Tutorial → 04.

(1) $T(n) = 3T(n/2) + n^2$

$a = 3,\ b = 2,\ f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ Case 3.

$\therefore$ according to master's theorem.

$T(n) = \Theta(n^2)$.

(2) $T(n) = 4T(n/2) + n^2$

$a = 4,\ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ Case 2.

$\therefore$ according Master's theorem.

$T(n) = \Theta(n^2 \log n)$

(3) $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ Case 3.

$T(n) = \Theta(2^n)$

(4) $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

Master theorem is not applicable.

(5) $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4,\ f(n) = n$

$n^{\log_b a} = n^{\log_4 16} \Rightarrow n^2,\ f(n) < n^2$

$T(n) = \Theta(n^2).$

(6) $T(n) = 2T(n/2) + n\log n.$

$a = 2,\ b = 2,\ f(n) = n\log n.$

$n^{\log_b a} = n^{\log_2 2} \Rightarrow n$

$f(n) > n$

$T(n) = \Theta(n\log n).$

(7) $T(n) = 2T(n/2) + \frac{n}{\log n}$

$a = 2,\ b = 2,\ K = 1,\ P = -1$

$a = b^K$, $\underline{\underline{P \neq -1}}$

$(2 = 2^1)$

$T(n) = \Theta(n^{\log_b a} \log\log n)$

$= \Theta(n\log\log n).$

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(8) $T(n) = 2T(n/4) + n^{0.51}$

$a=2,\ b=4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$T(n) = \Theta(n^{0.51})$

(9) $T(n) = 0.5T(n/2) + \frac{1}{n}$

$a<1$, not applicable Master theorem.

(10) $T(n) = 16T(n/4) + n!$

$a=16,\ b=4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$T(n) = \Theta(n!)$

(11) $T(n) = 4T(n/2) + \log n$

$a=4,\ b=2,\ f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$T(n) = \Theta(n^2)$

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(12) $T(n) = sqrt(n) + \frac{n}{2} + \log n$

Master's theorem not applicable

(13). $T(n) = 3T(n/2) + n$

$a = 3, b = 2, f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$T(n) = \Theta(n^{\log_2 3})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, b = 3, f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + Cn$

$a = 4, b = 2, f(n) = C * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > C * n$

$T(n) = \Theta(n^2)$

(19) $T(n) = 4T(n/2) + n\log n$

$a = 4,\ b = 2,\ f(n) = n\log n.$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n\log n$

$T(n) = \Theta(n^2)$

(20). $T(n) = 64T(n/8) - n^2\log n.$

Master's theorem is not applicable as $f(n)$ is not increasing function.

(21). $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$T(n) = \Theta(n^2)$,

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem is not applicable,

(16) $T(n) = 3T(n/4) + n\log n$

$a=3,\ b=4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n.$

$T(n) = \Theta(n\log n).$

(17). $T(n) = 3T(n/3) + (n/2)$

$a=3,\ b=3,\ f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$O(n) \cong O(n/2)$

$T(n) = \Theta(n\log n).$

(18). $T(n) = 6T(n/3) + n^2\log n$

$a=6,\ b=3,\ f(n) = n^2\log n.$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2\log n$

$T(n) = \Theta(n^2\log n).$