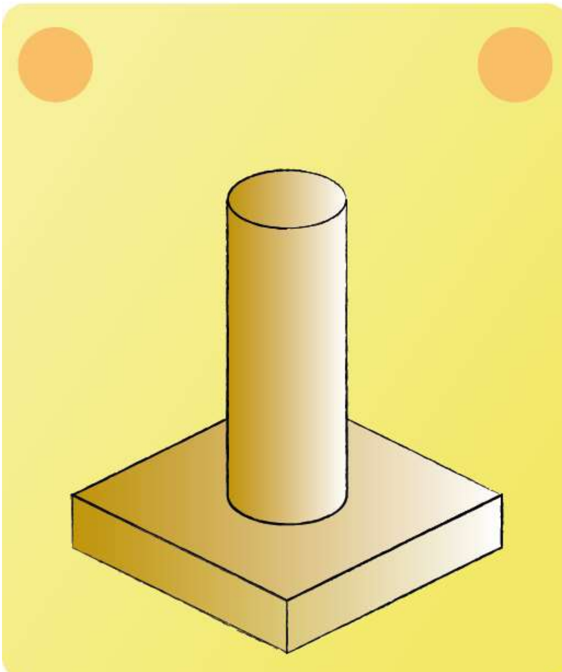
Lecture 11 - Question 4



After simplifications, the result is the following 2nd order homogeneous differential equation for fins: $\frac{\partial^2\theta}{\partial x^2} - m^2 \cdot \theta = 0$ Which of the following options describe possible solutions for the homogeneous differential equation?

$$\theta\left(x\right) = A \cdot sinh(m \cdot x) + B \cdot cosh(m \cdot x)$$

$$\theta\left(x\right) = C \cdot e^{mx} + D \cdot e^{-mx}$$

See the derivation:

$$Try: \ \theta(x) = e^{sx} \Rightarrow \frac{\partial^2\theta}{\partial x^2} = s^2 e^{sx}$$

$$s^2 e^{sx} - m^2 e^{sx} = 0$$

$$(s^2 - m^2)e^{sx} = 0, \ note \ that \ e^{sx} \neq 0, \quad for \ x \in R$$

$$s_{1,2} = \pm\sqrt{m^2}$$

$$\rightarrow \theta(x) = Ce^{s_1 x} + De^{s_2 x} = Ce^{mx} + De^{-mx}$$

Remember the mathematical transformations:

$$\sinh\left(x\right) = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$\cosh\left(x\right) = \frac{1}{2}\left(e^x + e^{-x}\right)$$

$$C = \frac{A+B}{2}, \qquad D = \frac{B-A}{2}$$

Resulting in a different form:

$$\theta\left(x\right) = A \cdot \sinh\left(m \cdot x\right) + B\cosh\left(m \cdot x\right)$$