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AE 3330 PROBLEM SET

Content:

Q 14,15,16,17,18,19,20,21,22,23l,24,25,

Q 14

Twin-turbofan aircraft.

$$S(to) = 1023\ ft^2, \quad C_{D0}(to) = 0.035, \quad e(to) = 0.75,$$

$$S(clean) = 979.5\ ft^2, \quad C_{D0}(clean) = 0.0182, \quad e(clean) = 0.85,$$

$$span = 93\ ft, \quad w = 100{,}000\ lb$$

$$k(to) = \frac{1}{\pi eAR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{1023}} = 0.0501994$$

$$k(clean) = \frac{1}{\pi eAR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{979.5}} = 0.0480648$$

$$T(S|L) = 12{,}000\ lbf, \quad T(5{,}000\ ft) = 10{,}000\ lbf$$

$$\rho_{SL} = 0.07647 \frac{lb}{ft^3}, \quad \rho(5000\ ft) = 0.0659\ lb/ft^3$$

➔ ROC (fpm) at SSL at 180 KTAS in takeoff config

$$V_{true} = 180\ kt\ (303.806\ ft/s)$$

$$D = 0.5\rho V^2 S C_{D0} + \frac{kw^2}{0.5\rho V^2 S}$$
$$= 0.5 \times 0.07647 \times 303.806^2 \times 1023 \times 0.035 + \frac{0.0501994 \times 100{,}000^2}{0.5 \times 0.07647 \times 303.806^2 \times 1023}$$
$$= 126495.51\ lbf$$

$$ROC = v\frac{T - D}{w} = 303.806 \frac{12{,}000 -}{100{,}000}$$

$$ROC = V\left[\frac{T}{w} - 0.5\rho V^2 \left(\frac{S}{w}\right) C_{D0} - \frac{w}{S}\frac{2K}{\rho V^2}\right]$$

$303.806 \left[\frac{24{,}000}{100{,}000} - 0.5 \times 0.07647 \times 303.806^{\ 2} \left(\frac{1023}{100{,}000}\right) 0.035 - \frac{100{,}000}{1023}\frac{2 \times 0.0501994}{0.07647 \times 303.806^{\ 2}}\right]$ yields negative sign as if the drag surpasses the thrust produced

Second attempt

We have the thrust and speed and need to find the rate of climb.

$$ROC = v\frac{T - D}{w} = 303.806 \left[\frac{24{,}000}{100{,}000} - 0.5 \times 0.07647 \times 303.806^{\ 2} \times \left(\frac{1023}{100{,}000}\right) 0.035 - \frac{100{,}000}{1023}\frac{2 \times 0.0501994}{0.07647 \times 303.806^2}\right] = -311.3\ ft/s$$

Second attempt

SSL at 180 KTAS () in takeoff configuration

Takeoff configuration:

$$S(to) = 1023\ ft^2, \quad C_{D0}(to) = 0.035, \quad e(to) = 0.75,$$

$$k(to) = \frac{1}{\pi eAR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{1023}} = 0.0501994$$

$$V = 180\ KTAS = 303.806\ ft/s$$

$ROC = \left(\frac{T-D}{W}\right) V = (T - D)\frac{v}{W} = (24{,}000 - A)\frac{303.806}{100{,}000}$ this equation does not work because the drag is gonna be negative so there has to be another way.

8365.02 the drag should be yet there is not a

Third attempt

Assuming mini power req

$$C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3\times 0.035}{0.0501994}} = 1.44626 \quad C_D = 4C_{D0} = 4\times 0.035 = 0.14$$

Assuming max range

$$\frac{L}{D}|_{max} = \frac{1}{\sqrt{4\times 0.035\times 0.0501994}}$$

$$ROC = v\sin\alpha = \sqrt{\frac{w}{0.5\,\rho C_L S}}\left[\frac{T}{W} - \frac{C_D}{C_L}\right] = V\left[\frac{T}{W} - \frac{C_D}{C_L}\right] = 303.806\left[\frac{24{,}000}{100{,}000} - 0.0838327\right]$$

$$= 47.44\frac{ft}{s} \sim 2847\ ftm$$

Assuming minimum drag

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.035}{0.0501994}} = 0.834997 \quad C_D = 2C_{D0} = 2\times 0.035 = 0.07$$

$$ROC = v\sin\alpha = \sqrt{\frac{w}{0.5\,\rho C_L S}}\left[\frac{T}{W} - \frac{C_D}{C_L}\right] = V\left[\frac{T}{W} - \frac{C_D}{C_L}\right] = 303.806\left[\frac{24{,}000}{100{,}000} - \frac{0.07}{0.834997}\right]$$

$$= 47.44\frac{ft}{s} \sim 2847\ ftm$$

So basically the ratio is kinda there, but has to be more accurately choosen, instead of random baseless assumptions.

Or in order to climb with a steep angle, we need the minimum drag possible.

➔ **Max ROC (fpm) at 5,000 ft alt. in clean config. Mach number and calibrated airspeed (kt)**

$$ROC_{max} = \left[\frac{\left(\frac{w}{S}\right)Z}{3\rho C_{D0}}\right]^{0.5}\left(\frac{T}{w}\right)^{\frac{3}{2}}\left[1 - \frac{Z}{6} - \frac{3}{2\left(\frac{T}{w}\right)^2\left(\frac{L}{D}\right)_{max}^2 Z}\right] \text{ for jet-propelled airplane}$$

$$Z = 1 + \sqrt{1 + \frac{3}{\left(\frac{L}{D}\right)_{max}^2\left(\frac{T}{W}\right)^2}} = 1 + \sqrt{1 + \frac{3}{\frac{1}{4\times 0.0182\times 0.0480648}\left(\frac{20{,}000}{100{,}000}\right)^2}} = 2.123581$$

$$ROC_{max} = \left[\frac{\left(\frac{W}{S}\right)Z}{3\rho C_{D0}}\right]^{0.5}\left(\frac{T}{w}\right)^{\frac{3}{2}}\left[1 - \frac{Z}{6} - \frac{3}{2\left(\frac{T}{w}\right)^2\left(\frac{L}{D}\right)_{max}^2 Z}\right]$$

$$V_{max\,ROC} = \sqrt{\frac{\left(\frac{T}{w}\right)\left(\frac{w}{S}\right)Z}{3\rho C_{D0}}} = \sqrt{\frac{\frac{20{,}000}{979.5}\times 2.123581}{3\times 1771\times 0.0182}} = 83.0640\frac{ft}{s}$$

Maximum rate of climb

$$C_L = \frac{-T}{2wk} \pm \left[\left(\frac{T}{2wk}\right)^2 + \frac{3C_{D0}}{k}\right]^{0.5}$$

$$= \frac{-20{,}000}{2\times 100{,}000\ \times 0.0480648} \pm \left[\left(\frac{20{,}000}{2\times 100{,}000\ \times 0.0480648}\right)^2 + \frac{3\times 0.0182}{0.0480648}\right]^{0.5}$$

$$= 0.257113, -4.41816$$

$$ROC = \sqrt{\frac{2w}{\rho S}}\left(\frac{T}{w}\frac{1}{\sqrt{C_L}} - \frac{C_{D0}}{C_L^{\frac{1}{3}}} - KC_L^2\right) = \sqrt{\frac{2\times 100{,}000}{0.0660585\ \times 979.5}}\left(\frac{20{,}000}{100{,}000}\frac{1}{\sqrt{4.41816}} - \frac{0.0182}{4.41816^{\frac{1}{3}}} - 0.0480648 \times 4.41816^2\right) = 20.1852$$ yields negative

$$ROC = \sqrt{\frac{2w}{\rho S}}\left(\frac{T}{w}\frac{1}{\sqrt{C_L}} - \frac{C_{D0}}{C_L^{\frac{1}{3}}} - KC_L^2\right)$$
$$= \sqrt{\frac{2 \times 100{,}000}{0.0660585\ \times 979.5}}\left(\frac{20{,}000}{100{,}000}\frac{1}{\sqrt{0.257113}} - \frac{0.0182}{0.257113^{\frac{1}{3}}} - 0.0480648 \times 0.257113^2\right) = 20.1852$$

Second attempt
Clean Configuration at 5000 fl.

$$S(clean) = 979.5\ ft^2, \qquad C_{D0}(clean) = 0.0182, \qquad e(clean) = 0.85,$$
$$k(clean) = \frac{1}{\pi eAR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{979.5}} = 0.0480648$$
$$\rho = 0.0660585\ psf$$

Find the max Rate of Climb
$$-\frac{3C_{D0}\rho}{2(w/S)}(v^4) + \frac{T}{w}(v^2) + \frac{2k(w/S)}{\rho} = 0$$
$$-\frac{3 \times 0.0182 \times 0.0660585}{2(100{,}000/979.5)}(v^4) + \frac{20{,}000}{100{,}000}(v^2) + \frac{2 \times 0.0480648 \times (100{,}000/979.5)}{0.0660585} = 0$$
The velocity would be 109.644 ft/s

Airspeed for maximum rate of climb for the case where thrust is independent of airspeed
$$v^2 = \frac{T}{6A} \pm \frac{1}{2}\sqrt{\left(\frac{T}{3A}\right)^2 + \frac{4B}{3A}}$$
$$= \frac{20{,}000}{6 \times 0.588809136825} \pm \frac{1}{2}\sqrt{\left(\frac{20{,}000}{3 \times 0.588809136825}\right)^2 + \frac{4 \times 1.485675587 \times 10^7}{3 \times 0.588809136825}}$$
$$A = 0.5\ \rho S C_{D0} = 0.5 \times 0.0660585\ \times 979.5 \times 0.0182 = 0.588809136825 \qquad B = \frac{kw^2}{0.5\rho S}$$
$$= \frac{0.0480648 \times 100{,}000^2}{0.5 \times 0.0660585\ \times 979.5} = 1.485675587 \times 10^7$$
$$C_L = \frac{1}{2k}\left[-\frac{T}{w} \pm \sqrt{\left(\frac{T}{w}\right)^2 + 12C_{D0}k}\right]$$
$$= \frac{1}{2 \times 0.0480648}\left[\frac{20{,}000}{100{,}000} \pm \sqrt{\left(\frac{20{,}000}{100{,}000}\right)^2 + 12 \times 0.0182 \times 0.0480648}\right]$$
$$= 4.41816, -0.257113$$
$$v = \sqrt{\frac{100{,}000}{0.5 \times 0.0660585\ \times 979.5 \times 0.257113}} = 109.644\ ft/s$$

Fourth attempt

$$v = \sqrt{\frac{100{,}000}{0.7 \times 1{,}771\ \times 979.5 \times A}} = 0.57$$

$$C_L = \frac{-T}{2wk} \pm \left[\left(\frac{T}{2wk}\right)^2 + \frac{3C_{D0}}{k}\right]^{0.5}$$

$$= \frac{-20{,}000}{2 \times 100{,}000\ \times 0.0480648} \pm \left[\left(\frac{20{,}000}{2 \times 100{,}000\ \times 0.0480648}\right)^2 + \frac{3 \times 0.0182}{0.0480648}\right]^{0.5}$$

$$= 0.257113, -4.41816$$

$$ROC = \sqrt{\frac{2w}{\rho S}}\left(\frac{T}{w}\frac{1}{\sqrt{C_L}} - \frac{C_{D0}}{C_L^{\frac{1}{3}}} - KC_L^2\right)$$

$$= \sqrt{\frac{2 \times 100{,}000}{1.4 \times 1771 \times 979.5}}\left(\frac{20{,}000}{100{,}000}\frac{1}{\sqrt{0.257113}} - \frac{0.0182}{0.257113^{\frac{1}{3}}} - 0.0480648 \times 0.257113^2\right)$$

Final Attempt

The above attempts produced max ROC off from the expected. It has to be through Cl that connects the dots which should be around 0.257.

$$C_L = \frac{-T}{2wk} \pm \left[\left(\frac{T}{2wk}\right)^2 + \frac{3C_{D0}}{k}\right]^{0.5}$$

$$= \frac{-20{,}000}{2 \times 100{,}000\ \times 0.0480648} \pm \left[\left(\frac{20{,}000}{2 \times 100{,}000\ \times 0.0480648}\right)^2 + \frac{3 \times 0.0182}{0.0480648}\right]^{0.5}$$

$$= 0.257113, -4.41816$$

If answer was found somehow to be 4454 fpm, then

$$m = \sqrt{\frac{100{,}000}{0.7 \times 1{,}771 \times 979.5 \times 0.257113}} = 0.565949 \sim 0.57\ Mach$$

$$V = 0.57 \times V_{sound} = 367.989085 \sim 368k\ n$$

Note: neglecting climb angle caused some deviation.

Q 15

propeller aircraft.

$S = 34.27\ m^2$, $span = 16.76m$, $C_{D0} = 0.02$, $e = 0.83$, $wing\ loading\ (w/S) = 1.23\ kPa$, $\eta_P = 0.8$

$$K = \frac{1}{\pi eAR} = \frac{1}{\pi \times 0.83 \times \frac{16.76^2}{34.27}} = 0.0467884$$

$$W = 42{,}152.1\ N$$

$$\rho(5{,}000) = 0.73647\frac{kg}{m^3}$$

At 185 KEAS and 5,000m alt, max power is applied with constant trim leading to 1000 fpm climb.

- **Climb angle.**

$$V_e = 185\ kt\ \left(95.172\frac{m}{s}\right)$$

$$V = \frac{1}{\sqrt{\sigma}}V_e = \frac{1}{\sqrt{0.6012}}95.172 = 122.744\frac{m}{s}$$

$$ROC = v\sin\gamma \therefore \gamma = \sin^{-1}\left(\frac{ROC}{v}\right) = \sin^{-1}\left(\frac{5.08}{122.744}\right) = 2.372°$$

- **Maximum shaft power**

$$\eta_P = \frac{TV}{P_{shaf}}$$

$$P_{shaft} = \frac{TV}{\eta_p} = \frac{\square}{\square}$$

Second attempt the thrust is around 6kN so to say

$$V = 122.744\frac{m}{s}$$

$$\gamma = \frac{T}{w} - \frac{C_D}{C_L}|_{SLF}$$

$$T = w\left(\gamma + \frac{C_D}{C_L}|_{SLF}\right) = 42{,}152.1\ (0.0419 + \sqrt{4 \times 0.02 \times 0.0467884})$$

$$P_{shaft} = \frac{TV}{\eta_p} = \frac{10{,}0000 \times 122.744}{0.8}$$

Third attempt derive from the maximum rate of climb

$$V_{ROC_{max}} = \sqrt{\frac{2}{\rho}\frac{w}{S}\sqrt{\frac{K}{3C_{D0}}}}$$

Or

$$ROC_{max} = \frac{\eta P}{w} - \frac{2}{\rho}\sqrt{\frac{K}{3C_{D0}}}\left(\frac{w}{S}\right)^{0.5}\frac{1.155}{\left(\frac{L}{D}\right)_{max}}$$

$$P = \frac{w}{\eta}\left(ROC_{max} + \frac{2}{\rho}\sqrt{\frac{K}{3C_{D0}}}\left(\frac{w}{S}\right)^{0.5}\frac{1.155}{\left(\frac{L}{D}\right)_{max}}\right)$$

$$= \frac{42{,}152.1}{0.8}\left(5.08 + \frac{2}{0.73647}\sqrt{\frac{0.0467884}{3 \times 0.02}}\left(\frac{42{,}152.1}{34.27}\right)^{0.5}\frac{1.155}{1/\sqrt{4 \times 0.02 \times 0.0467884}}\right)$$

$$= 58.08\ kwatt$$

Note: maximizing the second term in the above equation leads to higher shaft power.

Q 16

turbojet aircraft.

$S = 40.25\ m^2$, $K = 0.05$, $C_{D0} = 0.025$, $w = 20{,}000\ kg$, $T_{\max}(\text{SSL}) = 100\ \text{kN}$

In SLF, maximum thrust is applied with constant trim at 200 kt speed.

➔ **Climb angle and Mach number.**

Final Attempt

$$\boldsymbol{V = 200\ kt\ (102.889\frac{m}{s})}$$

$$\boldsymbol{T_A = 100\ kN}$$

The velocity is not correct for some reason (0.3 Mach but should be 0.286 Mach somehow
Find the angle of the maximum rate of climb for jet.

$$V_{ROC_{max}} = \sqrt{\left(\frac{\frac{T}{S}}{3\rho C_{D0}}\right)\left(1+\sqrt{1+\frac{3}{\left(\frac{L}{D}\right)_{max}^2\left(\frac{T}{W}\right)^2}}\right)}$$

$$= \sqrt{\left(\frac{\frac{100,000}{40.25}}{3\times 1.225\times 0.025}\right)\left(1+\sqrt{1+\frac{3}{\frac{1}{4\times 0.025\times 0.05}\left(\frac{100,000}{20,000\times 9.81}\right)^2}}\right)}$$

$$= 234.208\ m/s$$

That is off for sure and not correct.

➔ **ROC (fpm). No inertial correction.**

$$ROC = v\sin\gamma = 102.889\ \sin 26.9° = 46.5506\frac{m}{s}\ ()$$

Second attempt

$$C_{L_{Max\ ROC}} = -\frac{T}{2wK} \pm \left[\left(\frac{T}{2wK}\right)^2 + \frac{3C_{D0}}{K}\right]^{0.5}$$

$$= -\frac{100,000}{2\times 20,000\times 9.81\times 0.05} \pm \left[\left(\frac{100,000}{2\times 20,000\times 9.81\times 0.05}\right)^2 + \frac{3\times 0.025}{0.05}\right]^{0.5}$$

$$= 0.1451$$

$$ROC = \left(\frac{2W}{\rho S}\right)^{0.5}\left[\frac{T}{W}\frac{1}{\sqrt{C_L}} - \frac{C_{D0}}{C_L^{\frac{1}{3}}} - KC_L^2\right]$$

$$= \left(\frac{2\times 20,000\times 9.81}{1.225\times 40.25}\right)^{0.5}\left[\frac{100,000}{20,000\times 9.81}\frac{1}{\sqrt{0.1451}} - \frac{0.025}{0.1451^{1/3}} - 0.05\times 0.1451^2\right]$$

Final Attempt

Assuming angle is 26.9 and Mach speed is 0.286

$$v = 0.286\times 0.340.4 = 97.3544\frac{m}{s}\quad \therefore ROC = 97.3544\ \sin 26.9° = 44.05\frac{m}{s}\ (8,671.26\ fpm)$$

Q 17

Twin-turbofan aircraft.

$$S = 80\ m^2,\ K = 0.06,\ C_{D0} = 0.018,\ w = 20,000\ kg,\ T_{eng}(10\text{ km alt. } 0.6\text{M}) = 50\text{ kN}\quad 10\text{ km alt. } 0.6\text{M}$$

➔ **ROC and climb angle (inertially uncorrected & corrected) (Ans: 26.9, 15695 fpm, 16485 fpm, 27.7)**

Uncorrected

$$V = 0.6\times 299.8 = 179.88\frac{m}{s}$$

$$V_e = 0.6\times 343 = 205.8\frac{m}{s}$$

$$\theta = \sin^{-1}\left[\frac{T}{w} - 0.5\rho V^2\left(\frac{w}{S}\right)^{-1} C_{D0} - \frac{w}{S}\frac{2k}{\rho V^2}\right]$$

$$\theta = \sin^{-1}\left[\frac{100,000}{20,000\times 9.81} - 0.5\times 0.41356\times 179.88^2\left(\frac{20,000\times 9.81}{80}\right)^{-1} 0.018 - \frac{20,000\times 9.81}{80}\frac{2\times 0.06}{0.41356\times 179.88^2}\right] = 26.01°$$

$$ROC = v\sin\theta = 179.88\ \sin 26.01 = 78.8824\ m/s\ (15528\ fpm)$$

Corrected

climb at constant Mach => $1-0.133\,M^2 = 1-0.133\times 0.6^2 = 0.95212$

$$ROC_{corrected} = \frac{ROC_{uncorrected}}{1+\frac{V}{g}\frac{dV}{dh}} = \frac{78.8824\,fpm}{0.95212} = 82.85\ m/s\ (16308.9\ fpm)$$

$$\gamma = sin^{-1}\left(\frac{ROC}{v}\right) = sin^{-1}\left(\frac{82.85}{179.88}\right) = 27.42\,°$$

Q 18

turbofan aircraft.

$$S = 100m^2,\quad span = 25m,\quad e = 0.8,\quad K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.8\times\frac{25^2}{100}} = 0.0637,\quad C_{D0} = 0.015,\quad W = 200\ KN$$

$T_0 = 200\ KN$, net thrust at SSL.

$T(h) = T_0 - 10h(m)$, no A/B

$$T(5000) = 200{,}000 - 10\times 5{,}000 = 150{,}000\ N$$

$T(h) = 1.5\times T(h)$, with A/B

150 kt SLF at SSL

$$\rho(5{,}000m) = 0.73647\ kg/m^3$$

➔ **Climb angle and rate at 200 KEAS and 5000m alt. with no A/B. inertial correction error percentage.**

$$V_e = 200\ kt = 102.889\frac{m}{s}$$

$$V = \frac{1}{\sqrt{\sigma}}V_e = \frac{1}{\sqrt{0.6012}}102.889 = 132.696\frac{m}{s}$$

$$\theta = sin^{-1}\left[\frac{T}{w} - 0.5\rho V^2\left(\frac{w}{S}\right)^{-1}C_{D0} - \frac{w}{S}\frac{2k}{\rho V^2}\right]$$

$$= sin^{-1}\left[\frac{150{,}000}{200{,}000} - 0.5\times 0.73647\ \times 132.696^2\left(\frac{200{,}000}{100}\right)^{-1}0.015 - \frac{200{,}000}{100}\frac{2\times 0.0637}{0.73647\times 132.696^2}\right] = 42.98\,°$$

$$ROC = vsin\theta = 132.696\ sin\ 42.98° = 90.4646\frac{m}{s}\ (17808\ fpm)$$

Climb at constant V_e

$$\delta = \frac{P}{P_\infty} = \frac{0.5405}{1.01325} = 0.533$$

Correction Factor $1+\frac{v}{g}\frac{dv}{dh} = 1+4.9\times 10^{-6}\left(\frac{V_E^2}{\delta}\right) = 1+4.9\times 10^{-6}\left(\frac{102.889^2}{\frac{0.5405}{1.01325}}\right) = 1.09724$

$$ROC_{corrected} = \frac{ROC_{uncorrected}}{f} = \frac{17808}{1.09724}$$

Q 19

Piston-engine aircraft.

$$S = 16.7m^2,\quad span = 25m,\quad K = 0.066,\quad C_{D0} = 0.022,\quad W = 18\ KN$$

$P_{max.\ shaft}|_{SSL} = 435\ KW,\quad 1200\ rpm,\quad 3.43m\ diameter,$ control beta mode below full power.

150 kt SLF at SSL

➔ **Full power and no trim change, find climb angle and rate of climb. (Ans: 2900 fpm, 11°)**

Assumption max power is applied.

$$ROC = \frac{P}{w} - \left(\frac{2w}{\rho S}\right)^{0.5}\left[\frac{C_{D0}}{C_L^{1.3}} + k\sqrt{C_L}\right] = \frac{435000}{18000} - \left(\frac{2\times 18000}{1.225\times 16.7}\right)^{0.5}\left[\frac{0.022}{1^{1.3}} + 0.066\sqrt{1}\right] = 4030\ fpm$$

$$V_{max\ P} = \sqrt{2\frac{(W/S)}{\rho}\sqrt{\frac{K}{3C_{D0}}}} = \sqrt{2\times\frac{\frac{18000}{16}.7}{1.225}\sqrt{\frac{0.066}{3\times 0.022}}} = 41.9493\ m/s$$

$$= sin^{-1}\left[\frac{P}{wv} - \frac{1}{2}\rho V^2\left(\frac{w}{S}\right)^{-1}C_{d0} - \frac{w}{S}\frac{2k}{\rho v^2}\right]$$

$$= sin^{-1}\left[\frac{435{,}000}{18{,}000\times 77.17} - \frac{1}{2}\times 1.225\times 77.17^2\left(\frac{18{,}000}{16.7}\right)^{-1}0.022 - \frac{18{,}000}{16.7}\frac{2\times 0.066}{1.225\times 77.17^2}\right] = 12.66°$$

$$ROC = \frac{P}{w} - \left(\frac{2w}{\rho S}\right)^{0.5}\left[\frac{C_{D0}}{C_L^{1.3}} + k\sqrt{C_L}\right] = \frac{435{,}000}{18{,}000} - \left(\frac{2\times 18000}{1.225\times 16.7}\right)^{0.5}\left[\frac{0.022}{0.2955^{1.3}} + 0.066\sqrt{0.2955}\right] = 18.1594\ m/s$$

Final Attempt

Max power

$$ROC = \frac{P_{max}}{w}$$

$$V_0 = 150\ kt \sim 77.17\frac{m}{s}\ \ at\ SSL$$

$$P_{max} = 435\ kW$$

$$C_L = \frac{2w}{\rho SV^2} = \frac{2\times 18{,}000}{1.225\times 16.7\times 77.17^2} = 0.2955$$

$$ROC = \frac{P}{w} - V\left[\frac{1}{2}\rho v^2\left(\frac{w}{S}\right)^{-1}C_{D0} + \frac{w}{s}\frac{2k}{\rho v^2}\right] = \frac{435{,}000}{18{,}000} - 77.17\left[0.5\times 1.225\times 77.17^2\left(\frac{18{,}000}{16.7}\right)^{-1}0.022 + \frac{18{,}000}{16.7}\frac{2\times 0.066}{1.225\times 77.17^2}\right]$$

If somehow it was found that $\gamma = 11°$, then

$$ROC = vsin\gamma = 77.17\ sin\ 11° = 14.72\frac{m}{s}\ (2900\ fpm)$$

➔ **10° climb with no change to power, find climb airspeed. (Ans: 130 kt 66.88 m/s)**

$$\gamma = 10°\ and\ ROC = vsin\gamma \therefore v = ROC/sin\gamma$$

Roc should be 11.6.

$$C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3\times 0.022}{0.066}} = 1$$

$$ROC = \frac{P}{w} - \left(\frac{2w}{\rho S}\right)^{0.5}\left[\frac{C_{D0}}{C_L^{1.3}} + k\sqrt{C_L}\right] = \frac{435000}{18000} - \left(\frac{2\times 18000}{1.225\times 16.7}\right)^{0.5}\left[\frac{0.022}{1^{1.3}} + 0.066\sqrt{1}\right] = 20.4751\frac{m}{s}$$

$$V_{max\ P} = \sqrt{2\frac{(W/S)}{\rho}\sqrt{\frac{K}{3C_{D0}}}} = \sqrt{2\times\frac{\frac{18000}{16}.7}{1.225}\sqrt{\frac{0.066}{3\times 0.022}}} = 41.9493\ m/s$$

Maximum rate of climb for a propeller driven aircraft

Condition $\left(\frac{C_L^3}{C_D^2}\right)_{Max}$ where $C_L = \sqrt{\frac{3C_{D0}}{K}}$ $C_D = 4C_{D0}$ at minimum required power to max out the excess power

$$P_r = w\sqrt{\frac{w}{S}\frac{2}{\rho}\left(\frac{C_D^2}{C_L^3}\right)} = w\sqrt{\frac{w}{S}\frac{2}{\rho}\left(\frac{C_D^2}{C_L^3}\right)}$$

$$\therefore \frac{P_r}{w} = \sqrt{\frac{w}{S}\frac{2}{\rho}\left(\frac{C_D^2}{C_L^3}\right)} = \sqrt{\frac{18{,}000}{16.7}\frac{2}{1.225}\left(\frac{(4\times 0.022)^2}{\left(\sqrt{\frac{3\times 0.022}{0.066}}\right)^3}\right)} = 3.69154$$

$$ROC = \frac{P_A - P_r}{w} = \frac{435{,}000}{18{,}000} - 3.69154 = 20.475\frac{m}{s}$$

$$v = \frac{ROC}{sin\gamma} = \frac{20.475}{sin 10°} = 117.911$$

Second attempt

Airspeed at maximum rate of climb give 10 degree

$$V_{max\ P} = \sqrt{2\frac{(W/S)}{\rho}\sqrt{\frac{K}{3C_{D0}}}} = \sqrt{2\times\frac{\frac{18000}{16.7}}{1.225}\sqrt{\frac{0.066}{3\times 0.022}}} = 41.9493\ m/s$$

Maximum rate of climb then cuz there is no other way to get to the core of the problem.

$$ROC_{max} = v\sin\gamma \therefore v = \frac{ROC}{sin}$$

$$\sin\gamma = \frac{P}{wv} - \left[\frac{1}{2}\rho v^2\left(\frac{w}{S}\right)^{-1}C_{d0} + \frac{w}{S}\frac{2k}{\rho v^2}\right]$$

$$sin\ 10° = \frac{435{,}000}{18{,}000v} - \left[0.5\times 1.225v^2\left(\frac{18{,}000}{16.7}\right)^{-1}0.022 + \frac{18{,}000}{16.7}\frac{2\times 0.066}{1.225v^2}\right]$$

$$V = \frac{4(w/S)k}{\rho(P/w)} = \frac{4\left(\frac{18{,}000}{16.7}\right)0.066}{1.225\times(\frac{435{,}000}{18{,}000})}$$

Final attempt

$$\gamma = 10°$$

$$P = 435\ kW\ (kN\cdot m/s)$$

$$w = 18{,}000\ kN$$

$$v_0 = 150\ kt\ (77.17\ m/s)$$

Condition: max power and climb angle of 10

$$ROC = v\sin\gamma = v\sin 10° = 0.173648v$$

$$ROC = \frac{P}{w} - v\left[\frac{1}{2}\rho v^2\left(\frac{w}{S}\right)^{-1} C_{do} + \frac{w}{S}\frac{2k}{\rho v^2}\right]$$

$$v \sin 10° = \frac{435{,}000}{18{,}000} - v\left[\frac{1}{2} \times 1.225 \times 0.022 \times v^2\left(\frac{18{,}000}{16.7}\right)^{-1} + \frac{18{,}000}{16.7}\frac{2 \times 0.066}{1.225 \times v^2}\right] = 85.8403$$

$$\sin\gamma = \frac{\rho v^2 S}{4kw} - \left[\left(\frac{\rho v^2 S}{4kw}\right)^2 - \left[\frac{T\rho v^2 S}{2kw^2} - \frac{C_{d0}(\rho v^2 S)^2}{4kw^2} - 1\right]\right]^{0.5}$$

$$\sin 10° = \frac{1.225 \times 16.7v^2}{4 \times 0.066 \times 18{,}000} - \left[\left(\frac{1.225 \times 16.7v^2}{4 \times 0.066 \times 18{,}000}\right)^2 - \left[\frac{435{,}000 \times 1.225 \times 16.7v}{2 \times 0.066 \times 18{,}000^2} - \frac{0.022(1.225 \times 16.7v^2)^2}{4 \times 0.066 \times 18{,}000^2} - 1\right]\right]^{0.5}$$

$$v = 85.9341\frac{m}{s} \quad or\ 165.2\ kn$$

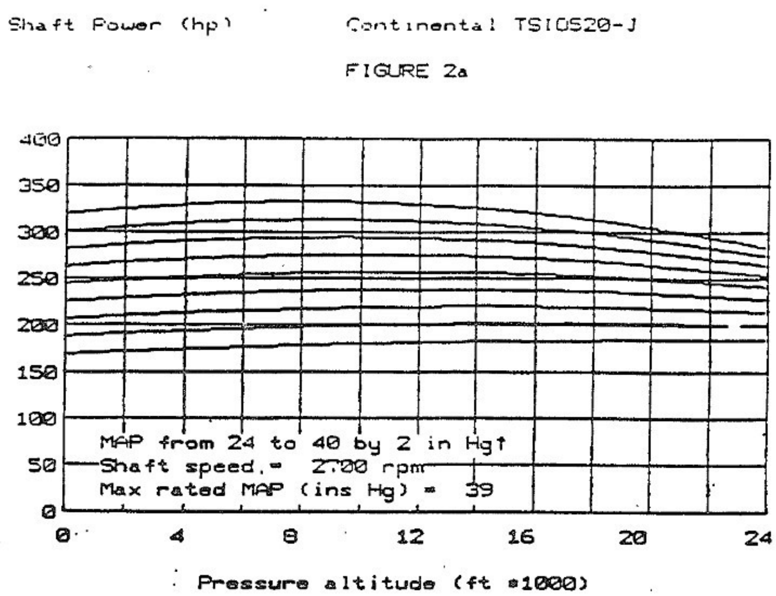
Q 20

Twin-piston engine aircraft.

$$S = 294\ ft^2, \quad span = 49.2\ ft, \quad e = 0.75, \quad W = 7865\ lbf$$

$$dia = 6.56\ ft, \quad 2700\ rpm$$

$$K = \frac{1}{\pi eAR} = \frac{1}{\pi \times 0.75 \times \frac{49.2^2}{294}} = 0.0515473$$



Maximum speed at 8,000 ft alt. and max rated MAP is 180 KIAS

➔ **Max ROC (fpm) at pressure alt. zero, OAT -20°C and 38 in Hg for MAP**

$$T = 15.1 - (20) = -4.9 = 268.25\ K$$

$$MAP = 38\ in\ Hg\ (128.683\ KPa\ )$$

$$P_a|_{38\ in\ Hg} = 300\ hp = 223.7\ kWatt$$

$$C_L = \sqrt{\frac{3C_{D0}}{K}}$$

$$ROC = \frac{P}{w} - \left(\frac{2w}{\rho S}\right)^{0.5}\left[\frac{C_{D0}}{C_L^{1.3}} + k\sqrt{C_L}\right] = \frac{300\ hp}{7865\ lbf} - \left(\frac{2 \times 7865}{1.225 \times 294}\right)^{0.5}\left[\frac{0.022}{1^{1.3}} + 0.0515473\sqrt{1}\right]$$
$$= 20.4751\frac{m}{s}$$

$$V_{max} = 180\ KIAS\ or\ 92.6\ m/s$$

$$\rho_{8000\ ft\ alt.} = 0.9636\ kg/m^3$$

$$P_{8000\ ft\ alt.} = 325\ hp$$

Final Attempt

$$DA = PA + 120(OAT - 20°C) = 0 + 120(15.1 - 20) = -588\ m$$

$$\rho = 1.0578232 \times 1.225 = 1.29583342\ kg/m^3 (0.104828\ lb/ft^3)$$

$$P|_{PA\ zero} = 300\ hp\ or\ 594\ klbf \cdot ft/s$$

$$ROC = v \sin\gamma$$

So, we have the power, density for a propeller aircraft.

The given condition that max speed at 8 kft elevation at max MAP is 180 KIAS (303.7795 ft/s)

$$V_{max} = 180\ KIAS\ or\ 92.6\ m/s$$

$$\rho_{8000\ ft\ alt.} = 0.9636\ kg/m^3$$

$$P_{8000\ ft\ alt.} = 325\ hp$$

We might need to find C_l and probably C_{d0} or speed.

$$D = w\frac{C_D}{C_L}$$

$$C_D = C_{D0} + kC_L^2$$

$$ROC = \frac{P}{w} - \left(\frac{2w}{\rho S}\right)^{0.5}\left[\frac{C_{D0}}{C_L^{1.3}} + k\sqrt{C_L}\right] = \frac{594\ lbf\frac{ft}{s}}{7865\ lbf} - \left(\frac{2 \times 7865}{0.104828 \times 294}\right)^{0.5}\left[\frac{0.022}{1^{1.3}} + 0.0515473\sqrt{1}\right]$$
$$= 20.4751$$

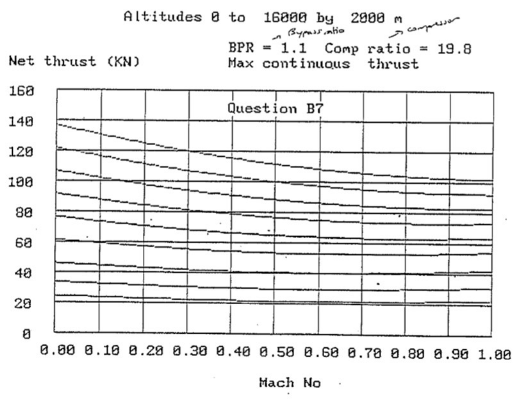
➔ **Climb/descend angle at IOE climb.**

Q 21

Twin-turbofan aircraft.

$$S = 80m^2, \quad K = 0.05, \quad C_{D0} = 0.018, \quad W = 150\ KN$$

SSL Landing at $20°$ *and* 20% over Stall Speed with full flaps and airbrakes



➔ **Aircraft is climbing at constant 0.5M. Find inertially corrected rate of climb and climb angle at 10 Km alt. (Ans: 20250 fpm, 43.5°)**

Final Attempt

$$T = 75\ kN$$

$$V = 0.5 \times 295.2 = 147.6 \frac{m}{s}$$

$$V_e = 0.5 \times 343 = 171.5 \frac{m}{s}$$

$$\theta = sin^{-1}\left[\frac{T}{w} - 0.5\rho V^2 \left(\frac{W}{S}\right)^{-1} C_{D0} - \frac{w}{S}\frac{2k}{\rho V^2}\right]$$

$$sin^{-1}\left[\frac{110{,}000}{150{,}000} - 0.5 \times 0.41356 \times 147.6^2 \left(\frac{150{,}000}{80}\right)^{-1} \times 0.018 - \frac{150{,}000}{80}\frac{2 \times 0.05}{0.41356 \times 147.6^2}\right] = 42.01°$$

$$ROC = v \sin\gamma = 147.6 \sin 42.01° = 98.7828 \frac{m}{s} \quad (19445.4\ fpm)$$

Inertial correction

climb at constant Mach => $1 - 0.133\ M^2 = 1 - 0.133 \times 0.5^2 = 0.96675$

$$ROC_{corrected} = \frac{ROC_{uncorrected}}{1 + \frac{V}{g}\frac{dV}{dh}} = \frac{19445.4\ fpm}{0.96675} = 102.2\ m/s\ (20{,}114\ fpm)$$

$$\gamma = sin^{-1}\left(\frac{ROC}{v}\right) = sin^{-1}\left(\frac{102.2}{147.6}\right) = 43.82\ °$$

Q 22

Military STOL aircraft.

$$S(full\ flaps) = 46.45\ m^2, \quad C_{D0}(flaps, gear\ down, without\ airbrakes) = 0.08, \quad C_{Lmax} = 2.5,$$
$$e = 0.85, \quad AR = 8, \quad W = 4536\ kg$$

$$K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.85 \times 8} = 0.0468$$

SSL Landing at $20°$ *and* 20% over Stall Speed with full flaps and airbrakes

➔ **Approach Speed (Kt) (Ans: 58.3 kt)**

$$V = 1.2\,V_{stall} = 1.2\sqrt{\frac{w}{0.5\rho SC_{Lmax}}} = 1.2\sqrt{\frac{4536\times 9.81}{0.5\times 1.225\times 46.45\times 2.5}} = 30.0149\frac{m}{s}\ (58.344\ kt)$$

$$C_L = 1.7361$$

➔ **ROD (fpm) (Ans: 2020 fpm)**

$$ROD = V\sin\gamma = 30.0149\sin(20) = 10.2657\frac{m}{s}\ (2020.8072\ fpm)$$

➔ **C_{D0} with airbrakes extended (Ans: 0.49)**

$$C_D(no\ brakes) = C_{D0} + KC_L^2 = 0.08 + 0.0468(2.5)^2 = 0.3725$$

$$ROD = \sqrt{\frac{w}{0.5\rho S(C_L^3/C_D^2)}}$$

$$C_D(with\ brakes) = \sqrt{\frac{10.2657^2(0.5\times 1.225\times 46.45\times 1.7361^3)}{4536\times 9.81}} = 0.593780$$

$$C_{D0}(with\ brakes) = C_D(with\ brakes) - KC_L^2 = 0.593 - 0.0468(1.736)^2 = 0.452722777772$$

It would be through the amount of drug added. Cd increases while maintaining constant Cl.

➔ **Reverse thrust substituting airbrakes (Ans: -10 KN)**

It would be through the difference in the additional drag to account for

$$\Delta(T-D)|_{reverse\ thrust} = \Delta(T-D)|_{airbrakes}$$

$$C_L = 1.7361$$

$$\therefore C_D = C_{D0} + kC_L^2 = 0.49 + 0.0468\times 1.7361^2 = 0.631057222228$$

$$D = w\frac{C_D}{C_L} = 4536\times 9.81\ \frac{0.631057222228}{1.7361} = 16{,}174.6877 \sim 16\ kN$$

$$\therefore C_D(without\ air\ brakes) = C_{D0} + kC_L^2 = 0.08 + 0.0468\times 1.7361^2 = 0.631057222228$$

$$D = w\frac{C_D}{C_L} = 4536\times 9.81\ \frac{0.221057}{1.7361} = 16{,}174.6877\ N = 5{,}665.9 \sim 6\ kN$$

$$\Delta T = 6 - 16 = -10\ kN$$

Q 23

Military STOL aircraft.

$S(full\ flaps) = 46.45\ m^2$, $C_{D0}(flaps, gear\ down, without\ airbrakes) = 0.08$, $C_{Lmax} = 2.5$, $e = 0.85$, $AR = 8$, $W = 4536\ kg$

$$K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.85\times 8} = 0.0468103\,,\qquad W = 200\ KN$$

SSL Landing at $20°$ *and* 20% over Stall Speed with full flaps and airbrakes

3g level turn at SSL, M0.78 (265.434 m/s).

➔ **Maximum time above 3000m (Ans: 1310 s)**

$$\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right) = \frac{1}{4}\left(\frac{3}{kC_{Do}^{1/3}}\right)^{\frac{3}{4}} = \frac{1}{4}\left(\frac{3}{0.0468103\times 0.03^{1/3}}\right)^{\frac{3}{4}} = 13.6064$$

$$\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right)^2 = 13.6064^2 = 185.134120$$

$$\dot{h} = \sqrt{\frac{2w}{\rho S C_L^3/C_D^2}} = \sqrt{\frac{2 \times 4536 \times 9.81}{0.58575 \times 1.225 \times 46.45 \times 185.135}} = 3.79774$$

$$\Delta t = -\frac{\Delta h}{\dot{h}} = \frac{5000}{3.7977} = 1316.586\ s$$

➔ **Range with a 10 kt tailwind (Ans: 69.1 km)**

$$C_L = \sqrt{\frac{3C_{DO}}{K}} = 2.27$$

$$V = \sqrt{\frac{4536 \times 9.81}{0.5 \times 0.58575 \times 1.225 \times 46.45 \times 2.278}} = 43.878\frac{m}{s}$$

$$R = (43.878 + 10.289)1310 = 70.96\ km$$

➔ **1000 kg weight reduced, extra time and distance. (Ans: 176s 1 km)**

$$\dot{h} = \sqrt{\frac{2w}{\rho S C_L^3/C_D^2}} = \sqrt{\frac{2 \times 3536 \times 9.81}{0.58575 \times 1.225 \times 46.45 \times 185.135}} = 3.35308$$

$$\Delta t = -\frac{\Delta h}{\dot{h}} = \frac{5000}{3.35308} = 1491.166\ s$$

$$\text{extra time } \Delta\text{t} = 1491.166 - 1316.586 = 174.58s$$

$$C_L = \sqrt{\frac{3C_{DO}}{K}} = 2.278$$

$$V = \sqrt{\frac{3536 \times 9.81}{0.5 \times 0.58575 \times 1.225 \times 46.45 \times 2.278}} = 38.741\frac{m}{s}$$

$$R = (38.741 + 10.289)1491.166\ = 73.11\ km$$

$$\text{extra range } \Delta\text{R} \approx 1 - 2km$$

➔ **At 8000m, trim up or down to maintain minimum descent rate while losing altitude. (Ans: hold trim constant)**

To maintain a mini rate of descent, changing the pitch angle will significantly affect the range and descend rate.

Trim down-> higher sink rate and less time.

Trim up -> loss of speed and less range.

Sustained trim -> desired range and sink rate.

It is usually the best to hold the trim.

Q 24

Glider

$$C_{D0} = 0.015, \quad AR = 12, \quad Span = 15m, \; K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.92 \times 12} = 0.0288,$$

$$W\ (inc. pilot\ \&\ 500N\ ballast) = 4\ KN, \quad S = \frac{15^2}{12} = 18.75\ m^2$$

$$\rho(2000m) = \left(0.8217 \times 1.225 \frac{kg}{m^3}\right) = 1.0065825 \frac{kg}{m^3}$$

$$\rho(1500m) = \left(0.8638 \times 1.225 \frac{kg}{m^3}\right) = 1.058155 \frac{kg}{m^3}$$

$$\rho(500m) = \left(0.9529 \times 1.225 \frac{kg}{m^3}\right) = 1.1673025 \frac{kg}{m^3}$$

$$\rho_{avg}\ (2000m, 1500m) = 1.03236875 \frac{kg}{m^3}$$

$$\rho_{avg}\ (1500m, 500m) = 1.11272875 \frac{kg}{m^3}$$

Pressure alt. of 2000m glide against 10kt (-5.144 m/s) headwind to 500m ASL. During release, pilot trim for maximum air range. At 1500m, all ballast is dropped.

➔ **Indicated airspeeds and ground range. (Ans: 22 m/s, 20.6 m/s, 27.7 km)**

$$q = 0,5 \times \rho(h) \times V_{true}^2$$

$$V_{IAS} = \sqrt{\frac{2q}{\rho}}$$

Maximum range $C_D = 2C_{D0} = 2 \times 0.015 = 0.03, \quad C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.015}{0.0288}} = 0.722$

$$V|_{2000m\ alt.} = \sqrt{\frac{w}{0.5\rho SC_L}} = \sqrt{\frac{4000}{0.5 \times 1.03236875 \times 18.75 \times 0.722}} = 23.9254\ m/s$$

$$V|_{1500m\ alt.} = \sqrt{\frac{w}{0.5\rho SC_L}} = \sqrt{\frac{3500}{0.5 \times 1.11272875 \times 18.75 \times 0.722}} = 21.5568\ m/s$$

$$h' = \sqrt{\frac{w}{\rho S\left(\frac{C_L^3}{C_D^2}\right)}} \quad i.e.$$

$$h'|_{2000m\ alt.} = \sqrt{\frac{2 \times 4000}{1.03236875 \times 18.75\left(\frac{0.7723^3}{0.03^2}\right)}} = 0.996 \frac{m}{s}$$

$$h'|_{1500m\ alt.} = \sqrt{\frac{2 \times 3500}{1.11272875 \times 18.75\left(\frac{0.7723^3}{0.03^2}\right)}} = 0.896 \frac{m}{s}$$

$\Delta t = \frac{\Delta h}{h'}$ i.e. $\quad \Delta t|_{2000m\ alt.} = \frac{500}{0.996} = 502s \quad \Delta t|_{1500m\ alt.} = \frac{1000}{0.896} = 1115\ s$

$$R_g = (\Delta V \Delta t)|_{2000m\ alt.} + (\Delta V \Delta t)|_{1500m\ alt.}$$

$$R_g = (23.9254 - 5.114)(502) + (21.5568 - 5.144)(1115) = 27.743\ km$$

➔ **Will speed for maximum still air range be optimum speed for ground range in a headwind? (Ans: optimum speed higher v=25.9 m/s at the start of descent v=24.3 m/s)**

The best action is to sustain the trim because adjusting the descend angle will yield undesirable outcomes. Optimal seed should be higher recommended 20%-75% of headwind to avoid negative net velocity and to adequately counter the headwind. $V_{opt} = 0.25V_{head} + V = 0.25 \times 5.144 + 23.937 = 25.223\ m/s$

Q 25

Glider.

$$S = 80m^2, \quad C_{D0} = 0.018, \quad K = 0.05, \quad W = 125\ KN$$

At 11 km, the engine fails. After trim, 20° flap is deployed and 20 KN fuel dumped. A tailwind of 20 kt (+10.289 m/s) appeared.

$$\rho(11\ km) = 0.2978 \times 1.225 = 0.364805\ kg/m^3$$

$$\rho(2\ km) = 0.8217 \times 1.225 = 1.0065825\ kg/m^3$$

$$\rho_{avg}(11\ km, 2\ km) = 0.68569375\ kg/m^3$$

➔ **Comment on the pilot's actions.**

the flaps contribute to higher C_L therefore approaching $L/D|_{max}$ where the range is maximum.

➔ **No flaps used, find time taken and ground distance up to 2000m ASL. (Ans: $\Delta t = 1700s$, $R_g = 167\ Km$)**

Maximum air range $C_D = \frac{4}{3}C_{D0} = \frac{4}{3}0.018 = 0.024$, $\quad C_L = \sqrt{\frac{C_{D0}}{3K}} = \sqrt{\frac{0.018}{3\times0.05}} = 0.346410$

$$C_L = \sqrt{\frac{C_{D0}}{3k}} \quad C_D = \frac{4}{3}C_{D0}$$

$$V = \sqrt{\frac{w}{0.5\rho SC_L}} = \sqrt{\frac{2 \times 105{,}000}{1.225 \times 0.46 \times 80 \times 0.346410}} = 105.125\frac{m}{s}$$

$$h' = \sqrt{\frac{2w}{\rho S\left(\frac{C_L^3}{C_D^2}\right)}} = \sqrt{\frac{2 \times 105{,}000}{0.68569375 \times 80 \times \left(\frac{0.346410^3}{0.024^2}\right)}} = 7.28325\frac{m}{s}$$

$$\Delta t = \frac{\Delta h}{h'} = \frac{9{,}000}{7.28325} = 1{,}877.879\ s$$

$$R_g = (\Delta V \Delta t)$$
$$R_g = (+10.289)(1700)$$

Final Attempt

In a glide scenario, where jettison occurred, tailwind of 20 kt existed, and descent from altitude of 10 km with zero thrust took place.

Time taken

$$C_L = \sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.018}{3 \times 0.05}} = 0.346410 \quad C_D = \frac{4}{3}C_{D0} = \frac{4}{3}0.018 = 0.024$$

$$V = \sqrt{\frac{2w}{\rho SC_L}} = \sqrt{\frac{2 \times 105{,}000}{1.0065825 \times 80 \times 0.346410}} = 86.7650 \sim 87\frac{m}{s}\ or\ 169\ kt$$

$$h' = \sqrt{\frac{2w}{\rho S\left(\frac{C_L^3}{C_D^2}\right)}} = \sqrt{\frac{2 \times 105{,}000}{1.225 \times 80 \times (\frac{0.346410^3}{0.024^2})}} = 5.449 \sim 5.4\,\frac{m}{s}$$

$$\Delta h = 9000\ m$$

$$\Delta t = \frac{\Delta h}{\dot{h}} = \frac{9000\ m}{5.4\ m/s} = 1666.6666 \sim 1700\ s$$

Ground Range

$$V = V_t + V_{tail} = 171 + 20 = 191\ kt \sim 98.2\ m/s$$

$$R_g = (V\Delta t) = 98.2 \times 1700 = \sim 167\ km$$