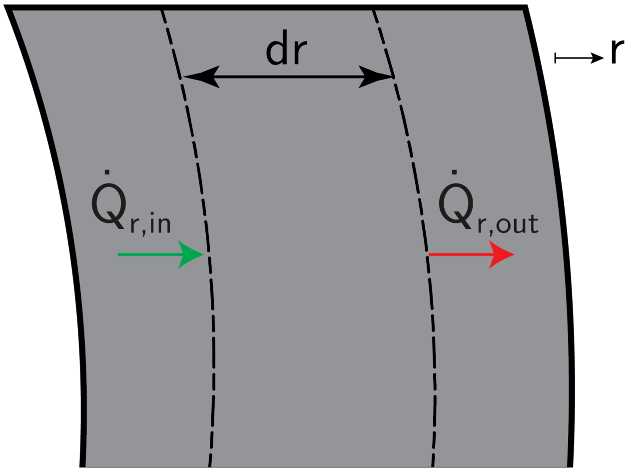
EB - Cond. - IE 9

Hot water flows through a long pipe of length L. The water temperature and external surface temperature of the pipe are constant and equal to T_∞ and T_1 respectively. Set up the energy balance for radial heat conduction in the pipe wall and give the appropriate boundary conditions.



Energy balance:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

Since the type of heat transfer is steady-state, the sum of the in- and outgoing heat fluxes of the control volume should equal zero.

Heat fluxes:

$$\dot{Q}_{r,in} = -\lambda A(r)\tfrac{\partial T}{\partial r} = -\lambda 2\pi r L\tfrac{\partial T}{\partial r}$$

$$\dot{Q}_{r,out} = \dot{Q}_{r,in} + \tfrac{\partial \dot{Q}_{r,in}}{\partial r}dr = -\lambda 2\pi r L\tfrac{\partial T}{\partial r} + \tfrac{\partial}{\partial r}\left(-\lambda 2\pi r L\tfrac{\partial T}{\partial r}\right)dr$$

The ingoing flux can be described by use of Fourier's law and the outgoing flux can be approximated by use of the Taylor series expansion.

Substituting and rewriting:

$$\dot{Q}_{r,in} - \dot{Q}_{r,out} = 0$$

$$-\lambda 2\pi r L\frac{\partial T}{\partial r} + \lambda 2\pi r L\frac{\partial T}{\partial r} - \frac{\partial}{\partial r}\left(-\lambda 2\pi r L\frac{\partial T}{\partial r}\right)dr = 0 \qquad (1)$$

$$\Rightarrow \frac{\partial}{\partial r}\left(\lambda 2\pi r L\frac{\partial T}{\partial r}\right) = 0 \qquad (2)$$

Even further simplifying:

$$\Rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \qquad (3)$$