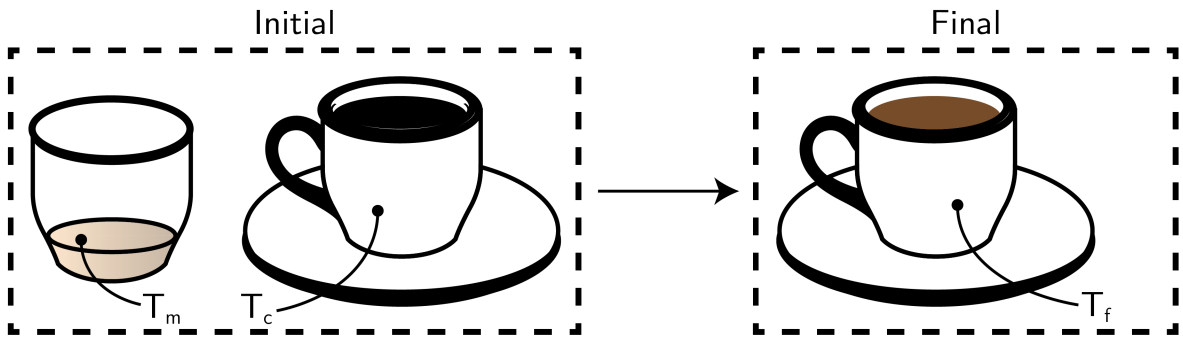
Mixing of Coffee

Derive the energy balance to determine the equilibrium mixing temperature.



1) Setting up an energy balance:

The energy balance can be described as:

$$U_\mathrm{i} = U_\mathrm{f}$$

2) Defining the internal energies:

Where the internal energy in the initial situation is described by the sum of the internal energies of the coffee and the milk:

$$U_\mathrm{i} = m_\mathrm{c} c_\mathrm{c} T_\mathrm{c} + m_\mathrm{m} c_\mathrm{m} T_\mathrm{m}$$

$$= \rho_\mathrm{c} V_\mathrm{c} c_\mathrm{c} T_\mathrm{c} + \rho_\mathrm{m} V_\mathrm{m} c_\mathrm{c} c_\mathrm{m} T_\mathrm{m}$$

In the final situation, the mixture of coffee will reach its equilibrium temperature T_f, and one can describe the internal again as the sum of internal energies of the coffee and the milk:

$$U_\mathrm{i} = m_\mathrm{c} c_\mathrm{c} T_\mathrm{f} + m_\mathrm{m} c_\mathrm{m} T_\mathrm{f}$$

$$= T_\mathrm{f} \left(\rho_\mathrm{c} V_\mathrm{c} c_\mathrm{c} + \rho_\mathrm{m} V_\mathrm{m} c_\mathrm{c} c_\mathrm{m} \right)$$

3) Inserting and rearranging:

Inserting the found energies into the energy balance yields:

$$\rho_\mathrm{c} V_\mathrm{c} c_\mathrm{c} T_\mathrm{c} + \rho_\mathrm{m} V_\mathrm{m} c_\mathrm{c} c_\mathrm{m} T_\mathrm{m} = T_\mathrm{f} \left(\rho_\mathrm{c} V_\mathrm{c} c_\mathrm{c} + \rho_\mathrm{m} V_\mathrm{m} c_\mathrm{c} c_\mathrm{m} \right)$$

With this energy balance, one would find the following equilibrium temperature T_f:

$$\boxed{\rightarrow T_\mathrm{f} = \frac{\rho_\mathrm{c} V_\mathrm{c} c_\mathrm{c} T_\mathrm{c} + \rho_\mathrm{m} V_\mathrm{m} c_\mathrm{c} c_\mathrm{m} T_\mathrm{m}}{\rho_\mathrm{c} V_\mathrm{c} c_\mathrm{c} + \rho_\mathrm{m} V_\mathrm{m} c_\mathrm{c} c_\mathrm{m}}}$$