Mass Transfer

Advective mass transport and derivation of conservation equations

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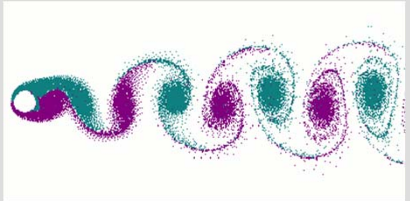
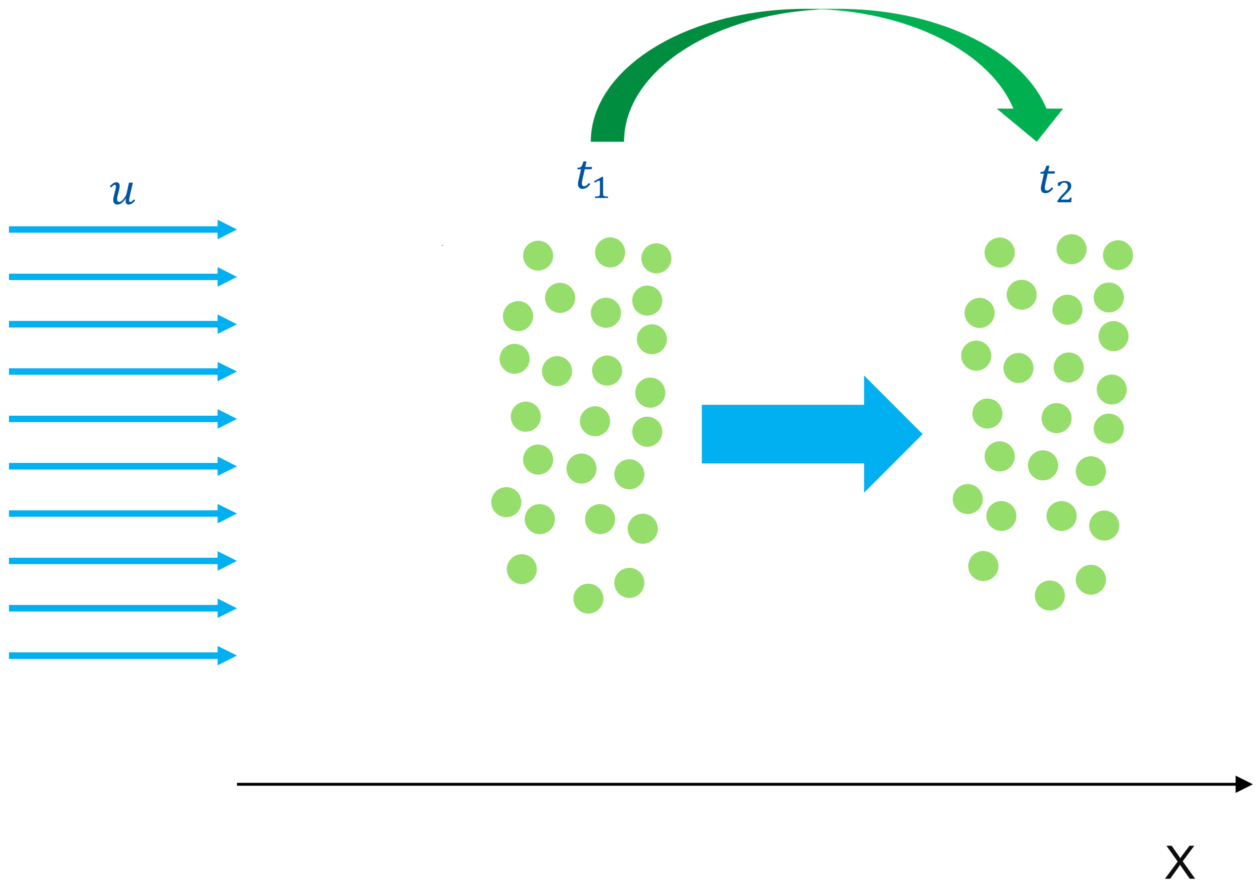
Learning goals

Mass transfer in a moving system:

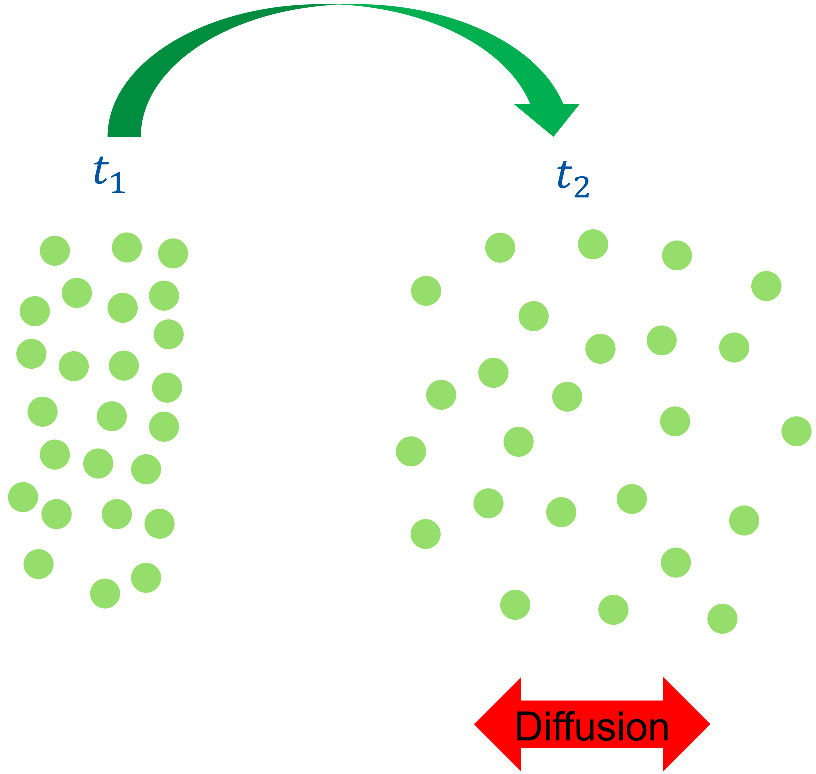


- Differentiation between diffusive and advective mass transport
- Understand the concept of mass average velocity and component velocity
- Learn the relevant dimensionless numbers and the analogy to heat transfer

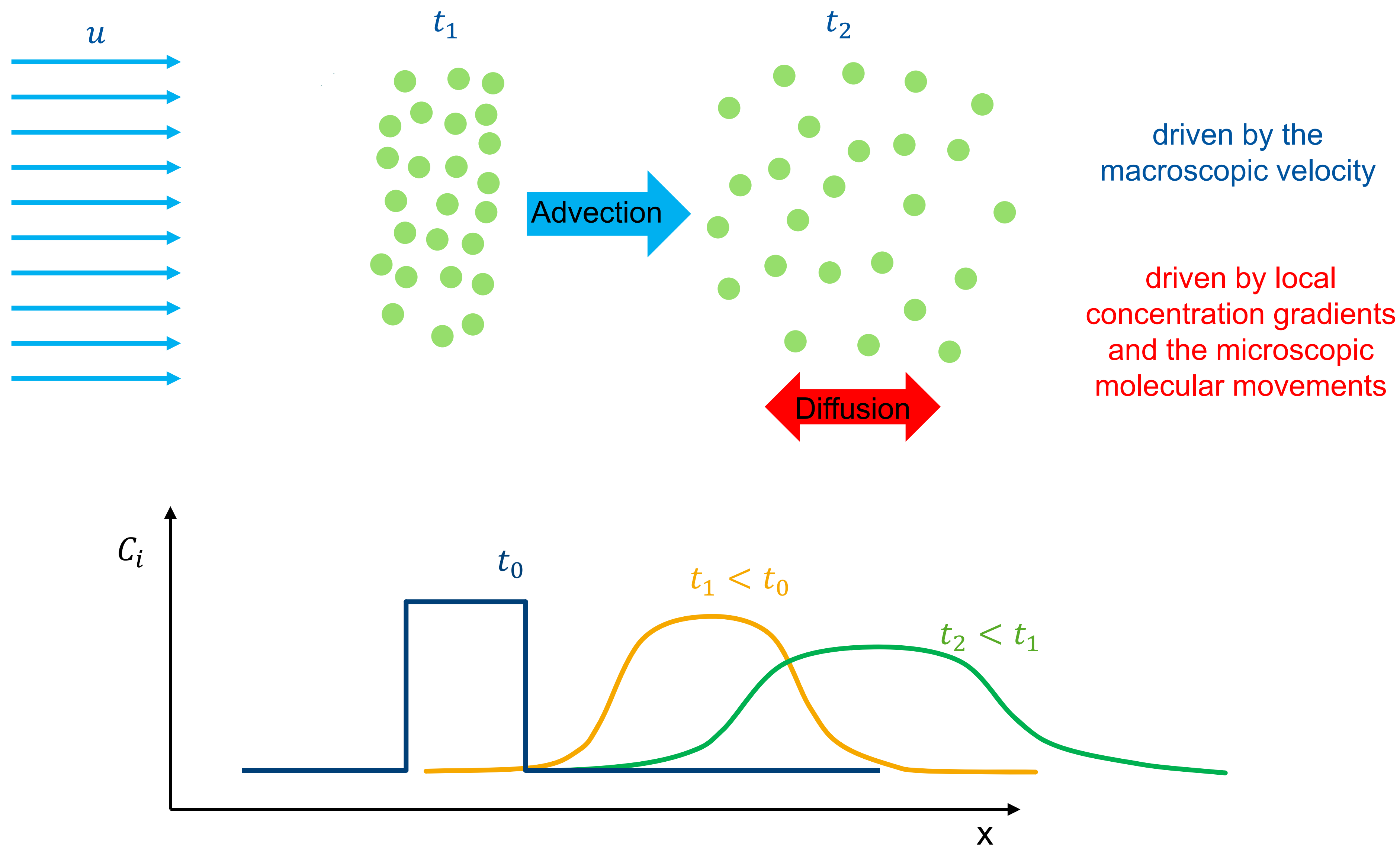
Mass transfer by advection



Mass transfer by diffusion



Mass transfer by advection and diffusion

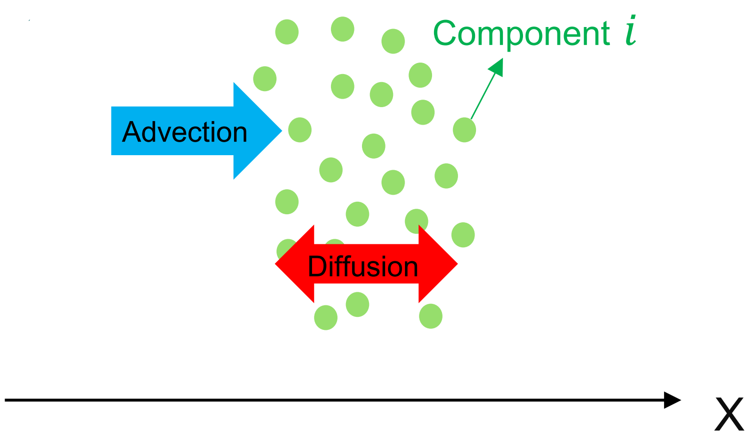


Mass transfer in a moving system

Transport mechanism:

Mass transport of component i = Mass transport of component i by advection + Mass transport of component i by diffusion

$$\boxed{\dot{m}_i'' = \dot{m}''_{i,\mathrm{Adv}} + j''_{i,\mathrm{Diff}}}$$

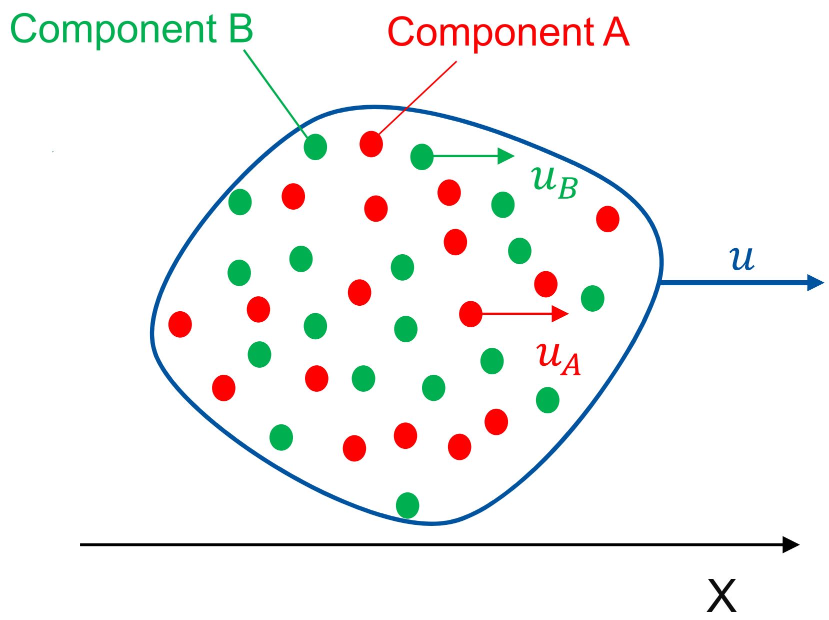


- **Advection**: Mass transport in flow direction as a result of fluid flow
- **Diffusion**: Mass transport in all directions as a result of concentration gradients and microscopic molecular movements

Mass transfer in a moving system

Assumptions:

- Consider mass flows component by component
- Distinguish between diffusive and advective transport



- u_A: Average velocity of the component A
- u_B: Average velocity of the component B
- u: Average flow velocity (velocity of the total mass flow)

Mass transfer in a moving system

Calculation of the average total velocity:

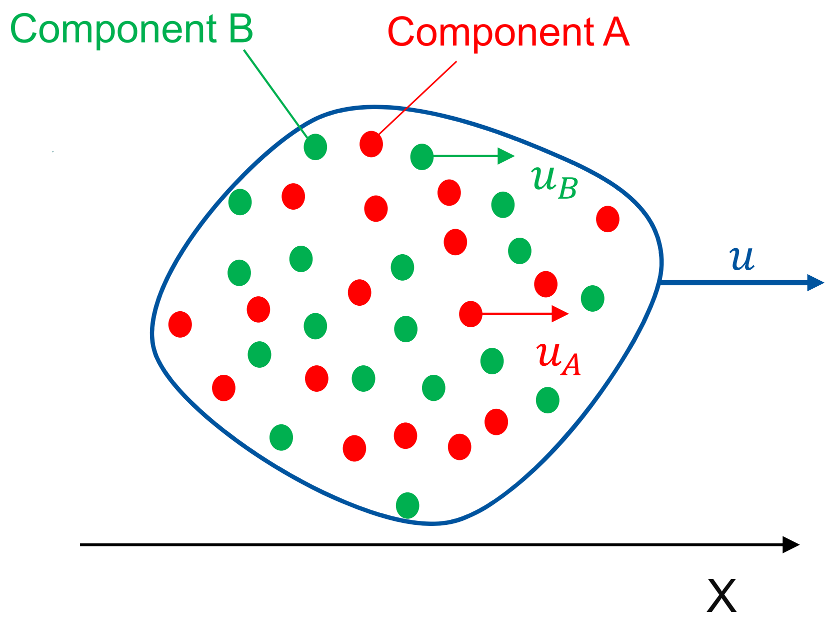
$$\dot{m}_{\mathrm{tot}} = \dot{m}_A + \dot{m}_B$$

$$\bar{\rho}\, u = \rho_A u_A + \rho_B u_B$$

$$u = \overbrace{\frac{\rho_A}{\bar{\rho}}}^{\xi_A} u_A + \overbrace{\frac{\rho_B}{\bar{\rho}}}^{\xi_B} u_B$$

$$\boxed{\mathrm{u} = \sum_{\mathrm{i}} \xi_{\mathrm{i}} \mathrm{u}_{\mathrm{i}}}$$

$\boldsymbol{u}$ is a mass averaged velocity



The total mass flow is the sum of advective and diffusive mass flow:

$$\dot{m}_i'' = \dot{m}''_{i,\mathrm{Adv}} + j''_{i,\mathrm{Diff}}$$

Mass transfer in a moving system

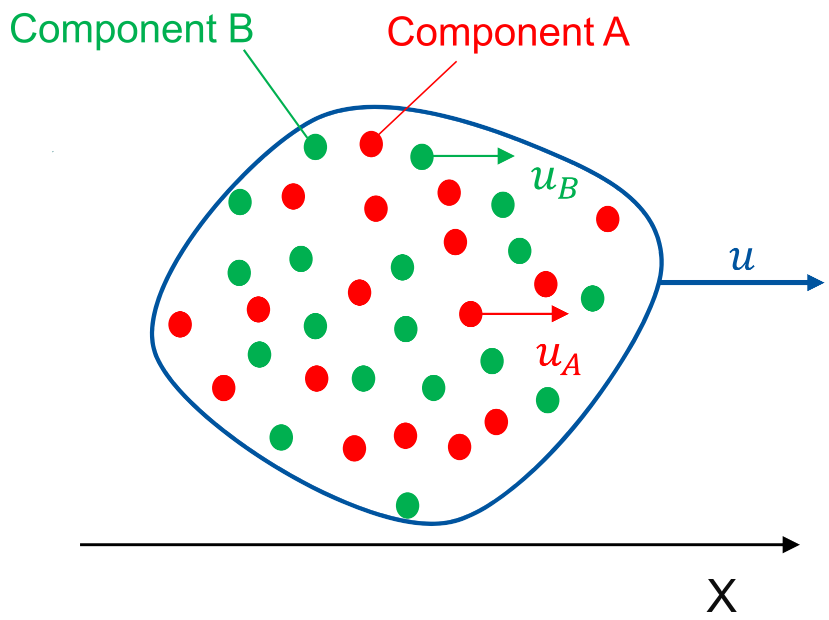
Determination of the diffusion flow:

$$\dot{m}''_i = \dot{m}''_{i,\mathrm{Adv}} + \dot{J}''_{i,\mathrm{Diff}}$$

$$\rho_i u_i = \rho_i u + j''_i$$

$$\boxed{j''_i = \rho_i (u_i - u)}$$

The diffusion velocity is the deviation of the component velocity from the average total velocity



Mass transfer in a moving system

Determination of the diffusion flow:

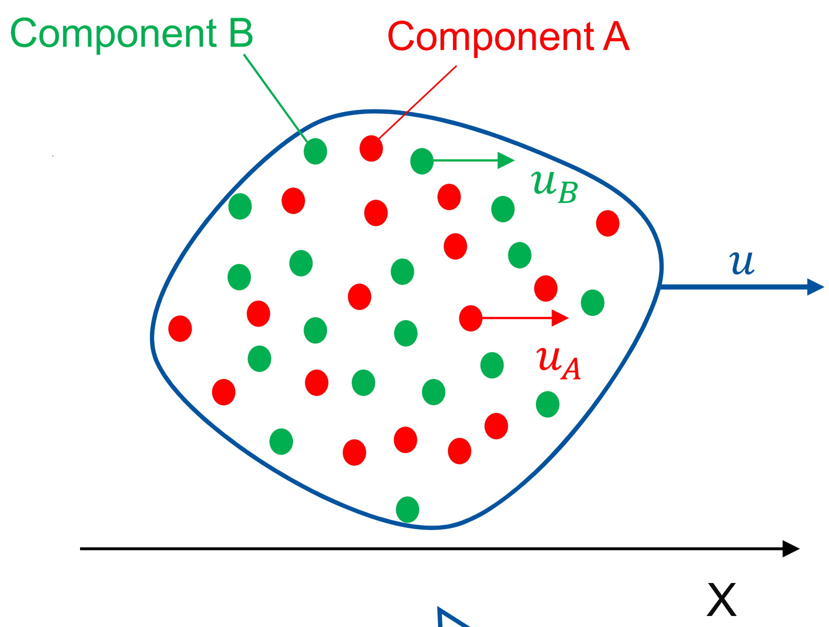
$$\dot{m}''_i = \dot{m}''_{i,\mathrm{Adv}} + j''_{i,\mathrm{Diff}}$$

$$\rho_i u_i = \rho_i u + j''_i$$

$$\boxed{j''_i = \rho_i (u_i - u)}$$

Total diffusion flow as the sum of all components:

$$\sum_{i=1}^{n} j''_i = \sum_{i=1}^{n} \rho_i \, u_i - \boxed{\sum_{i=1}^{n} \rho_i \, u} = \bar{\rho}\, u = \sum_i \rho_i \, u_i \Rightarrow \boxed{j''_i = \rho_i (u_i - u)}$$



All diffusion flows add up to zero

Example: 1D steady state flow without sources

Balance around the control volume:

$$0 = \dot{m}_{i,x} - \dot{m}_{i,x+dx}$$

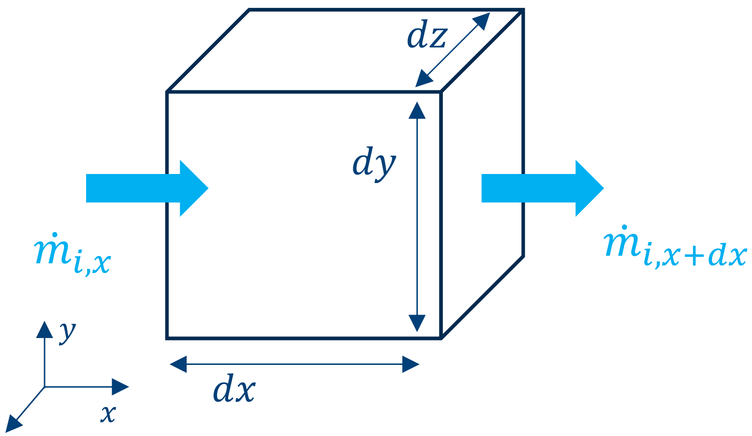
$$0 = \dot{m}''_{i,x} \cdot dy \cdot dz - \dot{m}''_{i,x+dx} \cdot dy \cdot dz$$

Taylor series expansion:

$$\dot{m}_{i,x+dx} = \left[\dot{m}''_{i,x} + \frac{\partial}{\partial x}\left(\dot{m}''_{i,x}\right) \cdot dx\right] \cdot dy \cdot dz$$

$$\cancel{\dot{m}}''_{i,x} \cdot \cancel{dy} \cdot \cancel{dz} - \left[\cancel{\dot{m}}''_{i,x} + \frac{\partial}{\partial x}\left(\dot{m}''_{i,x}\right) \cdot \cancel{dx}\right] \cdot \cancel{dy} \cdot \cancel{dz} = 0$$

$$-\frac{\partial}{\partial x}\left(\dot{m}''_{i,x}\right) = 0$$



Example: 1D steady state flow without sources

Balance around the control volume:

$$-\frac{\partial}{\partial x}\left(\dot{m}''_{i,x}\right) = 0$$

$$\dot{m}''_i = \xi_i \cdot \rho \cdot u + j_i''$$

Insert $\dot{\boldsymbol{m}}''_i$ in the balance:

$$\frac{\partial}{\partial x}\left(\xi_i \cdot \rho \cdot u + j''_{i,x}\right) = 0$$

Use sum rule and chain rule:

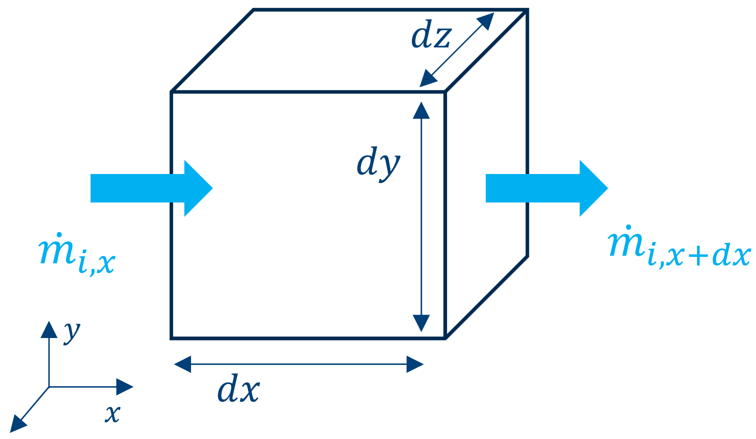
$$\rho\, u\, \frac{\partial \xi_i}{\partial x} + \xi_i \frac{\partial(\rho\, u)}{\partial x} + \frac{\partial j''_{i,x}}{\partial x} = 0$$

Fick's law:

$$j''_{i,x} = -\rho\, D\, \frac{\partial \xi_i}{\partial x}$$

$$\rho\, u\, \frac{\partial \xi_i}{\partial x} + \xi_i \cancel{\frac{\partial(\rho\, u)}{\partial x}} + \rho\, D\, \frac{\partial^2 \xi_i}{\partial x^2} = 0$$

$\boxed{=\ 0\text{: 1-D continuity equation}}$



Example: 3D steady state flow without sources

Balance around the control volume:

$$\frac{\partial}{\partial x}\left(\dot{m}''_{i,x}\right) + \frac{\partial}{\partial y}\left(\dot{m}''_{i,y}\right) + \frac{\partial}{\partial z}\left(\dot{m}''_{i,z}\right) = 0$$

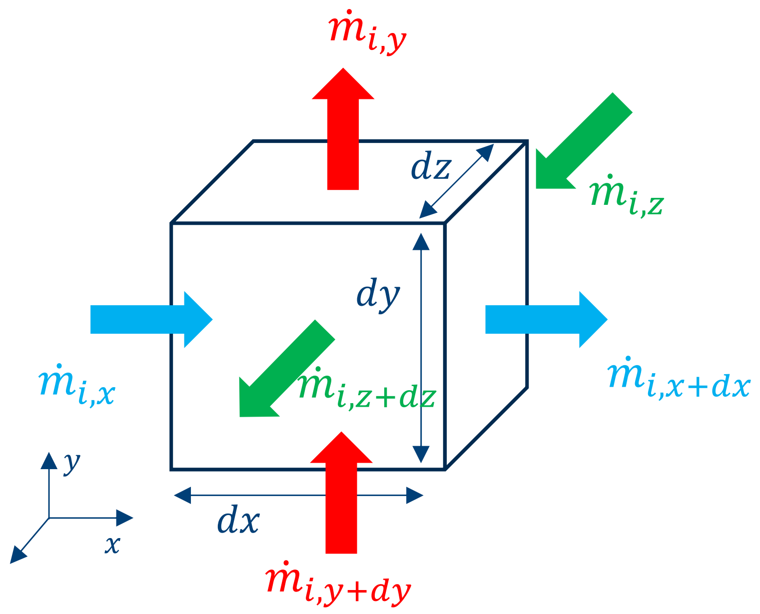
$$\dot{m}''_i = \xi_i \cdot \rho \cdot u + j_i''$$

Insert $\dot{m}''_i$ in the balance:

$$\frac{\partial}{\partial x}\left(\xi_i \cdot \rho \cdot u + j''_{i,x}\right) + \frac{\partial}{\partial y}\left(\xi_i \cdot \rho \cdot v + j''_{i,y}\right) + \frac{\partial}{\partial z}\left(\xi_i \cdot \rho \cdot w + j''_{i,z}\right) = 0$$

Separation of terms with the help of the sum rule and chain rule:

$$\rho\, u\, \frac{\partial \xi_i}{\partial x} + \xi_i \frac{\partial(\rho\, u)}{dx} + \frac{\partial j''_{i,x}}{\partial x} + \rho\, v\, \frac{\partial \xi_i}{\partial y} + \xi_i \frac{\partial(\rho\, v)}{\partial y} + \frac{\partial j''_{i,y}}{\partial y} + \rho\, w\, \frac{\partial \xi_i}{\partial z} + \xi_i \frac{\partial(\rho\, w)}{\partial z} + \frac{\partial j''_{i,z}}{\partial z} = 0$$



Example: 3D steady state flow without sources

Balance around the control volume:

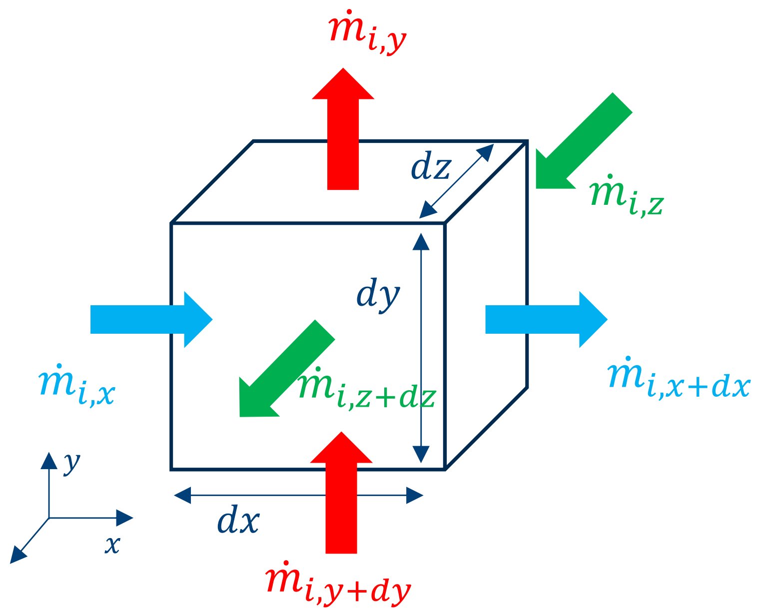
Rearrange Diff. equation:

$$\underbrace{\rho\, u\,\frac{\partial \xi_i}{\partial x} + \rho\, v\,\frac{\partial \xi_{i,y}}{\partial y} + \rho\, w\,\frac{\partial \xi_i}{\partial z} + \xi_i\left[\underbrace{\frac{\partial(\rho\, u)}{dx} + \frac{\partial(\rho\, v)}{\partial y} + \frac{\partial(\rho\, w)}{\partial z}}_{=\,0 \text{ (Continuity equation!)}}\right]}_{\text{Advective flow}}$$

$$= \underbrace{-\left(\frac{\partial j"_{i,x}}{\partial x} + \frac{\partial j"_{i,y}}{\partial y} + \frac{\partial j"_{i,z}}{\partial z}\right)}_{\text{Diffusive flow}}$$

Ficks' law: $j_{i,x} = -\rho\, D\,\frac{\partial \xi_i}{\partial x}$

$$\boxed{\rho\, u\,\frac{\partial \xi_i}{\partial x} + \rho\, v\,\frac{\partial \xi_{i,y}}{\partial y} + \rho\, w\,\frac{\partial \xi_i}{\partial z} = \rho\, D\left(\frac{\partial^2 \xi_i}{\partial x^2} + \frac{\partial^2 \xi_{i,y}}{\partial y^2} + \frac{\partial^2 \xi_i}{\partial z^2}\right)}$$



Analogy between energy and mass transport

Energy transport:

$$\cancel{\rho} u \cancel{c_p} \frac{\partial T}{\partial x} + \cancel{\rho} v \cancel{c_p} \frac{\partial T}{\partial y} + \cancel{\rho} w \cancel{c_p} \frac{\partial T}{\partial z} = \cancel{\lambda} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

$$\downarrow$$

Scaling down: $a = \frac{\lambda}{\rho c_p} = \frac{\nu}{Pr}$

Mass transport:

$$\cancel{\rho}\, u \frac{\partial \xi_i}{\partial x} + \cancel{\rho}\, v \frac{\partial \xi_{i,y}}{\partial y} + \cancel{\rho}\, w \frac{\partial \xi_i}{\partial z} = \cancel{\rho}\, D \left(\frac{\partial^2 \xi_i}{\partial x^2} + \frac{\partial^2 \xi_{i,y}}{\partial y^2} + \frac{\partial^2 \xi_i}{\partial z^2} \right)$$

$$\downarrow$$

Scaling down: $\mathrm{D} = \frac{\nu}{Sc}$

Prandtl number:

$$\mathrm{Pr} = \frac{\nu}{a} = \frac{\text{Diffusive momentum transport}}{\text{Diffusive heat transport}}$$

Schmidt number:

$$\mathrm{Sc} = \frac{\nu}{D} = \frac{\eta}{\rho D} = \frac{\text{Diffusive momentum transport}}{\text{Diffusive mass transport}}$$

Nusselt number:

$$Nu = \frac{\alpha \cdot L}{\lambda}$$

$\mathrm{Nu}(\mathrm{Re}, \mathrm{Pr}, \mathrm{Geometry}) \Longrightarrow \alpha$

Sherwood number:

$$Sh = \frac{g \cdot L}{\rho \cdot D}$$

$\mathrm{Sh}(\mathrm{Re}, \mathrm{Sc}, \mathrm{Geometry}) \Longrightarrow g$

Mass transport = mass transfer coefficient • area • driving potential

Comprehension questions

What is the name of the driving potential of diffusion and advective mass transfer?

Which mass transfer dimensionless number can be considered as an analogue to the Prandtl number in heat transfer?

Why is the sum of all diffusion flows equal to zero?