

SECOND SEMESTER EXAMINATION, 2022 – 23
IIInd Year, B.Tech. – CSE/ECE/ME/CE/EE/IT
Mathematics-II

Duration: 3:00 hrs

Max Marks: 100

Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

<p>Answer any four parts of the following.</p> <p>a) Find the value of λ for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x + y)x^2 dy = 0$ is exact.</p> <p>b) Solve $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \sin x$.</p> <p>c) Solve by the method of variations of parameters $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$.</p> <p>d) Solve $x^2 p^2 + xyp - 6y^2 = 0$</p> <p>e) Solve $x dx + y dy = \frac{a^2(x dy - y dx)}{x^2 + y^2}$</p> <p>f) Solve $\frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$.</p>	<p>5x4=20</p>
<p>Answer any four parts of the following.</p> <p>a) Solve by the method of variation of parameter $\frac{d^2 y}{dx^2} + y = \sec x$</p> <p>b) Express the algebraic polynomial $6x^3 + 15x^2 - x + 9$ as Legendre's polynomials.</p> <p>c) Prove that $x J'_n(x) = n J_n(x) - x J_{(n+1)}(x)$ for Bessel's functions.</p> <p>d) Solve $\frac{d^2 y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$ by changing the independent variable.</p> <p>e) Solve $(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$ in series.</p> <p>f) Solve $(x^2 y^2 + xy + 1)y dx + (x^2 y^2 - xy + 1)x dy = 0$.</p>	<p>5x4=20</p>
<p>Answer any two parts of the following.</p> <p>(a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$ by changing to polar coordinates. Hence show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.</p> <p>(b) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy dy dx$ by changing the order of integration.</p>	<p>10x2= 20</p>

(c) Evaluate $\iint_S \vec{F} \cdot \hat{n} dS$, where $\vec{F} = xy\hat{i} - x^2\hat{j} + (x+z)\hat{k}$ and S is the region of the plane $2x + 2y + z = 6$ in the first octant.

Answer any two parts of the following.

10x2= 20

(a) State and prove Cauchy Riemann equations in Cartesian coordinate system.

(b) Prove that the functions $u(x, y) = (x^2 - y^2)$ and $v = \frac{y}{x^2 + y^2}$ are harmonic function but not harmonic conjugate.

(c) Find the value of C_1 and C_2 such that the function $f(z) = x^2 + C_1y^2 - 2xy + i(C_2x^2 - y^2 + 2xy)$ is analytic. Also, find $f'(z)$.

Answer any two parts of the following.

10x2= 20

(a) (i) Using Cauchy integral formula to calculate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$

(ii) Define Poles and Residues.

(b) Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$ using Residue theorem.

(c) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series for (i) $1 < |z| < 3$ (ii) $|z| > 3$
