Exploring the Influence of Opinion Dynamics on Infectious Disease Spread

Project Seminar

M. Sc. Mathematical Modelling, Simulation and Optimization

Faculty 3: Mathematics
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Declaration of authorship

We hereby confirm that we have authored this paper together, did not use any resources other than those that we have cited — in particular no online sources not included in the bibliography section — and that we have not previously submitted this report in association with any other examination procedure.

Koblenz August 4, 2024

City, Date

Abstract

In recent times, the field of epidemiology has become increasingly significant to modern society, intersecting profoundly with mathematics. For mathematicians, epidemiology offers new and exciting branches of study, while for epidemiologists, mathematical modeling serves as a vital research tool for understanding disease evolution. One of the fundamental epidemiological models is the SIR model (susceptible-infected-recovered), which categorizes individuals into mutually exclusive compartments to represent the dynamics of disease spread. This model is widely applied across various fields, including health, marketing, informatics, and sociology, as a preliminary approach to understanding diverse situations. In this project, we aim to enhance the traditional SIR model by incorporating opinion dynamics, thereby increasing the model's complexity. This enhancement reflects how individuals' ideas and actions can influence the opinions and behaviors of others within a population. For instance, social media platforms, which facilitate the sharing and access to a vast array of information, play a significant role in shaping public opinion and behaviour. By integrating these factors, our enhanced model seeks to provide a more comprehensive understanding of the interplay between disease spread and opinion dynamics.

Contents

1 Introduction

The worldwide pandemic emerged at the beginning of 2020, deeply influencing certain behavioural practices, such as wearing masks, practicing social distancing, and accepting vaccines, plays a crucial role in impeding the spread of COVID-19 and reducing the severity of symptoms. For example, a large-scale randomized trial involving 350,000 people in 600 villages in Bangladesh has shown that the use of surgical masks impedes the spread of COVID-19 [1] . Observing isolation when exposed or infected also impedes the spread. Vaccines reduce the severity of symptoms, including hospitalization and death, and also the spread rates. Opinions regarding whether to observe behavioral patterns conducive to the containment of COVID-19 evolve over time through social exchanges via networks that overlap with but are not identical to the COVID-19 propagation networks. Thus, the biological and information contagion spread simultaneously and necessitate a joint investigation of the two phenomena. The joint investigation is yet to be studied for COVID-19, despite enormous progress in research on COVID-19 in the last 4 years. However, certain distinguishing characteristics of COVID-19 necessitate an investigation of the joint spread focusing on COVID-19 [1].

In this project, We develop a modified SIR model that can be easily adapted to a wide range of behavioral practices and captures in a computationally tractable manner, the joint evolution of the disease and relevant opinions in the population. Our model captures how different opinion dynamics, rates of transmission, and recovery, deviate from the basic model where no dynamics are involved. The model is flexible enough to capture the dynamics of different kinds of behavior, namely, wearing surgical masks and receiving vaccines. We consider the reality that in the age of social media, opinions regarding behavioural dynamics rapidly evolve, through social networks that overlap with but are not identical to biological networks [1]. In particular, during physical interactions, both disease and opinions may spread, whereas only opinions may spread through remote (e.g., electronic) interactions, and only the disease might spread when individuals share the same physical space (e.g., public spaces like beaches, parks, public transports) without engaging in social interactions [1]. We demonstrate how individuals from different compartmental states (susceptible/Infected/recovered) with some opinions goes to other states or remain in the same state but adapt their opinions through interactions.

1.1 Mathematical Modelling

Mathematical modelling is the process of describing a real-world problem in mathematical terms, usually in the form of equations, and then using these equations both to help understand the original problem and also to discover new features about the problem [2]. It helps in the prediction of future outputs, optimizes performance, and makes suitable measures. During this process, some assumptions are required to create an environment that is almost similar to real-world phenomena. However, assumptions can be responsible for inaccuracies and limitations. Variables also play an important role in a mathematical model, which can be either dependent or independent.

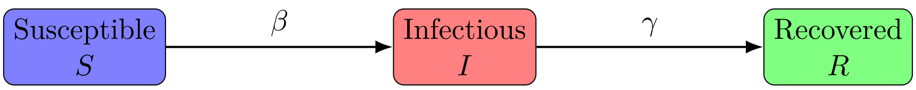
1.2 SIR Model

The SIR model, developed by Ronald Ross, William Hamer, and others in the early twentieth century, consists of a system of three coupled nonlinear ordinary differential equations. Theoretical papers by Kermack and McKendrinck, between 1927 and 1933 about infectious disease models[3], have had a great influence on the development of mathematical epidemiology models. Most of the basic theory had been developed during that time, but the theoretical progress has been steady since then [4].

The SIR model in epidemiology has played a crucial role in forecasting the patterns of infectious diseases, also known as Compartmental models. The model is a mathematical representation used to determine how infection spreads within the system of a population. The model is divided into three compartments, including susceptible, infected, and recovered, described as follows:

- **Susceptible (S)**: The susceptible individuals are prone to getting infected when they comes in contact with infected individuals.
- **Infected (I)**: These individuals are already infected with the disease and they can transmit it to others.
- **Recovered (R)**: These individuals have recovered from the disease and can no longer infect other individuals.

This epidemiological model captures the dynamics of acute infections that confers lifelong immunity once recovered. Diseases where individuals acquire permanent immunity, and for which this model may be applied, include measles, smallpox, chickenpox, mumps, typhoid fever and diphtheria. Generally, the total population size is considered constant, i.e., N = S + I + R. Then two cases should be studied, distinguished by the inclusion or exclusion of demographic factors. The dynamics are governed by two parameters: the infection rate (β) and the recovery rate (γ) [4].



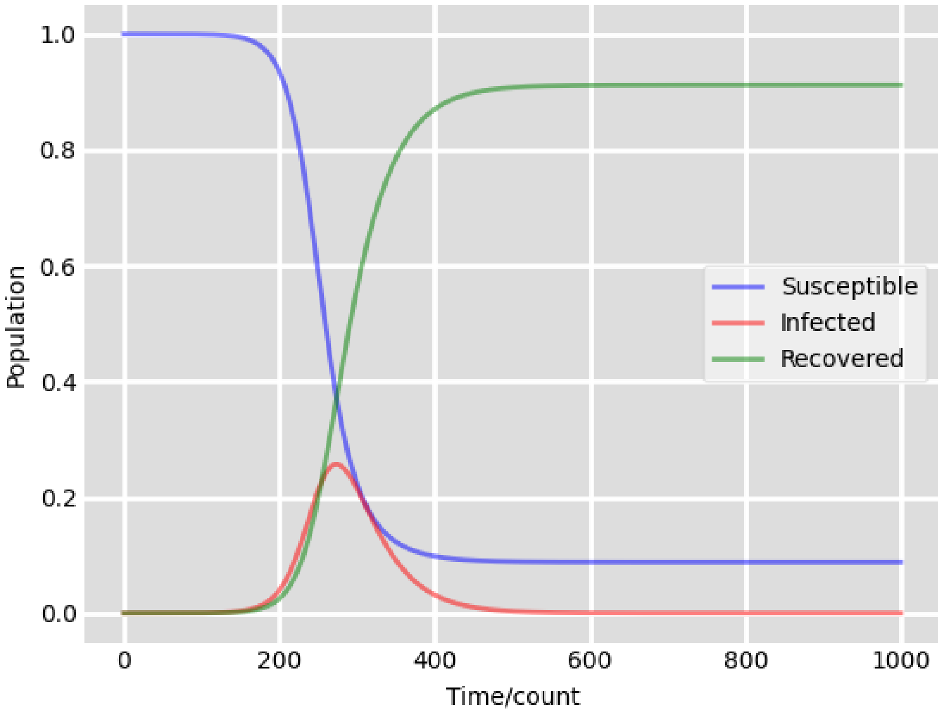


Figure 1: SIR model

1.3 Markov Chain and Transition Matrix

Markov Chain [5]

Let $\{X(t) : t \in T\}$ be random variables denoting the state of a system at time $t \in T = \{0, \Delta t, 2\Delta t, \dots\}$ with a discrete state space

$$\{i_0, i_{\Delta t}, i_{2\Delta t}, \dots, i_{t-\Delta t}, i_t, j\} = S \subset \{0, 1, 2, \dots, N\}.$$

A stochastic process is a Discrete Time Markov Chain (DTMC) if it satisfies the following equation

$$P[X(t+\Delta t) = j \mid X(t) = i, \dots, X(\Delta t) = i\Delta t, X(0) = i_0] = P[X(t+\Delta t) = j \mid X(t) = i] = p_{j\leftarrow i}(\Delta t).$$

A discrete-time process satisfies the Markov property, that is, the process at any time $t + \Delta t$ depends only on the state of the immediate past process in time t. As time goes on, the process does not require the information from further back for future transitions. The probability that the process will transition from state i at time t to state j at time $t + \Delta t$ is denoted by $p_{j\leftarrow i}(\Delta t)$. We assume that the process is time-homogeneous, that is, the transition probability does not change with time. The process is independent of t and

$$P[X(t + \Delta t) = j \mid X(t) = i] = P[X(\Delta t) = j \mid X(0) = i] = p_{j\leftarrow i}(\Delta t).$$

The one-step transition probability is the probability of transiting from state i to state j in one step, that is, in a period of Δt. We denote the one-step transition probability as $p_{j\leftarrow i}(\Delta t)$. It is sometimes denoted by $p_{ij}(\Delta t)$. The n-step transition probability is the probability of moving from state i to state j in n steps, that is, in a period of $n\Delta t$ and is given as

$$P[X(t + n\Delta t) = j \mid X(t) = i] = p^{(n)}_{j\leftarrow i}(n\Delta t).$$

The probability $p_{j\leftarrow i}(\Delta t)$ may sometimes be zero.

Transition Matrix [6]

In many dynamic systems, the state transitions can be represented using a transition matrix. This matrix captures the probabilities of moving from one state to another in a given time step. The matrix describing the Markov chain is called the transition matrix. It is the most important tool for analysing Markov chains.

$$\begin{array}{c}X_t\\ \text{list}\\ \text{all}\\ \text{states}\end{array}\left\{\overbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}}^{\substack{X_{t+1}\\ \text{list all states}}}\right\}\begin{array}{c}\text{rows add}\\ \text{to 1}\end{array}\quad\begin{array}{c}\text{rows add}\\ \text{to 1}\end{array}$$

insert
probabilities
p_{ij}

The transition matrix is usually given the symbol $P = (p_{ij})$.
In the transition matrix P:

- **the ROWS represent NOW, or FROM (X_t);**
- **the COLUMNS represent NEXT, or TO (X_{t+1});**
- **entry (i, j) is the CONDITIONAL probability that NEXT $= j$, given that NOW $= i$: the probability of going FROM state i TO state j.**

$$p_{ij} = \mathbb{P}(X_{t+1} = j \mid X_t = i).$$

The transition matrix approach extends the SIR model by incorporating opinion dynamics. Each individual is assigned an opinion, and the transition probabilities between states depend on these opinions. This method allows for the discrete-time simulation of the system using Markov chains.

1.4 System of Equations Generated by Transition Matrix

In such systems, the state at the next time step is determined by multiplying the transition matrix with the current state vector. This equation forms the basis for analyzing the evolution of states in the system over time. By iteratively applying the transition matrix, one can predict the future states based on the initial state vector.

$$\mathbf{x}(t + \Delta t) = P\mathbf{x}(t)$$

Where P is the transition matrix:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}$$

And $\mathbf{x}(t)$ is the state vector at time t:

$$\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

Thus, the state vector at the next time step $t + \Delta t$ is:

$$\mathbf{x}(t + \Delta t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}$$

1.5 Optimization Techniques for Parameter Fitting

Parameter fitting is a crucial task in many fields, such as machine learning, statistics, and system identification. It involves finding the set of parameters that best explains the observed data within a given model. Various optimization techniques are employed to achieve this goal, each with its strengths and weaknesses.

Mean Square Error (MSE) [7]

Mean Square Error is a common measure used to evaluate the accuracy of a model. It calculates the average of the squares of the errors between observed and predicted values.

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

where y_i are the observed values and $\hat{y}_i$ are the predicted values.

Gradient Descent [8]

Gradient descent is a first-order iterative optimization algorithm for finding the minimum of a function. To find a local minimum of a function using gradient descent, one takes steps proportional to the negative of the gradient (or approximate gradient) of the function at the current point. The algorithm is widely used for parameter fitting in machine learning models due to its simplicity and effectiveness.

$$\theta := \theta - \eta \nabla_\theta J(\theta)$$

where θ represents the parameters, η is the learning rate, and $J(\theta)$ is the cost function to be minimized.

Least Squares Method [9]

The least squares method is a standard approach in regression analysis to approximate the solution of overdetermined systems. It minimizes the sum of the squares of the residuals, which are the differences between observed and calculated values. This method is particularly useful for linear regression models.

$$\min_\theta \sum_{i=1}^{n} (y_i - f(x_i; \theta))^2$$

where y_i are the observed values, $f(x_i; \theta)$ are the predicted values, and θ are the parameters.

Maximum Likelihood Estimation (MLE) [10]

Maximum Likelihood Estimation is a method of estimating the parameters of a statistical model by maximizing the likelihood function. The estimates are those values of the parameters that maximize the likelihood that the process described by the model produced the observed data.

$$\hat{\theta} = \arg\max_{\theta} L(\theta; \mathbf{X})$$

where $L(\theta; \mathbf{X})$ is the likelihood function given the data $\mathbf{X}$.

Bayesian Optimization [11]

Bayesian optimization is an approach to optimizing objective functions that are expensive to evaluate. It builds a probabilistic model of the objective function and uses it to select the most promising parameters to evaluate in the real objective function. This is particularly useful in hyperparameter tuning for machine learning models.

$$\theta^* = \arg\max_{\theta} \mathbb{E}[f(\theta)]$$

where $f(\theta)$ is the objective function and $\mathbb{E}[f(\theta)]$ is the expected value.

Genetic Algorithms [12]

Genetic algorithms are search heuristics that mimic the process of natural evolution. They are used to find approximate solutions to optimization and search problems. These algorithms use techniques such as selection, crossover, and mutation to evolve solutions towards the best fit.

2 SIR Model with Opinion Dynamics

The Opinion Dynamics based SIR Model is a mathematical framework that is useful to simulate the spreading pattern of the infection from one opinion group to another and its impact on every opinion group within the population. This model for the spread of infectious diseases is a an important tool for investigating and quantifying such impacts, not least because the spread of disease among humans is not amenable to direct experimental study [13].

In the SIR model, the entire population is a homogeneous group. In addition, there are only three compartments, for example, for susceptible people, infected people, and recovered people. However, this extended SIR model, the population is also divided into opinion groups. For example Not Cautious, Less Cautious, More Cautious, complete cautious,etc.

Once we have the all opinion groups. The transition matrix captures the probabilities of transitioning between states (S, I, R) based on opinions. In addition, we can decide which opinion group will be affected more (if there would be any dominating one). Accordingly, we can take interventions to control the spread of the disease. This allows for a more nuanced understanding of how opinions affect disease dynamics.

2.1 Modeling Framework

Here, the total population (N) is given by,

N = S(t) + I(t) + R(t)

where S(Susceptible), I(Infected) and R(Recovered) are three compartmental/epidemiological states and Opinions array q given by,

q=[-q1,-q2,-q3,...,-qn]

Where the minus sign indicates the negative severity of getting infected, and q1, q2... magnitude represent the degree of cautiousness. Implying, the higher the magnitude, the lower will be the chances of getting infected. For modeling purposes, We take opinion as 0,1,2...etc(more like the index of this q-opinion array).

2.2 Assumptions

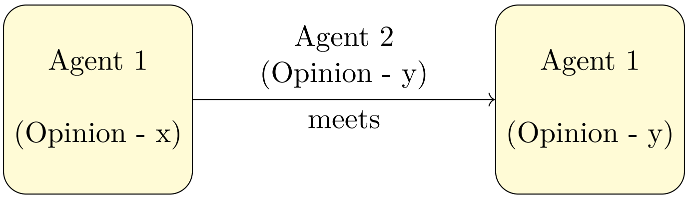
These assumptions are employed while modeling:

- One-Way Flow:- The flow of the population is from S to I to R. Implying Susceptible could get infected and eventually recover therefore achieving the immunity from the infection.

- Actions $(\pi_1), (\pi_2)$: Two action are defined as 1 – Doing Nothing, 2- Full Protection. Their effect can be calculated by:

$$\pi_1^q = (\frac{1}{1+e^{-q}}), \pi_2^q = (\frac{e^{-q}}{1+e^{-q}}) \tag{1}$$

- Opinion Adaption:- When an Agent (someone from any compartmental state with some opinion) meets/Interact with another agent, then the first agent takes the opinion of the second agent.



For example:

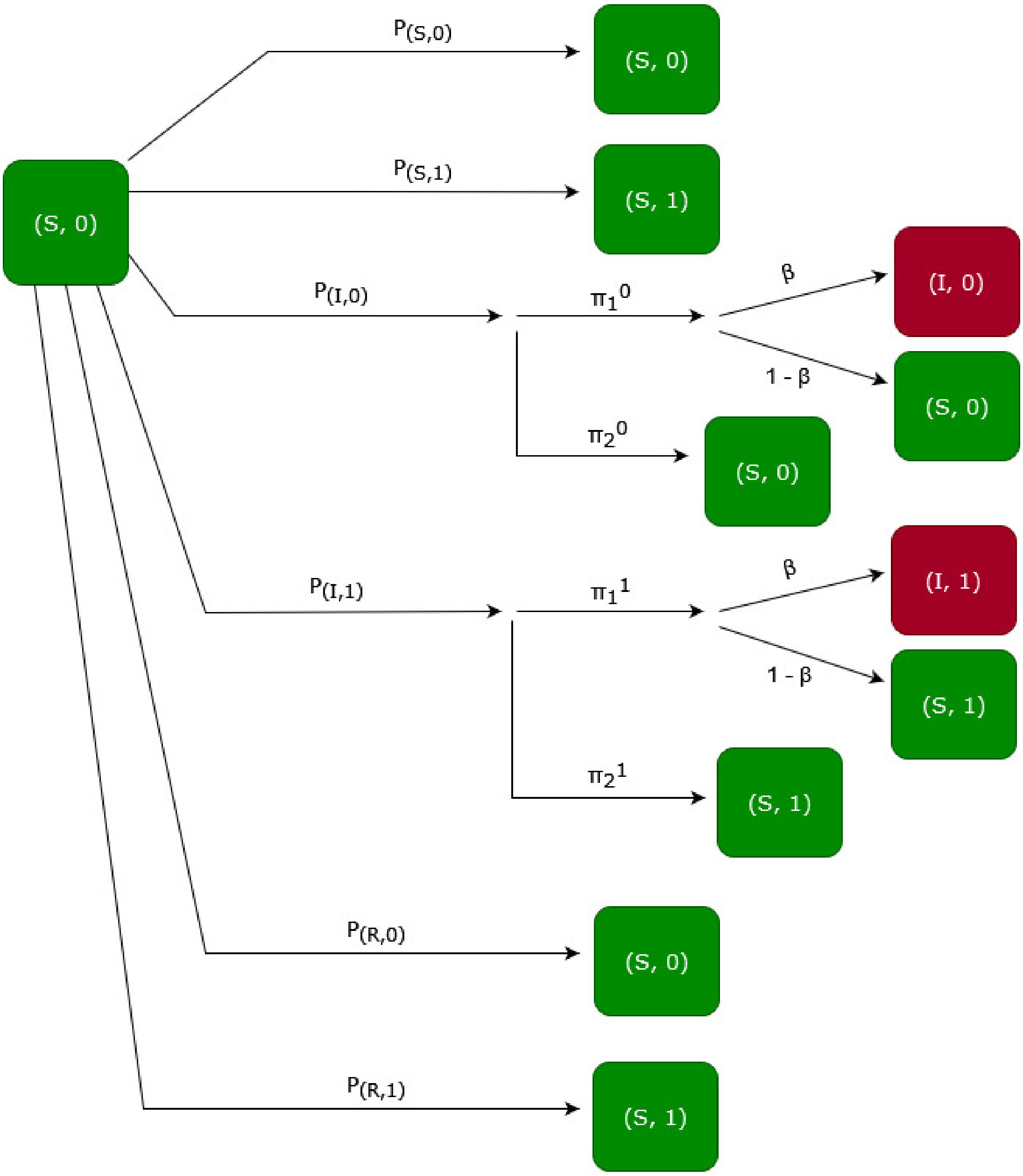


Figure 2: Susceptible with 0 opinion interactions

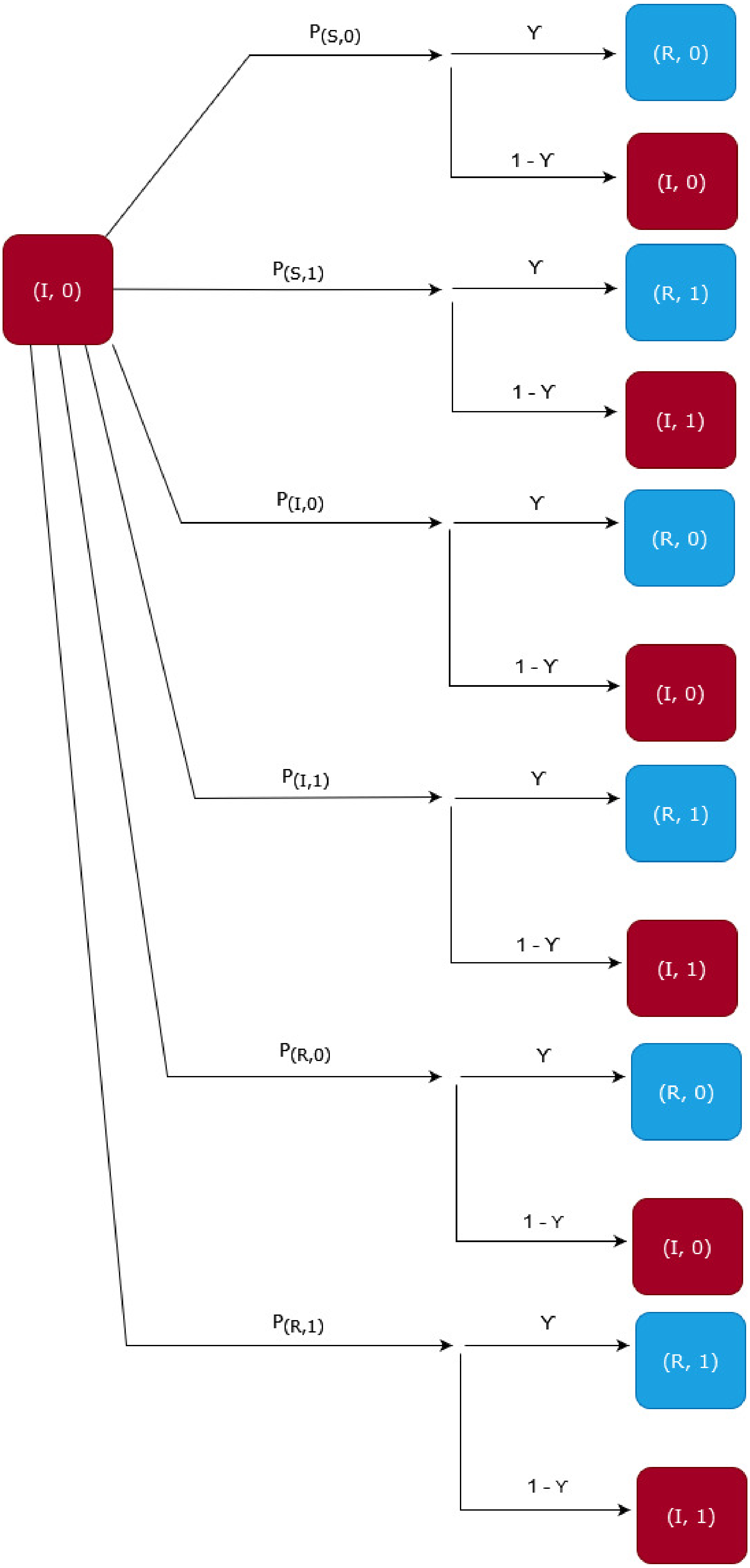


Figure 3: Infected with 0 opinion interactions

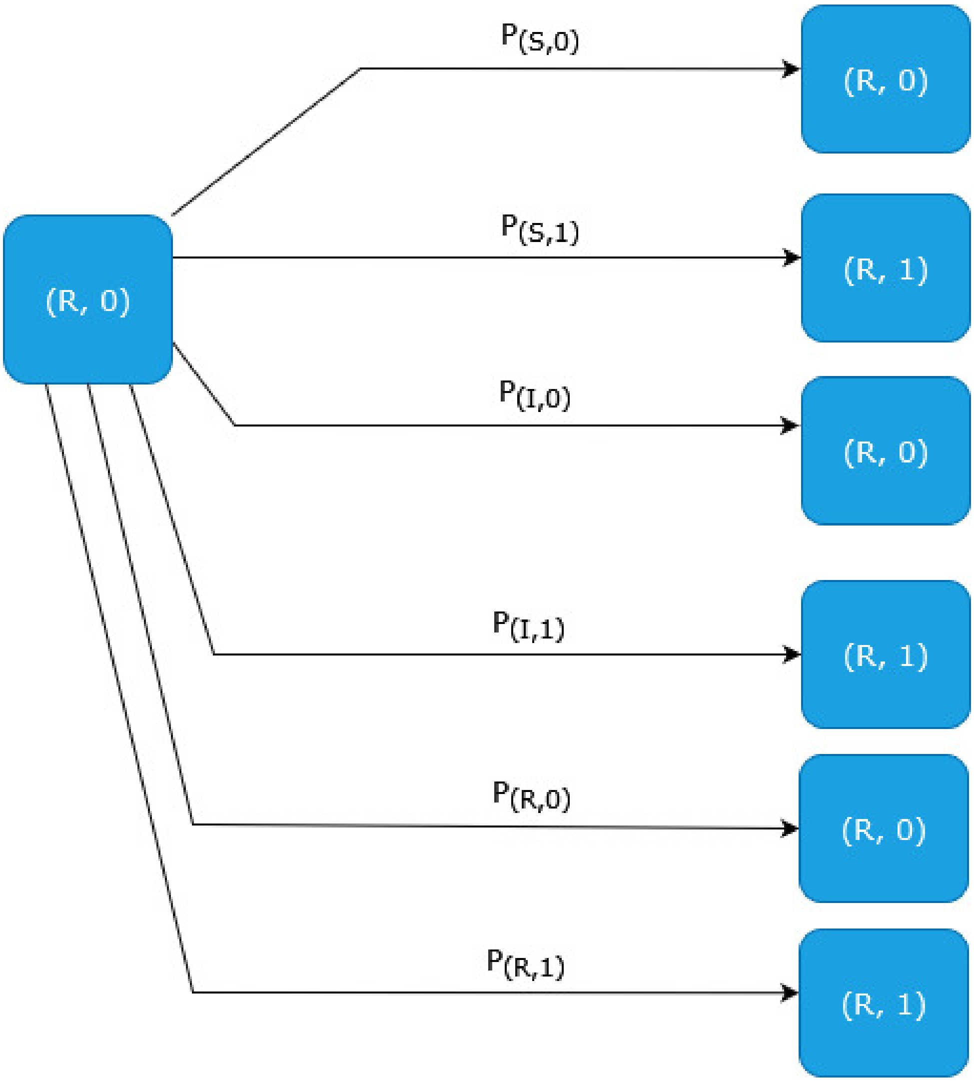


Figure 4: Recovered with 0 opinion interactions

2.3 Markov Chain and Transition Matrix

Here we have articulated all possible Interactions as Agent-1 meets Agent-2 :-

- Susceptible meets Susceptible :-

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains Susceptible with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then Agent-1 Remains Susceptible but with Agent-2's Opinion

- Susceptible meets Infected then Agent-1 can two actions pi1 and pi2:-

  If Agent-2 has Same Opinion as Agent-1 :-

  If pi1 is taken then:-

  Agent-1 gets infected with beta rate and has Same Opinion

  OR Agent-1 Remains Susceptible with (1-beta) rate with Same Opinion

  If pi2 is taken then:-

  Agent-1 Remains Susceptible with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then :-

  If pi1 is taken then:-

  Agent-1 gets infected with beta rate and has Agent-2's Opinion

  OR Agent-1 Remains Susceptible with (1-beta) rate with Agent-2's Opinion

  If pi2 is taken then:-

  Agent-1 Remains Susceptible with Agent-2's Opinion

- Susceptible meets Recovered:-

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains Susceptible with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then Agent-1 Remains Susceptible but with Agent-2's Opinion

- Infected meets Susceptible :-

  If Agent-2 has Same Opinion as Agent-1 then:-

  Agent-1 gets recovered with gamma rate and has Same Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Agent-2's Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Agent-2's Opinion

- Infected meets Infected:-

  If Agent-2 has Same Opinion as Agent-1 then:-

  Agent-1 get recovered with gamma rate and has Same Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Agent-2's Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Agent-2's Opinion

- Infected meets Recovered:-

  If Agent-2 has Same Opinion as Agent-1 then:-

  Agent-1 get recovered with gamma rate and has Same Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Agent-2's Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Agent-2's Opinion

- Recovered meets Susceptible :-

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains recovered with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then Agent-1 Remains recovered but with Agent-2's Opinion

- Recovered meets Infected:-

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains recovered with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then Agent-1 Remains recovered but with Agent-2's Opinion

- Recovered meets Recovered:-

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains recovered with Same Opinion

  If Agent-2 has different Opinion from Agent-1 then Agent-1 Remains recovered but with Agent-2's Opinion

Here is the example of Markov Chain for 2 Opinions 0 and 1 :-

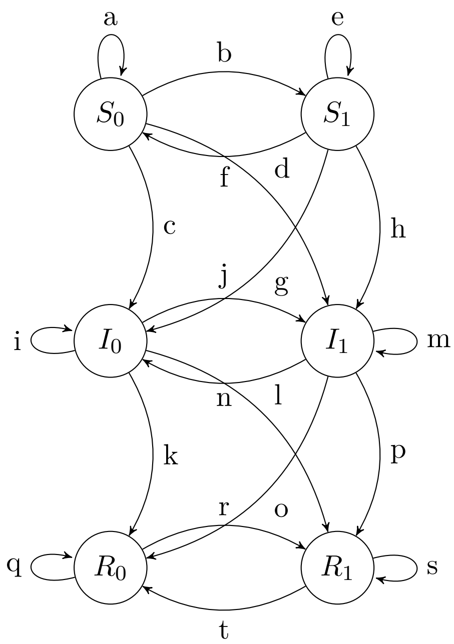


Figure 5: Markov Chain Diagram for 2 Opinions 0 and 1 [14]

Transition Probabilities

- **a**: $S_0 \to S_0$: $S_0 + I_0\pi_1^0(1-\beta) + I_0\pi_2^0 + R_0$
- **b**: $S_0 \to S_1$: $S_1 + I_1\pi_1^0(1-\beta) + I_1\pi_2^0 + R_1$
- **c**: $S_0 \to I_0$: $I_0\pi_1^0\beta$
- **d**: $S_0 \to I_1$: $I_1\pi_1^0\beta$
- **e**: $S_1 \to S_1$: $S_1 + I_1\pi_1^1(1-\beta) + I_1\pi_2^1 + R_1$
- **f**: $S_1 \to S_0$: $S_0 + I_0\pi_1^0(1-\beta) + I_0\pi_2^0 + R_0$
- **g**: $S_1 \to I_0$: $I_0\pi_1^1\beta$
- **h**: $S_1 \to I_1$: $I_1\pi_1^1\beta$
- **i**: $I_0 \to I_0$: $(1-\gamma)(S_0 + I_0 + R_0)$
- **j**: $I_0 \to I_1$: $(1-\gamma)(S_1 + I_1 + R_1)$
- **k**: $I_0 \to R_0$: $\gamma(S_0 + I_0 + R_0)$
- **l**: $I_0 \to R1$: $\gamma(S_1 + I_1 + R_1)$
- **m**: $I_1 \to I_1$: $(1-\gamma)(S_1 + I_1 + R_1)$
- **n**: $I_1 \to I_0$: $(1-\gamma)(S_0 + I_0 + R_0)$
- **o**: $I_1 \to R_0$: $\gamma(S_0 + I_0 + R_0)$
- **p**: $I_1 \to R_1$: $\gamma(S_1 + I_1 + R_1)$

- **q**: $R_0 \rightarrow R_0$: $S_0 + I_0 + R_0$
- **r**: $R_0 \rightarrow R_1$: $S_1 + I_1 + R_1$
- **s**: $R_1 \rightarrow R_1$: $S_1 + I_1 + R_1$
- **t**: $R_1 \rightarrow R_0$: $S_0 + I_0 + R_0$

Yields the Transition Matrix "T"

	S_0	S_1	I_0	I_1	R_0	R_1
S_0	$S_0 + I_0\pi_1^0(1-\beta)$ $+I_0\pi_2^0 + R_0$	$S_1 + I_1\pi_1^0(1-\beta)$ $+I_1\pi_2^0 + R_1$	$I_0\pi_1^0\beta$	$I_1\pi_1^0\beta$	0	0
S_1	$S_0 + I_0\pi_1^1(1-\beta)$ $+I_0\pi_2^1 + R_0$	$S_1 + I_1\pi_1^1(1-\beta)$ $+I_1\pi_2^1 + R_1$	$I_0\pi_1^1\beta$	$I_1\pi_1^1\beta$	0	0
I_0	0	0	$(1-\gamma)(S_0 + I_0 + R_0)$	$(1-\gamma)(S_1 + I_1 + R_1)$	$\gamma(S_0 + I_0 + R_0)$	$\gamma(S_1 + I_1 + R_1)$
I_1	0	0	$(1-\gamma)(S_0 + I_0 + R_0)$	$(1-\gamma)(S_1 + I_1 + R_1)$	$\gamma(S_0 + I_0 + R_0)$	$\gamma(S_1 + I_1 + R_1)$
R_0	0	0	0	0	$S_0 + I_0 + R_0$	$S_1 + I_1 + R_1$
R_1	0	0	0	0	$S_0 + I_0 + R_0$	$S_1 + I_1 + R_1$

Figure 6: Transition Matrix with Opinion index 0 and 1

and thereby producing the Linear System of equations as:-

$$\begin{pmatrix} S_0(t+\Delta t) \\ S_1(t+\Delta t) \\ I_0(t+\Delta t) \\ I_1(t+\Delta t) \\ R_0(t+\Delta t) \\ R_1(t+\Delta t) \end{pmatrix}^T = \begin{pmatrix} S_0(t) \\ S_1(t) \\ I_0(t) \\ I_1(t) \\ R_0(t) \\ R_1(t) \end{pmatrix}^T \begin{pmatrix} S_0 + I_0\pi_1^0(1-\beta) + I_0\pi_2^0 + R_0 & S_1 + I_1\pi_1^0(1-\beta) + I_1\pi_2^0 + R_1 & I_0\pi_1^0\beta & I_1\pi_1^0\beta & 0 & 0 \\ S_0 + I_0\pi_1^0(1-\beta) + I_0\pi_2^0 + R_0 & S_1 + I_1\pi_1^0(1-\beta) + I_1\pi_2^0 + R_1 & I_0\pi_1^0\beta & I_1\pi_1^0\beta & 0 & 0 \\ 0 & 0 & (1-\gamma)(S_0 + I_0 + R_0) & (1-\gamma)(S_1 + I_1 + R_1) & \gamma(S_0 + I_0 + R_0) & \gamma(S_1 + I_1 + R_1) \\ 0 & 0 & (1-\gamma)(S_0 + I_0 + R_0) & (1-\gamma)(S_1 + I_1 + R_1) & \gamma(S_0 + I_0 + R_0) & \gamma(S_1 + I_1 + R_1) \\ 0 & 0 & 0 & S_0 + I_0 + R_0 & S_1 + I_1 + R_1 & \\ 0 & 0 & 0 & S_0 + I_0 + R_0 & S_1 + I_1 + R_1 & \end{pmatrix}$$

Similarly for i Opinions, Transition Matrix "T"

	S_0	S_1	$\cdots$	S_i	I_0	I_1	$\cdots$	I_i	R_0	R_1	$\cdots$	R_i
S_0	S_0+ $I_0\pi_1^0(1-\beta)$ $+I_0\pi_2^0 + R_0$	S_1+ $I_1\pi_1^0(1-\beta)$ $+I_1\pi_2^0 + R_1$	$\cdots$	S_i+ $I_i\pi_1^0(1-\beta)$ $+I_i\pi_2^0 + R_i$	$I_0\pi_1^0\beta$	$I_1\pi_1^0\beta$	$\cdots$	$I_i\pi_1^0\beta$	0	0	$\cdots$	0
S_1	S_0+ $I_0\pi_1^1(1-\beta)$ $+I_0\pi_2^1 + R_0$	S_1+ $I_1\pi_1^1(1-\beta)$ $+I_1\pi_2^1 + R_1$	$\cdots$	S_i+ $I_i\pi_1^1(1-\beta)$ $+I_i\pi_2^1 + R_i$	$I_0\pi_1^1\beta$	$I_1\pi_1^1\beta$	$\cdots$	$I_i\pi_1^1\beta$	0	0	$\cdots$	0
$\vdots$			$\ddots$				$\ddots$				$\ddots$	
S_i	S_0+ $I_0\pi_1^i(1-\beta)$ $+I_0\pi_2^i + R_0$	S_1+ $I_1\pi_1^i(1-\beta)$ $+I_1\pi_2^i + R_1$	$\cdots$	S_i+ $I_i\pi_1^i(1-\beta)$ $+I_i\pi_2^i + R_i$	$I_0\pi_1^i\beta$	$I_1\pi_1^i\beta$	$\cdots$	$I_i\pi_1^i\beta$	0	0	$\cdots$	0
I_0	0	0	$\cdots$	0	$(1-\gamma)(S_0$ $+I_0 + R_0)$	$(1-\gamma)(S_1$ $+I_1 + R_1)$	$\cdots$	$(1-\gamma)(S_i$ $+I_i + R_i)$	$\gamma(S_0 + I_0 + R_0)$	$\gamma(S_1 + I_1 + R_1)$	$\cdots$	$\gamma(S_i + I_i + R_i)$
I_1	0	0	$\cdots$	0	$(1-\gamma)(S_0$ $+I_0 + R_0)$	$(1-\gamma)(S_1$ $+I_1 + R_1)$	$\cdots$	$(1-\gamma)(S_i$ $+I_i + R_i)$	$\gamma(S_0 + I_0 + R_0)$	$\gamma(S_1 + I_1 + R_1)$	$\cdots$	$\gamma(S_i + I_i + R_i)$
$\vdots$			$\ddots$				$\ddots$				$\ddots$	
I_i	0	0	$\cdots$	0	$(1-\gamma)(S_0$ $+I_0 + R_0)$	$(1-\gamma)(S_1$ $+I_1 + R_1)$	$\cdots$	$(1-\gamma)(S_i$ $+I_i + R_i)$	$\gamma(S_0 + I_0 + R_0)$	$\gamma(S_1 + I_1 + R_1)$	$\cdots$	$\gamma(S_i + I_i + R_i)$
R_0	0	0	$\cdots$	0	0	0	$\cdots$	0	$S_0 + I_0 + R_0$	$S_1 + I_1 + R_1$	$\cdots$	$S_i + I_i + R_i$
R_1	0	0	$\cdots$	0	0	0	$\cdots$	0	$S_0 + I_0 + R_0$	$S_1 + I_1 + R_1$	$\cdots$	$S_i + I_i + R_i$
$\vdots$			$\ddots$				$\ddots$				$\ddots$	
R_i	0	0	$\cdots$	0	0	0	$\cdots$	0	$S_0 + I_0 + R_0$	$S_1 + I_1 + R_1$	$\cdots$	$S_i + I_i + R_i$

Figure 7: Transition Matrix with i Opinions

2.4 Linear System of Equations for Opinion Based SIR Model for i Opinions

$$
\begin{pmatrix} S_0(t+\Delta t) \\ S_1(t+\Delta t) \\ \vdots \\ S_i(t+\Delta t) \\ I_0(t+\Delta t) \\ I_1(t+\Delta t) \\ \vdots \\ I_i(t+\Delta t) \\ R_0(t+\Delta t) \\ R_1(t+\Delta t) \\ \vdots \\ R_i(t+\Delta t) \end{pmatrix}^T =
$$

$$
\begin{pmatrix} S_0(t) \\ S_1(t) \\ \vdots \\ S_i(t) \\ I_0(t) \\ I_1(t) \\ \vdots \\ I_i(t) \\ R_0(t) \\ R_1(t) \\ \vdots \\ R_i(t) \end{pmatrix}^T
\begin{pmatrix}
S_0 + I_0\pi_1^0(1-\beta) + I_0\pi_2^0 + R_0 & \cdots & I_0\pi_1^0\beta & \cdots & 0 & \cdots \\
S_0 + I_0\pi_1^1(1-\beta) + I_0\pi_2^1 + R_0 & \cdots & I_0\pi_1^1\beta & \cdots & 0 & \cdots \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
S_0 + I_0\pi_1^i(1-\beta) + I_0\pi_2^i + R_0 & \cdots & I_0\pi_1^i\beta & \cdots & 0 & \cdots \\
0 & \cdots & (1-\gamma)(S_0 + I_0 + R_0 & \cdots & \gamma(S_0 + I_0 + R_0) & \cdots \\
0 & \cdots & (1-\gamma)(S_0 + I_0 + R_0) & \cdots & \gamma(S_0 + I_0 + R_0) & \cdots \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & (1-\gamma)(S_0 + I_0 + R_0) & \cdots & \gamma(S_0 + I_0 + R_0) & \cdots \\
0 & \cdots & 0 & \cdots & S_0 + I_0 + R_0 & \cdots \\
0 & \cdots & 0 & \cdots & S_0 + I_0 + R_0 & \cdots \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & S_0 + I_0 + R_0 & \cdots
\end{pmatrix}
$$

3 Results

To evaluate the results of the equations, we need to set some parameters according to our assumptions and , the infection rate, the recovery rate and opinion set q.

- Number of opinions: $n = 5$
- Total population: $N = 1000$
- Initial infected: $I_0 = 0.01 \times N = 10$
- Initial recovered: $R_0 = 0$
- Initial susceptible: $S_0 = N - I_0 - R_0 = 990$
- Initial state: $(S_0, I_0, R_0) = (990, 10, 0)$

The initial opinion state is given by:

$$\text{init\_opinion\_state} = \begin{bmatrix} \frac{S_0}{n}, & \frac{S_0}{n}, & \ldots, & \frac{S_0}{n}, & \frac{I_0}{n}, & \frac{I_0}{n}, & \ldots, & \frac{I_0}{n}, & \frac{R_0}{n}, & \frac{R_0}{n}, \\ \ldots, & \frac{R_0}{n} \end{bmatrix}$$

This can be simplified to:

$$\text{init\_opinion\_state} = \begin{bmatrix} 198, & 198, & 198, & 198, & 198, & 2, & 2, & 2, & 2, & 2, \\ 0, & 0, & 0, & 0, & 0 \end{bmatrix}$$

The opinion values are defined as:

$$q = \begin{bmatrix} -0, & -0.1, & -0.2, & -0.3, & -0.4 \end{bmatrix}$$

- Infection rate: $\beta = 0.7$
- Recovery rate: $\gamma = 0.1$
- Number of time steps to simulate: 100

3.1 Dynamics of Susceptible, Infected, and Recovered of different Opinion groups

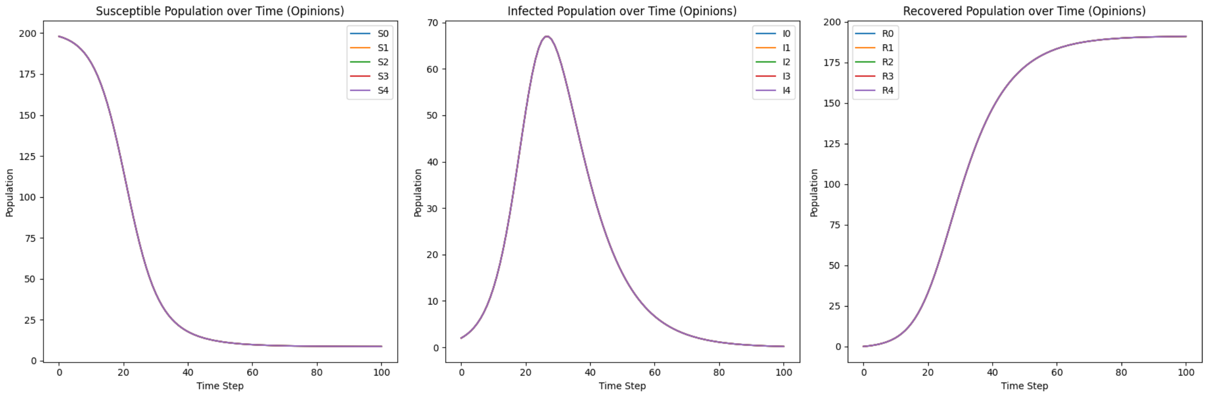


Figure 8: Susceptible, Infected, and Recovered of different Opinion groups

As all the opinions are equally valuable(no dominance) implies a same number of people have changed their opinion from one to another and as there was no difference in size of the population (Susceptible, Infected, and recovered) initially, the results in all opinion behave equally to the infection. Therefore all 5 curves of S, I, and R, are one another.

Proof

Let's see this analytically. In loose words, the transition matrix for each column represents the probabilities which come into the compartment.

Total Portion of People Coming/Staying into Compartment S_0

$$\begin{aligned}
S_0(t+\Delta t) &= S_0S_0 + S_0I_0\pi_1^0(1-\beta) + S_0I_0\pi_2^0 + S_0R_0 \\
&\quad + S_0S_1 + S_1I_0\pi_1^1(1-\beta) + S_1I_0\pi_2^1 + S_1R_0 \\
&\quad + \ldots + S_0S_i + S_iI_0\pi_1^i(1-\beta) + S_iI_0\pi_2^i + S_iR_0 \\
&= S_0(S_0 + S_1 + \ldots + S_i) + I_0(1-\beta)(S_0\pi_1^0 + S_1\pi_1^1 + \ldots + S_i\pi_1^i) \\
&\quad + I_0(S_0\pi_2^0 + S_1\pi_2^1 + \ldots + S_i\pi_2^i) + R_0(S_0 + S_1 + \ldots + S_i)
\end{aligned}$$

Total Portion of People Coming/Staying into Compartment S_1

$$\begin{aligned}
S_1(t+\Delta t) &= S_1S_0 + S_0I_1\pi_1^0(1-\beta) + S_0I_1\pi_2^0 + S_0R_1 \\
&\quad + S_1S_1 + S_1I_1\pi_1^1(1-\beta) + S_1I_1\pi_2^1 + S_1R_1 \\
&\quad + \ldots + S_1S_i + S_iI_1\pi_1^i(1-\beta) + S_iI_1\pi_2^i + S_iR_1 \\
&= S_1(S_0 + S_1 + \ldots + S_i) + I_1(1-\beta)(S_0\pi_1^0 + S_1\pi_1^1 + \ldots + S_i\pi_1^i) \\
&\quad + I_1(S_0\pi_2^0 + S_1\pi_2^1 + \ldots + S_i\pi_2^i) + R_1(S_0 + S_1 + \ldots + S_i)
\end{aligned}$$

Total Portion of People Coming/Staying into Compartment S_i

$$\begin{aligned}
S_i(t+\Delta t) &= S_iS_0 + S_0I_i\pi_1^0(1-\beta) + S_0I_i\pi_2^0 + S_0R_i \\
&\quad + S_iS_1 + S_1I_i\pi_1^1(1-\beta) + S_1I_i\pi_2^1 + S_1R_i \\
&\quad + \ldots + S_iS_i + S_iI_i\pi_1^i(1-\beta) + S_iI_i\pi_2^i + S_iR_i \\
&= S_i(S_0 + S_1 + \ldots + S_i) + I_i(1-\beta)(S_0\pi_1^0 + S_1\pi_1^1 + \ldots + S_i\pi_1^i) \\
&\quad + I_i(S_0\pi_2^0 + S_1\pi_2^1 + \ldots + S_i\pi_2^i) + R_i(S_0 + S_1 + \ldots + S_i)
\end{aligned}$$

Assumption of Initial Population

According to our assumption, the population is the same in each compartment initially (Say S, I, and R):

$$S_0 = S_1 = \ldots = S_i = S$$

$$I_0 = I_1 = \ldots = I_i = I$$

$$R_0 = R_1 = \ldots = R_i = R$$

And (we know),

$$\pi_1^i + \pi_2^i = 1$$

Now, Plugging in $S_0(t+\Delta t)$ we get:

$$\begin{aligned}
S_0(t+\Delta t) &= S(S + S + \ldots + S) + I(1-\beta)(S\pi_1^0 + S\pi_1^1 + \ldots + S\pi_1^i) \\
&\quad + I(S\pi_2^0 + S\pi_2^1 + \ldots + S\pi_2^i) + R(S + S + \ldots + S) \\
&= iS^2 - IS\beta(\pi_1^0 + \pi_1^1 + \ldots + \pi_1^i) \\
&\quad + IS(\pi_2^0 + \pi_2^1 + \ldots + \pi_2^i) + iRS
\end{aligned}$$

now in $S_1(t+\Delta t)$ we get:

$$\begin{aligned}
S_1(t+\Delta t) &= S(S + S + \ldots + S) + I(1-\beta)(S\pi_1^0 + S\pi_1^1 + \ldots + S\pi_1^i) \\
&\quad + I(S\pi_2^0 + S\pi_2^1 + \ldots + S\pi_2^i) + R(S + S + \ldots + S) \\
&= iS^2 - IS\beta(\pi_1^0 + \pi_1^1 + \ldots + \pi_1^i) \\
&\quad + IS(\pi_2^0 + \pi_2^1 + \ldots + \pi_2^i) + iRS
\end{aligned}$$

and in $S_i(t+\Delta t)$ we get:

$$
\begin{aligned}
S_i(t+\Delta t) = S(S+S+\ldots+S) &+ I(1-\beta)(S\pi_1^0 + S\pi_1^1 + \ldots + S\pi_1^i) \\
&+ I(S\pi_2^0 + S\pi_2^1 + \ldots + S\pi_2^i) + R(S+S+\ldots+S) \\
= iS^2 &- IS\beta(\pi_1^0 + \pi_1^1 + \ldots + \pi_1^i) \\
&+ IS(\pi_2^0 + \pi_2^1 + \ldots + \pi_2^i) + iRS
\end{aligned}
$$

Therefore:

$$S_0(t+\Delta t) = S_1(t+\Delta t) = S_i(t+\Delta t)$$

As the resulting equation of the S compartment for each opinion is the same, the proportions have the same result as time progresses.

3.2 Dynamics of Basic and Aggregated (Summed of Opinion) SIR

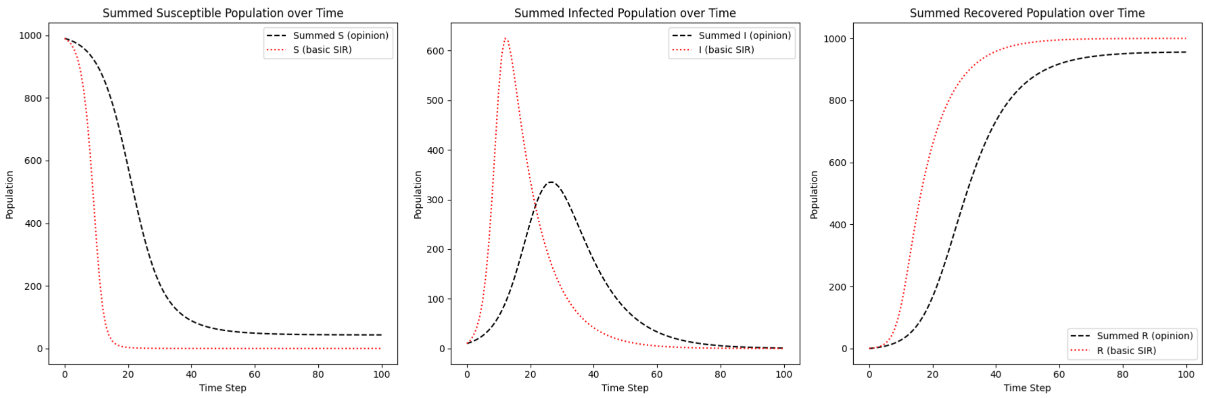


Figure 9: Basic and aggregated (Summed of Opinion) SIR

We can observe despite the trivial conditions, the aggregated model damps the spread of infection than the Basic SIR model.

3.3 Dynamics of Basic and Aggregated (Summed of Opinion) SIR in 3D Space

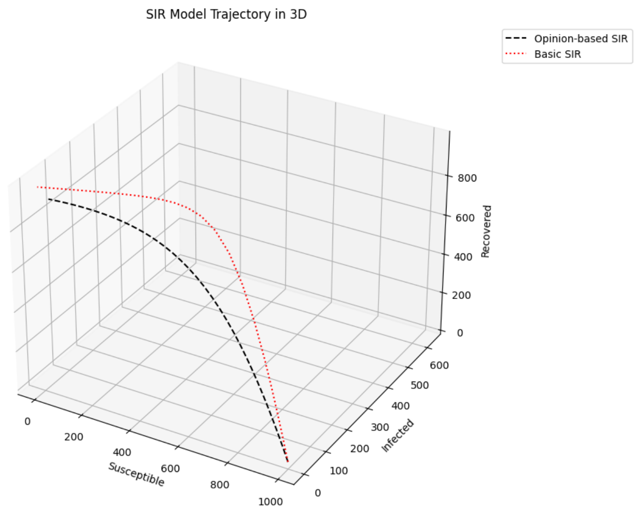


Figure 10: Basic and aggregated (Summed of Opinion) SIR in 3D Space

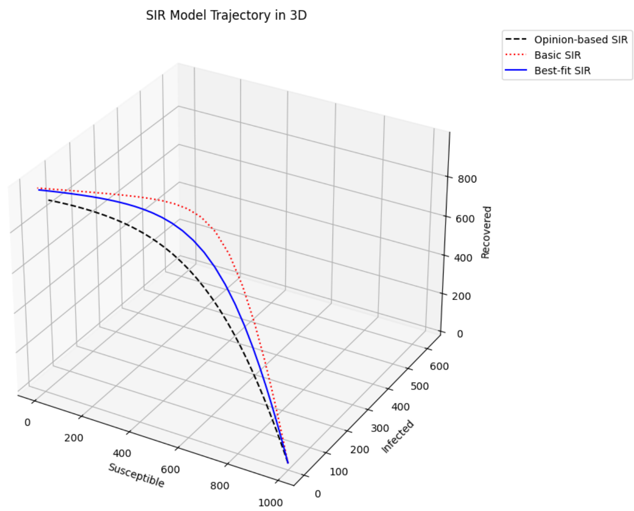


Figure 11: Basic, aggregated (Summed of Opinion), and fitted SIR in 3D Space

To find the best fit between the opinion-based SIR model and the basic SIR model, we optimize the parameters β and γ by minimizing the Mean Square Error. We can observe, that the fitted curve between Basic and Aggregated Opinion based SIR Model. For, $\beta = 0.7$ and $\gamma = 0.1$ has $\beta = 1$ and $\gamma =$0.099 . This implies, the aggregated model to perform as badly as the no-opinion model, the β should be as high as the optimized beta.

3.4 Error Calculation for Varying β and γ

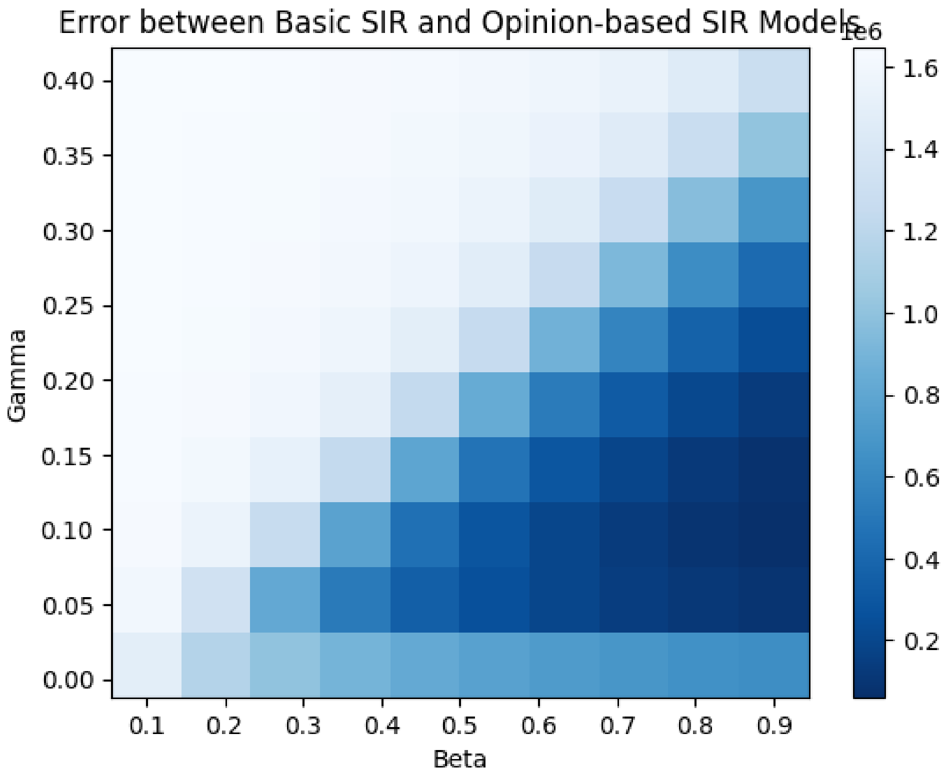


Figure 12: Error for different β and γ

We can observe that, as the aggregated model's response to the infection is slower and weaker than the basic model, the aggregated model differs positively from the basic model at lower infection rates or higher recovery rates.

3.5 Error Calculation for Varying q Values

Taking three q sets, implying 3 independent Populations with q3 having higher values than q2 than q1 :-

$$q_{\text{set1}} = \begin{bmatrix} 0 \\ -0.1 \\ -0.2 \\ -0.3 \\ -0.4 \end{bmatrix}, q_{\text{set2}} = \begin{bmatrix} 0 \\ -0.2 \\ -0.4 \\ -0.6 \\ -0.8 \end{bmatrix}, q_{\text{set3}} = \begin{bmatrix} 0 \\ -0.3 \\ -0.6 \\ -0.9 \\ -1.2 \end{bmatrix}$$

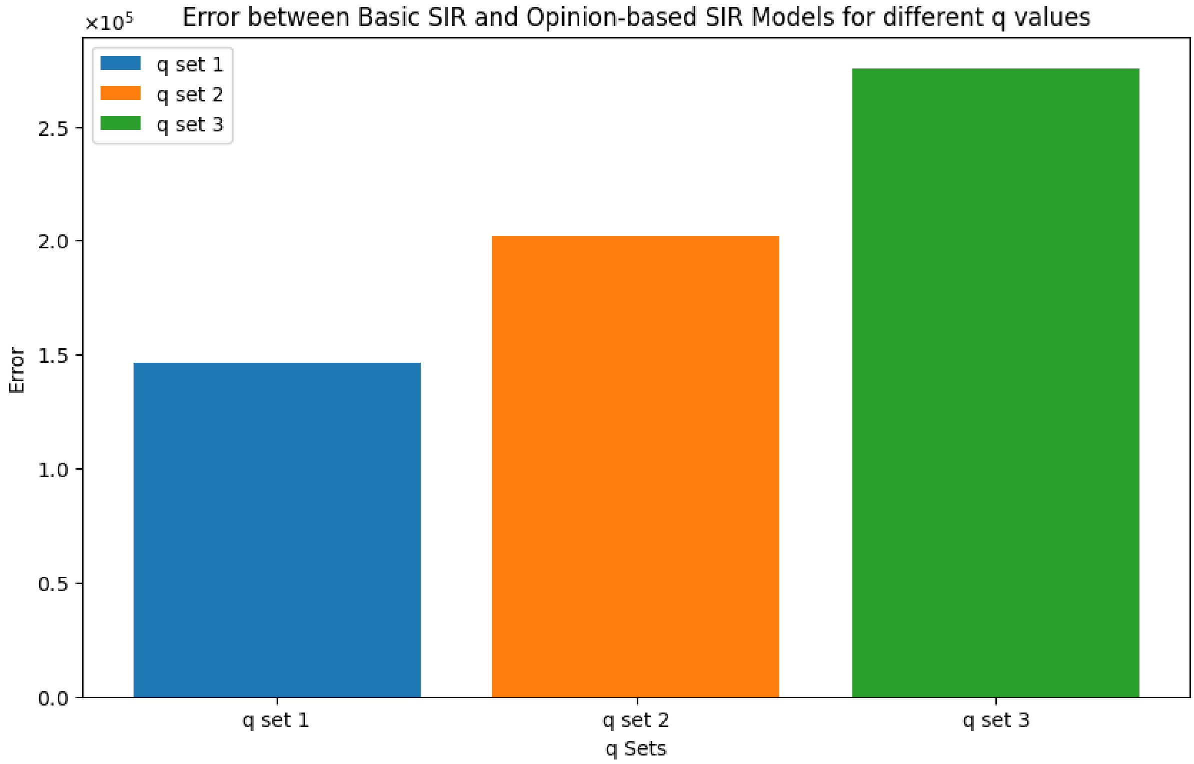


Figure 13: Error for different q

The result indicates that the population with a higher opinions can withstand higher infection rate (optimized β) than the population with no Opinion dynamics.

The infection spread for different q sets can be observed from this plot:-

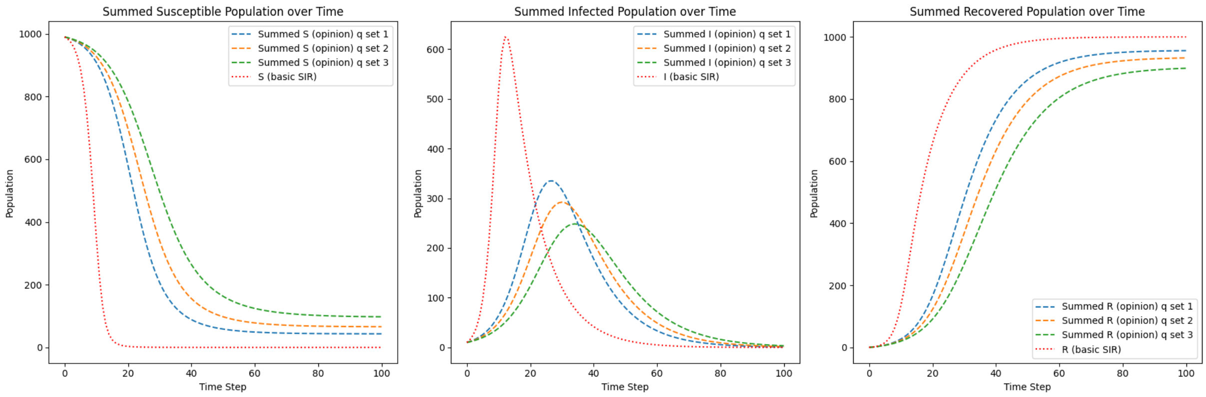


Figure 14: Error for different q

This result indicates that the higher the opinion group, the lower the peak in infection rates. Additionally, the timeframe is extended, with the peak occurring later for groups with higher opinions. This suggests that higher opinion groups not only experience a less intense peak in infection rates but also reach this peak at a later time, demonstrating a more gradual progression of the infection within these groups.

3.6 Error Calculation for Varying β, γ, and q

The combined effect of varying β, γ, and q is studied to understand their joint impact on the model's deviation.

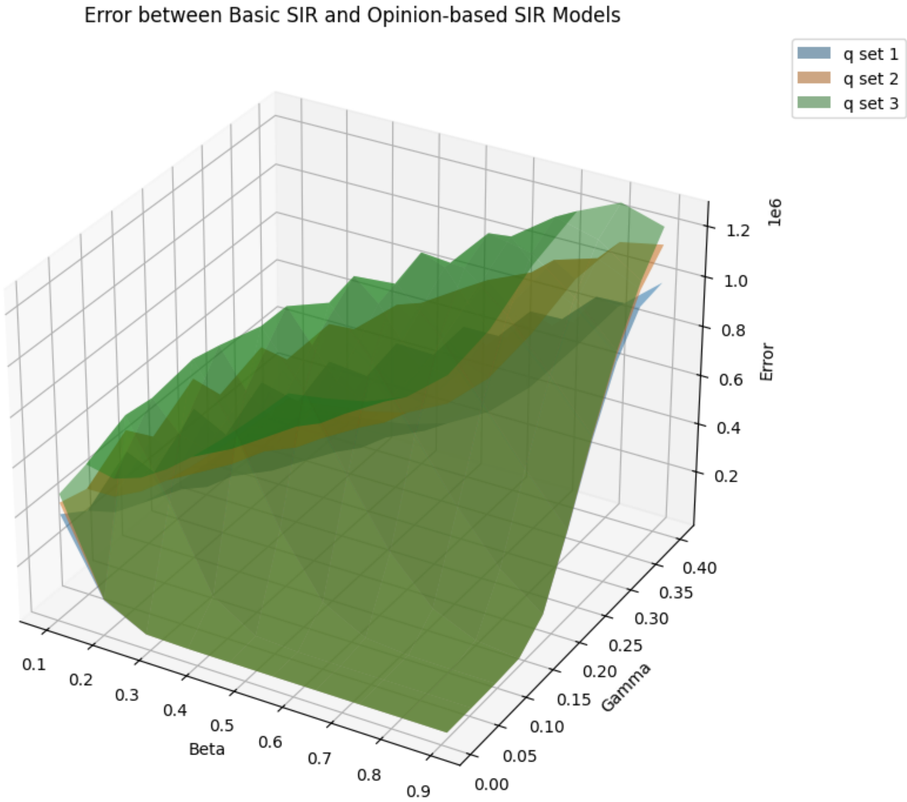


Figure 15: Error for different β, γ, and q

We can observe that for the highest opinion group, the error is the highest when varying β and γ values. This indicates the greatest deviation from the basic model, highlighting how significantly the infection dynamics differ for this group compared to the standard model.

4 Conclusion

The incorporation of opinion dynamics into the SIR model reveals intriguing patterns in disease spread. The transition matrix method effectively captures the influence of opinions on infection and recovery rates. Our analysis of errors indicates that varying q values significantly impacts the model's performance, underscoring the importance of considering opinion dynamics in epidemiological modeling.

The Opinion-based Model successfully captures the dynamics of infectious disease spread across different opinions within a population. Notably, the model demonstrates a reduction in infection spread, as increased public opinion (cautiousness) significantly decreases infection rates. This highlights the model's potential to offer valuable insights for public health across a wide range of infectious diseases.

By continuously adapting and refining this model, we can gain deeper insights into disease control and optimize public health interventions for future outbreaks. The findings of this project emphasize the critical role of incorporating social and opinion dynamics into epidemiological models to enhance their accuracy and effectiveness in predicting and managing disease spread.

5 Further Potential Research Topics

- **Impact of Population Size on Opinion Dynamics**
  - Investigate how different population sizes for each opinion group influence the overall dynamics of disease spread and opinion shifts.
- **Dynamics of Adapting Opinions Under Dominance**
  - Explore how the presence of dominant opinions can lead to different adaptation mechanisms for other opinions, resulting in significant changes in disease spread patterns.
- **Modeling Disease Spread with SIS Framework**
  - Analyze the dynamics of disease spread assuming a Susceptible-Infectious-Susceptible (SIS) model, and how opinion dynamics integrate within this framework.
- **Utilizing Softmax Functions for Opinion Calculation**
  - Evaluate the impact of using different softmax functions to calculate opinion strengths and transitions, and how these influence the spread of infectious diseases.

6 Appendix - I

Model with Opinion Dominance

In this case we are considering the dominance of one Opinion over the other, therefore changes the dynamics as:-

- Susceptible meets Susceptible :-

  If Agent-2 has lower Opinion than Agent-1 then Agent-1 Remains Susceptible with own Opinion

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains Susceptible with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then Agent-1 Remains Susceptible but with Agent-2's Opinion

- Susceptible meets Infected then Agent-1 can two actions pi1 and pi2:-

  If Agent-2 has lower Opinion than Agent-1 :-

  If pi1 is taken then:-

  Agent-1 get infected with beta rate and have own Opinion

  OR Agent-1 Remains Susceptible with (1-beta) rate with own Opinion

  If pi2 is taken then:-

  Agent-1 Remains Susceptible with own Opinion

  If Agent-2 has Same Opinion as Agent-1 :-

  If pi1 is taken then:-

  Agent-1 get infected with beta rate and have Same Opinion

  OR Agent-1 Remains Susceptible with (1-beta) rate with Same Opinion

  If pi2 is taken then:-

  Agent-1 Remains Susceptible with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then :-

  If pi1 is taken then:-

  Agent-1 get infected with beta rate and have Agent-2's Opinion

  OR Agent-1 Remains Susceptible with (1-beta) rate with Agent-2's Opinion

  If pi2 is taken then:-

  Agent-1 Remains Susceptible with Agent-2's Opinion

- Susceptible meets Recovered:-

  If Agent-2 has lower Opinion than Agent-1 then Agent-1 Remains Susceptible with own Opinion

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains Susceptible with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then Agent-1 Remains Susceptible but with Agent-2's Opinion

- Infected meets Susceptible :-

  If Agent-2 has lower Opinion than Agent-1 then:-

  Agent-1 get recovered with gamma rate and have own Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has Same Opinion as Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Same Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Agent-2's Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Agent-2's Opinion

- Infected meets Infected:-

  If Agent-2 has lower Opinion than Agent-1 then:-

  Agent-1 get recovered with gamma rate and have own Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has Same Opinion as Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Same Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Agent-2's Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Agent-2's Opinion

- Infected meets Recovered:-

  If Agent-2 has lower Opinion than Agent-1 then:-

  Agent-1 get recovered with gamma rate and have own Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has Same Opinion as Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Same Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then:-

  Agent-1 get recovered with gamma rate and have Agent-2's Opinion

  OR Agent-1 Remains infected with (1-gamma) rate with Agent-2's Opinion

- Recovered meets Susceptible :-

  If Agent-2 has lower Opinion than Agent-1 then Agent-1 Remains recovered with own Opinion

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains recovered with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then Agent-1 Remains recovered but with Agent-2's Opinion

- Recovered meets Infected:-

  If Agent-2 has lower Opinion than Agent-1 then Agent-1 Remains recovered with own Opinion

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains recovered with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then Agent-1 Remains recovered but with Agent-2's Opinion

- Recovered meets Recovered:-

  If Agent-2 has lower Opinion than Agent-1 then Agent-1 Remains recovered with own Opinion

  If Agent-2 has Same Opinion as Agent-1 then Agent-1 Remains recovered with Same Opinion

  If Agent-2 has higher Opinion than Agent-1 then Agent-1 Remains recovered but with Agent-2's Opinion

Here is the example of Markov Chain for 2 Opinions 0 and 1 :-

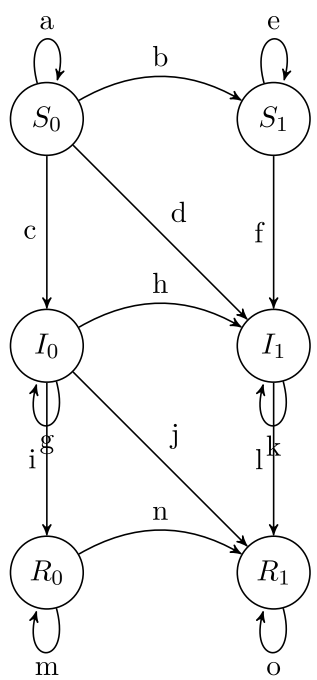


Figure 16: Markov Chain Diagram for 2 Opinions 0 and 1 [14]

Description of Transition Probabilities

- **a**: $S_0 \rightarrow S_0$ transition with probability $S_0 + I_0\pi_1^0(1-\beta) + I_0\pi_2^0 + R_0$
- **b**: $S_0 \rightarrow S_1$ transition with probability $S_1 + I_1\pi_1^0(1-\beta) + I_1\pi_2^0 + R_1$
- **c**: $S_0 \rightarrow I_0$ transition with probability $I_0\pi_1^0\beta$
- **d**: $S_0 \rightarrow I_1$ transition with probability $I_1\pi_1^0\beta$
- **e**: $S_1 \rightarrow S_1$ transition with probability $S_0 + S_1 + I_0\pi_1^1(1-\beta) + I_1\pi_1^1(1-\beta) + I_0\pi_2^1 + I_1\pi_2^1 + R_0 + R_1$
- **f**: $S_1 \rightarrow I_1$ transition with probability $I_0\pi_1^1\beta + I_1\pi_1^1\beta$
- **g**: $I_0 \rightarrow I_0$ transition with probability $(1-\gamma)(S_0 + I_0 + R_0)$
- **h**: $I_0 \rightarrow I_1$ transition with probability $(1-\gamma)(S_1 + I_1 + R_1)$
- **i**: $I_0 \rightarrow R_0$ transition with probability $\gamma(S_0 + I_0 + R_0)$
- **j**: $I_0 \rightarrow R_1$ transition with probability $\gamma(S_1 + I_1 + R_1)$
- **k**: $I_1 \rightarrow I_1$ transition with probability $(1-\gamma)(S_0 + S_1 + I_0 + I_1 + R_0 + R_1)$
- **l**: $I_1 \rightarrow R_1$ transition with probability $\gamma(S_0 + S_1 + I_0 + I_1 + R_0 + R_1)$
- **m**: $R_0 \rightarrow R_0$ transition with probability $S_0 + I_0 + R_0$
- **n**: $R_0 \rightarrow R_1$ transition with probability $S_1 + I_1 + R_1$
- **o**: $R_1 \rightarrow R_1$ transition with probability $S_0 + S_1 + I_0 + I_1 + R_0 + R_1$

	S_0	S_1	I_0	I_1	R_0	R_1
S_0	$S_0 + I_0\pi_1^0(1-\beta)$ $+I_0\pi_2^0 + R_0$	$S_1 + I_1\pi_1^0(1-\beta)$ $+I_1\pi_2^0 + R_1$	$I_0\pi_1^0\beta$	$I_1\pi_1^0\beta$	0	0
S_1	0	$S_0 + I_0\pi_1^1(1-\beta)$ $+I_0\pi_2^1 + R_0$ $+$ $S_1 + I_1\pi_1^1(1-\beta)$ $+I_1\pi_2^1 + R_0$	0	$I_0\pi_1^1\beta + I_1\pi_1^1\beta$	0	0
I_0	0	0	$(1-\gamma)(S_0 + I_0 + R_0)$	$(1-\gamma)(S_1 + I_1 + R_1)$	$\gamma(S_0 + I_0 + R_0)$	$\gamma(S_1 + I_1 + R_1)$
I_1	0	0	0	$(1-\gamma)(S_0 + I_0 + R_0 + S_1 + I_1 + R_1)$	0	$\gamma(S_0 + I_0 + R_0 + S_1 + I_1 + R_1)$
R_0	0	0	0	0	$S_0 + I_0 + R_0$	$S_1 + I_1 + R_1$
R_1	0	0	0	0	0	$S_0 + I_0 + R_0 + S_1 + I_1 + R_1$

Figure 17: Transition Matrix with Opinion index 0 and 1

and thereby producing the Linear System of Equations as:-

$$\begin{pmatrix} S_0(t+\Delta t) \\ S_1(t+\Delta t) \\ I_0(t+\Delta t) \\ I_1(t+\Delta t) \\ R_0(t+\Delta t) \\ R_1(t+\Delta t) \end{pmatrix}^T = \begin{pmatrix} S_0(t) \\ S_1(t) \\ I_0(t) \\ I_1(t) \\ R_0(t) \\ R_1(t) \end{pmatrix}^T \begin{pmatrix} S_0 + I_0\pi_1^0(1-\beta) + I_0\pi_2^0 + R_0 & S_1 + I_1\pi_1^0(1-\beta) + I_1\pi_2^0 + R_1 & I_0\pi_1^0\beta & I_1\pi_1^0\beta & 0 & 0 \\ 0 & S_0 + I_0\pi_1^1(1-\beta) + I_0\pi_2^1 + R_0 + S_1 + I_1\pi_1^1(1-\beta) + I_1\pi_2^1 + R_1 & 0 & I_0\pi_1^1\beta + I_1\pi_1^1\beta & 0 & 0 \\ 0 & 0 & (1-\gamma)(S_0 + I_0 + R_0) & (1-\gamma)(S_1 + I_1 + R_1) & \gamma(S_0 + I_0 + R_0) & \gamma(S_1 + I_1 + R_1) \\ 0 & 0 & 0 & (1-\gamma)(S_0 + I_0 + R_0 + S_1 + I_1 + R_1) & 0 & \gamma(S_0 + I_0 + R_0 + S_1 + I_1 + R_1) \\ 0 & 0 & 0 & 0 & S_0 + I_0 + R_0 & S_1 + I_1 + R_1 \\ 0 & 0 & 0 & 0 & 0 & S_0 + I_0 + R_0 + S_1 + I_1 + R_1 \end{pmatrix}$$

Result

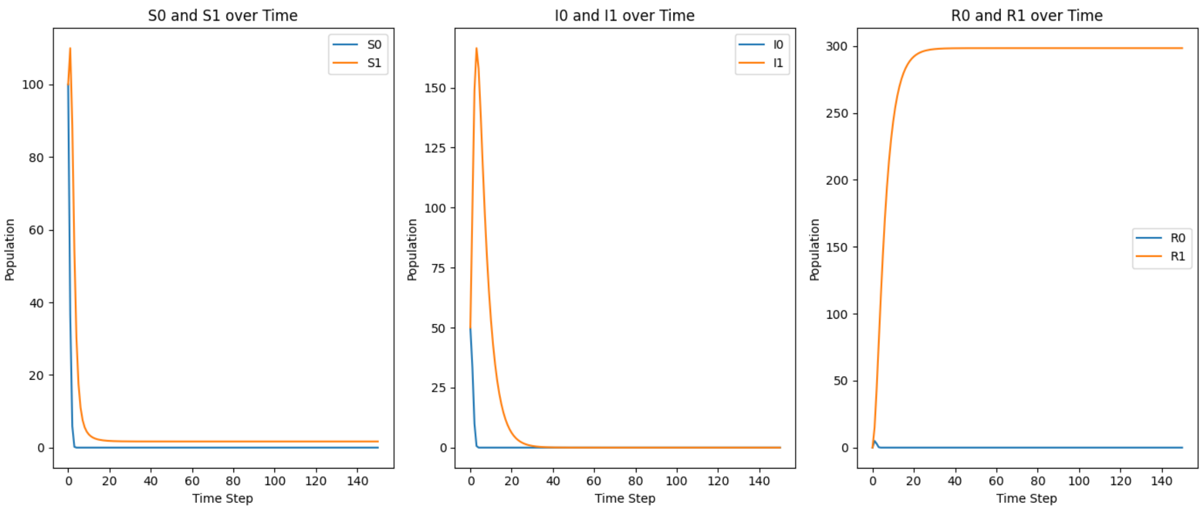


Figure 18: Dominance

The result implies that due to the dominating opinion, the lower opinion vanishes completely.

7 Appendix - II

Impact of Population Size on Opinion Dynamics

Here, we will again consider the trivial opinion adaptation, but this time with an irregular population distribution among opinions.

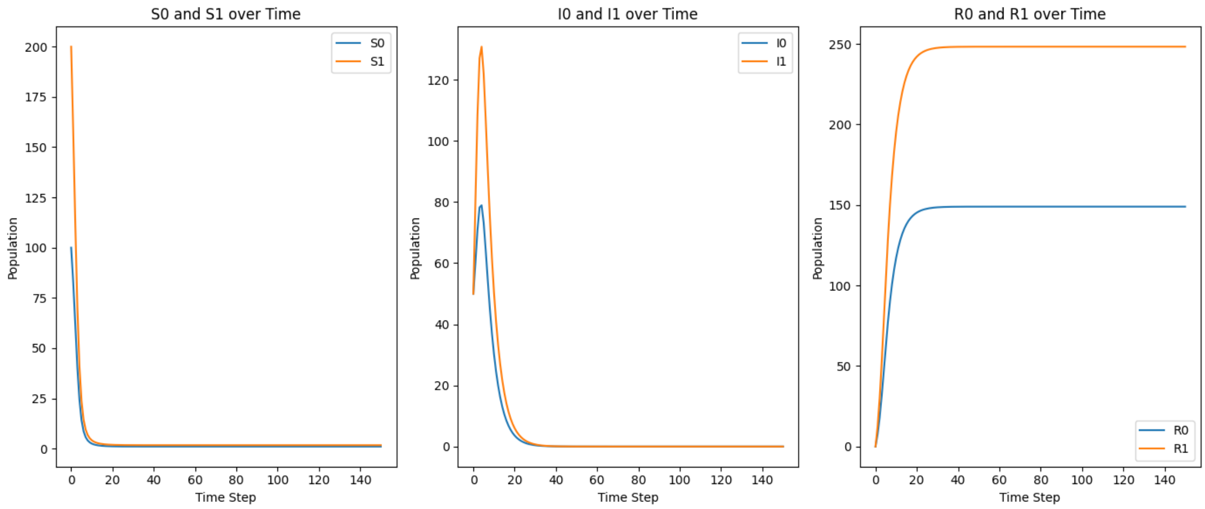


Figure 19: Dominance

The result implies that disease spread dynamics differ when the population sizes of Opinions vary.

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