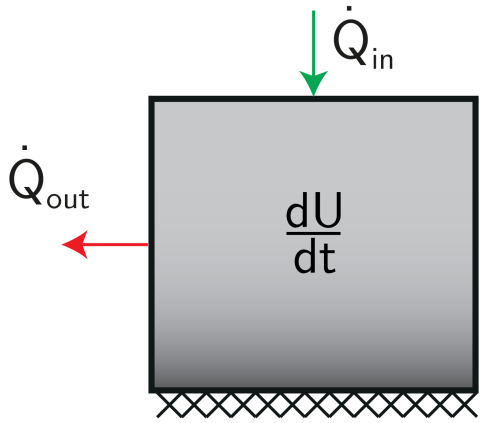
EB - Cond. - Body 10

All sides of a steel cube (except the bottom) are subject to convection. The bottom side is adiabatic, while the top side is subject to heat flow in addition to convection. Derive the governing energy equation that describes the time variation of the homogeneous temperature of the cube. Radiation does not contribute to the heat transfer from/to the cube.



Energy balance:

$$\frac{dU}{dt} = \dot{Q}_{in} - \dot{Q}_{out}$$

The heat transfer can be classified as transient, for that reason the change of internal energy over time equals the sum of the in and outgoing fluxes.

Change of internal energy over time:

$$\frac{dU}{dt} = \rho_2 \cdot c_{p2} \cdot L^3 \cdot \frac{dT_w}{dt}$$

The internal energy of the control volume can be described as: $U = m \cdot c_\mathrm{p} \cdot T$.

Heat fluxes:

$$\dot{Q}_{in} = \dot{q}'' L^2$$

$$\dot{Q}_{out} = 5\alpha \cdot L^2 \cdot (T_\mathrm{w} - T_\infty)$$

Substituting and rewriting:

$$\frac{dU}{dt} = \dot{Q}_{in} - \dot{Q}_{out}$$

$$\Rightarrow \rho c_\mathrm{p} L^3 \frac{\partial T_\mathrm{w}}{\partial t} = \dot{q}'' L^2 - 5\alpha L^2 (T_\mathrm{w} - T_\infty)$$