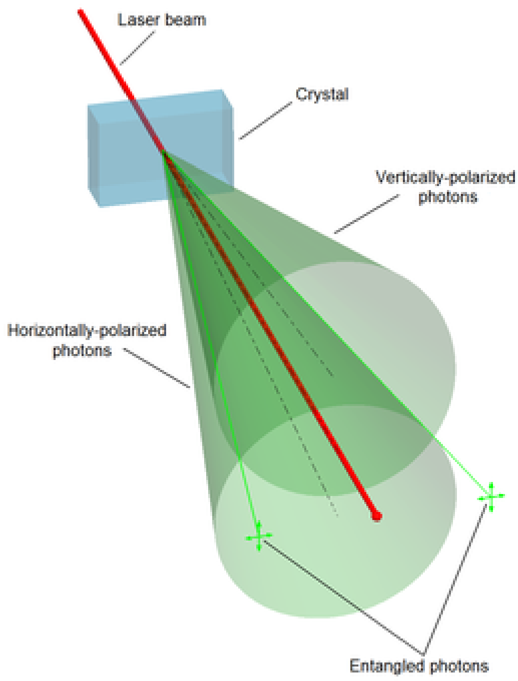
Quantum entanglement

Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles are generated, interact, or share spatial proximity in ways such that the quantum state of each particle cannot be described independently of the state of the others, even when the particles are separated by a large distance.



Spontaneous parametric down-conversion process can split photons into type II photon pairs with mutually perpendicular polarization.

Measurements of physical properties such as position, momentum, spin, and polarization, performed on entangled particles are found to be perfectly correlated. For example, if a pair of particles is generated in such a way that their total spin is known to be zero, and one particle is found to have clockwise spin on a certain axis, the spin of the other particle, measured on the same axis, will be found to be counterclockwise, as is to be expected due to their entanglement. However, this behavior gives rise to seemingly paradoxical effects: any measurement of a property of a particle performs an irreversible collapse on that particle and will change the original quantum state. In the case of entangled particles, such a measurement will be on the entangled system as a whole.

Such phenomena were the subject of a 1935 paper by Albert Einstein, Boris Podolsky, and Nathan Rosen,[1] and several papers by Erwin Schrödinger shortly thereafter,[2][3] describing what came to be known as the EPR paradox. Einstein and others considered such behavior to be impossible, as it violated the local realism view of causality (Einstein referring to it as "spooky action at a distance")[4] and argued that the accepted formulation of quantum mechanics must therefore be incomplete.

Later, however, the counterintuitive predictions of quantum mechanics were verified experimentally[5] in tests where the polarization or spin of entangled particles were measured at separate locations, statistically violating Bell's inequality. In earlier tests it couldn't be absolutely ruled out that the test result at one point could have been subtly transmitted to the remote point, affecting the outcome at the second location.[6] However so-called "loophole-free" Bell tests have been performed in which the locations were separated such that communications at the speed of light would have taken longer—in one case 10,000 times longer—than the interval between the measurements.[7][8]

According to *some* interpretations of quantum mechanics, the effect of one measurement occurs instantly. Other interpretations which don't recognize wavefunction collapse dispute that there is any "effect" at all. However, all interpretations agree that entanglement produces *correlation* between the measurements and that the mutual information between the entangled particles can be exploited, but that any *transmission* of information at faster-than-light speeds is impossible.[9][10]

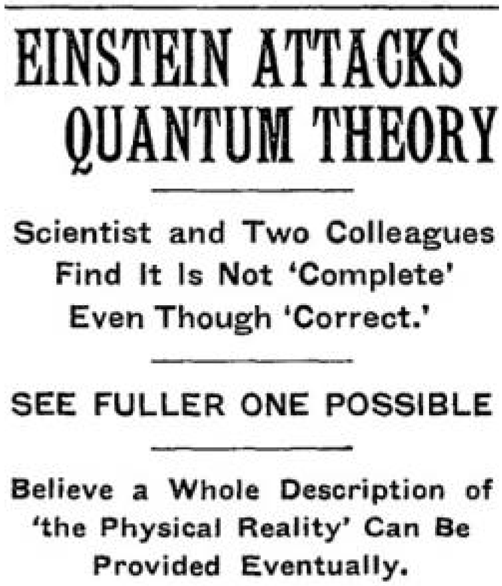
Quantum entanglement has been demonstrated experimentally with photons,[11][12][13][14] neutrinos,[15] electrons,[16][17] molecules as large as buckyballs,[18][19] and even small diamonds.[20][21] On 13 July 2019, scientists from the University of Glasgow reported taking the first ever photo of a strong form of quantum entanglement known as Bell entanglement.[22][23] The utilization of entanglement in communication and computation is a very active area of research.

Contents

History

The counterintuitive predictions of quantum mechanics about strongly correlated systems were first discussed by Albert Einstein in 1935, in a joint paper with Boris Podolsky and Nathan Rosen.[1] In this study, the three formulated the Einstein–Podolsky–Rosen paradox (EPR paradox), a thought experiment that attempted to show that quantum mechanical theory was incomplete. They wrote: "We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete."[1]

However, the three scientists did not coin the word *entanglement*, nor did they generalize the special properties of the state they considered. Following the EPR paper, Erwin Schrödinger wrote a letter to Einstein in German in which he used the word *Verschränkung* (translated by himself as *entanglement*) "to describe the correlations between two particles that interact and then separate, as in the EPR experiment."[24]



EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Article headline regarding the Einstein–Podolsky–Rosen paradox (EPR paradox) paper, in the May 4, 1935 issue of *The New York Times*.

Schrödinger shortly thereafter published a seminal paper defining and discussing the notion of "entanglement." In the paper he recognized the importance of the concept, and stated:[2] "I would not call [entanglement] *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought."

Like Einstein, Schrödinger was dissatisfied with the concept of entanglement, because it seemed to violate the speed limit on the transmission of information implicit in the theory of relativity.[25] Einstein later famously derided entanglement as "*spukhafte Fernwirkung*"[26] or "spooky action at a distance."

The EPR paper generated significant interest among physicists which inspired much discussion about the foundations of quantum mechanics (perhaps most famously Bohm's interpretation of quantum mechanics), but produced relatively little other published work. Despite the interest, the weak point in EPR's argument was not discovered until 1964, when John Stewart Bell proved that one of their key assumptions, the principle of locality, as applied to the kind of hidden variables interpretation hoped for by EPR, was mathematically inconsistent with the predictions of quantum theory.

Specifically, Bell demonstrated an upper limit, seen in Bell's inequality, regarding the strength of correlations that can be produced in any theory obeying local realism, and showed that quantum theory predicts violations of this limit for certain entangled systems.[27] His inequality is experimentally testable, and there have been numerous relevant experiments, starting with the pioneering work of Stuart Freedman and John Clauser in 1972[28] and Alain Aspect's experiments in 1982,[29] all of which have shown agreement with quantum mechanics rather than the principle of local realism.

For decades, each had left open at least one loophole by which it was possible to question the validity of the results. However, in 2015 an experiment was performed that simultaneously closed both the detection and locality loopholes, and was heralded as "loophole-free"; this experiment ruled out a large class of local realism theories with certainty.[30] Alain Aspect notes that the "setting-independence loophole" – which he refers to as "far-fetched", yet, a "residual loophole" that "cannot be ignored" – has yet to be closed, and the free-will / *superdeterminism* loophole is unclosable; saying "no experiment, as ideal as it is, can be said to be totally loophole-free."[31]

A minority opinion holds that although quantum mechanics is correct, there is no superluminal instantaneous action-at-a-distance between entangled particles once the particles are separated.[32][33][34][35][36]

Bell's work raised the possibility of using these super-strong correlations as a resource for communication. It led to the 1984 discovery of quantum key distribution protocols, most famously BB84 by Charles H. Bennett and Gilles Brassard[37] and E91 by Artur Ekert.[38] Although BB84 does not use entanglement, Ekert's protocol uses the violation of a Bell's inequality as a proof of security.

In October 2018, physicists reported that quantum behavior can be explained with classical physics for a single particle, but not for multiple particles as in quantum entanglement and related nonlocality phenomena.[39][40]

In July 2019 physicists reported, for the first time, capturing an image of quantum entanglement.[22][23]

Concept

Meaning of entanglement

An entangled system is defined to be one whose quantum state cannot be factored as a product of states of its local constituents; that is to say, they are not individual particles but are an inseparable whole. In entanglement, one constituent cannot be fully described without considering the other(s). The state of a composite system is always expressible as a sum, or superposition, of products of states of local constituents; it is entangled if this sum necessarily has more than one term.

Quantum systems can become entangled through various types of interactions. For some ways in which entanglement may be achieved for experimental purposes, see the section below on methods. Entanglement is broken when the entangled particles decohere through interaction with the environment; for example, when a measurement is made.[41]

As an example of entanglement: a subatomic particle decays into an entangled pair of other particles. The decay events obey the various conservation laws, and as a result, the measurement outcomes of one daughter particle must be highly correlated with the measurement outcomes of the other daughter particle (so that the total momenta, angular momenta, energy, and so forth remains roughly the same before and after this process). For instance, a spin-zero particle could decay into a pair of spin-½ particles. Since the total spin before and after this decay must be zero (conservation of angular momentum), whenever the first particle is measured to be spin up on some axis, the other, when measured on the same axis, is always found to be spin down. (This is called the spin anti-correlated case; and if the prior probabilities for measuring each spin are equal, the pair is said to be in the singlet state.)

The special property of entanglement can be better observed if we separate the said two particles. Let's put one of them in the White House in Washington and the other in Buckingham Palace (think about this as a thought experiment, not an actual one). Now, if we measure a particular characteristic of one of these particles (say, for example, spin), get a result, and then measure the other particle using the same criterion (spin along the same axis), we find that the result of the measurement of the second particle will match (in a complementary sense) the result of the measurement of the first particle, in that they will be opposite in their values.

The above result may or may not be perceived as surprising. A classical system would display the same property, and a hidden variable theory (see below) would certainly be required to do so, based on conservation of angular momentum in classical and quantum mechanics alike. The difference is that a classical system has definite values for all the observables all along, while the quantum system does not. In a sense to be discussed below, the quantum system considered here seems to acquire a probability distribution for the outcome of a measurement of the spin along any axis of the other particle upon measurement of the first particle. This probability distribution is in general different from what it would be without measurement of the first particle. This may certainly be perceived as surprising in the case of spatially separated entangled particles.

Paradox

The paradox is that a measurement made on either of the particles apparently collapses the state of the entire entangled system—and does so instantaneously, before any information about the measurement result could have been communicated to the other particle (assuming that information cannot travel faster than light) and hence assured the "proper" outcome of the measurement of the other part of the entangled pair. In the Copenhagen interpretation, the result of a spin measurement on one of the particles is a collapse into a state in which each particle has a definite spin (either up or down) along the axis of measurement. The outcome is taken to be random, with each possibility having a probability of 50%. However, if both spins are measured along the same axis, they are found to be anti-correlated. This means that the random outcome of the measurement made on one particle seems to have been transmitted to the other, so that it can make the "right choice" when it too is measured.[42]

The distance and timing of the measurements can be chosen so as to make the interval between the two measurements spacelike, hence, any causal effect connecting the events would have to travel faster than light. According to the principles of special relativity, it is not possible for any information to travel between two such measuring events. It is not even possible to say which of the measurements came first. For two spacelike separated events x_1 and x_2 there are inertial frames in which x_1 is first and others in which x_2 is first. Therefore, the correlation between the two measurements cannot be explained as one measurement determining the other: different observers would disagree about the role of cause and effect.

(In fact similar paradoxes can arise even without entanglement: the position of a single particle is spread out over space, and two widely separated detectors attempting to detect the particle in two different places must instantaneously attain appropriate correlation, so that they do not both detect the particle.)

Hidden variables theory

A possible resolution to the paradox is to assume that quantum theory is incomplete, and the result of measurements depends on predetermined "hidden variables".[43] The state of the particles being measured contains some hidden variables, whose values effectively determine, right from the moment of separation, what the outcomes of the spin measurements are going to be. This would mean that each particle carries all the required information with it, and nothing needs to be transmitted from one particle to the other at the time of measurement. Einstein and others (see the previous section) originally believed this was the only way out of the paradox, and the accepted quantum mechanical description (with a random measurement outcome) must be incomplete.

Violations of Bell's inequality

The hidden variables theory fails, however, when measurements of the spin of entangled particles along different axes are considered (e.g., along any of three axes that make angles of 120 degrees). If a large number of pairs of such measurements are made (on a large number of pairs of entangled particles), then statistically, if the local realist or hidden variables view were correct, the results would always satisfy Bell's inequality. A number of experiments have shown in practice that Bell's inequality is not satisfied. However, prior to 2015, all of these had loophole problems that were considered the most important by the community of physicists.[44][45] When measurements of the entangled particles are made in moving relativistic reference frames, in which each measurement (in its own relativistic time frame) occurs before the other, the measurement results remain correlated.[46][47]

The fundamental issue about measuring spin along different axes is that these measurements cannot have definite values at the same time—they are incompatible in the sense that these measurements' maximum simultaneous precision is constrained by the uncertainty principle. This is contrary to what is found in classical physics, where any number of properties can be measured simultaneously with arbitrary accuracy. It has been proven mathematically that compatible measurements cannot show Bell-inequality-violating correlations,[48] and thus entanglement is a fundamentally non-classical phenomenon.

Other types of experiments

In experiments in 2012 and 2013, polarization correlation was created between photons that never coexisted in time.[49][50] The authors claimed that this result was achieved by entanglement swapping between two pairs of entangled photons after measuring the polarization of one photon of the early pair, and that it proves that quantum non-locality applies not only to space but also to time.

In three independent experiments in 2013 it was shown that classically-communicated separable quantum states can be used to carry entangled states.[51] The first loophole-free Bell test was held in TU Delft in 2015 confirming the violation of Bell inequality.[52]

In August 2014, Brazilian researcher Gabriela Barreto Lemos and team were able to "take pictures" of objects using photons that had not interacted with the subjects, but were entangled with photons that did interact with such objects. Lemos, from the University of Vienna, is confident that this new quantum imaging technique could find application where low light imaging is imperative, in fields like biological or medical imaging.[53]

In 2015, Markus Greiner's group at Harvard performed a direct measurement of Renyi entanglement in a system of ultracold bosonic atoms.

From 2016 various companies like IBM, Microsoft etc. have successfully created quantum computers and allowed developers and tech enthusiasts to openly experiment with concepts of quantum mechanics including quantum entanglement.[54]

Mystery of time

There have been suggestions to look at the concept of time as an emergent phenomenon that is a side effect of quantum entanglement.[55][56] In other words, time is an entanglement phenomenon, which places all equal clock readings (of correctly prepared clocks, or of any objects usable as clocks) into the same history. This was first fully theorized by Don Page and William Wootters in 1983.[57] The Wheeler–DeWitt equation that combines general relativity and quantum mechanics – by leaving out time altogether – was introduced in the 1960s and it was taken up again in 1983, when Page and Wootters made a solution based on quantum entanglement. Page and Wootters argued that entanglement can be used to measure time.[58]

In 2013, at the Istituto Nazionale di Ricerca Metrologica (INRIM) in Turin, Italy, researchers performed the first experimental test of Page and Wootters' ideas. Their result has been interpreted to confirm that time is an emergent phenomenon for internal observers but absent for external observers of the universe just as the Wheeler-DeWitt equation predicts.[58]

Source for the arrow of time

Physicist Seth Lloyd says that quantum uncertainty gives rise to entanglement, the putative source of the arrow of time. According to Lloyd; "The arrow of time is an arrow of increasing correlations."[59] The approach to entanglement would be from the perspective of the causal arrow of time, with the assumption that the cause of the measurement of one particle determines the effect of the result of the other particle's measurement.

Emergent gravity

Based on AdS/CFT correspondence, Mark Van Raamsdonk suggested that spacetime arises as an emergent phenomenon of the quantum degrees of freedom that are entangled and live in the boundary of the space-time.[60] Induced gravity can emerge from the entanglement first law.[61][62]

Non-locality and entanglement

In the media and popular science, quantum non-locality is often portrayed as being equivalent to entanglement. While this is true for pure bipartite quantum states, in general entanglement is only necessary for non-local correlations, but there exist mixed entangled states that do not produce such correlations.[63] A well-known example is the Werner states that are entangled for certain values of p_{sym}, but can always be described using local hidden variables.[64] Moreover, it was shown that, for arbitrary numbers of parties, there exist states that are genuinely entangled but admit a local model.[65] The mentioned proofs about the existence of local models assume that there is only one copy of the quantum state available at a time. If the parties are allowed to perform local measurements on many copies of such states, then many apparently local states (e.g., the qubit Werner states) can no longer be described by a local model. This is, in particular, true for all distillable states. However, it remains an open question whether all entangled states become non-local given sufficiently many copies.[66]

In short, entanglement of a state shared by two parties is necessary but not sufficient for that state to be non-local. It is important to recognize that entanglement is more commonly viewed as an algebraic concept, noted for being a prerequisite to non-locality as well as to quantum teleportation and to superdense coding, whereas non-locality is defined according to experimental statistics and is much more involved with the foundations and interpretations of quantum mechanics.[67]

Quantum mechanical framework

The following subsections are for those with a good working knowledge of the formal, mathematical description of quantum mechanics, including familiarity with the formalism and theoretical framework developed in the articles: bra–ket notation and mathematical formulation of quantum mechanics.

Pure states

Consider two noninteracting systems A and B, with respective Hilbert spaces H_A and H_B. The Hilbert space of the composite system is the tensor product

$$H_A \otimes H_B.$$

If the first system is in state $|\psi\rangle_A$ and the second in state $|\phi\rangle_B$, the state of the composite system is

$$|\psi\rangle_A \otimes |\phi\rangle_B.$$

States of the composite system that can be represented in this form are called separable states, or product states.

Not all states are separable states (and thus product states). Fix a basis $\{|i\rangle_A\}$ for H_A and a basis $\{|j\rangle_B\}$ for H_B. The most general state in $H_A \otimes H_B$ is of the form

$$|\psi\rangle_{AB} = \sum_{i,j} c_{ij} |i\rangle_A \otimes |j\rangle_B.$$

This state is separable if there exist vectors $[c_i^A],[c_j^B]$ so that $c_{ij}=c_i^A c_j^B$, yielding $|\psi\rangle_A=\sum_i c_i^A|i\rangle_A$ and $|\phi\rangle_B=\sum_j c_j^B|j\rangle_B$. It is inseparable if for any vectors $[c_i^A],[c_j^B]$ at least for one pair of coordinates c_i^A,c_j^B we have $c_{ij}\neq c_i^A c_j^B$. If a state is inseparable, it is called an 'entangled state'.

For example, given two basis vectors $\{|0\rangle_A,|1\rangle_A\}$ of H_A and two basis vectors $\{|0\rangle_B,|1\rangle_B\}$ of H_B, the following is an entangled state:

$$\frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right).$$

If the composite system is in this state, it is impossible to attribute to either system A or system B a definite pure state. Another way to say this is that while the von Neumann entropy of the whole state is zero (as it is for any pure state), the entropy of the subsystems is greater than zero. In this sense, the systems are "entangled". This has specific empirical ramifications for interferometry.[68] The above example is one of four Bell states, which are (maximally) entangled pure states (pure states of the $H_A \otimes H_B$ space, but which cannot be separated into pure states of each H_A and H_B).

Now suppose Alice is an observer for system A, and Bob is an observer for system B. If in the entangled state given above Alice makes a measurement in the $\{|0\rangle,|1\rangle\}$ eigenbasis of A, there are two possible outcomes, occurring with equal probability:[69]

1. Alice measures 0, and the state of the system collapses to $|0\rangle_A|1\rangle_B$.
2. Alice measures 1, and the state of the system collapses to $|1\rangle_A|0\rangle_B$.

If the former occurs, then any subsequent measurement performed by Bob, in the same basis, will always return 1. If the latter occurs, (Alice measures 1) then Bob's measurement will return 0 with certainty. Thus, system B has been altered by Alice performing a local measurement on system A. This remains true even if the systems A and B are spatially separated. This is the foundation of the EPR paradox.

The outcome of Alice's measurement is random. Alice cannot decide which state to collapse the composite system into, and therefore cannot transmit information to Bob by acting on her system. Causality is thus preserved, in this particular scheme. For the general argument, see no-communication theorem.

Ensembles

As mentioned above, a state of a quantum system is given by a unit vector in a Hilbert space. More generally, if one has less information about the system, then one calls it an 'ensemble' and describes it by a density matrix, which is a positive-semidefinite matrix, or a trace class when the state space is infinite-dimensional, and has trace 1. Again, by the spectral theorem, such a matrix takes the general form:

$$\rho = \sum_i w_i |\alpha_i\rangle\langle\alpha_i|,$$

where the w_i are positive-valued probabilities (they sum up to 1), the vectors α_i are unit vectors, and in the infinite-dimensional case, we would take the closure of such states in the trace norm. We can interpret ρ as representing an ensemble where w_i is the proportion of the ensemble whose states are $|\alpha_i\rangle$. When a mixed state has rank 1, it therefore describes a 'pure ensemble'. When there is less than total information about the state of a quantum system we need density matrices to represent the state.

Experimentally, a mixed ensemble might be realized as follows. Consider a "black box" apparatus that spits electrons towards an observer. The electrons' Hilbert spaces are identical. The apparatus might produce electrons that are all in the same state; in this case, the electrons received by the observer are then a pure ensemble. However, the apparatus could produce electrons in different states. For example, it could produce two populations of electrons: one with state $|\mathbf{z}+\rangle$ with spins aligned in the positive $\mathbf{z}$ direction, and the other with state $|\mathbf{y}-\rangle$ with spins aligned in the negative $\mathbf{y}$ direction. Generally, this is a mixed ensemble, as there can be any number of populations, each corresponding to a different state.

Following the definition above, for a bipartite composite system, mixed states are just density matrices on $H_A \otimes H_B$. That is, it has the general form

$$\rho = \sum_i w_i \left[\sum_j \bar{c}_{ij} (|\alpha_{ij}\rangle \otimes |\beta_{ij}\rangle)\right] \otimes \left[\sum_k c_{ik} (\langle\alpha_{ik}| \otimes \langle\beta_{ik}|)\right]$$

where the w_i are positively valued probabilities, $\sum_j |c_{ij}|^2 = 1$, and the vectors are unit vectors. This is self-adjoint and positive and has trace 1.

Extending the definition of separability from the pure case, we say that a mixed state is separable if it can be written as[70]:131–132

$$\rho = \sum_i w_i \rho_i^A \otimes \rho_i^B,$$

where the w_i are positively valued probabilities and the ρ_i^A's and ρ_i^B's are themselves mixed states (density operators) on the subsystems A and B respectively. In other words, a state is separable if it is a probability distribution over uncorrelated states, or product states. By writing the density matrices as sums of pure ensembles and expanding, we may assume without loss of generality that ρ_i^A and ρ_i^B are themselves pure ensembles. A state is then said to be entangled if it is not separable.

In general, finding out whether or not a mixed state is entangled is considered difficult. The general bipartite case has been shown to be NP-hard.[71] For the 2×2 and 2×3 cases, a necessary and sufficient criterion for separability is given by the famous Positive Partial Transpose (PPT) condition.[72]

Reduced density matrices

The idea of a reduced density matrix was introduced by Paul Dirac in 1930.[73] Consider as above systems A and B each with a Hilbert space H_A, H_B. Let the state of the composite system be

$$|\Psi\rangle \in H_A \otimes H_B.$$

As indicated above, in general there is no way to associate a pure state to the component system A. However, it still is possible to associate a density matrix. Let

$$\rho_T = |\Psi\rangle\ \langle\Psi|.$$

which is the projection operator onto this state. The state of A is the partial trace of ρ_T over the basis of system B:

$$\rho_A \stackrel{\text{def}}{=} \sum_j \langle j|_B \left(|\Psi\rangle\langle\Psi|\right) |j\rangle_B = \mathrm{Tr}_B\ \rho_T.$$

ρ_A is sometimes called the reduced density matrix of ρ on subsystem A. Colloquially, we "trace out" system B to obtain the reduced density matrix on A.

For example, the reduced density matrix of A for the entangled state

$$\tfrac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right),$$

discussed above is

$$\rho_A = \tfrac{1}{2}\left(|0\rangle_A\langle 0|_A + |1\rangle_A\langle 1|_A\right)$$

This demonstrates that, as expected, the reduced density matrix for an entangled pure ensemble is a mixed ensemble. Also not surprisingly, the density matrix of A for the pure product state $|\psi\rangle_A \otimes |\phi\rangle_B$ discussed above is

$$\rho_A = |\psi\rangle_A\langle\psi|_A.$$

In general, a bipartite pure state ρ is entangled if and only if its reduced states are mixed rather than pure.

Two applications that use them

Reduced density matrices were explicitly calculated in different spin chains with unique ground state. An example is the one-dimensional AKLT spin chain:[74] the ground state can be divided into a block and an environment. The reduced density matrix of the block is proportional to a projector to a degenerate ground state of another Hamiltonian.

The reduced density matrix also was evaluated for XY spin chains, where it has full rank. It was proved that in the thermodynamic limit, the spectrum of the reduced density matrix of a large block of spins is an exact geometric sequence[75] in this case.

Entanglement as a resource

In quantum information theory, entangled states are considered a 'resource', i.e., something costly to produce and that allows to implement valuable transformations. The setting in which this perspective is most evident is that of "distant labs", i.e., two quantum systems labeled "A" and "B" on each of which arbitrary quantum operations can be performed, but which do not interact with each other quantum mechanically. The only interaction allowed is the exchange of classical information, which combined with the most general local quantum operations gives rise to the class of operations called LOCC (local operations and classical communication). These operations do not allow the production of entangled states between the systems A and B. But if A and B are provided with a supply of entangled states, then these, together with LOCC operations can enable a larger class of transformations. For example, an interaction between a qubit of A and a qubit of B can be realized by first teleporting A's qubit to B, then letting it interact with B's qubit (which is now a LOCC operation, since both qubits are in B's lab) and then teleporting the qubit back to A. Two maximally entangled states of two qubits are used up in this process. Thus entangled states are a resource that enables the realization of quantum interactions (or of quantum channels) in a setting where only LOCC are available, but they are consumed in the process. There are other applications where entanglement can be seen as a resource, e.g., private communication or distinguishing quantum states.[76]

Classification of entanglement

Not all quantum states are equally valuable as a resource. To quantify this value, different entanglement measures (see below) can be used, that assign a numerical value to each quantum state. However, it is often interesting to settle for a coarser way to compare quantum states. This gives rise to different classification schemes. Most entanglement classes are defined based on whether states can be converted to other states using LOCC or a subclass of these operations. The smaller the set of allowed operations, the finer the classification. Important examples are:

- If two states can be transformed into each other by a local unitary operation, they are said to be in the same *LU class*. This is the finest of the usually considered classes. Two states in the same LU class have the same value for entanglement measures and the same value as a resource in the distant-labs setting. There is an infinite number of different LU classes (even in the simplest case of two qubits in a pure state).[77][78]
- If two states can be transformed into each other by local operations including measurements with probability larger than 0, they are said to be in the same 'SLOCC class' ("stochastic LOCC"). Qualitatively, two states ρ_1 and ρ_2 in the same SLOCC class are equally powerful (since I can transform one into the other and then do whatever it allows me to do), but since the transformations $\rho_1 \to \rho_2$ and $\rho_2 \to \rho_1$ may succeed with different probability, they are no longer equally valuable. E.g., for two pure qubits there are only two SLOCC classes: the entangled states (which contains both the (maximally entangled) Bell states and weakly entangled states like $|00\rangle + 0.01|11\rangle$) and the separable ones (i.e., product states like $|00\rangle$).[79][80]
- Instead of considering transformations of single copies of a state (like $\rho_1 \to \rho_2$) one can define classes based on the possibility of multi-copy transformations. E.g., there are examples when $\rho_1 \to \rho_2$ is impossible by LOCC, but $\rho_1 \otimes \rho_1 \to \rho_2$ is possible. A very important (and very coarse) classification is based on the property whether it is possible to transform an arbitrarily large number of copies of a state ρ into at least one pure entangled state. States that have this property are called distillable. These states are the most useful quantum states since, given enough of them, they can be transformed (with local operations) into any entangled state and hence allow for all possible uses. It came initially as a surprise that not all entangled states are distillable, those that are not are called 'bound entangled'.[81][76]

A different entanglement classification is based on what the quantum correlations present in a state allow A and B to do: one distinguishes three subsets of entangled states: (1) the *non-local states*, which produce correlations that cannot be explained by a local hidden variable model and thus violate a Bell inequality, (2) the *steerable states* that contain sufficient correlations for A to modify ("steer") by local measurements the conditional reduced state of B in such a way, that A can prove to B that the state they possess is indeed entangled, and finally (3) those entangled state that are neither non-local nor steerable. All three sets are non-empty.[82]

Entropy

In this section, the entropy of a mixed state is discussed as well as how it can be viewed as a measure of quantum entanglement.

Definition

In classical information theory H, the Shannon entropy, is associated to a probability distribution, $p_1, \cdots, p_n$, in the following way:[83]

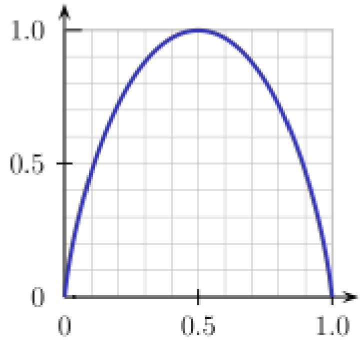
$$H(p_1,\cdots,p_n) = -\sum_i p_i \log_2 p_i .$$

Since a mixed state ρ is a probability distribution over an ensemble, this leads naturally to the definition of the von Neumann entropy:

$$S(\rho) = -\mathrm{Tr}\left(\rho \log_2 \rho\right).$$

In general, one uses the Borel functional calculus to calculate a non-polynomial function such as $\log_2(\rho)$. If the nonnegative operator ρ acts on a finite-dimensional Hilbert space and has eigenvalues $\lambda_1,\cdots,\lambda_n$, $\log_2(\rho)$ turns out to be nothing more than the operator with the same eigenvectors, but the eigenvalues $\log_2(\lambda_1),\cdots,\log_2(\lambda_n)$. The Shannon entropy is then:

$$S(\rho) = -\mathrm{Tr}\left(\rho \log_2 \rho\right) = -\sum_i \lambda_i \log_2 \lambda_i .$$



The plot of von Neumann entropy Vs Eigenvalue for a bipartite 2-level pure state. When the eigenvalue has value .5, von Neumann entropy is at a maximum, corresponding to maximum entanglement.

Since an event of probability 0 should not contribute to the entropy, and given that

$$\lim_{p\to 0} p \log p = 0,$$

the convention $0 \log(0) = 0$ is adopted. This extends to the infinite-dimensional case as well: if ρ has spectral resolution

$$\rho = \int \lambda dP_\lambda ,$$

assume the same convention when calculating

$$\rho \log_2 \rho = \int \lambda \log_2 \lambda dP_\lambda .$$

As in statistical mechanics, the more uncertainty (number of microstates) the system should possess, the larger the entropy. For example, the entropy of any pure state is zero, which is unsurprising since there is no uncertainty about a system in a pure state. The entropy of any of the two subsystems of the entangled state discussed above is $\log(2)$ (which can be shown to be the maximum entropy for 2×2 mixed states).

As a measure of entanglement

Entropy provides one tool that can be used to quantify entanglement, although other entanglement measures exist.[84] If the overall system is pure, the entropy of one subsystem can be used to measure its degree of entanglement with the other subsystems.

For bipartite pure states, the von Neumann entropy of reduced states is the unique measure of entanglement in the sense that it is the only function on the family of states that satisfies certain axioms required of an entanglement measure.

It is a classical result that the Shannon entropy achieves its maximum at, and only at, the uniform probability distribution $\{1/n,...,1/n\}$. Therefore, a bipartite pure state $\rho \in H_A \otimes H_B$ is said to be a **maximally entangled state** if the reduced state of ρ is the diagonal matrix

$$\begin{bmatrix} \frac{1}{n} & & \\ & \ddots & \\ & & \frac{1}{n} \end{bmatrix}.$$

For mixed states, the reduced von Neumann entropy is not the only reasonable entanglement measure.

As an aside, the information-theoretic definition is closely related to entropy in the sense of statistical mechanics (comparing the two definitions in the present context, it is customary to set the Boltzmann constant $k = 1$). For example, by properties of the Borel functional calculus, we see that for any unitary operator U,

$$S(\rho) = S\left(U\rho U^*\right).$$

Indeed, without this property, the von Neumann entropy would not be well-defined.

In particular, U could be the time evolution operator of the system, i.e.,

$$U(t) = \exp\left(\frac{-iHt}{\hbar}\right),$$

where H is the Hamiltonian of the system. Here the entropy is unchanged.

The reversibility of a process is associated with the resulting entropy change, i.e., a process is reversible if, and only if, it leaves the entropy of the system invariant. Therefore, the march of the arrow of time towards thermodynamic equilibrium is simply the growing spread of quantum entanglement.[85] This provides a connection between quantum information theory and thermodynamics.

Rényi entropy also can be used as a measure of entanglement.

Entanglement measures

Entanglement measures quantify the amount of entanglement in a (often viewed as a bipartite) quantum state. As aforementioned, entanglement entropy is the standard measure of entanglement for pure states (but no longer a measure of entanglement for mixed states). For mixed states, there are some entanglement measures in the literature[84] and no single one is standard.

- Entanglement cost
- Distillable entanglement
- Entanglement of formation
- Relative entropy of entanglement
- Squashed entanglement
- Logarithmic negativity

Most (but not all) of these entanglement measures reduce for pure states to entanglement entropy, and are difficult (NP-hard) to compute.[86]

Quantum field theory

The Reeh-Schlieder theorem of quantum field theory is sometimes seen as an analogue of quantum entanglement.

Applications

Entanglement has many applications in quantum information theory. With the aid of entanglement, otherwise impossible tasks may be achieved.

Among the best-known applications of entanglement are superdense coding and quantum teleportation.[87]

Most researchers believe that entanglement is necessary to realize quantum computing (although this is disputed by some).[88]

Entanglement is used in some protocols of quantum cryptography.[89][90] This is because the "shared noise" of entanglement makes for an excellent one-time pad. Moreover, since measurement of either member of an entangled pair destroys the entanglement they share, entanglement-based quantum cryptography allows the sender and receiver to more easily detect the presence of an interceptor.

In interferometry, entanglement is necessary for surpassing the standard quantum limit and achieving the Heisenberg limit.[91]

Entangled states

There are several canonical entangled states that appear often in theory and experiments.

For two qubits, the Bell states are

$$\begin{aligned}|\Phi^\pm\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B \pm |1\rangle_A \otimes |1\rangle_B)\\ |\Psi^\pm\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B \pm |1\rangle_A \otimes |0\rangle_B).\end{aligned}$$

These four pure states are all maximally entangled (according to the entropy of entanglement) and form an orthonormal basis (linear algebra) of the Hilbert space of the two qubits. They play a fundamental role in Bell's theorem.

For M>2 qubits, the GHZ state is

$$|\mathrm{GHZ}\rangle = \frac{|0\rangle^{\otimes M} + |1\rangle^{\otimes M}}{\sqrt{2}},$$

which reduces to the Bell state $|\Phi^+\rangle$ for $M = 2$. The traditional GHZ state was defined for $M = 3$. GHZ states are occasionally extended to qudits, i.e., systems of *d* rather than 2 dimensions.

Also for M>2 qubits, there are spin squeezed states.[92] Spin squeezed states are a class of squeezed coherent states satisfying certain restrictions on the uncertainty of spin measurements, and are necessarily entangled.[93] Spin squeezed states are good candidates for enhancing precision measurements using quantum entanglement.[94]

For two bosonic modes, a NOON state is

$$|\psi_{\text{NOON}}\rangle = \frac{|N\rangle_a|0\rangle_b + |0\rangle_a|N\rangle_b}{\sqrt{2}},$$

This is like the Bell state $|\Psi^+\rangle$ except the basis kets 0 and 1 have been replaced with "the *N* photons are in one mode" and "the *N* photons are in the other mode".

Finally, there also exist twin Fock states for bosonic modes, which can be created by feeding a Fock state into two arms leading to a beam splitter. They are the sum of multiple of NOON states, and can used to achieve the Heisenberg limit.[95]

For the appropriately chosen measure of entanglement, Bell, GHZ, and NOON states are maximally entangled while spin squeezed and twin Fock states are only partially entangled. The partially entangled states are generally easier to prepare experimentally.

Methods of creating entanglement

Entanglement is usually created by direct interactions between subatomic particles. These interactions can take numerous forms. One of the most commonly used methods is spontaneous parametric down-conversion to generate a pair of photons entangled in polarisation.[76] Other methods include the use of a fiber coupler to confine and mix photons, photons emitted from decay cascade of the bi-exciton in a quantum dot,[96] the use of the Hong–Ou–Mandel effect, etc., In the earliest tests of Bell's theorem, the entangled particles were generated using atomic cascades.

It is also possible to create entanglement between quantum systems that never directly interacted, through the use of entanglement swapping. Two independently-prepared, identical particles may also be entangled if their wave functions merely spatially overlap, at least partially.[97]

Testing a system for entanglement

A density matrix ρ is called separable if it can be written as a convex sum of product states, namely

$$\rho = \sum_j p_j \rho_j^{(A)} \otimes \rho_j^{(B)}$$

with $1 \geq p_j \geq 0$ probabilities. By definition, a state is entangled if it is not separable.

For 2-Qubit and Qubit-Qutrit systems (2 × 2 and 2 × 3 respectively) the simple Peres–Horodecki criterion provides both a necessary and a sufficient criterion for separability, and thus -inadvertently- for detecting entanglement. However, for the general case, the criterion is merely a necessary one for separability, as the problem becomes NP-hard when generalized.[98][99] Other separability criteria include (but not limited to) the range criterion, reduction criterion, and those based on uncertainty relations.[100][101][102][103] See Ref.[104] for a review of separability criteria in discrete variable systems.

A numerical approach to the problem is suggested by Jon Magne Leinaas, Jan Myrheim and Eirik Ovrum in their paper "Geometrical aspects of entanglement".[105] Leinaas et al. offer a numerical approach, iteratively refining an estimated separable state towards the target state to be tested, and checking if the target state can indeed be reached. An implementation of the algorithm (including a built-in Peres-Horodecki criterion testing) is "StateSeparator" (http://phweb.technion.ac.il/~stateseparator/) web-app.

In continuous variable systems, the Peres-Horodecki criterion also applies. Specifically, Simon [106] formulated a particular version of the Peres-Horodecki criterion in terms of the second-order moments of canonical operators and showed that it is necessary and sufficient for $1 \oplus 1$-mode Gaussian states (see Ref.[107] for a seemingly different but essentially equivalent approach). It was later found [108] that Simon's condition is also necessary and sufficient for $1 \oplus n$-mode Gaussian states, but no longer sufficient for $2 \oplus 2$-mode Gaussian states. Simon's condition can be generalized by taking into account the higher order moments of canonical operators [109][110] or by using entropic measures.[111][112]

In 2016 China launched the world’s first quantum communications satellite.[113] The $100m Quantum Experiments at Space Scale (QUESS) mission was launched on Aug 16, 2016, from the Jiuquan Satellite Launch Center in northern China at 01:40 local time.

For the next two years, the craft – nicknamed "Micius" after the ancient Chinese philosopher – will demonstrate the feasibility of quantum communication between Earth and space, and test quantum entanglement over unprecedented distances.

In the June 16, 2017, issue of *Science*, Yin et al. report setting a new quantum entanglement distance record of 1,203 km, demonstrating the survival of a two-photon pair and a violation of a Bell inequality, reaching a CHSH valuation of 2.37 ± 0.09, under strict Einstein locality conditions, from the Micius satellite to bases in Lijian, Yunnan and Delingha, Quinhai, increasing the efficiency of transmission over prior fiberoptic experiments by an order of magnitude.[114][115]

Naturally entangled systems

The electron shells of multi-electron atoms always consist of entangled electrons. The correct ionization energy can be calculated only by consideration of electron entanglement.[116]

Photosynthesis

It has been suggested that in the process of photosynthesis, entanglement is involved in the transfer of energy between light-harvesting complexes and photosynthetic reaction centers where the kinetic energy is harvested in the form of chemical energy. Without such a process, the efficient conversion of optical energy into chemical energy cannot be explained. Using femtosecond spectroscopy, the coherence of entanglement in the Fenna-Matthews-Olson complex was measured over hundreds of femtoseconds (a relatively long time in this regard) providing support to this theory.[117][118]

Living systems

In October 2018, physicists reported producing quantum entanglement using living organisms, particularly between living bacteria and quantized light.[119][120]

See also

- CNOT gate
- Concurrence (quantum computing)
- Einstein's thought experiments
- Entanglement distillation
- Entanglement witness
- Faster-than-light communication
- Ghirardi–Rimini–Weber theory
- Multipartite entanglement
- Normally distributed and uncorrelated does not imply independent
- Observer effect (physics)
- Quantum coherence
- Quantum discord
- Quantum phase transition
- Quantum computing
- Quantum pseudo-telepathy
- Quantum teleportation
- Retrocausality
- Separable state
- Squashed entanglement
- Ward's probability amplitude
- Wheeler–Feynman absorber theory

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External links

- The original EPR paper (http://prola.aps.org/abstract/PR/v47/i10/p777_1)
- Quantum Entanglement at Stanford Encyclopedia of Philosophy (http://plato.stanford.edu/entries/qt-entangle/)
- How to entangle photons experimentally (subscription required) (http://physicsworldarchive.iop.org/index.cfm?action=summary&doc=11%2F3%2Fphwv11i3a29%40pwa-xml&qt=)
- A creative interpretation of Quantum Entanglement (https://web.archive.org/web/20110220045318/http://www.physicaltv.com.au/DanceFilmEntanglementTheoryrichardJamesAllenkarenPearlmangaryHayesmixedRealityLiveActionsecondLifeMachinima_619_1307_3_0.html)
- Albert's chest: entanglement for lay persons (http://www.science20.com/hammock_physicist/einstein_got_it_wrong_can_you_do_better-85544)
- How Quantum Entanglement Works (https://web.archive.org/web/20080402000326/http://davidjarvis.ca/entanglement/)
- Explanatory video by *Scientific American* magazine (https://www.youtube.com/watch?v=xM3GOXaci7w)
- Hanson Lab – Loophole-free Bell test 'Spooky action at a distance', no cheating. (https://web.archive.org/web/20180704082456/http://hansonlab.tudelft.nl/loophole-free-bell-test/)
- Two Diamonds Linked by Strange Quantum Entanglement (https://news.yahoo.com/two-diamonds-linked-strange-quantum-entanglement-190805281.html)
- Entanglement experiment with photon pairs – interactive (http://www.didaktik.physik.uni-erlangen.de/quantumlab/english/index.html)
- Multiple entanglement and quantum repeating (http://www.physorg.com/news63037231.html)
- Quantum Entanglement and Bell's Theorem at MathPages (http://www.mathpages.com/home/kmath521/kmath521.htm)
- Audio – Cain/Gay (2009) Astronomy Cast (http://www.astronomycast.com/physics/ep-140-entanglement/) Entanglement
- Recorded research seminars at Imperial College relating to quantum entanglement (http://www.imperial.ac.uk/quantuminformation)
- Quantum Entanglement and Decoherence: 3rd International Conference on Quantum Information (ICQI) (http://www.osa.org/meetings/topicalmeetings/ICQI/default.aspx)
- Ion trapping quantum information processing (https://web.archive.org/web/20090214015126/http://www.npl.co.uk/server.php?show=ConWebDoc.433)
- IEEE Spectrum On-line: *The trap technique* (https://web.archive.org/web/20081005001623/http://www.spectrum.ieee.org/aug07/5378/1)
- Was Einstein Wrong?: A Quantum Threat to Special Relativity (http://www.sciam.com/article.cfm?id=was-einstein-wrong-about-relativity)
- *Spooky Actions At A Distance?*: Oppenheimer Lecture, Prof. David Mermin (Cornell University) Univ. California, Berkeley, 2008. Non-mathematical popular lecture on YouTube, posted Mar 2008 (https://www.youtube.com/watch?v=ta09WXiUqcQ)