# Portfolio Optimization Using ETFs

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# 1 Introduction

This report presents a detailed analysis of an ETF-based portfolio optimization project. It includes:

- An explanation of ETFs and their investment significance.

- Formulation of the optimization problem.
- Detailed implementation steps in Python.
- Insights into personalization for different client profiles.
- Interpretation of results and their alignment with client objectives.

The project demonstrates a real-world application of decision analysis and optimization, making it suitable for inclusion in a professional portfolio.

# 2 What is an ETF?

## 2.1 Definition and Overview

An Exchange-Traded Fund (ETF) is a type of investment fund traded on stock exchanges, similar to stocks. ETFs hold a collection of assets such as stocks, bonds, or commodities, and their value tracks the performance of the underlying assets.

## 2.2 Benefits of ETFs

- **Diversification:** By investing in a single ETF, investors gain exposure to a broad range of assets.
- **Liquidity:** ETFs are traded like stocks, providing high liquidity.
- **Cost Efficiency:** Lower management fees compared to mutual funds.
- **Transparency:** Holdings are usually disclosed daily.

## 2.3 Investment in ETFs

Investing in ETFs involves selecting funds that align with individual objectives, such as:

- Growth: ETFs tracking high-growth sectors like technology (e.g., QQQ).
- Income: ETFs focusing on dividends (e.g., VYM).
- Risk Management: Diversified bond ETFs (e.g., AGG).
- Sustainability: ETFs emphasizing ESG factors (e.g., ESGU).

# 3 Problem Formulation

## 3.1 What Makes It an Optimization Problem?

Portfolio optimization involves determining the best allocation of funds across a set of ETFs to achieve specific goals while satisfying constraints. This makes it a mathematical optimization problem:

- **Objective:** Maximize returns while minimizing risks.
- **Constraints:** Budget, risk tolerance, ESG preferences, and diversification.
- **Decision Variables:** Weights assigned to each ETF.

### 3.2 Objective Function

The optimization model maximizes a weighted combination of portfolio metrics:

$$f(w) = \alpha R(w) - \beta \Sigma(w) + \gamma M(w) + \delta E(w) - \zeta H(w)$$

where:

- $R(w)$: Portfolio return.
- $\Sigma(w)$: Portfolio risk.
- $M(w)$: Portfolio momentum.
- $E(w)$: Portfolio ESG score.
- $H(w)$: Portfolio concentration (diversification measure).
- $\alpha, \beta, \gamma, \delta, \zeta$: Coefficients reflecting client preferences.

### 3.3 Constraints

- **Budget:** $\sum_{i=1}^{N} w_i = 1$.
- **Risk:** $\Sigma(w) \leq \sigma_{\max}^2$.
- **Momentum:** $M(w) \geq M_{\min}$.
- **ESG:** $E(w) \geq E_{\min}$.
- **Diversification:** $H(w) \leq H_{\max}$.
- **Bounds:** $0 \leq w_i \leq 1, \forall i$.

## 4 Personalization for Different Client Profiles

### 4.1 Growth-Oriented Clients

- Prioritize high returns: $\alpha = 0.7$, $\beta = 0.2$.
- Less emphasis on ESG and diversification: $\delta, \zeta = 0.05$.

### 4.2 Risk-Averse Clients

- Minimize risks: $\beta = 0.5$, $\alpha = 0.3$.
- Strong diversification preferences: $\zeta = 0.1$.

### 4.3 Sustainability-Focused Clients

- Maximize ESG scores: $\delta = 0.5$, $\alpha = 0.3$.
- Risk and diversification moderately weighted: $\beta, \zeta = 0.1$.

# 5 Implementation Steps

## 5.1 Data Extraction

Historical data for 100 ETFs was extracted using `yfinance`. Metrics computed:

- **Return:** Annualized mean of daily returns.
- **Risk:** Annualized variance of daily returns.
- **Momentum:** Ratio of short-term to long-term moving averages.
- **ESG Scores:** Weighted averages of constituent scores.
- **Diversification:** Herfindahl-Hirschman Index.

## 5.2 Normalization

All metrics were normalized to [0, 1] to ensure comparability:

$$\text{Normalized Value} = \frac{\text{Metric Value}}{\text{Maximum Metric Value}}$$

## 5.3 Optimization

The `scipy.optimize.minimize` function solved the optimization problem with:

- **Objective:** Weighted combination of normalized metrics.
- **Method:** `trust-constr`.
- **Regularization:** Small term ($\epsilon = 10^{-6}$) added for stability.

## 5.4 Results Interpretation

- **Optimized Weights:** Allocations for each ETF.
- **Portfolio Metrics:** Computed using optimized weights.
- **Visualization:** Scatter plot of ETFs with significant weights.

# 6 Conclusion

This project demonstrated a systematic approach to portfolio optimization using ETFs. The detailed formulation, implementation, and results interpretation showcase its value for investors with diverse objectives. By incorporating personalized coefficients, the model aligns with varying client preferences.

# 7 Annex: Glossary and Concepts

## 7.1 Financial Terms

- **ETF:** A fund traded on stock exchanges.
- **ESG:** Environmental, social, and governance criteria.
- **Momentum:** A measure of price trends.

## 7.2 Mathematical Terms

- **Optimization:** Maximizing or minimizing a function.
- **Regularization:** Prevents overfitting in optimization problems.