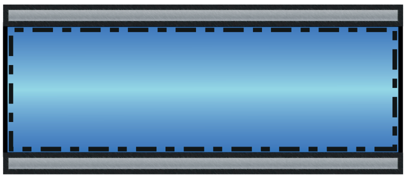
Control Volume - Conv. - Body 1

Water flows through a pipe with an average velocity u, inlet temperature T_1 and a constant wall temperature T_w. Assume steady-state conditions.
Pick the suitable control volume for determining the outlet's mean temperature T_2.

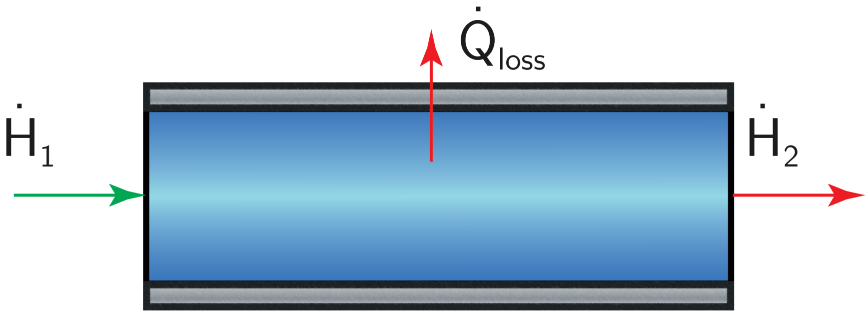


Defining the domain:

The outlet temperature needs to be determined. As we are dealing with steady-state conditions, the outlet temperature can be determined from a global energy balance having the in- and outlet at the boundaries of the domain. Therefore a global energy balance around the entire body is suitable for determining the outlet's mean temperature T_2.

EB - Conv. - Body 1

Water flows through a pipe an average velocity u and inlet temperature T_1. Provide the energy balance to determine the water temperature T_2.
Hint: $T_w < T_2 < T_1$



Energy balance:

$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

Definition of fluxes:

Enthalpies entering and leaving:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

Convective heat losses:

$$\dot{Q}_{\text{loss}} = \alpha \cdot \pi \cdot D \cdot L \cdot \Delta T$$

Logarithmic mean temperature difference:

$$\Delta T = \frac{\dot{m} \cdot c}{\alpha \cdot \pi \cdot D \cdot L}(T_1 - T_2) = \frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

Mass flow rate:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

Substituting and rewriting:

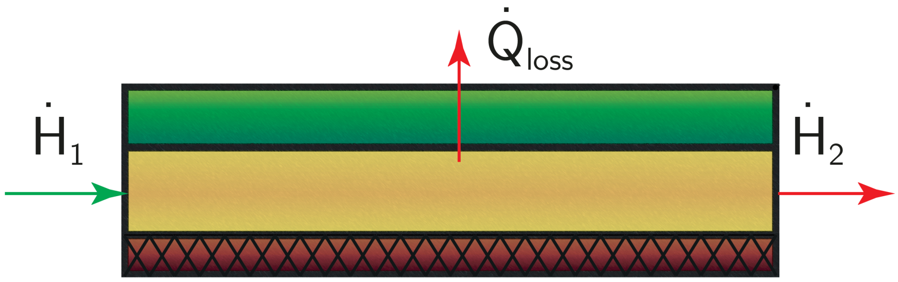
$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

$$u\,\frac{\pi\,D^2}{4}\,\rho\,c\,T_1 - u\,\frac{\pi\,D^2}{4}\,\rho\,c\,T_2 - \alpha\,\pi\,D\,L\,\frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

$$\Rightarrow 0 = \frac{u\rho c\pi D^2}{4}(T_1 - T_2) - \alpha\pi D L\frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

EB - Conv. - Body 2

A project consists to heat up the grass layer (width W, length L) by pumping warm water through a porous membrane. Derive an energy balance to calculate the exit temperature T_2.
Hint:
The overall heat transfer coefficient k fulfils a similar function as the convection heat transfer coefficient α.



Energy balance:

$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

Energy fluxes:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

$$\dot{Q}_{\text{loss}} = k \cdot W \cdot L \cdot \Delta T$$

Logarithmic mean temperature difference:

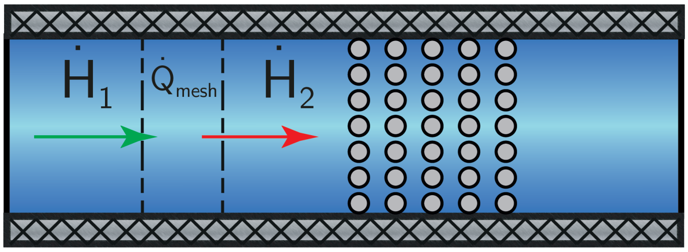
$$\Delta T = \frac{\dot{m} \cdot c}{k \cdot W \cdot L} \left(T_1 - T_2\right) = \frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

Mass flow rate:

$$\dot{m} = u \cdot H \cdot W \cdot \rho$$

EB - Conv. - Body 3

Water flows through a rectangular duct (height H, width W) with a velocity u and inlet temperature T_1. The water is heated with a wire mesh (heating power per surface $\dot{q}''$), located at the beginning of the duct. Behind the mesh a heat exchanger in a pipe bundle configuration is mounted perpendicularly to the flow. Provide the energy balance to determine the water temperature T_2 directly after the wire mesh.



Energy balance:

$$\dot{H}_1 - \dot{H}_2 + \dot{Q}_{\text{mesh}} = 0$$

Energy fluxes:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

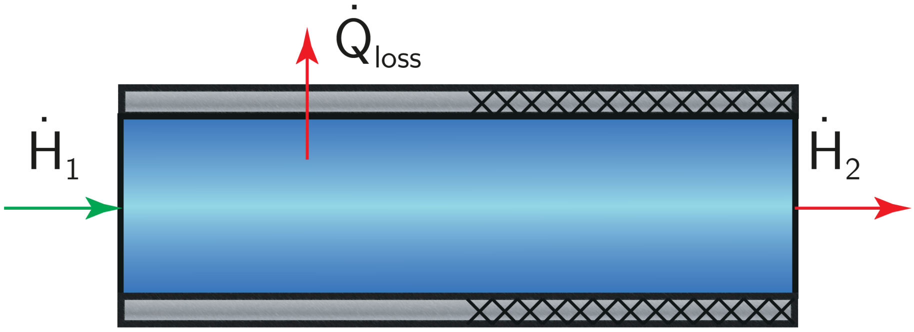
$$\dot{Q}_{\text{mesh}} = H \cdot W \cdot \dot{q}''$$

Mass flow rate:

$$\dot{m} = u \cdot H \cdot W \cdot \rho$$

EB - Conv. - Body 4

Water flows through a pipe an average velocity u and inlet temperature T_1. Provide the energy balance to determine the water temperature T_2.
Hint: $T_w < T_2 < T_1$



Energy balance:

$$\dot{H}_1 - \dot{H}_2 - \dot{Q}_{\text{loss}} = 0$$

Energy fluxes:

$$\dot{H}_1 = \dot{m} \cdot c \cdot T_1$$

$$\dot{H}_2 = \dot{m} \cdot c \cdot T_2$$

$$\dot{Q}_{\text{loss}} = \frac{1}{2} \cdot \alpha \cdot \pi D \cdot L \cdot \Delta T$$

Logarithmic mean temperature difference:

$$\Delta T = \frac{2 \cdot \dot{m} \cdot c}{\alpha \cdot \pi D \cdot L} (T_1 - T_2) = \frac{T_1 - T_2}{\ln\left(\frac{T_1 - T_{\text{w}}}{T_2 - T_{\text{w}}}\right)}$$

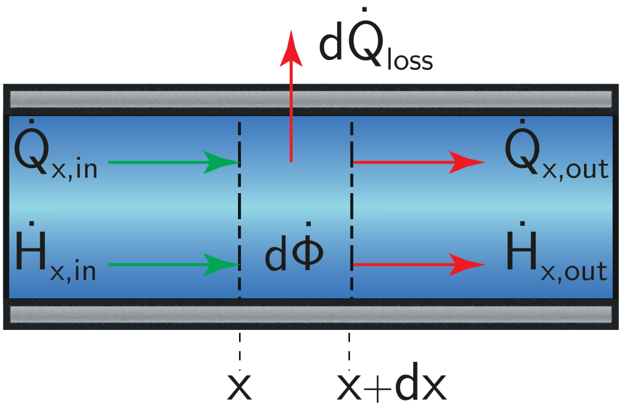
Mass flow rate:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

EB - Conv. - IE 1

Through a very long pipe with diameter D flows a heat generating fluid (homogeneous and constant source strength $\dot{\Phi}''' > 0$). In addition, the pipe has a uniform, constant wall temperature T_w.

Derive the differential equations for the temperature profile in the flow direction, not neglecting the diffusive heat transport in the direction of the flow.



Energy balance:

$$\dot{Q}_{x,in} + \dot{Q}_{x,out} + \dot{H}_{x,in} - \dot{H}_{x,out} + d\dot{Q}_\mathrm{loss} + d\dot{\Phi} = 0$$

Energy fluxes:

$$\dot{Q}_{x,in} = -\lambda \cdot \frac{\pi \cdot D^2}{4} \cdot \frac{\partial T}{\partial x}$$

$$\dot{Q}_{x,out} = \dot{Q}_{x,in} + \frac{\partial \dot{Q}_{x,in}}{\partial x} \cdot dx$$

$$\dot{H}_{x,in} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{x,out} = \dot{H}_{x,in} + \frac{\partial \dot{H}_{x,in}}{\partial x} \cdot dx$$

$$d\dot{Q}_\mathrm{loss} = \alpha \cdot \pi \cdot D \cdot dx \cdot (T - T_\mathrm{w})$$

$$d\dot{\Phi} = \dot{\Phi}''' \cdot \frac{\pi \cdot D^2}{4} \cdot dx$$

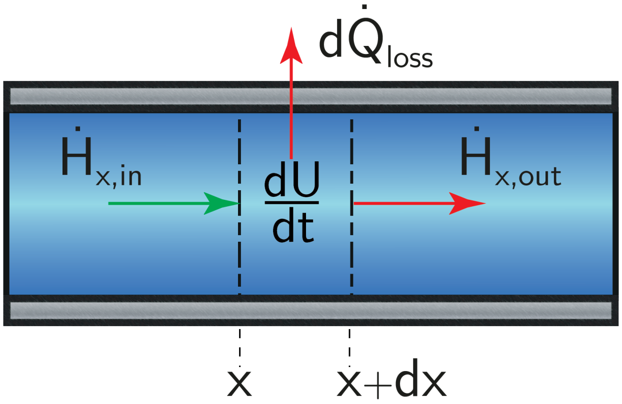
Mass flow rate:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$

EB - Conv. - IE 2

A fluid flows through a long cylindrical tube. A constant heat flux density $\dot{q}''$ is imposed on the fluid.

Derive the transient differential energy balance for the averaged temperature in the fluid, using a stationary coordinate system in the x–direction. Axial heat conduction is negligible in this case.



Energy balance:

$$\dot{H}_{x,in} - \dot{H}_{x,out} + d\dot{Q}_{\text{loss}} = \frac{\partial U}{\partial t}$$

Energy fluxes:

$$\dot{H}_{x,in} = \dot{m} \cdot c \cdot T$$

$$\dot{H}_{x,out} = \dot{H}_{x,in} + \frac{\partial \dot{H}_{x,in}}{\partial x} \cdot dx$$

$$d\dot{Q}_{\text{gain}} = \dot{q}'' \cdot \pi \cdot D \cdot dx$$

$$\frac{\partial U}{\partial t} = \frac{\pi \cdot D^2}{4} \cdot dx \cdot \rho \cdot c \cdot \frac{\partial T}{\partial t}$$

Mass flow rate:

$$\dot{m} = u \cdot \frac{\pi \cdot D^2}{4} \cdot \rho$$