Name → Mohd Nasir   Student Id → 200??5?8
Section → F   University Roll → 2016855
Roll No → 14

TUTORIAL-02

DESIGN AND ANALYSIS OF ALGORITHM

Ques 1.
```
void fun(int n){
  int i=0, j=1;
  while(i<n){
      i=i+j;
      j++;
  }
}
```

$j \to 0, 1, 3, 6, 10, \ldots\ldots, k$

$i \to 0, 1, 3, 6, \ldots, k$

$0 \quad 1 \quad 1+2 \quad 1+2+3 \quad 1+2+3+4+\cdots+k$

$(1+2+3+\cdots+k) = n$ (terminate the program)

$\frac{k(k+1)}{2} = n$

$k^2+k = 2n$

$k^2 = n$

$k = \sqrt{n}$

$T(n) = O(\sqrt{n})$

Ques 2. $T(n) = T(n-1) + T(n-2) + 1$ if$(n>0)$ otherwise 1.

$T(n-1) \simeq T(n-2)$

$T(n) = T(n-1) + T(n-1) + 1$

$T(n) = 2T(n-1) + 1 \quad —— (1)$

$T(n-1) = 2T(n-2) + 1$

From equ (1) :→

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$$T(n) = 2[2T(n-2)+1]+1$$
$$= 2^2T(n-2)+2 \quad \text{—— (2)}$$
$$T(n-2) = 2T(n-3)+1$$

from equ (2):

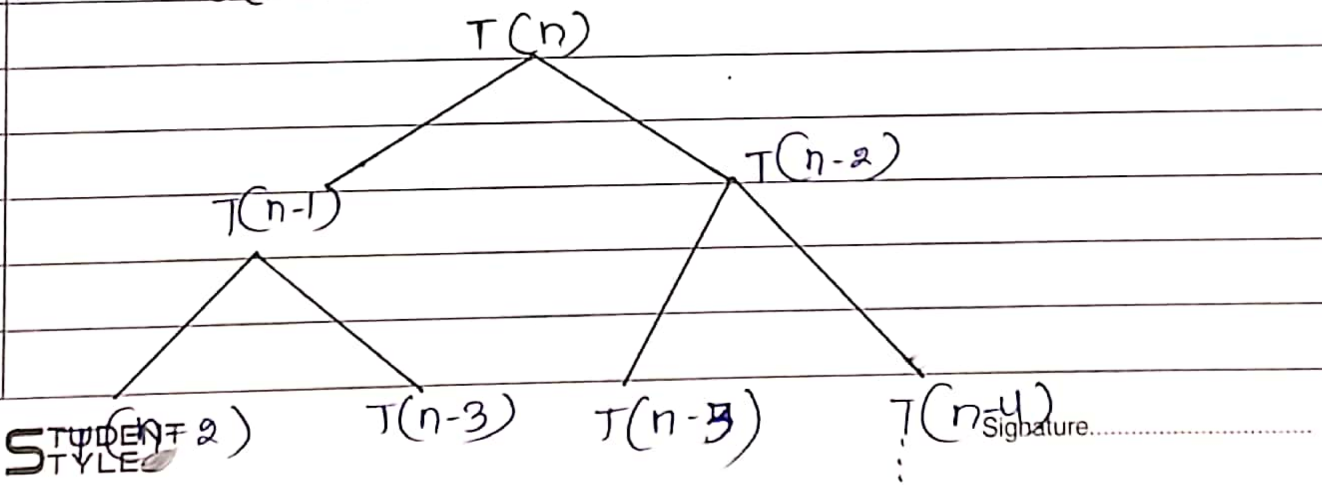
$$T(n) = 2^3T(n-3)+3$$
$$T(n) = 2^KT(n-K)+K \quad \text{—— (3)}$$
$$n-K=0$$
$$n=K$$

From equ (3):

$$T(n) = 2^nT(n-n)+n$$
$$= 2^nT(0)+n$$
$$T(n) = 2^n \times 1+n$$
$$= O(2^n)+O(n)$$

$$\boxed{T(n) = O(2^n)}$$

Space Complexity → If we draw the recursion tree of the fibonacci recursion then we found the maximum height of tree will be n and hence the space complexity of fibonacci recursion will be O(n).



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Ans→3 O(n logn), O(n^3), O(log(logn))

(i) n logn.

```
void fun (int a) {
  int i, j; sum=0; → if(n==0)
                       return
  for (i=0; i<a; i=(i*2)
        sum = sum + i;
      fun (n-1);
  }
```

$T(n) = (n-1) + n$

(ii)

```
void fun (int n) {
  int i, j; sum=0;   if (n==0)
                       return;
  for (i=0; i<n; i++)
    for (j=0; j<n; j++)
        sum = i+j;
    fun (n-1)
  }.
```

$T(n) = (n-1) + n^2$ if $n>0$ otherwise 1.

(iii) log(logn).

```
for (int i=2; i<=n; i=pow(i,k))
  {
      print ("i : , i);
  }.
```

$K \geq 2$

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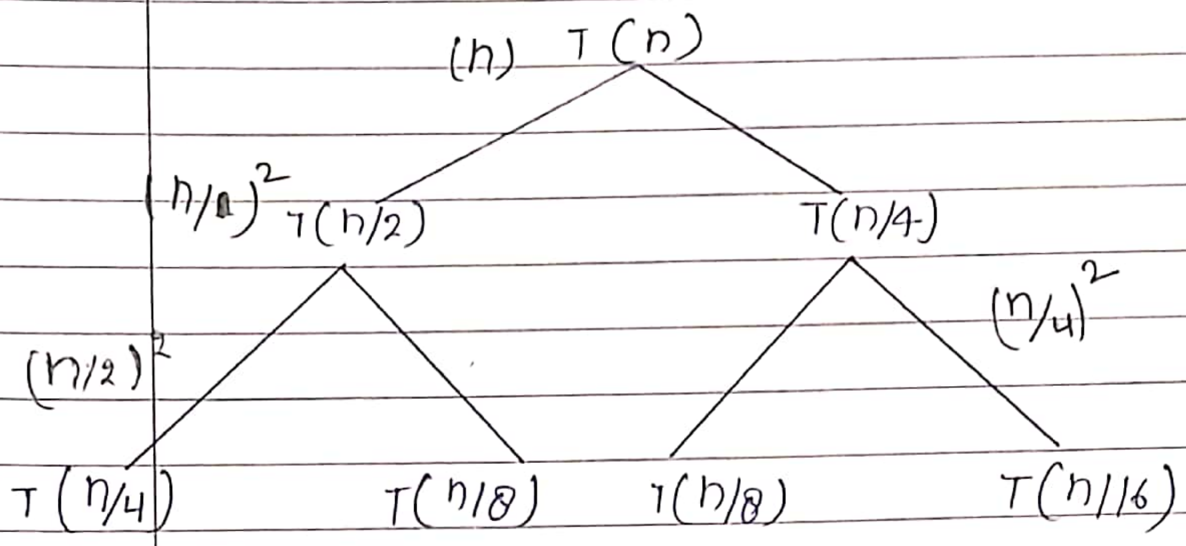
Ques→4 $T(n) = T(n/2) + T\left(\frac{n}{4}\right) + n^2$



$$T(n) = n^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 + \cdots + \cdots +$$

$$= n^2\left(1 + \frac{1}{2^2} + \frac{1}{2^2} + \cdots\right) \rightarrow 1$$

$$T(n) = n^2 \times 1$$

$$\boxed{T(n) = O(n^2)}$$

Or

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2$$

$$\alpha = \frac{1}{2},\ \beta = \frac{1}{4},\ f(n) = n^2$$

$$\alpha + \beta = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} < 1$$

$$T(n) = f(n) = \underline{O(n^2)}$$

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Ans → 5

```
int fun(int n) {
    for (int i=1; i<=n; i++) {
        for (int j=1; j<n; j+=i)   → j = i+j;
            Some O(1) task
}.
```

$i \to 01 \quad j \to 1, 2, 3, 4, 5, \dots, n$

$i \to 2 \quad j \to 1, 3, 5, 7, \dots, n/2$

$i \to 3 \quad j \to 1, 4, 7, 11, \dots, n/3$

$\vdots$

$i \to n \quad j \to 1 \dots, n/n$

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + \frac{n}{n}$$

$$= n\Big(1 + \underbrace{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}}_{\log n}\Big)$$

$$T(n) = n\log n$$

$$\boxed{T(n) = O(n\log n)}$$

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Ques→6. for (int i = 2 ; i <= n ; pow(i, k))
Some O(1) expression.

$i = 2, 2^k, (2^k)^k, (2^{k^2})^k, \ldots (2^{k^3})^k \ldots\ldots, 2^{k^k}$

$2^{k^2} \quad 2^{k^3} \quad 2^{k^4} \quad 2^{k^k}$

$2^{k^k} = n$

$\log_2 2^{k^k} = \log_2 n$

$k^k \log_2 2 = \log n$

$k^k = \log n$

$\log k^k = \log(\log n)$

$k \log_2 k = \log(\log n).$

$k = O(\log \log n).$

$\boxed{T(n) = O[\log(\log n)]}$

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Ques. 8 Arrange the following in increasing order of rate of growth.

(a) $n, n!, \log n, \log\log n, root(n), \log(n!), n\log n, \log^2 n$
$2^n, 2^{2^n}, 4^n, n^2, 100$

Ans $100 < \log\log n < \log n < \log^2 n < root(n) < n < n\log n < \log(n!)$
$< n^2 < 2^n < 4^n < 2^{2^n} < n!$.

(b). $2(2^n), 4n, 2n, 1, \log(n), \log\log(n), \sqrt{\log n}, \log 2n$.
$2\log n, n, \log(n!), n!, n^2, n\log(n)$.

Ans. $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < 2n < 4n$
$< n\log n < \log n! < n^2 < 2^{2^n} < n!$.

(c). $8^{2n}, \log_2(n), n\log_6(n), n\log_2(n), \log(n!), n!, \log_8(n)$
$96, 8n^2, 7n^3, 5n$.

Ans $96 < \log_8 n < \log_2 n < 5n < n\log_6 n < n\log_2 n < \log(n!) < 8n^2 <$
$7n^3 < n! < 8^{2n} < n!$.

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