Bayesian models and Markov chains

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Research topic: Sleep deprivation

Research Question
How does sleep deprivation impact reaction time?

The Study

- measure reaction time on Day 0
- restrict sleep to 3 hours per night
- measure reaction time on Day 3
- measure the change in reaction time

For subject i, let Y_i be the change in reaction time (in ms) after 3 sleep deprived nights. Of course, people react differently to sleep deprivation. It's reasonable to assume that Y_i are Normally distributed around some average m with standard deviation s

Y_i = change in reaction time(ms) for subject i

Assume
Y_i are Normally distributed around some average change in reaction time m with standard deviation s.

$Y_i \sim N(m, s^2)$

Prior model for parameter m
Y_i = change in reaction time (ms)
$Y_i \sim N(m, s^2)$
$m = averageY_i$

Prior information:

- with normal sleep, average reaction time is ~250 ms
- expect average to increase by ~50 m
- average is unlikely to decrease & unlikely to increase by more than ~150 ms

Thus, m ~ N(50, 25^2)

Also, * s>0 * with normal sleep, s.d. in reaction times is ~30 ms * s is equally likely to be anywhere from 0 to 200 ms

Thus, s ~ Unif(0,200)

Therefore, $Y_i \sim N(m, s^2)$ $m \sim N(50, 25^2)$ $s \sim Unif(0, 200)$

```r
library(ggplot2)
```

Normal-Normal priors

In the first step of your Bayesian analysis, you'll simulate the following prior models for parameters m and s:

- Use rnorm(n, mean, sd) to sample 10,000 draws from the m prior. Assign the output to prior_m.
- Use runif(n, min, max) to sample 10,000 draws from the s prior. Assign the output to prior_s.
- After storing these results in the samples data frame, construct a density plot of the prior_m samples and a density plot of the prior_s samples.

```r
# Take 10000 samples from the m prior
prior_m <- rnorm(n=10000, mean=50, sd=25)

# Take 10000 samples from the s prior
prior_s <- runif(n=10000, min=0, max=200)

# Store samples in a data frame
samples <- data.frame(prior_m, prior_s)

# Density plots of the prior_m & prior_s samples
ggplot(samples, aes(x = prior_m)) +
    geom_density()
```
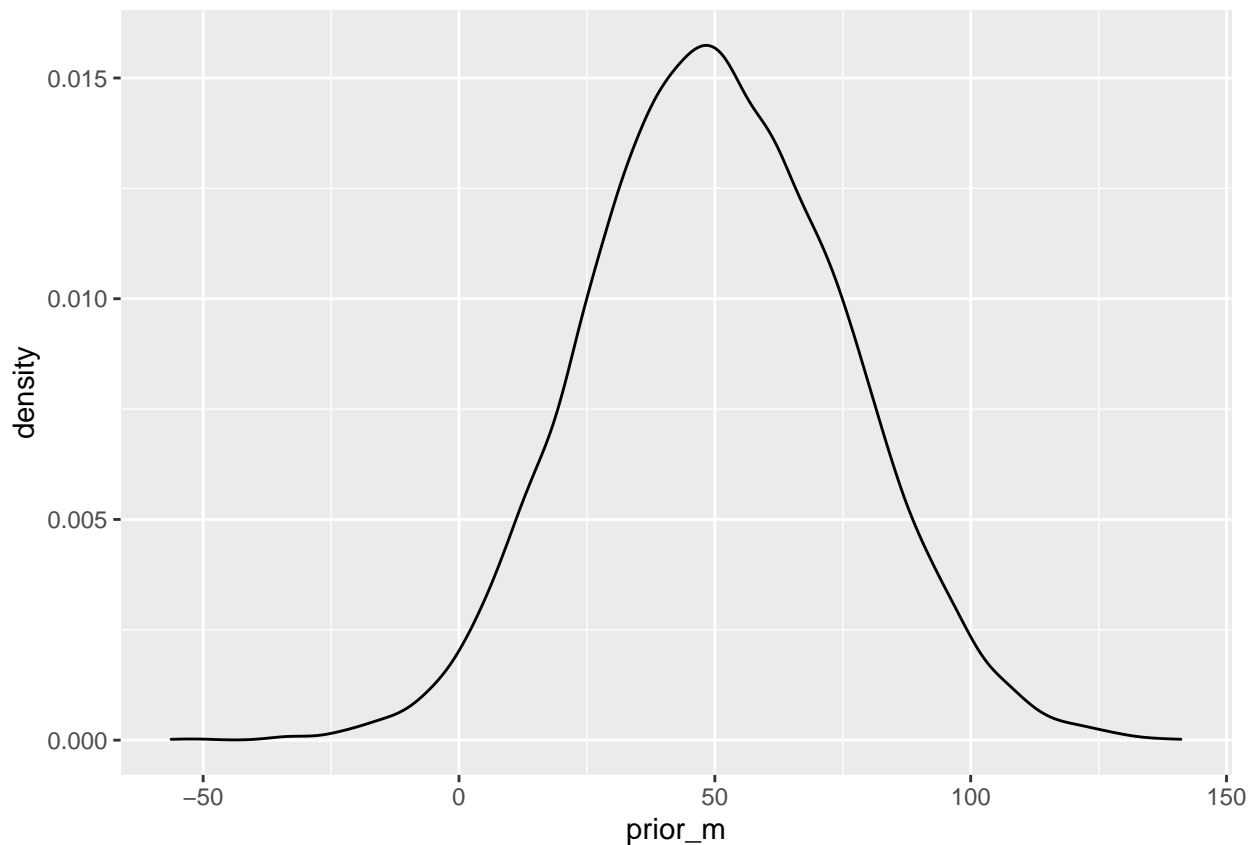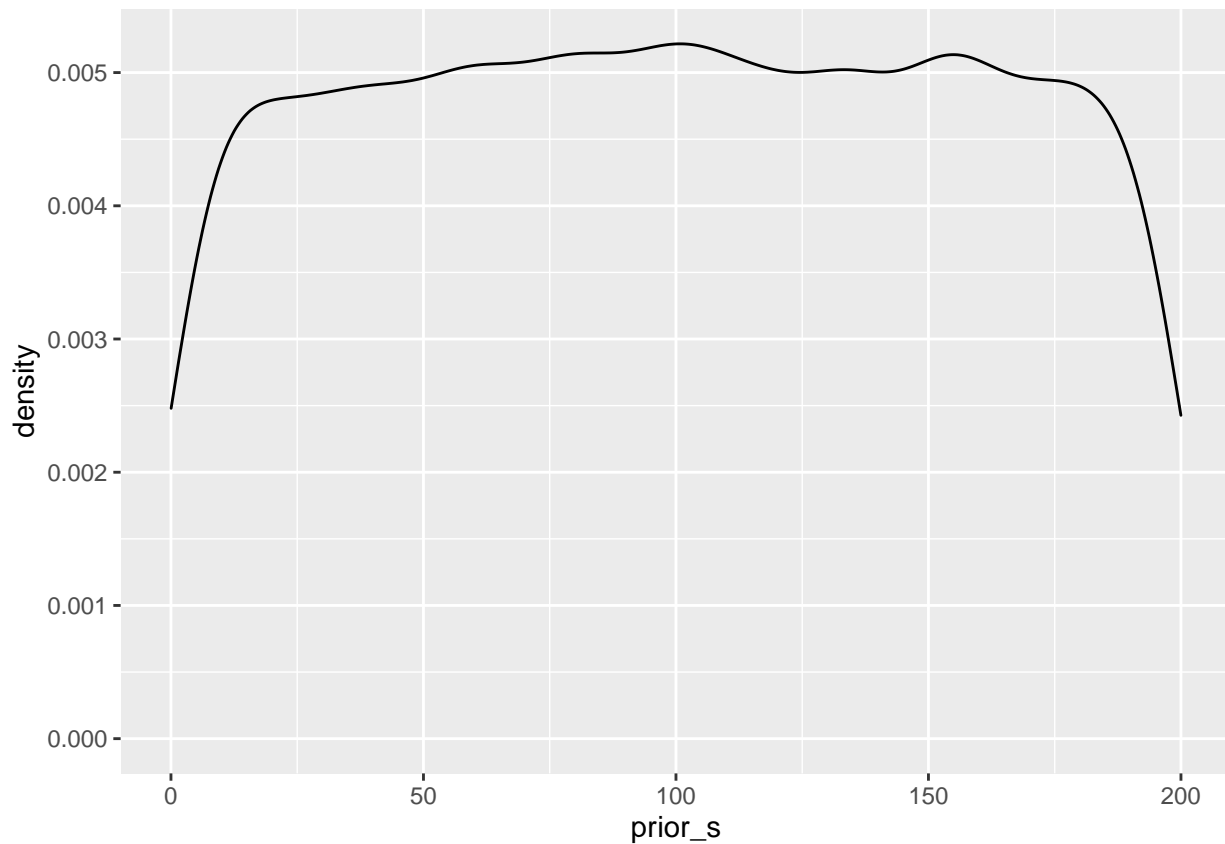
```
ggplot(samples, aes(x = prior_s)) +
    geom_density()
```


The distributions of these random samples approximate the features of your Normal prior for m and Uniform prior for s.

Sleep study data

Researchers enrolled 18 subjects in a sleep deprivation study. Their observed `sleep_study` data are loaded in the workspace. These data contain the `day_0` reaction times and day_3 reaction times after 3 sleep deprived nights for each subject.

You will define and explore `diff_3`, the observed difference in reaction times for each subject. This will require the `mutate()` & `summarize()` functions. For example, the following would add variable `day_0_s`, `day_0` reaction times in seconds, to sleep_study:

```
sleep_study <- sleep_study %>%
    mutate(day_0_s = day_0 * 0.001)
```

You can then summarize() the day_0_s values, here by their minimum & maximum:

```
sleep_study %>%
    summarize(min(day_0_s), max(day_0_s))
```

```
sleep_study <- readr::read_csv('data/sleep_study.csv')
library(dplyr)
```

Instructions

- sample
- Check out the first 6 rows of sleep_study.
- Define a new sleep_study variable diff_3, the day_3 minus the day_0 reaction times.
- Use `ggplot()` with a geom_histogram() layer to construct a histogram of the diff_3 data.
- `summarize()` the mean and standard deviation of the diff_3 observations.

```
# Check out the first 6 rows of sleep_study
head(sleep_study)
```

```
## # A tibble: 6 x 3
##    subject day_0 day_3
##      <dbl> <dbl> <dbl>
## 1     308  250.  321.
## 2     309  223.  205.
## 3     310  199.  233.
## 4     330  322.  285.
## 5     331  288.  320.
## 6     332  235.  310.
```

```
# Define diff_3
sleep_study <- sleep_study %>%
    mutate(diff_3 = day_3 - day_0)

# Histogram of diff_3
ggplot(sleep_study, aes(x = diff_3)) +
    geom_histogram(binwidth = 20, color = "white")
```
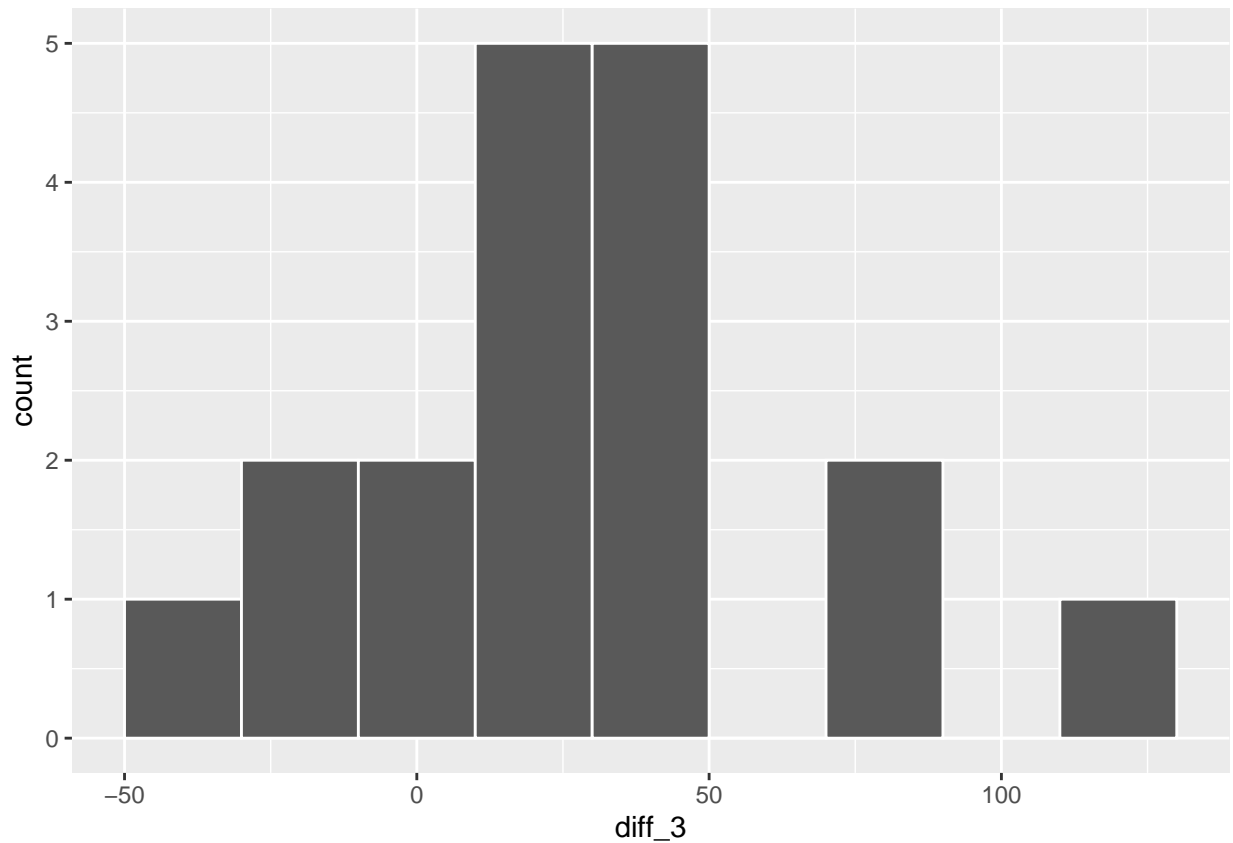
```r
# Mean and standard deviation of diff_3
sleep_study %>%
    summarize(mean(diff_3), sd(diff_3))
```

```
## # A tibble: 1 x 2
##   `mean(diff_3)` `sd(diff_3)`
##            <dbl>        <dbl>
## 1           26.3         37.2
```

Reaction times increased by an average of ~26 ms with a standard deviation of ~37 ms. Further, only 4 of the 18 test subjects had faster reaction times on day 3 than on day 0. Though not in perfect agreement about the degree to which the average reaction time changes under sleep deprivation, both the likelihood and prior are consistent with the hypothesis that the average increases relative to reaction time under normal sleep conditions.