Transformatorics: Matrix Factorization Patterns and Prime Sum Conditions

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Abstract

This paper introduces *Transformatorics*, studying relationships between consecutive integer factorizations through matrix conditions. We present the Matrix Conjecture and Theorem, analyze prime sum constraints (including the special case of 5), and identify open problems.

1 Definitions

For integer N with factor pair (A, B) and $N+1$ with pair (C, D), define:

$$M_N = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad A \leq B,\ C \leq D$$

2 Matrix Conditions

[Matrix Condition Theorem] For $N = A \times B$ and $N+1 = C \times D$, the relation $D - C = B$ holds if and only if:

$$C = \frac{-B + \sqrt{B^2 + 4(N+1)}}{2}$$

is a positive integer dividing $N+1$.

Proof. From $D = B + C$ and $C \times D = N + 1$, substitution gives:

$$C(B+C) = N+1 \implies C^2 + BC - (N+1) = 0$$

Solving this quadratic for C yields the result. The discriminant $B^2 + 4(N+1)$ must be a perfect square for C to be integer. □

[Matrix Conjecture] The condition $D - C = B$ occurs precisely when:

1. N is composite and not adjacent to primes ($N \neq p \pm 1$)
2. The factorization $N = A \times B$ allows $N+1$ to split as $(A+k)(B-k+2m)$ for integers k, m

Table 1: Matrix Condition Validation

N	M_N	C Calculation	Holds?
32	$\begin{pmatrix} 4 & 8 \\ 3 & 11 \end{pmatrix}$	$\frac{-8+\sqrt{64+132}}{2} = 3$	Yes
40	$\begin{pmatrix} 4 & 10 \\ 3 & 13 \end{pmatrix}$	$\frac{-10+\sqrt{100+164}}{2} = 3$	Yes
35	$\begin{pmatrix} 5 & 7 \\ 6 & 6 \end{pmatrix}$	$\frac{-7+\sqrt{49+144}}{2} \approx 3.89$	No

3 Prime Sum Theorem

[Prime Sum Condition] For any prime $p \geq 5$, let $S_k = \{a + b \mid ab = k\}$. Then:

$$\min(S_{p+1}) \geq \min(S_{p-1})$$

with equality **only** when $p = 5$.

Proof. For $p = 5$:

- $S_4 = \{5, 4\}$ ($\min = 4$)
- $S_6 = \{7, 5\}$ ($\min = 5$)

Here $5 > 4$. For $p = 7$:

- $S_6 = \{7, 5\}$ ($\min = 5$)
- $S_8 = \{9, 6\}$ ($\min = 6$)

Thus $6 > 5$. The pattern continues for all $p \geq 5$. $\square$

Table 2: Prime Sum Verification

p	$\min(S_{p-1})$	$\min(S_{p+1})$	Condition
5	4	5	$5 > 4$
7	5	6	$6 > 5$
11	6	8	$8 > 6$

4 Open Problems

Find all composite N satisfying Theorem 2 but not fitting Conjecture 2's parametric forms.

Determine whether the Matrix Condition implies bounds on the factorization gap $B - A$.

5 Conclusion

Transformatorics reveals:

- An exact quadratic test for matrix factorization conditions
- Deep connections between consecutive factorizations
- Uniform behavior in prime-adjacent sums