AE 3330 PROBLEM SET

Content

Q 26,27,28,29,30,31,32,33

Mohammed S. Al-Mahrouqi

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Q 26

Twin-turbojet aircraft.

$$S = 50\,m^2, \quad C_{D0} = 0.025, \quad K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.8 \times \frac{12^2}{50}} = 0.138\,, \quad W = 200\,KN,$$

$$C_{Lbuff} = 0.48\ at\ M0.78$$

$$T_A = 2 \times 75\ KN\ at\ SSL, \quad T|_{11km\ alt.} = 2 \times 37.5\ KN \quad \rho = 1.225\frac{kg}{m^3}\ at\ 0m\ alt. ISA$$

$$C_D = 0.025 + 0.138(0.48)^2 = 0.0568$$

3g level turn at SSL, M0.78 (265.434 m/s).

➔ **Level-Turn Turn Radius.**

Assumptions:

constant gross weight, $g = 9.81\ m/s^2$, **ISA conditions apply.**

$$R_{min} = \frac{1}{0.5\rho g C_{L_{max}}}\left(\frac{W}{S}\right)\left(\frac{n}{\sqrt{n^2-1}}\right) = \frac{1}{0.5(1.225)(9.81)(0.48)}\left(\frac{200,000}{50}\right)\left(\frac{3}{\sqrt{9-1}}\right) = (0.3467)(4,000)\left(\frac{3}{\sqrt{8}}\right)$$
$$= 1,471.024\ m$$

$$R = \frac{V^2}{g\sqrt{n^2-1}} = \frac{265.434^2}{9.81\sqrt{9-1}} = 2,539.2128\ \approx 2.539 \sim 2.54\ Km$$

$$\frac{R}{R} = \frac{\frac{1}{0.5\rho g C_L}\left(\frac{W}{S}\right)\left(\frac{n}{\sqrt{n^2-1}}\right)}{\frac{V^2}{g\sqrt{n^2-1}}} = 1 \qquad \frac{1}{0.5\rho C_L}\left(\frac{W}{S}\right)(n) = V^2 \therefore C_L = \frac{1}{0.5\rho V^2}\left(\frac{W}{S}\right)(n)$$
$$= \frac{1}{0.5(1.225)(265.434)^2}\left(\frac{200,000}{50}\right)(3) = 0.278 < C_{Lmax}$$

Q 27

twin-turbojet aircraft.

$$S = 50\,m^2, \quad C_{D0} = 0.025, \quad K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.8 \times \frac{12^2}{50}} = 0.138\,, \quad W = 200\,KN,$$

$$C_{Lbuff} = 0.48\ at\ M0.78$$

$$T_A = 2 \times 75\ KN\ at\ SSL, \quad T|_{11km\ alt.} = 2 \times 37.5\ KN$$

$$C_D = 0.025 + 0.138(0.48)^2 = 0.0568$$

➔ **Need for Airbrakes.**

Airbrakes (aka speed brakes) are typically used to reduce air speed.

$$n_{max} = \frac{T_a/w}{2\sqrt{kC_{D0}}} = \frac{2 \times /200}{2\sqrt{0.138 \times 0.025}} = 3.19221$$

If contrained by stall

$$n_{lim} = \frac{V^2}{W}(0.5\rho S C_{Lmax}) = \frac{(221.4)^2}{200,000}(0.5)(0.2978 \times 1.225)(50)(0.48) = 1.0729$$

Yes we need airbrakes to reduce the speed due → nonsense.

Attempt

Maneuver: Maximum-g turn at $M0.75$ $(0.75 \times 295.2\ m/s)$ given 30° dive angle and 11km.

$$T_{total} = 75\ kN$$

$$C_{L_{buff}} = 0.48$$

$$T - D - w\ sin\ \gamma = 0$$

$$n = \left(\frac{0.5\rho v^2}{k\left(\frac{W}{S}\right)}\left(\frac{T}{w} - 0.5\rho v^2 \frac{C_{D0}}{\left(\frac{W}{S}\right)}\right)\right)^{0.5} = \left(\frac{0.5 \times 0.364805 \times 221.4^2}{0.138\left(\frac{200000}{50}\right)}\left(\frac{75}{200} - 0.5 \times 0.364805 \times 221.4^2 \frac{0.025}{\left(\frac{200000}{50}\right)}\right)\right)^{0.5}$$

$$n_{max} = \frac{T_a/w}{2\sqrt{kC_{D0}}} = \frac{75/200}{2\sqrt{0.138 \times 0.025}} = 3.19221$$

Final Attempt

$$\boldsymbol{C_L = 0.61}$$

$$n_{max} = \frac{0.5\rho v^2 C_{L_{max}}}{w/S} = \frac{0.5 \times 0.364805 \times 221.4^2 \times 0.61}{200{,}000/50} = 1.36350226345725$$

$$\Delta T - D - w\sin\gamma = 0$$

$$D = wn\frac{C_D}{C_L} = 200{,}000 \times 1.36350226345725 \times \frac{0.025 + 0.138 \times 0.61^2}{0.61} = 34132.172168691261$$

$$D = \left(0.5\rho S C_{D0} v^2 + \left(\frac{2kw^2n^2}{\rho S v^2}\right)\right)$$

$$= \left(0.5 \times 0.364805 \times 50 \times 221.4^2 \times 0.025 + \left(\frac{2 \times 0.138 \times (200{,}000 \times 1.36350226345725)^2}{0.364805 \times 50 \times 221.4^2}\right)\right)$$

$$= 34132.172168691261.$$

$$(75{,}000 - T_{air\ brakes}) - 34132.172168691261 - 200{,}000\sin(-30°) = 0$$

$$T_{rev\ thrust_{eng}} = \frac{140867.827831308739}{2} = 70433.9139 \sim 71\ kN$$

➔ **Bank Angle and Turn Radius (Ans: 42.8°, $4.67\ Km$ n=1.363)**

$$n_{max} = \left(\frac{L}{D}\right)_{max} = \left(\frac{T}{W}\right)_{max} = \frac{T_a/W}{2\sqrt{KC_{D0}}} = \frac{75/200}{2\sqrt{0.138 \times 0.025}} = 3.192 \text{ ∴ maximum of the maximum values of load factor}$$

$$V_{stall} = \sqrt{\frac{2nw}{\rho S C_L}} = \sqrt{\frac{2 \times 3.192 \times 200{,}000}{0.2978 \times 1.225 \times 50 \times 0.48}} = 381.879\ m/s$$

failed

$$L = W = C_{L_{max}} 0.5\rho V_{stall}^2 S\ \ at\ stall \quad L = n_{max}W = C_{L_{max}}0.5\rho V^2 S \quad \text{confined by load factor.}$$

$$C_{L_{max}}\ is\ not\ quite\ there$$

$$n_{max} = \frac{V^2}{V_{stall}^2} = \frac{221.4^2}{\frac{W}{C_{L_{max}}0.5\rho S}} = \frac{49017.96}{\frac{200{,}000}{0.48(0.5)(0.2978 \times 1.225)(50)}} = 1.072919813868$$

$$n_{max} = \frac{V^2}{W}(0.5\rho S C_{Lmax}) = \frac{(221.4)^2}{200{,}000}(0.5)(0.2978 \times 1.225)(50)(0.48) = 1.1176$$

$$n_{max} = \frac{\frac{1}{2}\rho V^2 C_{Lmax}}{W/S} = \frac{0.5(0.2978 \times 1.225)(221.4)^2 0.48}{200{,}000/50} = 1.072$$

Second attempt

Find correct n_max

If stall limited, embrace the value of CL max

$$M = \sqrt{\frac{2w}{\rho S \ _{LMax}}} = \sqrt{\frac{2\times 200{,}000}{(0.2978\times 1.225)50\times 0.48}}$$

From M find CL bounds and then translate that into the value of corrected n_max.

$\frac{1}{\sqrt{1.4 p_{\mathrm{h}} S C_{D0}}}\left[D \pm \sqrt{D^2 + 4k(nw)^2 C_{D0}}\right]^{0.5} = \frac{1}{\sqrt{1.4\times\times 50\times 0.025}}[\pm\sqrt{75{,}000 + 4 \times 0.138}]$ can't use this formula because it depends on n in the first place.

It might be thrust limited.

Thirds attempt

Maximum velocity for a given thrust. then

Then translate the max velocity into n if given we are flying at maximum cL

$$C_L = \frac{T}{W} \pm \frac{\sqrt{\left(\frac{T}{W}\right) - 4C_{D0}k}}{2k} = \frac{75{,}000}{200{,}000} \pm \frac{\sqrt{\frac{75{,}000}{200{,}000} - 4 \times 0.025 \times 0.138}}{2 \times 0.138}$$

$$C_L = -1.8, \qquad 2.55\ (\therefore 0.48)$$

We need to reduce C_L to proper value which is $C_{L_{max}}$. Then find the respective T.

5855.072 N. The thrust should be.

Final Attempt

$\therefore n_{max}$ *should be* 1.363 from previous part

$$\phi = \cos^{-1}\left(\frac{1}{n_{max}}\right) = \cos^{-1}\left(\frac{1}{1.363}\right) = 42.8°$$

$$R = \frac{V_\infty^2 \cos\gamma}{g\sqrt{\left(\frac{n_{max}}{cos\ \gamma}\right)^2 - 1}} = \frac{(221.4)^2 \cos(-30°)}{9.81\sqrt{\left(1.363\frac{1}{cos\ (-30°)}\right)^2 - 1}} = 4.672\ Km$$

Q 28

Given twin-turbojet aircraft.

$$S = 50\ m^2, \qquad C_{D0} = 0.025, \qquad K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.8 \times \frac{12^2}{50}} = 0.138\ , \qquad W = 200\ KN,$$

$$C_{Lbuff} = 0.48\ at\ M0.78$$

$$T_A = 2 \times 75\ KN\ at\ SSL, \qquad T|_{11km\ alt.} = 2 \times 37.5\ KN$$

$$C_D = 0.025 + 0.138(0.48)^2 = 0.0568$$

3g coordinated turn at pressure altitude zero at $M0.78(265.512\ m/s)$ with full thrust.

➔ **Climb or Descend (Ans: Climb)**

The aircraft has a high thrust as well as Mach number, therefore it is inferred that the aircraft will climb.

T>D

$$\boldsymbol{T - D - w\sin\gamma = 0}$$

Final Attempt

$$M0.78(265.512\ m/s)$$

$$T = 150\ kN$$

$$K = \frac{1}{\pi eAR} = \frac{1}{\pi 0.8 \times \frac{12^2}{50}} = 0.138$$

$$C_L = 0.48, C_D = C_{D0} + kC_L^2 = 0.025 + 0.138 \times 0.6^2 = 0.07468$$

$$D = w\frac{C_D}{C_L}n = 200\frac{0.07468}{0.6}3 = 74.68{\sim}75\ kN$$

Since T>D, it incentives to initiate a climb

➔ **Climb Rate and Angle (Ans: 98.2 m/s, $\gamma = 21.4°$)**

$$ROC = V_\infty\left[\frac{T}{w} - 0.5\rho V^2\left(\frac{S}{w}\right)C_{D0} - \frac{w}{S}\frac{2K}{\rho V^2}\right]$$

$$265.512\left[\frac{2 \times 75{,}000}{200{,}000} - 0.5 \times 1.225 \times 265.512^2\left(\frac{50}{200{,}000}\right)0.025 - \frac{200{,}000}{50}\frac{2 \times 0.138}{1.225 \times 265.512^2}\right] = 124.086\ m/s$$

$$C_L = \frac{nw}{0.5\rho SV^2} = \frac{3 \times 200{,}000}{0.5 \times 1.225 \times 50 \times 265.512^2} = 0.2779$$

$$C_L = \frac{nw}{0.7pSM^2} = \frac{3 \times 200{,}000}{0.7 \times 101325\ \times 50 \times 0.78^2} = 0.27808$$

$$C_D = 0.025 + 0.138(0.27808)^2 = 0.03566$$

$$D = 0.5\rho V^2 C_D = 0.5 \times 1.225 \times 265.512^2 \times 0.03566 = 1539.77$$

Or another way

$ROC = \frac{T}{w} - \frac{C_D}{C_L} = \frac{2\times 75{,}000}{200{,}000} - \frac{0.03566}{0.2779}$ our angle is not that small so this assumption is not valid

$$ROC = V\left(\frac{T}{W} - \frac{D}{W}\right) = 265.512(\frac{150{,}000 - 1539.77}{200{,}000})$$

$$\mathrm{ROC} = \mathrm{v}\sin\gamma \therefore \gamma = \sin^{-1}\left(\frac{ROC}{v}\right) = \sin^{-1}\left(\frac{98.2}{265.51}\right) = 21.7°$$

Final Attempt

$$T - D - w\sin\gamma = 0$$
$$150 - 75 - 200\sin\gamma = 0$$

$$\gamma = 22.02°$$

$$ROC = V\left(\frac{T}{W} - \frac{D}{W}\right) = 265.512\left(\frac{150{,}000 - 75{,}000}{200{,}000}\right) = 99.567\ m/s$$

➔ **Bank Angle and Turn Radius (Ans: $\phi = 72°, R_{turn} = 2.18\ Km$)**

$n = \frac{\cos\gamma}{\cos\phi}$ simplified to $\left(\frac{n}{\cos\gamma}\right) = \frac{3}{\cos 22.02°} = \frac{1}{\cos\phi}$

$$\phi = \cos^{-1}\left(\frac{\cos\gamma}{n}\right) = \cos^{-1}\left(\frac{\cos 22.02°}{3}\right) = 72\,°$$

$$R=\frac{V_\infty^2\cos\gamma}{g\sqrt{\left(\frac{n}{\cos\gamma}\right)^2-1}}=\frac{265.51^2\cos 21.4°}{g\sqrt{\left(\frac{3}{\cos 22.02°}\right)^2-1}}=21749m\approx 2.174\ Km$$

Q 29

Given twin-turbojet aircraft.

$$S=50\ m^2,\quad C_{D0}=0.025,\quad K=\frac{1}{\pi eAR}=\frac{1}{\pi 0.8\times\frac{12^2}{50}}=0.138\,,\quad W=200\ KN,$$

$$C_{Lbuff}=0.48\ at\ M0.78$$

$$T_A=2\times 75\ KN\ at\ SSL,\quad T|_{11km\ alt.}=2\times 37.5\ KN$$

$$C_D=0.025+0.138(0.48)^2=0.0568$$

SSL

➔ **Minimum Turn Radius, and Mach Number at 3g** **(Ans: $R_{turn}=667\ m, 0.4\ Mach$)**

$$T_A=(C_{D0}0.7pS)M^2+\left(\frac{Kn^2w^2}{0.7pS}\right)\frac{1}{M^2}$$

$$0=(0.025\times 0.7\times 101{,}325\times 50)M^4-150{,}000M^2+\left(\frac{0.138\times 3^2\times 200{,}000^2}{0.7\times 101{,}325\times 50}\right)$$

$$0=(88659.375)M^4-150{,}000M^2+(14008.671)$$

$$M=0.315,\quad 1.262\ Mach$$

$C_L=\frac{nw}{0.7pM^2S}=\frac{3\times 200{,}000}{0.7\times 101{,}325\times 0.315^2\times 50}=1.7>C_{Lbuff}$ ∴ **stall limited**

From buffet limit chart C_L_{max} is 1.06

$$R_{min}=\frac{W}{0.5\rho gSC_{L_{max}}}\frac{n_{max}}{\sqrt{n_{max}^2-1}}=\frac{200{,}000}{0.5\times 1.225\times 9.81\times 50\times 1.06}\frac{3}{\sqrt{3^2-1}}=666.124\ m$$

$$R=\frac{V^2}{g\sqrt{n^2-1}}\ \therefore V=\sqrt{Rg\sqrt{n^2-1}}=\sqrt{666.124\times 9.81\sqrt{3^2-1}}=135.952\frac{m}{s}$$

$$M=\frac{V}{a}=\frac{135.952}{340.4}=0.3993\approx 0.4\ Mach$$

Q 30

Given twin-turbojet aircraft.

$$S=50\ m^2,\quad C_{D0}=0.025,\quad K=\frac{1}{\pi eAR}=\frac{1}{\pi 0.8\times\frac{12^2}{50}}=0.138\,,\quad W=200\ KN,$$

$$C_{Lbuff}=0.48\ at\ M0.78$$

$$T_A=2\times 75\ KN\ at\ SSL,\quad T|_{11km\ alt.}=2\times 37.5\ KN$$

$$C_D=0.025+0.138(0.48)^2=0.0568$$

At 11 Km alt ($0.3648\frac{kg}{m^3}$, $295.2\ m/s$) and 1.5 g with 10° Climb angle

➔ **Minimum Turn Radius and Mach Number at $1.5\ g$ & $\gamma = 10°$**

Assumptions: minimum power => minimal velocity => minimal turn reduce

$$\sigma = 0.2978 \therefore \rho|_{11km} = 0.3648\frac{Kg}{m^3} \quad a = 295.2\frac{m}{s}$$

$$R_{min} = \frac{W}{0.5\rho g S C_L}\frac{n_{max}}{\sqrt{n_{max}^2 - 1}}$$

From buffet limit chart C_L_{max} is 1.06

$$R_{min} = \frac{W cos\gamma}{0.5\rho g S C_{L_{max}}}\frac{n_{max}}{\sqrt{\left(\frac{n_{max}}{cos\gamma}\right)^2 - 1}} = \frac{200{,}000 cos10°}{0.5(0.3648)(9.81)(50)(1.06)}\frac{1.5}{\sqrt{\left(\frac{1.5}{cos10°}\right)^2 - 1}}$$

$$= \frac{200{,}000\ cos(10°)}{0.5 \times 0.3648 \times 9.81 \times 50 \times 1.06}\frac{1.5}{\sqrt{1.5^2 - 1}} = 666.124\ m$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} =$$

$$C_L = \sqrt{\frac{3C_{D0}}{K}} = \sqrt{\frac{3 \times 0.025}{0.138}} = 0.737$$

$$n = \frac{cos\ \gamma}{cos\ \mu}, \quad sin\ \mu = \frac{sin\phi}{cos\ \gamma} \therefore \mu = cos^{-1}\left(\frac{cos\gamma}{n}\right) = 48.964°, \phi = sin^{-1}(sin\ \mu\ \ cos\ \gamma) = 47.974°$$

$$V = \sqrt{\frac{2nW}{C_{Lmax}\rho S}} = \sqrt{\frac{2 \times 1.5 \times 200000}{1.05 \times 0.3648 \times 50}} = 211.26\frac{m}{s}(0.72\ Mach)$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} = \frac{211.26^2 cos10°}{9.81\sqrt{\left(\frac{1.5}{cos10}\right)^2 - 1}} = 3.9\ Km$$

Second attempt

$$(C_{D0} 0.7pS)M^2 + \left(\frac{Kn^2w^2}{0.7pS}\right)\frac{1}{M^2} = \mathrm{T_A}$$

$$(0.025 \times 0.7 \times 22{,}700 \times 50)M^2 + \left(\frac{0.138 \times 1.5^2 200{,}000^2}{0.7 \times 22{,}700 \times 50}\right)\frac{1}{M^2} = 75{,}000$$

$$(19862.5)M^4 - 75{,}000\mathrm{M}^2 + (15632.47) = 0$$

$$M = 0.47, 1.88\ Mach$$

$C_L = \frac{nw}{0.7pM^2S} = \frac{1.5 \times 200{,}000}{0.7 \times 22{,}700 \times 0.47^2 \times 50} = 1.7 > C_{Lbuff}$ ∴stalled limited

$$M = \sqrt{\frac{nw}{0.7pSC_L}} = \sqrt{\frac{1.5 \times 200{,}000}{0.7 \times 22{,}700 \times 1.05 \times 50}} = 0.6\ Mach$$

Final Attempt

$$D = T_{\mathrm{max}} - \mathrm{wsin\gamma} = 75{,}000 - 200{,}000\ \sin 10° = 40{,}270\ \mathrm{N}$$

$$v_{1,2} = \frac{1}{\sqrt{\rho S C_0}}\left[D \pm \sqrt{D^2 - 4k(nw)^2 C_{D0}}\,\right]^{0.5}$$

$$= \frac{1}{\sqrt{0.3648 \times 50 \times 0.025}}\left[40270 \pm \sqrt{40270^2 - 4 \times 0.138(1.5 \times 200{,}000)^2 0.025}\,\right]^{0.5}$$

$$= 1.4809[40270 \pm 20398]^{0.5} = 211.11\frac{m}{s}, \quad 366109\frac{m}{s}$$

$$C_L = \frac{nw}{0.5\rho V^2 S} = \frac{1.5 \times 200{,}000}{0.5 \times 0.3648 \times 50 \times 211.11^2} = 0.738 \ \ not\ limited$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} = \frac{211.11^2 cos\,10°}{9.81\sqrt{\left(\frac{1.5}{cos10°}\right)^2 - 1}} = 3894.2198 \approx 3.9\ Km$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} \therefore V = \frac{1}{cos\gamma} \times \sqrt{Rg\sqrt{\left(\frac{n}{cos\gamma}\right)^2 - 1}} = \frac{1}{cos10°} \times \sqrt{R \times 9.81\sqrt{\left(\frac{1.5}{cos10°}\right)^2 - 1}} = 136.041\frac{m}{s}$$

$$M = \frac{V}{a} = \frac{211.11}{295.2} = 0.715 \approx 0.72\ Mach$$

Q 31

Given twin-prop aircraft.

$$S = 27.3\ m^2, \quad C_{D0} = 0.03, \quad K = 0.047, \quad W = 35.5\ KN, \quad C_{Lbuff} = 1.9$$

$$P_A = 380\ KW\ at\ SSL$$

$$P_{max} = 2\sigma^1 P_e = 2(0.8217)^1(380) = 624.492\ KW\ at\ 2000\ m$$

$$2000\ rpm, dia\ 1.8m, \eta_p = 0.47(2.54J - J^2)^{0.5}$$

$$C_D = 0.03 + 0.047(1.9)^2 = 0.19967$$

min-radius turn at 2g, full throttle, at altitude of 2000m ($\rho = 1.00658\frac{kg}{m^3}, a = 332.5\ m/s$) and dive angle of 10 degrees.

➔ **Stall or thrust Limited. (Ans: stall-limited) assume initially that it is stall limited not thrust limited.**

$$v_{1,2} = \frac{1}{\sqrt{\rho S C_0}}\left[D \pm \sqrt{D^2 - 4k(nw)^2 C_{D0}}\,\right]^{0.5}$$

$$D = T_A - w sin\gamma = \frac{0.47\left(2.54\frac{1}{60V} - \left(\frac{1}{60}\right)^2\right)^{0.5} 624{,}492}{1} - 35{,}500\ sin10°$$

$$D = 293511.24\left(2.54\frac{1}{60V} - \left(\frac{1}{60}\right)^2\right) - 6164.51$$

$$v_{1,2} = \frac{1}{\sqrt{1.00658 \times 27.3 \times 0.03}}\left[D \pm \sqrt{D^2 - 4 \times 0.047(2 \times 35{,}500)^2 \times 0.03}\,\right]^{0.5}$$

Usnig to wolfram to solve for V, $v_{1,2} = 152.39\frac{m}{s}$

$$C_L = \frac{nw}{0.5\rho V^2 S} = \frac{2 \times 35{,}500}{0.5 \times 1.00658 \times 27.4 \times 152.39^2} =$$

$C_L = \frac{nw}{0.7pM^2S} = \frac{3 \times 200{,}000}{0.7 \times 101{,}325 \times 0.315^2 \times 50} = 1.7 > C_{Lbuff}$ ∴ stall limited

From tutorial

Thrust required

$$T_{req} = AM^2 + \frac{Bn}{M^2} \quad when\ you\ find\ M_1, M_2\ \ use\ the\ smalled$$

$C_L = \frac{nw}{0.7pM^2S}$ $\quad$ $and\ compare\ to\ C_L\ buffet$ if higher then stall limited at the chosen mach number we just found

Attempt

Minimum turn radius of 2-g

$$V_{R_{min}} < V_{T_{min}} < V_{n_{\square}{}_{min}}$$

$$P = \sigma P_o = 0.8217 \times 760{,}000 = 624{,}492\ watt$$

$$\eta = 0.47(2.54J - J^2)^{0.5}$$

$$J = \frac{v}{nD} = \frac{v}{\frac{2000}{60}1.8} = \frac{v}{60}$$

$$\eta = 0.47(2.54J - J^2)^{0.5} = 0.47\left(2.54\frac{V}{60} - \left(\frac{V}{60}\right)^2\right)^{0.5}$$

$$T - D - w\sin\gamma = 0 \therefore D = T - w\sin\gamma = wn\frac{C_D}{C_L}$$

$$\eta P_A = T_A V \rightarrow T_A = \frac{\eta P_A}{V} = \frac{0.47\left(2.54\frac{1}{60V} - (\frac{1}{60})^2\right)^{0.5} 624{,}492}{1}$$

$$T = (0.5\rho S C_{D0})v^2 + \left(\frac{2kn^2w^2}{\rho S}\right)1/v^2$$

$$T = (0.5 \times 1.0065825 \times 27.3 \times 0.02)v^2 + \left(\frac{2 \times 0.047 \times 2^2 \times 35{,}500^2}{1.0065825 \times 27.3}\right)1/v^2$$

$$T = (0.2747970225)v^2 + (1.724378218108240 \times 10\text{^}7)1/v^2$$

$$\frac{T_A}{T} = \frac{0.47\left(2.54\frac{1}{60V} - (\frac{1}{60})^2\right)^{0.5} 624{,}492}{(0.2747970225)V^2 + (1.7243782181082 \times 10\text{^}7)1/V^2}$$

$$v = \ 57.5247, 82.9806$$

Find the velocity then tie that to lift coefficient

$$C_L = \frac{nw}{0.5\rho V^2 S} = \frac{2 \times 35{,}500}{0.5 \times 1.00658 \times 27.4 \times 152.39^2}$$

Attempt

Minimum turn radius for prop aircraft with 2-g and full throttle and 10 degree dive

$C_L = \frac{nw}{0.5\rho V^2 S} = \frac{2\times 35{,}500}{0.5\times1.00658\times27.4\times152.39^2}$ find the velcotiy first which should be lower than 52

$$\frac{0.47\left(2.54\frac{\sqrt{\frac{2 \times 35500}{0.5 \times 1.00658 \times 27.3 \times C_L}}}{60} - \left(\frac{\sqrt{\frac{2 \times 35500}{0.5 \times 1.00658 \times 27.3 \times C_L}}}{60}\right)^2\right)^{0.5} 624{,}492}{\sqrt{\frac{2 \times 35500}{0.5 \times 1.00658 \times 27.3 \times C_L}}}$$

Final Attempt

$$T_A = \frac{\eta P_A}{V} = \frac{0.47\left(2.54\frac{V}{60} - \left(\frac{V}{60}\right)^2\right)^{0.5} 624{,}492}{V} = \frac{0.47\left(2.54\frac{1}{60V} - \left(\frac{1}{60}\right)^2\right)^{0.5} 624{,}492}{1}$$

$$V = \sqrt{\frac{nw}{0.5\rho S C_L}} = \sqrt{\frac{2 \times 35500}{0.5 \times 1.00658 \times 27.3 \times C_L}}$$

$$w \sin\gamma = 35500 \sin 10°$$

$$D = T - w\sin\gamma$$

$$2 \times 35500 \frac{0.03 + 0.047C_L^2}{C_L} = \frac{0.47\left(2.54\frac{1}{60\sqrt{\frac{2\times 35500}{0.5\times 1.00658\times 27.3\times C_L}}} - \left(\frac{1}{60}\right)^2\right)^{0.5} 624{,}492}{1} - 35500\sin 10°$$

$$n_{Rmin} = \sqrt{2 - \frac{4kC_{D0}}{(T/w)}} = \sqrt{2 - \frac{4\times 0.047\times 0.03}{(T/35500)}} = 2$$

$$V_{Rmin} = \sqrt{\frac{4k\left(\frac{w}{s}\right)}{\rho(T/w)}} = \sqrt{\frac{4\times 0.047\left(\frac{35500}{27.3}\right)}{1.00658(T/35500)}} = \sqrt{\frac{2\times 35500}{0.5\times 1.00658\times 27.3\times C_L}} = \sqrt{\frac{nw}{0.5\rho S C_L}}$$

$$\sqrt{\frac{4\times 0.047\left(\frac{35500}{27.3}\right)}{1.00658\left(\frac{0.47\left(2.54\frac{1}{60\sqrt{\frac{2\times 35500}{0.5\times 1.00658\times 27.3\times C_L}}} - \left(\frac{1}{60}\right)^2\right)^{0.5} 624{,}492}{35500}\right)}} = \sqrt{\frac{2\times 35500}{0.5\times 1.00658\times 27.3\times C_L}}$$

$$C_L = 6.00522, 0.22512$$

The higher term is in our interest and it seems to be overshooting, exceeding stall costraint

The higher term because acheiving corner velocity requires high CL and high n (given)

$$C_L > C_{Lbuff} \quad \therefore \textit{stall limited}$$

➔ **Turn Radius? (Ans: 155 m)**

$$v = \sqrt{\frac{nw}{0.5\rho S C_L}} = \sqrt{\frac{2\times 35{,}500}{0.5\times 1.00658\times 27.3\times 1.9}} = 52.15\frac{m}{s}$$

$$R = \frac{V^2 \cos\gamma}{g\sqrt{\left(\frac{n}{\cos\gamma}\right)^2 - 1}} = \frac{(52.15)^2 \cos 10}{9.81\sqrt{\left(\frac{2}{\cos 10}\right)^2 - 1}} = \frac{2678.399}{17.34} = 154.5 \sim 155\ m$$

Q 32

Given jet engine.

$$S = 20\ m^2, C_{D0} = 0.025, K = 0.071, W = 30\ KN$$

$$T_A = 16\ KN\ at\ SSL$$

$$T_{max} = \sigma^1 T_e = (0.6012)^1(16) = 9.6192\ KN\ at\ 5000m$$

$$C_D = 0.025 + 0.071\left(C_{Lbuff}\right)^2$$

Alt 5000m ($\rho = 0.6621125 \frac{kg}{m^3}, p = \ 54{,}100\ Pa,\ a = 320.5\ m/s$)

➔ **At 5000m SLF with 80% thrust, then pulled 3.5 g coordinated turn with max thrust and no change in Mach. (Ans: Descend at $0.9°$, banked at $73.4°$)**

$T_A = 7.695\ KN$ find Mach Number

$$M = \frac{1}{\sqrt{1.4pSC_{D0}}}\left[T_A \pm \sqrt{T_A^2 - 4kw^2C_{D0}}\right]$$

$$M = \frac{1}{\sqrt{1.4 \times 54100 \times 20 \times 0.025}}\left[7{,}695 \pm \sqrt{7{,}695^2 - 4 \times 0.071 \times 30{,}000^2 \times 0.025}\right]$$

$$= 0.00514[7{,}695 \pm 7267.94] = 2.1950884$$

$$V^2 = \frac{T}{C_{D0}\rho S} \pm \sqrt{\left(\frac{T}{C_{D0}\rho S}\right)^2 - \frac{4KW^2}{C_{D0}\rho^2 S^2}}$$

$$V^2 = \frac{7695}{0.025 \times 0.6621125 \times 20} \pm \sqrt{\left(\frac{7695}{0.025 \times 0.6621125 \times 20}\right)^2 - \frac{4 \times 0.071 \times 30{,}000^2}{0.025 \times 0.6621125^2 \times 20^2}}$$

$$V^2 = 23243.78 \pm \sqrt{23243.78^2 -} = 45197.6, 1289.98$$

$$sin\gamma = \frac{T - D}{W} = \frac{7695}{30{,}000}$$

Second attempt

$$M = \frac{1}{\sqrt{1.4 \times 54050 \times 20 \times 0.025}}\left[7695 \pm \sqrt{7695^2 - 4 \times 0.071 \times 30{,}000^2 \times 0.025}\right]$$

Those equations will not yield something useful.

$$n = \frac{cos\ \gamma}{cos\ \mu},\quad sin\ \mu = \frac{sin\phi}{cos\ \gamma}\ \therefore \gamma = cos^{-1}\left(\frac{sin\phi}{sin\ \mu}\right) i.e.$$

$$\frac{cos\gamma}{cos\gamma} = \frac{n\ cos\mu}{\frac{sin\phi}{sin\mu}} = \frac{n}{sin\phi \times tan\mu} = 1 \qquad \gamma = cos^{-1}\left(\frac{sin\phi}{sin(tan^{-1}(\frac{n}{sin\phi}))}\right) = 6.59°$$

Final Attempt

$$k = 0.071$$

SLF

$$T_{before} = 0.8 \times 9.6192 = 7.695\ kN$$

$$C_D = C_{D0} + kC_L^2 = 0.025 + 0.071C_L^2$$

$$T = w\frac{C_D}{C_L} = 30{,}000\frac{0.025 + 0.071C_L^2}{C_L} = 7695$$

$$C_L = 0.100248, 3.51243$$

Turn

Mach number at the specific elevation is unchanged so is Cl

$$T_{after} = 9.6192\ kN$$

$$D = nw\frac{C_D}{C_L} = 3.5 \times 30{,}000 \times \frac{0.025 + 0.071 \times\ 0.100248^2}{0.100248} = 26932.41$$

$D > T$ ∴Decent is anticipated

$\phi = cos^{-1}(1/3.5) = 73.398°$ bank angle

➔ **Mach number range at 5000m? (Ans: 0.36-0.84 Mach)**

$$T_A = (0.7pC_{D0}S)M^2 + \left(\frac{Kw^2}{0.7\mathrm{p}S}\right)\frac{1}{M^2}$$

$$(0.7 \times 54100 \times 0.025 \times 20)M^4 - 9619.2M^2 + \left(\frac{0.071 \times 3.5^2 \times 30{,}000^2}{0.7 \times 54100 \times 20}\right) = 0$$

$$(0.7 \times 54100 \times 0.025 \times 20)M^4 - 9619.2M^2 + \left(\frac{0.071 \times 3.5^2 \times 30{,}000^2}{0.7 \times 54100 \times 20}\right) = 0$$

$$18{,}935M^4 - 9619.2M^2 + 1033.403 = 0$$

The results are imaginary numbers not making sense.

Second attempt

$$D = T_{\max} - \mathrm{w}\sin\gamma = 9{,}619.2 - 30{,}000 \times \mathrm{si\,n}(0.9) = 9147.98\ N$$

$$m_{1,2} = \frac{1}{\sqrt{1.4p_{\mathrm{h}}SC_{D0}}}\left[D \pm \sqrt{D^2 + 4k(nw)^2C_{D0}}\right]^{0.5}$$

$$\frac{1}{\sqrt{1.4 \times 54{,}100 \times 20 \times 0.025}}\left[9147.98\ \pm \sqrt{9147.98\ ^2 + 4 \times 0.071(3.5 \times 30{,}000)^2 \times 0.025}\right]^{0.5}$$

$$m_{1,2} = 0.005139[9147.98 \pm 2325.519]^{0.5} = 0.4244 - 0.55$$

$$M = \sqrt{\frac{nw}{0.7pSC_{Lmax}}} = \sqrt{\frac{3.5 \times 30{,}000}{0.7 \times 54100 \times 20 \times 1.05}} = 0.363\ Mach$$

Third attempt

$$T_A = (0.7pC_{D0}S)M^2 + \left(\frac{Kw^2}{0.7\mathrm{p}S}\right)\frac{1}{M^2}$$

$$(0.7 \times 54100 \times 0.025 \times 20)M^4 - 9619.2M^2 + \left(\frac{0.071 \times 3.5^2 \times 30{,}000^2}{0.7 \times 54100 \times 20}\right) = 0$$

$$(0.7 \times 54100 \times 0.025 \times 20)M^4 - 9619.2M^2 + \left(\frac{0.071 \times 3.5^2 \times 30{,}000^2}{0.7 \times 54100 \times 20}\right) = 0$$

$$18{,}935M^4 - 9619.2M^2 + 1033.403 = 0$$

Final Attempt

$$C_L = \frac{\frac{T}{W} \pm \sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D0}K}}{2K} = \frac{\frac{9.6192}{30} \pm \sqrt{\left(\frac{9.6192}{30}\right)^2 - 4 \times 0.071 \times 0.025}}{2 \times 0.071}$$

$$= 4.35\ (reduced\ to\ C_{Lmax}), \qquad 0.2$$

$$M = \sqrt{\frac{nw}{0.7pSC_{Lmax}}} = \sqrt{\frac{3.5 \times 30{,}000}{0.7 \times 54100 \times 20 \times 1.05}} = 0.36336 \approx 0.36\ Mach$$

$$M = \sqrt{\frac{nw}{0.7pSC_{Lmin}}} = \sqrt{\frac{3.5 \times 30{,}000}{0.7 \times 54100 \times 20 \times 0.1617}} = 0.8325 \approx 0.83\ Mach$$

The mach speed range is [0.36, 0.83]

➔ **Possible to pull-up while stalling below structural limit? (Ans: $n = 8.5\ at\ C_{Lmax} < 12g$)**

First attempt (assume C_L max)

$$M = \sqrt{\frac{nW}{0.7pSC_{Lmax}}} \therefore n = \frac{0.7pM^2SC_{Lmax}}{W} = \frac{0.7 \times 54100 \times 0.36^2 \times 20 \times 1.05}{30{,}000} = 3.4$$

$$M = \sqrt{\frac{nw}{0.7pSC_{Lmax}}} = \sqrt{\frac{3.5 \times 30{,}000}{0.7 \times 54100 \times 20 \times 1.05}} = 0.36336 \approx 0.36\ Mach$$

$$V_{stall} = V\sqrt{n_z} \therefore n_z = \left(\frac{V_{stall}}{V}\right)^2 =$$

Max thrust

$$n = \frac{sin\phi}{cos\gamma} = \frac{sin73.4}{cos\ 0.9}$$

Attempt

Stated condition: stalling during a pull-up

$$M = \sqrt{\frac{nw}{0.7pSC_{Lmax}}} = 0.36\ Mach$$

$$T = 9.6192\ kN$$

$$C_{L_{max}} = 1.05$$

$$3.5g - coordinated\ turn, in\ addition\ to\ pull\ up\ turn$$

$$V = \sqrt{\frac{w}{0.5\rho SC_L}\frac{\cos\gamma}{\cos\mu}}$$

$$D = \frac{C_D}{C_L}\frac{w\cos\gamma}{\cos\mu}$$

$$\cos\mu = \cos(\sin^{-1}(\frac{\sin\phi}{\cos\gamma})) = \cos(\sin^{-1}(\frac{\sin 73.4}{\cos\gamma}))$$

$$C_D = C_{D0} + kC_L^2 = 0.025 + 0.071 \times 1.05^2 = 0.1032775$$

$$T - D - w\sin\gamma = 0$$

$$9619.2 - \frac{0.1032775}{1.05} \times \frac{30000 \cos \gamma°}{\cos(\sin^{-1}(\frac{\sin 73.4°}{\cos \gamma°}))} - 30000 \sin \gamma° = 0$$

114.592 (-0.116213 + 3.14159 n) 25 degree

114.592 (-0.0123519 + 3.14159 n)

$$\sin \mu = \frac{\sin \phi}{\cos \gamma} = \frac{\sin 73.4}{\cos 0.9} = 0.958441$$

$$\cos \mu = \cos(\sin^{-1}(\frac{\sin \phi}{\cos \gamma})) = \cos(\sin^{-1}(\frac{\sin 73.4}{\cos 25}))$$

$$n = \frac{\cos \gamma}{\cos \mu} = \frac{\cos 25°}{\cos(\sin^{-1}(\frac{\sin 73.4\ °}{\cos 25\ °}))}$$

$$n = \frac{\cos \gamma}{\cos \mu} = \frac{\cos 25°}{\cos(\sin^{-1}(\frac{\sin 73.4\ °}{\cos 25\ °}))}$$

$$8.5 = \frac{\cos \gamma}{\cos(\sin^{-1}(\frac{\sin 73.4\ °}{\cos \gamma}))}$$

Den 83.5

-15.302

15.3 turn angle should be

$$v = \sqrt{\frac{w}{0.5\ \rho S C_L} \frac{\cos \gamma}{\cos \mu}}$$

Attempt

$$V_{stall} = V_{SLF} \sqrt{n_z}$$

$$9619.2 - \frac{0.1032775}{1.05} \times 30000 \times n - 30000 \sin 0.9° = 0$$

End goal is finding n

$$65.6901 = \sqrt{\frac{30000}{0.5 \times 0.6621125 \times 20 \times 1.05} n}$$

$$n = \frac{\cos \gamma}{\cos \mu}$$

$$9619.2 - \frac{0.1032775}{1.05} \times 30000 \times n - 30000 \sin 15.3° = 0$$

$$\sin \gamma = \frac{T}{w} - \frac{C_D}{C_L} \frac{1}{\cos \phi} = \frac{9619.2}{30{,}000} - \frac{0.1032775}{1.05} \frac{1}{\cos 73.4°}$$

$$R = \frac{V^2}{g(n-1)}$$

Final Attempt

Pull-up characteristics

There is banking + pull-up

Forced pull-up turn leads to great loading on the wings

$$C_L = 1.05, C_D = 0.1032775$$

$$n = \frac{\cos\gamma}{\cos\mu} = \frac{\frac{\sin\phi}{\sin\mu}}{\cos\mu} = \frac{\sin\phi}{\sin\mu\cos\mu}$$

$$T = 9619.2$$

$$C_D = C_{D0} + kC_L^2$$

$$v = \sqrt{\frac{nw}{0.5\rho SC_L}} = \sqrt{\frac{3.5 \times 30{,}000}{0.5 \times 0.6621125 \times 20 \times 1.05}} = 122.8949696$$

$$v = \sqrt{\frac{w}{0.5\rho SC_L}} = \sqrt{\frac{30{,}000}{0.5 \times 0.6621125 \times 20 \times 0.1617}} = 167.394$$

$$v_{turn} = v_{SLF}\sqrt{n} \therefore n = \left(\frac{v_{turn}}{v_{SLF}}\right)^2 = \frac{C_{L_{SLF}}}{C_{L_{turn}}} = \frac{0.100248}{1.05}$$

$$x = 89.0292$$

Note: forcing pull-up when the natural behavior is decent means the higher the load factor is as I saw many times in War Thunder game where the pilot would be fatigued or subconcise with substatial g's, but I could not characterize the pull up features nor the default conditions. If the path angle is known that would make life easier since it would be clear how much force is experineced on the structure.

- **Max alt. at M0.63 and n 12g (Ans: 2300m)**

$$\boldsymbol{C_L|_{M=0.63} = 0.85}$$

$$\boldsymbol{p = \frac{nw}{0.7SM^2C_L} = \frac{12 \times 30{,}000}{0.7 \times 20 \times 0.63^2 \times 0.85} = 76220.965\ Pa}$$

At 2000m, p=79.5 Kpa. at 2500m, p=74.7 Kpa. using interpolation. p= 76.22 Kpa when alt. is 2341.67 m

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Q 33

Given 4-turbofan engines.

$$S = 150\ m^2, C_{D0} = 0.019, C_{Lmax} = 1.7, K = 0.057, W = 510\ KN$$

$$T_{max} = 4\sigma^{0.9}T_e = 4(0.69762)^{0.9}(70) = 202.495\ KN$$

$$C_D = 0.019 + 0.057(1.7)^2 = 0.18373$$

at 3600 m ISA ($\rho = 0.69762\frac{kg}{m^3}$,, $a = 326.2$) with 2.5 g Load Factor and climb angle 10 °

- **Needed Minimum Turn Radius and Mach Number (Ans: 662m, 0.38 Mach, not stall limited but thrust limited)**

Is thrust Limited ($T > T_A$)? Is stall limited ($C_L > C_{Lmax}$)?

$D = \frac{nw\,C_D}{C_L} = \frac{2.5(510{,}000)(0.18373)}{(1.7)} \rightarrow D = 137{,}797\ N < T_A = 202{,}495\ N$ ∴not thrust limited

$$T_A = (0.5\rho S C_{D0})V^2 + \left(\frac{Kn^2w^2}{0.5\rho S}\right)\frac{1}{V^2}$$

$$202.495 = ((0.5)(0.69762)(150)(0.019))V^2 + \left(\frac{(0.057)(2.5)^2(510{,}000)^2}{0.5(0.69762)(150)}\right)\frac{1}{V^2}$$

$$202{,}495 = 0.994V^2 + 1770985637\frac{1}{V^2}$$

$$V = 441.089\frac{m}{s}, \qquad 95.6947\frac{m}{s}$$

$$C_L = \frac{nw}{0.5\rho V^2 cos\gamma S} = \frac{2.5 \times 510{,}000}{0.5 \times 0.69762 \times 150 \times 95.6947^2} = 2.66 > C_{Lbuff}$$

∴The turn is lift/stall limited.

$$C_{L_{new}} = 1.05$$

$$C_L = \frac{nw}{0.5\rho V^2 S} \rightarrow V = \sqrt{\frac{nw}{0.5\rho S C_{Lbuff}}} = \sqrt{\frac{2.5 \times 510{,}000}{0.5 \times 0.69762 \times 150 \times 1.05}} = 150.342\frac{m}{s}\ (0.46\ Mach)$$

From the buffet boundary chart, $M = 0.3125 \Rightarrow\ C_{Lbuff} = 1.05$

$$R = \frac{V_\infty^2 \cos\gamma}{g\sqrt{\left(\frac{n}{cos\gamma}\right)^2 - 1}} = \frac{V_\infty^2 \cos 10}{9.81\sqrt{\left(\frac{2.5}{\cos 10}\right)^2 - 1}}$$

$$\text{R\_min} = \frac{V_n^2}{g\sqrt{n^2-1}} = \frac{M_{SL}^2}{g\sqrt{n^2-1}} = \frac{\left(0.4014 * 340.29\frac{m}{s}\right)^2}{9.81\sqrt{9-1}} = 672.4177\ m$$

$$V_{stall} = \sqrt{\frac{nw}{0.5\rho S C_{Lmax}}}$$

$$C_L = \frac{\frac{T}{W} \pm \sqrt{\left(\frac{T}{W}\right)^2 - 4C_{D0}K}}{2K} = \frac{\frac{202.495}{510} \pm \sqrt{\left(\frac{202.495}{510}\right)^2 - 4 \times 0.019 \times 0.057}}{2 \times 0.057} = 6.91758590681\ or\ 0.0481863670107$$

∴the turn is stall limited

$$R_{min} = \frac{V^2}{g\sqrt{n^2-1}}$$

$$R_{min} = \frac{W cos\gamma}{0.5\rho g S C_{L_{max}}}\frac{n_{max}}{\sqrt{n_{max}^2 - 1}} = \frac{510{,}000 \times cos10}{0.5 \times 0.69762 \times 9.81 \times 150 \times 0.0482}\frac{2.5}{\sqrt{2.5^2 - 1}}$$

$$Mach\ Number = \frac{V}{a} = \frac{124}{326.2} = 0.38\ Mach$$

Second attempt

$$D = T_{\max} - \mathrm{w}\sin\gamma = 202{,}495\ - 510{,}000\ \sin 10° = 113934.43\ \mathrm{N}$$

$$v_{1,2} = \frac{1}{\sqrt{\rho S C_0}}\left[D \pm \sqrt{D^2 - 4k(nw)^2 C_{D0}}\right]^{0.5}$$

$$= \frac{1}{\sqrt{0.69762 \times 150 \times 0.019}}\left[113934.43 \pm \sqrt{113934.43^2 - 4 \times 0.057(2.5 \times 510{,}000)^2 0.019}\right]^{0.5}$$

$$= 0.7092[113934.43 \pm 77063.914]^{0.5} = 136.178\frac{m}{s}, \qquad 309.944\frac{m}{s}$$

$$C_L = \frac{nw}{0.5\rho V^2 S} = \frac{2.5 \times 510{,}000}{0.5 \times 0.69762 \times 150 \times 136.178^2} = 1.314\ \ not\ stall\ limited$$

$$D = nw\left(\frac{C_D}{C_L}\right) = 2.5 \times 510{,}000 \times \left(\frac{0.18373}{1.314}\right) = 178276.8265\ N\ no\ thrust\ limited$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} = \frac{136.178^2 cos\ 10°}{9.81\sqrt{\left(\frac{2.5}{cos10°}\right)^2 - 1}} = 797.856$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} \therefore V = \frac{1}{cos\gamma} \times \sqrt{Rg\sqrt{\left(\frac{n}{cos\gamma}\right)^2 - 1}} = \frac{1}{cos10°} \times \sqrt{R \times 9.81\sqrt{\left(\frac{1.5}{cos10°}\right)^2 - 1}} = 136.041\frac{m}{s}$$

$$M = \frac{V}{a} = \frac{136.178}{326.2} = 0.4\ Mach$$

Third attempt

$$V_{stall} = \sqrt{\frac{nw}{0.5\rho SC_{Lmax}}} = \sqrt{\frac{2.5 \times 510{,}000}{0.5 \times 0.69762 \times 150 \times 1.7}} = 119.72\frac{m}{s}(0.37\ Mach)$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} = \frac{(119.72)^2 cos10}{9.81\sqrt{\left(\frac{2.5}{cos10}\right)^2 - 1}} = 616.65m$$

Our v should be 124.043 m/s

Fourth attempt (thrust limited)

Method is find T required to perform the maneuver for a given nz

$$T_A = (C_{D0} 0.7pS)M^2 + \left(\frac{Kn^2w^2}{0.7pS}\right)\frac{1}{M^2}$$

$$202{,}495 = (0.019 \times 0.7 \times 64{,}980 \times 150)M^2 + \left(\frac{0.057 \times 2.5^2 \times 510{,}000^2}{0.7 \times 64{,}980 \times 150}\right)\frac{1}{M^2}$$

$$202495 = 129635M^2 + 13580.83\frac{1}{M^2}$$

$$M = 0.265, 1.2214\ Mach$$

$$202{,}495 = (0.019 \times 0.69762 \times 1.225 \times 150)V^2 + \left(\frac{0.057 \times 2.5^2 \times 510{,}000^2}{0.69762 \times 1.225 \times 150}\right)\frac{1}{V^2}$$

$$V = 61.1372\frac{m}{s}(0.1875\ Mach \quad 5.3221\ C_L), 281.786\frac{m}{s}\ (0.864374\ Mach \quad 0.250527\ C_L)$$

$$C_L = \frac{nw}{0.7pM^2S} = \frac{2.5 \times 510{,}000}{0.7 \times 64{,}980 \times 150 \times 0.265^2} = 2.1288 \therefore \text{stall limited}$$

$$M = \sqrt{\frac{nw}{0.7pSC_{Lbuff}}} = \sqrt{\frac{2.5 \times 510{,}000}{0.7 \times 64{,}980 \times 150 \times 1.7}} = 0.33$$

$$C_D = 0.18373$$

$$D = \frac{nw\ C_D}{C_L} = \left(\frac{2.5 \times 510{,}000 \times 0.18373}{\square}\right)$$

Final Attempt (unsolved, thrust limited C_L needed is lower than stall limit)

$$C_L = \frac{nw}{0.7pSM^2} = \frac{(2.5 \times 510{,}000)}{0.7 \times 64{,}980 \times 150 \times 0.38^2}$$

$$C_{L_{max}} = \frac{nw}{0.7pSM_{min}^2}$$

$$M = \sqrt{\frac{nw}{0.7pSC_{Lmax}}} = \sqrt{\frac{2.5 \times 510{,}000}{0.7 \times 64980 \times 150 \times 1.7}}$$

$$D = T_{\max} - \mathrm{wsin\gamma} = 202{,}495 - 510{,}000 sin 10° = 113{,}934.43\ N$$

$$m_{1,2} = \frac{1}{\sqrt{1.4p_{\mathrm{h}}SC_{D0}}}\left[D \pm \sqrt{D^2 - 4k(nw)^2C_{D0}}\right]^{0.5}$$

$$= \frac{1}{\sqrt{1.4 \times 64{,}980 \times 150 \times 0.019}}\left[113{,}934.43 \pm \sqrt{113{,}934.43^2 - 4 \times 0.057(2.5 \times 510{,}000)^2 0.019}\right]^{0.5}$$

$$m_{1,2} = 0.001964[113{,}934.43^2 \pm 77063.914]^{0.5} = 0.3771 \approx 0.38\ Mach$$

$$R = \frac{V^2 cos\gamma}{g\sqrt{(\frac{n}{cos\gamma})^2 - 1}} = \frac{(0.38 \times 326.2)^2 \cos 10°}{9.81\sqrt{(\frac{2.5}{cos 10°})^2 - 1}} = 661.1\ m$$

$$C_L = \frac{nw}{0.7pSM^2} = \frac{2.5 \times 510{,}000}{0.7 \times 64{,}980 \times 150 \times 0.38^2} = 1.294 < C_{Lmax} \therefore not\ stall\ limited$$

$$C_D = 0.019 + 0.057(1.294)^2 = 0.11444$$

$$D = \frac{nw\, C_D}{C_L} = \frac{2.5 \times 510{,}0000 \times 0.11444}{1.294} = 4 = 181032.26$$

Minimum turn radius

$$V_{Rmin} = \sqrt{\frac{4k\left(\frac{w}{s}\right)}{\rho(T/w)}} = \sqrt{\frac{nw}{0.5\rho SC_L}} = V$$

$$\frac{4k}{(T/w)} = \frac{n}{0.5C_L} \therefore C_L = \frac{n\, T/w}{2k} = \frac{2.5 \times \frac{202{,}495}{510{,}000}}{2 \times 0.057} = 8.707215$$

$$M = \sqrt{\frac{nw}{0.7pSC_L}} = \sqrt{\frac{2.5 \times 510{,}000}{0.7 \times 64{,}980 \times 150 \times 1.7}}$$

Thrust limited is the ultimate condition.

$$D = nw\frac{C_D}{C_L} = 2.5 \times 510{,}000 \times \frac{0.019 + 0.057 \times 1.7^2}{1.7} = 137797.5\ N$$

$$\mathrm{D} = T_{\max} - \mathrm{wsin\gamma} = 202{,}495 - 510{,}000 sin 10° = 113934.4293$$

The needed coeffiicent of lift is around 1.3 $< C_{Lmax}$, thus thrust-limited is the ultimate condition.

$$\frac{T - D}{w} = \sin\gamma$$

$$\frac{202{,}495 - D}{510{,}000} = \sin 10°$$

$$= \sqrt{\frac{2.5 \times 510{,}000}{0.5 \times 0.8545845 \times 150 \times 1.7}}$$

$$D = C_D\ 0.5\rho \mathrm{V}^2 S = (C_{D0} + kC_L^2)(\frac{\mathrm{nw}}{C_L})$$