Heat Transfer: Radiation

Example: Radiation transfer between two self-enclosed grey bodies

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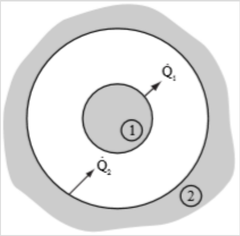
Learning goals

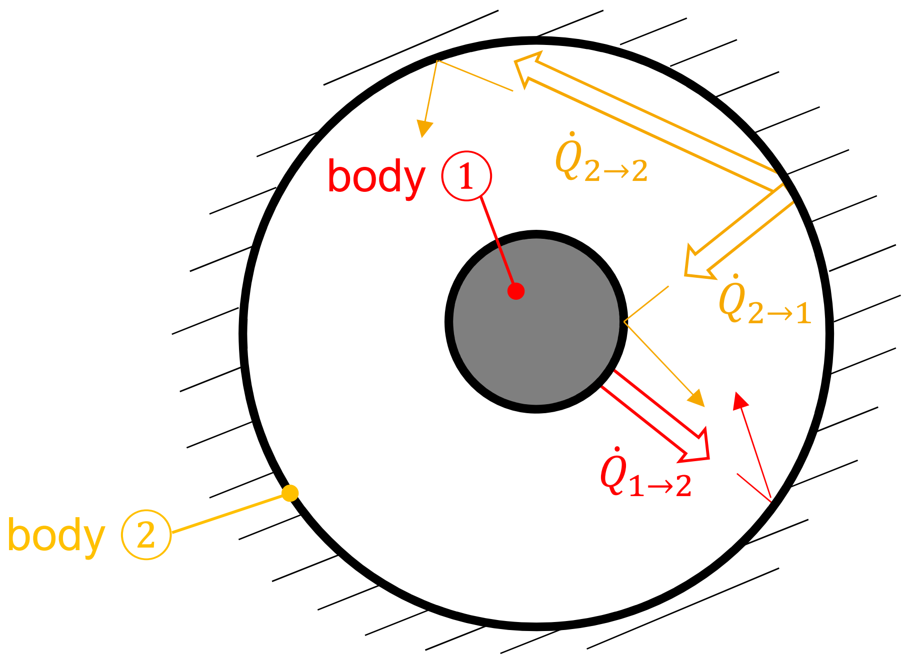
Self-enclosed bodies:

- Learn: Calculation of the radiation exchange for enclosed bodies
- to practice / to use: Approach for solving radiation tasks



[1] Max Planck

Radiation exchange between two self-enclosed grey bodies



Question:

How large is the heat exchange between two self-enclosed grey bodies with respective temperatures T_1 and T_2?

What occurs here?

The radiation emitted and reflected by body 1 (surface brightness body 1) hits body 2.

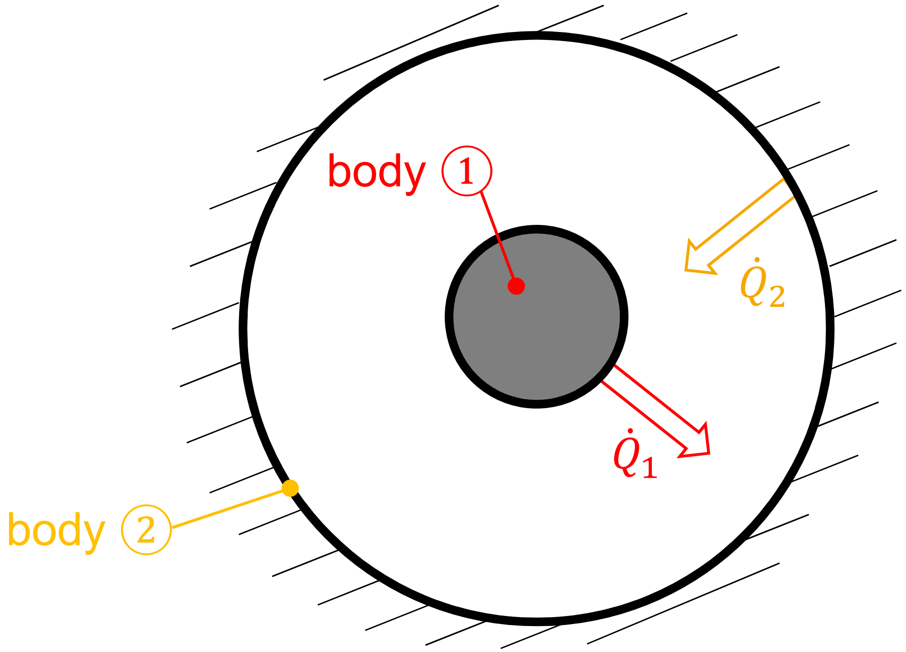
A portion of this radiation is reflected.

All the radiation emitted from body 1 hits body 2.

The radiation emitted by body 2 (surface brightness body 2) hits partially body 1 and body 2.

A portion of this radiation is also reflected.

Approach to solution



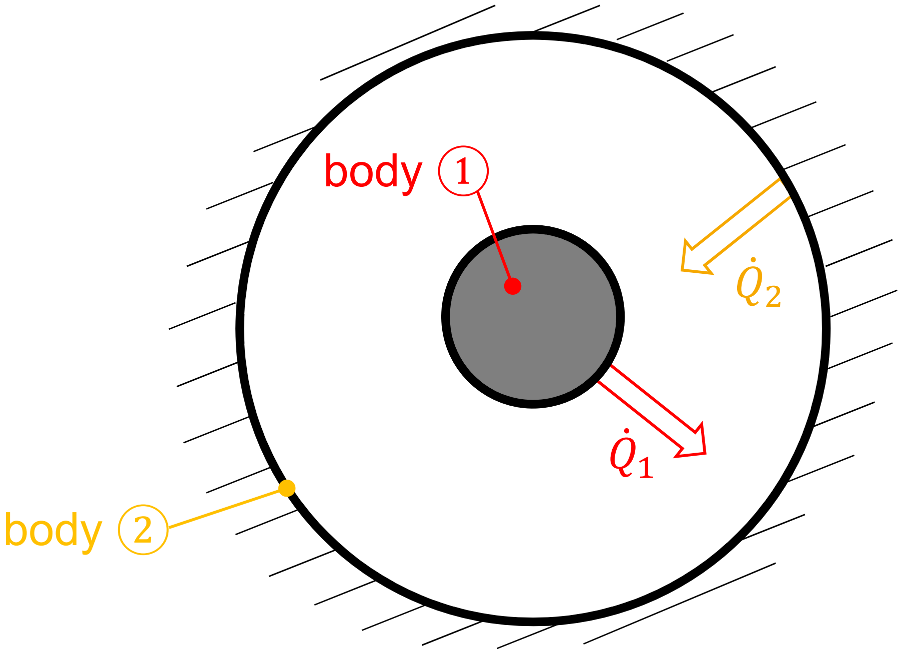
1: Set up the balance

$$\dot{Q}_{1\rightleftarrows 2} = \phi_{12}\dot{Q}_1 - \phi_{21}\dot{Q}_2$$

view factors?

surface brightness?

Approach to solution



1: Set up the balance

$$\dot{Q}_{1\rightleftarrows 2} = \phi_{12}\dot{Q}_1 - \phi_{21}\dot{Q}_2$$

2: Determine the view factors

$$\phi_{11} = 0 \qquad \phi_{12} = 1$$

Reciprocal rule:

$$A_1 \cdot \phi_{12} = A_2 \cdot \phi_{21}$$

$$\phi_{21} = \cancelto{= 1}{\phi_{12}} \cdot \frac{A_1}{A_2}$$

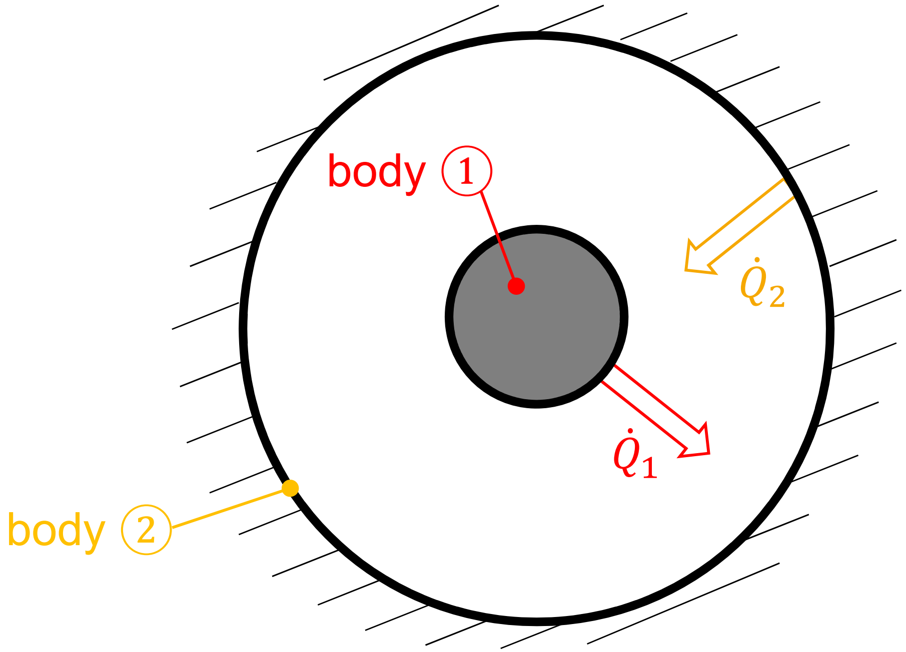
Summation rule:

$$1 = \phi_{21} + \phi_{22}$$

$$\phi_{22} = 1 - \phi_{21}$$

$$\phi_{22} = 1 - \frac{A_1}{A_2}$$

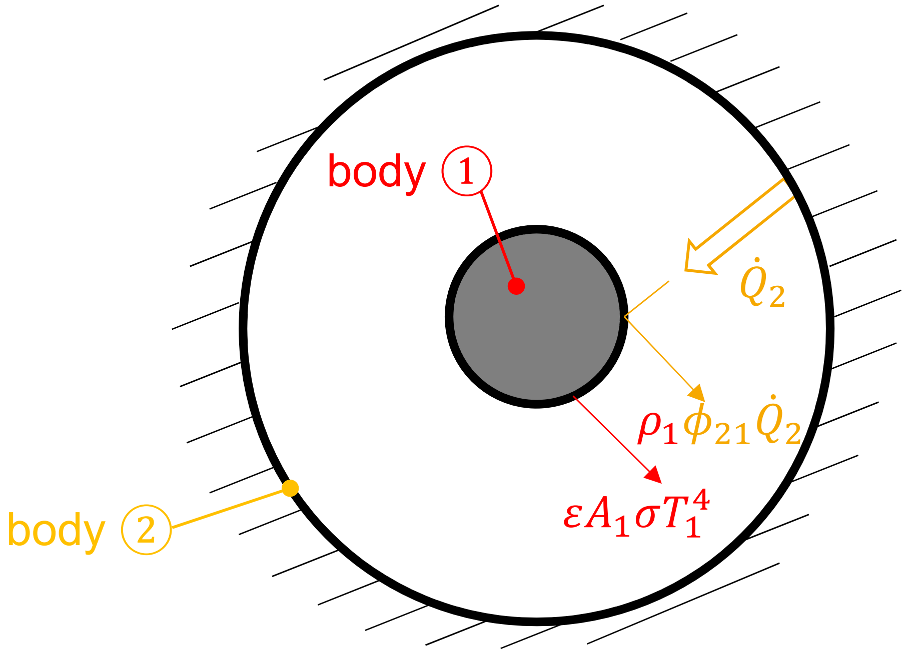
Approach to solution



The surface brightness $\dot{Q}_1$ is the **sum** of the
emission from body 1
and the
(complete or partial) reflection of the
radiation directed
from body 2 to body 1

3: Surface brightness body 1

Approach to solution



The surface brightness $\dot{Q}_1$ is the **sum** of the
emission from body ①
and the
(complete or partial) reflection of the
radiation directed
from body ② to body ①

3: Surface brightness body ①

$$\dot{Q}_1 = \dot{Q}_{1,\varepsilon} + \dot{Q}_{1,\rho}$$

$$= \varepsilon_1 A_1 \dot{q}''_{b1} + \rho_1 \Phi_{21} A_2 \dot{q}''_2$$

not transparent, grey Body $(1-\varepsilon_1)$

$\frac{A_1}{A_2}$

$$\alpha + \rho + \tau = 1$$

↳ $\tau = 0$

$$\dot{Q}_1 = \varepsilon_1 A_1 \dot{q}''_{b1} + (1-\varepsilon_1) \frac{A_1}{A_2} A_2 \dot{q}''_2$$

$\rho = (1-\alpha)$

$$\dot{Q}_1 = \varepsilon_1 A_1 \dot{q}''_{b1} + (1-\varepsilon_1) \frac{A_1}{\cancel{A_2}} \cancel{A_2} \dot{q}''_2$$

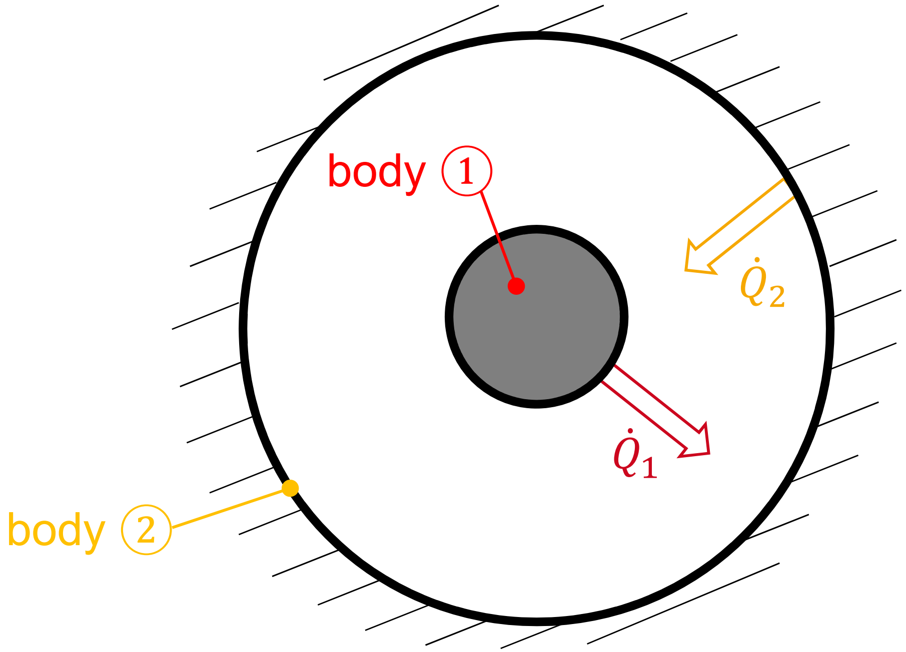
$Kirchhoff; grey$

↳ $\alpha = \varepsilon$

$$\dot{Q}_1 = A_1 [\varepsilon_1 \dot{q}''_{b1} + (1-\varepsilon_1) \dot{q}''_2]$$

↳ $\rho = (1-\varepsilon)$

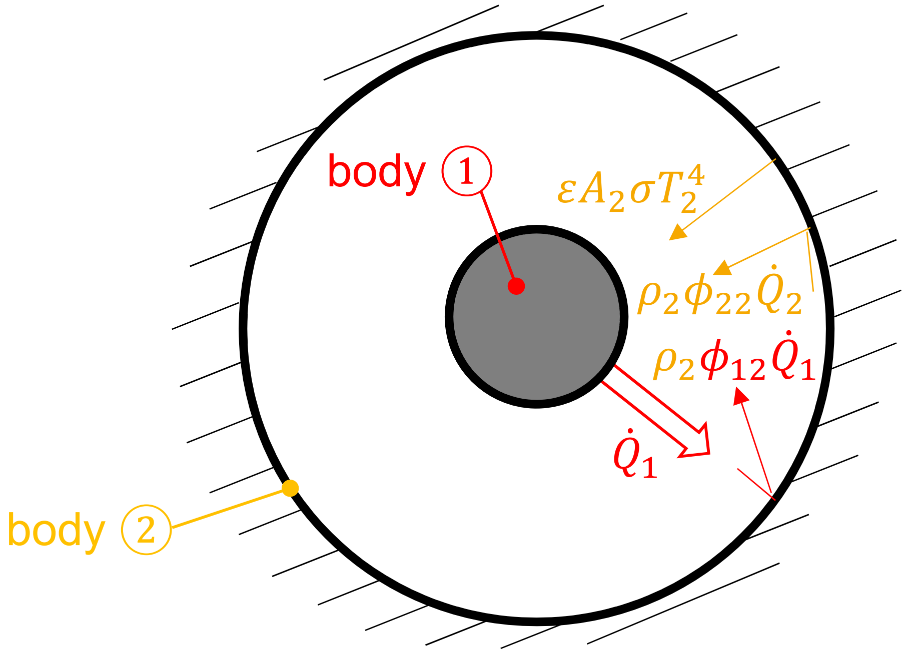
Radiation exchange between two self-enclosed grey bodies



The surface brightness $\dot{Q}_2$ is the sum of the
emission from body 2
and the
partial reflection from body 1 to body 2
and the
partial reflection body 2 to body 2
directed radiation

4: Surface brightness body 2

Radiation exchange between two self-enclosed grey bodies



The surface brightness $\dot{Q}_2$ is the sum of the
emission from body ②
and the
partial reflection from body ① to body ②
and the
partial reflection body ② to body ②
directed radiation

4: Surface brightness body ②

$$\dot{Q}_2 = \dot{Q}_{2,\varepsilon} + \dot{Q}_{2,\rho}$$

$$= \varepsilon_2 A_2 \dot{q}''_{b2} + \underbrace{\rho_2}_{(1-\varepsilon_2)}(\underbrace{\phi_{12}}_{=1} A_1 \dot{q}''_1 + \underbrace{\phi_{22}}_{\left(1-\frac{A_1}{A_2}\right)} A_2 \dot{q}''_2)$$

$$= \varepsilon_2 A_2 \dot{q}''_{b2} + (1-\varepsilon_2) A_1 \dot{q}''_1$$

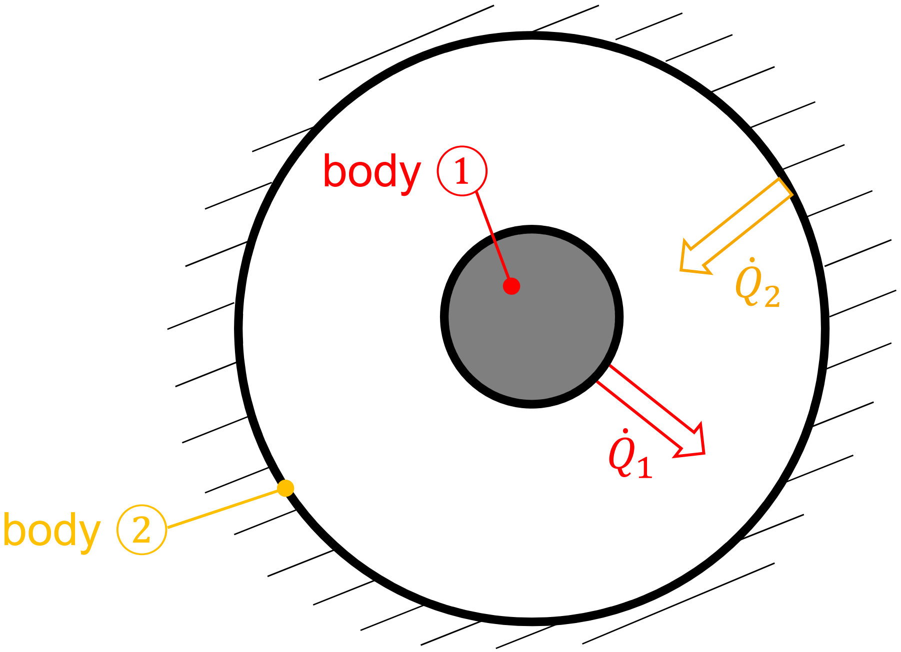
$\boxed{\text{from step 3.}}$

$$+(1-\varepsilon_2)\left(1-\frac{A_1}{A_2}\right) A_2 \dot{q}''_2$$

⋮

$$\dot{Q}_2 = \frac{\varepsilon_2 A_2 \dot{q}''_{b2} + (1-\varepsilon_2) A_1 \dot{q}''_{b1}}{1-(1-\varepsilon_2)(1-\varepsilon_1)\frac{A_1}{A_2} - (1-\varepsilon_2)\left(1-\frac{A_1}{A_2}\right)}$$

Radiation exchange between two self-enclosed grey bodies



1: Balance

$$\dot{Q}_{1\rightleftarrows 2} = \phi_{12}\dot{Q}_1 - \phi_{21}\dot{Q}_2$$

2: View factors

Body 1: $\phi_{11} = 0 \qquad \phi_{12} = 1$

Body 2: $\phi_{21} = \dfrac{A_1}{A_2} \qquad \phi_{22} = 1 - \dfrac{A_1}{A_2}$

3: Surface brightness body 1

$$\dot{Q}_1 = A_1\left[\varepsilon_1 \dot{q}''_{b1} + (1-\varepsilon_1)\frac{A_1}{A_2}\dot{Q}_2\right]$$

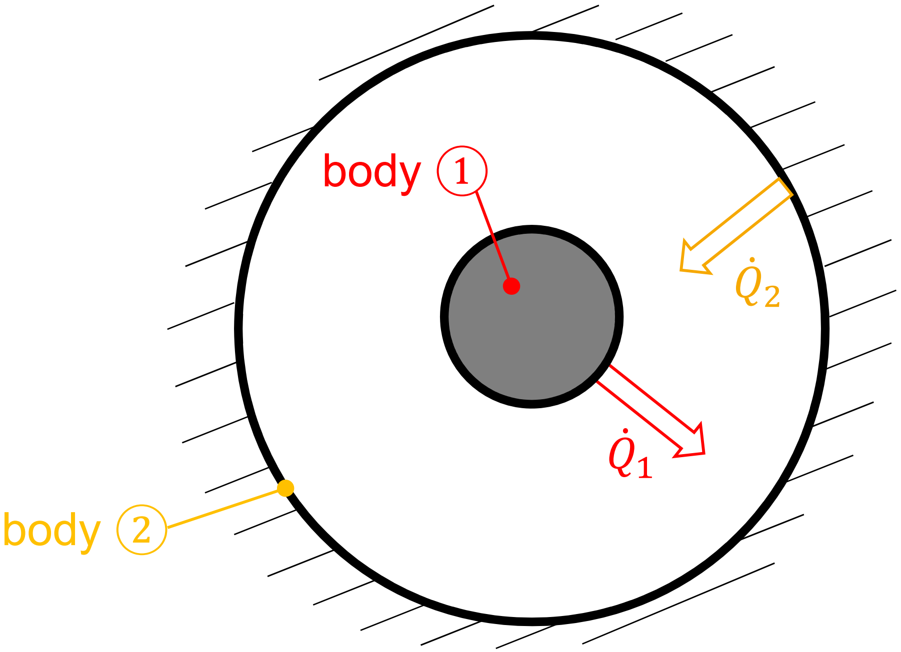
4: Surface brightness body 2

$$\dot{Q}_2 = \frac{\varepsilon_2 A_2 \dot{q}''_{b2} + (1-\varepsilon_2) A_1 \dot{q}''_{b1}}{1-(1-\varepsilon_2)(1-\varepsilon_1)\frac{A_1}{A_2} - (1-\varepsilon_2)\left(1-\frac{A_1}{A_2}\right)}$$

5: Insert step 2, 3 and 4 in 1

$$\dot{Q}_{1\rightleftarrows 2} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

Radiation exchange between two self-enclosed grey bodies



If environment is "large" compared to the enclosed object
→ environment radiates **black**

Marginal cases:

a) Plane geometry $\rightarrow \frac{A_1}{A_2} \approx 1$ → Equation for parallel plates

b) Body 1 ≪ body 2 $\rightarrow \frac{A_1}{A_2} \approx 0 \rightarrow \dot{Q}_{1\rightleftarrows 2} = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4)$

c) Body 2 is black $\rightarrow \varepsilon_2 = 1 \rightarrow \dot{Q}_{1\rightleftarrows 2} = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4)$

Comprehension Questions

For self-enclosed grey bodies, which properties may contribute to increase radiative exchange?

Which marginal cases exist and what is their meaning?