TMA-201 | Printed Pages : 4

Paper ID and Roll No. to be filled in your Answer Book

Roll No. ☐☐☐☐☐☐☐☐☐☐☐

B. Tech.

(SEM. II) EXAMINATION, 2011

MATHEMATICS - II

Time : 3 Hours] | [Total Marks : 100

Attempt all questions.

1 Attempt any **four** of following : **5×4=20**

(i) Solve : $x\,dx + y\,dy = \dfrac{a^2(x\,dy - y\,dx)}{x^2 + y^2}$.

(ii) Solve $\dfrac{dz}{dx} + \dfrac{z}{x}\log z = \dfrac{z}{x^2}(\log z)^2$.

(iii) Solve : $\left(D^2 + a^2\right)y = \sec ax$.

(iv) Solve the following simultaneous equations

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$

(v) Solve $\dfrac{d^2y}{dx^2} + \dfrac{2}{x}\dfrac{dy}{dx} + \dfrac{a^2}{x^4}y = 0$.

(vi) Apply the method of variation of parameters to solve $\dfrac{d^2y}{dx^2} + y = \tan x$.

2 Attempt any two of the following $10\times2=20$

(i) (a) If $f(t)$ is continuous and of exponential order a as $t\to\infty$, then prove that Laplace transform of $f(t)$ exists for $s>a$.

(b) Find the Laplace transform of the following periodic function

$$f(t)=\frac{kt}{T} \text{ for } 0<t<T$$

(ii) If $\bar{f}_1(s)$ and $\bar{f}_2(s)$ are the Laplace transforms of $f_1(t)$ and $f_2(t)$ respectively, then prove that

$$\bar{f}_1(s)\cdot\bar{f}_2(s)=L\left\{\int_0^t f_1(u)\cdot f_2(t-u)\,du\right\}$$

Hence evaluate $L^{-1}\left\{\dfrac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right\}$

(iii) Solve the following simultaneous differential equations by using Laplace transformation :

$$Dx-y=e^t,\ Dy+x=\sin t,\ x_0=1,\ y_0=0.$$

3 Attempt any two of the following $10\times2=20$

(i) Test the convergence or divergence of the following series :

(a) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}\ (p>0)$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{(n+1)}}$

(ii) State and prove the Cauchy's root test. Hence test for convergence the following series

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$

(iii) (a) Show that the following series converges uniformly in any interval

$$\sum_{n=1}^{\infty}\frac{\cos nx}{n^p}$$

(b) Prove that

$$\int_0^1\left(\sum_{n=1}^{\infty}\frac{x^n}{n^2}\right)dx=\sum\frac{1}{n^2(n+1)}$$

4 Attempt any four of the following : 5×4=20

(i) Find the Fourier series for the function $f(x)=x+x^2$ in the interval $-\pi<x<\pi$. Hence show that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$

(ii) Obtain Fourier series for the function $f(x)$ given by $f(x)=1+2x/\pi,\ -\pi\le x\le 0$

$=1-2x/\pi,\ 0\le x\le\pi$

(iii) Express $f(x)=x$ as a half-range sine series in $0<x<2$

(iv) Eliminate the arbitrary function f and g from the following equation

$y=f(x+iy)+g(x-iy)$, where $i=\sqrt{(-1)}$

(v) Find the particular integral of

$$(D-D'-1)(D-D'-2)z=\sin(2x+3y)$$

where $D\equiv\frac{\partial}{\partial x},\ D'\equiv\frac{\partial}{\partial y}$.

(vi) Show that the equation

$$\frac{\partial^2 z}{\partial x^2}+2x\frac{\partial^2 z}{\partial x\,\partial y}+\left(1-y^2\right)\frac{\partial^2 z}{\partial y^2}=0$$

is elliptic for all values of x and y in the region $x^2+y^2<1$, parabolic on the boundary and hyperbolic out side the region.

5 Attempt any two of the following : 10×2=20

(i) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0)=y_0\sin(\pi x/l)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.

(ii) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, $0<x<\pi$, $0<y<\pi$, which satisfies the conditions

$u(0,y)=u(\pi,y)=u(x,\pi)=0$ and

$u(x,0)=\sin^2 x$.

(iii) A rod of length L has its ends A and B kept at $0^\circ C$ and $100^\circ C$, respectively, until steady state conditions prevail. If the temperature of B is then reduced suddenly to $0^\circ C$ and kept so, while that of A is maintained, find the temperature $u(x,t)$ at distance x from A at time t.

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TMA-201

Printed Pages : 4

Paper ID and Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B. Tech.

(SEM. II) EXAMINATION, 2012

MATHEMATICS - II

Time : **3** Hours] [Total Marks : **100**

Note : Attempt all questions. All questions carry equal marks.

1 Attempt any four parts of the following : **5×4=20**

(a) Solve :

$$\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$$

(b) Solve :

$$\left(1+y^2\right)dx = \left(\tan^{-1} y - x\right)dy$$

(c) Solve :

$$\left(D^2 - 2D + 1\right)y = x\sin x$$

(d) $\dfrac{dx}{dt} + 2x - 3y = t, \dfrac{dy}{dt} - 3x + 2y = e^{2t}$

(e) Solve :

$$\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + 4y\ \mathrm{cosec}^2 x = 0$$

by changing the independent variable.

(f) Apply the method of variation of parameters to solve

$$\frac{d^{2y}}{dx^2}+n^2=\sec nx$$

2 Attempt any two parts of the following : **10×2=20**

(a) Find Laplace Transform of

(i) te^{at} sin at

(ii) (cos at − cos bt)/t

(b) (i) Find $L^{-1}\left[\log\frac{S+1}{S-1}\right]$

(ii) Find $L^{-1}\left[\frac{1}{S(s+a)^3}\right]$

(c) Using Laplace Transform solve the following equations :

$$\frac{d^2x}{dt^2}+x=t\cos 2t$$

Given x(0) = x'(0) = 0

3 Attempt any two parts of the following : **10×2=20**

(a) (i) Test for convergence the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+------\infty$$

(ii) Test for convergence the series

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}------\infty$$

(b) Discuss the convergence of the series.

$$\sum_{2}^{\infty} \frac{1}{n(\log n)^p}\text{; p>0, by Integral Test.}$$

(c) Show that the Geometric series $\sum_{n=0}^{\infty} r^n$, (r>0) is convergent when r<1 and divergent when $r \geq 1$

4 Attempt any two parts of the following :

(a) Find the Fourier series expansion for f(x), if

$$f(x)\begin{cases} -\pi, -\pi < x < 0 \\ x, 0 < x < \pi \end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+---=\frac{\pi^2}{8}$

(b) Expand $f(x)=\begin{cases} \frac{1}{4}-x, 0 < x < 1/2 \\ x-3/4, 1/2 < x < 1 \end{cases}$

as the Fourier sine series

(c) Solve $(D^3-4D^2D' + 4DD'^2)z = 6 \sin(3x+2y)$.

5 Attempt any two parts of the following : **10×2=20**

(a) A tightly stretched string with fixed end points x=0 nd x=1 is initially in a position given by $y(x,0) = y_0 \sin(\pi x / l)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.

(b) A homogeneous rod of conducting material of length 'l' has its ends kept at zero temperature. The temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature distribution.

(c) Solve $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given $u(0,y) = 4e^{-y} - e^{-5y}$, by the method of separation of variables.

Paper Code & Roll No. to be filled in your Answer Book

Roll No.

Uttrakhand Technical University

B.Tech. (II Sem.) Examination-2013-14

Mathematics

Time : 3 Hrs. ***Max. Marks :100***

Note : Attempt all questions, the marks assigned to each question is indicated at question itself.

Q1. Attempt any four (5*4)

(a) Solve $dy/dx = \sin(x+y) + \cos(x+y)$

(b) Solve $y(\log y)dx + (x \log y)dy = 0$

(c) Solve $(y^2+2x^2y)dx + (2x^3-xy)dy = 0$

(d) Solve $(D^2-4D+3)y = \sin 3x \cos 2x$

(e) Using Method of variation of parameters solve

$$d^2y/dx^2 + 4y = \tan 2x$$

(f) Solve $d^2y/dx^2 + a^2y = \sec(ax)$

Q2. Attempt any four (5*4)

(a) Find the Laplace transformation of function $\sin 2t \cos 3t$ and $(1-\cos t)/t^2$.

(b) Find the Laplace transformation of function $t.e^{2t}\sin 3t$.

(c) Find the Inverse Laplace transformation of the function

$$\left\{\frac{2\left(s^{2}-2\right)}{(s+1)\,(s-2)\,(s-3)}\right\}$$

(d) Apply convolution theorem to Evaluate

$$L^{-1}\frac{s^{2}}{\left(s^{2}+a^{2}\right)\left(s^{2}+b^{2}\right)}$$

(e) Evaluate $\int_0^\infty te^{-3t}\sin t\,dt$.

(f) Solve

$xd^2y/dx^2+dy/dx+xy=0$, $y(0)=2$, $y'(0)=0$

Q3. Attempt any two (10*2)

(a) State and prove the D'alembert's ratio test

(b) Disscuss the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{(\log n)^n}$

(c) Examine the geometric series

$1+x+x^2+x^3+\ldots\ldots.x^{n-1}+\ldots\ldots\infty$ For uniform convergence, Q4. Attempt <u>any two</u> (10*2)

(a) Expand $f(x) = x\sin x, 0<x<2\pi$ as fourier series.

(b) Develop $f(x)= (\sin \pi x/l)$ in half range cosine series in the range $0<x<l$. Graph the corresponding periodic continuation of $f(x)$.

(c) Solve the partial differential equation

$$r-t = \tan^3 x.\tan y - \tan x.\tan^3 y$$

Q5. Attempt <u>any two</u> (10*2)

(a) Solve the equation using separation of variables $py^3 + qx^2 = 0$

(b) Solve the equation $\partial u/\partial t = \partial^2 u/\partial x^2$ with boundary condition $u(x,0)=3 \sin(n\pi x)$, $u(0,t)=0$ $U(-,t)=0$, where $0<x<-$ $t>0$

(c) Solve the Laplace equation $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0$ subject to condition $u(0,y)=u(l,y)=u(x,0)=0$ and $u(x,a) = \sin(n\pi x/l)$.

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PAPER ID : 9917

TMA-201 Printed Pages : 4

Paper ID and Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B. Tech.

(SEM. II) (EVEN SEM.) EXAMINATION, 2013

MATHEMATICS - II

Time : **3** Hours] [Total Marks : **100**

Note : Attempt all questions, the marks assigned to each question is indicated at question itself.

1 Attempt any four : 5×4

(a) Solve $(1+e^{\frac{x}{y}})\,dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$

(b) Is the differential equation $xy\,dx + \left(2x^2 + 3y^2 - 20\right)dy = 0$ exact ? If not find a suitable integrating factor and solve it.

(c) Solve $4\frac{d^2y}{dx^2} + 36y = \operatorname{cosec} 3x$ by method of variation of parameters.

(d) Find the solution of $\left(D^2 + 4\right)y = \sinh\left(2x\right) + \pi$.

(e) Solve $\frac{dx}{dt} + y = \sin t$; $\frac{dy}{dt} + x = \cos t$.

(f) Solve the differential equation

$$x^2y'' - xy' + y = \log\left(x\right) + \pi$$

2 Attempt any four : 5×4

(a) Find the Laplace transform of

$$f(t)=\frac{\cos 2t-\cos 3t}{t}.$$

(b) Evaluate $L^{-1}\left\{\frac{3s^2+16s+26}{s\left(s^2+4s+13\right)}\right\}$.

(c) Find the Laplace transform of

$$f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases} \text{ with } f(t+2a)=f(t).$$

(d) Sate the convolution theorem of Laplace transform and using convolution theorem

evaluate $\int_0^{\infty}\frac{e^{-t}-e^{-3t}}{t}dt$.

(e) Solve, by Laplace transform,

$y''+y=\sqrt{2}\sin\sqrt{2}\,t$, $y(0)=10$, $y'(0)=0$.

(f) Solve $\frac{dx}{dt}-y=e^t$; $\frac{dy}{dt}+x=\sin t$ given that

$x(0)=1$, $y(0)=0$ by Laplace transform.

3 Attempt any two : 10×2

(a) Test the convergence of the series

$$1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+\ldots$$

(b) Test the convergence and absolute convergence of the series

$$\sum_1^{\infty}(-1)^{n+1}\left[\sqrt{n^2+1}-n\right].$$

(c) (1) State Weierstress's M –test for uniform convergence. Using M-test check the uniform convergence of the series

$$\frac{1}{(1+x)^3}+\frac{2}{(2+x)^3}+\frac{3}{(3+x)^3}+\ldots, x\geq 0$$

(2) Prove that the sequence $\{x_n\}$ defined by $x_1=\sqrt{2}\,;\, x_{n+1}=\sqrt{2+x_n}$ converges to the positive root of the equation $x^2-x-2=0$.

4 Attempt any two : **10×2**

(a) Find the Fourier series of periodicity 2π for $f(x)=\begin{cases} x, & (0,\pi) \\ 2\pi-x, & (\pi,2\pi) \end{cases}$ and hence deduce $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$

(b) (1) Find the half range sine series for $f(x)=\left(x-x^2\right)$ in $0<x<\pi$ **7**

(2) Find the differential equation of all spheres whose centers lie on the z-axis. **3**

(c) (1) Expand $f(x)=x\sin x$ as a cosine series in $0<x<\pi$ and show that **7**

$$1+\frac{2}{1.3}-\frac{2}{3.5}+\frac{2}{5.7}+\ldots=\frac{\pi}{2}$$

(2) Classify the partial differential equation **3**

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} + z = x^2 + y^2 .$$

5 Attempt any two : **10×2**

(a) A taut string of length 2l is fastened at both ends. The mid point of the string is taken to a height b and then released from the rest in that position. Find the displacement of the string.

(b) Solve $\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2}$ for $t > 0$ and $0 < x < 1$,

θ begin the temperature. The initial and boundary conditions are $\theta(0,t) = 0, t > 0$;

$\frac{\partial \theta(x,t)}{\partial t} = 0$ at $x = 1, t > 0$;

$\theta(x, 0) = x$ for $0 < x < 1$.

(c) A rectangular plate with insulated surfaces is l cm. wide and so long compared to its width that it may be considered infinite in the length without introducing an appreciable error. If the temperature along one short edge $y = 0$ is given by $u(x, 0) = 100 \sin \frac{\pi x}{l}, 0 < x < l$, while the two long edges $x = 0$ and $x = l$ as well as the other short edges, are kept at zero temperature. Show that the steady state temperature $u(x, y) = 100 \sin \frac{\pi x}{l} e^{-\frac{\pi x}{l}}$.

Paper Code & Roll No. to be filled in your Answer Book

Roll No. :

Even Semester Examination-2017

B.Tech. (Semester - II)

MATHEMATICS-II

(TMA-201)

Time : 3 Hours **Maximum Marks: 100**

Note: Attempt all questions, the marks assigned to each question is indicated at question itself.

1. Attempt *any four* questions : [4x5=20]

(i) Solve : $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

(ii) Solve : $(D^2 + 2D + 1)y = x\cos x$

(iii) Solve : $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} + 9y = \log(3x+2)\sin\{\log(3x+2)\}$.

(iv) Solve the following simultaneous differential equation

$$\frac{dx}{dt} = 2y, \qquad \frac{dy}{dt} = 2z, \qquad \frac{dz}{dt} = 2x$$

(v) Solve by method of variation of parameters

$$x^2y'' + xy' - y = x^2e^x$$

(vi) Solve$(x + 1)y'' - 2(x + 3)y' + (x + 5)y = e^x$

2. Attempt ***any four*** questions : [4 x 5 = 20]

(i) Find the Laplace transform of $\frac{1-\cos t}{t^2}$,

(ii) Evaluate the following integral

$$\int_0^\infty \frac{e^{-\sqrt{2}\,t}\sinh t \sin t}{t} dt$$

(iii) Find the Laplace transform of the following periodic function with period 2c.

$$f(t) = \begin{cases} t, & 0 < t < c \\ 2c - t, & c < t < 2c \end{cases}$$

(iv) Find the Inverse Laplace transform of

$$f(s) = \frac{s + 2}{(s^2 + 4s + 8)^2}$$

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(v) Find the Inverse Laplace transform of the following using convolution theorem $\frac{1}{(s+4)\sqrt{s}}$,

(vi) Solve the following differential equations using Laplace transform

$$y'' + 9y = \cos 2t, \quad y(0) = 1, \; y\left(\frac{\pi}{2}\right) = -1$$

3. Attempt *any two* questions : [2x10= 20]

(i) Find Fourier series of $f(x) = x - x^2$ in $(-\pi, \pi)$.

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \infty$

(ii) Find the half range cosine series of the function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 2(2-x), & 1 < x < 2 \end{cases}$$

(iii) Solve the following differential equation

$$(D - 3D' - 2)^2 z = 2e^{2x} \sin(y + 3x)$$

4. Attempt *any two* questions : [2x10 = 20]

(i) (a) Test the convergence of the series whose nth term is given by $\frac{\sqrt{n}}{n^2+1}$

(b) Test the convergence of the series

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\cdots\infty$$

(ii) Test the following series for convergence

$$\frac{1^2}{2^2}+\frac{1^2.3^2}{2^2.4^2}x+\frac{1^2.3^2.5^2}{2^2.4^2.6^2}x^2+\cdots\infty$$

(iii) Test the convergence of following series

$$1+\frac{1}{2}\frac{x^2}{4}+\frac{1.3.5}{2.4.6}\frac{x^4}{8}+\frac{1.3.5.7.9}{2.4.6.8.10}\frac{x^6}{12}+\cdots\infty$$

5. Attempt ***any two*** questions : [2x10=20]

(i) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state condition prevails. If and B is reduced to 0°C and maintained at that temperature, find the temperature of the rod at a distance x from A after time t.

(ii) A string of length l has its ends fastened at x = 0, and x = l. The midpoint of the string is then taken to height h and then released from rest from that position. Find the displacement.

(iii) A rectangular plate with insulated surfaces is 10cm wide and infinitely long. If the temperature along one short edge $y = 0$ is given by $100 \sin\left(\frac{\pi x}{10}\right)$, while all the other edges are at 0°C. Find the steady state temperature at any point of the plate.

TMA-201 Roll No. ....

Even Semester Examination Back Paper 2019-20

B.Tech, Semester - II

Mathematics-II

Time: 03:00 MM. 100

Note: i) Attempt all questions

ii) The choice of questions is internal as indicated

Q1. Attempt any four of the following: **[4x5=20]**

(a) $y^2 y dx - (x^3 + y^3) dy = 0$

(b) Solve $\frac{dx}{dt} + y = \sin t \quad ; \quad \frac{dy}{dt} + x = \cos t$

(c) Apply the method of variation of parameter to solve

$$\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x$$

(d) Solve $[D^2 - 2D + 1]y = x\sin x$

(e) Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$

(f) Find the values of λ for which the differential equation is exact. Also find its solution

$$(xy^2 + \lambda x^3 y^2)dx + (x^3 y + yx)xdy = 0$$

Q.2 Attempt any two of the following: (10X2=20)

(a) Find the Laplace Transform of the function

(i) $F(t) = te^{-t}\sin 2t$

(ii) $F(t) = \frac{1 - \cos t}{t^2}, \quad t > 0$

(b) Find the inverse Laplace Transform of the function

(i) $\frac{p^2}{(p^2 + a^2)(p^2 + b^2)}$

(ii) $\frac{1}{p(p + a)^3}$

(c) Solve the following simultaneous differential equation using Laplace transformation:

$$\frac{dx}{dt} + y = \sin t, \quad \frac{dy}{dt} + x = \cos t$$

Given that x=2, y=0 at t=0.

Q.3 Attempt any two of the following: (10X2=20)

(a) Find the Fourier series of the function

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = \frac{\pi^2}{8}$

(b) Solve $(D-D'-1)(D-D'-2)z=\sin(2x+3y)$

(c) Find the Half Range Sine Series for $f(x) = (x-x^2)$ in $0 < x < \pi$

Q.4 Attempt any two of the following: (10X2=20)

(a) Discuss the convergence of the following series

(i) $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \ldots\ldots\ldots$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(b) Define the following:

(i) Comparison Test

(ii) P-Series Test

(iii) Cauchy's Integral Test

(iv) Raabe's Test

(c) Discuss the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots\ldots\ldots$$

Q.5 Attempt any two of the following: (10X2=20)

(a) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \ where u(x,t) = 6e^{-3x} \ when t = 0.$$

(b) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance x from one end at any time t.

(c) Find the temperature in a bar of length 2 whose ends are kept at zero and internal surface insulated if the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$.

BAST-105

Roll No.

Even Semester Examination, 2021-22

Course Name : B.TECH

Branch : CSE/ECE/EEE/ME/CIVIL

Semester: II

Subject : Mathematics-II

Time: 3 Hours **Max Marks: 100**

Number of Printed pages: 2

Note: - Attempt all questions: All Questions carry equal marks

Q 1. Attempt any four parts of the following (5 x 4 = 20)

(a) Solve $x\log x\frac{dy}{dx} + y = 2\log x$.

(b) Find the value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x + y)x^2 dy = 0$ is exact. Solve the equation for this value of λ.

(c) Solve the differential equation $(D^2 - 5D + 6)y = e^x$.

(d) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x\log x$.

(e) Determine the solution of the differential equation $p - \frac{1}{p} - \frac{x}{y} + \frac{y}{x} = 0$.

Q 2. Attempt any four parts of the following (5 x 4 = 20)

(a) **Form the partial differential equation from** $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

(b) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$.

(c) Solve the partial differential equation $p^2 + q^2 = 2$.

(d) **Form the partial differential equation from** $z = f(x^2 - y^2)$.

(e) Determine the solution of $p^2 + q^2 = x + y$.

Q 3. Attempt any two parts of the following (10 x 2 = 20)

(a) State and prove the orthogonal properties of Legendres polynomials.

(b) Apply the method of variation of parameter to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

(c) Apply the method of changing the independent variable to solve $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2 y = x^4$.

Q4. Attempt any two parts of the following: (10 x 2 = 20)

(a) (i) Test the convergence of the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots \ldots\ldots\ldots, \infty$.

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$.

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(b) Find the Fourier series expansion for the function $f(x) = x\sin x\ , -\pi < x < \pi$.

(c) If $f(x) = \begin{cases} x & 0 \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}$, then show that

$(i)\ f(x) = \frac{4}{\pi}[Sinx - \frac{1}{3^2}Sin3x + \frac{1}{5^2}Sin5x - \cdots \ldots\ldots\ldots\ldots]$

$(ii)\ f(x) = \frac{\pi}{4} - \frac{2}{\pi}[Cos2x + \frac{1}{3^2}Cos6x + \frac{1}{5^2}Cos10x - \cdots \ldots\ldots\ldots\ldots]$

Q5. Attempt any two parts of the following: (10 x 2 = 20)

(a) (i) Determine p such that the function $f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{px}{y}$ be an analytic function.

(ii) Prove that the function $u = e^x(x\cos y - y\sin y)$ satisfies Laplace equation.

(b) (i) *Differentiate between Cauchy Goursat Theorem and Cauchy Integral Formula.*

(ii) Evaluate $\int_C \frac{3z^2+z}{z^2-1}dz$, **where C is the circle** $|z| = 2$.

(c) Using contour integration method to evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta}d\theta$.

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TMA-201

Roll No. | | | | | | | | | | |

Even Semester Back Paper Examination, 2021-22

Course Name : B.TECH

Semester: II

Branch : .....

Subject : Mathematics-II

Time: 3 Hours **Max Marks: 100**

Number of Printed pages: 2

Note: - Attempt all questions: All Questions carry equal marks

Q 1. Attempt any four parts of the following (5x4=20)

(a) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$

(b) Solve the differential equation $(D^2 + 4)y = \sin 3x + \cos 2x$; $D = \frac{d}{dx}$

(c) Solve the following homogenous differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$$

(d) Solve the equation $(y - xy^2)dx - (x + x^2y)dy = 0$

(e) Solve the Simultaneous differential equations:

$$\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t \ ; \quad \frac{dx}{dt} + y - x = \cos t$$

(f) Solve: $(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2)dy = 0$

Q 2. Attempt any four parts of the following (5x4=20)

(a) Find the Laplace transform of $\sin 2t \cos 3t$.

(b) State Convolution theorem and hence evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$

(c) Apply Laplace transform to solve the equation

$\frac{d^2y}{dt^2} + y = \sin 3t, \quad t > 0$; given that $y = \frac{dy}{dt} = 0$ at t=0

(d) Find the Laplace transform of $t e^{-t} \sin 2t$.

(e) Evaluate $L^{-1}\left\{\log \frac{s+1}{s-1}\right\}$

Q 3. Attempt any two parts of the following (10x2=20)

(a) Test the convergence and divergence of the following series $\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{5 + n^5}$

(b) Using Cauchy Root test, examine the convergence of the series $\sum \left(\frac{n}{n+1}\right)^{n^2}$

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(c) Test convergence of the series $1-\frac{1}{3}+\frac{1}{2}-\frac{1}{3^2}+\frac{1}{2^2}-\frac{1}{3^5}+$........................

Q4. Attempt any two parts of the following: (10x2=20)

(a) Expand $f(x)=x^2$, $-\pi \le x \le \pi$ as a Fourier series.

(b) Expand $f(x)=\pi x - x^2$ in a half range sine series in the interval $(0,\pi)$ upto the first three terms.

(c) Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x \partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y$

Q5. Attempt any two parts of the following: (10x2=20)

(a) Use the method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+4, \quad \text{where } u(x,0)=6e^{-3x}$$

(b) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary conditions

$$u(x,0)=3\sin n\pi x,\ u(0,t)=0,\ u(l,t)=0 \ \ where\ 0<x<l$$

(c) If a string of length l is initially at rest in equilibrium position and each of its points given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0}=b\sin^3\frac{\pi x}{l}$, then find the displacement $y\ (x,t)$.

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BSCT-201

Roll No. | | | | | | | | | | |

Even Semester Back Paper Examination, 2021-22

Course Name : B.TECH

Semester: II

Branch : Common to All Branches

Subject : Mathematics-II

Time: 3 Hours **Max Marks: 100**

Number of Printed pages: 2

Note: - Attempt all questions: All Questions carry equal marks

Q 1. Attempt any four parts of the following: (5 x 4 =20)

(a) Solve the differential equation $cos^3x\frac{dy}{dx} + y\,cosx = sinx$.

(b) Solve $(e^y + 1)cosx\,dx + e^y sinx\,dy = 0$.

(c) Solve $p^2 + 2pycotx - y^2 = 0$.

(d) Solve $\frac{dy}{dx} + \frac{y}{x}logy = \frac{y}{x^2}(logy)^2$.

(e) Solve $y = 2px + yp^2$.

Q 2. Attempt any four parts of the following: (5 x 4 = 20)

(a) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2Sin(logx)$.

(b) Using the method of variation of parameter, solve $\frac{d^2y}{dx^2} + y = secx$.

(c) Apply the method of changing the independent variable to

solve $\frac{d^2y}{dx^2} + Cotx\frac{dy}{dx} + 4yCosec^2x = 0$.

(d) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre's polynomials.

(e) Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$.

Q 3. Attempt any two parts of the following: (10 x 2 = 20)

(a) *Change the order of integration and evaluate* $\int_0^1\int_{x^2}^{2-x} xydx\,dy$.

(b) Evaluate $\iint_S (yz\,\hat{\imath} + zx\hat{\jmath} + xy\hat{k}).\hat{n}\,ds$ *where S is the surface of the sphere* $x^2 + y^2 + z^2 = a^2$ *in the first octant.*

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(c) $\textit{Use Stoke'stheorem to evaluate} \int_C \left[(x+2y)dx + (x-z)dy + (y-z)dz\right.$,

where C is

$\textit{the boundary of the triangle with vertices } (2,0,0), (0,3,0) \textit{ and } (0,0,6)$

oriented in the anti -clockwise direction.

Q4. Attempt any two parts of the following: (10 x 2 = 20)

(a) $\textit{Determine p such that the function } f(z) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{px}{y} \textit{ be an analytic function.}$

(b) Determine the analytic function $f(z) = u + iv$ such that $u - v = e^x(\cos y - \sin y)$.

(c) State and prove cauchy – Riemann equations for cartesian coordinate system.

Q5. Attempt any two parts of the following: (10 x 2 = 20)

(a) (i) **Evaluate** $\int_C \frac{dz}{z(z+i\pi)}$, **where** C **is** $|z+3i| = 1$.

(ii) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ **along the path** $y = x^2$.

(b) Expand the function $f(z) = \frac{1}{z(z-1)(z-2)}$ *for* $|z-1| < 1$ *in a Laurent's series.*

(c) Using Cauchy Residue theorem, evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ **where** C **is** $|z| = 3$.

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