Heat transfer: Radiation

Lecture notes

08/2021

Contents

Chapter 1

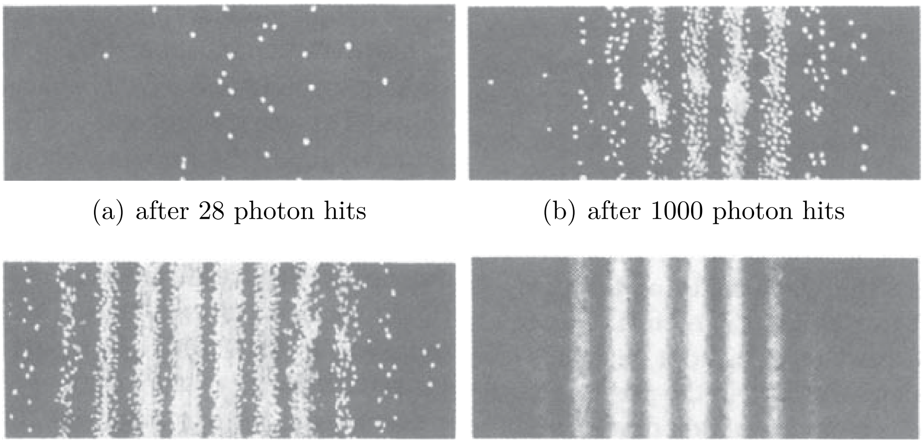
Properties of radiating bodies

Conduction and convection are heat transfer mechanisms, where energy is transported through molecular processes respectively macroscopic movement of fluids, whereas heat radiation does not require any medium, since it is based on electromagnetic processes. The intensity and the sort of radiation, emitted from a gaseous, liquid or solid body depends on the surface properties of the body and its temperature, yet they are independent of its surroundings. If not only the emitted radiation is to be considered, but also the heat exchange between the body and its surroundings, then the type, temperature and geometrical orientation in space of the surrounding bodies must be taken into account. In spite of the fact that in most heat transfer problems energy is simultaneously transported through conduction and/or convection and radiation, henceforth only radiation will be regarded, as far as possible.

1.1 Physics of radiation: Wave/Quantum dualty

Physics teaches us that radiation can be explained both using the wave as well as the quantum mechanics theory. This dual character has been impressively proven experimentally, see fig. 2.1, ?. Exposing a photographic plate with low light intensity through an aperture with two thin slits, the following interference patterns, Fig. 1.1(a), 1.1(b), 1.1(c) and 1.1(d), which depend on the exposure time can be observed.

After a very short time interval, Fig. 1.1(a), distinct light points can be recognized, which lead to the conclusion that individual light quanta - photons - hit the plate.



(a) after 28 photon hits (b) after 1000 photon hits

(c) after 10000 photon hits (d) double-slit interference pattern

Figure 1.1: Double-slit experiment by E.R. Huggins, Physics I, W.A. Benjamin, Inc., Menlo Park, California, USA, 1968.

At longer time periods it happens that the photons do not hit the screen at random, but form a striped pattern, Fig. 1.1(b) and 1.1(c). These stripes form an interference pattern, which can also be observed during superposition of circular waves passing through a diffraction grating and striking a dark screen behind it, Fig. 1.1(d). This demonstrates the wave character of radiation. The following figure shows the electromagnetic spectrum and the wavelengths at which heat radiation takes place Fig. 1.2.

The wavelength λ, usually given in the unit $[\mu m]$ is related to the frequency ν through the speed of light c as shown in the following equation

$$\lambda = \frac{c}{\nu} \tag{1.1}$$

The wave-like nature of radiation shows its character when the size of the "slits" through which the rays are refracted has the same order of magnitude as the wavelength of the radiation.

The spectrum shown above shows clearly that the radiation wavelengths are much smaller than the size of a typical "slit". Thus, the quantum character dominates

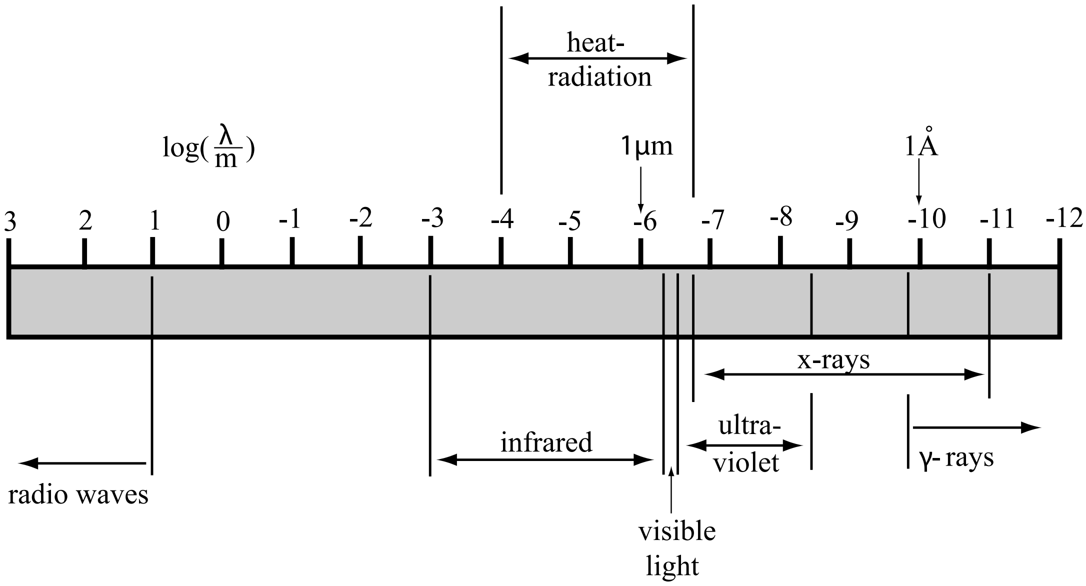


Figure 1.2: Electromagnetic spectrum

during heat radiation. According to Planck, each quantum or photon has the energy

$$E = h\,\nu = \frac{hc}{\lambda} \tag{1.2}$$

where Planck's constant is $h = 6{,}626\ 10^{-34}\ [Js]$.

Regarding equation (1.2) it is obvious that the transported energy of each quantum relates inversely proportional to the wavelength λ. This relation leads to the definition of the wavenumber η:

$$\eta = \frac{1}{\lambda} \tag{1.3}$$

In contrast to the wavelength, the wavenumber offers the advantage of a proportional relation to the transported energy of a quantum. This is only valid if the propagation velocity of the radiation does not change, see equation (1.2). If the velocity changes - e.g. by entering into a medium with a different refraction index - the wavelength changes as well as the wavenumber, see equation (1.2). So, for problems with different refraction indexes it is better to consider the frequency ν, because it does not depend on the refraction index.

1.2 Intensity distribution of radiation and entire radiation intensity

The radiation emitted from a body surface covers the whole spectrum of wavelengths. The intensity of radiation depends on the absolute temperature of a body. A simple model defines the so-called "black body": this body absorbs all the radiation falling upon it.

1.2.1 Planck's distribution law

Max Planck derived from the theory of quantum mechanics a relationship for the distribution of radiation intensity over the wavelength of a black body. **Planck's distribution law** describes the emitted radiation intensity $\dot{q}''_{b\lambda}$ in an infinetisimal wavelength range $d\lambda$, the so-called monochrome or spectral emissive power. It is defined as follows:

$$\dot{q}''_{b\lambda} = \frac{c_1 \lambda^{-5}}{\exp\left[\frac{c_2}{\lambda T}\right] - 1} \quad \left[\frac{W}{m^2}\frac{1}{m}\right] \tag{1.4}$$

where b indicates the black body. The spectral emissive power is a function of temperature of the emitting body.
The constants c_1 and c_2 can be determined using Planck's constant h, the Boltzmann constant k and the value for the speed of light in vacuum c_o. Hence, their values are

$$c_1 = 2\pi h c_o^2 = 2\pi \cdot\ 6.6256 \cdot\ 10^{-34} \left(2.9979 \cdot 10^8\right)^2 = 3.741 \cdot 10^{-16} \ \ [\mathrm{Wm}^2]$$

and

$$c_2 = h\frac{c_o}{k} = 6.6256 \cdot\ 10^{-34} \frac{2.9979 \cdot 10^8}{1.3805 \cdot 10^{-23}} = 1.439 \cdot 10^{-2} \ \ [\mathrm{mK}]$$

with

$$k = \frac{R_m}{N_L} = \frac{8.3143}{6.0225 \cdot 10^{23}} = 1.3805 \cdot 10^{-23}\ [\mathrm{J/K}],$$

and N_L as the Loschmidt number.

Figure 1.3 shows Planck's distribution law for selected temperatures and radiation wavelengths.

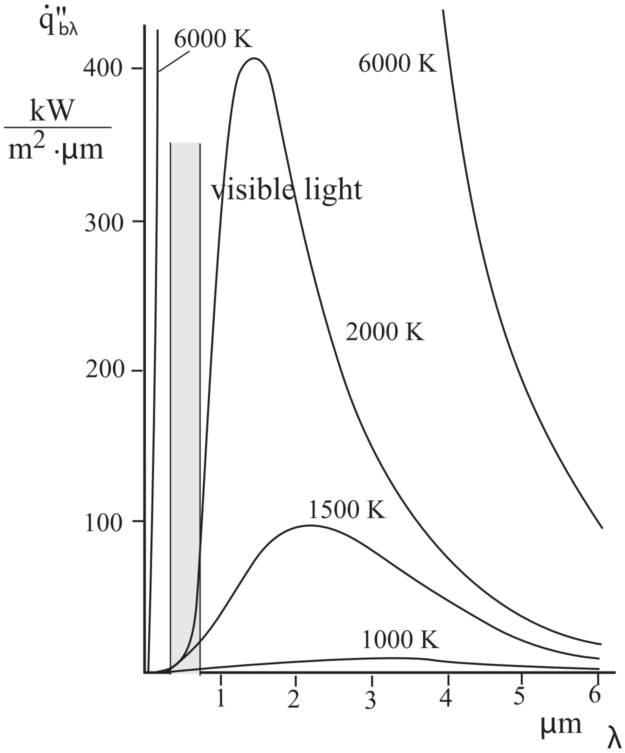


Figure 1.3: Spectral radiant flux density of a black body

Apparently the emitted radiation intensity increases with increasing temperature and the corresponding maximum shifts to smaller wavelengths as the temperature increases.

1.2.2 Wien's Law of displacement

The derivative of equation (1.4) gives the position of the maxima of the curve. This is the so-called **Wien's Law of displacement**

$$\lambda_{\max} = \frac{2898 \mu m K}{T} \tag{1.5}$$

1.2.3 Stefan-Boltzmann's law

In order to determine the entire emitted radiation power per area, Planck's distribution law (1.4) will be integrated over the entire wavelength range. An illustrative example is given in Figure 1.4, with the area below the intensity curve being equal to the emitted radiation power. This integration leads to the **Stefan-Boltzmann law**.

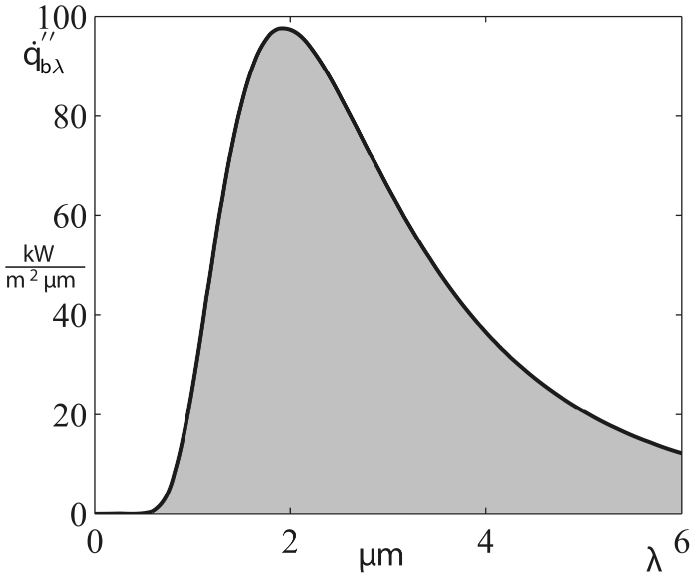


Figure 1.4: Entire emitted radiation, shown as the integral of the Planck-distribution

$$\dot{q}_b'' = \int_{\lambda=0}^{\infty} \dot{q}_{b\lambda}'' d\lambda = \int_{\lambda=0}^{\infty} \frac{c_1 \lambda^{-5}}{\exp\left[\frac{c_2}{\lambda T}\right] - 1} d\lambda = \sigma T^4 \tag{1.6}$$

The Stefan-Boltzmann constant σ has the value $\sigma = 5.67 \cdot 10^{-8} \left[\frac{W}{m^2 K^4}\right]$.

1.2.4 Radiative emissions within a spectral range

Occasionally, instead of considering the radiation emissions of the entire spectrum, it is necessary to consider the emissions only in a specific range between the wavelengths λ_1 and λ_2. The radiation density of this interval can be determined by integration of the Planck-distribution:

$$\dot{q}_{b,\lambda_1 \rightarrow \lambda_2}'' = \int_{\lambda_1}^{\lambda_2} \dot{q}_{b\lambda}'' d\lambda \tag{1.7}$$

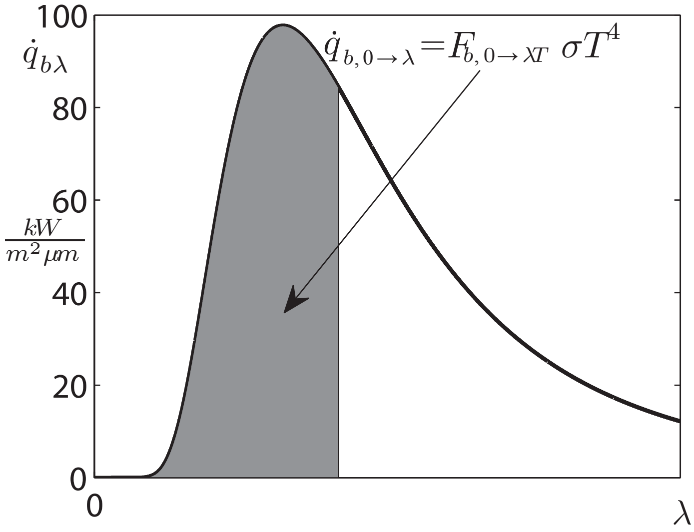


Figure 1.5: Emitted radiation power between $0 \rightarrow \lambda$

The solution of this integral can be described by the following series approach:

$$F_{0\rightarrow\lambda T} = \frac{15}{\pi^4} \sum_{n=1}^{\infty} \left[\frac{e^{-n\xi}}{n} \left(\xi^3 + \frac{3\xi^2}{n} + \frac{6\xi}{n^2} + \frac{6}{n^3} \right) \right] \tag{1.8}$$

with

$$\xi = \frac{c_2}{\lambda T}$$

The factor $F_{0\rightarrow\lambda T}$ describes the ratio between the entire emitted radiation intensity and the emitted radiation intensity in the spectral range between 0 and λ. By using Stefan-Boltzmann law the actual emitted radiation intensity, shown in Figure 1.5 can be calculated:

$$\dot{q}''_{b,0\rightarrow\lambda} = F_{0\rightarrow\lambda T} \cdot \left(\sigma T^4 \right) \tag{1.9}$$

The radation intensity between any interval of the spectrum - shown in Figure 1.6 - can be described as follows:

$$\dot{q}''_{b,\lambda_1\rightarrow\lambda_2} = (F_{0\rightarrow\lambda_2 T} - F_{0\rightarrow\lambda_1 T}) \cdot \sigma T^4 \tag{1.10}$$

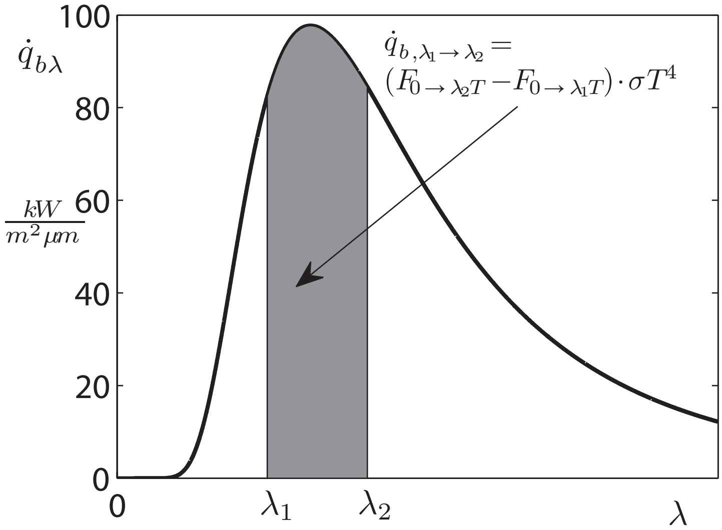


Figure 1.6: Emitted radiation power between $\lambda_1 \to \lambda_2$

For simple calculations the series approach, equation (1.8), is not practical; therefore tables with exemplary values for the factor $F_{0\to\lambda T}$ are used. The series approach is more common for computer-based calculations. Even with a small number of links n, results of sufficient accuracy can be obtained.

1.3 Gray and real bodies: Reflection, Absorption, and Transmission

When radiation of a specific wavelength falls upon the surface of a (non black) body, this radiation is either reflected off the surface, absorbed by the body or transmitted through it, whereas in case of reflection, it can be distinguished between fully reflective (angle of incidence is equal to the angle of reflection) or diffuse, in which case the radiation is equally distributed in all directions. The reflected component of the radiation is called reflectivity $\rho(\lambda)$, the absorbed portion absorptivity $\alpha(\lambda)$ and the transmitted part of radiation transmissivity $\tau(\lambda)$.

$$\rho(\lambda) \equiv \frac{\dot{q}''_{\lambda\rho}}{\dot{q}''_{\lambda o}}\ ;\ \alpha(\lambda) \equiv \frac{\dot{q}''_{\lambda\alpha}}{\dot{q}''_{\lambda o}}\ ;\ \tau(\lambda) \equiv \frac{\dot{q}''_{\lambda\tau}}{\dot{q}''_{\lambda o}} \tag{1.11}$$

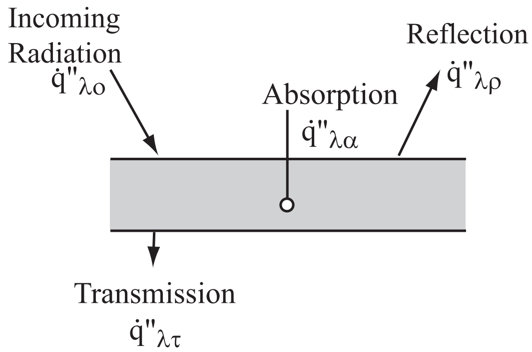


Figure 1.7: Contributions to radiation transport

So that

$$\rho(\lambda) + \alpha(\lambda) + \tau(\lambda) = 1 \tag{1.12}$$

We can distinguish between the following special cases:

- Since most solids are opaque to radiation, the incident radiation is partly reflected and partly absorbed in thin layers of a few μm (electric conductors) up to 2 mm (electric insulators)

$$\rho(\lambda) + \alpha(\lambda) = 1 \tag{1.13}$$

- Solids, which fully absorb all radiation have already been defined as "black bodies"

$$\alpha(\lambda) = \alpha = 1 \tag{1.14}$$

- "Grey" bodies, on the other hand, are bodies with radiation properties that are independent of the wavelength and which radiate in all directions (diffuse radiation) ($\alpha(\lambda) = \alpha,\ \rho(\lambda) = \rho,\ \tau(\lambda) = \tau$)

$$\rho + \alpha + \tau = 1 \tag{1.15}$$

- Gases normally do not reflect radiation, hence

$$\alpha(\lambda) + \tau(\lambda) = 1 \tag{1.16}$$

The following diagram shows experimental results of measurements of the spectral absorptivity of a polished and an anodised aluminium plate for wavelengths in the range of heat radiation.

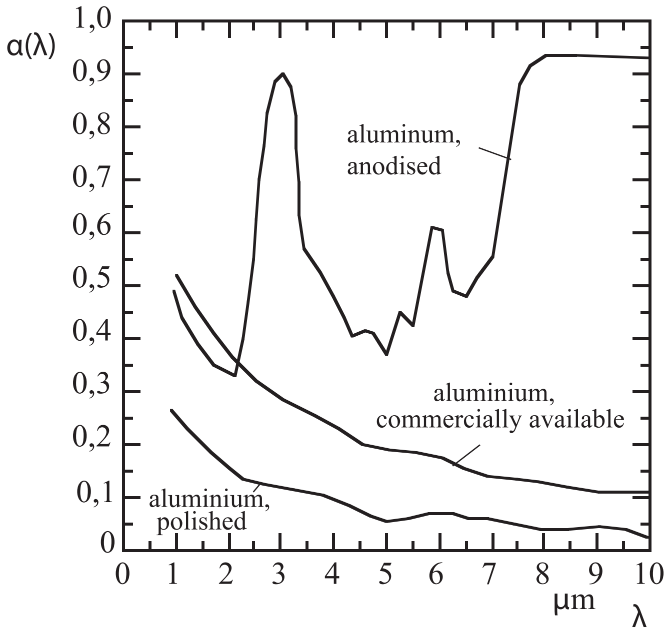


Figure 1.8: Absorptivity of a polished and an anodised aluminium plate. Source: Whitaker, 1977. **?** at 0°C

The reduction of the absorptivity at increasing wavelengths of both the polished and the typical (commercially available) aluminium is characteristic for all electric conductors. Isolators, such as the surface layer of the anodised aluminium, show the opposite trend.

In correspondence to the spectral properties of a body, the total reflectivity, the total absorptivity and the total transmissivity can be defined as mean over all wavelengths, respectively

$$\rho = \frac{\int_0^\infty \dot{q}''_{\lambda\rho} d\lambda}{\int_0^\infty \dot{q}''_{\lambda o} d\lambda}, \tag{1.17}$$

$$\alpha = \frac{\int_0^\infty \dot{q}''_{\lambda\alpha} d\lambda}{\int_0^\infty \dot{q}''_{\lambda o} d\lambda}, \tag{1.18}$$

$$\tau = \frac{\int_0^\infty \dot{q}''_{\lambda\tau} d\lambda}{\int_0^\infty \dot{q}''_{\lambda o} d\lambda}, \tag{1.19}$$

which again yields

$$\rho + \alpha + \tau = 1. \tag{1.20}$$

Unlike the spectral values, the total values are not only dependent on the temperature of the body in question, but they also depend on the body that emits the incident radiation. Since usually very little is known about the dependence of the radiation properties on the wavelength and, in general, their determination proves to be very difficult, normally only constant mean values are used in practice.

1.3.1 Kirchhoff's law

Kirchhoff's law states the relationship between the absorbed and the emitted radiation of a body. As shown in the following experiment (see figure below), two bodies with evacuated cavities and of different materials, adiabatic to the outside, have been positioned so that a small opening connects the cavities. Radiation coming from cavity 1 passes through the orifice into cavity 2, where it is being absorbed after a number of reflections. The same applies for radiation coming from cavity 2

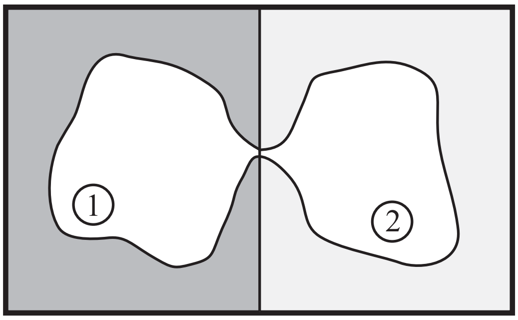


Figure 1.9: Radiation exchange between two evacuated cavities

and arriving in cavity 1. In thermal equilibrium, i.e. when temperature compensation between the two bodies has reached, the following must apply according to the second law of thermodynamics

$$\dot{q}''_{1\to 2} = \dot{q}''_{2\to 1}$$

Therefore, it can be concluded that the difference of the wall properties of the cavities does not influence the radiation flow between them.

If, for example, one of the two cavities is a black body, then the above stated still applies. This yields

$$\dot{q}''_{1\to 2} = \dot{q}''_{b} \text{ or in general } \dot{q}''_{C} = \dot{q}''_{b}$$

hence, it can be stated that the radiation of any cavity is equivalent to the radiation of a black body.

If in one of the cavities C a small, non-black body 1 with the surface area A1 is placed, then in thermal equilibrium, the absorbed portion of radiation from the cavity by body 1 has to be equal to the emitted radiation of the cavity. Furthermore, having in mind that the cavity radiates as a black body, it follows

$$\alpha_1 \dot{q}''_b A_1 = \dot{q}''_{\epsilon 1} A_1 \tag{1.21}$$

Which yields

$$\alpha = \frac{\dot{q}''_{\epsilon}}{\dot{q}''_{b}} \tag{1.22}$$

which is a restatement of Kirchhoff's law that the absorptivity of a body is equal to the relationship of the emitted radiation to that of a black body with the same temperature. This relationship is defined as emissivity ϵ

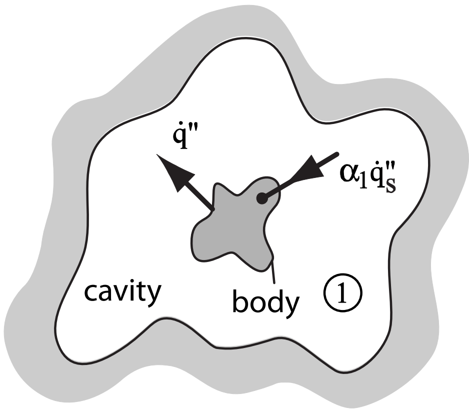


Figure 1.10: Radiation exchange between a body and a cavity

$$\epsilon \equiv \frac{\dot{q}''_{\epsilon}}{\dot{q}''_{b}} \tag{1.23}$$

Or rewritten in terms of Kirchhoff's law

$$\alpha = \epsilon. \tag{1.24}$$

Strictly speaking, Kirchhoff's law is valid only for monochromatic radiation, i.e.

$$\alpha(\lambda) = \epsilon(\lambda). \tag{1.25}$$

Yet, for many practical purposes equation (1.24) must be employed to avoid unnecessary complex computations or also because of the absence of spectral data. The necessary requirements for the validity of Kirchhoff's law will be discussed next. For this purpose the total emissivity ϵ has to be determined by integrating over the spectral radiation power.

$$\epsilon = \frac{\dot{q}''_{\epsilon}}{\dot{q}''_{b}} = \frac{\int\limits_0^\infty \dot{q}''_{\lambda\epsilon} d\lambda}{\int\limits_0^\infty \dot{q}''_{\lambda b} d\lambda}. \tag{1.26}$$

Defining the corresponding spectral emissivity as in equation (1.23)

$$\epsilon(\lambda) \equiv \frac{\dot{q}''_{\lambda\epsilon}}{\dot{q}''_{\lambda b}}, \tag{1.27}$$

equation (1.26) yields a relationship that depends on the type of the body and its temperature

$$\epsilon = \frac{\int_0^\infty \epsilon(\lambda)\dot{q}''_{\lambda b}d\lambda}{\int_0^\infty \dot{q}''_{\lambda b}d\lambda} = \epsilon\left(T_{\text{body}}\right). \tag{1.28}$$

The total absorption α can be written as

$$\alpha = \frac{\dot{q}''_{\alpha}}{\dot{q}''_{o}} = \frac{\int_0^\infty \dot{q}''_{\lambda\alpha}d\lambda}{\int_0^\infty \dot{q}''_{\lambda o}d\lambda}. \tag{1.29}$$

$$\alpha \equiv \frac{\dot{q}''_{\lambda\alpha}}{\dot{q}''_{\lambda o}} \tag{1.30}$$

Equation (1.29) leads to the following relationship, which depends on the type of both radiating bodies under investigation and their corresponding temperatures

$$\alpha = \frac{\int_0^\infty \alpha(\lambda)\dot{q}''_{\lambda o}d\lambda}{\int_0^\infty \dot{q}''_{\lambda 0}d\lambda} = \alpha\left(T_{\text{body}}, T_{\text{rad}}\right). \tag{1.31}$$

Comparison of equation (1.28) and (1.31) makes clear that Kirchhoff's law (1.24), defined for the total emissivity ϵ and total absorptivity α , may not always be valid since the incident and emitted radiation are not equally dependent on the temperature and spectral distribution.

Two special cases should be mentioned for which Kirchhoff's law for the total radiation properties remains valid and which can be derived from equation (1.28) and (1.31)

- The radiating body is a black or grey body whose temperature is equal to that of the investigated body, $T_{\text{rad}} = T_{\text{body}}$

- The surfaces of the body are grey, i.e. their absorptivity is independent from the wavelength.

Especially the second case is important, since it is valid for many practical cases.

If a high level of accuracy is required, then the exact knowledge of the monochromatic radiation properties is necessary.

Figure 1.11 shows the radiant flux density of an actual real body at T = 2000K. The results of Planck's distribution law, equation (1.4), for a black and a grey body are shown as a comparison. A detailed summary of emissivity values of different materials is given in the Appendix A6.

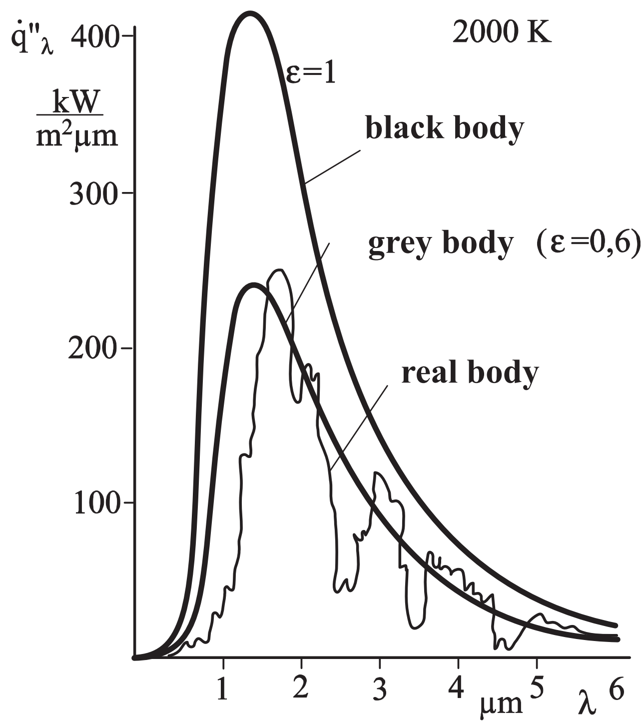


Figure 1.11: Spectral radiant flux density of a black, grey and a real body surface

1.3.2 Radiation from a diffuse surface and direction-dependent radiation

Emissivity data for different materials, see e.g. tables in Appendix, are usually determined by measurements in the entire hemisphere. Most of the real surfaces used in practice have an direction dependent emissivity, so that only ideal surfaces as that of the black or grey body can be considered diffuse, i.e. independent of the direction.

Measurement results, as shown in the following figure, show that for example electric conductors have direction-dependent emissivity, which increases with increasing viewing angle φ. On the other hand, insulators possess nearly constant and relatively large emissivity over a wide range of viewing angles.

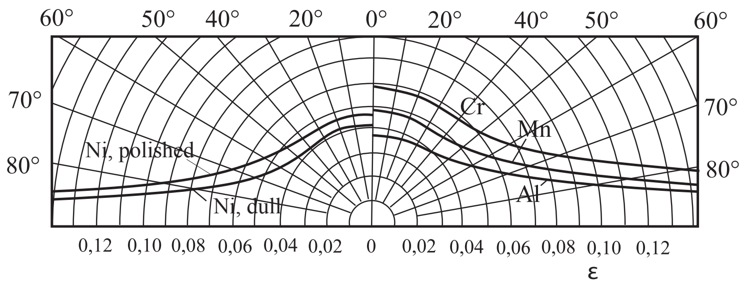


Figure 1.12: Electric conductors

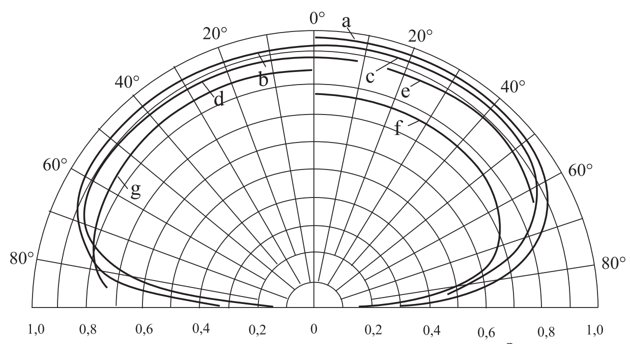


Figure 1.13: Isolators: (a) Wet ice, (b) wood, (c) glass, (d) paper, (e) chalk, (f) copper oxide, (g) aluminium oxide at room temperature

Chapter 2

View factors

2.1 Radiation flux - radiosity

The radiation properties necessary to describe the heat flow from a body have been discussed so far. If not the total emitted heat flow from a body, but the heat exchange between the body and a nearby object is regarded, it is necessary to know the exact radiation amount in direction to the object as well as the radiation absorbed by this object.

To gain better insight, we imagine a hemisphere of radius r over a small surface element dA, positioned at the centre of the hemisphere. In this case the entire heat flow emitted from the surface element passes through the hemisphere.

Considering a surface element on the hemisphere da, then the energy flow from dA to da is defined as radiation flux or radiosity L by the following equation

$$d\dot{Q}(\varphi,\psi) = L\ d\Omega\ dA\ \cos\varphi, \tag{2.1}$$

The radiosity L is thus that portion of the energy flux that is emitted from the projected area $dA\ \cos\varphi$ in direction of the radiation per unit angle $d\Omega$, where the solid angle $d\Omega$ is measured in units of "steradian" $[sr]$ and describes the ratio of the surface element on the hemisphere to the radius of the sphere squared

$$d\Omega = \frac{da\,(\varphi,\psi)}{r^2} \tag{2.2}$$

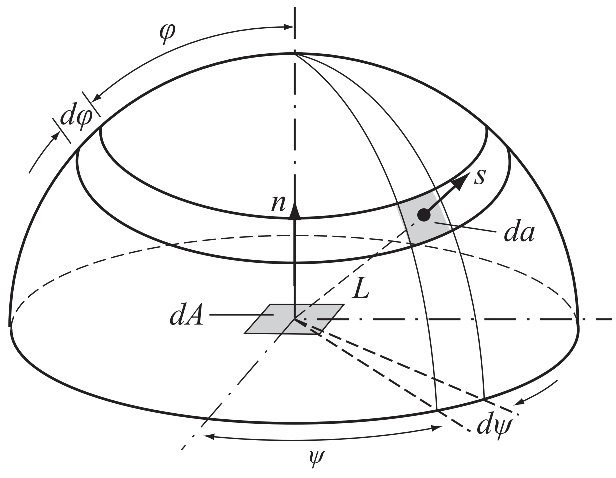


Figure 2.1: Radiation between a surface element and a hemisphere

Diffuse surfaces, i.e. surfaces whose radiation is direction-independent, such as a grey or black body, have constant radiation density. The energy flux to the solid angle $d\dot{Q}(\varphi,\psi)$ is thus proportional to the projected area $dA\cos\varphi$. The radiation density L is that intensity of the radiation of a surface that is readily perceived by the eye, which in turn is independent of the viewing angle in the case of diffusely radiating bodies. The radiation can have its origin both in the emissive body or can also consist of reflected or transmitted parts of another radiation source. The solid angle, which describes the surface element da can also be expressed in geometrical terms as

$$d\Omega = \frac{da(\phi,\psi)}{r^2} = \frac{r\ \sin\varphi d\psi r d\varphi}{r^2} = \sin\varphi\ d\varphi\ d\psi \tag{2.3}$$

Using equation (2.1) leads to

$$d\dot{Q}(\varphi,\psi) = L\ \sin\varphi\ \cos\varphi\ d\phi\ d\psi\ dA \tag{2.4}$$

which after integration over the hemisphere yields the relationship between the radiation density and the emissive power of the surface element

$$\dot{q}'' = \pi L \tag{2.5}$$

This particular form of the emissive power is often called "surface brightness".

2.2 Radiation transfer between two bodies

If radiation transfer between two diffusely radiating surfaces of two bodies with different temperatures occurs, then the hotter body emits more heat to the colder one, so that the net heat flow is from the hotter to the colder body. In an arbitrary orientation in space, heat $d\dot{Q}_{1\to 2}$ flows from the surface area dA_1 of body 1 to the surface area dA_2 of body 2.

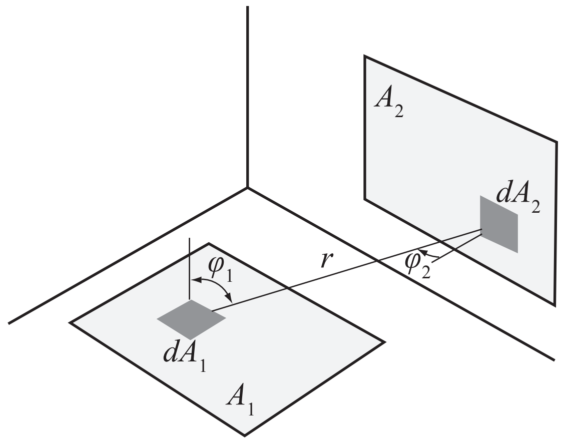


Figure 2.2: Radiation transfer between two surfaces

Using equation (2.1), it is

$$d\dot{Q}_{1\to 2} = L_1 \; \cos\varphi_1 \; d\Omega_1 \; dA_1 \tag{2.6}$$

Since the surface element dA_2 is not perpendicular to the connecting line r, only the projected portions of the area dA_2 should be used to determine the solid angle, i.e.

$$d\Omega_1 = \frac{dA_2 \; \cos\varphi_2}{r^2} \tag{2.7}$$

Which yields

$$d\dot{Q}_{1\to 2} = L_1 \frac{\cos\varphi_1 \cos\varphi_2}{r^2} dA_1 dA_2 \tag{2.8}$$

We get the corresponding relationship for the heat $d\dot{Q}_{2\to1}$ from the surface element dA_2 radiated to the surface element dA_1.

$$d\dot{Q}_{2\to1} = L_2 \frac{\cos\varphi_1 \cos\varphi_2}{r^2} dA_1 dA_2 \tag{2.9}$$

Hence, the net radiation transfer between the two bodies is

$$d\dot{Q}_{1\rightleftharpoons2} = \int\limits_{A_2}\int\limits_{A_1} (L_1 - L_2) \frac{\cos\varphi_1 \cos\varphi_2}{r^2} dA_1 dA_2 \tag{2.10}$$

Since only diffusely radiating surfaces with homogeneous radiation properties over the body surface are considered, it can be simplified to

$$d\dot{Q}_{1\rightleftharpoons2} = (L_1 - L_2) \int\limits_{A_2}\int\limits_{A_1} \frac{\cos\varphi_1 \cos\varphi_2}{r^2} dA_1 dA_2 \tag{2.11}$$

Replacing the radiosity L by the radiation power $\dot{q}''$ using equation (2.5)

$$d\dot{Q}_{1\rightleftharpoons2} = \left(\dot{q}_1'' - \dot{q}_2''\right) \int\limits_{A_2}\int\limits_{A_1} \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2 \tag{2.12}$$

The double integration over the surfaces of the two bodies gives an expression which contains only geometrical quantities and none of the radiation properties, but on the other hand such integration is normally too complex and is usually replaced by numerical, graphic or photographic methods. The results of the integration are readily available for a number of typical geometrical orientations in space and can be found in other literature. Usually, these results are given as the so-called "view factors".

The view factor Φ_{12} is that portion of radiation that originates from the area A_1 and hits the surface area A_2

$$\Phi_{12} = \frac{\dot{Q}_{1\to2}}{\dot{q}_1'' A_1} \tag{2.13}$$

If a direct radiation transfer between the surface area A_1 and n other surfaces exists,

$$\Phi_{11} + \Phi_{12} + \Phi_{13} + ... + \Phi_{1n} = 1 \tag{2.14}$$

is valid.
Integrating equation (2.8) and (2.13) yields a relationship for the view factor Φ_{12}, which depends only on the geometric values

$$\Phi_{12} = \frac{1}{A_1} \int\limits_{A_2} \int\limits_{A_1} \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2 \tag{2.15}$$

And for Φ_{21}

$$\Phi_{21} = \frac{1}{A_2} \int\limits_{A_2} \int\limits_{A_1} \frac{\cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2 \tag{2.16}$$

Thus, the double integrals of equation (2.15) and (2.16) are equal, which in turn leads to the reciprocal rule

$$A_1\Phi_{12} = A_2\Phi_{21} \tag{2.17}$$

and finally, using equation (2.12)

$$\dot{Q}_{1\rightleftharpoons 2} = A_1\Phi_{12}\left(\dot{q}_1^{''} - \dot{q}_2^{''}\right) \tag{2.18}$$

$$= A_2\Phi_{21}\left(\dot{q}_1^{''} - \dot{q}_2^{''}\right) \tag{2.19}$$

If only black bodies are involved, then the emissive power consists only of emitted radiation, for which the Stefan-Boltzmann law equation (1.6), is valid. Hence

$$\dot{Q}_{1\rightleftharpoons 2} = A_1\Phi_{12}\sigma\left[(T_1)^4 - (T_2)^4\right] \tag{2.20}$$

$$= A_2\Phi_{21}\sigma\left[(T_1)^4 - (T_2)^4\right] \tag{2.21}$$

As an example, the following diagram shows how the view factors can be determined for radiation transfer between parallel or perpendicular plates, according to Holman, 1976. This may be used for example, in the determination of direct radiation transfer between two walls of a room.

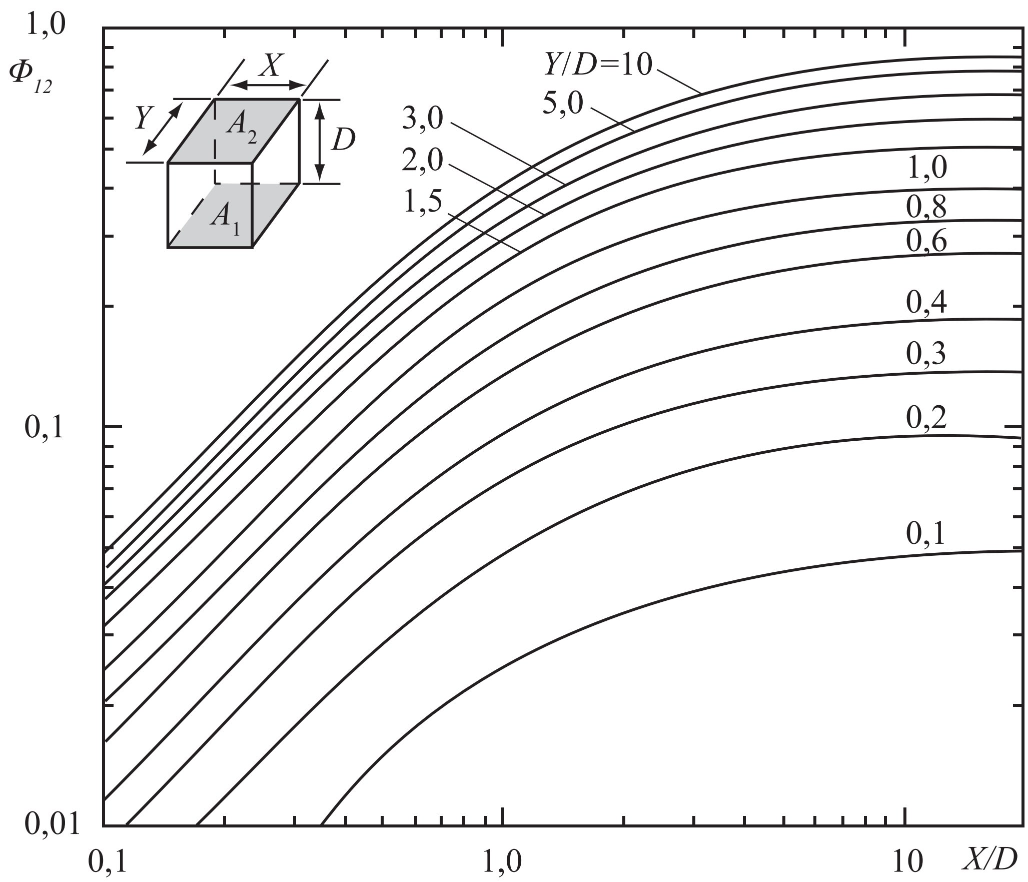


Figure 2.3: View factor of the radiation transfer between parallel, rectangular plates

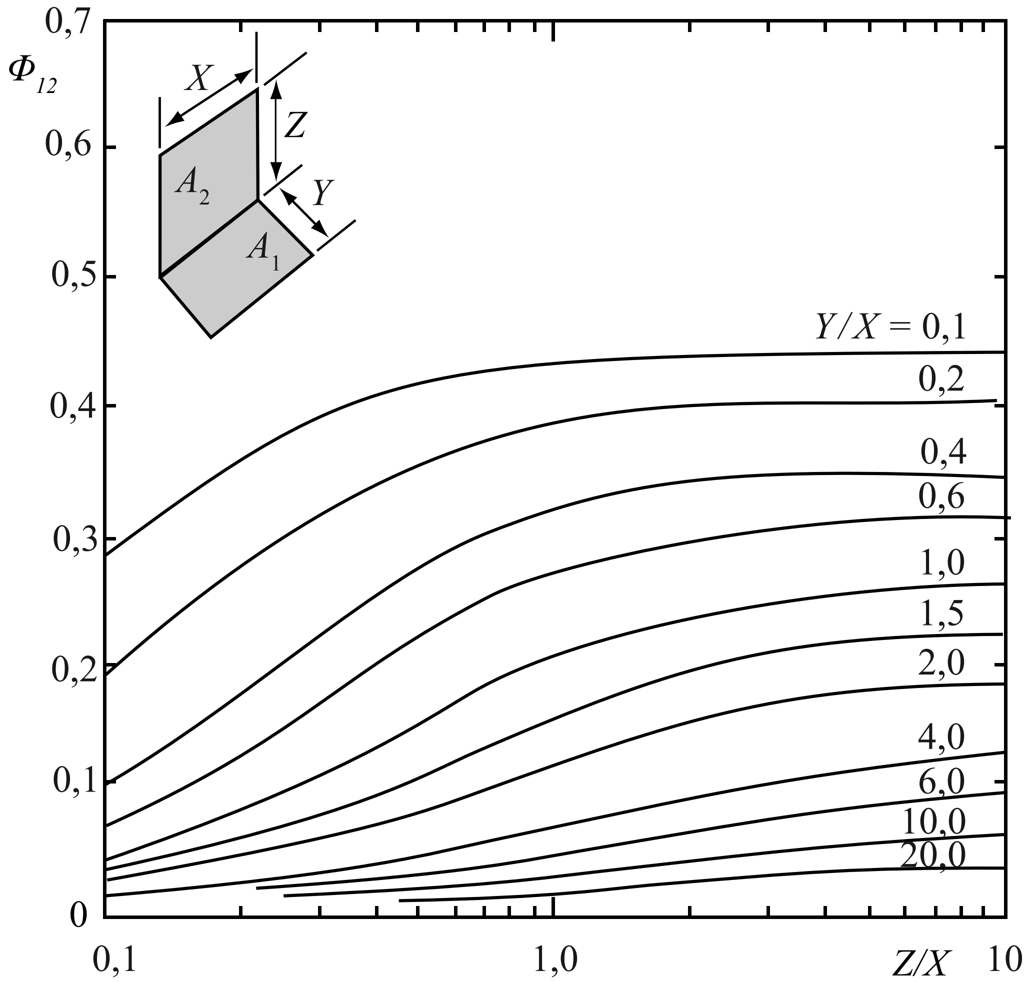


Figure 2.4: View factor of the radiation transfer between perpendicular, rectangular plates

Chapter 3

Radiative transport

This chapter containing the concept of surface brightness and the fundamentals of energy balances (inner and outer balance, etc.) is still missing and requires to be written.

Chapter 4

Examples for radiative transport

4.1 Radiation transfer between two grey surfaces

If, as a special case, radiation transfer between two black bodies occurs, then knowing the view factor and introducing Stefan-Boltzmann's law in equation (2.18) and (2.19), the relationship of the net radiation transfer as a function of the temperature of the bodies, equation (2.20) and (2.21) can be determined.

For grey bodies on the other hand, the surface brightness includes not only the emitted, but also portions of the reflected and transmitted radiation, since unlike black bodies, the incident radiation is not fully absorbed.

Thus, for the radiation transfer between two non-transmissive solids

$$\dot{Q}_{1\rightleftharpoons 2} = \Phi_{12}\dot{Q}_1 - \Phi_{21}\dot{Q}_2, \tag{4.1}$$

where e.g. $\dot{Q}_1 = A_1\ \dot{q}_1'' = \dot{Q}_{1,\epsilon} + \dot{Q}_{1,\rho}$ is valid.
The solution methods for these equations are usually rather complex. Yet, for a number of simple, often encountered geometries, complete relationships for the radiation transfer can be derived.

4.1.1 Radiation transfer between two infinitely long grey plates

By using the already derived euqations, the heat radiation of a plate can be expressed

$$\dot{Q}_1 = A_1 \dot{q}_1^{''} = \dot{Q}_{1,\epsilon} + \dot{Q}_{1,\rho} = \epsilon_1 A_1 \dot{q}_{b1}^{''} + \rho_1 A_2 \dot{q}_2^{''} \tag{4.2}$$

With $\rho + \alpha = 1$ and applying Kirchhoff's law, $\alpha = \epsilon$, yields

$$\dot{q}_1^{''} = \epsilon_1 \dot{q}_{b1}^{''} + (1 - \epsilon_1)\, \dot{q}_2^{''} \tag{4.3}$$

and

$$\dot{q}_2^{''} = \epsilon_2 \dot{q}_{b2}^{''} + (1 - \epsilon_2)\, \dot{q}_1^{''}. \tag{4.4}$$

respectively.

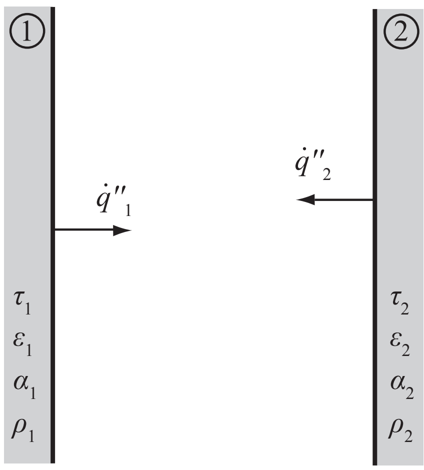


Figure 4.1: Radiation transfer between respectively. two grey surfaces

From equation (4.3) and (4.4) the surface brightnesses $\dot{q}_1^{''}$ and $\dot{q}_2^{''}$ are

$$\dot{q}_1^{''} = \frac{\epsilon_1 \dot{q}_{b1}^{''} + (1 - \epsilon_1)\epsilon_2 \dot{q}_{b2}^{''}}{1 - (1 - \epsilon_1)(1 - \epsilon_2)}, \tag{4.5}$$

$$\dot{q}_2^{''} = \frac{\epsilon_2 \dot{q}_{b2}^{''} + (1 - \epsilon_2)\epsilon_1 \dot{q}_{b1}^{''}}{1 - (1 - \epsilon_1)(1 - \epsilon_2)}. \tag{4.6}$$

Substituting in equation (2.42) and knowing that $\Phi_{12} = \Phi_{21} = 1$ we get the following relationship for the net radiation transfer,

$$\dot{q}''_{1\rightleftharpoons 2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \left(\dot{q}''_{b1} - \dot{q}''_{b2} \right), \tag{4.7}$$

which can also be written as a function of the body temperature using the Stefan-Boltzmann law, equation (1.6)

$$\dot{q}''_{1\rightleftharpoons 2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \sigma \left(T_1^4 - T_2^4 \right). \tag{4.8}$$

4.1.2 Radiation transfer between two self-enclosed grey bodies

In this case, it holds

$$\dot{Q}_1 = A_1 \dot{q}''_1 = \dot{Q}_{1,\epsilon} + \dot{Q}_{1,\rho} = \epsilon_1 A_1 \dot{q}''_{b1} + \rho_1 \left(\Phi_{21} \dot{q}''_2 A_2 \right) \tag{4.9}$$

or using Kirchhoff's law and the reciprocal rule $\Phi_{12} A_1 = \Phi_{21} A_2$

$$\dot{Q}_1 = A_1 \dot{q}''_1 = A_1 \left(\epsilon_1 \dot{q}''_{b1} + (1 - \epsilon_1) \Phi_{12} \dot{q}''_2 \right). \tag{4.10}$$

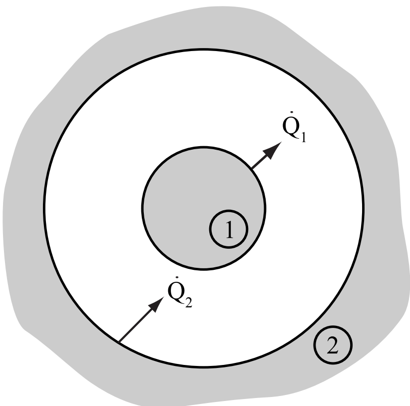


Figure 4.2: Radiation Transfer between two enclosed bodies

In the case of enclosed bodies, the view factor Φ_{12} has the value 1 and thus

$$A_1 \dot{q}''_1 = A_1 \left(\epsilon_1 \dot{q}''_{b1} + (1 - \epsilon_1) \dot{q}''_2 \right). \tag{4.11}$$

For the heat emitted from body 2 we get additionally parts of the reflected radiation, which originate from the surface brightness of the body itself - in other words “the body sees itself”.

$$\dot{Q}_2 = A_2\dot{q}_2^{''} = \epsilon_2 A_2 \dot{q}_{b2}^{''} + \rho_2 \left(\Phi_{12}\dot{q}_1^{''} A_1\right) + \rho_2 \left(\Phi_{22}\dot{q}_2^{''} A_2\right) \tag{4.12}$$

Knowing that $\Phi_{12} = 1$ and $\Phi_{21} = \frac{A_1\Phi_{12}}{A_2} = \frac{A_1}{A_2}$ yields,

$$A_2\dot{q}_2^{''} = \epsilon_2 A_2 \dot{q}_{b2}^{''} + (1-\epsilon_2)\,\dot{q}_1^{''} A_1 + (1-\epsilon_2)\left(1 - \frac{A_1}{A_2}\right)\dot{q}_2^{''} A_2. \tag{4.13}$$

Equation (4.11) and (4.13) are used to determine the surface brightness of both bodies, which after substituting in equation (4.1) and several transformations yield a relationship for the radiation transfer

$$\dot{Q}_{1\rightleftharpoons 2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)} A_1 \sigma \left(T_1^4 - T_2^4\right). \tag{4.14}$$