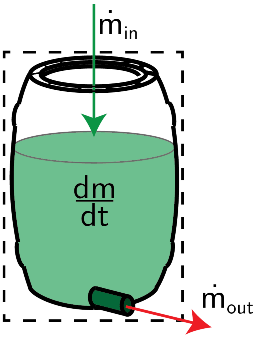
Filling of a Barrel

Derive the mass balance describing the height $h(t)$ of the fluid over time.



1) Setting up a mass balance:

The standard energy balance that describes the change in mass of a control volume over time can be expressed as

$$\frac{dm}{dt} = \sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}}$$

Which for the given scenario will be:

$$\frac{dU}{dt} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}$$

2) Defining the fluxes:

The change of mass in the system over the course of our control volume can be expressed as:

$$\frac{dm}{dt} = \frac{d}{dt}(\rho V) = \rho\pi \frac{D^2}{4}\frac{dh(t)}{dt}$$

The mass entering the system can be described by the multiplication of the volume flow rate ($=A_\text{c} \cdot u_{\text{avg}}$) and the density:

$$\dot{m}_{\text{in}} = \rho\dot{V} = \rho u_{\text{in}} \pi \frac{d^2}{4}$$

Similarly, mass leaving the system can be described by the multiplication of the volume flow rate and the density:

$$\dot{m}_{\text{out}} = \rho\dot{V} = \rho u_{\text{out}} \pi \frac{d^2}{4}$$

3) Inserting and rearranging:

Inserting the found fluxes into the balance yields:

$$\rho\pi \frac{D^2}{4}\frac{dh(t)}{dt} = \rho u_{\text{in}} \pi \frac{d^2}{4} - \rho u_{\text{out}} \pi \frac{d^2}{4}$$

$$\boxed{\rightarrow \frac{dh}{dt} = \frac{d^2}{D^2}\left(u_{\text{in}} - u_{\text{out}}\right)}$$