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AE 3330 – Spring 24

AE 3330 PROBLEM SET

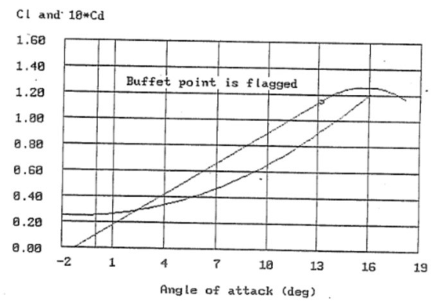
Content:

Q 1,2,3,4,5,6,7,8,9,10,11,12,13

Q 1

aircraft.

$\boldsymbol{w = 30\ KN,\ s = 25m^2}$ **at 11 km.** $\boldsymbol{\rho_{sea\ level} = 1.225\frac{kg}{m^3},\ \rho_{11km} = 0.364805,\ C_{Lbuff} = 1.15}$



$$\boldsymbol{C_{D0} = 0.025}$$

➔ **True and Equivalent speeds (kt).**

True ➔ real atmo

$$V = \sqrt{\frac{2w}{\rho S C_L}} = \sqrt{\frac{2 \times 30{,}000}{0.364805 \times 25 \times 1.15}} = 75.636\frac{m}{s}\ (147\ kt)$$

$$V_e = \sqrt{\frac{2w}{\rho_{SSL} S C_L}} = \sqrt{\frac{2 \times 30{,}000}{1.225 \times 25 \times 1.15}} = 41.275\frac{m}{s}\ (80.232\ kt)$$

➔ **Thrust power (kW) at 125 kt EAS. (Ans: 265 kW)**

$$V_e = 125\ kt\ (64.306\ m/s)$$

$$V = \sqrt{\sigma}\, V_e = \sqrt{0.2978} \times 64.306 = 35.0925\ m/s$$

First attempt

$$T = \frac{w}{(L/D)} = \frac{30{,}000}{(\frac{0.1}{2.5})}$$

Second attempt

$$P = TV = V_\infty \omega \frac{C_D}{C_L}|_{\alpha=0^\circ} = 35.0925 \times 30{,}000 \times \frac{0.025}{0.1} = 263193.75\ watt \approx 263.2\ kW$$

➔ **Thrust power at minimum drag speed.**

At minimum drag

Minimum Drag

need k to move forward.

$$k = \frac{1}{\pi e AR}$$

$$C_D = C_{D0} + kC_L^2\ \therefore C_{D0} = kC_L^2$$

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{2.5}{\square}} \qquad C_D = 2C_{D0} = 2 \times 0.025 = 0.05$$

$$C_L = \frac{2}{\rho V^2}\left(\frac{w}{S}\right)$$

$$V = \sqrt{\frac{2w}{\rho S C_L}} = \sqrt{\frac{2w}{\rho S}\sqrt{\frac{K}{C_{D0}}}} =$$

$$P = TV = V_\infty \omega \frac{C_D}{C_L}|_{\min drag} = \sqrt{\frac{2w}{\rho S C_L}} \times w \times \frac{2C_{D0}}{C_L} = \frac{w^{\frac{3}{2}}}{\sqrt{0.5\rho S}}\, 2\frac{C_{D0}}{C_L^{3/2}}$$

Flying lower than minimum drag speed, the drag will increase drastically thus not advised.

Second attempt

Minimum Drag speed.

$$C_L = \sqrt{\frac{C_{D0}}{K}} \qquad C_D = 2C_{D0}$$

Such that $T = D_{min} = \frac{w}{\left(\frac{L}{D}\right)_{max}} = w\sqrt{4C_{D0}K}$

$$P = TV = w\sqrt{4C_{D0}K\frac{2w}{\rho S C_L}}$$

I can't get rid of the k

Third attempt

K should be 0.166

$$\frac{L}{D}|_{md} = \frac{1}{\sqrt{2kC_{D0}}}$$

$$V_{md} = \left(\frac{2w}{\rho S}\sqrt{\frac{K}{C_{D0}}}\right)^{0.5}$$

$$P = TV = \omega\frac{C_D}{C_L}\left(\frac{2w}{\rho S}\sqrt{\frac{K}{C_{D0}}}\right)^{0.5} = w\sqrt{2kC_{D0}}\left(\frac{2w}{\rho S}\sqrt{\frac{K}{C_{D0}}}\right)^{0.5}$$

$$C_D = C_{D0} + KC_L^2 \therefore C_{D0} = KC_L^2 \quad then \quad K = \frac{C_{D0}}{C_L^2} = \frac{0.025}{0.15} = \frac{1}{6}$$

$$w\sqrt{2kC_{D0}}\left(\frac{2w}{\rho S}\sqrt{\frac{K}{C_{D0}}}\right)^{0.5} = 30{,}000\sqrt{2\times\frac{1}{6}\times 0.025\left(\frac{2\times 30{,}000}{1.225\times 25}\right)\sqrt{\frac{1}{6\times 0.025}}} = 194.78\ kW$$

Fourth attempt

$$C_D = 2C_{D0} = 2\times 0.025 = 0.05$$

From the graph

$$C_L|_{C_D=0.05} = 0.75$$

$$K = \frac{C_{D0}}{C_L^2} = \frac{0.025}{0.75^2}$$

$$C_D = 0.05$$

$$P = TV = 30{,}000\times\frac{0.05}{0.75}\left(\frac{2\times 30{,}000}{1.225\times 25\times 0.75}\right)^{0.5} = 102220$$

Fifth attempt

$$C_{Dbufft}$$

$$C_{D_{buf}} = C_{D0} + kC_{L_{buf}}^2$$

$$k = \frac{C_{D_{buf}} - C_{D0}}{C_{L_{buf}}^2} = \frac{0.09 - 0.025}{1.15^2} = 0.0491$$

$$0.09 = 0.025 + 0.0491\times 1.15^2$$

$$P = TV = w\sqrt{2kC_{D0}}\left(\frac{2w}{\rho S}\sqrt{\frac{K}{C_{D0}}}\right)^{0.5} = 30{,}000\sqrt{2\times 0.0491\times 0.025}\left(\frac{2\times 30{,}000}{1.225\times 25}\sqrt{\frac{0.0491}{0.0491}}\right)^{0.5} = 65793.$$

$$\approx 65.793\ kW$$

Sixth attempt

$$C_{D_{buf}} = C_{D0} + kC_{L_{buf}}^2$$

$$k = \frac{C_{D_{buf}} - C_{D0}}{C_{L_{buf}}^2} = \frac{0.09 - 0.025}{1.15^2} = 0.0491$$

Minimum Drag speed.

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.025}{0.0491}} = 0.713558 \quad C_D = 2C_{D0} = 2 \times 0.025 = 0.05$$

$$V_{md} = \left(\frac{2w}{\rho S C_{L_{md}}}\right)^{0.5} = \left(\frac{2 \times 30{,}000}{0.2978 \times 1.225 \times 25 \times 0.713558}\right)^{0.5} = 96.0198 \frac{m}{s}$$

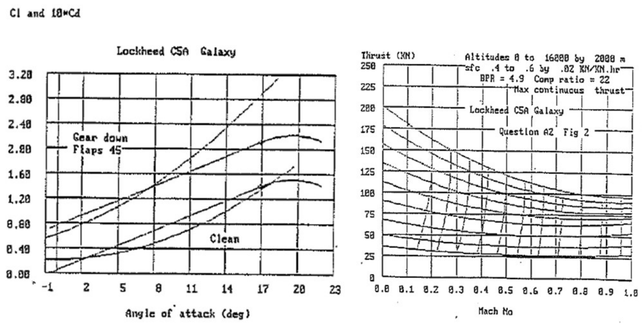
$$P = TV = \omega \frac{C_D}{C_L} V_{md} = 30{,}000 \times \frac{0.05}{0.713558} \times 96.0198 = 201847.2219\ Watt \approx 201.847\ kWatt$$

Q 2 (unlisted)

Four-turbofan aircraft.

$$\boldsymbol{w_{gross} = 3.25\ MN, \quad w_{max\ payload} = 980\ KN, \quad w_{max\ fuel} = 1.33\ MN,}$$

$$\boldsymbol{S = 576\ m^2, \quad A_{max\ flap\ per\ wing} = 27.9\ m^2}$$



➔ **Thrust power and % max thrust during M0.75 cruising at 11 km alt. with full payload and half full tanks.**

$$Mach\ 0.75\ at\ 100\ km\ alt.\ 221.4\ m/s$$

$$P = TV = 80{,}000 \times 221.4$$

83% of max thrust and 40.2 mw

$$4.895$$

$$P = TV = V_\infty \omega \frac{C_D}{C_L} = 221.4 \times (3.25 - 0.665) \times \frac{0.02}{0.1}$$

➔ **Indicated stall speed (kt), clean at 11 km alt. with 900 KN fuel and full payload.**

$$V_{stall} = \sqrt{\frac{2w}{\rho S C_L}} = \sqrt{\frac{2(3.25 - 1.33 + 0.9) \times 10^6}{0.2978 \times 1.225 \times 576 \times 1.4}}$$

➔ **Indicated stall speed, gear and flaps down, with 200 KN fuel, half payload at SSL.**

$$V_{stall} = \sqrt{\frac{2w}{\rho S C_L}} = \sqrt{\frac{2 \times (3.25 - 1.33 + 0.2 - 0.98 + 0.49)}{0.2978 \times 1.225 \times 576 \times 2.1}}$$

➔ **Pitch altitude as the main gear contacts a horizontal sea level runway given full payload, 200 N fuel, airspeed 15% above 1-g stall.**

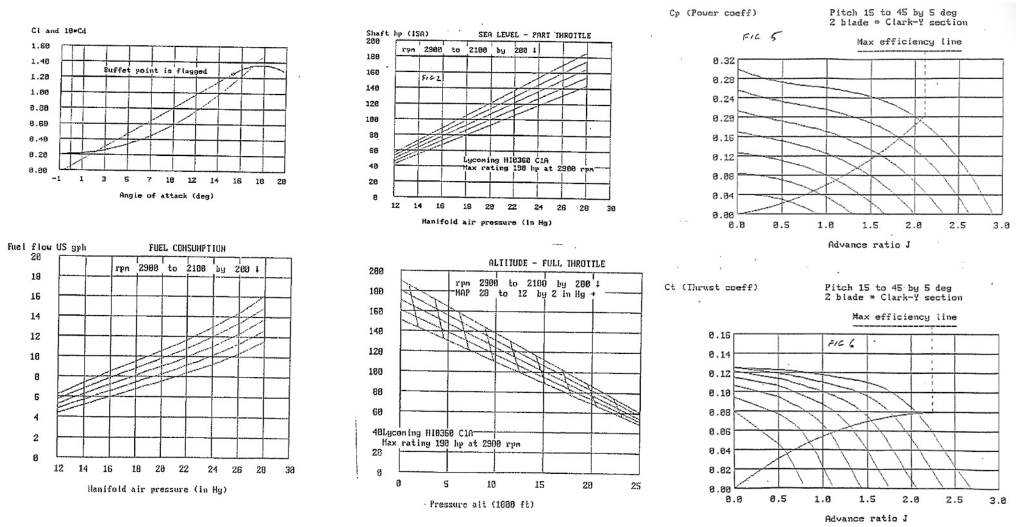
➔ **% max thrust use on approach to the landing on a 3-degree glideslope at V 15% above 1-g stall.**

Q 3

Piston-engine aircraft.

$$S = 16.72\ m^2, \quad dia = 1.75\ m, \quad fixed\ pitch\ 21°$$

Cruise at 1525 m alt. $OAT\ ISA\ +\ 10°$ $MAP\ 24\ in\ Hg$, prop shaft speed2800 rpm.



➔ **Cruise speed (kt) 120 kt (61.73)**

$$p = 24\ hg = 81\ kPa$$

Density altitude = Pressure altitude + [120 x (OAT - ISA temperature)]

$$1525\ m\ alt. \approx 5{,}000\ ft$$

$$5{,}000\ + [120 \times (5.3 + 10)] = 6836\ ft \approx 2084\ m$$

$$\rho_{den\ alt.} = 0.8147328 \times 1.225 = 0.99804768\ kg/m^3$$

$$P_{shaft} = \rho n = 150\ hp$$

$$\rho = \frac{P_{shaft}}{n} = \frac{111855\ watt}{2800/60}$$

$$\dot{m}_f = 12.5\ US\ gph$$

$$C_p = \frac{P_s}{\rho n_s^3 d^5} = \frac{111855\ watt}{0.99804768 \left(\frac{2800}{60}\right)^3 1.75^5} = 0.067188 \approx 0.067$$

$$C_p = \frac{P_s}{\rho n_s^3 d^5} = \frac{119312\ watt}{0.99804768 \left(\frac{2800}{60}\right)^3 1.75^5} = 0.071667 \approx 0.07$$

$$J \approx 0.75$$

Cruise speed

$$V = JnD = 0.75 \times \frac{2800}{60} \times 1.75 = 61.25\ m/s\ (119\ kt)$$

➔ **Weight (kN)**

Advance ratio from the power coefficient

$$C_t|_{J=0.75} = 0.062$$

$$T = D = nw\frac{C_D}{C_L} \approx w\frac{C_D}{C_L} \therefore w = T\frac{C_L}{C_D} = C_t \rho n_s^2 d^4 \frac{C_L}{C_D} = 0.062 \times 0.99804768 \times \left(\frac{2800}{60}\right)^2 \times 1.75^4 \frac{0.05}{0.02}$$
$$= 3350.01$$

$$C_L = \frac{2w}{V^2 S}$$

$$C_D = C_{D0} + KC_L^2$$

Second attempt

$$P = TV = w\frac{C_D}{C_L}V \therefore w = \frac{P}{V}\frac{C_L}{C_D} = \frac{111855}{61.25}\frac{0.05}{0.02}$$

$$w = T\frac{C_L}{C_D} = C_t\rho n^3 d^5 \frac{C_L}{C_D} =$$

Second attempt

By relying on the thrust only and neglecting the thrust power

$$w = V^2 0.5\rho S C_L = 61.25^2 \times 0.5 \times 0.99804768 \times 16.72 \times 0.2$$

C_L should be 0.655.

Thirst attempt.

At buffet.

$$k = \frac{C_D - C_{D0}}{C_L^2} = \frac{0.11 - 0.02}{1.25^2} = 0.0576$$

$$\sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.02}{3 \times 0.0576}} = 0.340207$$

$$\sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.02}{0.0576}} = 0.589256 \text{ at minimum drag}$$

$$\sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3 \times 0.02}{0.0576}} =$$

$$w = V^2 0.5\rho S C_L = 61.25^2 \times 0.5 \times 0.99804768 \times 16.72 \times 0.589256$$

$$w = T\frac{C_L}{C_D} = C_t\rho n_s^2 d^4 \frac{C_L}{C_D} =$$

C_L is not confined by something specifically (mini drag, mini power, max range)

Fourth attempt

We know ct and cp and the advance ratio as well as k

If we assume that the power is 160 hp

What would be the weight?

$$P = C_P\rho n^3 d^5 = \frac{119312 \ watt}{0.99804768\left(\frac{2800}{60}\right)^3 1.75^5} = 0.071667 \approx 0.07$$

$$P = TV \therefore T = \frac{P}{V} = \frac{119312}{61.25} = 1947.951 \ N$$

$$T = D = w\frac{C_D}{C_L} \therefore w = T\frac{C_L}{C_D} = 1947.951 \times \frac{0.05}{0.02}$$

Fifth attempt

Use the required thrust equation.

$$T = 0.5\rho S C_{D0} V^2 + \frac{2kw^2}{\rho S}\frac{1}{V^2} \rightarrow 1947.951$$

$$= 0.5 \times 0.99804768 \times 16.72 \times 0.02 \times 61.25^2 + \frac{2 \times 0.0576 \times w^2}{0.99804768 \times 16.72}\frac{1}{61.25^2}$$

$$= 626.0366353164 + 1.84015 \times 10^{-6} w^2$$

$$\therefore w = \sqrt{(1947.951 - 626.0366353164) 1.84015 \times 10^6} = 49320.6$$

$$P = TV = 0.5\rho S C_{D0} V^3 + \frac{2kw^2}{\rho S}\frac{1}{V}$$

$$119312 = 0.5 \times 0.99804768 \times 16.72 \times 0.02 \times 61.25^3 + \frac{2 \times 0.0576 \times w^2}{0.99804768 \times 16.72}\frac{1}{61.25^1}$$

$$w = 26.8\ kN$$

The answer is roughly 30% off.

➔ **Fuel used for 100 nm. (imperial gal)**

Fuel used for 100 nm kt=1 nm/hr.

$$\dot{m}_f = 12.5\ US\ gph \approx 10.416666\ imp\ gph$$

$$V = 119\ kt\ or\ 120\ kt\ \ i.e.kt = nm\ /hr$$

$$m_f = \dot{m}_f \Delta t|_{hour} = 10.416666 \frac{gal}{h} \times \frac{100\ nm}{120\ nm/h} = 8.680555\ imp.gal$$

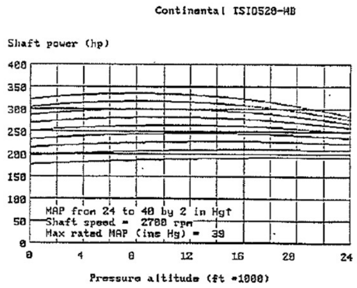
Q 4

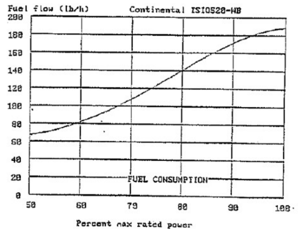
Two piston-engine aircraft.

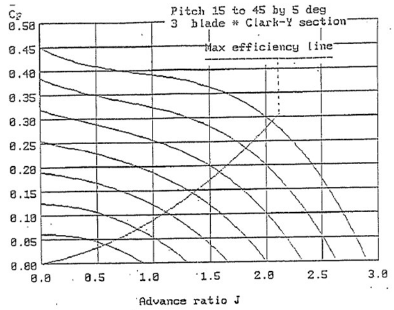
$$\boldsymbol{C_{D0} = 0.025, \quad S = 17.5\ m^2, \quad span = 11.5m, \qquad e = 0.75, \quad w = 26.7\ kN}$$

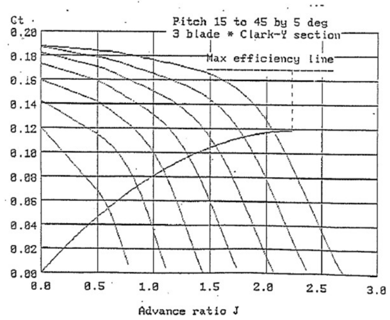
$$\boldsymbol{dia = 2m}$$

At 5000m alt., MAP and engine speed are set to maintain 250 kt in SLF with 20 kt headwind. Prop speed 2700 rpm









➔ **MAP and fuel (kg/nm) against 20-kt headwind.**

For the MAP, we need shaft power.

$$J = \frac{V}{nD} = \frac{128.6}{\frac{2700}{60} \times 2} = 1.43$$

We need the pitch angle to move on

$$1\ \frac{kg}{nm} = \frac{2.205\ lb/hr}{\frac{nm}{hr}} \quad so\ basically\quad m_{f1}\left(\frac{kg}{nm}\right) = \frac{2.205\ \dot{m}_f\ (lb/hr)}{V\ (kt)}$$

$$m_{f1}\left(\frac{kg}{nm}\right) = \frac{2.205 \times 150}{250 - 20} = 1.323\ kg/nm$$

Should be 0.75 1/SGR.

Second attempt

For the MAP, we need shaft power.

$$\rho(500\ m) = 0.73647\ kg/m^3$$

$$J = \frac{V}{nD} = \frac{128.6}{\frac{2700}{60} \times 2} = 1.43$$

Assuming operating at maximum efficiency

$$C_p = 0.15$$

Expected horsepower is 325 hp

$$P_s = C_p \rho n^3 d^5 = 0.15 \times 0.73647 \times \left(\frac{2700}{60}\right)^3 (2^5) = 322131.978\ watt$$

$$k = \frac{1}{\pi eAR} = \frac{1}{\pi 0.75 \times \frac{11.5^2}{17.5}} = 0.0561605$$

$$P = 0.5\rho S C_{D0} V^3 + \frac{2kw^2}{\rho S}\frac{1}{V^1} = 0.5 \times 0.73647 \times 17.5 \times 0.025 \times 128.6^3 + \frac{2 \times 0.0561605 \times 26{,}700^2}{0.73647 \times 17.5}\frac{1}{128.6}$$

$P_r = 390940\ watts = 524.3\ hp$ beyond the available

For the fuel used SGR at 100% max rated power

$$1\frac{kg}{nm} = \frac{2.205\ lb/hr}{\frac{nm}{hr}} \quad so\ basically \quad m_{f1}\left(\frac{kg}{nm}\right) = 2\frac{\dot{m}_f\ (lb/hr)}{2.205\ V\ (kt)}$$

$$\frac{1}{SGR} = m_{f1}\left(\frac{kg}{nm}\right) = 2\frac{190}{2.205(250-20)} = 0.7492 \approx 0.75\ kg/nm$$

Final Attempt

Step 1: find the MAP

MAP is tied to rpm, pressure altitude, and shaft power. If known the shaft power, the MAP can be found.

Solve for Shaft Power at 5000 m ➔ $\rho = 0.73647\frac{kg}{m^3},\ v = 250\ kt\ \left(128.611\frac{m}{s}\right)$

$$k = \frac{1}{\pi eAR} = \frac{1}{\pi 0.75 \times \frac{11.5^2}{17.5}} = 0.0561605$$

Range of Engine Shaft Power: 190 to 320 hp

Range of MAP: 24 to 40

Advance Ratio irrelevant ➔A **constant speed propeller** is a propeller that is designed to automatically change its blade pitch to allow it to maintain a constant RPM, irrespective of the amount of engine torque being produced or the airspeed or altitude at which the aircraft is flying. [https://skybrary.aero/articles/constant-speed-propeller]

$$J = \frac{V}{nD} = \frac{128.611}{\frac{2700}{60} \times 2} = 1.43$$

Shaft Power for a propeller engine

$$V = 270\ kt\ (138.9\ m/s)$$

$$P = DV = C_{d0}\ 0.5\rho S V^3 + \frac{kw^2}{0.5\rho S V}$$

$$0.025 \times\ 0.5 \times 0.73647 \times 17.5 \times 138.9^3 + \frac{0.0561605 \times 26700^2}{0.5 \times 0.73647 \times 17.5 \times 138.9}$$

$$476.46\ kW \sim 638.9\ hp \therefore P_{eng} = 319.45\ hp$$

From the chart, max MAP (39 in Hg) coincides with 320 hp at 16.4 kilofeet.

Step 2: SRG

$$1\frac{kg}{nm} = \frac{2.205\ lb/hr}{\frac{nm}{hr}} \quad so\ basically \quad m_{f1}\left(\frac{kg}{nm}\right) = 2\frac{\dot{m}_f\ (lb/hr)}{2.205\ V\ (kt)}$$

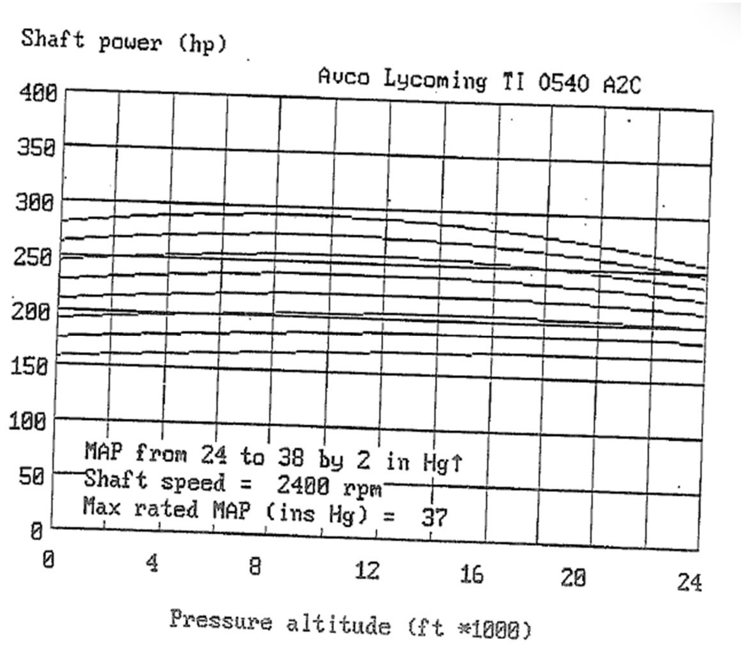
$$\frac{1}{SGR} = m_{f1}\left(\frac{kg}{nm}\right) = 2\frac{190}{2.205(250-20)} = 0.7492 \approx 0.75\ kg/nm$$

Q 5

Two piston aircraft.

$$S = 200\ ft^2,\ \ span = 41'\,5",\quad e = 0.7,\quad C\_D0 = 0.025,\quad w = 5500\ lb$$

Cruise at 10 kft pressure alt., OAT -15° C, 34 in Hg MAP, 2400 rpm prop shaft



➔ **True and equivalent cruise speed (kt) 220 kt, 189 kt, SAR = 0.75 nm/lb**

$$P_{shaft} = 255\ hp\ (190.153\ kW)$$

$$P = TV$$

$$T = D = w\ C_D/C_L$$

$$V = \frac{P}{T}$$

$$C_p = \frac{P_s}{\rho n_s^3 d^5} = \frac{255 \times 550\ ft\cdot lb/s}{0.05677\frac{lb}{ft^3} \times \left(\frac{2400}{60}\right)^3 6.3^5} = 0.06$$

$$J \approx 0.75$$

Cruise speed

$$V = JnD = 0.75 \times \frac{2800}{60} \times 1.75 = 61.25$$

Get the $\dot{m}_f$ from the shaft power.

$$\dot{m}_f = BSFC \cdot P_s$$

Final Attempt

Step 1: find Shaft Power

From the chart, given rpm, alt, MAP, the shaft power can be found so can as a result the velocity

$$Density\ alt. = pressure\ alt. + 120(OAT_{10,000ft} - 15°C) = 10,000 + 120(-4.8 - 15) = 7,624\ ft$$

$$\therefore \rho_{corrected} = 0.97\frac{kg}{m^3} = 0.060555\ lb/ft^3$$

$$P_{shaft} = 255\ hp\ (190.153\ kW)$$

$$C_p = \frac{P_s}{\rho n_s^3 d^5} = \frac{190153\ watts}{0.97\ kg/m^3 \times \left(\frac{2400}{60}\right)^3 1.9202^5} = 0.117$$

$$J = 1.47$$

$$V_t = JnD = 1.47 \times \frac{2400}{60} \times 6.3 = 370.44\frac{ft}{s}\ \ or\ 219.48\ kt$$

$$V = \sqrt{\sigma}\, V_t = \sqrt{0.7423} \times 219.48 = 189.097\ kt$$

➔ **Specific air range (nm/lb)**

Final Attempt

$$\dot{m}_f = 35\ US\ gph\ (292.075\ lb/hr)$$

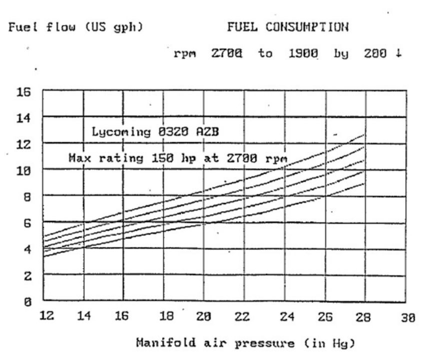
$$SAR = \frac{v(kt\ or\ \frac{nm}{hr})}{\dot{m}_f\ (lb/hr)} = \frac{220}{8.345 \times 35} = 0.75$$

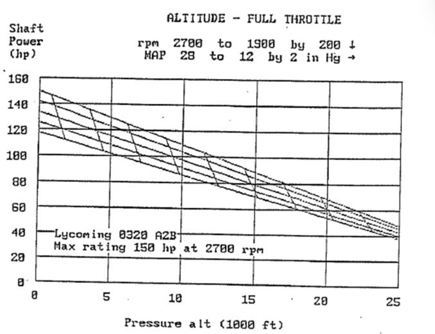
Q 6

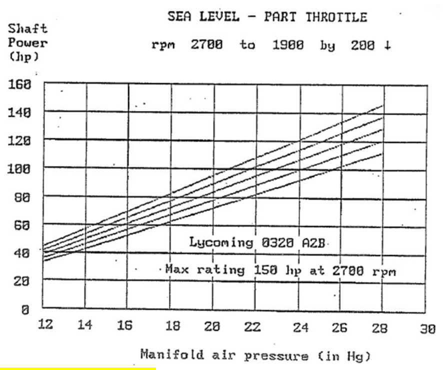
Two piston aircraft.

$$S = 16.59\ m^2, \quad span = 10.76\ m, \quad e = 0.814, \quad w = 7.5\ kN$$

Cruise at 1520 m pressure alt., OAT + 10° C, 21 in Hg MAP, 2500 rpm prop shaft







➔ **True speed (kt)**

$$\dot{m}_f = 8\ US\ gph$$

$$P_{shaft} = 100\ hp\quad - full\ throttle$$

$$P_{shaft} = 96\ hp - part\ throttle\ at\ sea\ level$$

Final Attempt

At the pitch angle 19°, the intersection with max efficiency line is chosen where $C_p = 0.4,\ J = 0.75$

$$J = 0.75$$

$$V = JnD = 0.75\left(\frac{2500}{60}\right)1.8 = 56.25\frac{m}{s} \sim 109.34\ kt$$

➔ **Zero-lift drag coefficient.**

First Attempt

$$C_L = \frac{2w}{\rho S V^2} = \frac{2 \times 7500}{0.877 \times 16.58 \times 56.25^2} = 0.326$$

$$C_D =$$

$$C_{D0} = C_D - kC_L^2 = -0.056 \times 0.326^2$$

$$C_{D0} = 0.031$$

Final attempt

$$DA = 1520 + (120\ x\ (5.272° + 10°)) = 3352.64\ m$$

$$\rho|_{84\quad alt.} = 0.71585 \times 1.225 = 0.877\ kg/m^3$$

$$k = \frac{1}{\pi e AR} = \frac{1}{\pi 0.814 \times \frac{10.76^2}{16.58}} = 0.056$$

$$P = TV = \frac{1}{2}\rho V^3 S C_{D0} + k\frac{2w^2}{\rho V S} \rightarrow C_{D0} = \left(P - k\frac{2w^2}{\rho V S}\right)\frac{1}{\frac{1}{2}\rho V^3 S}$$

$$= \left(P - 0.056 \times \frac{2 \times 7500^2}{0.877 \times 56.25 \times 16.58}\right)\frac{1}{\frac{1}{2}0.877 \times 56.25^3 \times 16.58} = 0.031$$

➔ **Specific range (nm/imp gal.) against 10-kft headwind**

Final Attempt

$$\boldsymbol{v = v_t - v_{headwind} = 110 - 10 = 100kt}$$

$$\boldsymbol{\dot{m}_f = 8\ US\ gal = 6.661 \sim 6.7\ imp\ gal}$$

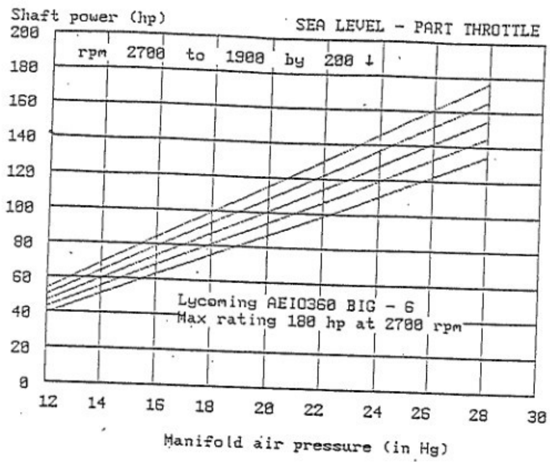
$$\boldsymbol{d\left(\frac{nm}{imp\ gal}\right) = \frac{v\left(\frac{nm}{hr} = kt\right)}{\dot{m}_f\left(\frac{imp\ gal}{hr}\right)} = \frac{100}{6.7} = 14.925\ nm/g}$$

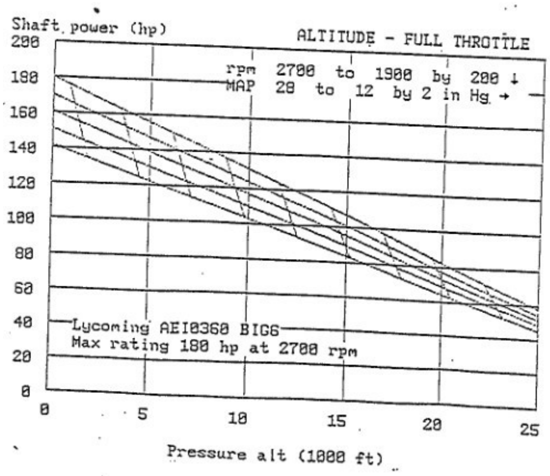
Q 7

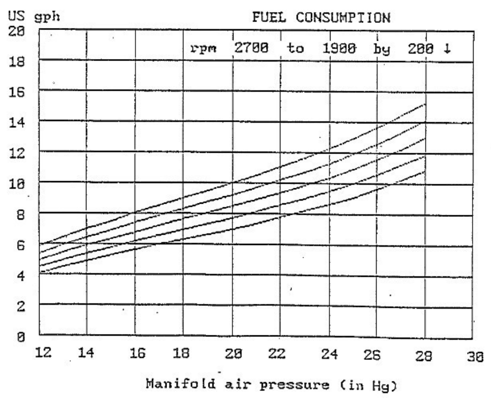
One piston aircraft.

$$S = 15.8\,m^2, \qquad span = 10.76\,m, \qquad e = 0.77, \qquad w = 11\,kN$$

Cruise at 1000 m pressure alt., OAT ISA - 10° C, 23.5 in Hg MAP, 2600 rpm prop shaft speed







➔ **True speed (kt), zero-lift drag coefficient, fuel consumption (US gal/nm) with 10-kt tailwind.**

Final attempt

$$\dot{m}_f = 11.5\ gph$$

$$DA = PA + \left(120\,x\,(OAT - 10°C)\right)$$

$$DA = 1000 + \left(120\,x\,(8.7° - 10°)\right) = 844\,m$$

$$\rho|_{844m\ alt.} = 0.9216648 \times 1.225 = 1.12903938\ kg/m^3$$

$$alt: 1000\ m\ or\ 3281\ ft$$

$$20°\ Pitch\ Angle$$

From the intersection of the max efficiency line and the pitch degree line, J=0.8

$$J = \frac{V}{nD} \therefore V = JnD = 0.8\left(\frac{2600}{60}\right)1.85 = 64.133\frac{m}{s}\ or \sim 124\ kt$$

$$K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.77 \frac{10.7^2}{15.8}} = 0.0570491$$

The last graph is not telling much because of low MAP

$$P = C_P \rho n^3 d^5 = 0.044 \times 1.12903938\left(\frac{2600}{60}\right)^3 (1.85)^5$$

$$P = TV = \frac{1}{2}\rho V^3 S C_{D0} + k\frac{2w^2}{\rho VS} \rightarrow C_{D0} = \left(P - k\frac{2w^2}{\rho VS}\right)\frac{1}{\frac{1}{2}\rho V^3 S}$$

$$= \left(P - 0.0570491 \times \frac{2 \times 11{,}000^2}{1.12903938 \times 64.1 \times 10.7}\right)\frac{1}{\frac{1}{2}1.12903938 \times 64.1^3 \times 10.7} = 0.0278$$

$$V = V_t + V_{tailwind} = 124 + 10 = 134\ kt$$

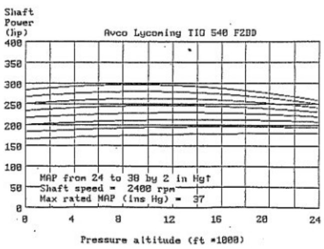
$$fuel\ \left(\frac{gal}{nautical\ mile = kt \cdot hr}\right) = \frac{\dot{m}(gal/hr)}{V} = \frac{11.5}{134} = 0.0858\ gal/nm$$

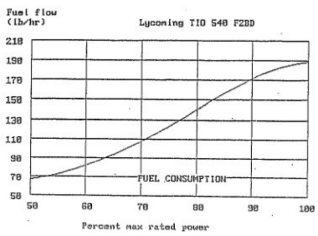
Q 8

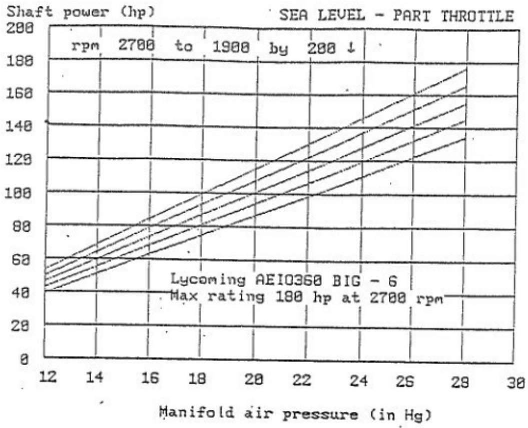
Three piston engine aircraft.

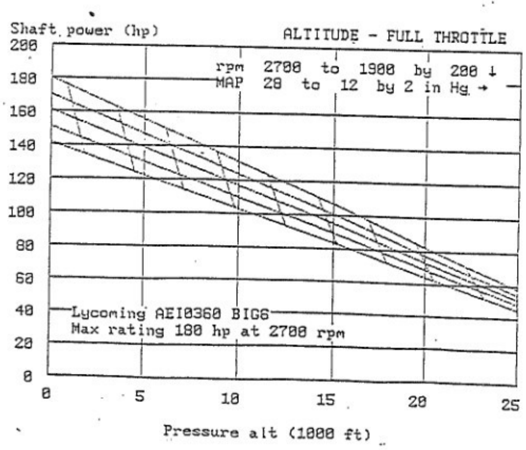
$$S = 29\ m^2, \qquad K = 0.055, \qquad C_{D0} = 0.024, \qquad w = 60\ kN$$

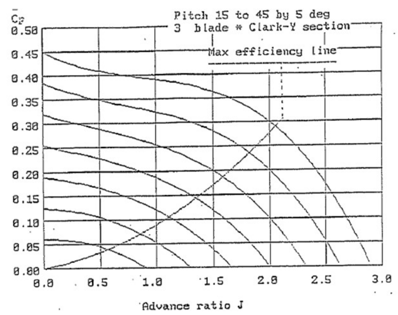
$dia = 2\ m.$ 2750 m alt., 200 kt true airspeed, 2400 rpm.

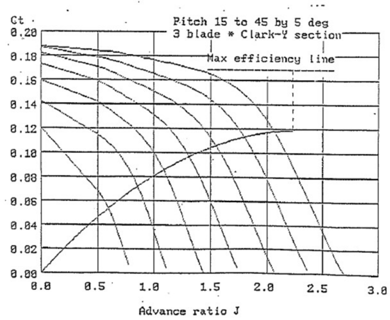












➔ **MAP**

MAP is found through the pressure altitude vs shaft power chart

$$v_t = 200\ kt\ or\ 102.9\ m/s$$

Alt: 2750 m or 9022 ft

Final Attempt

The only way to find MAP is through the shaft power

Alt: 2750 m or 9022 ft

Shaft power should be around 900 hp, 300 per engine

$$J = \frac{V}{nD} = \frac{102.9}{\frac{2400}{60}2} = 1.28625$$

Passing through the max efficiency line

$$C_p = 0.12$$

$$P_{eng} = C_p \rho n^3 d^5 = 0.12 \times 0.93314375(2400/60)^3(2)^5$$

$$P = 3C_p \rho n^3 d^5 = 3 \times 0.12 \times 0.93314375(2400/60)^3(2)^5 = 687988.224\ watt \sim 922.6\ hp$$

$$\therefore MAP\ is\ 38\ in\ Hg$$

➔ **Fuel (kg) to fly 500 km with 20-kt tailwind.**

First Attempt

$$d = 500\ km$$

$$V = V_t - V_{tailwind} = -200 \times 0.5144$$

$$t(s) = \frac{d(m)}{V\left(\frac{m}{s}\right)} = \frac{500{,}000}{\square}$$

$$P = C_P \rho n^3 d^5$$

$$P\% = \left(\frac{n_0}{n_{max}}\right)^3 = \left(\frac{2400}{2700}\right)^3 = 70.233\%$$

$$\dot{m}_f|_{100\%\ P} = 190\ lb/hr$$

$$\dot{m}_f|_{100\%\ P} = 190\ lb/hr$$

$$m(lb) = \dot{m}_f\left(\frac{lb}{hr}\right) * t(hr)$$

Quick Note

$$1\ kg = 2.205\ lb$$

Final Attempt

$$V = V_t - V_{tailwind} = 200 + 20 = 220\ kt \sim 407.4\ km/hr$$

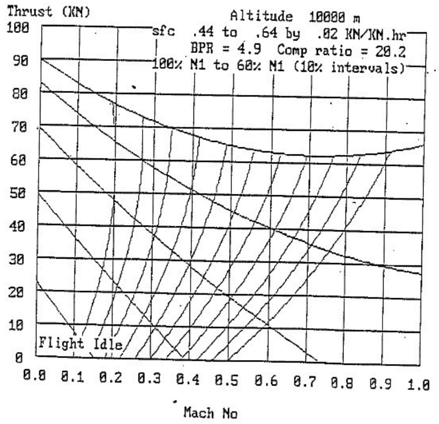
$$\dot{m}_{f_{eng}} = 190\frac{lb}{hr} \therefore Total\ \dot{m}_f = 570\frac{lb}{hr} \sim 258.5\ kg/hr$$

$$m(kg) = \frac{\dot{m}_f\left(\frac{kg}{hr}\right)}{v\left(\frac{km}{hr}\right)} \times d(km) = \frac{258.5}{407.4} \times 500 = 317.256\ kg$$

Q 9

Three turbofan aircraft.

$$S = 338.8\ m^2, \qquad K = 0.06, \qquad w = 1600\ kN$$



Cruise at M0.82 (0.82x299.8=245.836 m/s) at 10 km alt. against 20 kt headwind.

➔ **Specific ground range (km/kg) if engine speed 95% of Max N_1 (SGR =0.096)**

Specific fuel consumption aka power specific fuel consumption

Flight idle: minimal power output

$$SFC = \frac{\dot{m}_f}{P} = \frac{\dot{m}_f}{TV}$$

$$TSFC = \frac{\dot{m}_f}{T}$$

$$SFC = TSFC/V_{exhaust}$$

$$\rho_{10\ km\ alt.} =$$

$$SGR = \frac{V}{\dot{m}_f} = \frac{245.836 - 10.29}{\square}$$

$$TSFC = \frac{\dot{m}_f}{T} \therefore \dot{m}_f(hr\ kg\frac{km}{hr^2}) = TSFC\left(\frac{kN}{kN}hr\right) \cdot T(kN) = 0.64 \times 42{,}000$$

Final attempt

$$V = V_t - V_{headwind} = 0.82 \times 299.8 - 20 \times 0.514444 = 235.54712\frac{m}{s} \sim 847.9696\ km/hr$$

$$TSFC = \frac{\dot{m_f} g}{T} \therefore \dot{m}_f = TSFC\left(\frac{kN}{kN \cdot hr}\right) \cdot \frac{T\left(kg\frac{m}{s^2}\right)}{g\left(\frac{m}{s^2}\right)}$$

$$\dot{m}_{f_{eng}} = 0.64 \times \frac{45{,}000}{9.81} = 2935.7798\ kg/hr$$

$$\dot{m}_{f_{total}} = 3\dot{m}_{f_{eng}} = 8807.339\ kg/hr$$

$$SGR\left(\frac{km}{kg}\right) = \frac{V(km/hr)}{\dot{m}_f(kg/hr)} = \frac{847.9696}{8807.339} = 0.09627\ km/kg$$

➔ **Minimum rate of fuel burn (kg/hr) and % engine speed at the same altitude and weight (7200 kg/hr 91% N1)**

Minimum burn rate

$$TSFC = \frac{\dot{m}_f}{T}\ (\frac{kg/hr}{N})$$

$$SFC(\frac{kN}{kN \cdot hr})$$

Flight idle

10 km alt.

1600 kN weight

To minimize the burn/consumption rate, we need to minimize the specific fuel consumption and thrust produced.

$$\dot{m}_{f_{min}} = TV * SFC$$

Final attempt

Altitude: 10 km

Weight: 1600 kN

Minimum $\dot{m}_f$ implies minimum power condition

$$M|_{10km} = \frac{1}{299.8}\sqrt{\frac{w}{0.5\rho S}\sqrt{\frac{K}{3C_{D0}}}} = \sqrt{\frac{1600{,}000}{0.5 \times 0.41356 \times 338.8}\sqrt{\frac{0.06}{3C_{D0}}}}$$

$$\dot{m}_f = TSFC \cdot \frac{T}{g}$$

7200 kg/hr   91%

Quick note $\dot{m}_f = TSFC \cdot \frac{T}{g}$

To minimize $\dot{m}_f$, lowering TSFC then T should do it.

$$TSFC = 0.44\frac{kN}{kN \cdot hr} \quad T = 68\ kN$$

$$\dot{m}_{f_{eng}} = TSFC \cdot \frac{T}{g} = 0.44\frac{54{,}000}{9.81} = 2422.0183 \sim 2400\ kg/hr$$

$$\dot{m}_{f_{total}} = 3\dot{m}_{f_{eng}} = 7200\ kg/hr$$

At Mach No 0.2, The N1 appears at roughly 85% (between 80% & 90%)

Q 10

Jet aircraft.

$S = 85\ m^2$, $w = 25{,}000\ kgf\ (245166.25\ N)$, $e = 0.85$, $C_{D0} = 0.016$, $C_{Lbuff} = 1.25$, $span = 27.5\ m$

For SLF at 10 km alt. $\rho(10\ km) = 0.41356\ kg/m^3$ $k = \frac{1}{0.85 \times \pi \times \frac{27.5^2}{85}} = 0.0421$

➔ Maximum L/D and minimum thrust required.

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{\sqrt{4C_{D0}K}} = \frac{1}{\sqrt{4 \times 0.016 \times 0.0421}} = 19.2650$$

$$T_{min} = \frac{w}{\left(\frac{L}{D}\right)_{max}} = \frac{245166.25}{19.2650} = 12726 \approx 12.7\ kN$$

➔ Minimum drag V_e (kt)

Minimum Drag $C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.016}{0.0421}} = 0.616480$ $\quad C_D = 2C_{D0}$

$$V = \sqrt{\frac{2w}{\rho_{SL} S C_L}} = \sqrt{\frac{2 \times 245166.25}{1.225 \times 85 \times 0.616480}} = 87.399\frac{m}{s}\ (170\ kt)$$

➔ L/D at 200 kt V_e

$$V_e = 200\ kt(102.889\ m/s)$$

$$C_L = \frac{2w}{\rho_{SL} S V_e^2} = \frac{2 \times 245166.25}{1.225 \times 85 \times 102.889^2} = 0.445$$

$$C_D = C_{D0} + K C_L^2 = 0.016 + 0.0421 \times 0.445^2 = 0.0243$$

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.445}{0.0243} = 18.313$$

➔ **Thrust and thrust power at 200 kt V_e**

$$T = \frac{w}{\frac{L}{D}} = \frac{245166.25}{18.313} = 13388 \approx 13.4\ kN$$

$$V = \frac{1}{\sqrt{\sigma}} V_e = \frac{1}{\sqrt{0.3376}} 102.889 = 177.079 \frac{m}{s}$$

$$P = TV = 13388 \times 177.079 = 2370733.652 \approx 2.371\ MW$$

➔ **Maximum Mach number at $T_{\max A} = 20\ kN$ and constant alt.**

$$V_{max} = \sqrt{\frac{\frac{T_A}{w}\frac{w}{S} + \left(\frac{w}{S}\right)\sqrt{\left(\frac{T_A}{w}\right)^2 - 4C_{D0}K}}{\rho_{10\ km} C_{D0}}}$$

$$= \sqrt{\frac{\frac{20{,}000}{245166.25}\frac{245166.25}{85} + \left(\frac{245166.25}{85}\right)\sqrt{\left(\frac{20{,}000}{245166.25}\right)^2 - 4 \times 0.016 \times 0.0421}}{0.41356 \times 0.016}}$$

$$= 250.980 \frac{m}{s}$$

$$a(10\ km) = 299.8 \frac{m}{s}$$

$$M = \frac{V}{a} = \frac{250.980}{299.8} = 0.837\ Mach$$

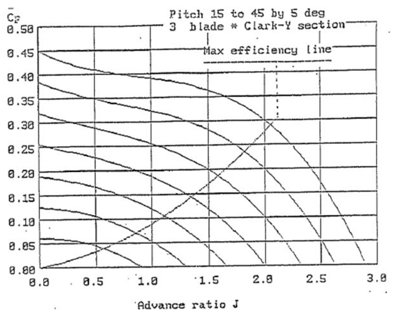
Q 11

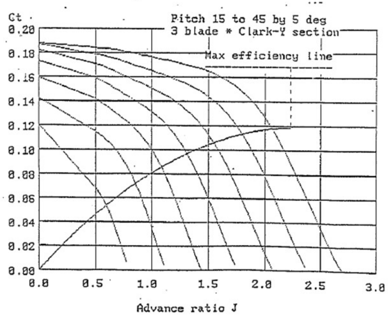
Two piston engine aircraft.

$$S = 915\ ft^2, \quad AR = 8.8, \quad C_{Lbuff} = 1, \quad e = 0.8, \quad w = 22{,}000\ lb$$

$$per\ engine\ max. P_{cruise} = 1350\ hp\ at\ 16{,}000\ ft\ (max. cruise\ speed\ 215\ kt\ equivalent)$$

1800 rpm, dia = 10 ft at 16,000 ft $\rho(16\ kft) = 0.73647\ kg/m^3 (0.04598 lb/ft^3)$





➔ **Maximum L/D and speed in SLF at minimum required power (17.5 108 kt)**

Minimum power trace the maximum efficiency line

First attempt

$$K = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.8 \times 8.8} = 0.0452145$$

Minimum Power $C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3\times 0.025}{0.0452145}} = 1.28793 \quad C_D = 4C_{D0} = 4 \times 0.025 = 0.1$

Advance ratio should be around 0.6.

The thrust should be around 1257.

$$T = D = w\frac{C_D}{C_L} \therefore \frac{L}{D} = \frac{C_L}{C_D} = \frac{w}{T} = \frac{22{,}000}{C_t \rho n^2 D^4} = \frac{22{,}000}{0.055 \times 0.04598 \times \left(\frac{1800}{60}\right)^2 10^4}$$

$$V = JnD = 1 \times \frac{1800}{60} \times 10$$

$$V = \sqrt{\frac{2w}{\rho S C_L}} = \sqrt{\frac{2 \times 22{,}000}{0.04598 \times 915 \times}} =$$

At Minimum power

$$\left(\frac{L}{D}\right) = \frac{\sqrt{\frac{3C_{D0}}{K}}}{4C_{D0}} = \frac{\sqrt{\frac{3C_{D0}}{K}}}{4 \times}$$

Second attempt

$$K = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.8 \times 8.8} = 0.0452145$$

Minimum Power $\frac{C_L}{C_D} = \frac{1}{\sqrt{4 \times 0.025 \times 0.0452145}}$

Third attempt

$$C_L = \frac{C_{Lbuf}}{\sqrt{1 + \frac{AR}{e}\pi}} = \frac{1}{\sqrt{1 + \frac{8.8}{0.8}\pi}}$$

Foruth attempt

$$C_L = \sqrt{\frac{3C_{D0}}{K}} \qquad C_D = 4C_{D0}$$

$$\frac{C_L}{C_D} = \frac{\sqrt{\frac{3C_{D0}}{K}}}{4C_{D0}}$$

Fifth attempt

Given power, max speed, power, prop charts

$$K = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.8 \times 8.8} = 0.0452145$$

Find C_{D0}

$$C_D = C_{D0} + kC_L^2 \therefore C_{D0} = C_D - kC_L^2$$

$$V_{stall} = \sqrt{\frac{22000}{0.5 \times 0.04598 \times 915 \times 1}} = 32.3394\ ft/s$$

$$\frac{C_L}{C_D} = \frac{P}{wv} = \frac{742500\ lbf\ ft/s}{22000\ lbf \times 362.9\ ft/s}$$

$$\frac{L}{D}\Big|_{max} = \frac{1}{\sqrt{4kC_{D0}}} = \frac{1}{\sqrt{4 \times 0.016 \times 0.0452145}}$$

$$C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3 \times 0.0180545}{0.0452145}} = 1.09450\ reduced\ to\ 1$$

$$V = \sqrt{\frac{2\omega}{\rho S C_L}} = \sqrt{\frac{2\times 22{,}000}{0.04598\times 915\times 1}} =$$

Cl should be the lowest

$$V = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{k}{3C_{D0}}}} = \sqrt{\frac{2}{0.04598}\left(\frac{22{,}000}{915}\right)\sqrt{\frac{0.0452145}{3\times 0.0180858}}}$$

Final Attempt

Maximum L/D

$$K = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.8 \times 8.8} = 0.0452145$$

$$C_D = C_{D0} + kC_L^2 \therefore C_{D0} = C_{D_{buff}} - kC_{L_{buff}}^2 \rightarrow C_{D0} = 3C_{D0} - kC_{L_{buff}}^2$$

$$\therefore C_{D0} = 0.0452145 \times \frac{1^2}{5}$$

$$C_{D0} = \frac{7}{2} C_{D0} - kC_{L_{buff}}^2 \therefore C_{D0} = \frac{2}{5} kC_{L_{buff}}^2 = \frac{2}{5} \times 0.0452145 \times 1^2 = 0.0180858$$

$$\frac{L}{D}|_{max} = \frac{1}{\sqrt{4kC_{D0}}} = \frac{1}{\sqrt{4 \times 0.0452145 \times 0.0180858}} = 17.4849 \sim 17.5$$

Using the advance ration to relate that to the velocity

Minimum power entails running at **max efficiency**

The intersection of 15° Pitch Angle with the max efficiency line suffice the requirements

$$\therefore J = 0.6 \quad for\ 15°\ Pitch\ Angle$$

$$V = JnD = 0.6 \times \frac{1800}{60} \times 10 = 180\frac{\text{ft}}{\text{s}} \approx 106.6 \sim 107\ knot$$

$$C_L = \frac{\omega}{0.5\rho S V^2} = \frac{22{,}000}{0.5 \times 0.04598 \times 915 \times 180^2} = 0.0322 < C_{L_{buff}}$$

➔ **Minimum required power per engine (356 hp)**

Let's assume we know the speed already

$$P_r = c_p \rho n^3 d^5 = 0.04 \times 0.0452145 \times \left(\frac{1800}{60}\right)^3 (10)^5$$

$$\frac{C_L}{C_D} = \frac{\sqrt{\frac{3C_{D0}}{K}}}{4C_{D0}}$$

Second attempt

From before

$$\left(\frac{L}{D}\right)_{max} = 17.5 = \frac{1}{\sqrt{4 \times C_{D0} \times 0.0452145}} \therefore C_{D0} = 0.0180545$$

Minimum Power

$$C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3 \times 0.0180545}{0.0452145}} = 1.09450 \text{ stall limited reduce to } C_{L_{max}} \quad C_D = 4C_{D0} = 4 \times 0.0180545 = 0.072218$$

$$P = 0.5\sqrt{\frac{2w^3 C_D^2}{\rho S C_L^3}} = 0.5\sqrt{\frac{2 \times 22000^3 \times 0.072218^2}{0.0452145 \times 915 \times 1^3}} = 25906.9\ lbf\frac{ft}{s} \quad (47.103\ hp)$$

Way too low

Third attempt (using charts)

$$v = 182.3\ ft/s$$

$$J = \frac{V}{nD} = \frac{182.3}{(\frac{1800}{60})10} = 0.61$$

$$P = C_p \rho n^3 d^5 = \times\ 0.0452145 \times \left(\frac{1800}{60}\right)^3 10^5$$

$$P = TV = C_t \rho n^2 d^4 V = 0.00355878 \times 0.0452145 \times \left(\frac{1800}{60}\right)^2 10^4 \times 182.283$$

$$P_{eng} = 0.5 C_p \rho n^3 d^5 = 0.5 \times 0.00160388 \times 0.0452145 \times \left(\frac{1800}{60}\right)^3 10^5 = 195800.03$$

Final Attempt

Quick note: $1\ hp = 550\ ft \cdot lb/s$

$$P_{eng} = TV = 0.5 C_t \rho n^2 d^4 V = 0.5 \times 0.0053 \times 0.0452145 \times \left(\frac{1800}{60}\right)^2 10^4 \times 180 = 194105.8485\ ft \cdot \frac{lb}{s}$$

$$\approx 352.91972 \sim 353\ hp$$

Quick analysis

GIVEN max power stats

$$P_{eng} = 1350\ hp\ or\ 742{,}500\ lb \cdot ft/s$$

$$V_e = 215\ kt\ or\ \ 362.9\ ft/s$$

$$V = \sqrt{\sigma}\, V_e \therefore\ V = \sqrt{0.6012} \times 362.9 = 281.382\ ft/s$$

$$P_{eng} = C_t \rho n^2 d^4 V$$

$$742{,}500\ = C_t \times 0.0452145 \times \frac{1800^2}{60}\ 10^4 \times 281.382$$

$$C_t = 0.00648455$$

$$J = \frac{V}{nD} = \frac{281.382}{\frac{1800}{60} \times 10} = 0.93794$$

The intersection of J and Ct is kinda oddly interesing

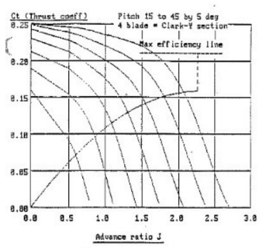
Q 12

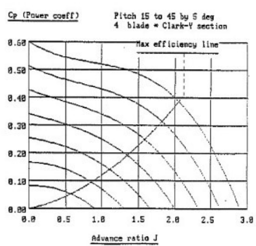
Twin-piston aircraft.

$$S = 85\ m^2, \qquad span = 25.25m, \qquad C_{D0} = 0.03, \qquad C_{Lbuf} = 1.25, \qquad e = 0.85, \qquad w = 10194\ kgf$$

$$K = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.85 \times \frac{25.25^2}{85}} = 0.0499261$$

$$dia = 3\ m, 2400\ rpm$$





➔ **$V'_e s$ at minimized thrust and power**

For minimum thrust

$$V_{T_{min}} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{k}{C_{D0}}}} = \sqrt{\frac{2}{1.225}\left(\frac{99968.9901}{85}\right)\sqrt{\frac{0.0499261}{0.03}}} = 49.7705\ m/s$$

For minimum power

$$V_{p_{min}} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{k}{3C_{D0}}}} = \sqrt{\frac{2}{1.225}\left(\frac{99968.9901}{85}\right)\sqrt{\frac{0.0499261}{3\times 0.03}}} = 37.8174\ m/s$$

➔ **Shaft power at 200 kt V_e and 6,000 m alt.**

$$V = \sqrt{\sigma}V_e = \sqrt{0.5389}\times 102.889 = 75.5306\frac{m}{s}$$

$$J = \frac{V}{nD} = \frac{75.5306}{\frac{2400}{60}\times 3} = 0.629421$$

$$P_s = C_p\rho n_s^3 d^5 = 0.26\times 1.225\times 0.5389\times\left(\frac{2400}{60}\right)^3 (3)^5$$

$$P_{s_{eng}} = \frac{P_s}{2} =$$

Second attempt

$$V = \frac{1}{\sqrt{\sigma}}V_e = \frac{1}{\sqrt{0.5389}}\times 102.889 = 140.157\frac{m}{s}$$

$$J = \frac{V}{nD} = \frac{140.157}{\frac{2400}{60}\times 3} = 1.167975 \approx 1.7$$

$$C_t = \frac{T}{\rho n^2 d^4} = \frac{\square}{0.5389\times 1.225\times\left(\frac{2400}{60}\right)^2 3^4}$$

$$P_s = C_p\rho n_s^3 d^5 = \times 0.5389\times 1.225\times\left(\frac{2400}{60}\right)^3 3^5$$

Third attempt

$$V = \frac{1}{\sqrt{\sigma}}V_e = \frac{1}{\sqrt{0.5389}}\times 102.889 = 140.157\frac{m}{s}$$

$$J = \frac{V}{nD} = \frac{140.157}{\frac{2400}{60}\times 3} = 1.167975 \approx 1.7$$

$$\rho(6000\ m) = 0.6601525\ kg/m^3$$

Need to find the pitch angle.

$$\alpha = 45°$$

$$C_L = \frac{99968.9901}{0.5\times 1.225\times 85\times 102.889^2} = 0.181385$$

$$C_D = C_{D0} + kC_L^2 = 0.03 + 0.0499261\times 0.181385^2 = 0.0316426$$

$$C_t = \frac{w\frac{C_D}{C_L}}{\rho n^2 d^4} = \frac{99968.9901\frac{0.0316426}{0.181385}}{0.6601525\times\left(\frac{2400}{60}\right)^2 3^4} = 0.2038$$

$$\alpha = 45°$$

$$P_s = C_p\rho n_s^3 d^5 = 0.45\times 0.6601525\left(\frac{2400}{60}\right)^3 3^5$$

Not sure why the numbers are off.

It can be figured out in two ways. One of them is tying the pitch angle with the thrust and power coefficients starting at the ct figure to solve for the angle then cp figure to find the shaft power. The other way that can be done simpler.

$$V = \frac{1}{\sqrt{\sigma}}V_e = \frac{1}{\sqrt{0.5389}}\times 102.889 = 140.157\frac{m}{s}$$

$$P_{total} = TV = w\frac{C_D}{C_L}v = 99968.9901\frac{0.0316426}{0.181385}140.157$$

$$P_{eng} = 0.5\times 2.444\times 10^6 = 1.22\ MW$$

➔ **Minimum shaft power at 6,000m alt. and 1800 rpm in SLF**

$$V = \sqrt{\sigma}V_e = \sqrt{0.5389}\times 37.8174 = 27.7617$$

$$J = \frac{V}{nD} = \frac{27.7617}{\frac{1800}{60}\times 3} = 0.308$$

$$P_s = C_p \rho n_s^3 d^5 = 0.15 \times 1.225 \times 0.5389 \times \left(\frac{1800}{60}\right)^3 (3)^5$$

Second attempt

$$\rho(6000\ m) = 0.6601525\ kg/m^3$$

Minimum Power

$$C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3 \times 0.03}{0.0499261}} = 1.34263 \quad C_D = 4C_{D0} = 4 \times 0.03 = 0.12$$

$$J = \frac{V}{nD} = \frac{\sqrt{\frac{99968.9901}{0.5 \times 0.6601525 \times 85 \times 1.34263}}}{\left(\frac{1800}{60}\right)3} = 0.572394 \approx 0.57$$

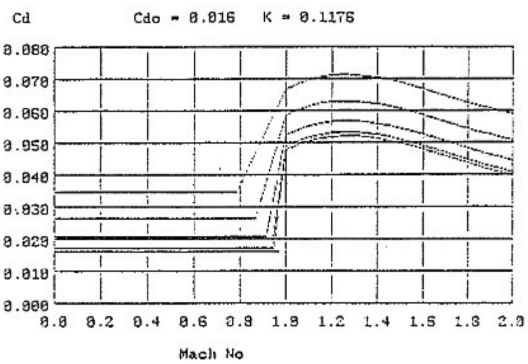
$C_p|_{\alpha=15°} = 0.065$ lowest possible power coefficient

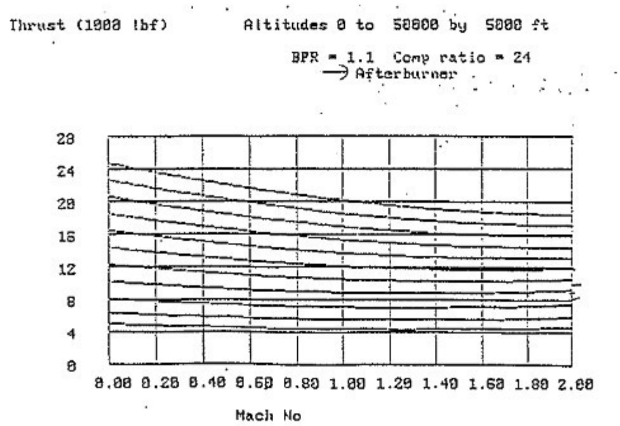
$$P_{s\ eng} = 0.5 C_p \rho n_s^3 d^5 = 0.065 \times 0.6601525 \times \left(\frac{1800}{60}\right)^3 3^5 = 0.281 \times 10^6\ watt \approx 0.3\ MW$$

Q 13

Twin-turbofan aircraft.

$$S = 400\ ft^2,\ span = 37.5\ ft,\ C_{D0} = 0.016,\ C_{Lbuff} = 1.1,\ e = 0.77,\ w = 25{,}000\ lb$$





➔ **Mach number and % full thrust (no afterburner) at minimum drag speed at 36,000 ft.**

Ans v = 209.521 m/s

$$K = \frac{1}{\pi e AR} = \frac{1}{\pi 0.77 \frac{37.5\text{^}2}{400}} = 0.117586$$

First attempt

Minimum drag speed

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.016}{0.117586}} = 0.368878 \quad C_D = 2C_{D0} = 2 \times 0.016$$

$$V = \sqrt{\frac{2w}{\rho S C_L}} = \sqrt{\frac{2 \times 25{,}000}{0.0228 \ \times 400 \times 0.368878}} = 121.912$$

Second attempt

$$M = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25{,}000}{0.7 \times 0.2243 \times 101{,}325 \times 400 \times 0.368878}}$$

Third attempt

$$K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.77 \frac{37.5\hat{}2}{400}} = 0.117586$$

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.016}{0.117586}} = 0.368878 \quad C_D = 2C_{D0} = 2 \times 0.016 = 0.032$$

$$M = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25{,}000\ lbf}{0.7 \times 480.4\ psf \times 400\ ft^2 \times 0.368878}} = 0.709820 \approx 0.71\ Mach$$

$$T = w\frac{C_D}{C_L} = 25{,}000\frac{0.032}{0.368878} = 2168.7387\ lbf$$

$$\text{max}\ cruise\ thrust\!: \frac{2}{3}\ 16{,}000 = 10666.67\ lb$$

$$\%T_{max} = \frac{T}{T_A} = \frac{2168.7387}{10666.67} = 0.2033\ or \approx 20\%\ T_A$$

➔ Thrust limited or stall limited, at 0.5M. What is the 0.5M ceiling?

Stall limited.

$$0.1527 \times 101{,}325 = 15472.3275$$

$$p = \frac{w}{0.7 \times S \times M^2 \times C_L} = \frac{25{,}000}{0.7 \times 400 \times 0.5^2 \times 0.008}$$

service ceiling.

Second attempt

$$C_L = \frac{w}{0.7pSM^2} = \frac{25{,}000}{0.7 \times 2116 \times 400 \times 0.5^2}$$

$$C_L = \frac{w}{0.7\rho S V^{2^2}} = \frac{25{,}000}{0.5 \times 0.07647 \times 400 \times 170.15^2}$$

$$M_{stall} = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25{,}000}{0.7 \times 2116 \times 400 \times 1.1}}$$

Third attempt

$$M = \sqrt{\frac{w}{0.7pSC_L}}$$

$$p = \frac{w}{0.7 \times S \times M^2 \times C_L} = \frac{25{,}000}{0.7 \times 1.1 \times 400 \times 0.5^2} = 324.6753\ psf$$

$$\frac{P}{P_{sea}} = \frac{324.6753}{2116} = 0.153 \quad at\ 44{,}000\ ft\ alt.\ \ 0.153$$

Mach number is a function of lift coefficient thus constrained by only the stall, regardless of thrust.

➔ Max Mach numbers at 35000ft with and without afterburner

First attempt

$$V_{max} = \sqrt{\frac{\frac{T_A}{w}\frac{w}{S} + \left(\frac{w}{S}\right)\sqrt{\left(\frac{T_A}{w}\right)^2 - 4C_{D0}K}}{\rho_{10\,km}C_{D0}}}$$

$$p(10668\; m\; or\; 35\,000\; ft) = 0.2353 \times 2116.7 = 498.05951\; psf$$

$$M_{max} = \sqrt{\frac{\frac{T_A}{w}\frac{w}{S} + \left(\frac{w}{S}\right)\sqrt{\left(\frac{T_A}{w}\right)^2 - 4C_{D0}K}}{1.4p_{10\,km}C_{D0}}} = \sqrt{\frac{\frac{6000}{400} + \left(\frac{25{,}000}{400}\right)\sqrt{\left(\frac{6000}{25{,}000}\right)^2 - 4 \times 0.016 \times 0.117586}}{1.4 \times 498.05951 \times 0.016}}$$

$$T = 6000\; lb$$

Attempt (irrational intuition)

Expectation: with afterburner the Mach number is higher than the without.

$$M_{drag\;div} = 1.25\; Mach$$

Drag divergence M is where the peak of the drag occurs due to an increase in parasite drag. As per the following relation, presuming constant coefficient of lift and T=D, the M limit is 1.25 Mach at DD as shown in the 1st graph.

$$T = \frac{w}{(L/D)} \therefore T_{max} = w\frac{C_{D_{max}}}{C_L}$$

As far as I could comprehend, albeit traditional subsonic equations do not include parasite drag, the aircraft encounters substantial sustained drag after hitting the sonic M– proportionally correlated.

The afterburner, the aircraft experiences the lowest thrust required (D=T) at roughly 1.6 Mach.

$$\sqrt{\frac{\frac{2\frac{2}{3}6000}{400} + \left(\frac{25000}{400}\right)\sqrt{\left(\frac{2\frac{2}{3}6000}{25000}\right)^2 - 4 \times 0.016 \times 0.117586}}{1.4 \times 498.05951 \times 0.016}}$$

Third attempt

$$D = T = w\frac{C_D}{C_L} = 25000\frac{0.071}{0.4} = 4437.5\; lbf$$

$$\sqrt{\frac{\frac{4437.5}{400} + \left(\frac{25000}{400}\right)\sqrt{\left(\frac{4437.5}{25000}\right)^2 - 4 \times 0.016 \times 0.117586}}{1.4 \times 498.05951 \times 0.016}} = 1.36451\; Mach$$

$$C_L = \sqrt{\frac{C_{D0}}{k}} = \sqrt{\frac{0.016}{0.117586}} = 0.368878$$

$$M = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25000}{0.7 \times 498.05951 \times 400 \times 0.368878}}$$

Fourth attempt

$$T_A = (0.7pC_{D0}S)M^2 + \left(\frac{Kw^2}{0.7pS}\right)\frac{1}{M^2}$$

$$4437.5 = (0.7 \times 498.05951 \times\ 0.016 \times 400)M^2 + \left(\frac{0.117586 \times 25000^2}{0.7 \times 498.05951 \times 400}\right)\frac{1}{M^2}$$

$$M = 0.356156, \qquad 1.36451$$

$$C_L = \frac{w}{0.7pSM^2} = \frac{25000}{0.7 \times 498.05951 \times 400 \times 1.36451^2} = 0.310294$$

$$\sqrt{\frac{25000}{0.7 \times 498.05951 \times 400 \times 0.356156^2}} = 1.18880\ (stall - limited\ \ reduce\ to\ C_{Lmax} = 1.1)$$

The numbers are nonsense – imaginary.

Fifth attempt

From figure, the mimimum C_L happens to be 0.1. minimizing C_L leads to higher Mach

$$M = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25000}{0.7 \times 498.05951 \times 400 \times 0.1}}$$

Sixth attempt

$$C_L = \sqrt{\frac{C_{D0}}{k}} = \sqrt{\frac{0.016}{0.117586}} = 0.368878$$

$$M = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25000}{0.7 \times 498.05951 \times 400 \times 0.368878}} = 0.697122$$

$$C_L = \sqrt{\frac{C_{D0}}{3k}} = \sqrt{\frac{0.016}{3 \times 0.117586}} = 0.212972$$

$$M = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25000}{0.7 \times 498.05951 \times 400 \times 0.212972}} = 0.917464$$

$$M = \sqrt{\frac{w}{0.7pSC_L}} = \sqrt{\frac{25000}{0.7 \times 498.05951 \times 400 \times 0.1}} = 1.33891$$

$$C_D = C_{D0} + kC_L^2 + C_{D\ div} \therefore C_L = \sqrt{\frac{C_D - C_{D0} - C_{D\ div}}{k}} = \sqrt{\frac{C_D - 0.016 - C_{D\ div}}{0.117586}}$$

Seventh attempt

Expectation: with afterburner the Mach number is higher than the without.

$$C_D = C_{D0} + kC_L^2 + C_{D\ div} \therefore C_L = \sqrt{\frac{C_D - C_{D0} - C_{D\ div}}{k}}$$

$$M_{max} = \sqrt{\frac{w}{0.7pSC_{L_{min}}}}$$

$$M_{drag\ div} = 1.25\ Mach$$

Drag divergence M is where the peak of the drag occurs due to an increase in transonic drag. As per the following relation, lift is inversely correlated with the M and transonic drag, so to achieve max M we need to minimize lift coefficient or maximize drag divergences if and only if L/D is constant. Comment: this is nonsense because the difference of net nominator value can remain constant so can the lift coefficient even though the drag terms can vary freely.

The M limit is 1.25 Mach at peak DD as shown in the 1st graph.

$$T = \frac{w}{(L/D)} \therefore T_{max} = w\frac{C_{D_{max}}}{const\ C_L}$$

As far as I understood, albeit traditional subsonic equations do not include transonic/supersonic drag, the aircraft encounters substantial incrementing drag near the sonic M– proportionally correlated.

The afterburner, the aircraft experiences the lowest thrust required (D=T) at roughly 1.6 Mach. Comment: this is nonsense too.

$$M_{max} = \sqrt{\frac{\frac{T_A}{w}\frac{w}{S} + \left(\frac{w}{S}\right)\sqrt{\left(\frac{T_A}{w}\right)^2 - 4C_{D0}K}}{1.4p_{10\ km}C_{D0}}}$$

$$= \sqrt{\frac{\frac{\frac{2}{3}9000}{400} + \left(\frac{25000}{400}\right)\sqrt{\left(\frac{\frac{2}{3}9000}{25000}\right)^2 - 4 \times 0.016 \times 0.117586}}{1.4 \times 498.05951 \times 0.016}} = 1.61186$$

Q 14

Twin-turbofan aircraft.

$$S(to) = 1023\ ft^2, \quad C_{D0}(to) = 0.035, \quad e(to) = 0.75,$$

$$S(clean) = 979.5\ ft^2, \quad C_{D0}(clean) = 0.0182, \quad e(clean) = 0.85,$$

$$span = 93\ ft, \quad w = 100{,}000\ lb$$

$$k(to) = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{1023}} = 0.0501994$$

$$k(clean) = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{979.5}} = 0.0480648$$

$$T(S|L) = 12{,}000\ lbf, \quad T(5{,}000\ ft) = 10{,}000\ lbf$$

$$\rho_{SL} = 0.07647\frac{lb}{ft^3}, \quad \rho(5000\ ft) = 0.0659\ lb/ft^3$$

➔ **ROC (fpm) at SSL at 180 KTAS in takeoff config**

$$V_{true} = 180\ kt\ (303.806\ ft/s)$$

$$D = 0.5\rho V^2 S C_{D0} + \frac{kw^2}{0.5\rho V^2 S} = 0.5 \times 0.07647 \times 303.806^2 \times 1023 \times 0.035 + \frac{0.0501994 \times 100{,}000^2}{0.5 \times 0.07647 \times 303.806^2 \times 1023} = 126495.51\ lbf$$

$$ROC = v\frac{T - D}{w} = 303.806\ \frac{12{,}000 -}{100{,}000}$$

$$ROC = V\left[\frac{T}{w} - 0.5\rho V^2\left(\frac{S}{w}\right)C_{D0} - \frac{w}{S}\frac{2K}{\rho V^2}\right]$$

$303.806\left[\frac{24{,}000}{100{,}000} - 0.5 \times 0.07647 \times 303.806^2\left(\frac{1023}{100{,}000}\right)0.035 - \frac{100{,}000}{1023}\frac{2\times0.0501994}{0.07647\times303.806^2}\right]$ yields negative sign as if the drag surpasses the thrust produced

Second attempt

We have the thrust and speed and need to find the rate of climb.

$$ROC = v\frac{T - D}{w} = 303.806\left[\frac{24{,}000}{100{,}000} - 0.5 \times 0.07647 \times 303.806^2 \times \left(\frac{1023}{100{,}000}\right)0.035 - \frac{100{,}000}{1023}\frac{2 \times 0.0501994}{0.07647 \times 303.806^2}\right] = -311.3\ ft/s$$

Second attempt

SSL at 180 KTAS () in takeoff configuration

Takeoff configuration:

$$S(to) = 1023\, ft^2, \qquad C_{D0}(to) = 0.035, \qquad e(to) = 0.75,$$

$$k(to) = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{1023}} = 0.0501994$$

$$V = 180\, KTAS = \; 303.806\, ft/s$$

$ROC = \left(\frac{T-D}{W}\right)V = (T-D)\frac{v}{W} = (24{,}000 - A)\frac{303.806}{100{,}000}$ this equation does not work because the drag is gonna be negative so there has to be another way.

8365.02 the drag should be yet there is not a

Third attempt

Assuming mini power req

$$C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3 \times 0.035}{0.0501994}} = 1.44626 \quad C_D = 4C_{D0} = 4 \times 0.035 = 0.14$$

Assuming max range

$$\frac{L}{D}|_{max} = \frac{1}{\sqrt{4 \times 0.035 \times 0.0501994}}$$

$$ROC = v \sin\alpha = \sqrt{\frac{w}{0.5\,\rho C_L S}}\left[\frac{T}{W} - \frac{C_D}{C_L}\right] \; = \; V\left[\frac{T}{W} - \frac{C_D}{C_L}\right] = 303.806\left[\frac{24{,}000}{100{,}000} - 0.0838327\right] = 47.44\frac{ft}{s} \sim 2847\, ftm$$

Assuming minimum drag

$$C_L = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.035}{0.0501994}} = 0.834997 \quad C_D = 2C_{D0} = 2 \times 0.035 = 0.07$$

$$ROC = v \sin\alpha = \sqrt{\frac{w}{0.5\,\rho C_L S}}\left[\frac{T}{W} - \frac{C_D}{C_L}\right] \; = \; V\left[\frac{T}{W} - \frac{C_D}{C_L}\right] = 303.806\left[\frac{24{,}000}{100{,}000} - \frac{0.07}{0.834997}\right] = 47.44\frac{ft}{s} \sim 2847\, ftm$$

So basically the ratio is kinda there, but has to be more accurately choosed, instead of random baseless assumptions.

Or in order to climb with a steep angle, we need minimum drag possible.

➔ Max ROC (fpm) at 5,000 ft alt. in clean config. Mach number and calibrated airspeed (kt)

$ROC_{max} = \left[\frac{\left(\frac{W}{S}\right)Z}{3\rho C_{D0}}\right]^{0.5}\left(\frac{T}{w}\right)^{\frac{3}{2}}\left[1 - \frac{Z}{6} - \frac{3}{2\left(\frac{T}{w}\right)^2\left(\frac{L}{D}\right)^2_{max} Z}\right]$ for jet-propelled airplane

$$Z = 1 + \sqrt{1 + \frac{3}{\left(\frac{L}{D}\right)^2_{max}\left(\frac{T}{W}\right)^2}} = 1 + \sqrt{1 + \frac{3}{\frac{1}{4 \times 0.0182 \times 0.0480648}\left(\frac{20{,}000}{100{,}000}\right)^2}} = 2.123581$$

$$ROC_{max} = \left[\frac{\left(\frac{W}{S}\right)Z}{3\rho C_{D0}}\right]^{0.5}\left(\frac{T}{w}\right)^{\frac{3}{2}}\left[1 - \frac{Z}{6} - \frac{3}{2\left(\frac{T}{w}\right)^2\left(\frac{L}{D}\right)^2_{max} Z}\right]$$

$$V_{max\,ROC} = \sqrt{\frac{\left(\frac{T}{w}\right)\left(\frac{w}{S}\right)Z}{3\rho C_{D0}}} = \sqrt{\frac{\frac{20{,}000}{979.5} \times 2.123581}{3 \times 1771 \times 0.0182}} = 83.0640\frac{ft}{s}$$

Maximum rate of climb

$$C_L = \frac{-T}{2wk} \pm \left[\left(\frac{T}{2wk}\right)^2 + \frac{3C_{D0}}{k}\right]^{0.5} = \frac{-20{,}000}{2 \times 100{,}000 \times 0.0480648} \pm \left[\left(\frac{20{,}000}{2 \times 100{,}000 \times 0.0480648}\right)^2 + \frac{3 \times 0.0182}{0.0480648}\right]^{0.5} = 0.257113, -4.41816$$

$ROC = \sqrt{\frac{2w}{\rho S}}\left(\frac{T}{w}\frac{1}{\sqrt{C_L}} - \frac{C_{D0}}{C_L^{\frac{1}{3}}} - KC_L^2\right) = \sqrt{\frac{2\times 100{,}000}{0.0660585 \times 979.5}}\left(\frac{20{,}000}{100{,}000}\frac{1}{\sqrt{4.41816}} - \frac{0.0182}{4.41816^{\frac{1}{3}}} - 0.0480648 \times 4.41816^2\right) = 20.1852$ yields negative

$$ROC = \sqrt{\frac{2w}{\rho S}}\left(\frac{T}{w}\frac{1}{\sqrt{C_L}} - \frac{C_{D0}}{C_L^{\frac{1}{3}}} - KC_L^2\right) = \sqrt{\frac{2 \times 100{,}000}{0.0660585 \times 979.5}}\left(\frac{20{,}000}{100{,}000}\frac{1}{\sqrt{0.257113}} - \frac{0.0182}{0.257113^{\frac{1}{3}}} - 0.0480648 \times 0.257113^2\right) = 20.1852$$

Second attempt
Clean Configuration at 5000 fl.

$$S(clean) = 979.5\, ft^2, \qquad C_{D0}(clean) = 0.0182, \qquad e(clean) = 0.85,$$

$$k(clean) = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.75 \times \frac{93^2}{979.5}} = 0.0480648$$

$$\rho = 0.0660585\, psf$$

Find the max Rate of Climb

$$-\frac{3C_{D0}\rho}{2(w/S)}(v^4) + \frac{T}{w}(v^2) + \frac{2k(w/S)}{\rho} = 0$$

$$-\frac{3\times 0.0182\times 0.0660585}{2(100{,}000/979.5)}(v^4)+\frac{20{,}000}{100{,}000}(v^2)+\frac{2\times 0.0480648\times(100{,}000/979.5)}{0.0660585}=0$$

The velocity would be 109.644 ft/s

Airspeed for maximum rate of climb for the case where thrust is independent of airspeed

$$v^2=\frac{T}{6A}\pm\frac{1}{2}\sqrt{\left(\frac{T}{3A}\right)^2+\frac{4B}{3A}}=\frac{20{,}000}{6\times 0.588809136825}\pm\frac{1}{2}\sqrt{\left(\frac{20{,}000}{3\times 0.588809136825}\right)^2+\frac{4\times 1.485675587\times 10^7}{3\times 0.588809136825}}$$

$$A=0.5\,\rho S C_{D0}=0.5\times 0.0660585\ \times 979.5\times 0.0182=0.588809136825\qquad B=\frac{kw^2}{0.5\rho S}=\frac{0.0480648\times 100{,}000^2}{0.5\times 0.0660585\ \times 979.5}=1.485675587\times 10^7$$

$$C_L=\frac{1}{2k}\left[-\frac{T}{w}\pm\sqrt{\left(\frac{T}{w}\right)^2+12C_{D0}k}\right]=\frac{1}{2\times 0.0480648}\left[\frac{20{,}000}{100{,}000}\pm\sqrt{\left(\frac{20{,}000}{100{,}000}\right)^2+12\times 0.0182\times 0.0480648}\right]=4.41816,-0.257113$$

$$v=\sqrt{\frac{100{,}000}{0.5\times 0.0660585\ \times 979.5\times 0.257113}}=109.644\ ft/s$$

Fourth attempt

$$v=\sqrt{\frac{100{,}000}{0.7\times 1{,}771\ \times 979.5\times A}}=0.57$$

$$C_L=\frac{-T}{2wk}\pm\left[\left(\frac{T}{2wk}\right)^2+\frac{3C_{D0}}{k}\right]^{0.5}=\frac{-20{,}000}{2\times 100{,}000\ \times 0.0480648}\pm\left[\left(\frac{20{,}000}{2\times 100{,}000\ \times 0.0480648}\right)^2+\frac{3\times 0.0182}{0.0480648}\right]^{0.5}=0.257113,-4.41816$$

$$ROC=\sqrt{\frac{2w}{\rho S}}\left(\frac{T}{w}\frac{1}{\sqrt{C_L}}-\frac{C_{D0}}{C_L^{\frac{1}{3}}}-KC_L^2\right)=\sqrt{\frac{2\times 100{,}000}{1.4\times 1771\times 979.5}}\left(\frac{20{,}000}{100{,}000}\frac{1}{\sqrt{0.257113}}-\frac{0.0182}{0.257113^{\frac{1}{3}}}-0.0480648\times 0.257113^2\right)$$

$$m=\sqrt{\frac{100{,}000}{0.7\times 1{,}771\times 979.5\times 0.257113}}=0.565949\sim 0.57\ Mach$$

$$V=367.989085\sim 368k\ n$$