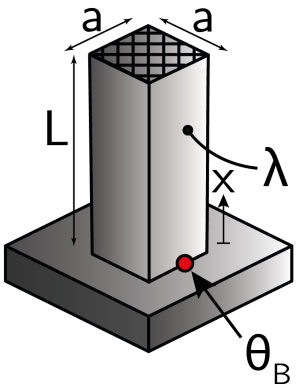
Fins - Flux 2

Calculate the rate of heat transfer for a fin with an adiabatic head.



Given the fin temperature profile:

$$\Theta(x) = \Theta_{\mathrm{B}} \frac{\cosh\left(m\left(L-x\right)\right)}{\cosh\left(mL\right)}$$

Where $\Theta(x) = T(x) - T_{\infty}$ and $m = \sqrt{\frac{4\alpha}{\lambda a}}$.

The rate of heat transfer through the fin can be expressed as:

$$\dot{Q} = -\lambda A_{\mathrm{c}} \frac{\partial T}{\partial x}|_{x=0} = -\lambda a^2 \frac{\partial \Theta}{\partial x}|_{x=0}$$

Differentiation of $\Theta(x)$ with respect to x yields:

$$\frac{\partial \Theta}{\partial x} = -m\Theta_{\mathrm{B}} \frac{\sinh\left(m\left(L-x\right)\right)}{\cosh\left(mL\right)}$$

There for at $x = 0$ gives:

$$\frac{\partial \Theta}{\partial x}|_{x=0} = -\Theta_{\mathrm{B}} \frac{\sinh\left(mL\right)}{\cosh\left(mL\right)} = -\Theta_{\mathrm{B}} \tanh\left(mL\right)$$

Substitution into the expression for the rate of heat transfer yields:

$$\dot{Q} = \lambda a^2 m \Theta_{\mathrm{B}} \tanh\left(mL\right)$$