

Q2) The series is given as:

$$x_t = w_t w_{t-1}$$

1. Mean of x_t

The mean of x_t is:

$$E[x_t] = E[w_t w_{t-1}]$$

Since w_t and w_{t-1} are independent, we have:

$$E[x_t] = E[w_t] \cdot E[w_{t-1}]$$

Given that w_t is white noise with $E[w_t] = 0$, the mean of x_t is:

$$E[x_t] = 0 \cdot 0 = 0$$

2. Autocovariance Function $\gamma_x(h)$

The autocovariance function is given by:

$$\gamma_x(h) = \text{Cov}(x_t, x_{t+h}) = E[x_t x_{t+h}] - E[x_t] E[x_{t+h}]$$

Since $E[x_t] = 0$, this simplifies to:

$$\gamma_x(h) = E[x_t x_{t+h}]$$

Now we compute $\gamma_x(h)$ for different values of h :

- For $h = 0$:

$$\gamma_x(0) = E[x_t^2] = E[(w_t w_{t-1})^2] = E[w_t^2 w_{t-1}^2]$$

Since w_t and w_{t-1} are independent:

$$E[w_t^2 w_{t-1}^2] = E[w_t^2] \cdot E[w_{t-1}^2] = \sigma_w^2 \cdot \sigma_w^2 = \sigma_w^4$$

Thus, the variance is:

$$\gamma_x(0) = \sigma_w^4$$

- For $h = 1$:

$$\gamma_x(1) = E[x_t x_{t+1}] = E[w_t w_{t-1} w_{t+1} w_t]$$

Since w_t , w_{t-1} , and w_{t+1} are independent:

$$\gamma_x(1) = E[w_t^2] \cdot E[w_{t-1}] \cdot E[w_{t+1}] = \sigma_w^2 \cdot 0 \cdot 0 = 0$$

- For $h = 2$:

$$\gamma_x(2) = E[x_t x_{t+2}] = E[w_t w_{t-1} w_{t+2} w_{t+1}]$$

Again, all terms are independent, so:

$$\gamma_x(2) = E[w_t] \cdot E[w_{t-1}] \cdot E[w_{t+2}] \cdot E[w_{t+1}] = 0$$

- For $|h| \geq 1$:

For all $h \geq 1$, the autocovariance is zero due to the independence of the white noise series:

$$\gamma_x(h) = 0 \quad \text{for} \quad |h| \geq 1$$

Final Conclusion:

The series has a mean of 0 and an autocovariance function that is:

$$\gamma_x(0) = \sigma_w^4 \quad \text{and} \quad \gamma_x(h) = 0 \quad \text{for} \quad |h| \geq 1$$

Thus, the series x_t is stationary.