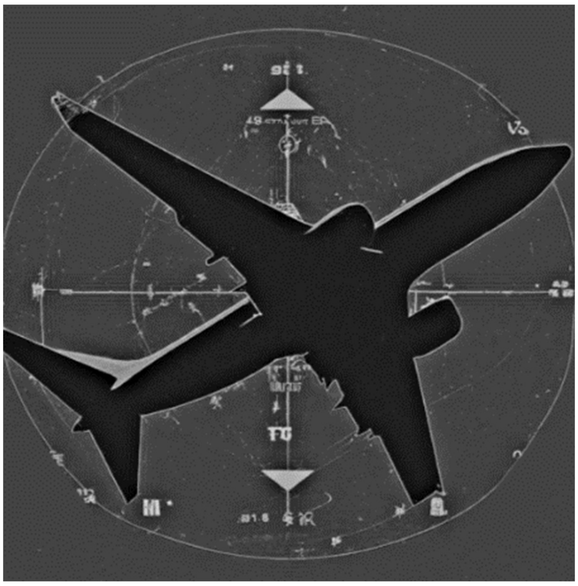
Flight Envelope Project



Abstract

This paper examines the different concepts tied to flight envelopes. Sequential Steps were followed to observe how they tie to each other logically. Two types of turbofans are used in two configurations (AEO, OEI). The difference between these configurations is rather clear, yet the findings faced some obstacles to substantially support this claim given the complexity of the problem. Aside from, a clear breakdown of steps and profiles. The drag profile is effortless to implement and integrate given some pre-established relations, and the thrust is not as straightforward as anticipated, since it represents the complexity of the engine itself which requires knowledge on the mass flow rate, manifold air pressure, and many others often times. Speaking of the velocity profile, the thrust impacted the velocity profile to establish clear borderlines for where ceilings ought to be. Yet some concepts were included in a figure in the velocity profile section.

Introduction

This paper seeks to explore the many concepts behind a what is called flight envelope. A flight envelope, simply put, defines the boundaries of operational speeds and altitudes such that beyond these boundaries an aircraft more likely may experience control difficulties, structural damages, or a fatigued pilot. In other words, it guides toward the safe practices during a flight, either in civil or combat fields. Below, the varying notions and formulas are introduced using sample cases of two different Pratt and Whitney Turbofans in order to demonstrate the tight connections between the different performance elements (lift, drag, speed, thrust, range, etc.).

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Theory and Method

Terminology: -

Flight Envelope: graphical representation of the safely achievable cruising Mach speeds given the altitude.
Flight Envelope Constraints: include stall, thrust/power available, structural strength,
All Engines Operative (AEO): all engines perform as intended in accordance with the manufacturer's specifications.
One Engine Inoperative (OEI): one engine fails due to natural causes, fuel feed system failure, power outage, or expulsion.
Absolute Ceiling: the maximum attained altitude that the aircraft can operate at.
Service Ceiling: high attainable altitude with a buffer zone (factory of safety) from the Absolute Ceiling. More specifically, it is determined when the best rate of climb is 100 ft/min.
Performance Ceiling: high attainable altitude and is determined when the best rate of climb is 150-200 ft/min.
Drag Divergence: condition in which the drag rises drastically due to reaching transonic or supersonic speeds.
Specific Fuel Consumption: the amount of fuel consumed per second per thrust - interpreted as such, fuel amount used (kg/s) to produce a unit of thrust/power.

Cruise Flight Conditions: -

Maximum Range: it is aimed to lengthen the range of a flight.
Minimum Power: it is aimed at conserving the remaining power where the power produced matches the power required, the bare minimum, to keep on cruising.
Minimum Drag: it is aimed to minimize drag to achieve a better aerodynamic performance.

Breakdown of Methodology: -

Aircraft Specifications

Detail	Description
Engines (two config.)	Two PW JT9D-3 or Two PW 4056
AEO vs OEI	Two Engines ($T_{total} = 2T$) vs. One Engine ($T_{total} = T$
Wingspan	$44.84\ m$
Wing Reference Area	$260\ m^2$
Weight	$160000 \times 9.81\ N$
$C_{L_{max}}$	1.7
C_{D0}	0.016
$C_{D0_{OEI}}$	0.02
K	$\frac{1}{\pi 0.85 \times \frac{44.48^2}{260}} = 0.0492125$
K_{OEI}	$\frac{1}{\pi 0.75 \times \frac{44.48^2}{260}} = 0.0557741$
Drag Divergence Limit	$M > 0.85$

- ❖ **Drag Profile**

Maximum Range

$$C_L = \sqrt{\frac{C_{D0}}{3K}} \qquad C_D = \frac{4}{3}C_{D0}$$

Minimum Power

$$C_L = \sqrt{\frac{3C_{D0}}{K}} \qquad C_D = 4C_{D0}$$

Minimum Drag

$$C_L = \sqrt{\frac{C_{D0}}{K}} \qquad C_D = 2C_{D0}$$

Cofficient of Drag

$$C_D = C_{D0} + kC_L^2$$

Coefficient of Drag with Drag divergence

$$C_D = \begin{cases} C_{D0} + K\dfrac{C_{D0}}{K}, & 0 \leq M_{max} \leq 0.85 \\ \mathrm{CDo_{sub}}\left(1 + {}^{(\mathrm{M} - 0.85)}/_{0.1}\right) + K\dfrac{C_{D0}}{K}, & M_{max} > 0.85 \end{cases} \qquad where, M_{max} = \frac{V_{max}}{a}|_{alt.}$$

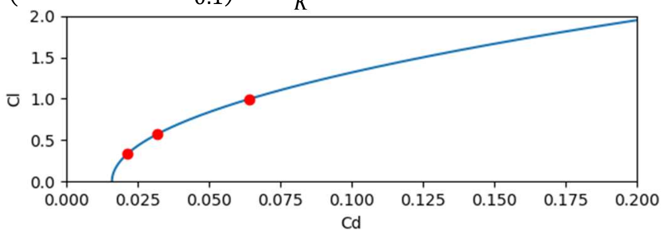


Figure 1 CL v CD 1) Min-Drag, 2) Max-range, 3) Min-Power

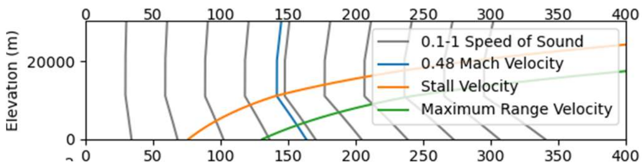


Figure 2 Various Velocity Terms

Quick note: *due to the drag coefficient quadratic formula, it assumes a continuous increase, yet the drag curve starts at C_{D0} then decreases for a short period then is followed by an increase as prescribed in the formula.*

- **Thrust Profile**

Original Chart

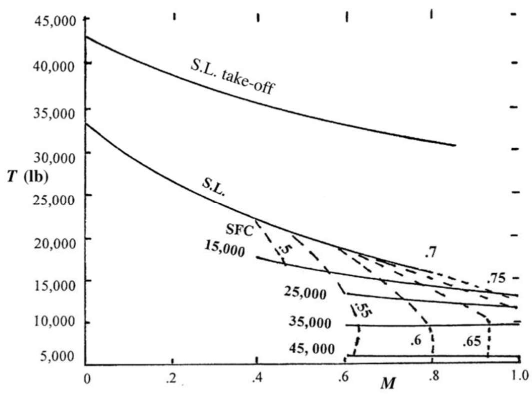


Figure D.9 Pratt and Whitney JT9D-3 Turbofan Engine

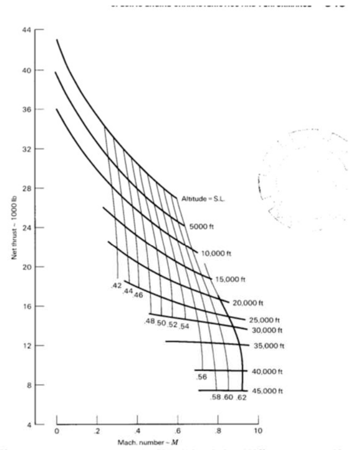
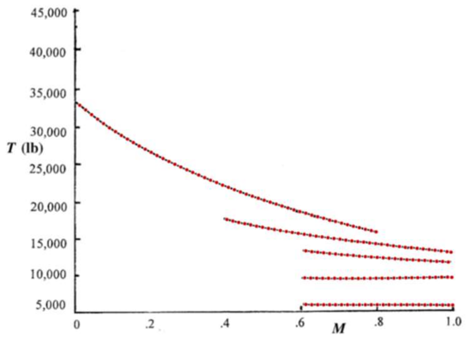
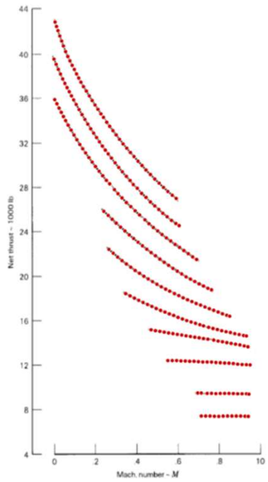


Figure 6.43 Maximum cruise thrust for the PW 4056 turbofan. 100% ram recovery. No airbleed. No power extraction. Standard day. (Courtesy, Pratt & Whitney)

Curated Charts





Quick note: *WebPlotDigitizer (apps.automeris.io/wpd4/) was utilized to reverse engineer the charts and extract precise datapoints at each altitude.*

Processing of Datasets (code steps)

- *Set up brackets between each altitude limit*
- *Use built-in functions, to obtain thrust polynomial models as functions of Mach at each altitude*
- *At a given altitude, set up an interpolation function, known altitude brackets and their thrust models to return a thrust value.*

Absolute Ceiling (Approximation – not used)

$$H = -19{,}867 \left(\frac{w}{(P_{A,0})_{max}}\right)\sqrt{\frac{2}{\rho_0}\left(\frac{w}{S}\right)}\left[\frac{0.7436\, C_{D,0}^{0.25}}{(eAR)^{\frac{3}{4}}}\right] \quad \text{(E 6.15.8)}$$

adopted from Anderson's Introduction to Flight

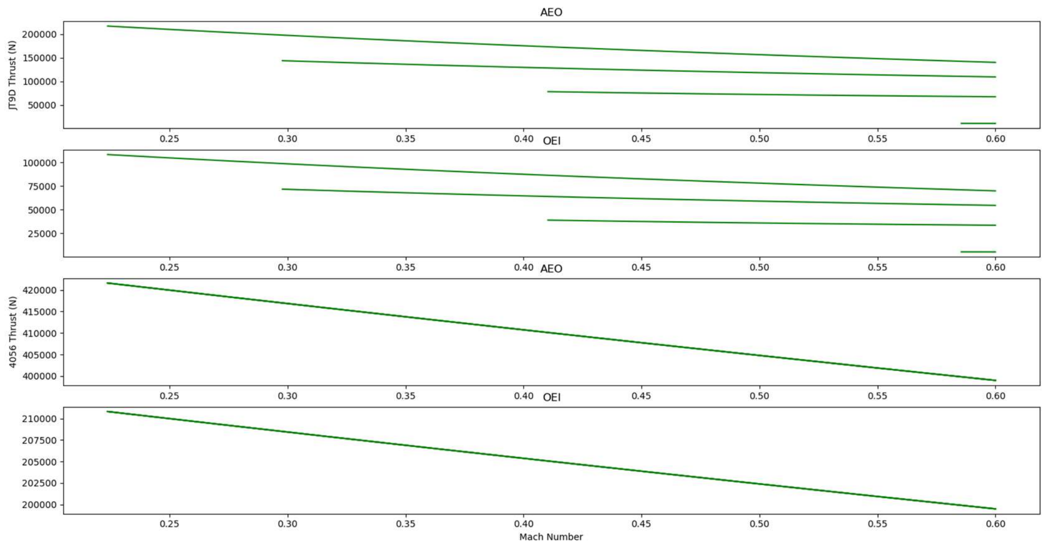


Figure 3 Thrust Vs Mach at 0,15,30,45 kft

❖ **Velocity Profile**

Speed of Sound *(code steps)*

- 0,0.1,0.2 to 1 Mach vs elevation.
- From ambiance module, get speed of sound (m/s) at an array of altitude.
- Multiply by 0.1 to 1 in 0.1 increments, to establish gridlines.
- Add 0.48 Mach distinguishable gridline.

Stall Velocity

$$V_{stall}(h) = \sqrt{\frac{w}{0.5\ \rho(h)SC_{L_{max}}}}$$

Max. Range Velocity

$$V_{max\ Range}(h) = \sqrt{\frac{w}{0.5\rho(h)S}\sqrt{\frac{k}{C_{D0}}}}$$

Discussion Questions

For each engine/aircraft configuration:

1. What is the absolute ceiling? (all engines operating)

Configuration (AEO)	Absolute Ceiling (estimate)
Two PW JT9D-3	45
Two PW 4056	30

2. What is the absolute ceiling? (OEI condition)

Configuration (OEI)	Absolute Ceiling (estimate)
One PW JT9D-3	45
One PW 4056	30

3. The current AWACS platform has an operational ceiling of 30,000 ft. Which of the engine/aircraft configurations is closest to this with all engines operating.

PW JT9D-3 twin configuration performs way better, but PW 4056 twin configuration is closer to 30 ft operating ceiling.

4. For each operating condition, what is the maximum cruise Mach number and the corresponding altitude.

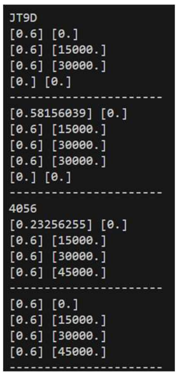


Figure 4 screenshot from VS Code

The connection does not seem to be right between the Mach number and altitude. Given two different config of the same engine type ought not yield identical readings to each other. I suspect an error somewhere complicating the matter.

Configuration (AEO)	Max. Mach	Altitude
Two PW JT9D-3	0.6	30 kft
Two PW 4056	0.6	45 kft
Configuration (OEI)	**Max. Mach**	**Altitude**
One PW JT9D-3	0.6	30 kft
One PW 4056	0.6	45 kft

5What are your observations about high subsonic cruise in both all engines operational and in the OEI condition.

Optimal cruising occurs often just below transonic range to avoid the implications of shocks presence at the intake or nose. Cruising faster is not always ideal, albeit more efficient some time.

Conclusions

There was multiple error factors present resulting in unclear findings such as the different between AEO and OEI where intuitively we expect that higher thrust for the same at a certain altitude leads to more flexibility towards the absolute ceiling, up to a point where thrust production is scarce. overall, key conclusions

1) flight envelope is an interesting concept to observe and test aircraft to ensure they meet the expectations
2) twin-engine configuration is better than one since rationally relying on a single engine is too risky in case of failure and such.
3) Thrust variation in real life can be unpredictable since it counts on the surrounding conditions as well as engine performance at these conditions.

Code

Attached is a copy of the code implemented.

Inputs.py: contains all aircraft specifications and important functions to run preliminary calculations.
Analysis.py: contains and ordered sequence of formulas utilized to generate plots and extract the paper's findings.
Below is a simplified overview of the code structure.

Inputs.py

Contains all aircraft specifications, functions to get M from CL and vice versa, along with the thrust profile datasets. They are contained in Thrust_Profile and accessed by Thrust function to be processed.

Analysis.py

Contains all necessary plotting and is intended to finalize findings' values which could be translated into meaningful conclusions.

PW JT9D-3.csv

Contains readable T vs. M datasets at different altitudes of the engine. It is used essentially to fill in the values for the thrust profile in Inputs.py.

PW 4056.csv

Contains readable T vs. M datasets at different altitudes of the engine. It is used essentially to fill in the values for the thrust profile in Inputs.py.