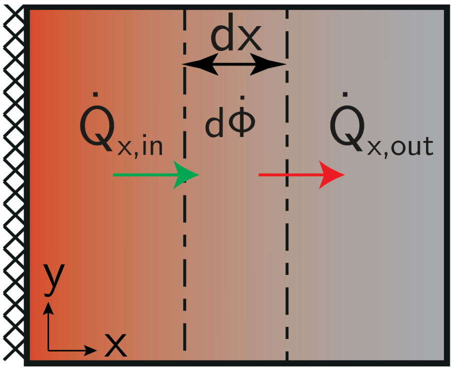
Temperature Function - Cond. - IE 15

The plate is losing heat to the environment. Assume one-dimensional steady-state heat without a source. Provide the temperature function derived from the heat conduction equation.



Given the differential equation and boundary conditions:

$$0 = \lambda \frac{\partial^2 T}{\partial x^2} + \dot{\Phi}'''$$

$$\frac{\partial T}{\partial x}(x = 0) = 0 \qquad\qquad T(x = L) = T_1$$

Solving the equation:

Integrating once yields:

$$\frac{\partial T}{\partial x} = -\frac{\dot{\Phi}'''}{\lambda} \cdot x + C_1$$

Substitution of the first boundary condition at $x = 0$ yields:

$$\frac{\partial T}{\partial x}(x = 0) = C_1 = 0$$

Second time integrating:

$$T(x) = -\frac{\dot{\Phi}'''}{2\lambda} \cdot x^2 + C_2$$

Substitution of the second boundary condition at $x = L$ yields:

$$T(L) = -\frac{\dot{\Phi}'''}{2\lambda} \cdot L^2 + C_2 = T_1 \qquad\qquad \Rightarrow C_2 = T_1 + \frac{\dot{\Phi}'''}{2\lambda} \cdot L^2$$

Substitution of the found values for the integration constants gives:

$$T(x) = \frac{\dot{\Phi}'''}{2\lambda} \left(L^2 - x^2\right) + T_1$$