Filter No (m) = 46

Butterworth filter :-

Passband tolerance = 0.15
Stopband tolerance = 0.15
Analog Signal freq range = 140 kHz
Sampling freq (Fs) = 320

$q(46) = 46 \times 0.1 = 4$

$\mu(46) = m - 10 \times q(m)$
$= 46 - 10 \times 4$
$= 6$

1) $B_L(46) = 5 + 1.4 \times q(m) + 4 \times \mu(m)$
$= 5 + 1.4 \times 4 + 4 \times 6$
$= 34.6$

2) $B_H(46) = 34.6 + 10 = 44.6$

3) $\omega = \frac{\Omega \times 2\pi}{\Omega_s}$, Sampling freq = 320

$\omega_{P1} = \frac{34.6 \times 2\pi}{320} = 0.216\pi$

$\omega_{P2} = \frac{44.6 \times 2\pi}{320} = 0.278\pi$

$\omega_{S1} = \omega_{P1} - \omega_T = 0.216\pi - 0.0125\pi$
$= 0.203\pi$

$\omega_T = \frac{2 \times 2\pi}{320} = 0.0125\pi$

Transition band for bandpass filter = 2.

$$\omega_{s2} = \omega_{p2} + \omega_T$$
$$= 0.290\pi.$$

Equivalent Analog filter specifications.

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

$$\Omega_{p1} = \tan\left(\frac{0.216\pi}{2}\right) = 0.352$$

$$\Omega_{p2} = \tan\left(\frac{\omega_{p2}}{2}\right) = \tan\left(\frac{0.278\pi}{2}\right)$$
$$= 0.466.$$

$$\Omega_{s1} = \tan\left(\frac{\omega_{s1}}{2}\right) = \tan\left(\frac{0.203\pi}{2}\right)$$
$$= 0.330$$

$$\Omega_{s2} = \tan\left(\frac{\omega_{s2}}{2}\right) = 0.489.$$

Frequency transformation for BPF to LPF conversion :-

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B \times \Omega}$$

$$B = \Omega_{p2} - \Omega_{p1} = 0.466 - 0.352$$
$$= 0.114$$

$$\Omega_0^2 = \Omega_{p1} \times \Omega_{p2} = 0.164.$$

Frequency transformed LPF Specifications :-

$$\Omega_L = \frac{\Omega^2 - 0.0164}{0.114 \times \Omega}$$

$$\Omega_{P1} = \frac{(0.352)^2 - 0.164}{0.114 \times 0.352} = -0.99$$

$$\Omega_{P2} = \frac{(0.466)^2 - 0.164}{0.114 \times 0.466} = 1.00$$

$$\Omega_{S1} = \frac{(0.330)^2 - 0.164}{0.114 \times 0.330} = -1.46$$

$$\Omega_{S2} = \frac{(0.489)^2 - 0.164}{0.114 \times 0.489} = 1.34.$$

Passband edge (Ω_{Pe}) = 1

Stopband edge (Ω_{Se}) = min (+1.46, 1.34)
= 1.34

Passband and Stopband tolerance = 0.15

for Nmin, obtaining the essential parameter of LPF.

Let,

$$D_1 = \frac{1}{(1-s^2)} - 1 = 0.384$$

&

$$D_2 = \frac{1}{s^2} - 1 = 43.44.$$

$$N = \left[\frac{\log\left(\sqrt{\frac{D2}{D1}}\right)}{\log\left(\frac{\Omega se}{\Omega pe}\right)}\right] = \frac{\log(10.635)}{\log(1.34)} = 8.07$$

$$N \approx 8$$

$$\frac{\Omega pe}{D1^{1/2N}} \leq \Omega c \leq \frac{\Omega se}{D2^{1/2N}}$$

$$\frac{1}{(0.384)^{\frac{1}{2\times 8}}} \leq \Omega c \leq \frac{1.34}{(43.44)^{\frac{1}{2\times 8}}}$$

$$\frac{1}{0.384^{1/16}} \leq \Omega c \leq \frac{1.34}{(43.44)^{1/16}}$$

$$1.061 \leq \Omega c \leq 1.058$$

$$[\Omega c = 1.0599.]$$

For Poles :-

$$Sk = \Sigma k + j\Omega k$$

No of Poles = 8 , $\therefore k = 0$ to 7.

$$Ak = \left(k + \frac{1}{2}\right)\frac{\pi}{N}$$

→ $A_0 = \left(0 + \frac{1}{2}\right)\frac{\pi}{8} = 0.196$

$A_1 = \left(1 + \frac{1}{2}\right)\frac{\pi}{8} = 0.58$, $A_6 = \left(6 + \frac{1}{2}\right)\frac{\pi}{8} = 2.55$

$A_2 = \left(2 + \frac{1}{2}\right)\frac{\pi}{8} = 0.981$, $A_7 = \left(7 + \frac{1}{2}\right)\frac{\pi}{8}$

$A_3 = \left(3 + \frac{1}{2}\right)\frac{\pi}{8} = 1.37$ $= 2.94$.

$A_4 = \left(4 + \frac{1}{2}\right)\frac{\pi}{8} = 1.76$

$A_5 = \left(5 + \frac{1}{2}\right)\frac{\pi}{8} = 2.15$

→ $\Sigma k = -\Omega c \times \sin A_k$

$\Sigma k_0 = -[1.0599 \times \sin(0.196)]$
$= -[0.206]$

$\Sigma k_1 = -[1.0599 \times \sin(0.58)] = -0.580$

$\Sigma k_2 = -[1.0599 \times \sin(0.981)] = -0.880$

$\Sigma k_3 = -1.038$ $\Sigma k_6 = -0.591$
$\Sigma k_4 = -1.040$ $\Sigma k_7 = -0.212$
$\Sigma k_5 = -0.887$

→ $\Omega k = \Omega c \times \cos A k$

$\Omega_0 = 1.0599 \times \cos(0.196) = 1.039$
$\Omega_1 = 0.886$
$\Omega_2 = 0.589$ $\Omega_5 = -0.580$
$\Omega_3 = 0.211$ $\Omega_6 = -0.879$
$\Omega_4 = -0.199$ $\Omega_7 = -1.038$.

∴ Poles are

$$S_k = \Sigma_k + j\Omega_k$$

$P_0 = -0.206 + j1.039$
$P_1 = -0.580 + j0.886$
$P_2 = -0.880 + j0.589$
$P_3 = -1.038 + j0.211$
$P_4 = -1.040 - j0.199$
$P_5 = -0.887 - j0.580$
$P_6 = -0.591 - j0.879$
$P_7 = -0.212 - j1.038$

Chebyscher filter :-

Passband tolerance = 0.15
Stopband tolerance = 0.15
Analog freq range = 140 kHz
Sampling freq = 250

Transition band on either side of Stopband = 2kHz
Assigned filter = 46.

$q(46) = 4$, $r(46) = 6$.

$$B_L(46) = 5 + 1.2 \times q(m) + 2.5 \times r(m)$$
$$= 5 + 1.2 \times 4 + 2.5 \times 6$$
$$= 5 + 4.8 + 15$$
$$= \underline{24.8}$$

$$B_H = B_L + 6$$
$$= 24.8 + 6 = \underline{30.8}$$

Normalised Digital filter Specifications :-

$$\omega = \frac{\Omega \times 2\pi}{\Omega_s}$$

$$\omega_{s1} = \frac{24.8 \times 2\pi}{250} = 0.198\pi$$

$$\omega_{s2} = \frac{30.8 \times 2\pi}{250} = 0.246\pi$$

$$\omega_T = \frac{2 \times 2\pi}{250} = 0.016\pi$$

$$\omega_{p1} = \omega_{s1} - \omega_T = 0.182\pi$$

$$\omega_{p2} = \omega_{s2} + \omega_T = 0.262\pi.$$

Equivalent analog filter Specifications :-

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

$$\Omega_{p1} = \tan\left(\frac{\omega_{p1}}{2}\right) = 0.293$$

$$\Omega_{p2} = \tan\left(\frac{\omega_{p2}}{2}\right) = 0.436$$

$$\Omega_{s1} = \tan\left(\frac{\omega_{s1}}{2}\right) = 0.321$$

$$\Omega_{s2} = \tan\left(\frac{\omega_{s2}}{2}\right) = 0.406.$$

Frequency transformed ~~LPF~~ Specifications :-

For BRF to LPF Conversion

$$\Omega_L = \frac{B \times \Omega}{\Omega_0^2 - \Omega^2}$$

$$B = \Omega_{p2} - \Omega_{p1} = 0.143$$

$\Omega_0^2 = \Omega_{p1} \times \Omega_{p2} = 0.127.$

Frequency transform LPF Specifications:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B \times \Omega}$$

$$\Omega_{p1} = \frac{(0.293)^2 - 0.127}{0.143 \times 0.293} = -0.98$$

$$\Omega_{p2} = \frac{(0.436)^2 - 0.127}{0.143 \times 0.436} = 1.01$$

$$\Omega_{s1} = \frac{(0.321)^2 - 0.127}{0.143 \times 0.321} = -0.52$$

Passband edge = 1.01

Stopband edge = ?

$$\Omega_{s2} = \frac{(0.406)^2 - 0.127}{0.143 \times 0.406} = 0.651$$

Stopband edge = min (0.52, 0.65)
= 0.52

Passband tolerance = 0.15
Stopband —11— = 0.15

for Nmin,

Let $D_1 = \frac{1}{(1-\delta)^2} - 1 = 0.384$

$D_2 = \frac{1}{\delta^2} - 1 = 43.444$

$$N = \left[\frac{\cosh^{-1}\left(\sqrt{\frac{D_2}{D_1}}\right)}{\cosh^{-1}\left(\frac{\Omega_{s_2}}{\Omega_{p_2}}\right)} \right]$$

$$= \frac{\cosh^{-1}(10.635)}{\cosh^{-1}(0.514)}$$

$$N_{min} = \underline{4}$$

For poles :-

$$s_k = \Sigma_k + j\Omega_k$$

No. of poles = 4, $\therefore k = 0$ to 3.

$$A_k = \left(k + \frac{1}{2}\right)\frac{\pi}{N}$$

$$A_0 = \left(0 + \frac{1}{2}\right)\frac{\pi}{4} = 0.392, \quad A_1 = 1.178$$

$$A_2 = 1.963, \quad A_3 = 2.748.$$

→ $\cosh B = 1.497$ } given.
$\sinh B = 0.319$ }

→ $\Sigma_k = \Omega_{p_1} \sinh B \cdot \sin A_k$

$\Sigma_0 = 1.01 \times 0.319 \times \sin(0.392)$
$= 0.1231.$

$\Sigma_1 = 1.01 \times 0.319 \times \sin(1.178)$
$= 0.297$

$\Sigma_2 = 0.297$

$\Sigma_3 = 0.123.$

→ $\Omega_k = \Omega_{pL} \cosh B \cos Ak$

$\Omega_{k0} = 1.01 \times 1.497 \times \cos(0.392)$

$= 1.397$

$\Omega_1 = 1.01 \times 1.497 \times \cos(1.178)$

$= 0.578$

$\Omega_2 = -0.577$

$\Omega_3 = -1.396$.

Poles :- $P_0 = 0.123 + j\,1.397$

$P_1 = 0.297 + j\,0.578$

$P_2 = 0.297 - j\,0.577$

$P_3 = 0.123 - j\,1.396$.