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DESIGN AND ANALYSIS OF ALGORITHM

TUTORIAL-01.

Ques-1 Asymptotic Notation :→ Asymptotic notations are the mathematical notations used to describe the complexity (running time) of an algorithm.

(i) Big-O :→ Big O notation specifically describes worst case scenario. It represents the tight upper bound running time complexity of an algorithm.

$$f(n) \leq C \cdot g(n) \quad \forall n \geq n_0 \quad C \geq 0$$

Ex : $O(1), O(n), O(n \log n)$.

for ($i=0; i < n; i++$)
 $\text{Sum} = \text{Sum} + i;$

The time complexity of above example is $O(n)$.

(ii) Omega (Ω) :- Omega Notation specifically describes best case scenario. It represents the tight lower bound running time of an algorithm.

$$f(n) \geq C \cdot g(n) \quad \forall n \geq n_0 \quad C \geq 0$$

Example :- $\Omega(1)$, $\Omega(\log n)$.

```
void search (int size){  
    int element, i;  
    if (arr.size == 1){  
        printf ("found");  
        return;  
    }  
    for (i=0; i<size; i++)  
    {  
        if (arr[i] == element){  
            printf ("found");  
            return;  
        }  
    }  
    printf ("Not found");  
}
```

The Best Complexity of above Example is : $\Omega(1)$.

(iii) Theta(Θ) :- This Notation describes both tight upper bound and tight lower bound of an algorithm. In real scenario the algorithm not always run on best and worst cases, the avg running lies between best and worst and can be represent by ' Θ ' Notation.

$$C_1 g(n) \leq f(n) \leq C_2 g(n), \quad \forall n \geq \max(n_1, n_2)$$

$C_1 > 0$ and $C_2 > 0$

Ques. 10:- For the function $n^k R$ and C^n , what is the asymptotic relationship b/w these function?

assume that $k \geq 1$ of $C > 1$ are constants find out the value of c . and no. of which relationship hold?

as given n^k and C^n

Relationship between n^k and C^n is.

$$n^k = O(C^n)$$

$$n^k \leq a(C^n)$$

$\forall n \geq n_0$ constant $a > 0$

for $n_0 = 1$: $C = 2$

$$1^k \leq a^1$$

$$\Rightarrow n_0 = 1 \text{ and } C = 2$$

Ques 2 for ($i=1$ to n)

$$\{ i = i \times 2 \}$$

$$i \rightarrow 1, 2, 2^2, 2^3, 2^4, \dots, 2^K.$$

$$2^K = n$$

take \log_2 both side.

$$\log_2 2^K = \log n$$

$$K \log_2 = \log n$$

$$K = O(\log n)$$

Ques 3 $T(n) = 3T(n-1)$ $n > 0$, otherwise 1.

$$T(n) = 3T(n-1) \quad \text{--- (1)}$$

$$T(n-1) = 3T(n-2)$$

From equ (1) \Rightarrow

$$T(n) = 3[3T(n-2)]$$

$$= 3^2 T(n-2) \quad \text{--- (2)}$$

$$T(n-2) = 3T(n-3)$$

From equ (2) \Rightarrow

$$T(n) = 3^2 [3T(n-3)]$$

$$= 3^3 T(n-3)$$

⋮

$$T(n) = 3^K T(n-K) \quad \text{--- (3)}$$

$$n - K = 0$$

$$n = K$$

From equ ③ \Rightarrow

$$T(n) = 3^k T(n-n)$$

$$= 3^k T(0)$$

$$T(0) = 1.$$

$$T(n) = 3^k$$

$$\boxed{T(n) = O(3^k)}.$$

Ques:- 8 $T(n) = 2T(n-1) - 1$

(1) $T(n)$

1

$1(2^0)$

(1) $T(n-1)$

(1) $T(n-1)$

$2(2^1)$

(1)

$T(n-2)$

(1) $T(n-2)$

(1) $T(n-2)$

(1) $T(n-2)$

$4(2^2)$

(1)

$T(n-3)$

$T(n-3)$

$T(n-3)$

$T(n-3)$

$T(n-3)$

$T(n-3)$

$T(n-3)$

$8(2^3)$

$T(n-K)$

$2^0, 2^1, 2^2, 2^3, \dots, 2^K$

(2^K)

$$T(n) = T(n-K) * 2^K$$

$$n-K=0$$

$$n=K$$

$$T(n) = T(0) * 2^K$$

$$\boxed{T(n) = O(2^n)}$$

Ques 5
`int i=1, s=1;
while (s <= n){
 i++;
 s = s + i;
 cout << "#";
}`

S (Value of S) :-

1, 3, 6, 10, ..., n
 (1+2) (1+2+3) (1+2+3+4) ... (1+2+3+...+K)

Termination Condition.

$$(1+2+3+\dots+K) = n$$

$$\frac{K(K+1)}{2} = n$$

$$K^2 + K = 2n$$

$$K^2 = n$$

$$K = \sqrt{n}$$

$$T(n) = O(\sqrt{n}).$$

Ques 6. void function (int n){

`int i, count=0;
for (i=1; i*i <= n; i++)
 count++;`

$$i \rightarrow 1, 2, 3, \dots, K^2$$

$K^2 = n$ (Terminate the program)
 $K = \sqrt{n}$ $K^2 > n$

**STUDENT
STYLE** $T(n) = O(\sqrt{n})$

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Ques 7 void fun (int n) {

int i, j, k, count = 0;

for (i = n/2; i <= n; i++) { loop 1.

 for (j = 1; j <= n; j = j * 2) { loop 2

 for (k = 1; k <= n; k = k * 2) { loop 3

 Count++;

}

}

loop 1 :-

$i \rightarrow \frac{n}{2}, \frac{n}{2} + 1, \frac{n}{2} + 2, \frac{n}{2} + 3, \dots, \frac{n}{2} + K$

$K > n$ (Terminate pro). $\hookrightarrow K = \frac{n}{2}$

$K = n$

$T(n) = O(n)$

For loops 1 and 2 we know time complexity.

$T(n) = \log n$.

Total time complexity.

$$T(n) = n * \log n + \log n$$

$$T(n) = O(n \log^2 n)$$

Ques 8 fun (int n) {

if (n == 0)
 return;

for (i=1 to n)
 for (j=1 to n)
 printf("#");

fun (n-3)

}

$$T(n) = (n-3) + O(n^2)$$

$$= T(n-3) + h^2 \quad \text{--- ①}$$

$$T(n-3) = T(n-6) + (n-3)^2$$

From equ ① :-

$$T(n) = T(n-6) + (n-3)^2 + h^2$$

$$T(n-6) = T(n-9) + (n-6)^2$$

$$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n^2$$

$$T(n) = T(n-3k) + (n-3k+3)^2 + [n-(3k-6)]^2 + n^2$$

$$n-3k=0$$

$$n=3k$$

$$k=n/3$$

$$T(n) = T(n-n) + 1^2 + 2^2 + 3^2 + \dots + n^2$$

$$= T(0) + \frac{n(n+1)(n+2)}{6}$$

$$T(n) = n \times n \times n$$

$$T(n) = O(n^3)$$

Ques - void fun(int n) {

 for (i=1 to n)

 for (j=1 ; j <= n ; j=j+i)

 printf("*");

}

i = 0, j → 1, 2, 3, 4, ..., n

i = 2, j → 1, 3, 5, 7, ..., n/2

i = 3, j → 1, 4, 7, 10, ..., n/3

⋮

i = n, j → 1, ..., n/n

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + \frac{n}{n}$$

$$= n \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right)$$

→ log n.

$$T(n) = n \log n$$

$T(n) = O(\log n)$

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TUTORIAL - 02

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DESIGN AND ANALYSIS OF ALGORITHM

Ques 1. Void Function n>?

```
int i=0, j=1;  
while (i < n) {  
    i = i + j;  
    j++;  
}
```

$i \rightarrow 0, 1, 3, 6, 10, \dots, k$
 $i \rightarrow 0, 1, 3, 6, \dots, k$
0 1 1+2 1+2+3 $1+3+4+\dots+k$

$(1+2+3+\dots+k) = n$ (to terminate the program)

$$\frac{k(k+1)}{2} = n$$

$$k^2 + k = 2n$$

$$k^2 = n$$

$$k = \sqrt{n}$$

$$T(n) = O(\sqrt{n})$$

Ques 2 $T(n) = T(n-1) + T(n-2) + 1$ if ($n > 0$) otherwise 1.
 $T(n-1) \asymp T(n-2)$

$$T(n) = T(n-1) + T(n-2) + 1$$

$$T(n) = 2T(n-1) + 1 \quad \text{--- (1)}$$

$$T(n-1) = 2T(n-2) + 1$$

From equ (1) \therefore

$$\begin{aligned} T(n) &= 2[T(n-2)] + 1 \\ &= 2^2 T(n-2) + 2 - \textcircled{2} \end{aligned}$$

$$T(n-2) = 2T(n-3) + 1$$

From equ $\textcircled{1}::$

$$T(n) = 2^3 T(n-3) + 3$$

$$T(n) = 2^K T(n-K) + K - \textcircled{3}$$

$$n-K=0$$

$$n=K$$

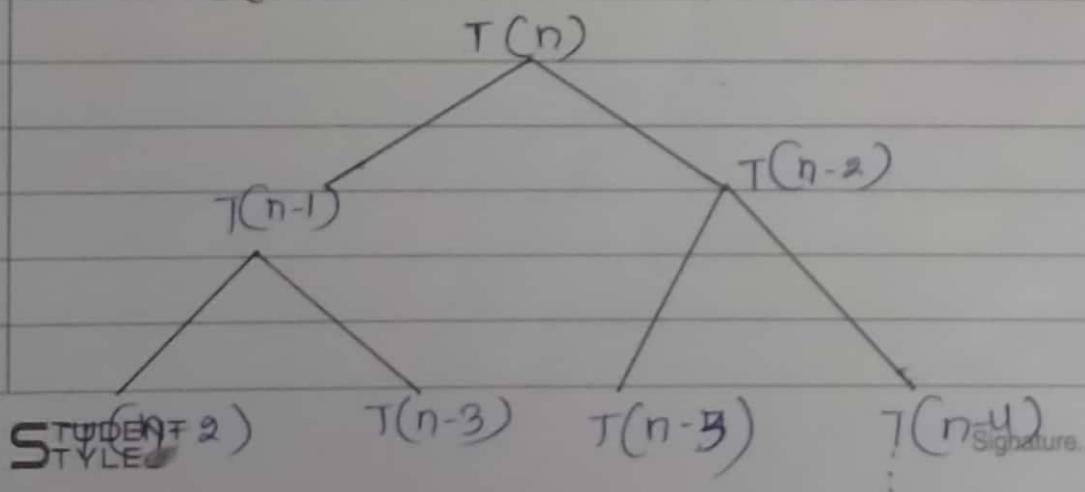
From equ $\textcircled{3}::$

$$\begin{aligned} T(n) &= 2^n T(n-n) + n \\ &= 2^n T(0) + n \end{aligned}$$

$$\begin{aligned} T(n) &= 2^n \times 1 + n \\ &= O(2^n) + O(n) \end{aligned}$$

$$T(n) = O(2^n)$$

Space Complexity \rightarrow If we draw the recursion tree of the Fibonacci recursion then we found the maximum height of tree will be n and hence the space complexity of Fibonacci recursion will be $O(n)$.



Ques 3 $O(n \log n)$, $O(n^3)$, $O(\log(\log n))$

(i) $n \log n$.

`void fun(int a){`

$\int \text{ if } (n=0) \text{ return }$

`for (i=0; i<a; i+=2)`

`sum = sum + i;`

`fun(n-1);`

}

$$T(n) = (n-1) + n$$

(iv) `void fun(int n){`

$\int \text{ if } (n=0) \text{ return }$

`for (i=0; i<n; i++)`

`for (j=0; j<n; j++)`

`sum = i+j;`

`fun(n-1);`

).

$$T(n) = (n-1) + n^2 \text{ if } n > 0 \text{ otherwise } 1.$$

(iii) $\log(\log n)$.

`for (int i=2; i<=n; i = pow(i, k))`

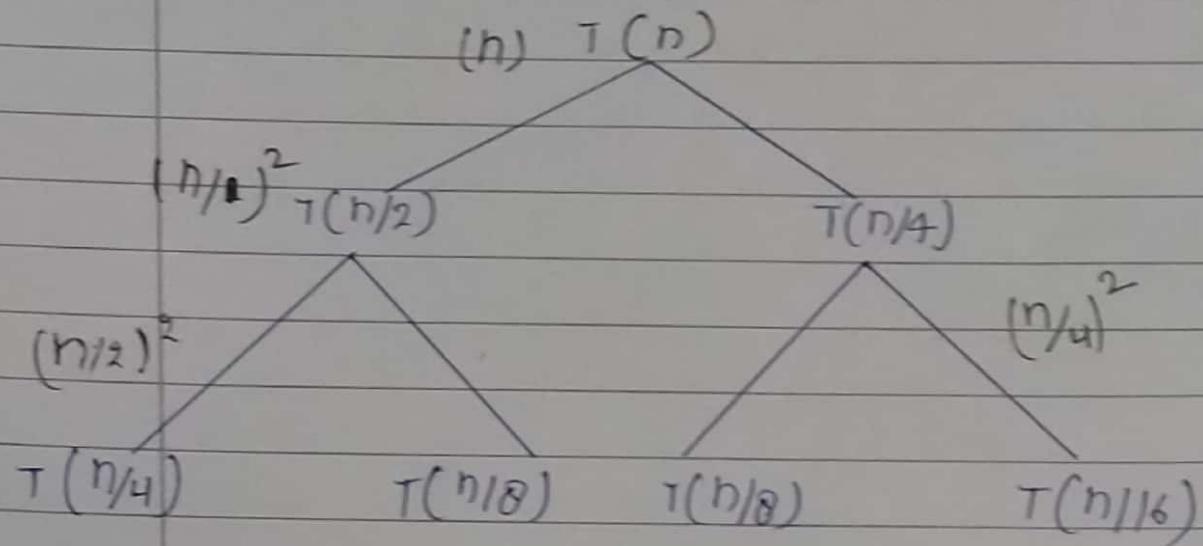
{

`point("i:", i);`

).

$$\frac{k}{=} \geq 2$$

Ques 4 $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2$



$$\begin{aligned} T(n) &= n^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 + \dots + \dots + \\ &= n^2 \left(1 + \frac{1}{2^2} + \frac{1}{4^2} + \dots \right) \end{aligned}$$

$$T(n) = n^2 \times 1$$

$$T(n) = O(n^2)$$

Or

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2$$

$$\alpha = \frac{1}{2}, \beta = \frac{1}{4}, f(n) = n^2$$

$$\alpha + \beta = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} < 1$$

$$T(n) = f(n) = O(n^2)$$

Ques 5 int fun(int n) {

for (int i=1; i<=n; i++) {

for (int j=1; j<n; j+=i) \rightarrow $j = i+j$;

Some $O(1)$ task

3.

$i \rightarrow 0$ $j \rightarrow 1, 2, 3, 4, 5, \dots, n$

$i \rightarrow 2$ $j \rightarrow 1, 3, 5, 7, \dots, \frac{n}{2}$

$i \rightarrow 3$ $j \rightarrow 1, 4, 7, 11, \dots, \frac{n}{3}$

⋮

$i \rightarrow n$ $j \rightarrow 1, \dots, \frac{n}{n}$

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + \frac{n}{n}$$

$$= n \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right)$$

$\curvearrowright \log n$

$$T(n) = n \log n$$

$T(n) = O(n \log n)$

Ques → 6. for (int i=2 ; i<=n ; Pow(i,k))
 some O(1) expression.

$$i = 2, 2^k, (2k)^k, \frac{(2^{k^2})^k}{2^{k^2}}, \dots, \frac{(2^{k^3})^k}{2^{k^3}}, \dots, \frac{2^{k^k}}{2^{k^k}}$$

$$2^{k^k} = n$$

$$\log_2 2^{k^k} = \log n$$

$$k^k \log_2 2 = \log n$$

$$k^k = \log n$$

$$\log k^k = \log(\log n)$$

$$\frac{k^k}{\log 2} = \log(\log n).$$

$$k = O(\log \log n).$$

$$T(n) = O[\log(\log n)]$$

Ques 8 Arrange the following in increasing order of growth.

- (a) $n, n!, \log n, \log \log n, \sqrt{100+n}, \log(n!), n \log n, \log^2 n$
 $2^n, 2^{2^n}, 4^n, n^2, 100$

Ans $100 < \log \log n < \log n < \log^2 n < \sqrt{100+n} < n < n \log n < \log(n!)$
 $< n^2 < 2^n < 4^n < 2^{2^n} < n!$

- (b) $2(2^n), 4n, 2n, 1, \log(n), \log \log(n), \sqrt{\log n}, \log n$
 $2 \log n, n, \log(n!), n!, n^2, n \log(n).$

Ans $1 < \log \log n < \sqrt{\log n} < \log n < \log_2 n < 2 \log n < n < 2n < 4n$
 $< n \log n < \log n! < n^2 < 2^{2^n} < n!.$

- (c) $8^{2^n}, \log_2(n), n \log_6(n), n \log_2(n), \log(n!), n! \cdot \log_8(n)$
 $96, 8n^2, 7n^3, 5n.$

Ans $96 < \log_2(n) < \log_8(n) < 5n < n \log_6(n) < n \log_2(n) < \log(n!) < 8n^2 <$
 $7n^3 < n! < 8^{2^n} < n!.$

Tutorial → 3

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Ans - 01

while (low ≤ high)

{

 mid = low + (high - low) / 2;

 if (arr[mid] == key)

 {

 Print("Found");

 Print(count);

}

 else if (arr[mid] > key)

 high = mid - 1;

 else

 low = mid + 1;

 count++;

}

 Print("Not found");

Ans: 02

Iterative Insertion Sort :-

```
for (int i=1; i<n; i++)
```

```
{   j = i-1;
```

```
x = arr[i];
```

```
while (j>-1 and arr[j]>x)
```

```
{   A[j+1] = A[j];
```

```
    j--;
```

```
}
```

```
arr[j+1] = x;
```

```
}
```

Recursive :-

Void InsertionSort (int arr[], int n)

```
{
```

```
if (n≤1)
```

```
return;
```

```
insertionSort(arr, n-1);
```

```
int last = arr[n-1],
```

```
j=n-2;
```

```
while (j>=0 and arr[j]>last)
```

```
{
```

```
arr[j+1] = arr[j];
```

```
j--;
```

```
arr[j+1] = last.
```

```
.
```

Insertion sort is online sorting because whenever a new element come, insertion sort define its right place.

Ans :- 03

Bubble sort $\rightarrow O(n^2)$
Insertion sort $\rightarrow O(n^2)$
Selection sort $\rightarrow O(n^2)$
Merge sort $\rightarrow O(n \log n)$
Quick sort $\rightarrow O(n \log n)$
Count sort $\rightarrow O(n)$
Bucket sort $\rightarrow O(n)$.

Ans :- 04

Online sorting \rightarrow Insertion sort.
Stable sorting \rightarrow Merge sort, Bubble sort.
Inplace sorting \rightarrow bubble sort, selection sort.

Ans :- 05

Iterative Binary Search.

```
while (low <= high)
{
    mid = low + (high - low) / 2;
    if (arr[mid] == key)
        return true;
    else if (arr[mid] > key)
        high = mid - 1;
    else
        low = mid + 1;
}
```

$O(n \log n)$

return false;

Recursive Binary Search.

```
if (low <= high)
{
    int mid = low + (high - low) / 2;
    if (arr[mid] == key)
        return true;
    else if (arr[mid] > key)
        BinarySearch(arr, low, mid - 1);
    else
        BinarySearch(arr, mid + 1, high);
}
return false;
```

Ans: 6 $T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + C$

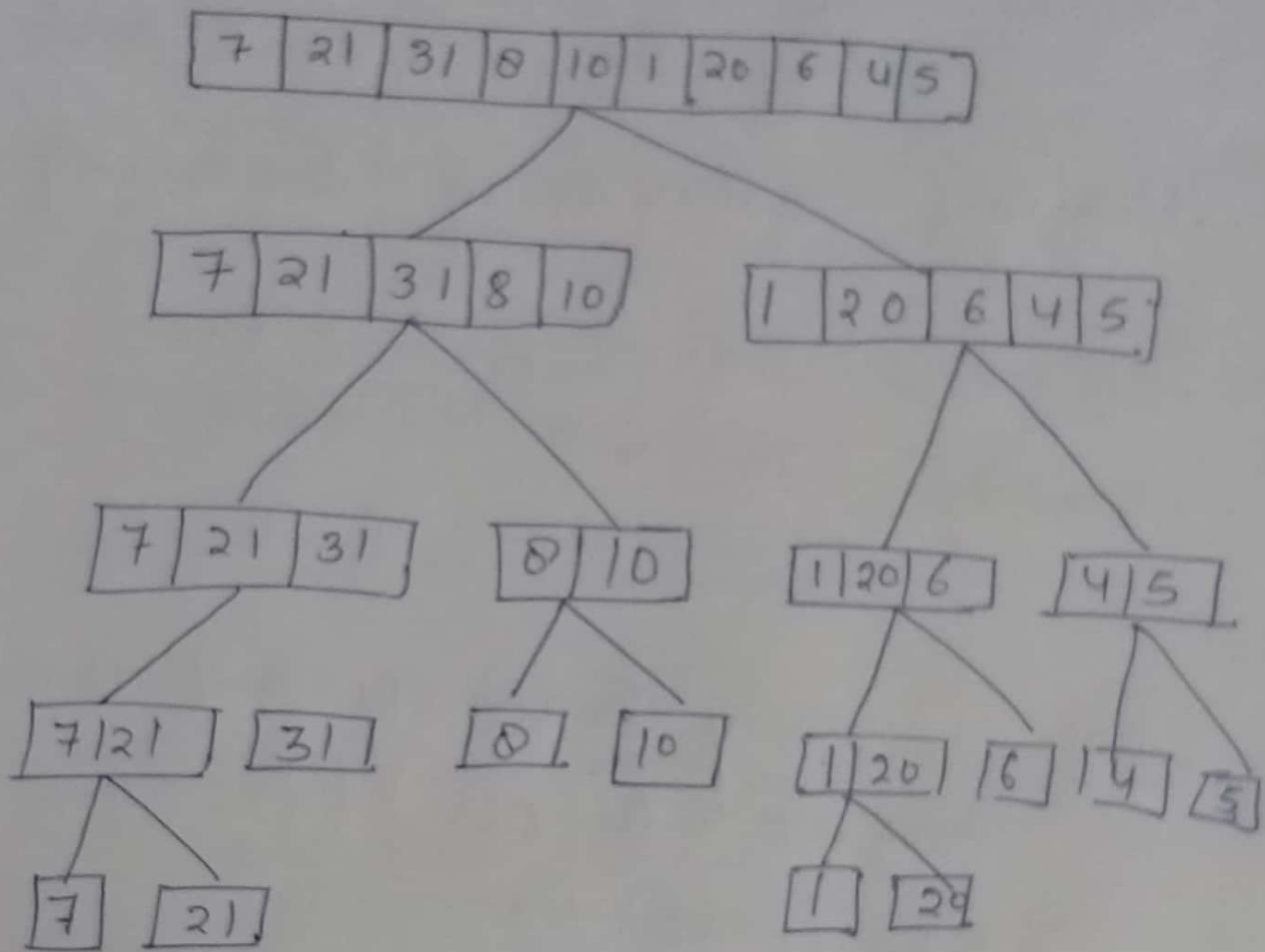
$$T(n) = T(n/2) + 1$$

Ans: 7 void (vector<int> arr, int key)

```
{  
    sort (arr.begin(), arr.end());  
    int low = 0, high = arr.size() - 1;  
    while (low < high)  
    {  
        if (arr[low] + arr[high] == key)  
            print (low, high);  
        break;  
        else if (arr[low] + arr[high] > key)  
            high--;  
        else  
            low++;  
    }  
    print ("No found");  
}
```

Ans:-08 Quick Sort is the fastest general purpose sort. In most practical solution, quicksort is the method of choice if stability is important and space is available, merge sort might be best.

Ans:-09 Inversion indicates - how far are close the array is from being sorted.



Inversion $\rightarrow 31$

Ques 11 Merge sort

Best Case $\rightarrow 2T(n/2) + O(n)$

Worst Case $\rightarrow 2T(n/2) + n$

Quick sort.

Best Case $\rightarrow 2T(n/2) + n$

Worst Case $\rightarrow T(n-1) + n$.

Basis	Quick sort	Merge sort.
• Position	Splitting done in any ratio.	Array divided in two equal parts.
• Works well on	smaller array.	fine on any size of array.
• Sorting Method	Internal	External.
• Stability	Not Stable	stable.

Ques 10 If array is already sorted than Quick Sort will take $O(n^2)$ time. otherwise it takes $(n \log n)$ time.

Ans 12:- void StableSelectionSort (int arr[], int n)

```
    for (int i=0; i<n-1; i++)  
    {  
        int pos=i;  
        int min=i;  
        for (int j=i+1; j<n; j++)  
        {  
            if (arr[j]<min)  
            {  
                min=arr[j];  
                pos=j;  
            }  
        }  
        if (pos!=i)  
        {  
            while (min pos>i)  
            {  
                arr[pos]=arr[pos-1];  
                pos--;  
            }  
            arr[i]=min;  
        }  
    }
```

Ques 13 We will use the merge sort because we can divide the 4 GB data into 4 parts of 1 GB and sort them separately and combine them latter.

Internal Sorting \rightarrow All the data is stored in memory at all times (Sorting is in progress).

External Sorting \rightarrow all the data is stored outside memory and only load in small parts.

Tutorial 04.

$$\textcircled{1} \quad T(n) = 3T(n/2) + n^2$$

$$a=3, b=2, f(n)=n^2$$

$$n^{\log_b a} = n^{\log_2 3}$$

comparing $n^{\log_2 3}$ and n^2

$$n^{\log_2 3} < n^2 \text{ Case 3.}$$

\therefore according to master's theorem.

$$T(n) = \Theta(n^2).$$

$$\textcircled{2} \quad T(n) = 4T(n/2) + n^2$$

$$a=4, b=2$$

$$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n) \text{ Case 2.}$$

\therefore according Master's theorem.

$$T(n) = \Theta(n^2 \log n)$$

$$\textcircled{3} \quad T(n) = T(n/2) + 2^n$$

$$a=1, b=2$$

$$n^{\log_b 1} = n^0 = 1$$

$$1 < 2^n \text{ Case 3.}$$

$$T(n) = \Theta(2^n)$$

(4)

$$T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$$

Master theorem is not applicable.

(5)

$$T(n) = 16 T\left(\frac{n}{16}\right) + n$$

$$a=16, b=4, f(n)=n$$

$$n^{\log_b a} = n^{\log_4 16} \Rightarrow n^2, f(n) < n^2$$

$$T(n) = \Theta(n^2).$$

(6)

$$T(n) = 2T\left(\frac{n}{2}\right) + n \log n.$$

$$a=2, b=2, f(n)=n \log n.$$

$$n^{\log_b a} = n^{\log_2 2} \Rightarrow n$$

$$f(n) > n$$

$$T(n) = \Theta(n \log n).$$

(7)

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

$$a=2, b=2, K=1, P=-1$$

$$a = b^K, \underline{P \neq -1}$$

$$T(n) = \Theta(n^{\log_b a} \log \log n)$$

$$= \Theta(n \log \log n).$$

⑧ $T(n) = 2T(n/4) + n^{0.51}$
 $a=2, b=4, f(n) = n^{0.51}$
 $n^{\log_2 a} = n^{\log_4 2} = n^{0.5}$
 $n^{0.5} < f(n)$

$$T(n) = \underline{\Theta(n^{0.51})}$$

⑨ $T(n) = 0.5T(n/2) + \frac{1}{n}$
 $a < 1$, not applicable Master theorem.

⑩ $T(n) = 16T(n/4) + n!$
 $a=16, b=4, f(n) = n!$
 $n^{\log_2 a} = n^{\log_4 16} = n^2$
 $n^2 < n!$
 $T(n) = \underline{\Theta(n!)} \quad \text{Note: } n! \neq O(n^2)$

⑪ $T(n) = 4T(n/2) + \log n$
 $a=4, b=2, f(n) = \log n$
 $n^{\log_2 a} = n^{\log_2 4} = n^2$
 $n^2 > f(n)$
 $T(n) = \underline{\Theta(n^2)}$

$$(12) \quad T(n) = \text{sqrt}(n) + \frac{n}{2} + \log n$$

Master's theorem not applicable.

$$(13). \quad T(n) = 3T(n/2) + n$$

$$a=3, b=2, f(n)=n$$

$$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$$

$$n^{1.58} > f(n)$$

$$T(n) = \Theta(n^{\log_2 3})$$

$$(14) \quad T(n) = 3T(n/3) + \sqrt{n}$$

$$a=3, b=3, f(n)=\sqrt{n}$$

$$n^{\log_b a} = n^{\log_3 3} = n$$

$$n > \sqrt{n}$$

$$\therefore T(n) = \Theta(n)$$

$$(15) \quad T(n) = 4T(n/2) + cn$$

$$a=4, b=2, f(n)=cn$$

$$n^{\log_b a} = n^{\log_2 4} = n^2$$

$$n^2 > cn$$

$$T(n) = \Theta(n^2)$$

(19) $T(n) = 4T(n/2) + n \log n$
 $a=4, b=2, f(n) = n \log n.$
 $n^{\log_2 4} = n^{\log_2 4} = n^2$
 $n^2 > n \log n$
 $T(n) = \Theta(n^2)$

(20). $T(n) = 64T(n/8) - n^2 \log n.$
Masters theorem is not applicable as $f(n)$ is not increasing function.

(21). $T(n) = 7T(n/3) + n^2$
 $a=7, b=3, f(n) = n^2$
 $n^{\log_3 7} = n^{\log_3 7} = n^{1.7}$
 $n^{1.7} < n^2$
 $T(n) = \Theta(n^2),$

(22) $T(n) = T(n/2) + n(2 - \alpha n)$
Masters theorem is not applicable

(16)

$$T(n) = 3T(n/4) + n \log n$$

$$a=3, b=4, f(n) = n \log n$$

$$n^{\log_6 3} = n^{\log_4 3^3} = n^{0.79}$$

$$n^{0.79} < n \log n.$$

$$T(n) = \Theta(n \log n).$$

(17)

$$T(n) = 3T(n/3) + (n/2)$$

$$a=3, b=3, f(n) = n/2$$

$$n^{\log_6 3} = n^{\log_3 3^3} = n$$

$$O(n) \cong O(n/2)$$

$$T(n) = \Theta(n \log n).$$

(18)

$$T(n) = 6T(n/3) + n^2 \log n$$

$$a=6, b=3, f(n) = n^2 \log n.$$

$$n^{\log_6 6} = n^{\log_3 6^3} = n^{1.63}$$

$$n^{1.63} < n^2 \log n$$

$$T(n) = \Theta(n^2 \log n).$$

TUTORIAL:05

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Ques: What is difference between DFS and BFS. Write application both the algorithm.

BFS

- (1). It's stands for Breath First Search.
- (2). In this searching we use Queue data structure.
- (3). It give 100% result.
- (4). It is more suitable for searching vertices are closer to given source.
- (5). There is no concept of backtracking.

DFS

- (1) It's stand for Depth first search.
- (2) In this searching we use stack datastructure.
- (3) It does not give 100% result.
- (4) It is more suitable when there is away solution away from source.
- (5) It is a recursive algorithm that use backtracking.

Applications:

- (a) BFS → Bipartite graph and minimum no of Nodes Path, networking, and GPS.
- (b) DFS → acyclic graph, topological Sort, Scheduling Problems.

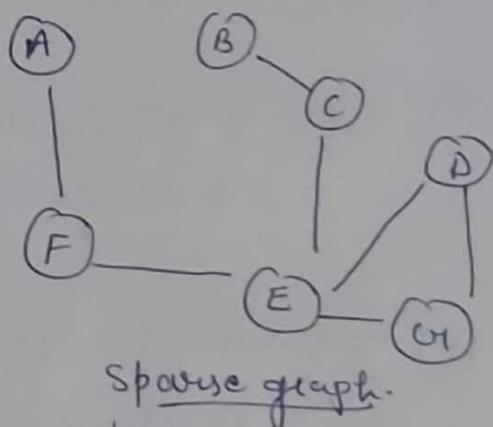
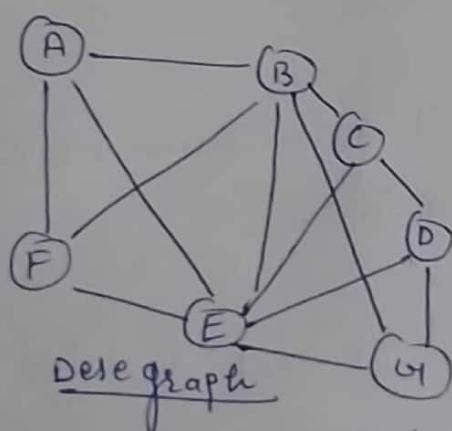
Ques 02 Which data structure are used to implement BFS and DFS and Why?

↳ For implementing BFS we use a Queue data-structure for finding minimum no. of nodes path between source node to destination node. We use queue because things don't have to be processed immediately. but have to be processed in FIFO order. like. BFS :- Searching for nodes level traverse. ex. it search nodes w.r.t their distance from source. For this Queue is better in any case to use BFS.

For implementing DFS we use stack data-structure if traverse a graph in depthward motion and stack to remember to get the next nodes to start a search when dead end occurs in any iteration.

Ques 03 What do you mean by sparse and dense graphs? Which representation of graph is better for sparse and dense graph?

- ↳ Dense graph is a graph in which no. of nodes is close to maximal no. of edges.
- ↳ Sparse graph is a graph in which no. of edges is very less.



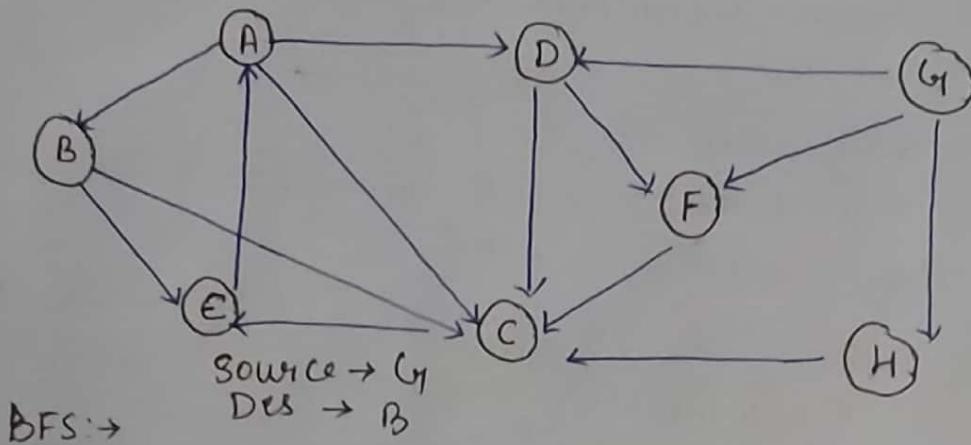
- .) For sparse graph it is preferred to use Adjacency list.
- .) For dense graph it is preferred to Adjacency Matrix.

(2) Union: \rightarrow It takes two elements as input and find superset of this sets using the find operation, and finally puts either one of the tree under set node of other tree effectively merging the trees and set.

void union(int a, int b)

$$\begin{cases} \text{if } arr[a] = b \\ \text{else } arr[a] = arr[b] \end{cases}$$

Ques 6 Run BFS and DFS on graph.



child	G	H	D	F	C	E	A	B
parent	-	G	G	G	H	C	E	A

Path \rightarrow G \rightarrow H \rightarrow C \rightarrow E \rightarrow A \rightarrow B

DFS \rightarrow

PUSH	POP
G	G
D	F
H	C
F	E
G	A
E	
A	
B	

Path \rightarrow G \rightarrow F \rightarrow C \rightarrow E \rightarrow A \rightarrow B.

Aus-04: How can you detect a cycle in graph using BFS and DFS.

Ans: For detecting cycle in a graph using BFS we need to use Kahn's algorithm for topological sorting.

The steps required are:-

- (1). Compute in-degree (no of incoming edges) for each of vertex Parent in graph and initialise count of visited node as 0.
- (2) Pick all vertices with in-degree as 0 and add them in queue.
- (3) Remove a vertex from queue and then:
 - increment count of visited nodes by 1.
 - Decrease in-degree by 1 for all neighbouring nodes.
 - If in-degree of neighbouring nodes reduced to zero then add it in queue.
- (4). Repeat 3. until queue is empty.
- (5). If count of visited nodes is not equal to no of nodes in graph, has cycle, otherwise not.

Aus-05: What do you mean by disjoint set data structure? Explain and ~~operations~~ along with example which can be performed on disjoint set.

Ans: A disjoint set is a data structure that keeps track of set of elements partitioned into several disjoint subsets. In other words, a disjoint set is a group of sets where no item can be in more than one set.

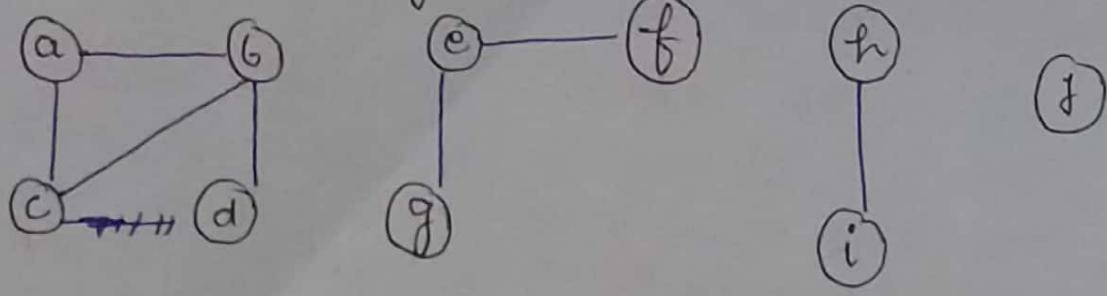
3 operations→

- (i) Find → can be implemented by recursively traversing the parent array until we hit a node who is parent to itself.

```
int find(int v){  
    if(v == arr[v])  
        return v;  
    else  
        return find(arr[v]);  
}
```

Ans → 7 find all no. of connected components and vertices in each component using disjoint set data structure.

Ans →

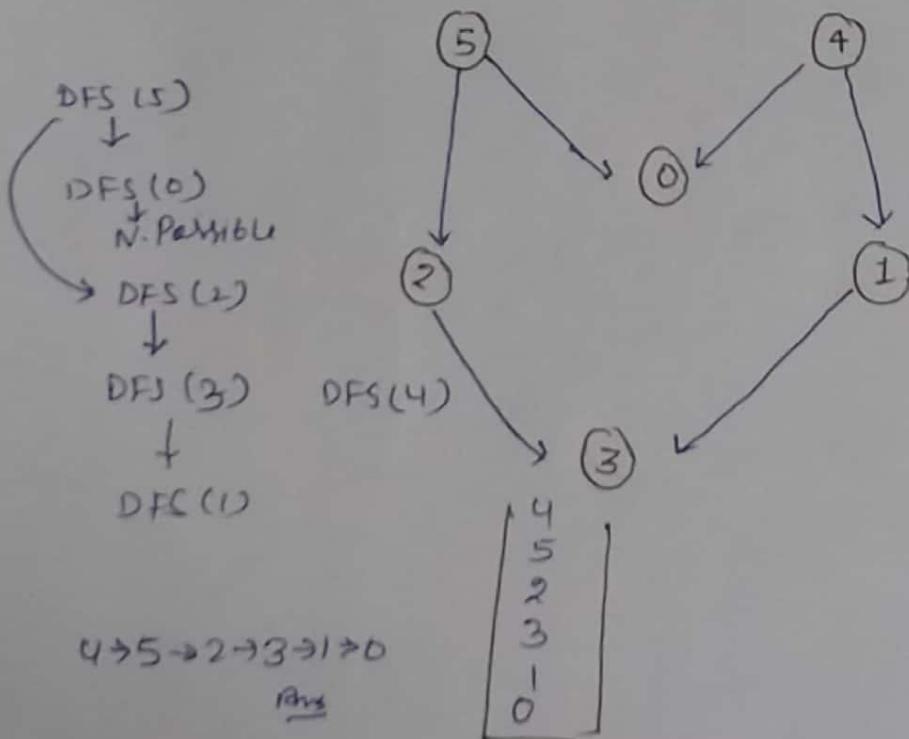


$$\begin{aligned}V &= \{a\} \cup \{b\} \cup \{c\} \cup \{d\} \cup \{e\} \cup \{f\} \cup \{g\} \cup \{h\} \cup \{i\} \cup \{j\} \\&= \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{e, f\}, \{e, g\}, \{h, i\}, \{j\}\end{aligned}$$

(a,b)	$\{a, b\} \cup \{c\} \cup \{e\} \cup \{f\} \cup \{g\} \cup \{h\} \cup \{i\} \cup \{j\}$
(a,c)	$\{a, b, c\} \cup \{d\} \cup \{e\} \cup \{f\} \cup \{g\} \cup \{h\} \cup \{i\} \cup \{j\}$
(b,c)	$\{a, b, c\} \cup \{d\} \cup \{e\} \cup \{f\} \cup \{g\} \cup \{h\} \cup \{i\} \cup \{j\}$
(b,d)	$\{a, b, c, d\} \cup \{e\} \cup \{f\} \cup \{g\} \cup \{h\} \cup \{i\} \cup \{j\}$
(e,g)	$\{a, b, c, d\} \cup \{e, f\} \cup \{g\} \cup \{h\} \cup \{i\} \cup \{j\}$
(h,i)	$\{a, b, c, d\} \cup \{e, f, g\} \cup \{h, i\} \cup \{j\}$

No. of connected components $\rightarrow 03$.

Ans → 8 Apply topological sort and DFS on graph having vertices from 0 to 5.



Ques → 3. Heap data structure can be used to implement priority queue. Name few graph algorithm where you need to use Priority queue and why?

Ans Yes, heap data structure can be used to implemented priority queue. It will take $O(\log n)$ time to insert and delete each element in priority queue. Based on heap structure priority queue has two types max-priority queue based on max heap and min heap priority queue based on min heap heaps provide better performance than do array etc.

Tutorial 6

DATE _____
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1.

Minimal spanning tree

A minimal spanning tree or minimum weight weight spanning tree is a subset of the edges of the connected, edge-weighted and undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.

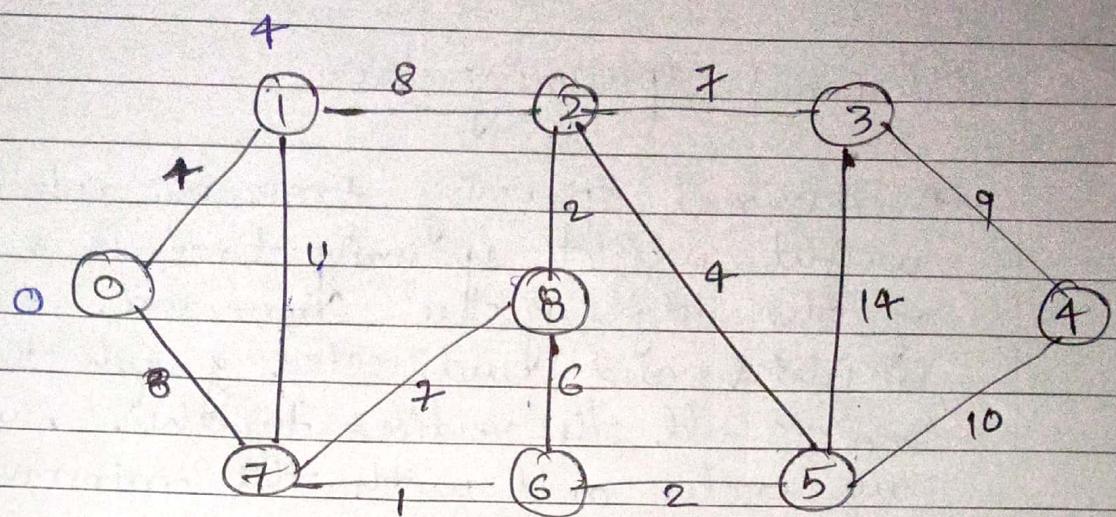
Application:-

1. Design local area network.
2. In constructing highways or railways spanning
3. Laying pipelines.
4. Telecommunication networks.

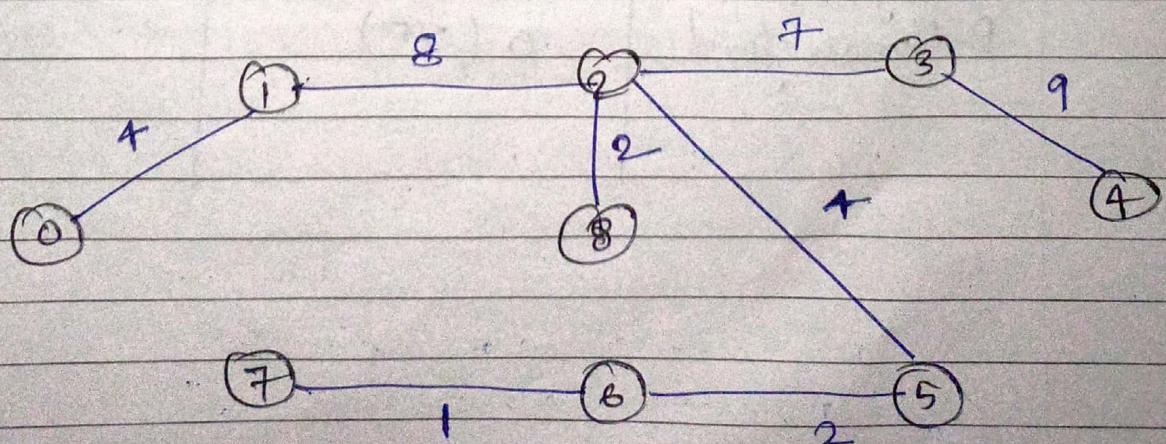
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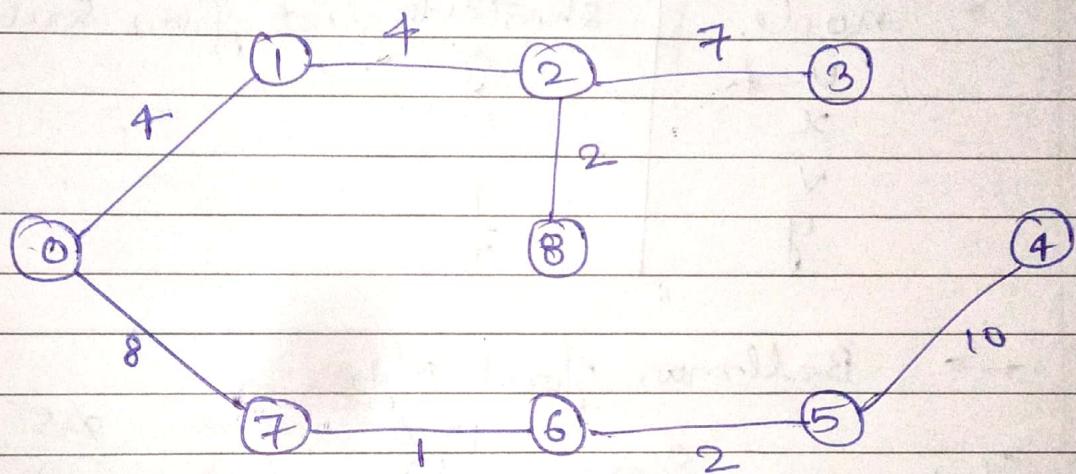
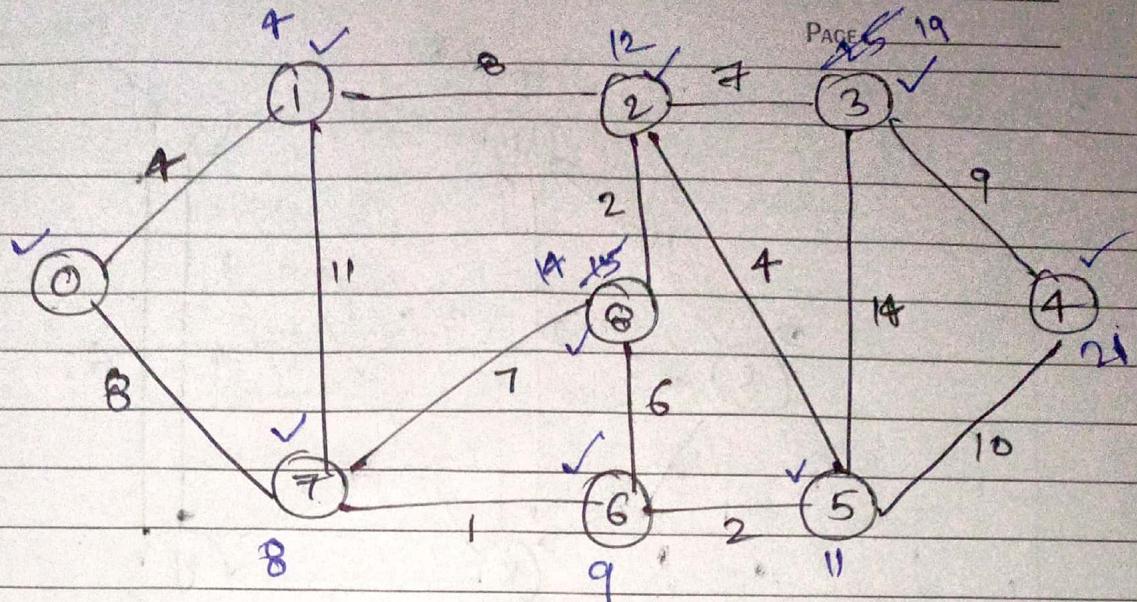
<u>Algo's.</u>	<u>Time Complexity</u>	<u>Space Complexity</u>
Prims	$O(V^2)$	$O(V+E)$
Kruskal	$O(E \log V)$	$O(\log E)$
Dijkstra	$O(V+E)$	$O(V+E)$
Bellman Ford	$O(VE)$	$O(N)$

3



Path	Weight
7 → 6	1
6 → 5	2
2 → 8	2
0 → 1	4
2 → 5	4
8 → 6	6
2 → 3	7
7 → 8	7
1 → 2	8
3 → 4	9.
5 → 4	10
1 → 7	11
3 → 5	14

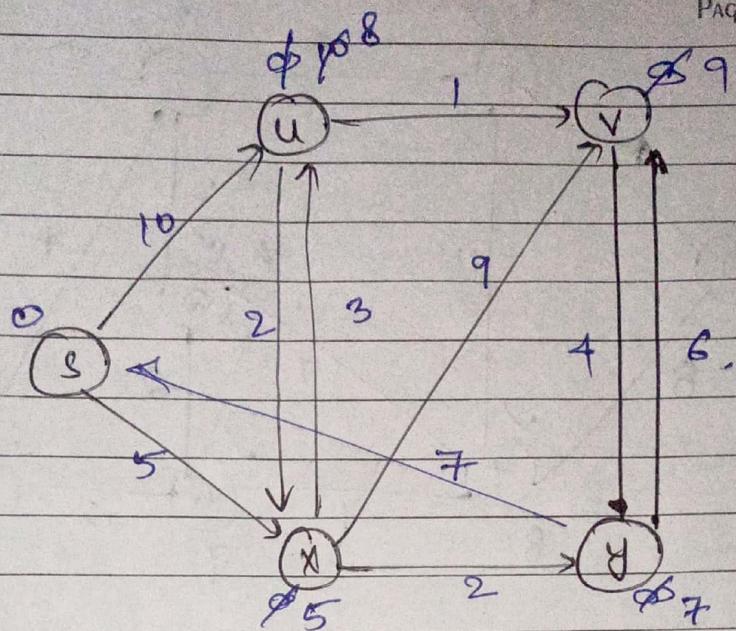




Q (i) The shortest path may change. The reason is there may be different no. of edges in different paths from s to t . For shortest path of weight 15 and 5 edge. and another 2 edge and 25 weight. weight of shortest path inc. by 5×10 and become $15 + 50$ and similarly other path become $25 + 20$, so shortest path weight as 45.

(ii) If we multiply all edges weight by 10, the shortest path does not change. The no. of edge on path does not matter.

5



node.	shortest dist from source node
u	8
x	5
v	9
y	7

→ Bellman Ford algo.

		1st	2nd	3rd	4th
		∞	10	8	8
		∞	∞	9	9
		∞	∞	7	7
		∞	∞	∞	9

Final graph:-

