Homework 1

ESE 5420/4020

Due Tuesday, Sept. 17, 2024 at 11:59pm

Type or scan your answers as a single PDF file and submit on Gradescope.

Tips:

- Don't worry too much about precision of numerical answers (3-4 decimal places should be enough).
- When asked to compute confidence intervals, Central Limit Theorem is your friend. Computing exact distributions of the sample mean can often be hard.
- Linearity of expectation is your friend.

Problem 1. Let X denote the temperature at which a certain chemical reaction takes place. Suppose the pdf of X, denoted by $f(x)$, has the following form:

$$f(x) = \begin{cases} \frac{1}{9}(4 - x^2) & \text{if } x \in [-1, 2], \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of $f(x)$.

(b) Determine the cdf and sketch it.

(c) Find the mean and variance of X.

Problem 2. Two components of a minicomputer have the following joint pdf for their useful lifetimes X and Y:

$$f(x, y) = \begin{cases} xe^{-x(1+y)} & \text{if } x \geq 0, y \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the probability that the lifetime X of the first component exceeds 3?

(b) What are the marginal pdf's of X and Y? Are the two lifetimes independent? Explain.

(c) What is the probability that the lifetime of at least one component exceeds 3?

Problem 3. An independent random sample of 50 students was taken to estimate the average number of hours they study per week. The sample mean was found to be 15 hours with a standard deviation of 4 hours.

(a) Calculate the 95% confidence interval for the population mean.

(b) The university claims that students study an average of 14 hours per week. Based on your confidence interval in part (b), would you consider this claim to be plausible? Why or why not?

(c) Suppose you wanted a 99% confidence interval instead. Would this interval be wider or narrower than the one in part (b)? Calculate the 99% confidence interval to confirm.

Problem 4. A particular area contains 8,000 condominium units. In a survey of the occupants with sample size 100, 12% of the respondents said they planned to sell their condos within the next year.

(a) Compute the 95% confidence interval for the estimated probability of people planning to sell. (Hint: The variance of the Bernoulli distribution with parameter p is given by $p(1-p)$).

(b) Suppose that another survey is done of another condominium project of 12,000 units. The sample size is 200, and the proportion planning to sell in this sample is .18. What is the standard error of this estimate? Give a 90% confidence interval.

(c) Suppose we use the notation $\hat{p}_1 = .12$ and $\hat{p}_2 = .18$ to refer to the proportions in the two samples. Let $\hat{d} = \hat{p}_1 - \hat{p}_2$ be an estimate of the difference, d, of the two population proportions p_1 and p_2. Using the fact that $\hat{p}_1$ and $\hat{p}_2$ are independent random variables, calculate the variance and standard error of $\hat{d}$.

(d) Because $\hat{p}_1$ and $\hat{p}_2$ are approximately normally distributed, so is $\hat{d}$. Use this fact to construct 99%, 95%, and 90% confidence intervals for d. Is there clear evidence that p_1 is really different from p_2?

Problem 5. There are $p = 25\%$ of the total population that will vote for candidate A in a coming election. In a survey of sample size $n = 100$, $\hat{p}$ of interviewees said they would vote for A. Assume that the votes of interviewees are i.i.d.

(a) Find δ such that $P(|\hat{p} - p| \geq \delta) \approx 0.025$.

(b) If, in the sample, $\hat{p} = 0.25$, will the 95% confidence interval for p contain the true value of p?

Problem 6. The Monkey-Corona-Pox pandemic has ended and you are planning to host a party to celebrate. You are inviting 64 friends over and plan to buy sandwiches for them. You know that each person (independently of others) eats either 0, 1, or 2 sandwiches with probability $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ respectively. How many sandwiches should you order so that with probability at least 0.95, there will be no shortage? (Hint: Use the Central Limit Theorem)