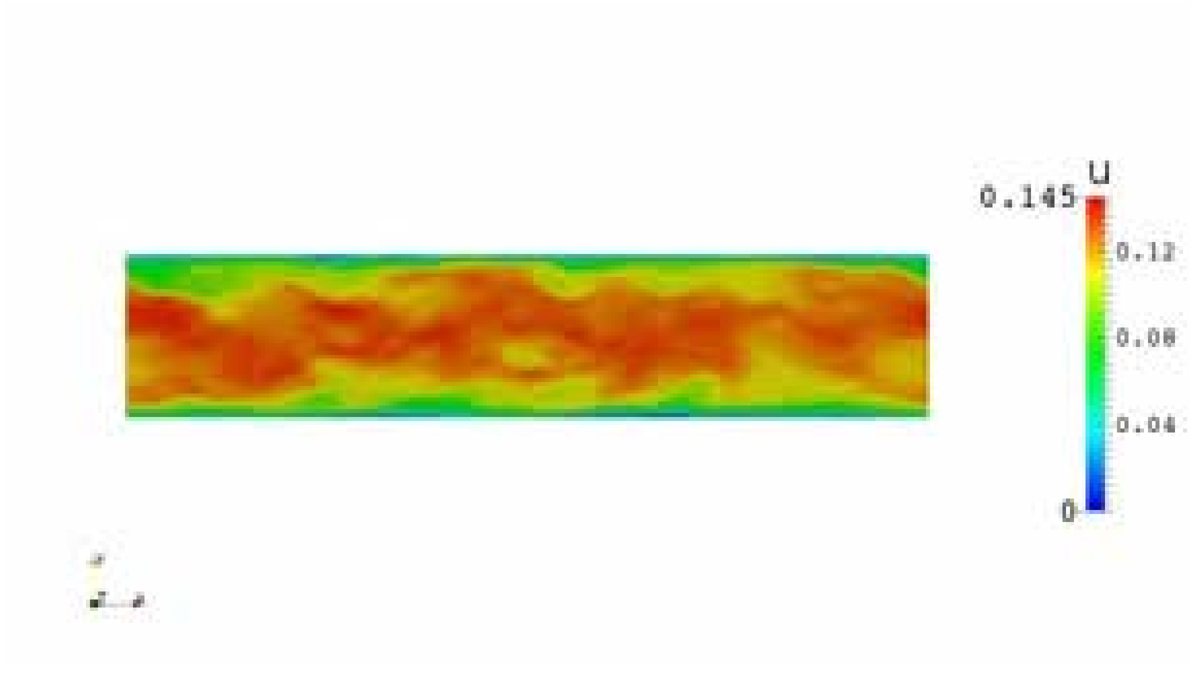
Turbulent Flows



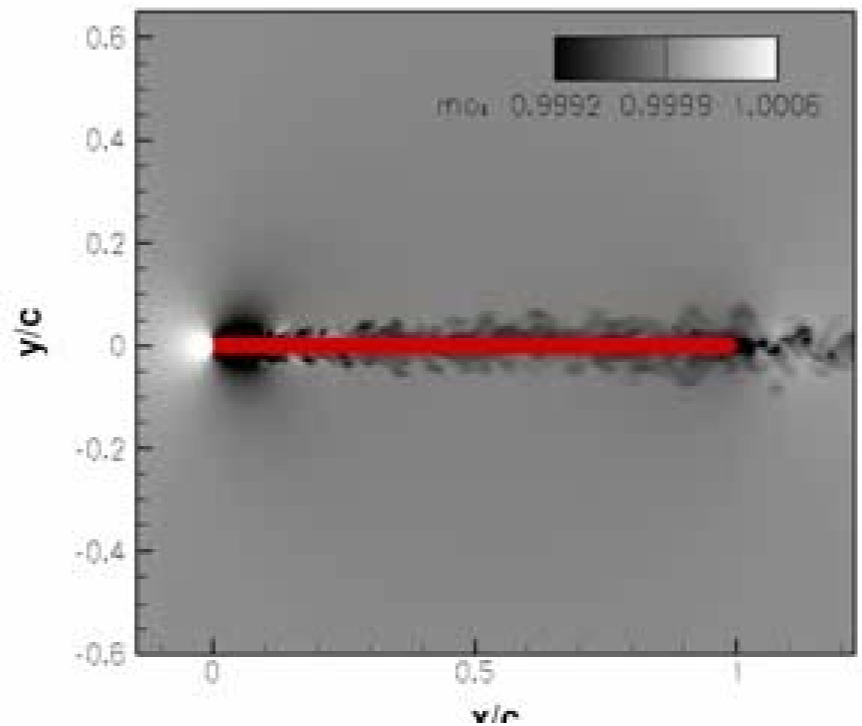
Source: Van Dyke, Handbook of Fluid Motion

Turbulent Flows

Internal Flow



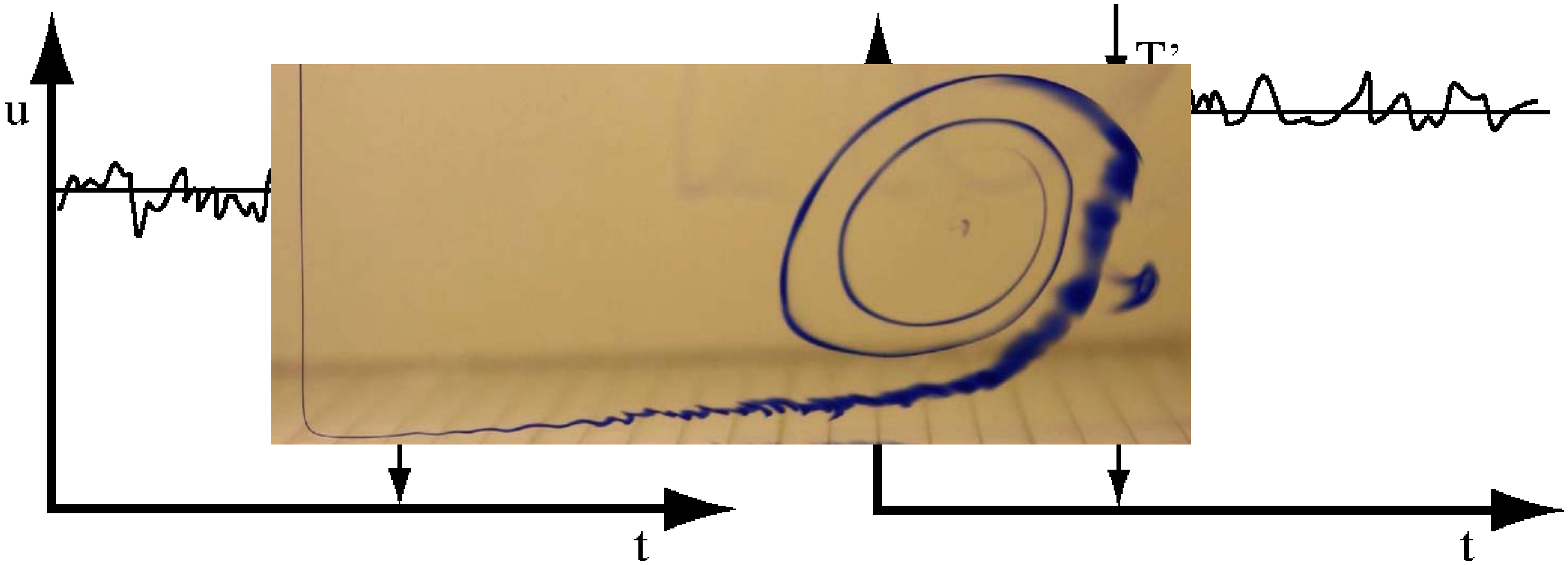
External Flow



Source: AIA, RWTH Aachen University

Turbulent Flows

- **Cause**
  - Inertial Forces (Momentum Flows) can no longer be stabilized by viscous forces = high Reynolds number
- **Effect**
  - Turbulent Fluctuations overlap the averaged flow velocity
  - Additional exchange mechanisms



$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$

$$T = \bar{T} + T'$$
$$p = \bar{p} + p'$$

Review: Conservation Equations (2D, steady state, incompressible)

Continuity equation

Mass Flows

$$\frac{\partial \overline{\bar{u} + u'}}{\partial x} + \frac{\partial \overline{\bar{v} + v'}}{\partial y} = 0$$

Eliminate

$\overline{u'} = 0 \qquad \overline{v'} = 0 \qquad \overline{p'} = 0 \qquad \overline{T'} = 0$

Momentum equation

Momentum Flows | Pressure | Shear stresses

$$\overline{\bar{u} + u'}\,\frac{\partial \overline{\bar{u} + u'}}{\partial x} + \overline{\bar{v} + v'}\,\frac{\partial \overline{\bar{u} + u'}}{\partial y} = -\frac{1}{\rho}\frac{\partial \overline{\bar{p} + p'}}{\partial x} + \nu\left(\frac{\partial^2 \overline{\bar{u} + u'}}{\partial x^2} + \frac{\partial^2 \overline{\bar{u} + u'}}{\partial y^2}\right)$$

$$\overline{\bar{u} + u'}\,\frac{\partial \overline{\bar{v} + v'}}{\partial x} + \overline{\bar{v} + v'}\,\frac{\partial \overline{\bar{v} + v'}}{\partial y} = -\frac{1}{\rho}\frac{\partial \overline{\bar{p} + p'}}{\partial y} + \nu\left(\frac{\partial^2 \overline{\bar{v} + v'}}{\partial x^2} + \frac{\partial^2 \overline{\bar{v} + v'}}{\partial y^2}\right)$$

Energy equation

Enthalpy Flows | Heat Conduction

$$\overline{\bar{u} + u'}\,\frac{\partial \overline{\bar{T} + T'}}{\partial x} + \overline{\bar{v} + v'}\,\frac{\partial \overline{\bar{T} + T'}}{\partial y} = \frac{\nu}{Pr}\left(\frac{\partial^2 \overline{\bar{T} + T'}}{\partial x^2} + \frac{\partial^2 \overline{\bar{T} + T'}}{\partial y^2}\right)$$

Conservation Equations (2D, steady state, incompressible)

Conclusion

Turbulent fluctuation act macroscopically like diffusion

Continuity equation

Mass Flows

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$$

Momentum equation

Momentum Flows | Pressure | Shear stresses | Turbulent Shear stresses

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \nu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right) - \left(\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y}\right)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} + \nu\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2}\right) - \left(\frac{\partial \overline{u'v'}}{\partial x} + \frac{\partial \overline{v'^2}}{\partial y}\right)$$

Energy equation

Enthalpy Flows | Heat Conduction | Turbulent Heat conduction

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{\nu}{Pr}\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right) - \left(\frac{\partial \overline{u'T'}}{\partial x} + \frac{\partial \overline{v'T'}}{\partial y}\right)$$

Conservation Equations (2D, steady state, incompressible)

Mass Flows

Continuity equation

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$$

Momentum Flows | Pressure | Shear stresses | Turbulent Shear stresses

Momentum equation

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \nu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right) + \frac{\eta_t}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} + \nu\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2}\right) + \frac{\eta_t}{\rho}\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2}\right)$$

Enthalpy Flows | Heat Conduction | Turbulent Heat Conduction

Energy equation

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{\nu}{Pr}\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right) + \frac{\lambda_t}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right)$$

Conservation Equations (2D, steady state, incompressible)

Definieren

$$\tau_t = -\rho \begin{pmatrix} \overline{u'^2} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'^2} \end{pmatrix} \equiv \eta_t \nabla \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \qquad \dot{q}''_t = \rho c_p \begin{pmatrix} \overline{u'T'} \\ \overline{v'T'} \end{pmatrix} \equiv -\lambda_t \nabla \bar{T}$$

Momentum Equation

Momentum flows | Pressure | Shear stresses | Turbulent Shear stresses

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \nu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right) + \frac{\eta_t}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} + \nu\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2}\right) + \frac{\eta_t}{\rho}\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2}\right)$$

Energy Equation

Enthalpy flows | Heat conduction | Turbulent Heat conduction

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{\nu}{Pr}\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right) + \frac{\lambda_t}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right)$$

Conservation Equations (2D, steady state, incompressible)

Mass Flows

Continuity equation

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$$

Momentum Flows | Pressure | Shear stresses

Momentum equation

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \frac{\eta_{\mathrm{eff}}}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} + \frac{\eta_{\mathrm{eff}}}{\rho}\left(\frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2}\right)$$

Effective viscosity

$$\eta_{\mathrm{eff}} = \eta + \eta_{\mathrm{t}} > \eta$$

Enthalpy Flows | Heat Conduction

Energy equation

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{\lambda_{\mathrm{eff}}}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right)$$

Effective thermal conductivity

$$\lambda_{\mathrm{eff}} = \lambda + \lambda_{\mathrm{t}} > \lambda$$

Comprehension Questions

How does turbulence affect heat transfer?