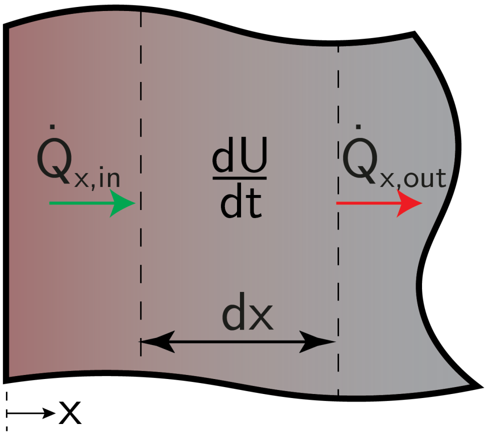
EB - Cond. - IE 17

A very thick wall $\mathrm{Fo} << 1$**, initially at a homogeneous temperature** T_0**, is heated up at the left-hand side. Give the energy balance to derive the heat conduction equation. Assume one-dimensional transient conditions in x-direction at constant atmospheric pressure.**



Energy balance:

$$\frac{\partial U}{\partial t} = \dot{Q}_{x,in} - \dot{Q}_{x,out}$$

For unsteady heat transfer the internal energy will change over time and equals the sum of the in- and outgoing heat fluxes.

Definition of terms:

$$\frac{\partial U}{\partial t} = \rho \cdot c_p \cdot dx \cdot A \cdot \frac{\partial T}{\partial t}$$

The internal energy of a constant volume can be described as: $U = m \cdot c_p \cdot T$.

$$\dot{Q}_{x,in} = -\lambda A \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = -\lambda A \frac{\partial T}{\partial x} + \frac{\partial \dot{Q}_{x,in}}{\partial x} dx$$

The ingoing flux can be described by use of Fourier's equation. The outgoing flux can be approximated by the use of the Taylor series expansion.

Substituting and rewriting:

$$\frac{\partial U}{\partial t} = \dot{Q}_{x,in} - \dot{Q}_{x,out}$$

$$\rho \cdot c_p \cdot dx \cdot A \cdot \frac{\partial T}{\partial t} = -\lambda A \frac{\partial T}{\partial x} + \lambda A \frac{\partial T}{\partial x} - \frac{\partial}{\partial x}\left(-\lambda A \frac{\partial T}{\partial x}\right) dx$$

$$\Rightarrow \rho c_{\mathrm{p}} \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2}$$