Heat Transfer: Convection

Forced Convection in Internal Flows – HTC in laminar fully developed flows

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HEATQUIZ

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

RWTH AACHEN UNIVERSITY

Learning goals

Forced convection in internal flows:

- Ability to calculate the heat transfer coefficient in laminar flows under fully developed conditions
- Ability to distinguish between different flow configurations and to choose the proper correlation for the HTC

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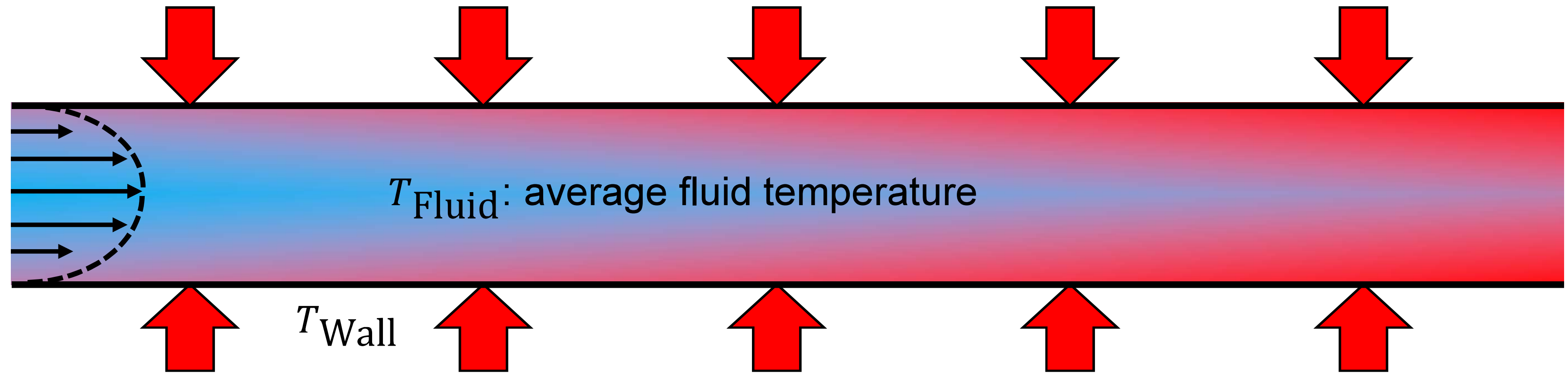
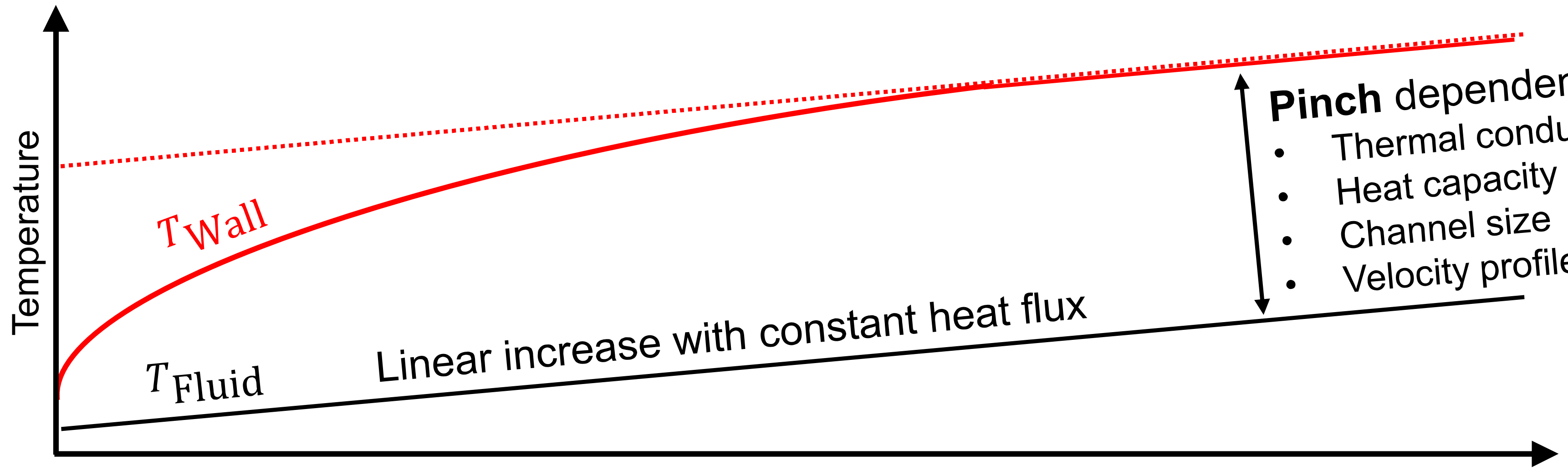
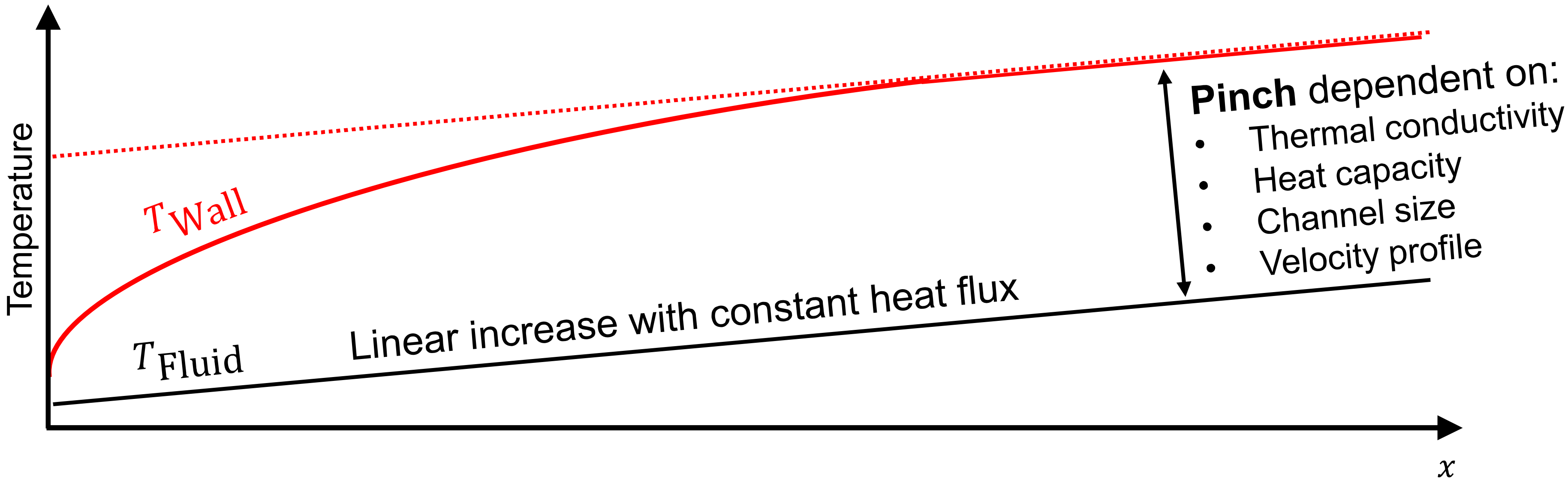
Determine the heat transfer coefficient in a fully developed flow with constant heat flux

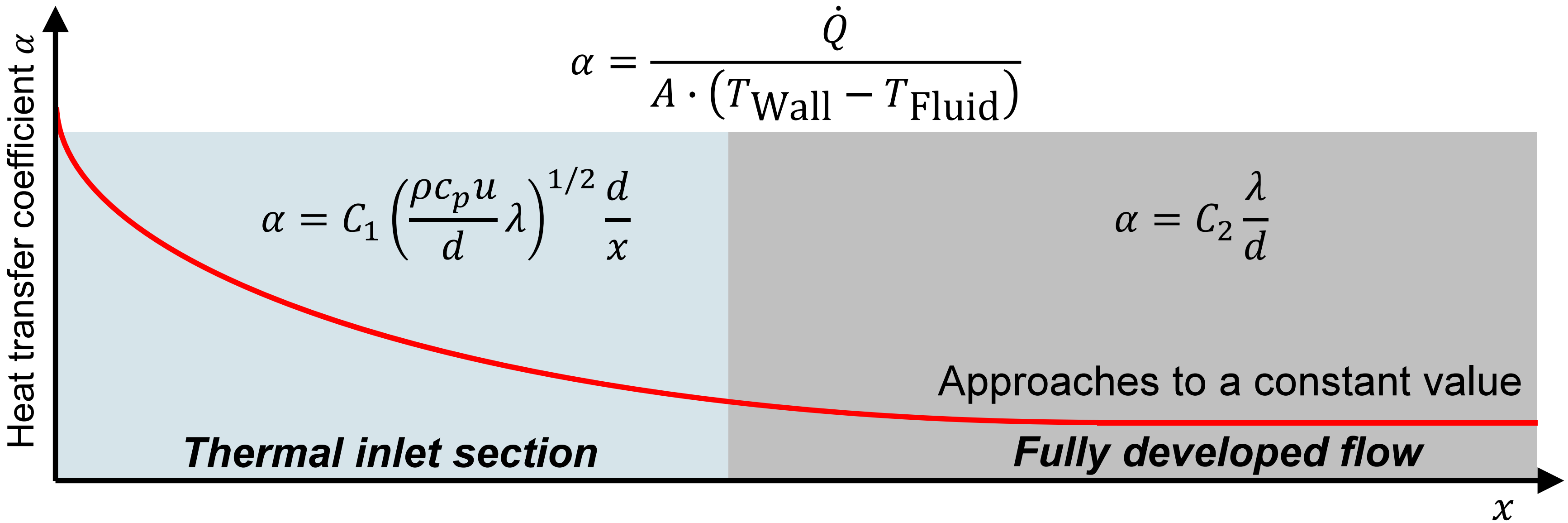




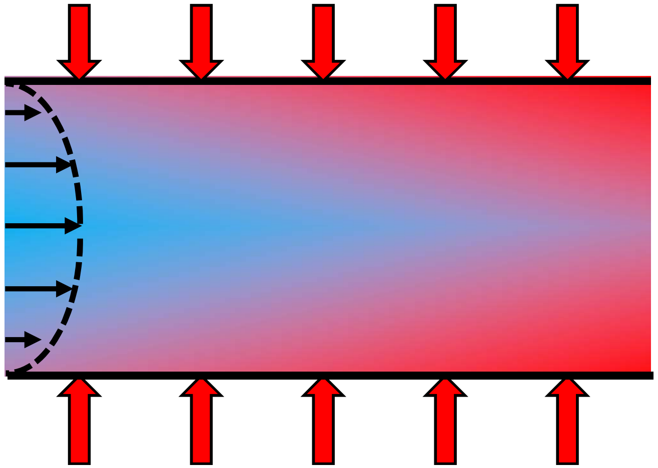
$$\dot{Q} = \alpha \cdot A \cdot (T_{\mathrm{Wall}} - T_{\mathrm{Fluid}}) \qquad \alpha = \frac{\dot{Q}}{A \cdot (T_{\mathrm{Wall}} - T_{\mathrm{Fluid}})}$$

Determine the heat transfer coefficient in a fully developed flow with constant heat flux





Determine the heat transfer coefficient in a fully developed flow with constant heat flux



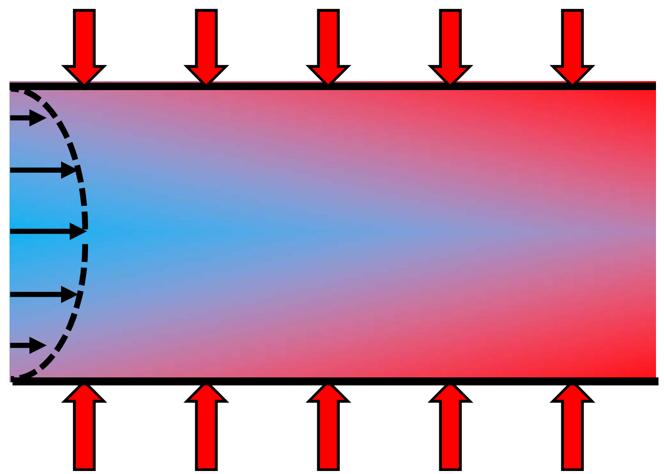
$$\alpha = \frac{\dot{Q}}{A \cdot (T_{\mathrm{Wall}} - T_{\mathrm{Fluid}})}$$

- Heat flux is constant
- T_{Wall} and T_{Fluid} need to be determined

Procedure:

1. Determine the radial temperature profile in the pipe flow
2. Calculate the **caloric mean fluid temperature** T_{Fluid}

Determine the heat transfer coefficient in a fully developed flow with constant heat flux



- ▶ Heat flux is constant
- → Caloric mean temperature increases linearly in axial direction
- → Temperature increases linearly with the same slope at any radial location
- → Temperature gradient in r-direction needs to be constant
- → $\frac{\partial T}{\partial x} = \frac{\dot{q}'' \pi D}{\dot{m} c_p}$ at any location r with $\dot{m} = \rho U \pi R^2$

U = mean velocity

Equation:

- ▶ Governing energy conservation equation (steady state, advection in x-direction, conduction in r-direction):

$$u \rho c_p \frac{\partial T}{\partial x} = \frac{\lambda}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right)$$

1) Temperature profile in a fully-developed pipe with constant heat flux

Equation:

$$u\rho c_p \frac{\partial T}{\partial x} = \frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \qquad \text{with} \qquad \frac{\partial T}{\partial x} = \frac{\dot{q}''2\pi}{\rho U \pi R c_p}$$

And the velocity profile in a laminar pipe flow: $u = 2U\left(1 - \frac{r^2}{R^2}\right)$

Mathematical steps (1):

$$2U\left(1 - \frac{r^2}{R^2}\right)\frac{\dot{q}''\rho c_p \pi 2}{\rho U \pi R c_p} = \frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

U, c_p, π and ρ cancel out

$$4\left(1 - \frac{r^2}{R^2}\right)\frac{\dot{q}''}{R\lambda} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$4\left(r - \frac{r^3}{R^2}\right)\frac{\dot{q}''}{\lambda R} = \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

First integration: $\boxed{\left(r\frac{\partial T}{\partial r}\right) = 4\left(\frac{1}{2}r^2 - \frac{r^4}{4R^2} + C_1\right)\frac{\dot{q}''}{\lambda R}}$

1) Temperature profile in a fully-developed pipe with constant heat flux

Radial temperature gradient:

$$\left(r\frac{\partial T}{\partial r}\right) = 4\left(\frac{1}{2R}r^2 - \frac{r^4}{4R^3} + C_1\right)\frac{\dot{q}''}{\lambda}$$

Boundary condition (symmetry): $\frac{\partial T}{\partial r}(r = 0) = 0 \rightarrow C_1 = 0$

$$\frac{\partial T}{\partial r} = 4\left(\frac{1}{2R}r^1 - \frac{r^3}{4R^3}\right)\frac{\dot{q}''}{\lambda} \Rightarrow \frac{\partial T}{\partial r} = \left(\frac{2}{R}r^1 - \frac{r^3}{R^3}\right)\frac{\dot{q}''}{\lambda}$$

Radial temperature distribution:

Second integration: $T(r) = \frac{\dot{q}''}{\lambda}\left(\frac{1}{R}r^2 - \frac{r^4}{4R^3} + C_2\right)$

Boundary condition (wall temperature): $T(r = R) = T_\mathrm{w}$

$$T_\mathrm{w} = \frac{\dot{q}''}{\lambda}\left(\frac{1}{R}R^2 - \frac{R^4}{4R^3} + C_2\right) \Rightarrow \mathrm{C}_2 = \mathrm{T_W}\frac{\lambda}{\dot{q}''} - R + \frac{R}{4} = \mathrm{T_W}\frac{\lambda}{\dot{q}''} - \frac{3}{4}R$$

1) Temperature profile in a fully-developed pipe with constant heat flux

Radial temperature distribution:

$$T(r) = \frac{\dot{q}''}{\lambda}\left(\frac{1}{R}r^2 - \frac{r^4}{4R^3} + C_2\right) \qquad \mathrm{C}_2 = \mathrm{T_W}\frac{\lambda c_p}{\dot{q}''} - \frac{3}{4}R$$

$$T(r) = T_W + \frac{\dot{q}''}{\lambda}\left(\frac{1}{R}r^2 - \frac{r^4}{4R^3} - \frac{3}{4}R\right)$$

$$\boxed{T(r) = T_W - \frac{\dot{q}''R}{\lambda}\left(\frac{3}{4} - \frac{1}{R^2}r^2 + \frac{r^4}{4R^4}\right)}$$

Radial temperature profile depending on wall temperature

2) Caloric mean temperature in the pipe flow:

- Determine mean temperature of the fluid (mass flow averaged temperature):

$$T_\mathrm{m} = \frac{\int u(r)A(r)T(r)\rho c_p}{\int u(r)A(r)\rho c_p} \quad \text{with} \quad A(r) = 2\pi r dr$$

2) Mass flow averaged temperature in the pipe flow

Mathematical calculations:

$$T_m = \frac{\int \left(T_w - \frac{\dot{q}''R}{\lambda}\left(\frac{3}{4} - \frac{1}{R^2}r^2 + \frac{r^4}{4R^4}\right)\right) \; 2U\left(1 - \frac{r^2}{R^2}\right) 2\pi r dr}{\int \; 2U\left(1 - \frac{r^2}{R^2}\right) 2\pi r dr}$$

$$T_m = \frac{\int \left(T_w - \frac{\dot{q}''R}{\lambda}\left(\frac{3}{4} - \frac{1}{R^2}r^2 + \frac{r^4}{4R^4}\right)\right) \; \left(r - \frac{r^3}{R^2}\right) dr}{\int \left(r - \frac{r^3}{R^2}\right) dr} = T_w \frac{\int \left(r - \frac{r^3}{R^2}\right) dr}{\int \left(r - \frac{r^3}{R^2}\right) dr} - \frac{\dot{q}''R}{\lambda} \frac{\int_0^R \left(\frac{3}{4} - \frac{1}{R^2}r^2 + \frac{r^4}{4R^4}\right)\left(r - \frac{r^3}{R^2}\right) dr}{\int_0^R \left(r - \frac{r^3}{R^2}\right) dr}$$

Integration boundaries:
$r = 0$ and $r = R$

$$T_m = T_w - \frac{11}{24}\frac{\dot{q}''R}{\lambda}$$

Heat transfer coefficient and Nusselt number

Final steps:

$$T_\mathrm{m} = T_\mathrm{w} - \frac{11}{24}\frac{\dot{q}''R}{\lambda}$$

The mean temperature is directly coupled to the wall temperature through the heat flux

$$\dot{q}'' = \frac{24}{11}\frac{\lambda}{R}(T_\mathrm{w} - T_\mathrm{m})$$

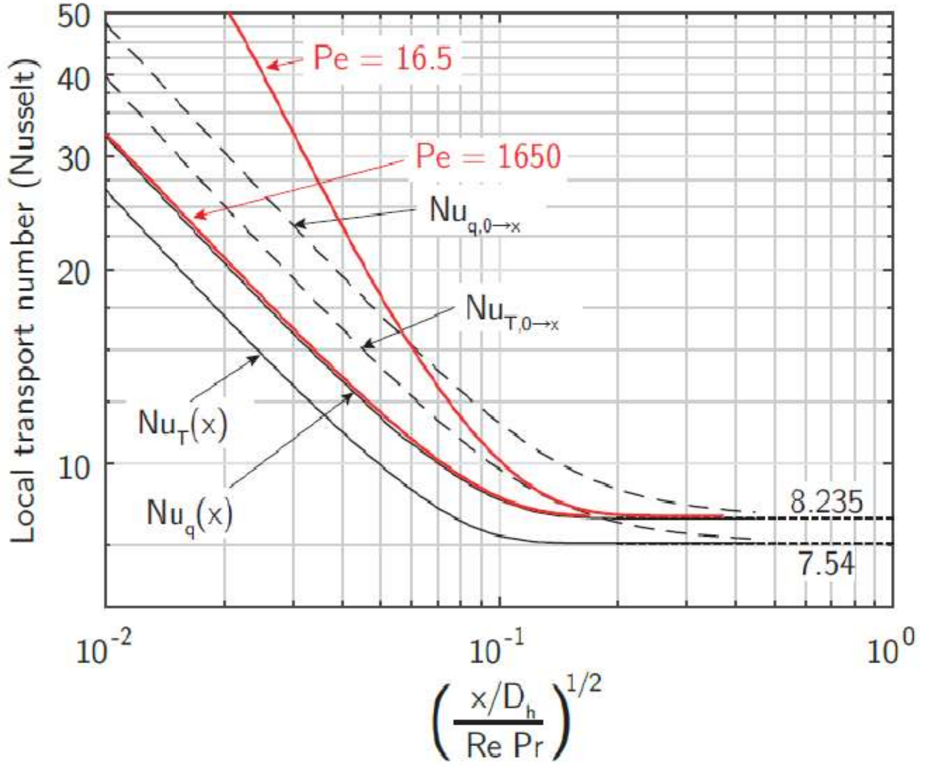
Rearrangement yields for the local heat flux

$$\alpha = \frac{24}{11}\frac{\lambda}{R} = 2.18\frac{\lambda}{R}$$

$$Nu = \frac{\alpha D}{\lambda} = 4.36$$

The fully developed velocity and temperature profile gives a **constant Nusselt number**!

Dimensionless heat transfer laws for a flow between two parallel plates



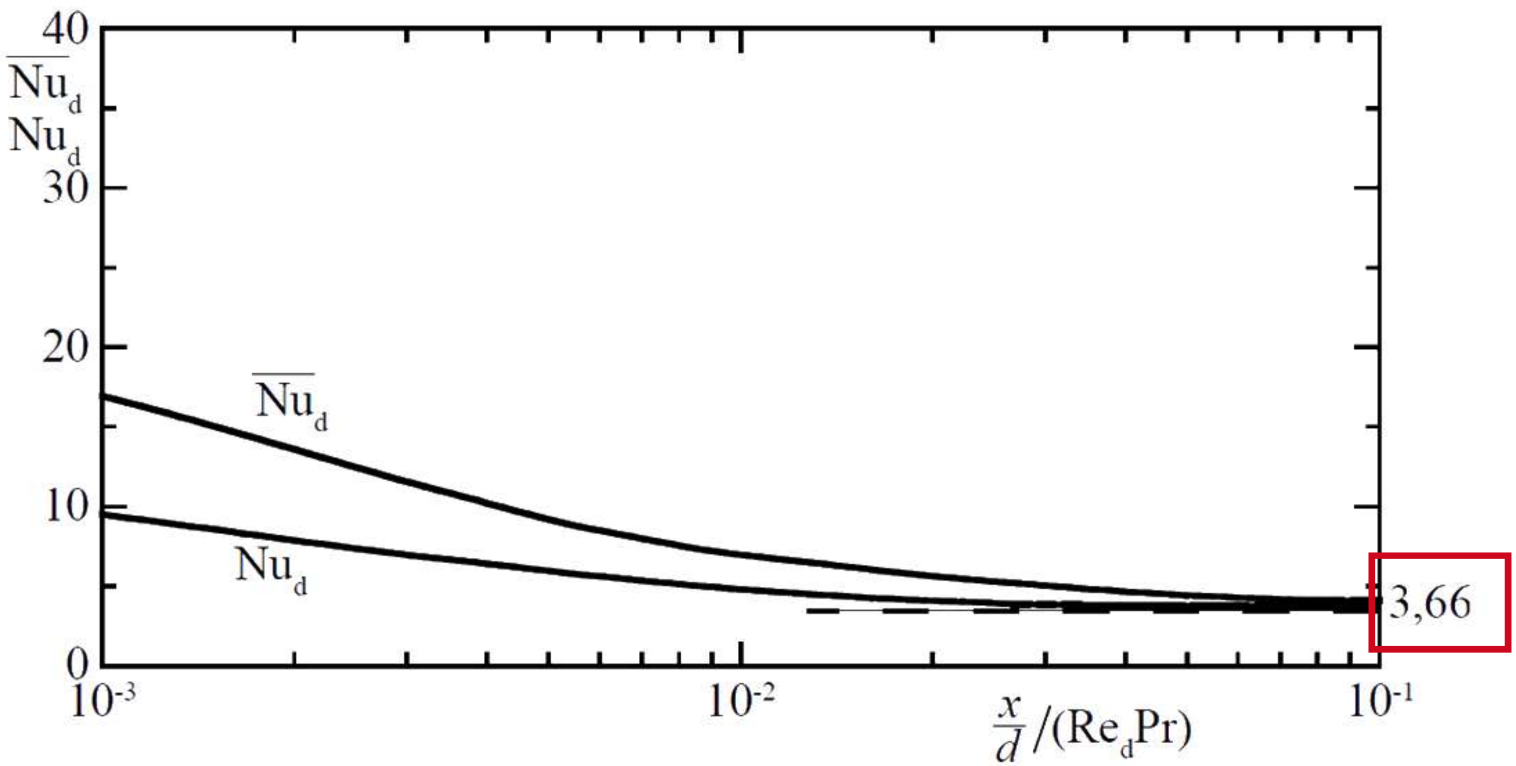
Dimensionless heat transfer laws:

- Nusselt number: $\mathrm{Nu} = \frac{\alpha d}{\lambda}$
- Reynolds number: $\mathrm{Re} = \frac{\rho u d}{\eta}$
- Prandtl number: $\mathrm{Pr} = \frac{\eta c_p}{\lambda}$
- Péclet number: $\mathrm{Pe} = \mathrm{Re} \cdot Pr$

Necessary assumptions:

- Constant fluid properties
  - → no shear thinning
  - → no temperature dependencies

Laminar pipe flow with hydrodynamically developed flow at the beginning of the heated/cooled pipe section and isothermal surface



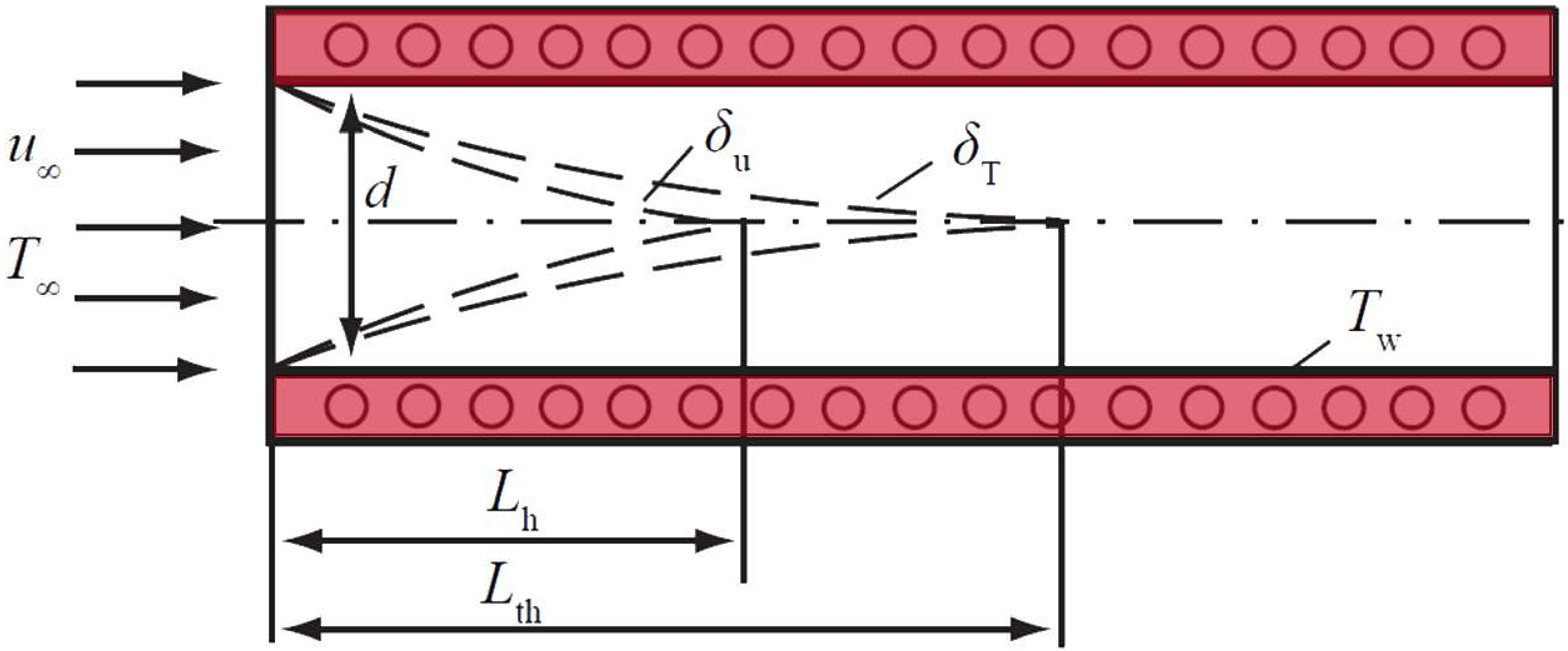
$$\overline{Nu_d} = \left(3.66 + \frac{0.19\left(Re_d Pr \frac{d}{L}\right)^{0.8}}{1 + 0.117\left(Re_d Pr \frac{d}{L}\right)^{0.467}}\right)\left(\frac{\eta}{\eta_w}\right)^{0.14} \qquad \text{(HTC. 12)}$$

Laminar pipe flow with hydrodynamically developed flow at the beginning of the heated/cooled pipe section and constant heat flux

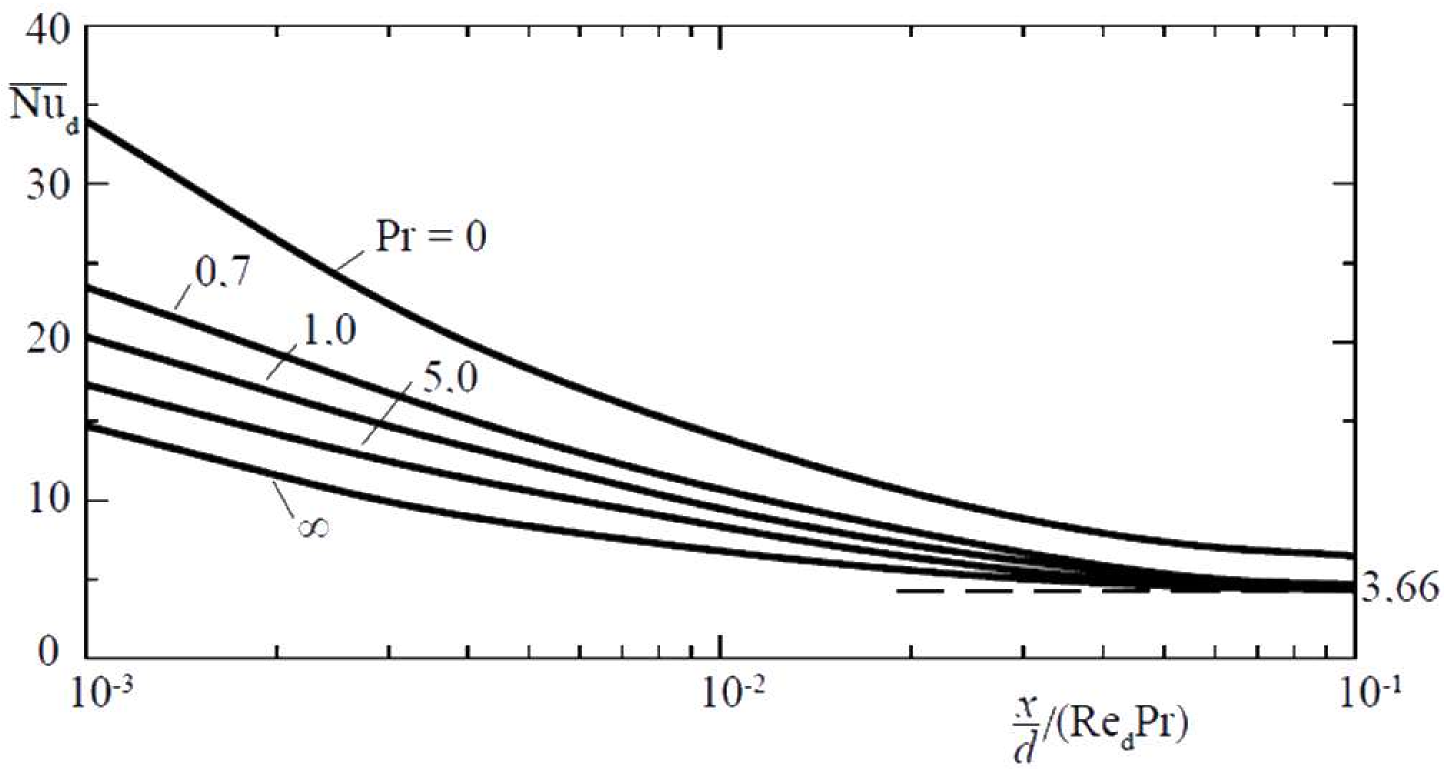
- If the heat flow density at the wall is kept constant instead of the wall temperature, the heat transfer coefficients are about 20% higher

- The final value for long pipes in this case is $\mathrm{Nu}_{\mathrm{d},\infty} = 4.36\left(\frac{\eta}{\eta_{\mathrm{W}}}\right)^{0.14}$

Laminar pipe flow with simutaneous hydrodynamic and thermal inlet and isothermal surface



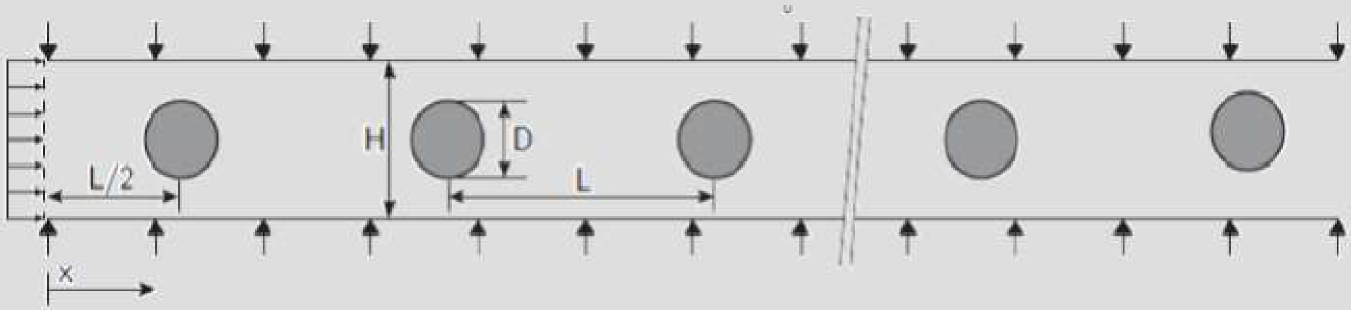
Laminar pipe flow with simutaneous hydrodynamic and thermal inlet and isothermal surface



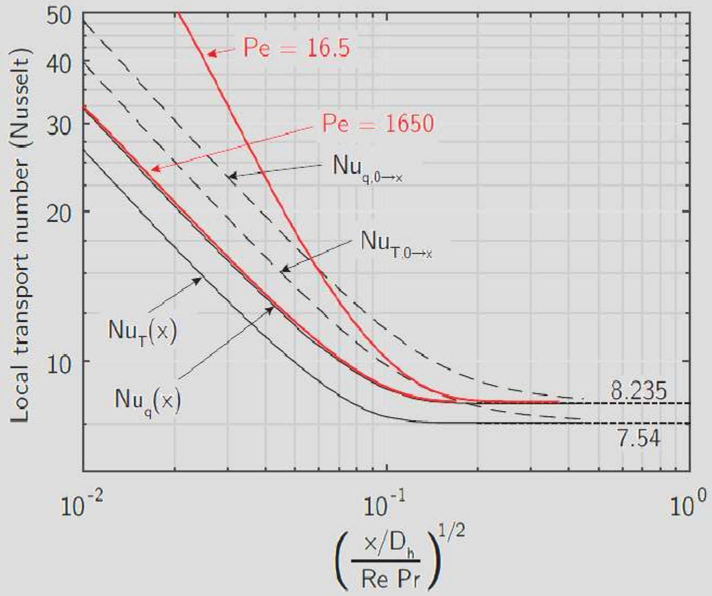
$$\overline{\mathrm{Nu}_\mathrm{d}} = \left(3.66 + \frac{0.0677\left(\mathrm{Re_d}\mathrm{Pr}\frac{d}{L}\right)^{1.33}}{1+0.1\left(\mathrm{Re_d}\mathrm{Pr}\frac{d}{L}\right)^{0.83}}\right)\left(\frac{\eta}{\eta_\mathrm{W}}\right)^{0.14}$$

Influence of complex obstacles on the heat transfer of laminar internal flow

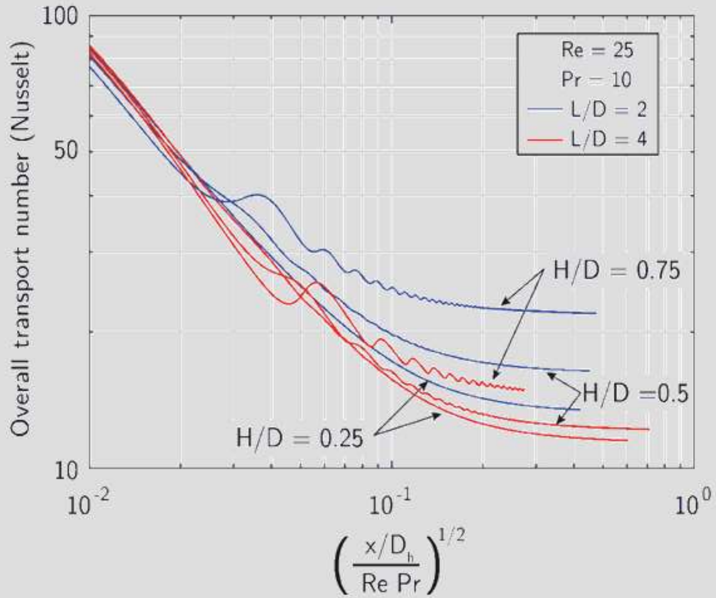
Simulation of a long internal flow - detection of inlet length effects:



Steady state, laminar conditions without obstacles

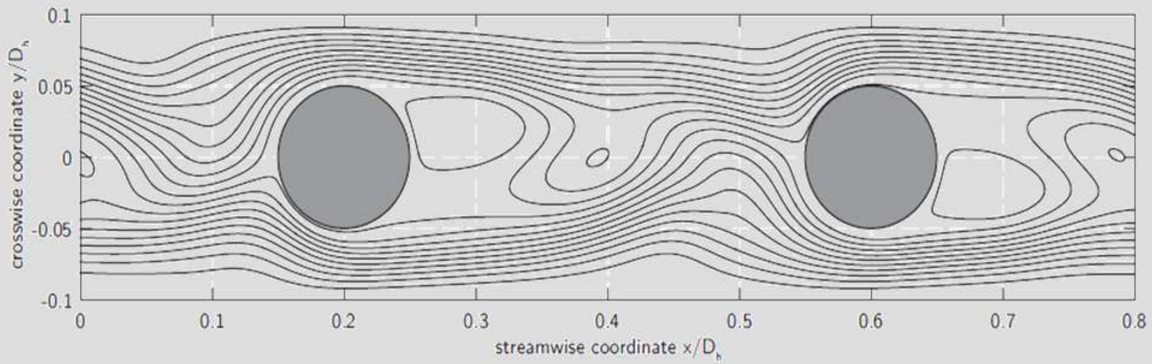


Steady state, laminar flow with obstacles

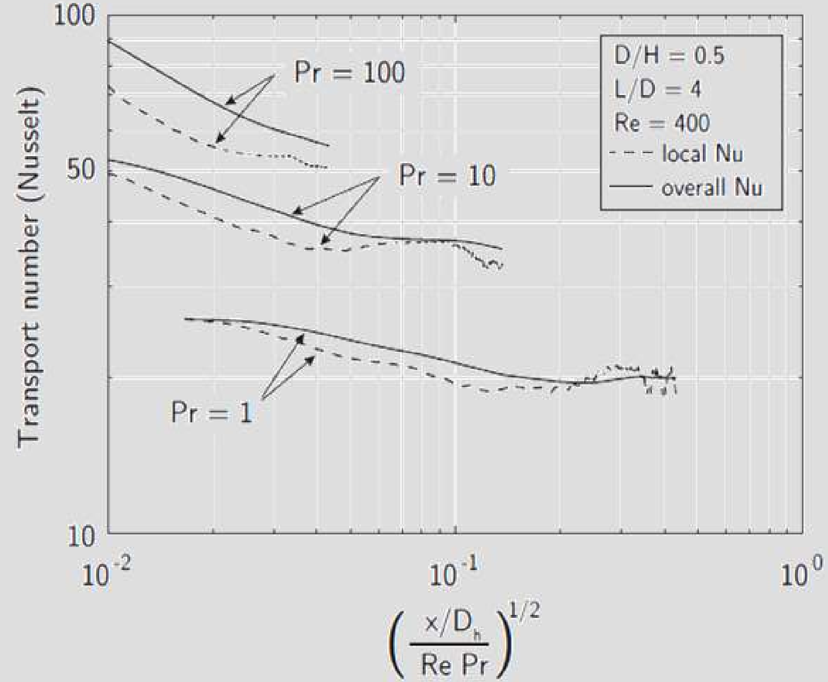


Loss of self-similarity due to transient flow behavior

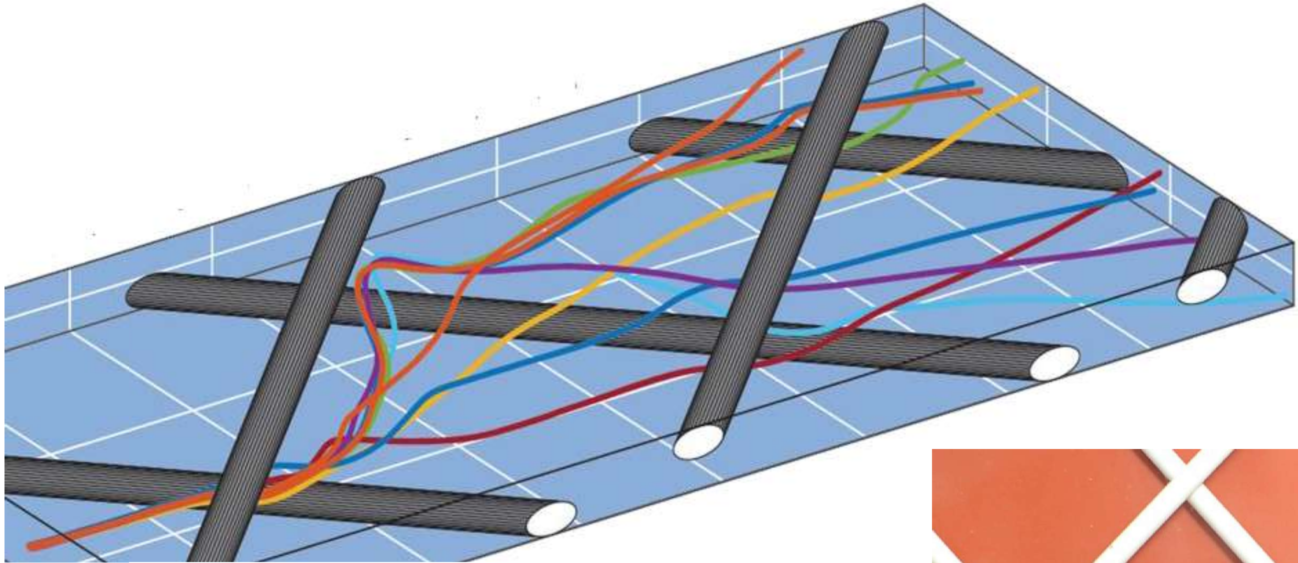
Transient flow behavior and the loss of self-similarity:

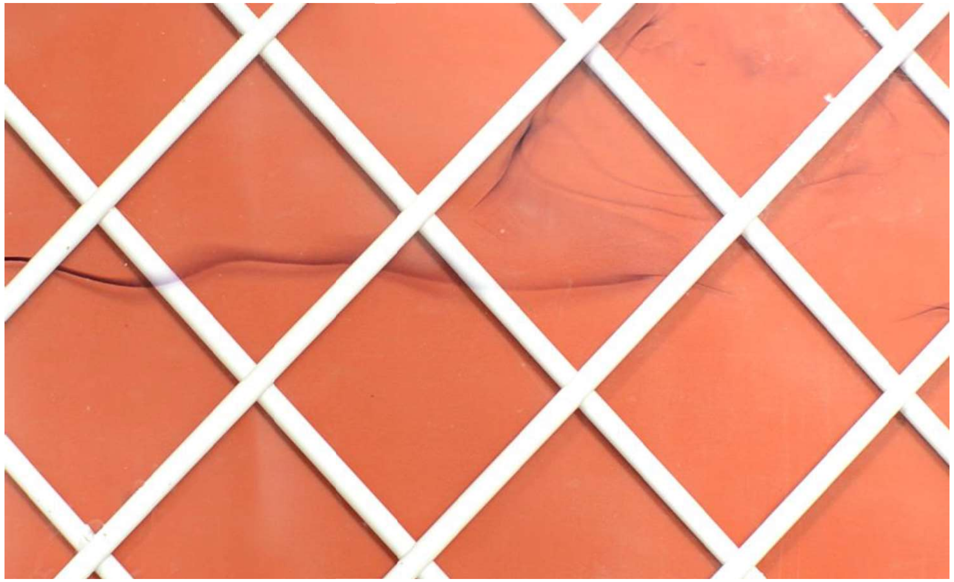


Transient laminar flow with vortex shedding



Loss of self-similarity due to transient flow behavior





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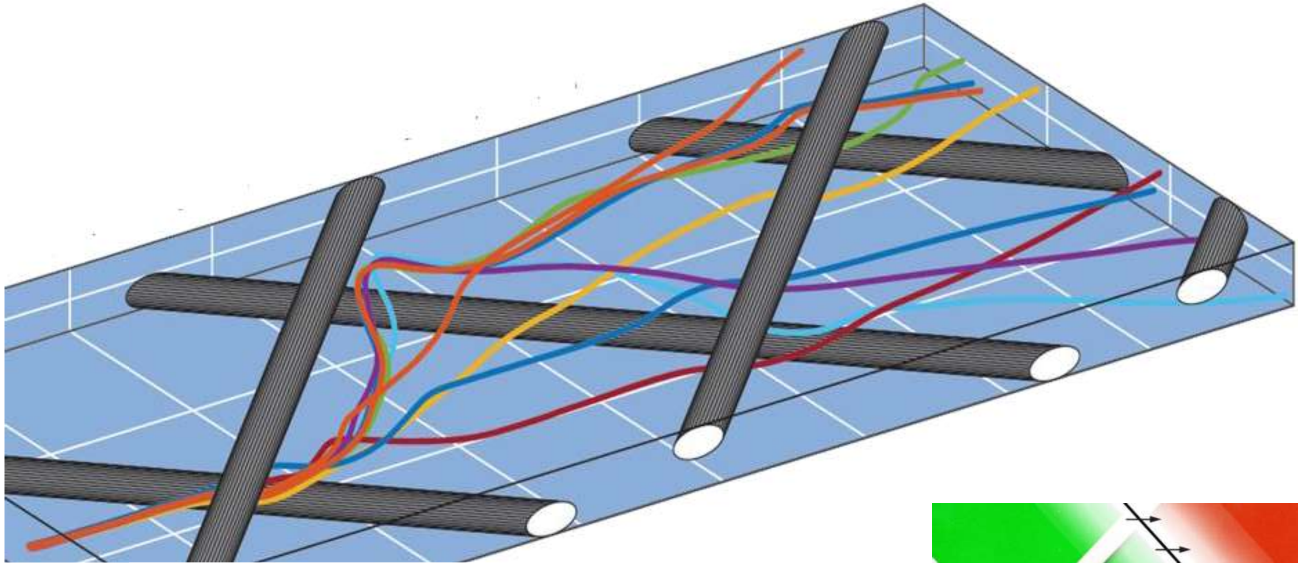
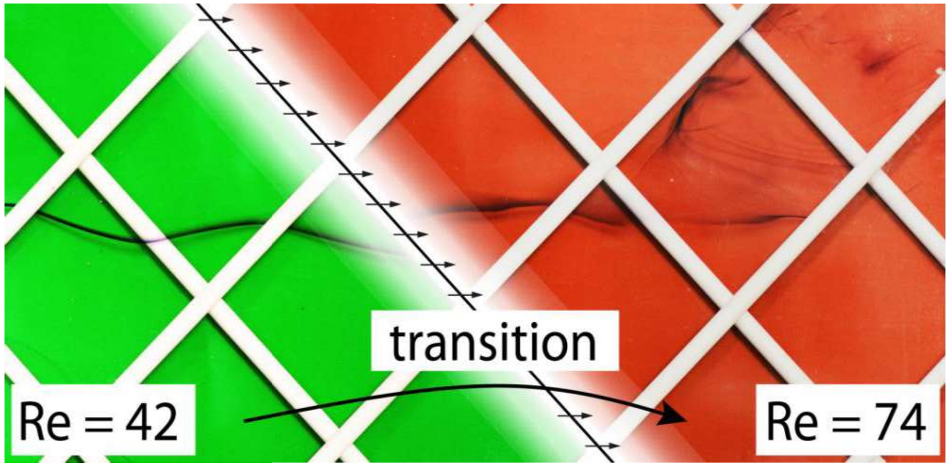
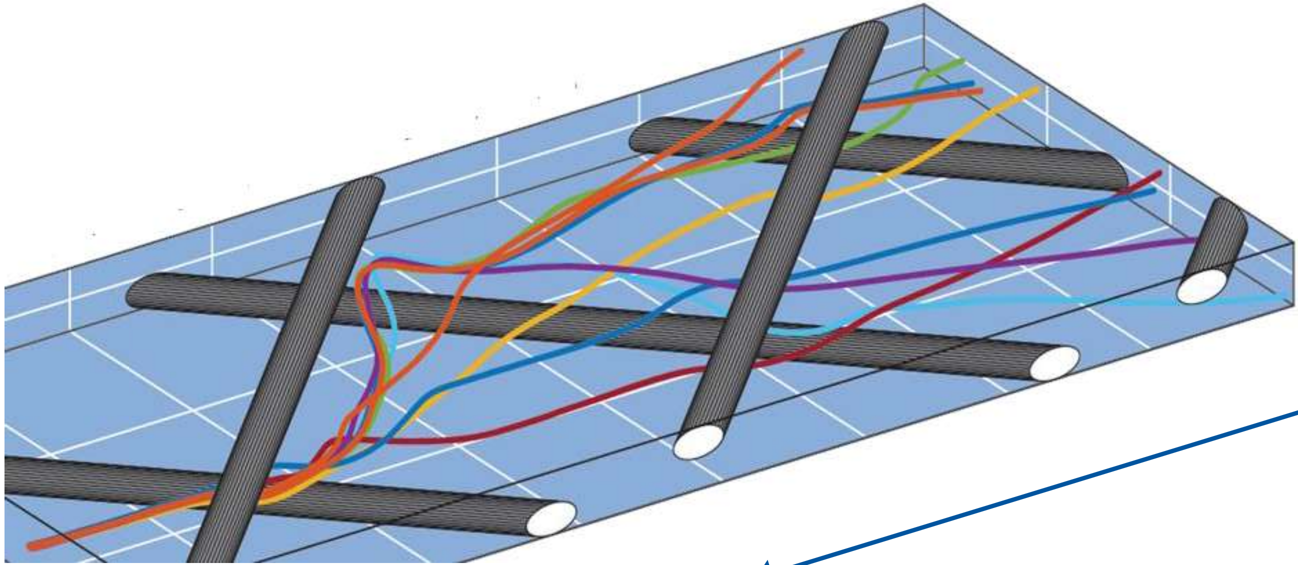
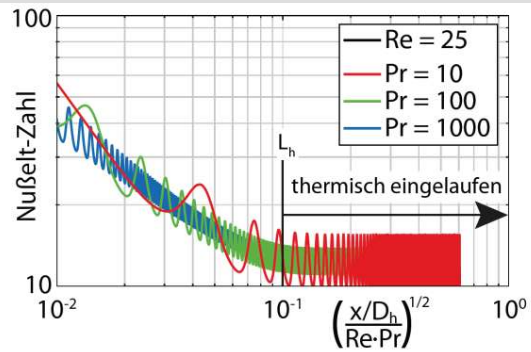




Loss of self-similarity due to transient flow behavior



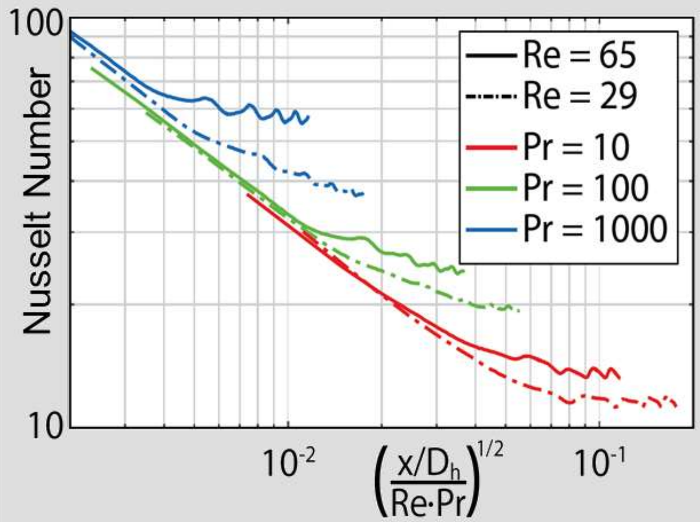
2D Channel:



- ▶ "classical" scaling is lost
  - → additional degree of freedom
  - → diverging paths

Local heat transfer:

- ▶ Dependent on Prandtl number



Turbulent pipe flow at isothermal wall temperature

$$\overline{\mathrm{Nu}_\mathrm{d}} = 0.0235\left(\mathrm{Re}_d^{0.8} - 230\right)\left(1.8\mathrm{Pr}^{0.3} - 0.8\right)\left(1 + \left(\frac{d}{L}\right)^{\frac{2}{3}}\right)\left(\frac{\eta}{\eta_\mathrm{W}}\right)^{0.14} \qquad \text{(HTC. 14)}$$

For $0.6 < \mathrm{Pr} < 500$ $\quad \frac{L}{d} > 1$

and $\mathrm{Re}_\mathrm{d} > 2300$

- In many cases, instead of the equation (HTC.14), the simpler relationship is sufficient:

$$\overline{\mathrm{Nu}_\mathrm{d}} = 0.027\mathrm{Re}_\mathrm{d}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\eta}{\eta_\mathrm{W}}\right)^{0.14} \qquad \text{(HTC. 15)}$$

For $\mathrm{Re}_\mathrm{d} > 10^5;\ 0.7 \leq \mathrm{Pr} \leq 16{,}700\ ;\ \frac{L}{d} > 10$

Comprehension questions

Why is the HTC constant in the fully developed region of an internal flow?

What are the major steps to calculate the HTC in the fully developed region?

What can result in a loss of self-similarity of the heat transfer behavior?

Extra tasks:

Proof that the Nusselt number for a laminar flow between two parallel plates with a constant heat flux boundary condition is Nu = 8.235.

Think about another geometry/flow configuration for which you can determine a laminar velocity profile analytically and calculate the Nusselt number.