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TUTORIAL-02

DESIGN AND ANALYSIS OF ALGORITHM

Ques 1. Void fun(int n){

```
int i=0, j=1;
while(i<n){
    i=i+j;
    j++;
}.
```

$j \to 0, 1, 3, 6, 10, \ldots\ldots, k$

$i \to 0, \quad 1, \quad 3, \quad 6, \quad \ldots, \quad k$

$0 \quad 1 \quad 1+2 \quad 1+2+3 \quad 1+2+3+4+\cdots+k$

$(1+2+3+\cdots+k) = n$ (terminate the program)

$\frac{k(k+1)}{2} = n$

$k^2+k = 2n$

$k^2 = n$

$k = \sqrt{n}$

$T(n) = O(\sqrt{n})$

Ques 2. $T(n) = T(n-1) + T(n-2) + 1$ if $(n>0)$ otherwise 1.

$T(n-1) \simeq T(n-2)$

$T(n) = T(n-1) + T(n-1) + 1$

$T(n) = 2T(n-1) + 1$ —— ①

$T(n-1) = 2T(n-2) + 1$

From equ ① :→

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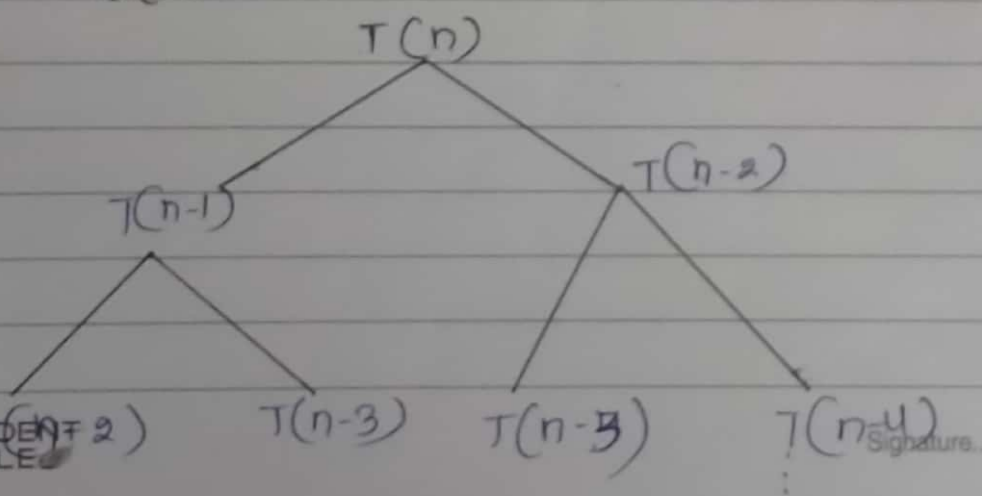
$$T(n) = 2[2T(n-2)+1]+1$$
$$= 2^2 T(n-2) + 2 \quad \text{--- } (2)$$
$$T(n-2) = 2T(n-3)+1$$
from equ (2):
$$T(n) = 2^3 T(n-3) + 3$$
$$T(n) = 2^K T(n-K) + K \quad \text{--- } (3)$$
$$n-K = 0$$
$$n = K$$
from equ (3):
$$T(n) = 2^n T(n-n) + n$$
$$= 2^n T(0) + n$$
$$T(n) = 2^n \times 1 + n$$
$$= O(2^n) + O(n)$$
$$\boxed{T(n) = O(2^n)}$$

Space Complexity → If we draw the recursion tree of the fibonacci recursion then we found the maximum height of tree will be n and hence the space complexity of fibonacci recursion will be O(n).



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**Ques→3** $O(n \log n)$, $O(n^3)$, $O(\log(\log n))$

(i) $n \log n$.

```
void fun (int a) {
    int i, j, sum=0; -> if(n==0)
                         return
    for (i=0; i<a; i= i*2)
            sum = sum + i;
        fun (n-1);
    }
```

$T(n) = (n-1) + n$

(ii)

```
void fun (int n) {
    int i, j; sum=0;   if (n==0)
                          return;
    for (i=0 ; i<n ; i++)
        for (j=0; j<n; j++)
            sum = i+j;
        fun(n-1)
}.
```

$T(n) = (n-1) + n^2$ if $n>0$ otherwise 1.

(iii) $\log(\log n)$.

```
for (int i=2 ; i<=n; i= pow(i, k))
    {
        print("i", i);
    }
```

$\underline{K} \geq 2$

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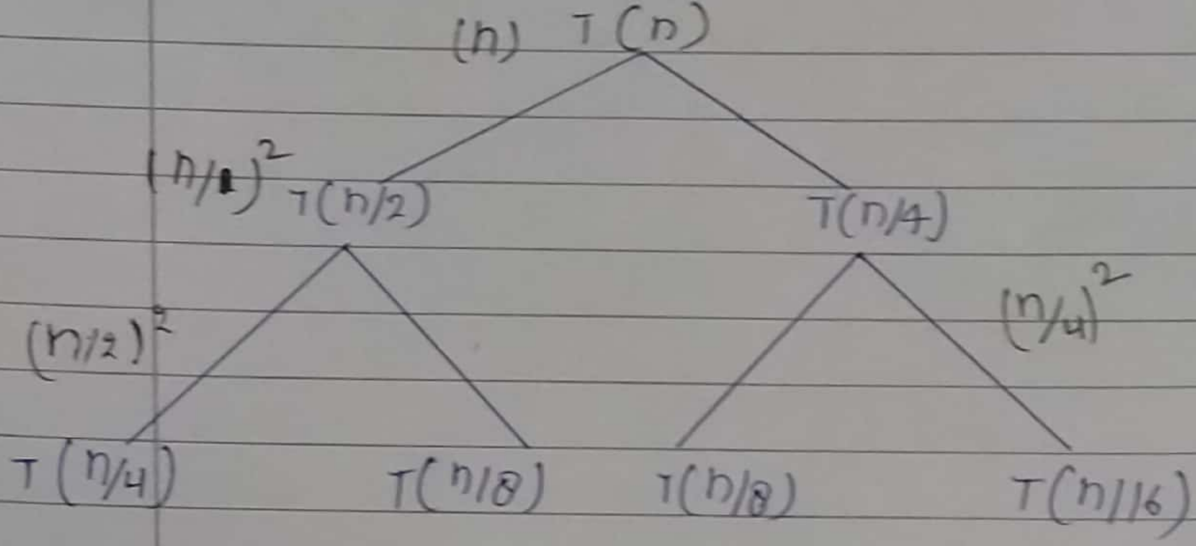
Ques 4 $T(n) = T(n/2) + T\left(\frac{n}{4}\right) + n^2$




$$T(n) = n^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 + \cdots \cdots + \cdots + $$

$$= n^2\left(1 + \frac{1}{2^2} + \frac{1}{2^2} + \cdots\cdots\right) \rightarrow 1$$

$$T(n) = n^2 \times 1$$

$$\boxed{T(n) = O(n^2)}$$

Or

$$F(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2$$

$$\alpha = \frac{1}{2},\ B = \frac{1}{4},\ f(n) = n^2$$

$$\alpha + B = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} < 1$$

$$T(n) = f(n) = \underline{O(n^2)}$$



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Ans→5 int fun(int n) {

for (int i=1 ; i<= n; i++) {

for (int j = 1; j< n; ~~j+=i~~ → j = i+j ;

Some O(1) task

}.

i→01 j→ 1, 2, 3, 4, 5, - - - - , n

i→2 j→ 1, 3, 5, 7, - - - - - , n/2

i→3 j→ 1, 4, 7, 11, - - - - - , n/3

⋮

i→n j→ 1 - - - - - , n/n

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cdots + \frac{n}{n}$$

$$= n\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}\right)$$

→ logn

$T(n) = n\log n$

$$\boxed{T(n) = O(n\log n)}$$

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Ques→6. for (int i = 2 ; i <= n ; pow(i, k))
Some O(1) Expression.

$i = 2,\ 2^k,\ (2^k)^k,\ (2^{k^2})^k,\ \dots\ (2^{k^3})^k \dots\dots,\ 2^{k^k}$

$\phantom{i = 2,\ 2^k,\ } 2^{k^2} \quad 2^{k^3} \quad\quad 2^{k^4} \quad\quad\quad 2^{k^k}$

$2^{k^k} = n$

$\log_2 2^{k^k} = \log_2 n$

$k^k \log_2 2 = \log n$

$k^k = \log n$

$\log k^k = \log(\log n)$

$k \log_2 k = \log(\log n).$

$k = O(\log \log n).$

$$\boxed{T(n) = O[\log(\log n)]}$$

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Qus. 8 Arrange the following in increasing order of rate of growth.

(a) n, n!, logn, log logn, root(n), log(n!), nlogn, $\log^2 n$
$2^n$, $2^{2^n}$, $4^n$, $n^2$, 100

Ans 100 < log logn < logn < $\log^2 n$ < root(n) < n < nlogn < log(n!) < $n^2$ < $2^n$ < $4^n$ < $2^{2^n}$ < n!.

(b). $2(2^n)$, 4n, 2n, 1, log(n), log log(n), $\sqrt{\log n}$, log2n, 2logn, n, log(n!), n!, $n^2$, nlog(n).

Ans 1 < log logn < $\sqrt{\log n}$ < logn < log2n < 2logn < n < 2n < 4n < nlogn < log n! < $n^2$ < $2^{2^n}$ < n!.

(c). $8^{2n}$, $\log_2(n)$, $n\log_6(n)$, $n\log_2(n)$, log(n!), n!, $\log_8(n)$, 96, $8n^2$, $7n^3$, 5n.

Ans 96 < $\log_8 n$ < log2n < 5n < $n\log_6 n$ < $n\log_2 n$ < log(n!) < $8n^2$ < $7n^3$ < n! < $8^{2n}$ < n!.

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