# Heat Transfer: Conduction

## Introduction to the transient heat conduction


Prof. Dr.-Ing. Reinhold Kneer

Prof. Dr.-Ing. Dr. rer. pol. Wilko Rohlfs


HEATQUIZ

UNIVERSITY OF TWENTE.

Lehrstuhl für Wärme- und Stoffübertragung

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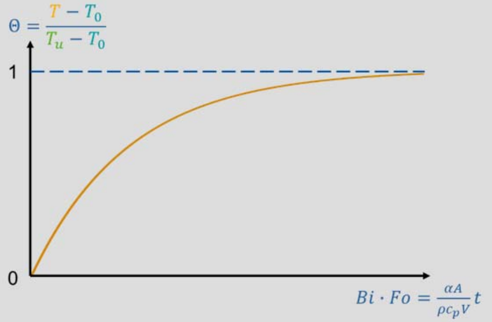
# Learning goals

## Categories of transient problems:

- Understanding and abstraction of the problem
- Problem reduction and selection of the appropriate solution strategy

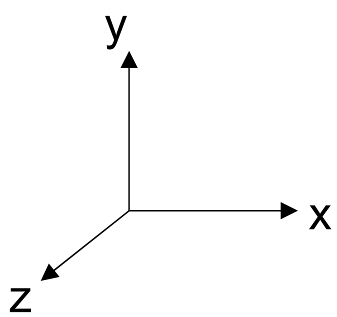
## Body with homogeneous temperature:

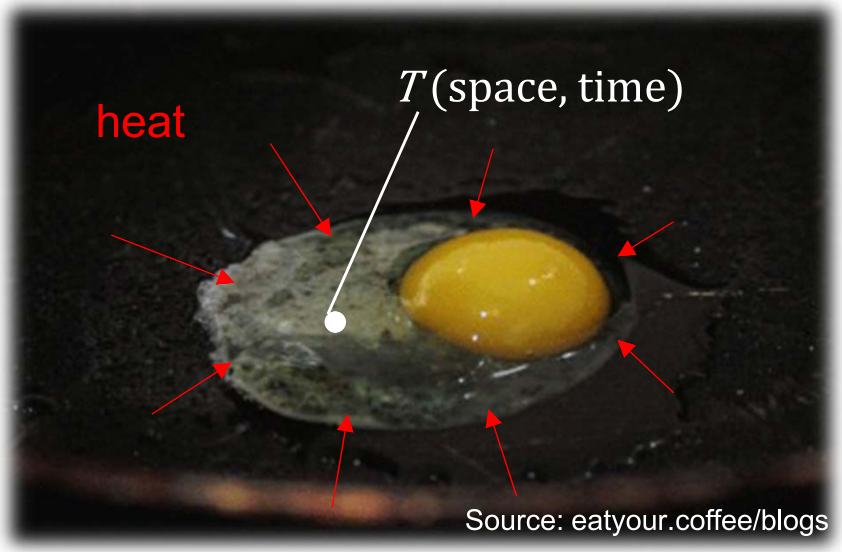
- Down-scaling of the problem
- Dimensionless numbers
- Mathematical solution of the differential equation



# Review: Fourier's differential equation

**„fried egg"**




**Transient heat conduction:**

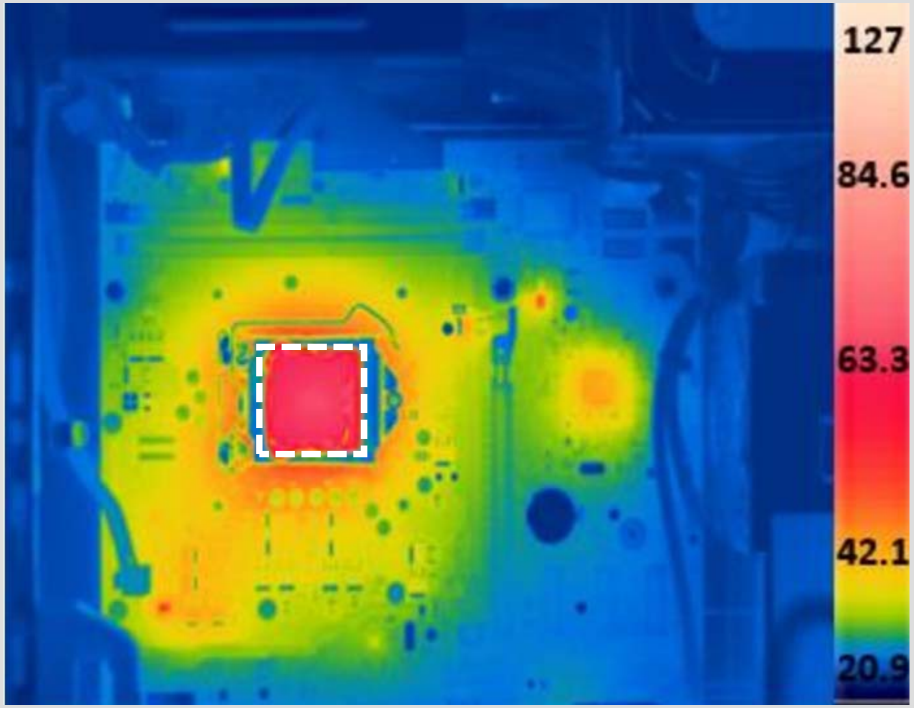
3-D conservation equation without advection and source

$$\boxed{\rho c_p \frac{\partial T}{\partial t}} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right)$$

# How can transient problems be categorized?

## Temperature of the body:

**Homogeneous (Bi ≪ 1):**




**Not homogeneous:**

[1] Lee, Soochan et al. "Hot Spot Cooling and Harvesting CPU Waste Heat Using Thermoelectric Modules." (2014).
[2] www.travelportal.cz

# How can transient problems be categorized?

**Temperature of the body:**

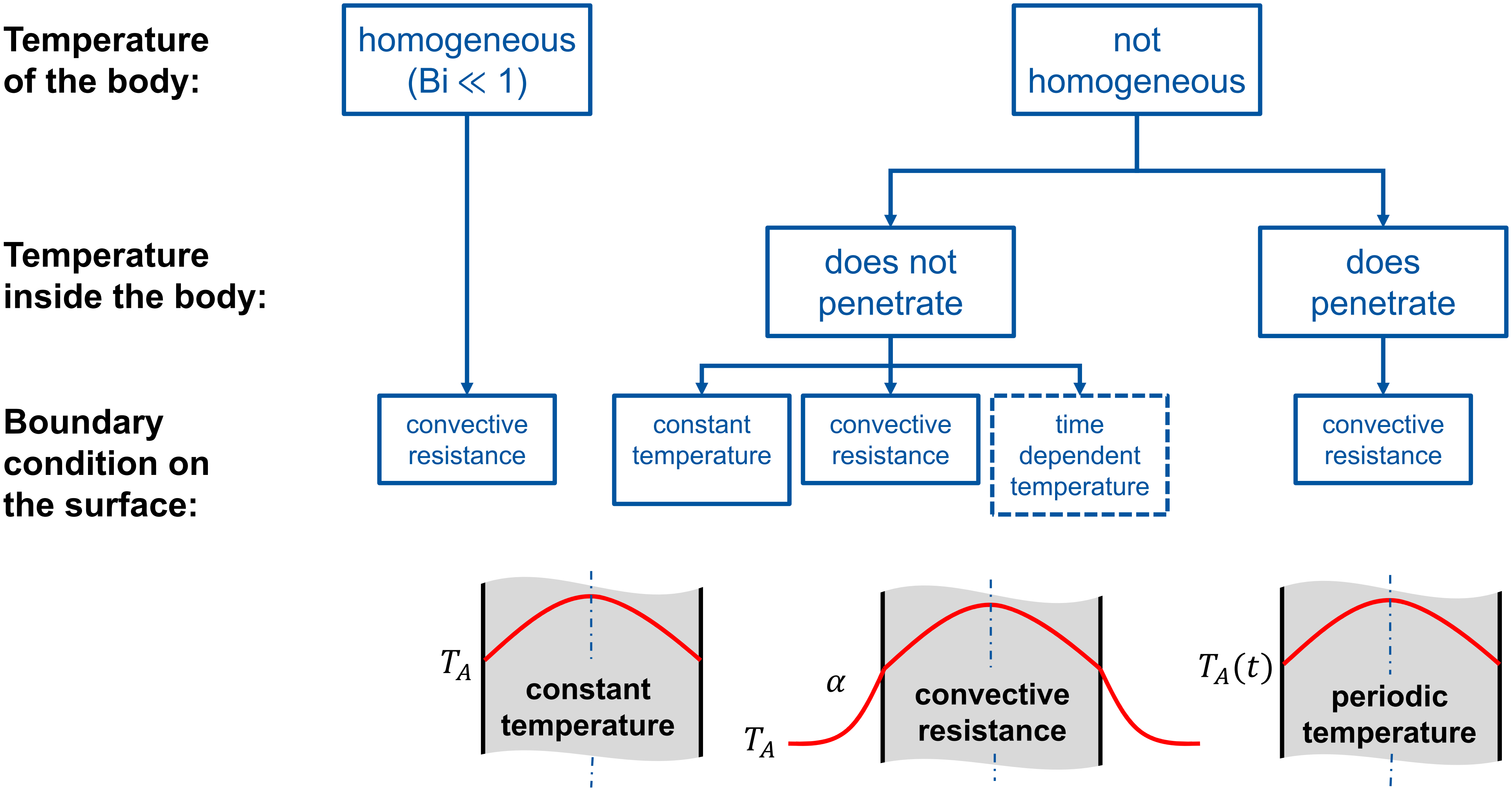
homogeneous (Bi ≪ 1) | not homogeneous

**Temperature inside the body:**

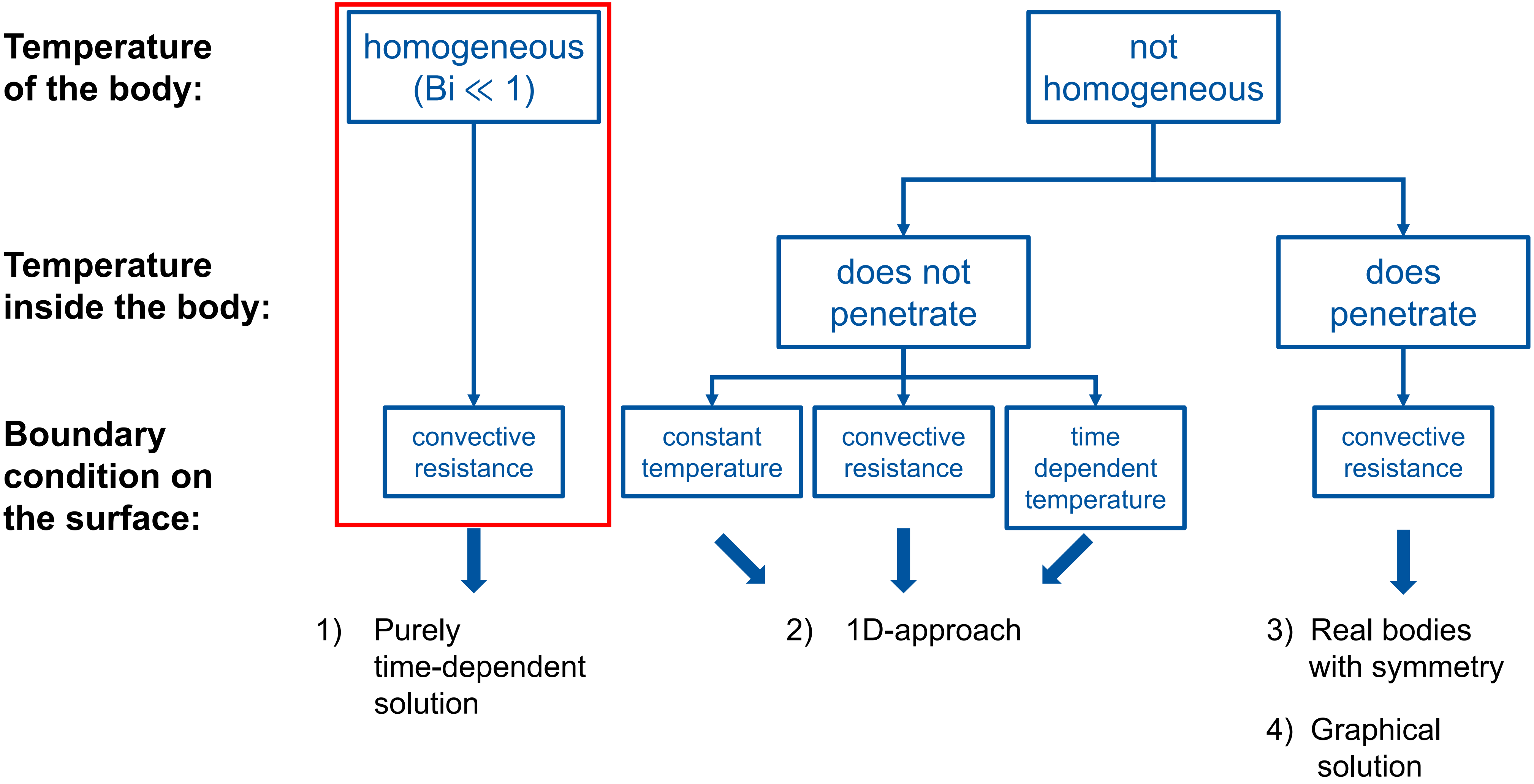
not homogeneous → does not penetrate | does penetrate

[1] www.mychicagosteak.com/steak-university/done-perfection-guide-steak-doneness/

# How can transient problems be categorized?

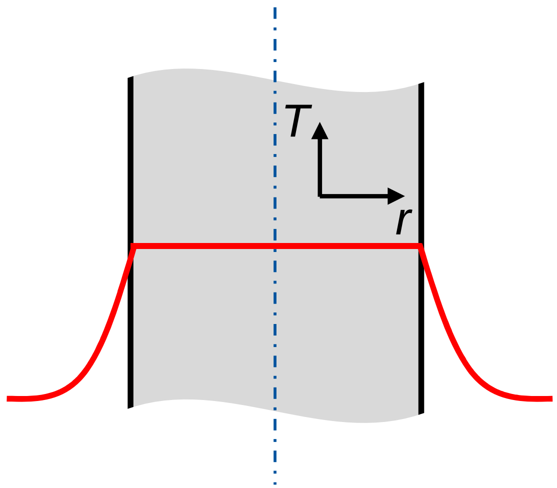


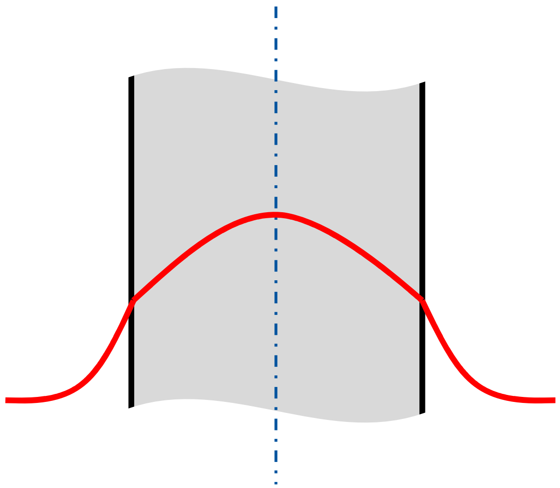
# How to simplify the problem?

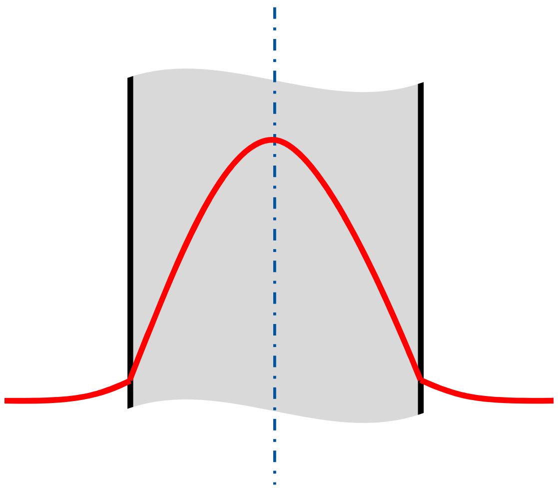


# Review of the Biot number

## → What conditions must be met for a homogeneous body temperature?





**Bi ≪ 1:**

- Homogeneous temperature in the body
- $R_\lambda$ negligible
- Small bodies or bodies with high thermal conductivity

**Bi ≈ 1:**

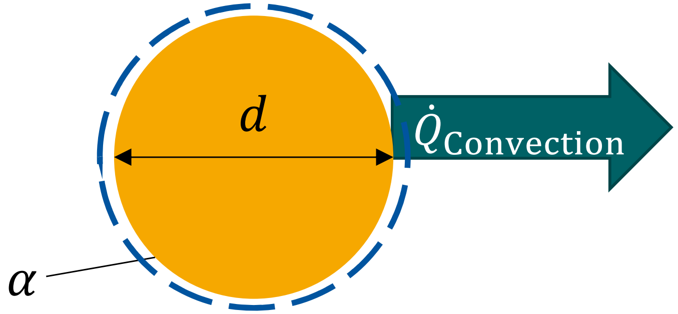
- Similar contributions of heat conduction and convection
- $R_\lambda \approx R_\alpha$

**Bi ≫ 1:**

- Higher thermal resistance
- $R_\lambda \gg R_\alpha$
- Frequent in bodies with low thermal conductivity

# Body with „high“ thermal conductivity (block capacity or lumped capacity model)

Temperature of a copper sphere $T(t)$
with initial temperature $T_0$




ambient $T_A$

## Energy balance:

Change of internal energy = heat flux entering control volume

$$\frac{dU}{dt} = -\dot{Q}_{\mathrm{convection}}$$

$$\rho c_p V \frac{dT}{dt} = -\alpha A (T - T_A)$$

## Dimensionless temperature:

$$\Theta^* = \frac{T(t) - T_0}{T_A - T_0} \qquad \frac{d\Theta^*}{dt} = \frac{1}{T_A - T_0}\frac{dT}{dt}$$

## Substituting:

$$\frac{d\Theta^*}{dt} + \frac{\alpha A}{\rho c_p V} \cdot \underbrace{\frac{(T - T_0) - (T_A - T_0)}{T_A - T_0}}_{\Theta^* - 1} = 0$$

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Temperature of a copper sphere $T(t)$
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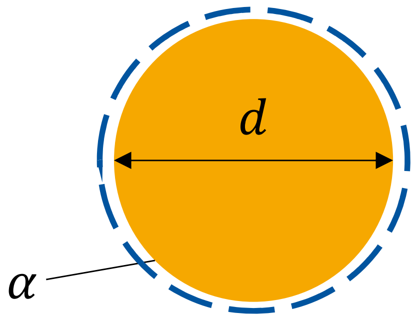



ambient $T_A$

**Equation for the temperature $T(t)$ :**

$$\frac{d\Theta^*}{\Theta^* - 1} = -\frac{\alpha A}{\rho c_p V} dt$$

**Integration with initial condition:**

$$\Theta_0^* = \frac{T_0 - T_0}{T_A - T_0} = 0$$

$$\int_{\Theta_0^*}^{\Theta^*} \frac{d\Theta^*}{\Theta^* - 1} = -\frac{\alpha A}{\rho c_p V} \int_0^t dt$$

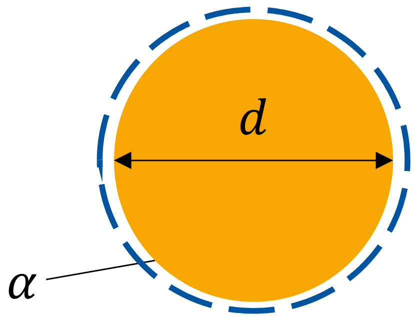
**Result:**

$$\ln\left(\frac{\Theta^* - 1}{\Theta_0^* - 1}\right) = \ln(1 - \Theta^*) = -\frac{\alpha A}{\rho c_p V} t$$

$$\Theta^* = 1 - e^{-\frac{\alpha A}{\rho c_p V} t}$$

# Body with „high“ thermal conductivity (block capacity or lumped capacity model)

Temperature of a copper sphere $T(t)$
with initial temperature $T_0$




ambient $T_A$

**Result:**

$$\Theta^* = 1 - e^{-\frac{\alpha A}{\rho c_p V} t}$$

**Review: Biot number**

$$Bi = \frac{\alpha L}{\lambda} \quad \text{with} \quad L = \frac{d}{2}$$

$$Bi = \frac{\text{outer resistance}}{\text{inner resistance}} \ll 1$$

**Definition: Fourier number**

$$Fo = \frac{\lambda t}{\rho c_p L^2} = \frac{a t}{L^2} \quad \text{with} \quad a = \frac{\lambda}{\rho c_p}$$

# Insertion: Fourier number

## Non-dimensionalization of the conservation equation:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right)$$

$$\frac{\rho c_p T_{\mathrm{ref}}}{t_{\mathrm{ref}}} \frac{\partial T^*}{\partial t^*} = \frac{T_{\mathrm{ref}} \lambda}{L^2} \frac{\partial}{\partial x^*}\left(\frac{\partial T^*}{\partial x^*}\right)$$

$$\frac{\partial T^*}{\partial t^*} = \boxed{\frac{\lambda t_{\mathrm{ref}}}{\rho c_p L^2}} \frac{\partial}{\partial x^*}\left(\frac{\partial T^*}{\partial x^*}\right)$$

$$\frac{\partial T^*}{\partial t^*} = Fo \frac{\partial^2 T^*}{\partial x^{*2}}$$

$$T = T^* \cdot T_{\mathrm{ref}}$$

$$t = t^* \cdot t_{\mathrm{ref}}$$

$$x = x^* \cdot L$$

Note: Quantities marked with * are dimensionless

$T_{\mathrm{ref}}$, $t_{\mathrm{ref}}$ and $L$ are reference quantities

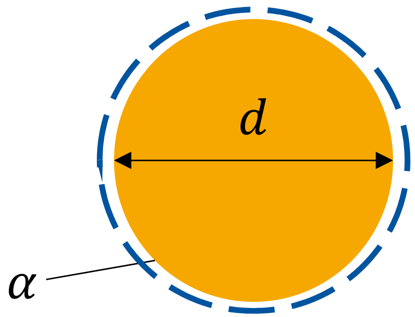
## Definition: Fourier number:

The Fourier number is a dimensionless time parameter.

It describes the duration of a thermal process in relation to the duration of heat transport.

# Body with „high“ thermal conductivity (block capacity or lumped capacity model)

Temperature of a copper sphere $T(t)$ with initial temperature $T_0$

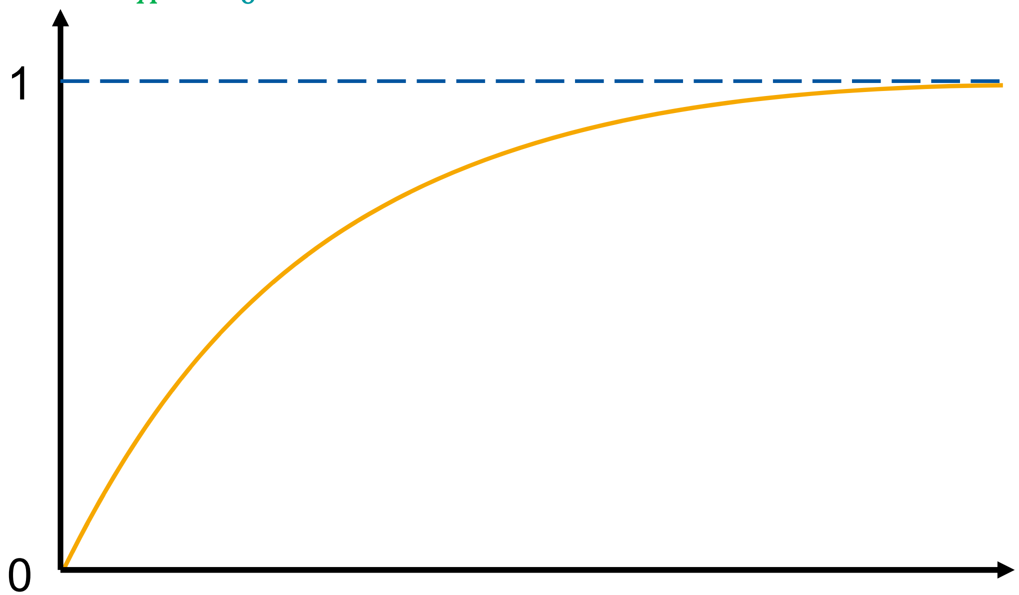



ambient $T_A$

**Result:**

$$\Theta^* = 1 - \mathrm{e}^{-[Bi \cdot Fo]}$$

$$\Theta^* = \frac{T - T_0}{T_A - T_0}$$




$$Bi \cdot Fo = \frac{\alpha A}{\rho c_p V} t$$



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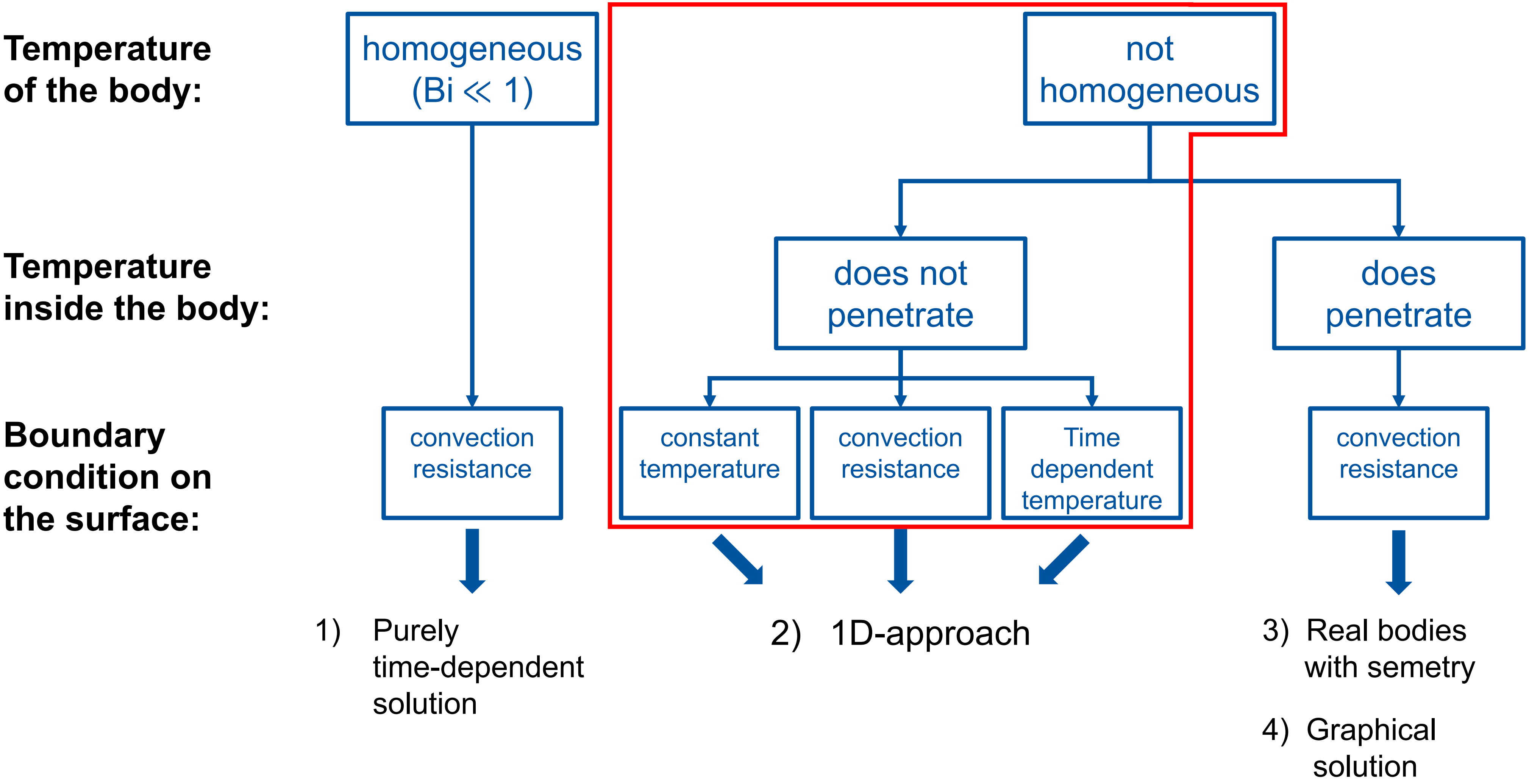
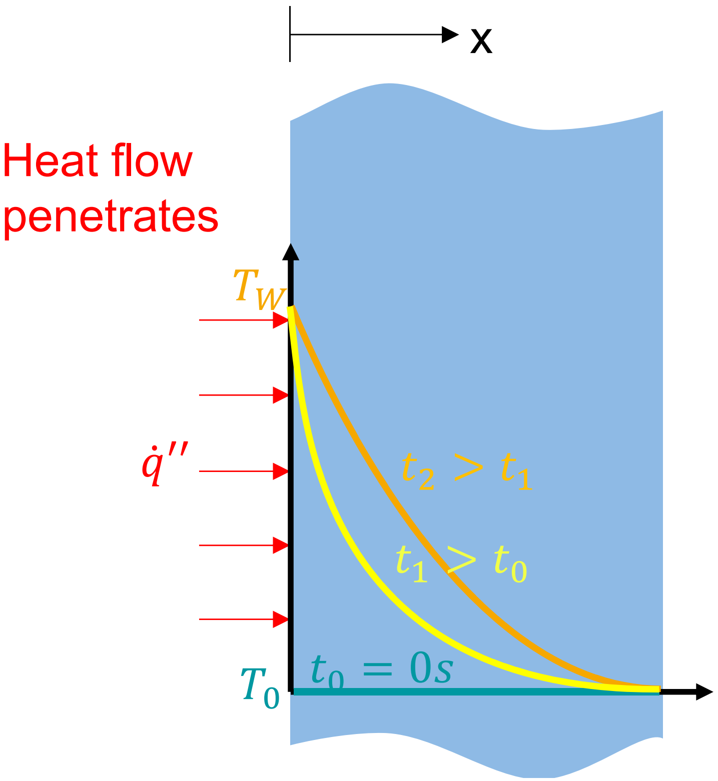
# How to simplify the problem?



# Semi-infinite bodies




Plate

Where does the temperature change at the beginning of the experiments?
→ **mathematically everywhere!**

As long as the temperature "on the right" has not changed significantly, the boundary condition on the right side is irrelevant

**Definition: Semi-infinite body:**

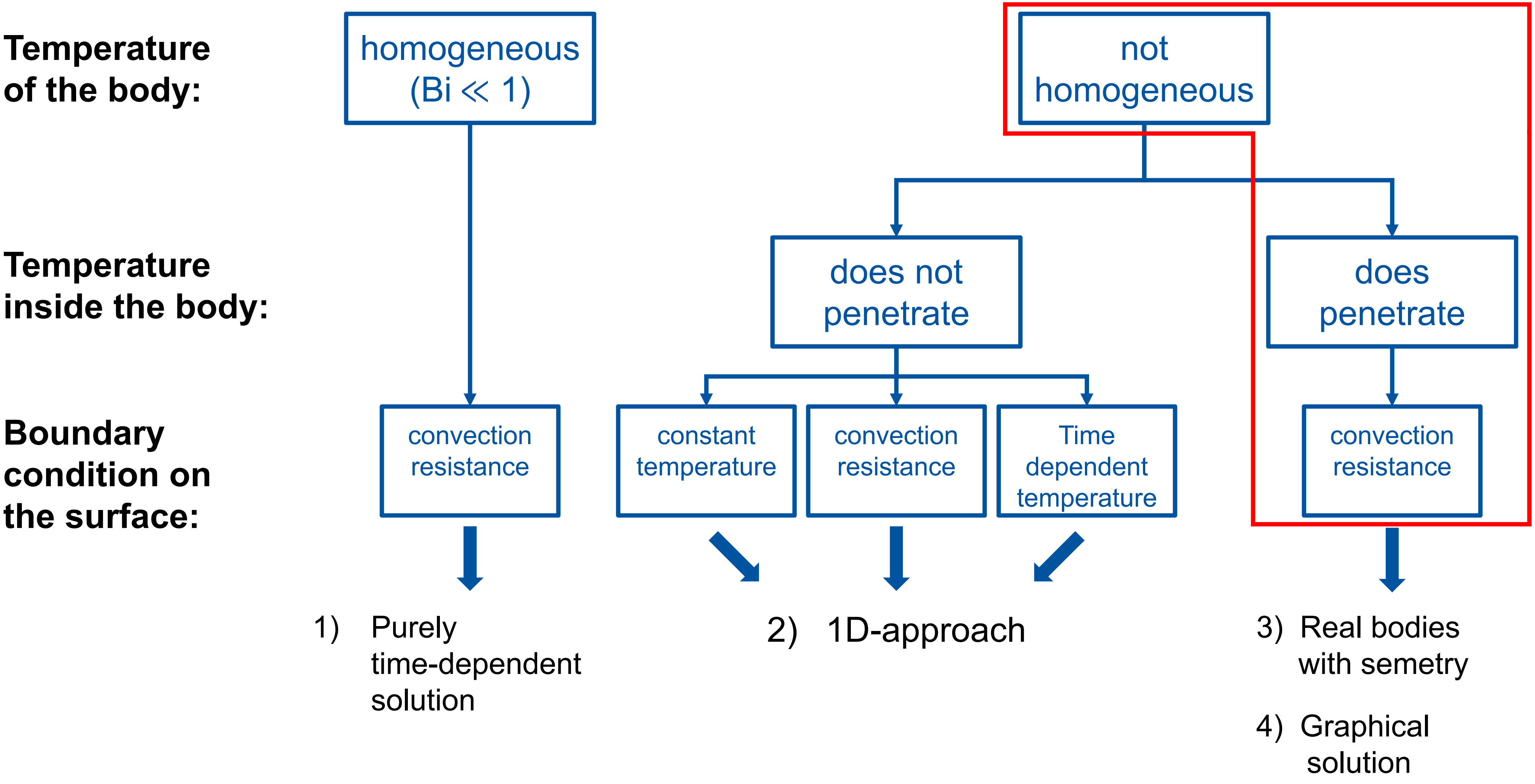
Object where a temperature change imposed on one side has not significantly propagated to the other side.

**Differential equation in 1D:**

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} \rightarrow \text{analytical Solution}$$

(Video „Semi-infinite Plate“)

# How to simplify the problem?

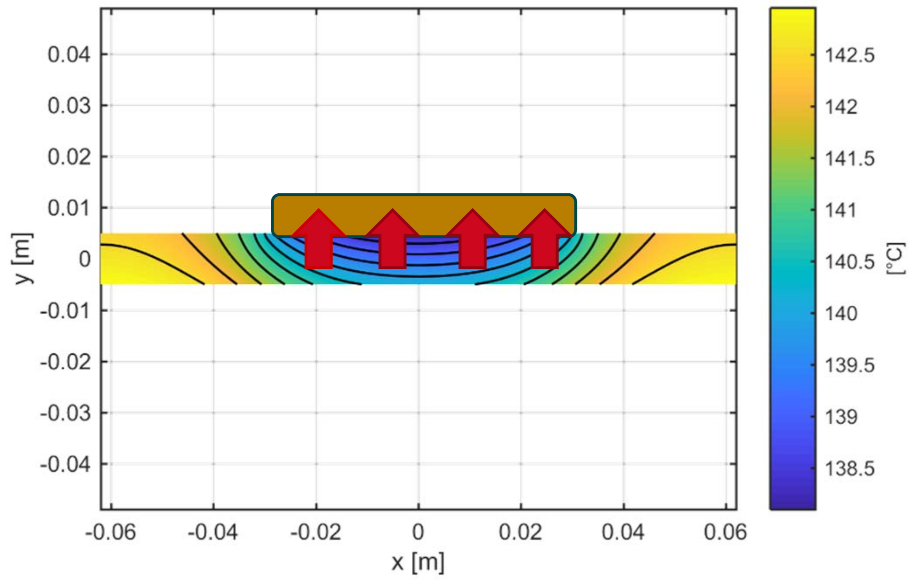


# Prebuilt solutions: Heisler diagram in dimensionless form

(in the Video „Dimensionless numbers and Heisler diagram“)

If the problem cannot be simplified, numerical methods are the methods of choice.

**Heating up a pan with food**



## Comprehension questions

**Under what conditions can the temperature within a body be assumed to be homogeneous?**

**Which dimensionless numbers can be used for this purpose?**

**What does the Fourier number describe?**