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Qubit

In quantum computing, a **qubit** (/ˈkjuːbɪt/) or **quantum bit** (sometimes **qbit**) is the basic unit of quantum information—the quantum version of the classical binary bit physically realized with a two-state device. A qubit is a two-state (or two-level) quantum-mechanical system, one of the simplest quantum systems displaying the peculiarity of quantum mechanics. Examples include: the spin of the electron in which the two levels can be taken as spin up and spin down; or the polarization of a single photon in which the two states can be taken to be the vertical polarization and the horizontal polarization. In a classical system, a bit would have to be in one state or the other. However, quantum mechanics allows the qubit to be in a coherent superposition of both states/levels simultaneously, a property which is fundamental to quantum mechanics and quantum computing.

Contents

Etymology

The coining of the term *qubit* is attributed to Benjamin Schumacher.[1] In the acknowledgments of his 1995 paper, Schumacher states that the term *qubit* was created in jest during a conversation with William Wootters. The paper describes a way of compressing states emitted by a quantum source of information so that they require fewer physical resources to store. This procedure is now known as Schumacher compression.

Bit versus qubit[2]

A binary digit, characterized as 0 and 1, is used to represent information in classical computers. A binary digit can represent up to one bit of Shannon information, where a bit is the basic unit of information. However, in this article, the word bit is synonymous with binary digit.

In classical computer technologies, a *processed* bit is implemented by one of two levels of low DC voltage, and whilst switching from one of these two levels to the other, a so-called forbidden zone must be passed as fast as possible, as electrical voltage cannot change from one level to another *instantaneously*.

There are two possible outcomes for the measurement of a qubit—usually taken to have the value "0" and "1", like a bit or binary digit. However, whereas the state of a bit can only be either 0 or 1, the general state of a qubit according to quantum mechanics can be a coherent superposition of both.[3] Moreover, whereas a measurement of a classical bit would not disturb its state, a measurement of a qubit would destroy its coherence and irrevocably disturb the superposition state. It is possible to fully encode one bit in one qubit. However, a qubit can hold more information, e.g. up to two bits using superdense coding.

For a system of n components, a complete description of its state in classical physics requires only n bits, whereas in quantum physics it requires 2^n-1 complex numbers.[4]

Standard representation

In quantum mechanics, the general quantum state of a qubit can be represented by a linear superposition of its two orthonormal basis states (or basis vectors). These vectors are usually denoted as $\boxed{|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$. They are written in the conventional Dirac—or "bra–ket"—notation; the $|0\rangle$ and $|1\rangle$ are pronounced "ket 0" and "ket 1", respectively. These two orthonormal basis states, $\{|0\rangle, |1\rangle\}$, together called the computational basis, are said to span the two-dimensional linear vector (Hilbert) space of the qubit.

Qubit basis states can also be combined to form product basis states. For example, two qubits could be represented in a four-dimensional linear vector space spanned by the following product basis states: $|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $|01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $|10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, and $|11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

In general, n qubits are represented by a superposition state vector in 2^n-dimensional Hilbert space.

Qubit states

A pure qubit state is a coherent superposition of the basis states. This means that a single qubit can be described by a linear combination of $|0\rangle$ and $|1\rangle$:

$$\boxed{|\psi\rangle = \alpha|0\rangle + \beta|1\rangle}$$

where α and β are probability amplitudes and can in general both be complex numbers. When we measure this qubit in the standard basis, according to the Born rule, the probability of outcome $|0\rangle$ with value "0" is $|\alpha|^2$ and the probability of outcome $|1\rangle$ with value "1" is $|\beta|^2$. Because the absolute squares of the amplitudes equate to probabilities, it follows that α and β must be constrained by the equation

$$|\alpha|^2 + |\beta|^2 = 1.$$

Note that a qubit in this superposition state does not have a value in between "0" and "1"; rather, when measured, the qubit has a probability $|\alpha|^2$ of the value “0” and a probability $|\beta|^2$ of the value "1". In other words, superposition means that there is no way, even in principle, to tell which of the two possible states forming the superposition state actually pertains. Furthermore, the probability amplitudes, α and β, encode more than just the probabilities of the outcomes of a measurement; the *relative phase* of α and β is responsible for quantum interference, *e.g.*, as seen in the two-slit experiment.

Bloch sphere representation

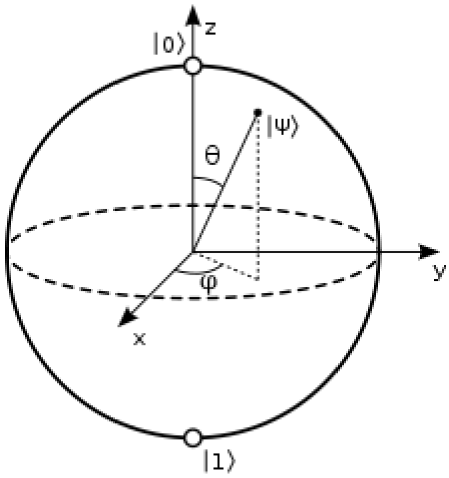
It might, at first sight, seem that there should be four degrees of freedom in $\boxed{|\psi\rangle = \alpha|0\rangle + \beta|1\rangle}$, as α and β are complex numbers with two degrees of freedom each. However, one degree of freedom is removed by the normalization constraint $|\alpha|^2 + |\beta|^2 = 1$. This means, with a suitable change of coordinates, one can eliminate one of the degrees of freedom. One possible choice is that of Hopf coordinates:

$$\boxed{\begin{aligned} \alpha &= e^{i\psi}\cos\frac{\theta}{2}, \\ \beta &= e^{i(\psi+\phi)}\sin\frac{\theta}{2}. \end{aligned}}$$

Additionally, for a single qubit the overall phase of the state $e^{i\psi}$ has no physically observable consequences, so we can arbitrarily choose α to be real (or β in the case that α is zero), leaving just two degrees of freedom:

$$\boxed{\begin{aligned} \alpha &= \cos\frac{\theta}{2}, \\ \beta &= e^{i\phi}\sin\frac{\theta}{2}, \end{aligned}}$$

where $e^{i\phi}$ is the physically significant *relative phase*.



Bloch sphere representation of a qubit. The probability amplitudes for the superposition state, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, are given by $\alpha = \cos\left(\frac{\theta}{2}\right)$ and $\beta = e^{i\phi}\sin\left(\frac{\theta}{2}\right)$.

The possible quantum states for a single qubit can be visualised using a Bloch sphere (see diagram). Represented on such a 2-sphere, a classical bit could only be at the "North Pole" or the "South Pole", in the locations where $|0\rangle$ and $|1\rangle$ are respectively. This particular choice of the polar axis is arbitrary, however. The rest of the surface of the Bloch sphere is inaccessible to a classical bit, but a pure qubit state can be represented by any point on the surface. For example, the pure qubit state $((|0\rangle + i|1\rangle)/\sqrt{2})$ would lie on the equator of the sphere at the positive y-axis. In the classical limit, a qubit, which can have quantum states anywhere on the Bloch sphere, reduces to the classical bit, which can be found only at either poles.

The surface of the Bloch sphere is a two-dimensional space, which represents the state space of the pure qubit states. This state space has two local degrees of freedom, which can be represented by the two angles ϕ and θ.

Mixed state

A pure state is one fully specified by a single ket, $\boxed{|\psi\rangle = \alpha|0\rangle + \beta|1\rangle}$, a coherent superposition as described above. Coherence is essential for a qubit to be in a superposition state. With interactions and decoherence, it is possible to put the qubit in a mixed state, a statistical combination or incoherent mixture of different pure states. Mixed states can be represented by points *inside* the Bloch sphere (or in the Bloch ball). A mixed qubit state has three degrees of freedom: the angles ϕ and θ, as well as the length r of the vector that represents the mixed state.

Operations on pure qubit states

There are various kinds of physical operations that can be performed on pure qubit states.

- Quantum logic gates, building blocks for a quantum circuit in a quantum computer, operate on one, two, or three qubits: mathematically, the qubits undergo a (reversible) unitary transformation under the quantum gate. For a single qubit, unitary transformations correspond to rotations of the qubit (unit) vector on the Bloch sphere to specific superpositions. For two qubits, the Controlled NOT gate can be used to entangle or disentangle them.

- Standard basis measurement is an irreversible operation in which information is gained about the state of a single qubit (and coherence is lost). The result of the measurement will be either $|0\rangle$ (with probability $|\alpha|^2$) or $|1\rangle$ (with probability $|\beta|^2$). Measurement of the state of the qubit alters the magnitudes of α and β. For instance, if the result of the measurement is $|1\rangle$, α is changed to 0 and β is changed to the phase factor $e^{i\phi}$ no longer experimentally accessible. When a qubit is measured, the superposition state collapses to a basis state (up to a phase) and the relative phase is rendered inaccessible (i.e., coherence is lost). Note that a measurement of a qubit state that is entangled with another quantum system transforms the qubit state, a pure state, into a mixed state (an incoherent mixture of pure states) as the relative phase of the qubit state is rendered inaccessible.

Quantum entanglement

An important distinguishing feature between qubits and classical bits is that multiple qubits can exhibit quantum entanglement. Quantum entanglement is a nonlocal property of two or more qubits that allows a set of qubits to express higher correlation than is possible in classical systems.

The simplest system to display quantum entanglement is the system of two qubits. Consider, for example, two entangled qubits in the $|\Phi^+\rangle$ Bell state:

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

In this state, called an *equal superposition*, there are equal probabilities of measuring either product state $|00\rangle$ or $|11\rangle$, as $|1/\sqrt{2}|^2 = 1/2$. In other words, there is no way to tell if the first qubit has value “0” or “1” and likewise for the second qubit.

Imagine that these two entangled qubits are separated, with one each given to Alice and Bob. Alice makes a measurement of her qubit, obtaining—with equal probabilities—either $|0\rangle$ or $|1\rangle$, i.e., she can now tell if her qubit has value “0” or “1”. Because of the qubits' entanglement, Bob must now get exactly the same measurement as Alice. For example, if she measures a $|0\rangle$, Bob must measure the same, as $|00\rangle$ is the only state where Alice's qubit is a $|0\rangle$. In short, for these two entangled qubits, whatever Alice measures, so would Bob, with *perfect* correlation, in any basis, however far apart they may be and even though both can not tell if their qubit has value “0” or “1” — a most surprising circumstance that can *not* be explained by classical physics.

Controlled gate to construct the Bell state

Controlled gates act on 2 or more qubits, where one or more qubits act as a control for some specified operation. In particular, the controlled NOT gate (or CNOT or cX) acts on 2 qubits, and performs the NOT operation on the second qubit only when the first qubit is $|1\rangle$, and otherwise leaves it unchanged. With respect to the unentangled product basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, it maps the basis states as follows:

$$\begin{aligned} |00\rangle &\mapsto |00\rangle \\ |01\rangle &\mapsto |01\rangle \\ |10\rangle &\mapsto |11\rangle \\ |11\rangle &\mapsto |10\rangle. \end{aligned}$$

A common application of the C_{NOT} gate is to maximally entangle two qubits into the $|\Phi^+\rangle$ Bell state. To construct $|\Phi^+\rangle$, the inputs A (control) and B (target) to the C_{NOT} gate are:

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_A$ and $|0\rangle_B$

After applying C_{NOT}, the output is the $|\Phi^+\rangle$ Bell State: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

Applications

The $|\Phi^+\rangle$ Bell state forms part of the setup of the superdense coding, quantum teleportation, and entangled quantum cryptography algorithms.

Quantum entanglement also allows multiple states (such as the Bell state mentioned above) to be acted on simultaneously, unlike classical bits that can only have one value at a time. Entanglement is a necessary ingredient of any quantum computation that cannot be done efficiently on a classical computer. Many of the successes of quantum computation and communication, such as quantum teleportation and superdense coding, make use of entanglement, suggesting that entanglement is a resource that is unique to quantum computation.[5] A major hurdle facing quantum computing, as of 2018, in its quest to surpass classical digital computing, is noise in quantum gates that limits the size of quantum circuits that can be executed reliably.[6]

Quantum register

A number of qubits taken together is a qubit register. Quantum computers perform calculations by manipulating qubits within a register. A **qubyte** (quantum byte) is a collection of eight qubits.[7]

Variations of the qubit

Similar to the qubit, the qutrit is the unit of quantum information that can be realized in suitable 3-level quantum systems. This is analogous to the unit of classical information trit of ternary computers. Note, however, that not all 3-level quantum systems are qutrits.[8] The term "**qu-*d*-it**" (***qu**antum* **d**-*git*) denotes the unit of quantum information that can be realized in suitable *d*-level quantum systems.[9] In 2017, scientists at the National Institute of Scientific Research constructed a pair of qudits with 10 different states each, giving more computational power than 6 qubits.[10]

Physical implementations

Any two-level quantum-mechanical system can be used as a qubit. Multilevel systems can be used as well, if they possess two states that can be effectively decoupled from the rest (e.g., ground state and first excited state of a nonlinear oscillator). There are various proposals. Several physical implementations that approximate two-level systems to various degrees were successfully realized. Similarly to a classical bit where the state of a transistor in a processor, the magnetization of a surface in a hard disk and the presence of current in a cable can all be used to represent bits in the same computer, an eventual quantum computer is likely to use various combinations of qubits in its design.

The following is an incomplete list of physical implementations of qubits, and the choices of basis are by convention only.

Physical support	Name	Information support	$\|0\rangle$	$\|1\rangle$
Photon	Polarization encoding	Polarization of light	Horizontal	Vertical
	Number of photons	Fock state	Vacuum	Single photon state
	Time-bin encoding	Time of arrival	Early	Late
Coherent state of light	Squeezed light	Quadrature	Amplitude-squeezed state	Phase-squeezed state
Electrons	Electronic spin	Spin	Up	Down
	Electron number	Charge	No electron	One electron
Nucleus	Nuclear spin addressed through NMR	Spin	Up	Down
Optical lattices	Atomic spin	Spin	Up	Down
Josephson junction	Superconducting charge qubit	Charge	Uncharged superconducting island (Q=0)	Charged superconducting island (Q=2e, one extra Cooper pair)
	Superconducting flux qubit	Current	Clockwise current	Counterclockwise current
	Superconducting phase qubit	Energy	Ground state	First excited state
Singly charged quantum dot pair	Electron localization	Charge	Electron on left dot	Electron on right dot
Quantum dot	Dot spin	Spin	Down	Up
Gapped topological system	Non-abelian anyons	Braiding of Excitations	Depends on specific topological system	Depends on specific topological system

Qubit storage

In a paper entitled "Solid-state quantum memory using the ^{31}P nuclear spin", published in the October 23, 2008, issue of the journal *Nature*,[11] a team of scientists from the U.K. and U.S. reported the first relatively long (1.75 seconds) and coherent transfer of a superposition state in an electron spin "processing" qubit to a nuclear spin "memory" qubit. This event can be considered the first relatively consistent quantum data storage, a vital step towards the development of quantum computing. Recently, a modification of similar systems (using charged rather than neutral donors) has dramatically extended this time, to 3 hours at very low temperatures and 39 minutes at room temperature.[12] Room temperature preparation of a qubit based on electron spins instead of nuclear spin was also demonstrated by a team of scientists from Switzerland and Australia.[13]

See also

- Two-state quantum system
- Ancilla bit
- Photonic computer
- W state
- Physical and logical qubits

Further reading

- A good introduction to the topic is the book by Nielsen and Chuang.[3]
- An excellent treatment of two-level quantum systems, decades before the term “qubit” was coined, is found in the third volume of The Feynman Lectures on Physics (2013 ebook edition) (http://www.feynmanlectures.caltech.edu/III_toc.html).
- A non-traditional motivation of the qubit aimed at non-physicists is found in Quantum Computing Since Democritus, by Scott Aaronson, Cambridge University Press (2013).
- A good introduction to qubits for non-specialists, by the person who coined the word, is found in Lecture 21 of ‘‘The science of information: from language to black holes’’, by Professor Benjamin Schumacher, The Great Courses, The Teaching Company (4DVDs, 2015).
- A picture-book introduction to entanglement, contrasting classical systems and a Bell state, is found in “Quantum entanglement for babies“, by Chris Ferrie (2017).

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