

Heat Transfer: Radiation

Example: Three-body problem

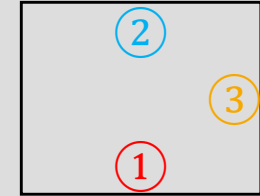
Prof. Dr.-Ing. Reinhold Kneer

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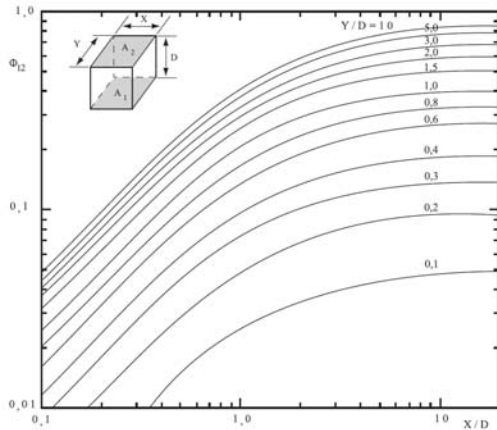
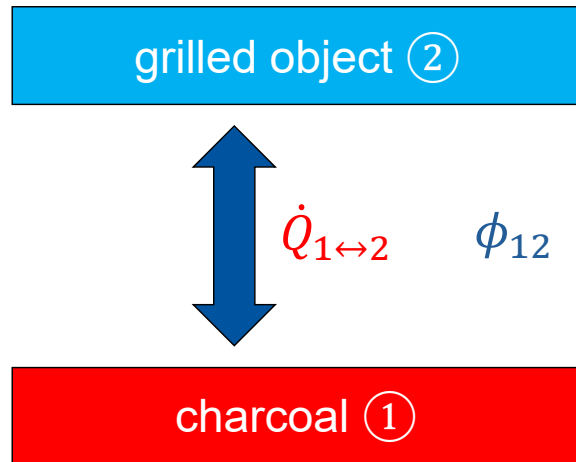
Learning goals

Three bodies:

- ▶ Expansion of the balances from two-body to multi-body problems
- ▶ Learning Approaches to solve Radiation tasks using the example of a three-body problem



Two-body problem



Question:

- ▶ Which heat flow goes from the charcoal ① by radiation to the grilled object ② ?

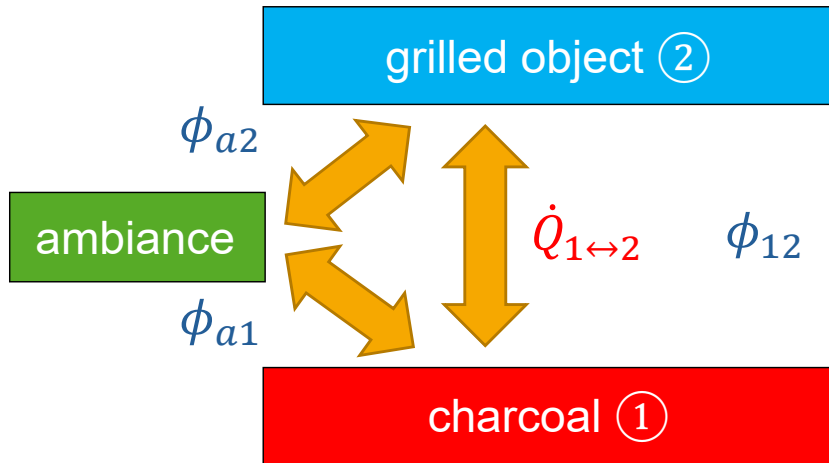
Initial situation:

- ▶ The charcoal has a higher temperature than the grilled object
- ▶ Charcoal and grilled object irradiate black

Approach:

- ▶ 1 energy balance
(around charcoal or grilled object)
- ▶ 2 surface brightnesses
(charcoal and grilled object)
- ▶ 1 relevant view factor (ϕ_{12})

Two-body problem



Question:

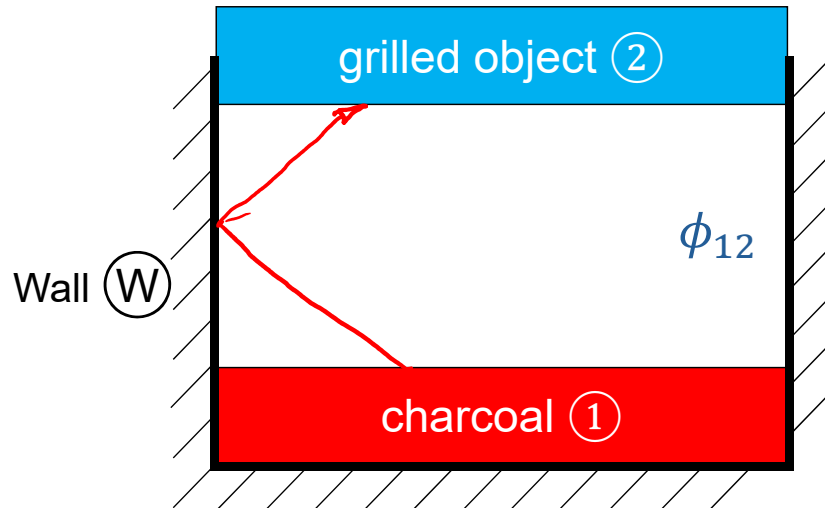
- ▶ Which heat flow goes from the charcoal ① by radiation to the grilled object ② ?

Initial situation:

- ▶ The heat transfer from the charcoal to the grilled object remains unchanged as long as the ambient radiation is not influenced by the radiation of objects ① and ②.

• https://www.amazon.de/dp/B00AC9B256/ref=asc_df_B00AC9B2561624469460000/?tag=lionshome-21&creative=22662&creativeASIN=B00AC9B256&linkCode=df0

Three-body problem



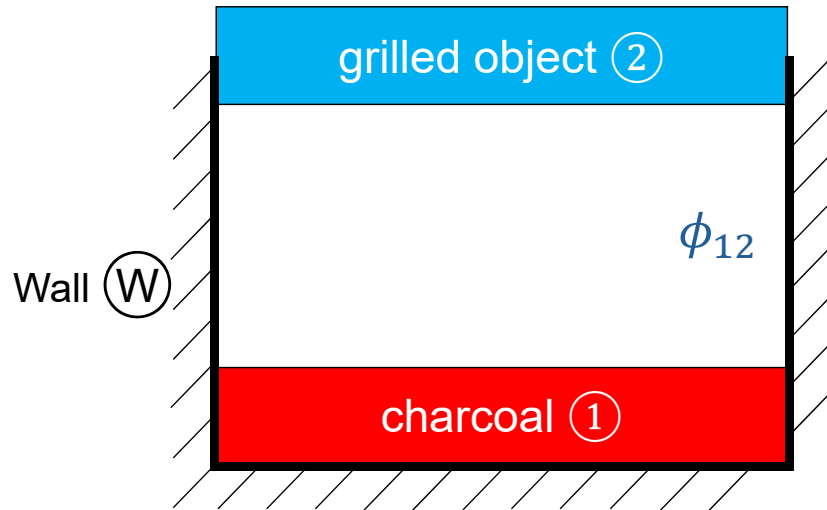
Question:

- ▶ Which heat flow goes from the charcoal ① by radiation to the grilled object ② ?

Considerations:

- ▶ Radiation transfer takes place between charcoal and wall as well as between grilled object and wall
- ▶ Charcoal, grilled object and wall irradiate black
- ▶ The wall is adiabatic towards the back, so all absorbed energy is released again by self-emission, this results in the wall temperature
- ▶ The radiant heat flow from the charcoal to the wall is larger than the one from the grilled object to the wall
- ▶ The wall irradiates evenly (symmetry) on charcoal and grilled object
- ▶ The wall causes a net radiant heat flow from the charcoal indirectly to the grilled object

Procedure to solve the task



Problem analysis:

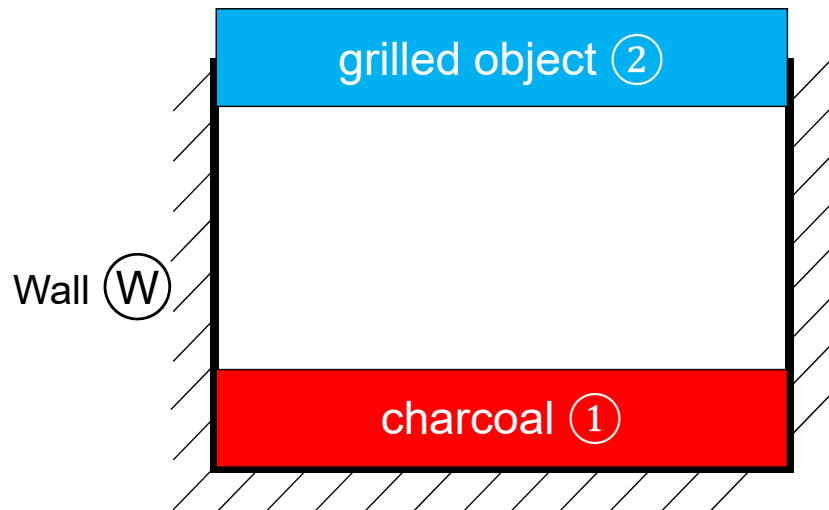
- ▶ Which heat flow goes from the charcoal ① by radiation to the grilled object ② ?

Procedure:

- ▶ Setting up the balances with **surface brightness** and **view factors**
 - Balance around grilled object
 - Balance around charcoal
 - Balance around wall

} one of two balances is enough.
- ▶ Define all **surface brightnesses** (implicit)
 - Charcoal $\Rightarrow f(T_1, \dot{Q}_2 \text{ and } \dot{Q}_W)$
 - Grilled body $\Rightarrow f(T_2, \dot{Q}_1 \text{ and } \dot{Q}_W)$
 - Wall $\Rightarrow f(T_W, \dot{Q}_1, \dot{Q}_2)$
- ▶ Define all view factors
- ▶ **Solve it**

Solution of the Two-body problem



Reciprocal rule:

$$A_1 \phi_{12} = A_2 \phi_{21}$$

Assumptions:

- ▶ Walls remain unconsidered
- ▶ All bodies irradiate black
- ▶ T_1, T_2 are known

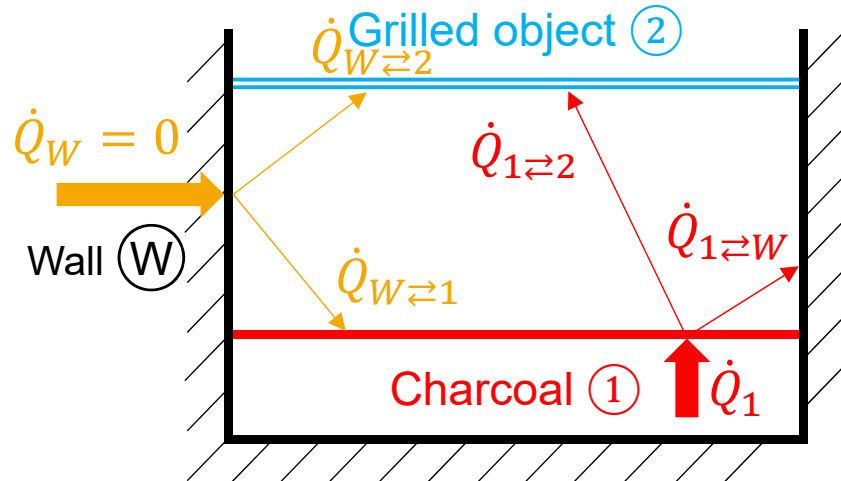
Procedure:

$$\dot{Q}_{1\rightleftharpoons 2} = \dot{Q}_{1\rightarrow 2} - \dot{Q}_{2\rightarrow 1}$$

$$\dot{Q}_{1\rightleftharpoons 2} = A_1 \phi_{12} (\dot{q}_1'' - \dot{q}_2'')$$

$$\dot{Q}_{1\rightleftharpoons 2} = A_1 \phi_{12} \sigma (T_1^4 - T_2^4)$$

Three-body problem



Energy Balance 1 (Charcoal):

$$\begin{aligned}\dot{Q}_1 &= \dot{Q}_{1 \rightleftharpoons 2} + \dot{Q}_{1 \rightleftharpoons W} \\ &= A_1 \phi_{12} \sigma (T_1^4 - T_2^4) + A_1 \phi_{1W} \sigma (T_1^4 - T_W^4)\end{aligned}$$

unknown ←

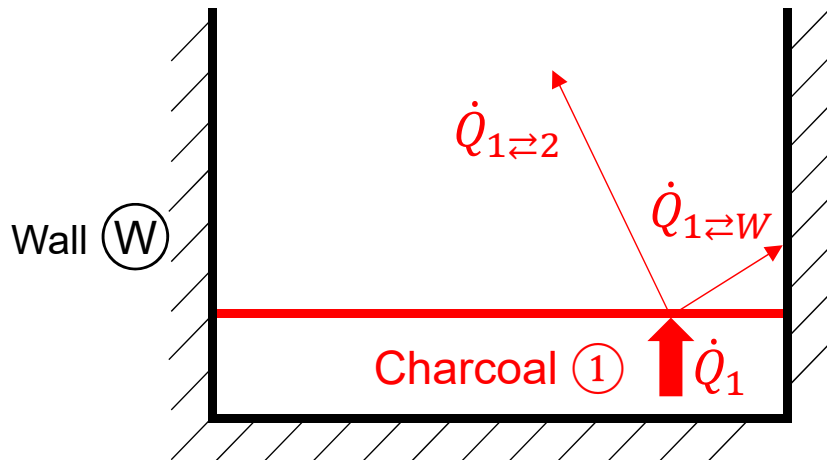
Energy balance 2 (Wall):

$$\begin{aligned}\dot{Q}_W = 0 &= \dot{Q}_{W \rightleftharpoons 1} + \dot{Q}_{W \rightleftharpoons 2} + \cancel{\dot{Q}_{W \rightleftharpoons W}} \\ &= \underbrace{A_W \phi_{W1} \sigma (T_W^4 - T_1^4)}_{A_1 \phi_{1W}} + \underbrace{A_W \phi_{W2} \sigma (T_W^4 - T_2^4)}_{A_2 \phi_{2W}}\end{aligned}$$

Wall Temperature:

$$T_W^4 = \frac{A_1 \phi_{1W} T_1^4 + A_2 \phi_{2W} T_2^4}{A_1 \phi_{1W} + A_2 \phi_{2W}}$$

Three-body problem



Assumptions:

- ▶ Walls remain ~~unconsidered~~ *considered*
- ▶ All bodies irradiate black
- ▶ T_1, T_2 are known

Energy Balance 1 (Charcoal):

$$\dot{Q}_1 = \dot{Q}_{1 \rightarrow 2} + \dot{Q}_{1 \rightarrow W}$$

$$= A_1 \phi_{12} \sigma (T_1^4 - T_2^4) + A_1 \phi_{1W} \sigma (T_1^4 - T_W^4)$$

Energy Balance 1 (Charcoal):

$$\dot{Q}_1 = \dot{Q}_{1 \rightarrow 2} + \dot{Q}_{1 \rightarrow W}$$

$$= A_1 \phi_{12} \sigma (T_1^4 - T_2^4) + A_1 \phi_{1W} \sigma (T_1^4 - T_W^4)$$

$$= \sigma T_1^4 \left[A_1 \phi_{12} + \frac{(A_1 \phi_{1W})(A_2 \phi_{2W})}{A_1 \phi_{1W} + A_2 \phi_{2W}} \right]$$

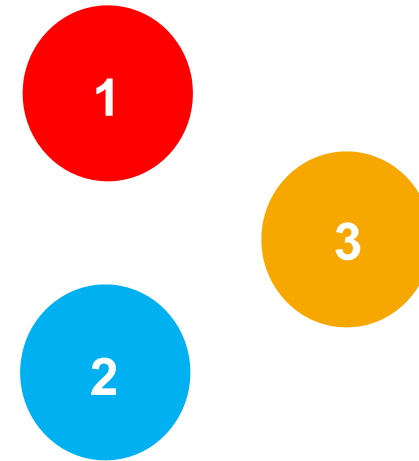
Wall Temperature:

$$T_W^4 = \frac{A_1 \phi_{12} T_1^4 + A_2 \phi_{21} T_2^4}{A_1 \phi_{1W} + A_2 \phi_{2W}}$$

Three-body problem: summary of the assumptions

Assumptions:

- ▶ All bodies irradiate black
- ▶ The radiation transfer takes place exclusively between the bodies (sum of the View Factors is 1)
- ▶ The temperature of Body ① and Body ② is given
- ▶ Body ③ emits all received Radiation energy (adiabatic)



Solution:

$$\begin{aligned}\dot{Q}_1 &= \dot{Q}_{1 \rightarrow 2} + \dot{Q}_{1 \rightarrow 3} \\ &= A_1 \sigma (T_1^4 - T_2^4) \left[\phi_{12} + \frac{1}{\frac{A_1}{A_2 \phi_{23}} + \frac{1}{\phi_{13}}} \right]\end{aligned}$$

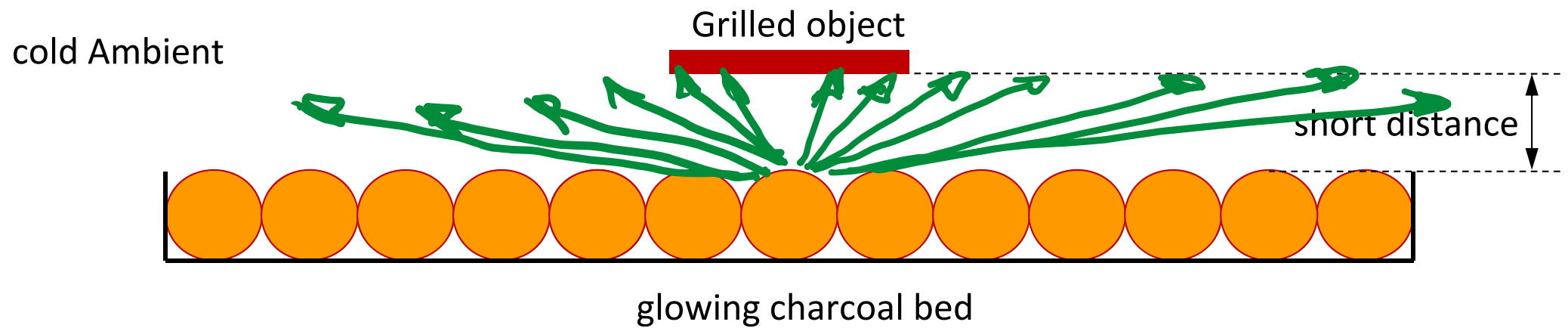
Grilling on a kettle grill

Tips for the next Barbecue:

- ▶ Lifting the grill hardly slows down the cooking process
- ▶ Black as color and spherical as a shape are extremely unfavorable

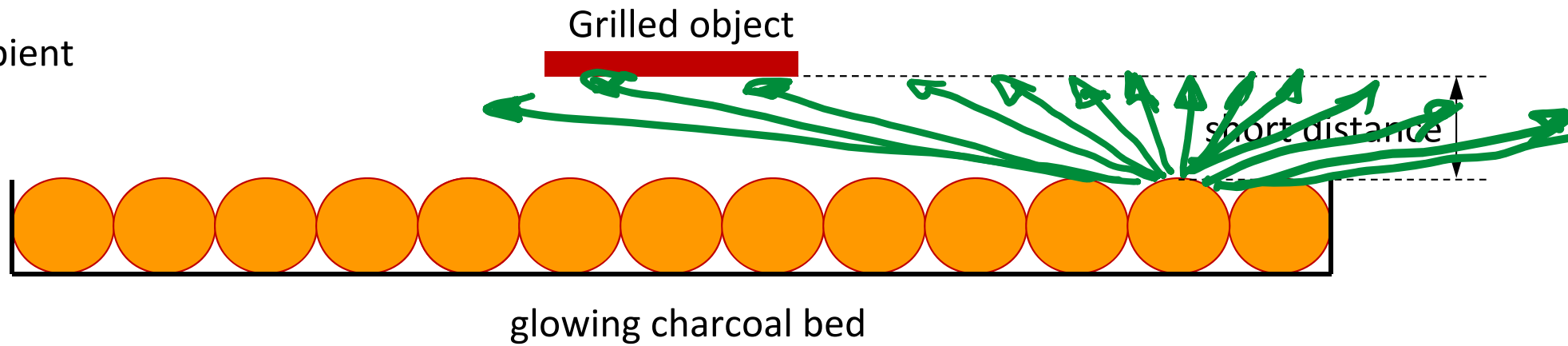


Lifting the grill grate during grilling

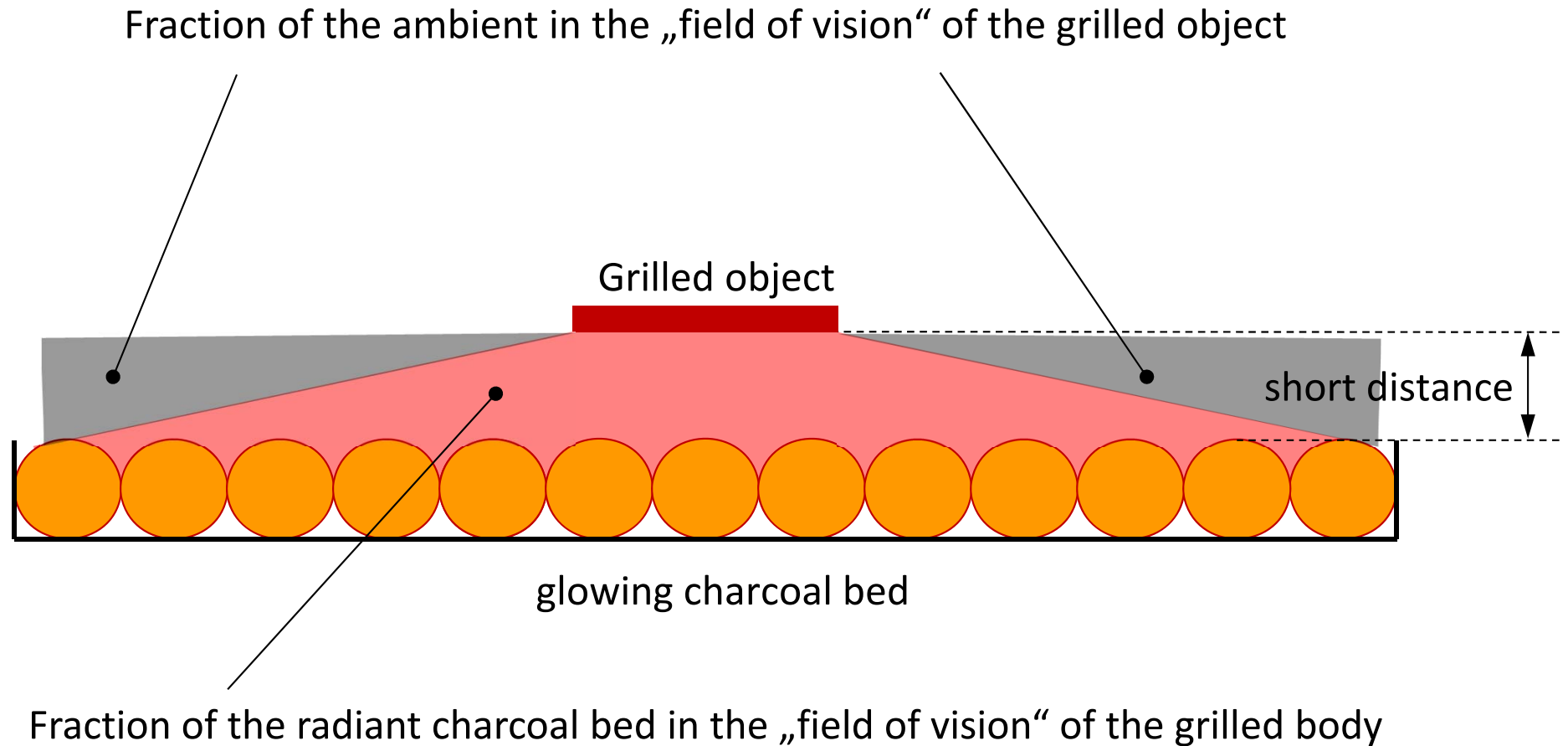


Lifting the grill grate during grilling

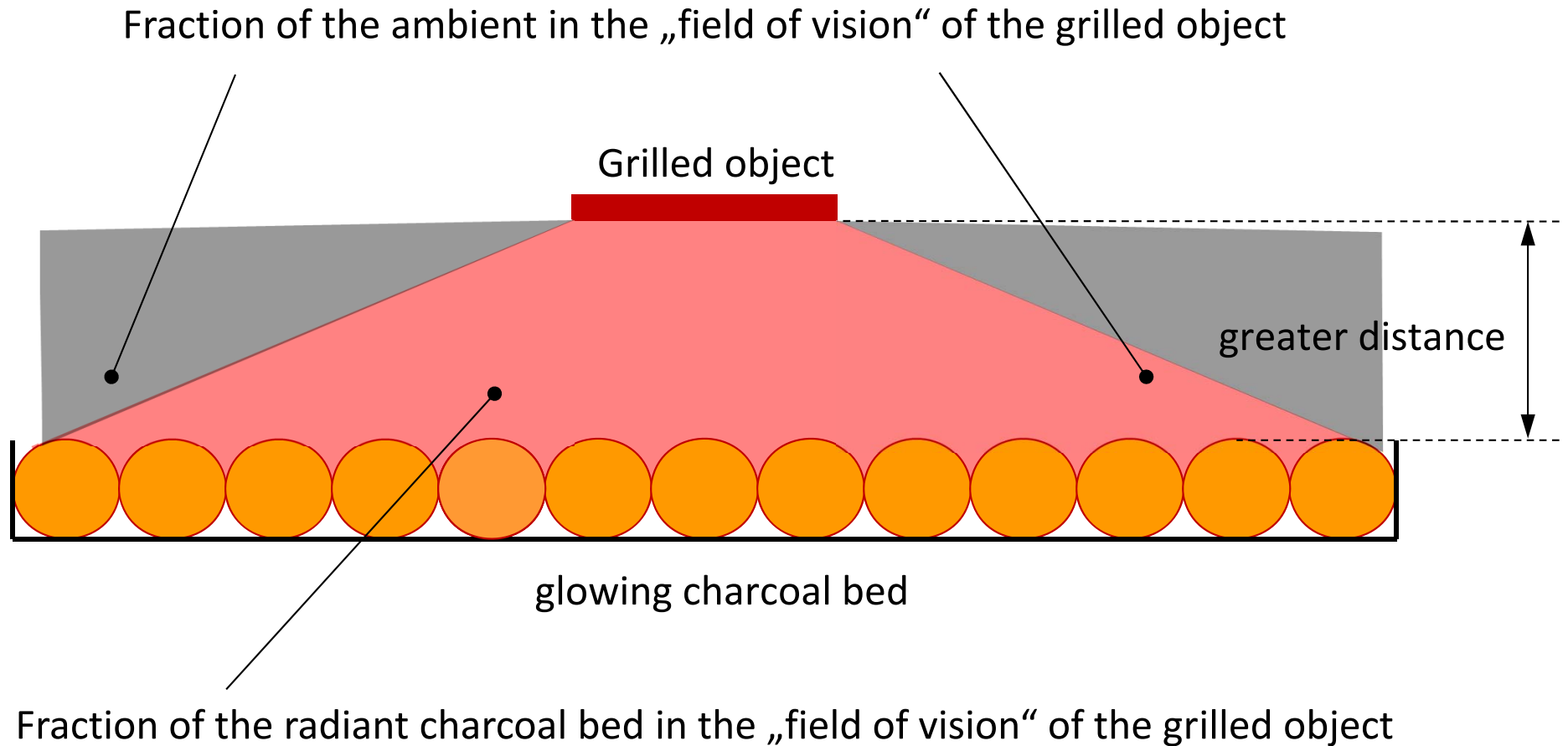
cold Ambient



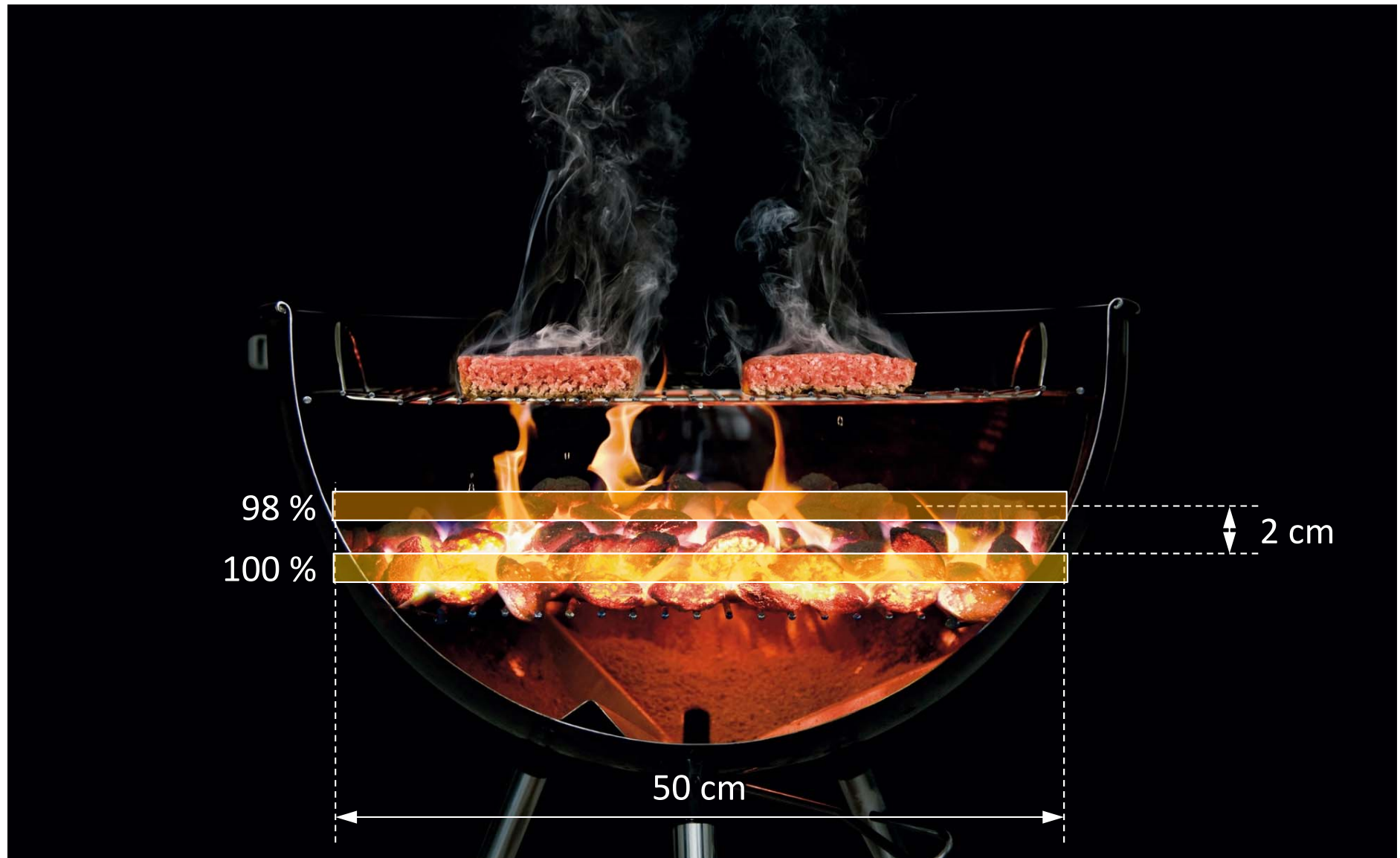
Lifting the grill grate during grilling



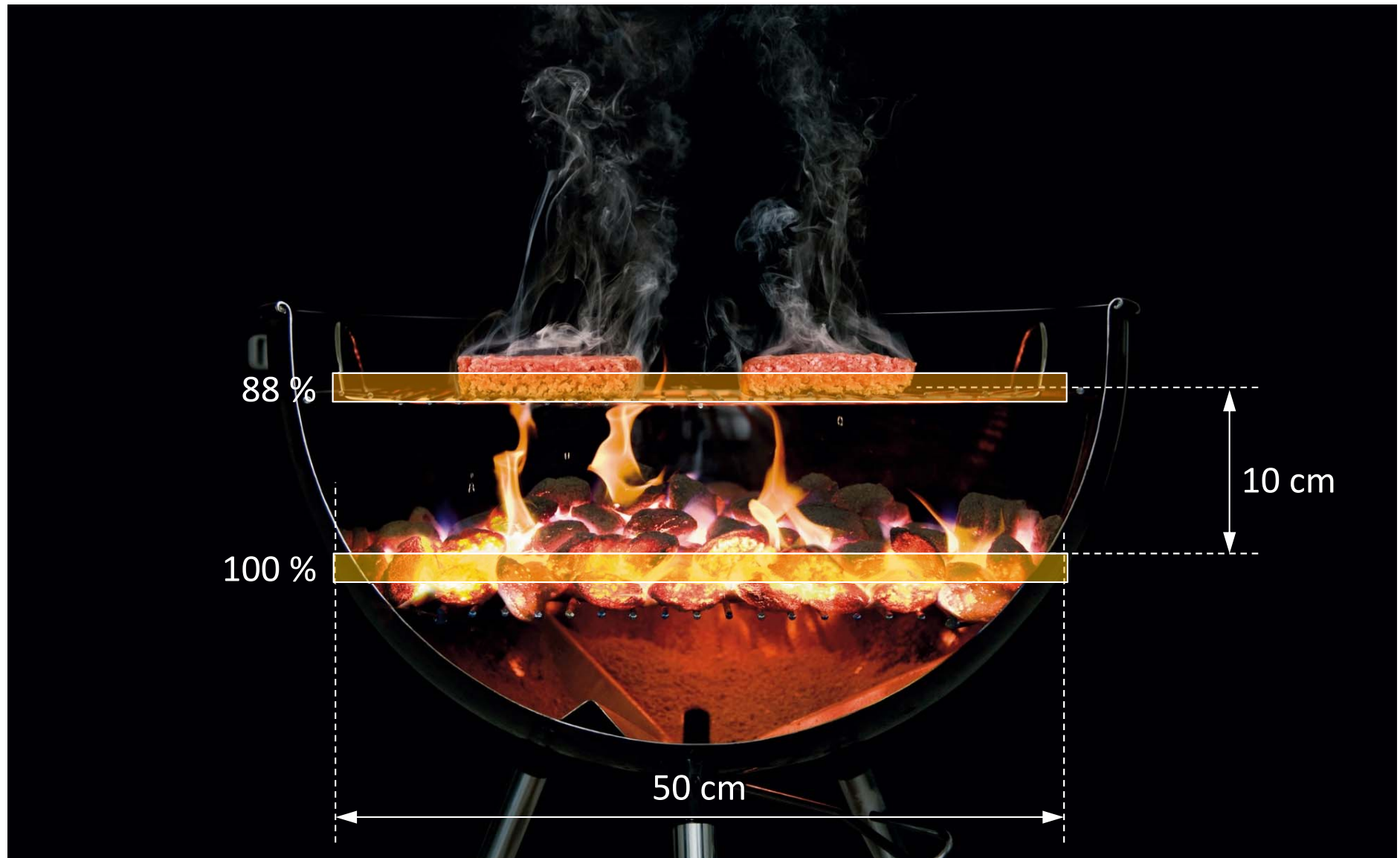
Lifting the grill grate during grilling



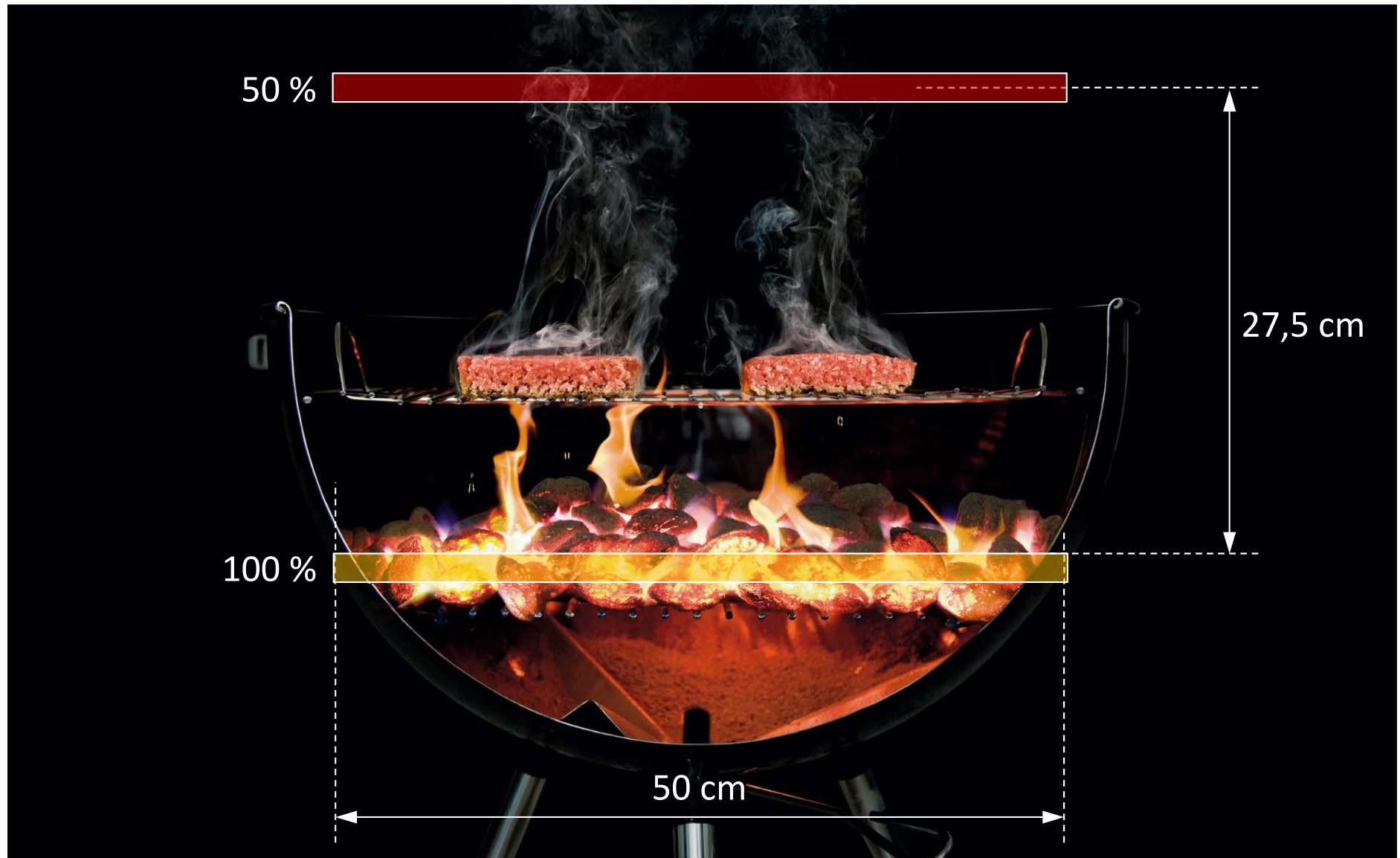
Lifting the grill grate during grilling



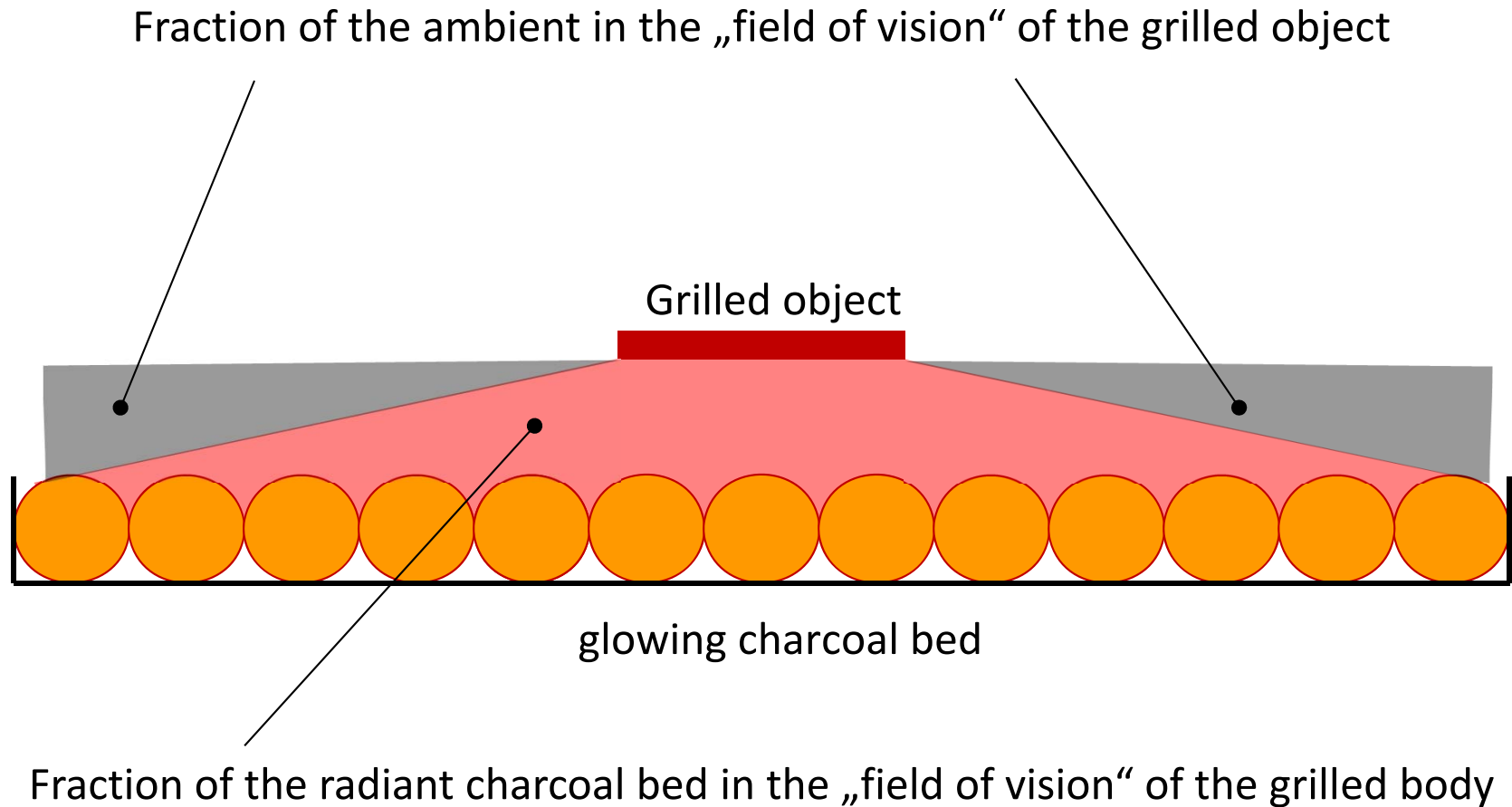
Lifting the grill grate during grilling



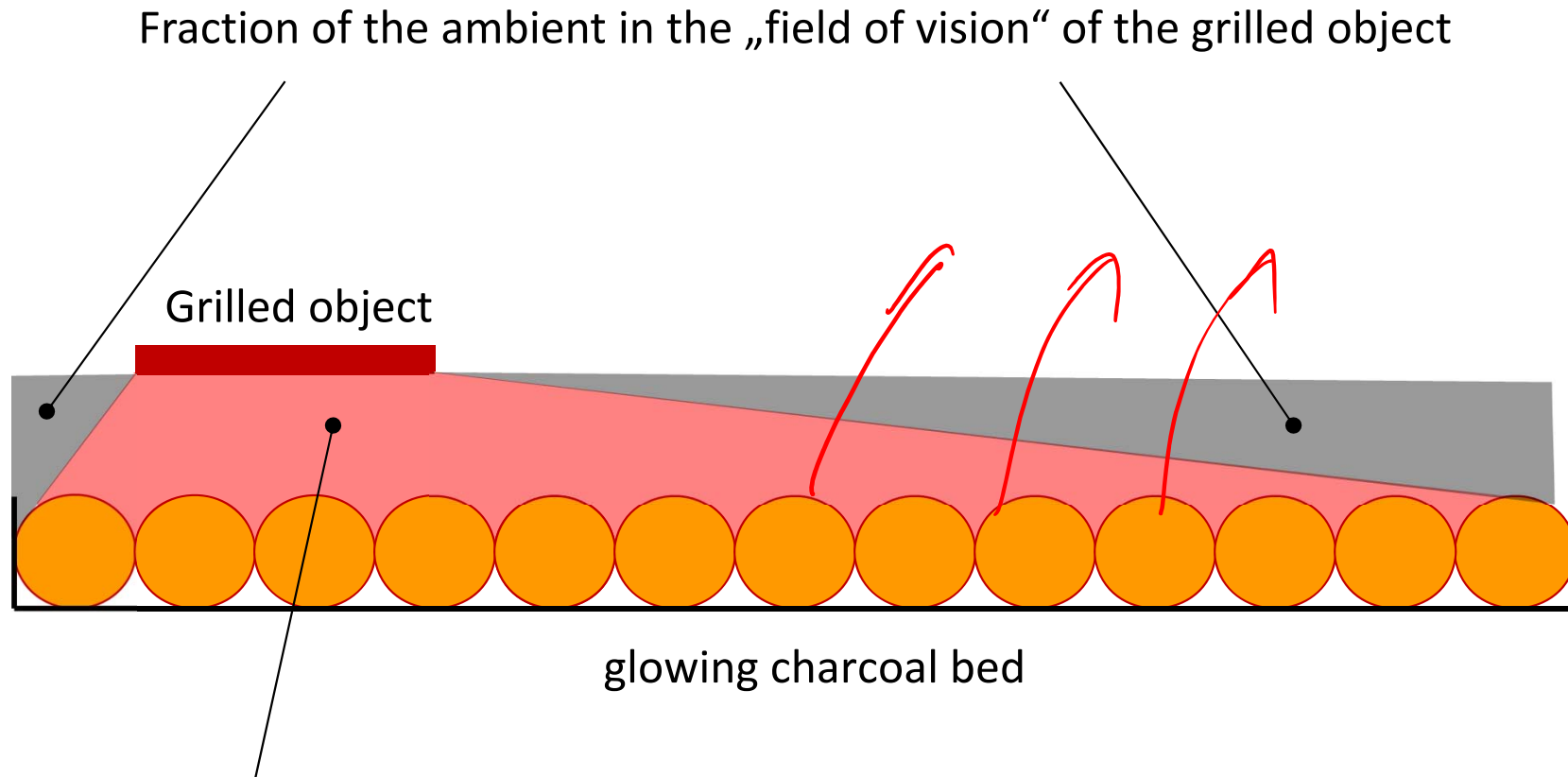
Lifting the grill grate during grilling



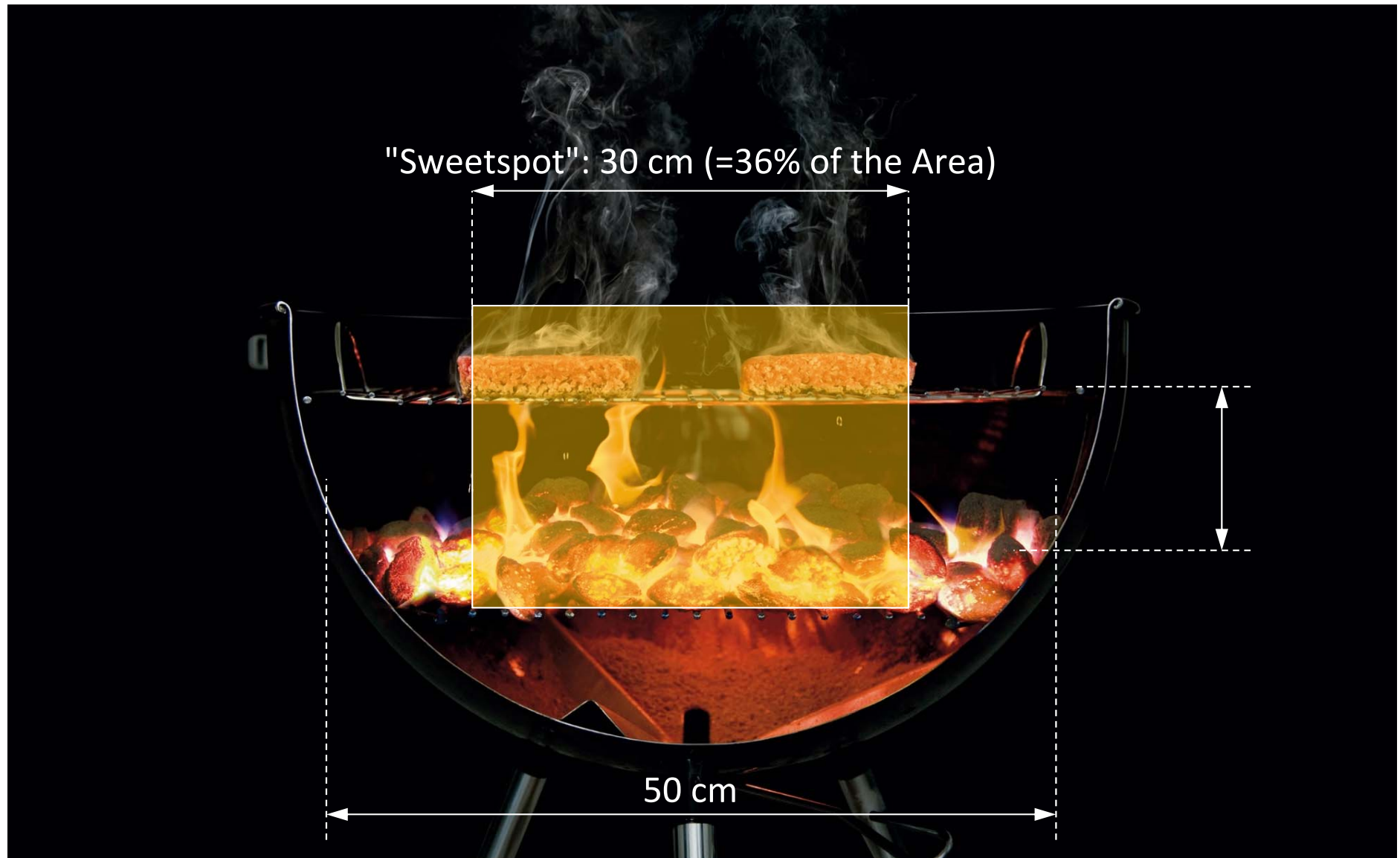
Influence of shape and optical properties of the grill



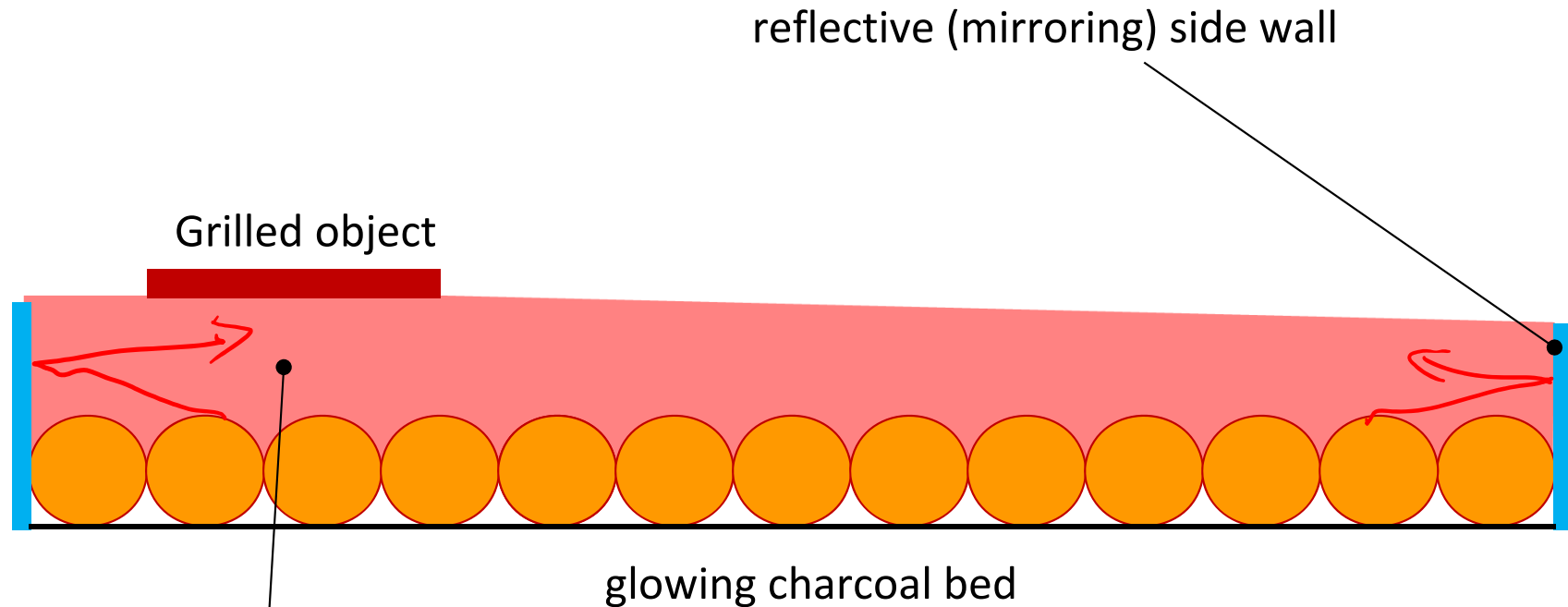
Influence of shape and optical properties of the grill



Lifting the grill grate during grilling

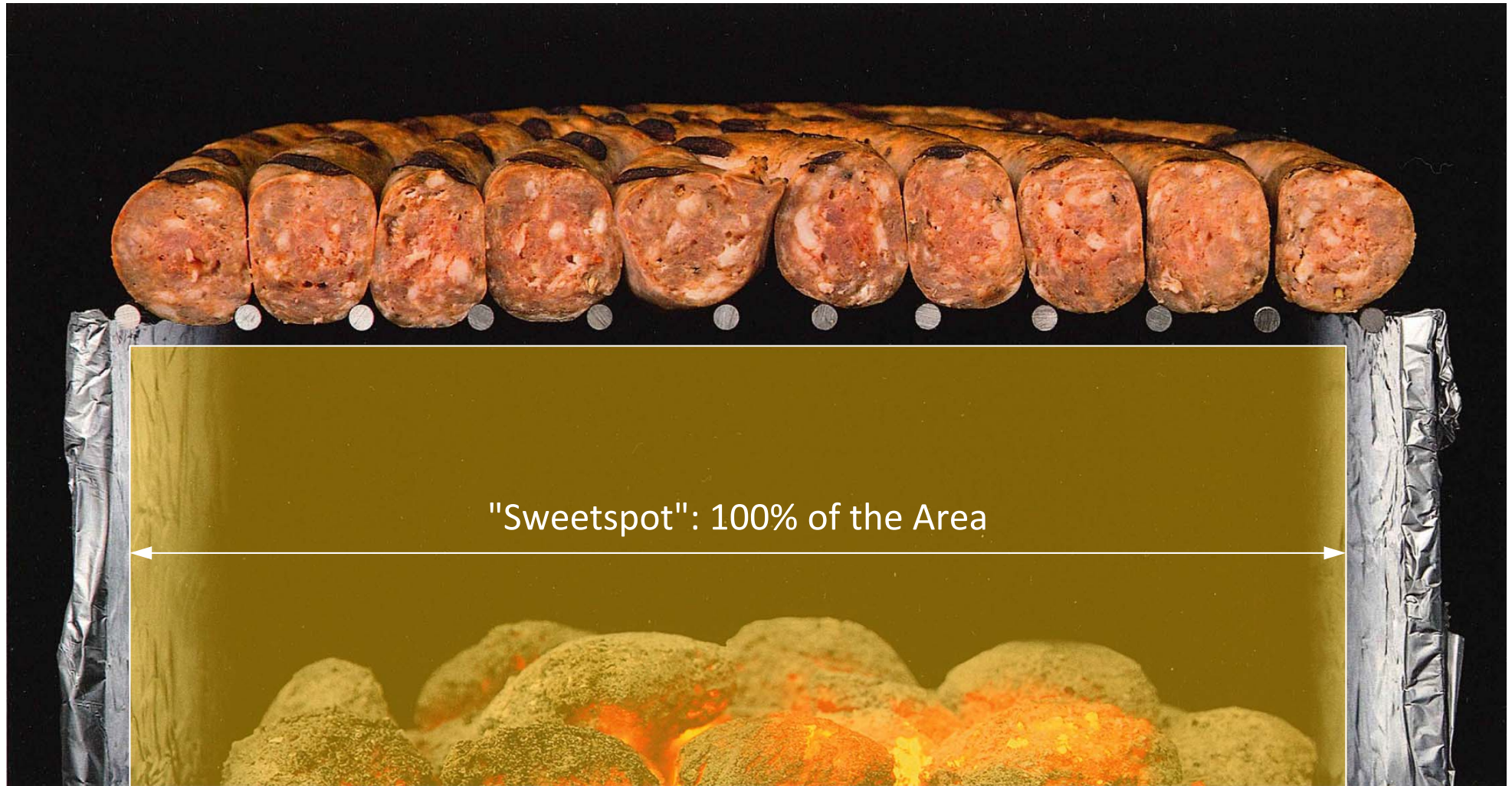


Influence of shape and optical properties of the grill



Fraction of the radiant charcoal bed in the „field of vision“ of the grilled object:
almost 100%

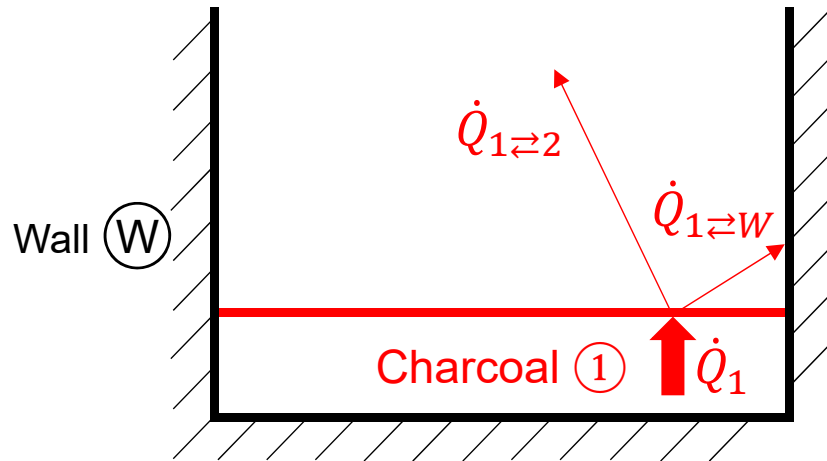
Influence of shape and optical properties of the grill



Influence of shape and optical properties of the grill



Discussion of assumptions and material properties



Comprehension Questions

Why is calculation of radiation transfer much more complicated when a third object is added?

**If several bodies are involved in the radiation transfer, can certain bodies be combined?
In which case can bodies be combined?**

Literature: Modernist Cuisine published by TASCHEN

All non-referenced images have been taken from this book*
Modernist Cuisine by Nathan Myhrvold et al.



*Which I acquired some years ago and, which provides a wealth of information