Digital Signal Processing
Filter Design Assignment

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1 Assignment Details

Student Name:
Rachna Pawar
Mohammad Shahbaz Alam
Filter Number: 46

2 Filter - I: Bandpass Filter Design(Butterworth)

2.1 Un-normalized Discrete Time Filter Specifications

Passband Tolerance = 0.15 (i.e 0.85-1.15)
Stopband Tolerance = 0.15 (i.e 0-0.15)
Analog Signal frequency range = 140 kHz
Sampling Frequency(f_s) = 320 kHz
Transition band on either side of Passband = 2 kHz
Passband Nature : Monotonic
Stopband Nature : Monotonic

Assigned Filter Number = 46
q(46) = 4, r(46) = 6
$B_L(46)$ = 5 + 1.4*4 + 4*6 = 34.6 kHz
$B_H(46)$ = 34.6 + 10 = 44.6 kHz

2.2 Normalized Digital Filter Specifications

Sampling Frequency = 320 kHz
Therefore, normalized frequency can be calculated as,

$$\omega = \frac{\Omega * 2\pi}{\Omega_s} \tag{1}$$

Making calculations as per above equation, the modified specifications are,

Lower Passband Edge (ω_{p1}) = 0.216π
Upper Passband Edge (ω_{p2}) = 0.278π
Transition band on either side of Passband (ω_T) = 0.0125π
Upper stopband Edge (ω_{s2}) = 0.290π
Lower stopband Edge (ω_{s1}) = 0.203π
Passband Nature : Monotonic
Stopband Nature : Monotonic
Passband Tolerance = 0.15 (i.e 0.85-1.00)
Stopband Tolerance = 0.15 (i.e 0-0.15)

2.3 Equivalent Analog filter Specifications

The analog filter specifications(Ω) can be determined using bilinear transformation on normalized specifications(ω) as,

$$\Omega = tan(\frac{\omega}{2}) \tag{2}$$

Lower Passband Edge (Ω_{p1}) = 0.352
Upper Passband Edge (Ω_{p2}) = 0.466
Upper stopband Edge (Ω_{s2}) = 0.489
Lower stopband Edge (Ω_{s1}) = 0.330

$$tan(\frac{0}{2}) = 0 \tag{3}$$

$$tan(\frac{\pi}{2}) = \infty \tag{4}$$

2.4 Frequency Transformation for BPF to LPF conversion

The frequency transformation employed for Band pass filter to equivalent low pass filter conversion,

$$\Omega_L = \frac{\Omega^2 - \Omega_o^2}{B * \Omega} \tag{5}$$

where,
B = $\Omega_{p2} - \Omega_{p1} = 0.114$
$\Omega_o^2 = \Omega_{p1} * \Omega_{p2} = 0.164$

2.5 Frequency transformed LPF Specifications

$$\Omega_L = \frac{\Omega^2 - 0.164}{0.114 * \Omega} \tag{6}$$

Lower Passband Edge (Ω_{p1}) = -0.99
Upper Passband Edge (Ω_{p2}) = 1.00
Upper stopband Edge (Ω_{s2}) = 1.34
Lower stopband Edge (Ω_{s1}) = -1.46

Passband Edge (Ω_{pl}) = 1
Stopband Edge (Ω_{sl}) = min(1.46,1.34) = 1.34
Passband and Stopband Tolerance = 0.15
Passband Nature = Monotonic
Stopband Nature = Monotonic

2.6 Analog Lowpass Transfer Function

The magnitude squared response of the butterworth filter in frequency domain,

$$|H_{analog,LPF}(j\Omega)|^2 = \frac{K}{1 + (\frac{\Omega}{\Omega_c})^{2N}} \tag{7}$$

where, K is the normalization constant.
Passband Tolerance(δ_1) = 0.15

Stopband Tolerance(δ_2) = 0.15
Defining,

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384 \tag{8}$$

$$D_2 = \frac{1}{\delta_2^2} - 1 = 43.44 \tag{9}$$

Taking minimum value of N,

$$N_{min} = \lceil \frac{log(D_2/D_1)}{2log(\Omega_s l/\Omega_p l)} \rceil = \lceil 8.07 \rceil = 8 \tag{10}$$

For Ω_c,

$$\frac{\Omega_p}{D_1^{1/(2N_{min})}} <= \Omega_c <= \frac{\Omega_s}{D_2^{1/(2N_{min})}} \tag{11}$$

$$1.061 <= \Omega_c <= 1.063 \tag{12}$$

Taking $\Omega_c = 1.062$ Therefore, in s-domain,

$$|H_{analog,LPF}(s)|^2 = \frac{K}{1 + (\frac{s_l}{j1.062})^8} \tag{13}$$

Solving for the poles of $|H_{analog,LPF}(s)|^2$, we get,

$$s_k = 1.062e^{j((\frac{2k+1}{8})\pi + \frac{\pi}{2})} \tag{14}$$

pole 1(s_{l1})	-0.206 + j1.039
pole 2(s_{l2})	-0.580 + j0.886
pole 3(s_{l3})	-0.880 + j0.589
pole 4(s_{l4})	-1.038 + j0.211
pole 5(s_{l5})	-1.040 - j0.199
pole 6(s_{l6})	-0.887 - j0.580
pole 7(s_{l7})	-0.591 - j0.879
pole 8(s_{l8})	-0.212 - j1.038

Table 1: Table for pole values

Stable filter implies all poles in left half plane, therefore required in $H_{analog,LPF}(s)$ are s_{l1}, s_{l2}, s_{l3}, s_{l4}, s_{l5}, s_{l6}, s_{l7}, s_{l8},.

$$H_{analog,LPF}(s_l) = \frac{(1.062)^8}{(s_l - s_{l1})(s_l - s_{l2})(s_l - s_{l3})(s_l - s_{l4})(s_l - s_{l5})(s_l - s_{l6})(s_l - s_{l7})(s_l - s_{l8})} \tag{15}$$

2.7 Analog Transfer Function of BPF Filter

In $H_{analog,LPF}(s_l)$, substituting,

$$s_l = \frac{s^2 + 0.164}{0.114s} \tag{16}$$

We get, $H_{analog}(s)$ with mentioned coefficients,
Numerator,

s^8
0.4616

Table 2

Denominator,

s^{16}	s^{15}	s^{14}	s^{13}	s^{12}	s^{11}	s^{10}
1.000	0.6195	1.5041	0.7498	0.9476	0.3822	0.3282

s^9	s^8	s^7	s^6	s^5	s^4	s^3
0.1063	0.0685	0.0174	0.0088	0.0017	0.0007	0.001

Table 3

2.8 Discrete Time Filter Transfer Function

Analog to discrete time conversion is made using the bilinear transformation,

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \tag{17}$$

We get, $H_{discrete,BPF}(z)$ with mentioned coefficients, Numerator,

z^0	z^{-2}	z^{-4}	z^{-6}	z^{-8}	z^{-10}	z^{-12}	z^{-14}	z^{-16}
0.0080	-0.0644	0.2254	-0.4507	0.5634	-0.4507	0.2254	-0.0644	0.0080

Table 4

Denominator,

z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
0.0010	-0.0107	0.0573	-0.2006	0.5115	-1.0044	1.5679	-1.9815	2.0474

z^{-9}	z^{-10}	z^{-11}	z^{-12}	z^{-13}	z^{-14}	z^{-15}	z^{-16}
-1.7349	1.2019	-0.6742	0.3006	-0.1032	0.0258	-0.0042	0.0003

Table 5

2.9 Direct Form II Realization

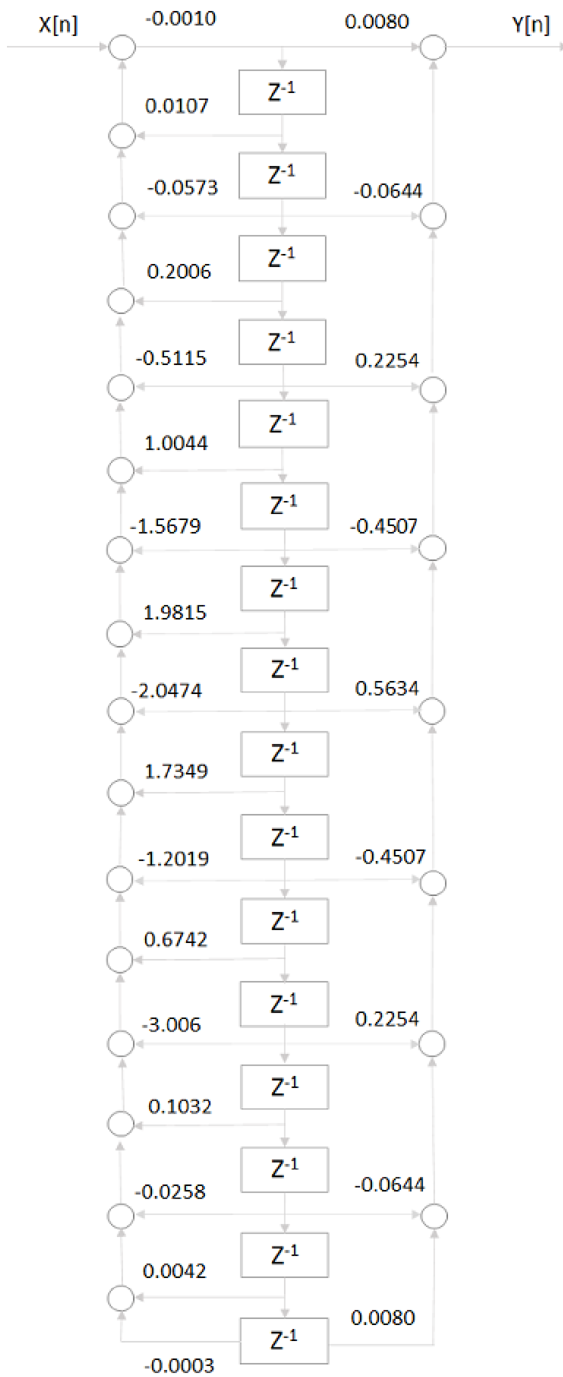

Figure 1: Direct Form II Block Diagram for Hdiscrete;BSF (z)

2.10 FIR BPF Filter Design using Kaiser Window

Stopband and Passband Tolerance(δ) = 0.15
Therefore, Kaiser parameters,
$\Delta\omega_T = \omega_{s2}$ - $\omega_{p2} = \omega_{p1}$ - ω_{s1} = 0.03768
A = -20log_{10} δ = 16.4782

$$N_{min} = \lceil \frac{A-8}{2.285 * \Delta\omega_T} \rceil = \lceil \frac{16.4782-8}{2.285 * 0.03768} \rceil = 98.47 \quad (18)$$

As A < 21, therefore, shape parameter in Kaiser window, $\alpha = \beta = 0$ (Rectangular Window).
The Kaiser window has to operate on ideal BPF filter, which is approximated as a separate function using truncated time domain response of ideal LPF filter. This introduces **non-ideality** which results in **increased order** of Kaiser window than necessary.
Therefore, length of Kaiser Window (L) = $N_{min} + 1 + 16 = 115.47$.
The transfer function of resultant BPF filter is given by the coefficients mentioned, from zero to increasing time delay (in Z-domain).

fir_h =

```
Columns 1 through 12

  0.0020   -0.0001   -0.0002    0.0012    0.0019   -0.0000   -0.0040   -0.0071   -0.0061   -0.0003    0.0076    0.0123

Columns 13 through 24

  0.0100    0.0008   -0.0099   -0.0156   -0.0123   -0.0015    0.0103    0.0159    0.0121    0.0018   -0.0084   -0.0123

Columns 25 through 36

 -0.0087   -0.0014    0.0041    0.0047    0.0019   -0.0001    0.0019    0.0064    0.0083    0.0029   -0.0090   -0.0201

Columns 37 through 48

 -0.0209   -0.0069    0.0162    0.0347    0.0346    0.0120   -0.0226   -0.0485   -0.0479   -0.0175    0.0272    0.0598

Columns 49 through 60

  0.0590    0.0227   -0.0295   -0.0670   -0.0666   -0.0269    0.0294    0.0694    0.0694    0.0294   -0.0269   -0.0666

Columns 61 through 72

 -0.0670   -0.0295    0.0227    0.0590    0.0598    0.0272   -0.0175   -0.0479   -0.0485   -0.0226    0.0120    0.0346

Columns 73 through 84

  0.0347    0.0162   -0.0069   -0.0209   -0.0201   -0.0090    0.0029    0.0083    0.0064    0.0019   -0.0001    0.0019

Columns 85 through 96

  0.0047    0.0041   -0.0014   -0.0087   -0.0123   -0.0084    0.0018    0.0121    0.0159    0.0103   -0.0015   -0.0123

Columns 97 through 108

 -0.0156   -0.0099    0.0008    0.0100    0.0123    0.0076   -0.0003   -0.0061   -0.0071   -0.0040   -0.0000    0.0019

Columns 109 through 112

  0.0012   -0.0002   -0.0001    0.0020
```

Figure 2: Coefficients of Transfer Function in Z-domain

3 Filter - II: Bandstop Filter Design(Chebyschev)

3.1 Un-normalized Discrete Time Filter Specifications

Passband Tolerance = 0.15 (i.e 0.85-1.15)
Stopband Tolerance = 0.15 (i.e 0-0.15)
Analog Signal frequency range = 140 kHz
Sampling Frequency(f_s) = 250 kHz
Transition band on either side of stopband = 2 kHz
Passband Nature : Equiripple
Stopband Nature : Monotonic

Assigned Filter Number = 46
q(46) = 4, r(46) = 46 - 10*4 = 6
$B_L(46)$ = 5+ 1.2*4 + 2.5*6 = 24.8 kHz
$B_H(46)$ = 24.8 + 6 = 30.8 kHz

3.2 Normalized Digital Filter Specifications

Sampling Frequency = 250 kHz
Therefore, normalized frequency can be calculated as,

$$\omega = \frac{\Omega * 2\pi}{\Omega_s} \tag{19}$$

Making calculations as per above equation, the modified specifications are,

Lower stopband Edge (ω_{s1}) = 0.198π
Upper stopband Edge (ω_{s2}) = 0.246π
Transition band on either side of Passband (ω_T) = 0.016π
Upper passband Edge (ω_{p2}) = 0.182π
Lower passband Edge (ω_{p1}) = 0.262π
Passband Nature : Equiripple
Stopband Nature : Monotonic
Passband Tolerance = 0.15 (i.e 0.85-1.00)
Stopband Tolerance = 0.15 (i.e 0-0.15)

3.3 Equivalent Analog filter Specifications

The analog filter specifications(Ω) can be determined using bilinear transformation on normalized specifications(ω) as,

$$\Omega = tan(\frac{\omega}{2}) \tag{20}$$

Lower Passband Edge (Ω_{p1}) = 293
Upper Passband Edge (Ω_{p2}) = 0.436
Upper stopband Edge (Ω_{s2}) = 0.321
Lower stopband Edge (Ω_{s1}) = 0.406

$$tan(\frac{0}{2}) = 0 \tag{21}$$

$$tan(\frac{\pi}{2}) = \infty \tag{22}$$

3.4 Frequency Transformation for BRF to LPF conversion

The frequency transformation employed for Band stop filter to equivalent low pass filter conversion,

$$\Omega_L = \frac{B * \Omega}{\Omega_o^2 - \Omega^2} \tag{23}$$

where,
B = $\Omega_{p2} - \Omega_{p1}$ = 0.143
$\Omega_o^2 = \Omega_{p1} * \Omega_{p2}$ = 0.127

3.5 Frequency transformed LPF Specifications

$$\Omega_L = \frac{\Omega^2 - 0.220}{0.154 * \Omega} \tag{24}$$

Upper Passband Edge (Ω_{p1}) = -0.98
Lower Passband Edge (Ω_{p2}) = 1.01
Lower stopband Edge (Ω_{s2}) = 0.651
Upper stopband Edge (Ω_{s1}) = -0.52

Passband Edge (Ω_{pl}) = 1.01
Stopband Edge (Ω_{sl}) = min(-0.52,0.65) = 0.52
Passband and Stopband Tolerance = 0.15
Passband Nature = Equiripple
Stopband Nature = Monotonic

3.6 Analog Lowpass Transfer Function

The magnitude squared response of the Chebyschev filter in frequency domain,

$$|H_{analog,LPF}(\Omega)|^2 = \frac{K}{1+\epsilon^2 C_N^2(\frac{\Omega}{\Omega_{pl}})} \tag{25}$$

where, K is the normalization constant.
Passband Tolerance(δ_1) = 0.15
Stopband Tolerance(δ_2) = 0.15
Defining,

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384 \tag{26}$$

$$D_2 = \frac{1}{\delta_2^2} - 1 = 43.444 \tag{27}$$

Taking minimum value of N,

$$N_{min} = \lceil \frac{cosh^{-1}(\sqrt{D2/D1})}{cosh^{-1}(\Omega_{sl}/\Omega_{pl})} \rceil = 4 \tag{28}$$

$\epsilon = \sqrt{D_1} = 0.6197$
Therefore, in s-domain,

$$|H_{analog,LPF}(s)|^2 = \frac{K}{1+\epsilon^2 C_N^2(\frac{s}{j\Omega_{pl}})} \tag{29}$$

Solving for the poles of $|H_{analog,LPF}(s)|^2$, we get,

$$s_k = \Omega_p l sin(A_k) sinh(B) + j\Omega cos(A_k) cosh(B) \tag{30}$$

pole 1(s_{l1})	0.123 + j1.397
pole 2(s_{l2})	0.297 + j0.578
pole 3(s_{l3})	0.297 - j0.577
pole 4(s_{l4})	0.123 - j1.396

Table 6: Table for pole values

Stable filter implies all poles in left half plane. But there no any poles in left half plane.Hence it is unstable.

3.7 Analog Transfer Function of BRF Filter

In $H_{analog,LPF}(s_l)$, substituting,

$$s_l = \frac{0.143s}{s^2 + 0.127} \tag{31}$$

We get, $H_{analog}(s)$ with mentioned coefficients,
Numerator,

s^8	s^6	s^4	s^2	s^0
1	0.5153	0.0996	0.0085	0.0003

Table 7

Denominator,

s^8	s^7	s^6	s^5	s^4	s^3	s^2	s^1	s^0
1	0.3752	0.6308	0.1552	0.1311	0.0200	0.0105	0.0008	0.0003

Table 8

3.8 Discrete Time Filter Transfer Function

Analog to discrete time conversion is made using the bilinear transformation,

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \tag{32}$$

We get, $H_{discrete,BRF}(z)$ with mentioned coefficients,

Numerator,

z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
0.6987	-4.3141	12.7834	-23.2207	28.1357	-23.2207	12.7834	-4.3141	0.6987

Table 9

Denominator,

z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
1.0000	-5.5926	15.0457	-24.9315	27.7116	-21.1188	10.8170	-3.4267	0.5257

Table 10

3.9 Direct Form II Realization

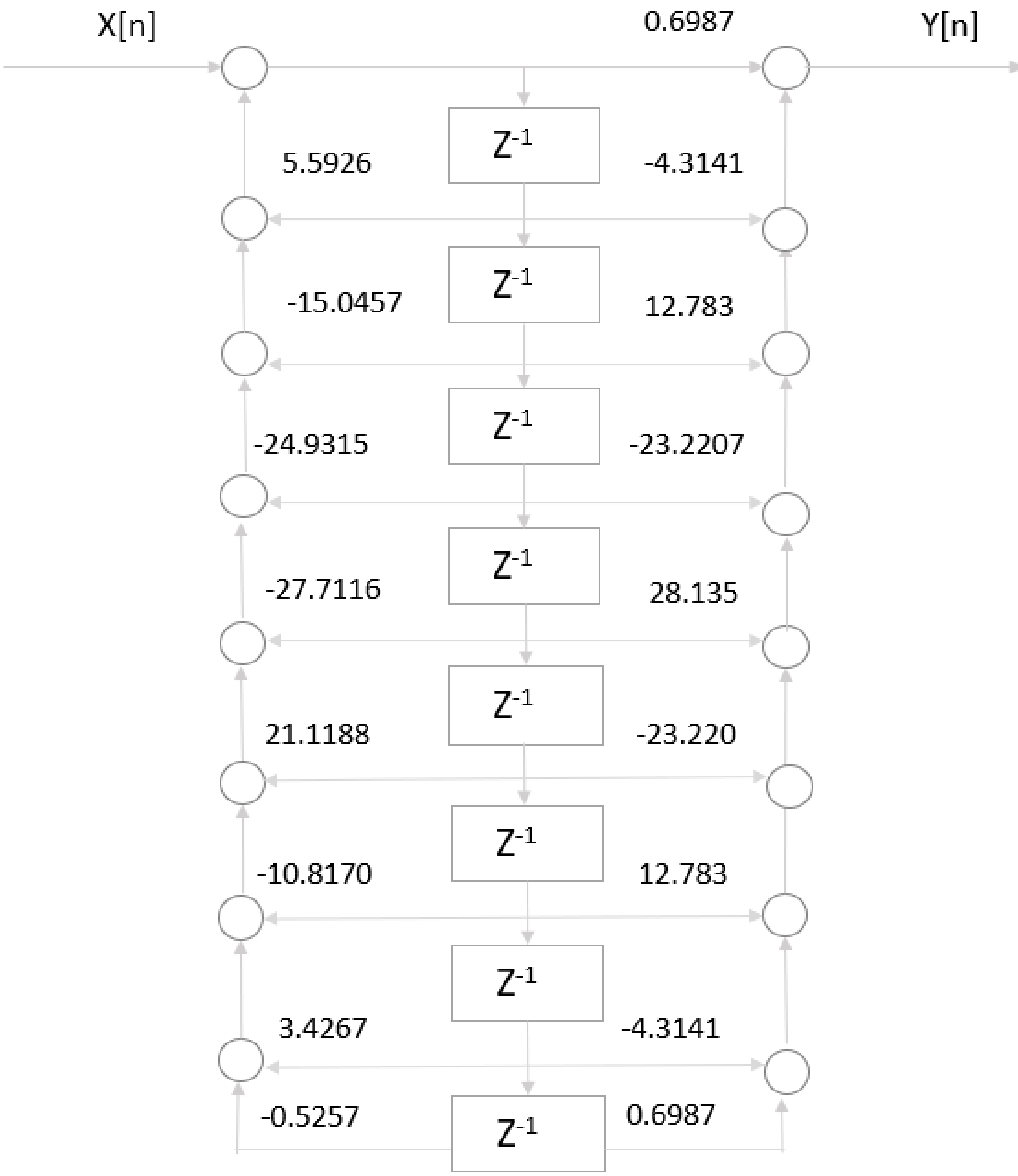

Figure 3

3.10 FIR Filter Design using Kaiser Window

Stopband and Passband Tolerance(δ) = 0.15
Therefore, Kaiser parameters,
$\Delta\omega_T = \omega_{p2}$ - $\omega_{s2} = \omega_{s1}$ - $\omega_{p1} = 0.20096$
A = -20log_{10} δ = 16.4782

$$N_{min} = \lceil \frac{A - 8}{2.285 * \Delta\omega_T} \rceil = \lceil \frac{16.4782 - 8}{2.285 * 0.20096} \rceil = 19 \tag{33}$$

As A $<$ 21, therefore, shape parameter in Kaiser window, $\alpha = \beta = 0$ (Rectangular Window).

The Kaiser window has to operate on ideal BRF filter, which is approximated as a separate function using truncated time domain response of ideal LPF and HPF filter. This introduces **non-ideality** which results in **increased order** of Kaiser window than necessary.

Therefore, length of Kaiser Window (L) = $N_{min} + 1 + 12 = 32$.

The transfer function of resultant BRF filter is given by the coefficients mentioned, from zero to increasing time delay (in Z-domain).

```
Columns 1 through 12

  -0.0064   -0.0120   -0.0116   -0.0059    0.0016    0.0071    0.0081    0.0052    0.0010   -0.0016   -0.0014    0.0005

 Columns 13 through 24

   0.0014   -0.0008   -0.0055   -0.0098   -0.0096   -0.0028    0.0086    0.0191    0.0219    0.0133   -0.0047   -0.0240

 Columns 25 through 36

  -0.0344   -0.0286   -0.0073    0.0209    0.0420    0.0445    0.0251   -0.0087   -0.0410   -0.0556   -0.0441   -0.0104

 Columns 37 through 48

   0.0299    0.0576    0.0586    0.0316   -0.0110   -0.0489    0.9360   -0.0489   -0.0110    0.0316    0.0586    0.0576

 Columns 49 through 60

   0.0299   -0.0104   -0.0441   -0.0556   -0.0410   -0.0087    0.0251    0.0445    0.0420    0.0209   -0.0073   -0.0286

 Columns 61 through 72

  -0.0344   -0.0240   -0.0047    0.0133    0.0219    0.0191    0.0086   -0.0028   -0.0096   -0.0098   -0.0055   -0.0008

 Columns 73 through 84

   0.0014    0.0005   -0.0014   -0.0016    0.0010    0.0052    0.0081    0.0071    0.0016   -0.0059   -0.0116   -0.0120

 Columns 85 through 86

  -0.0064    0.0027
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Figure 4: Coefficients of Transfer Function in Z-domain

4 Matlab Plot

4.1 Bandpass Filter (Monotonic)

4.1.1 IIR Design

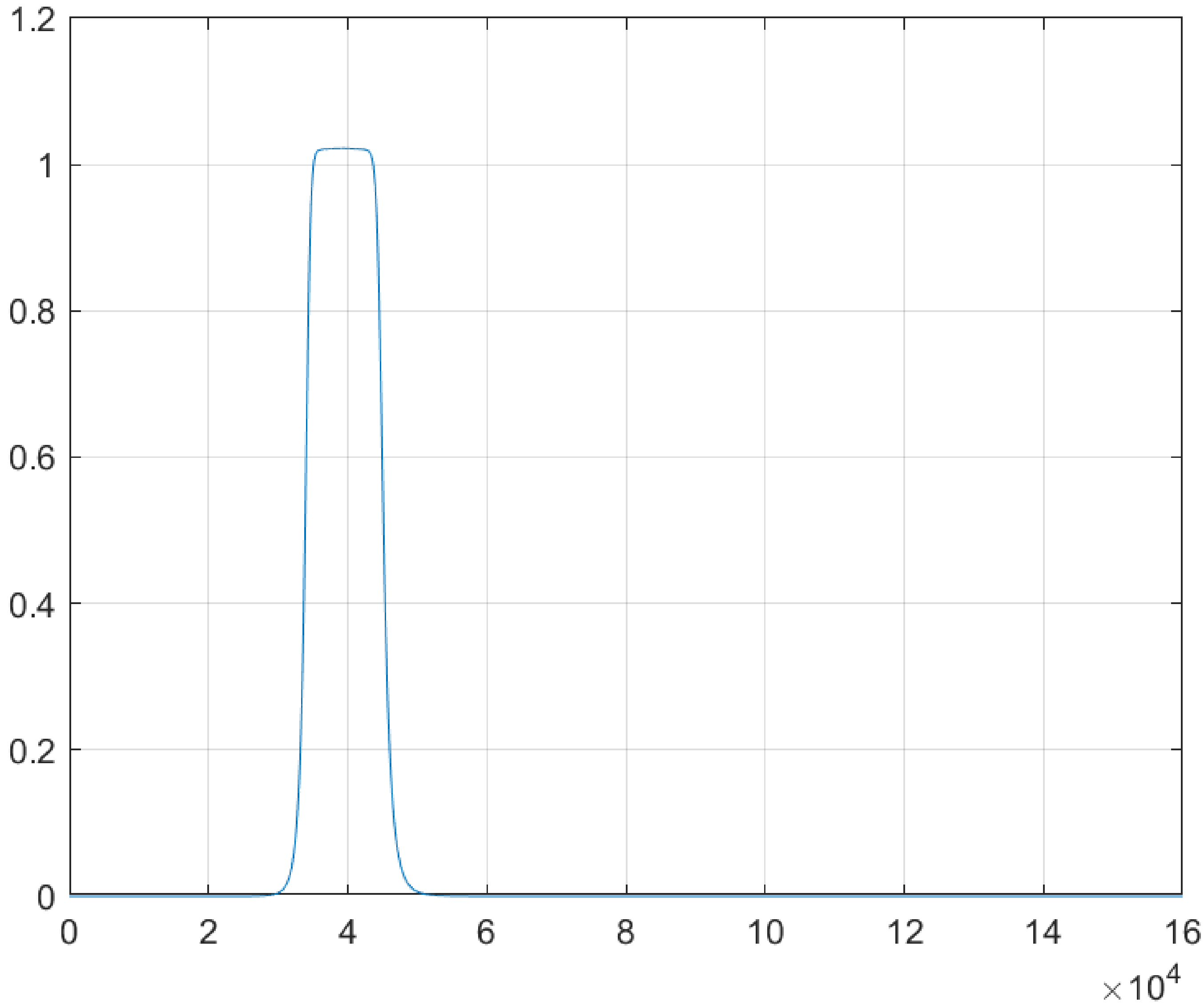

Figure 5: Magnitude Response

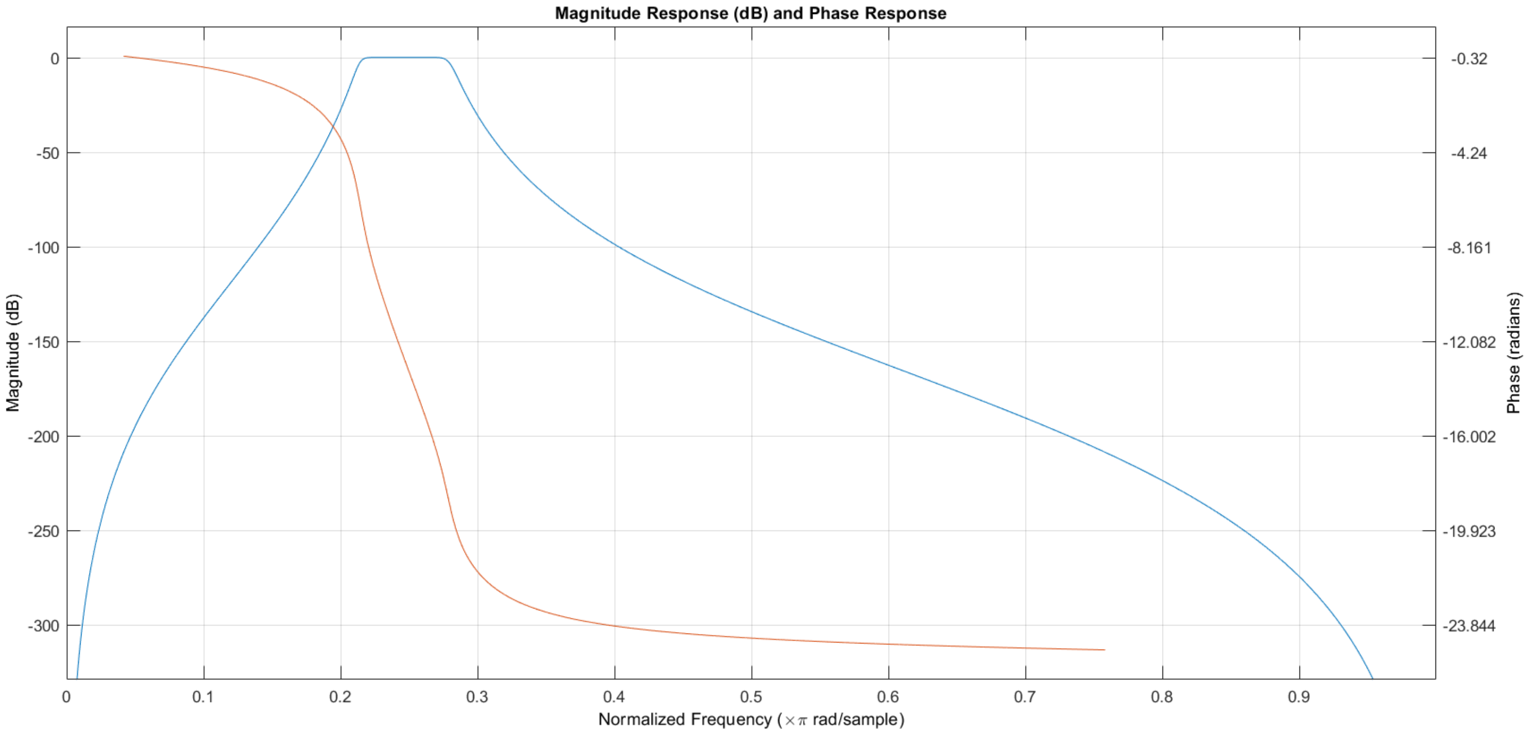

Figure 6: Attenuation Phase Response

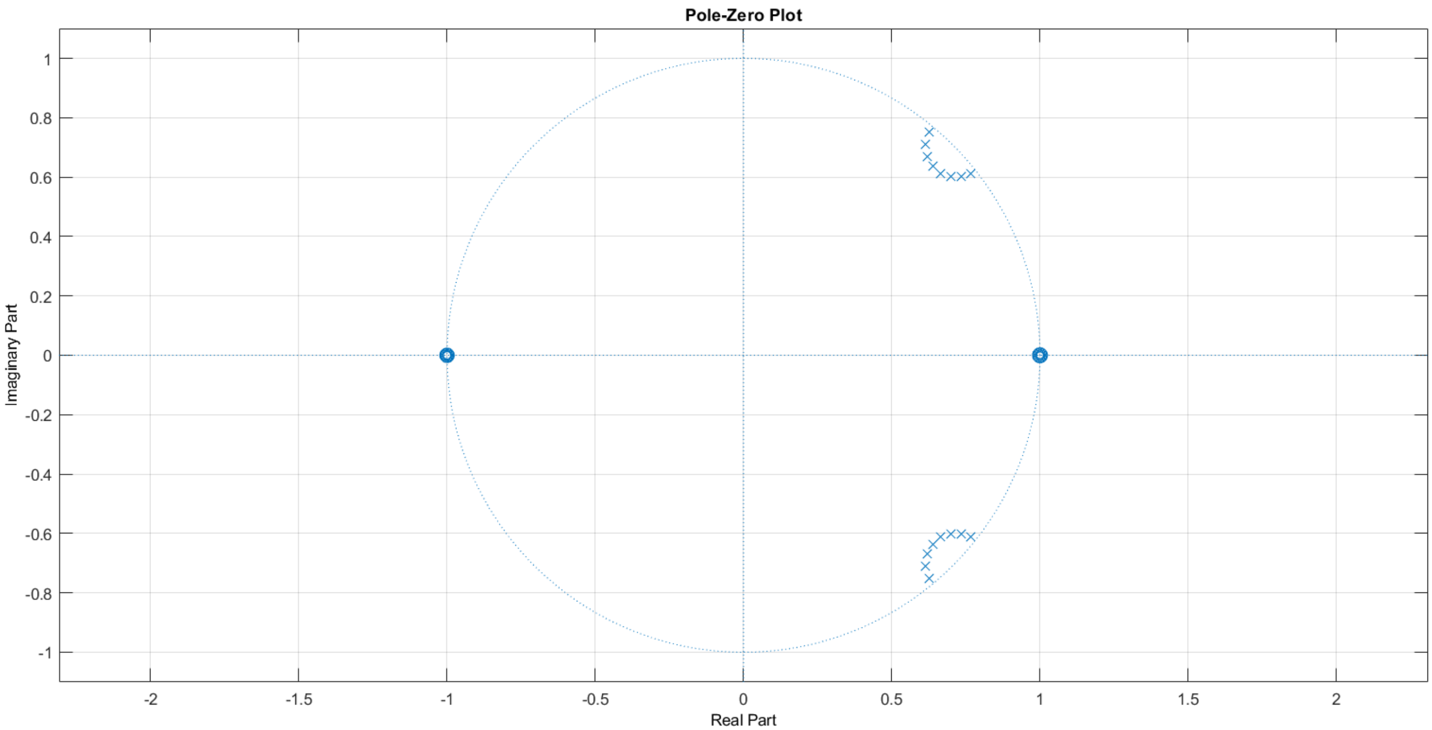

Figure 7: Pole/Zero Plot

The Magnitude plot justifies that the constraints on tolerances are met, the phase plot is non-linear as expected. The pole-zero plot shows all poles(CROSSES) are within the unit circle, hence the filter transfer function is stable.

4.1.2 FIR Design

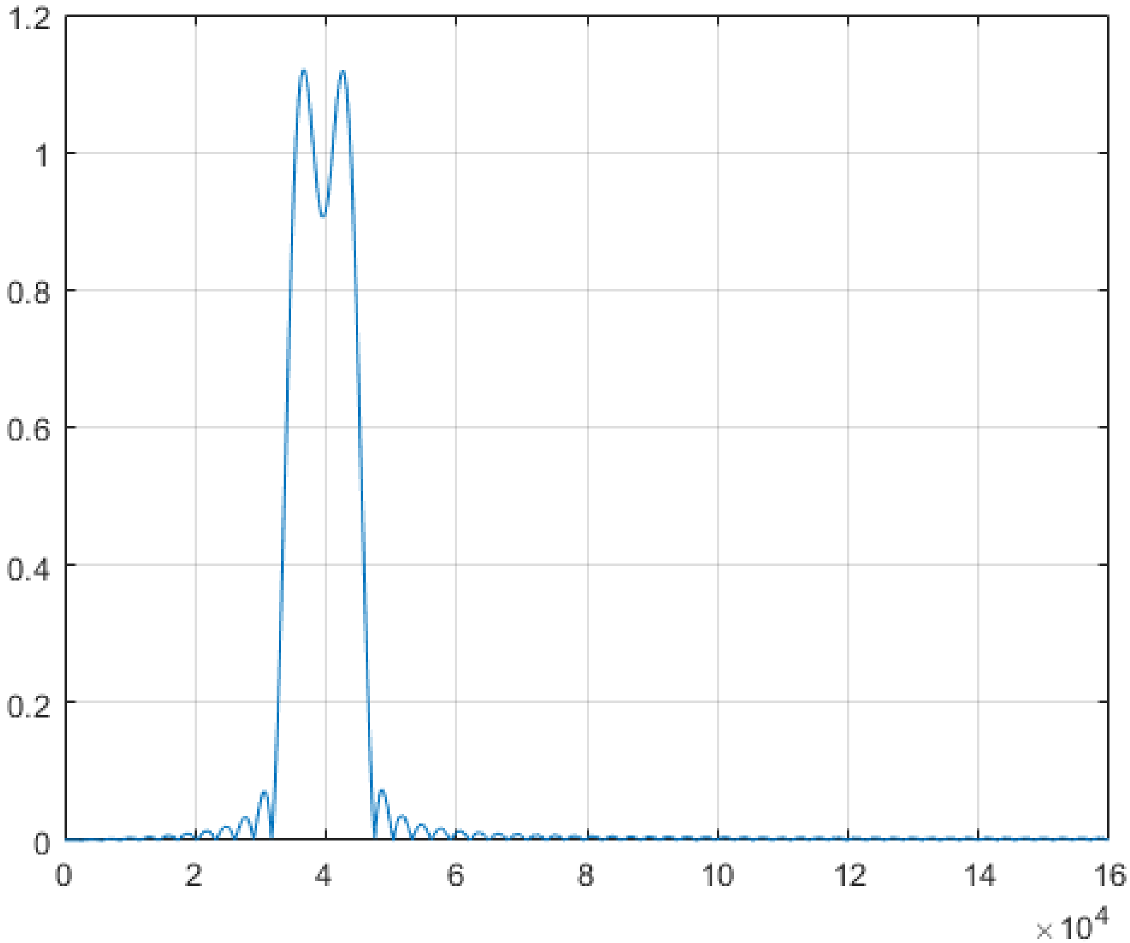

Figure 8: Magnitude Response

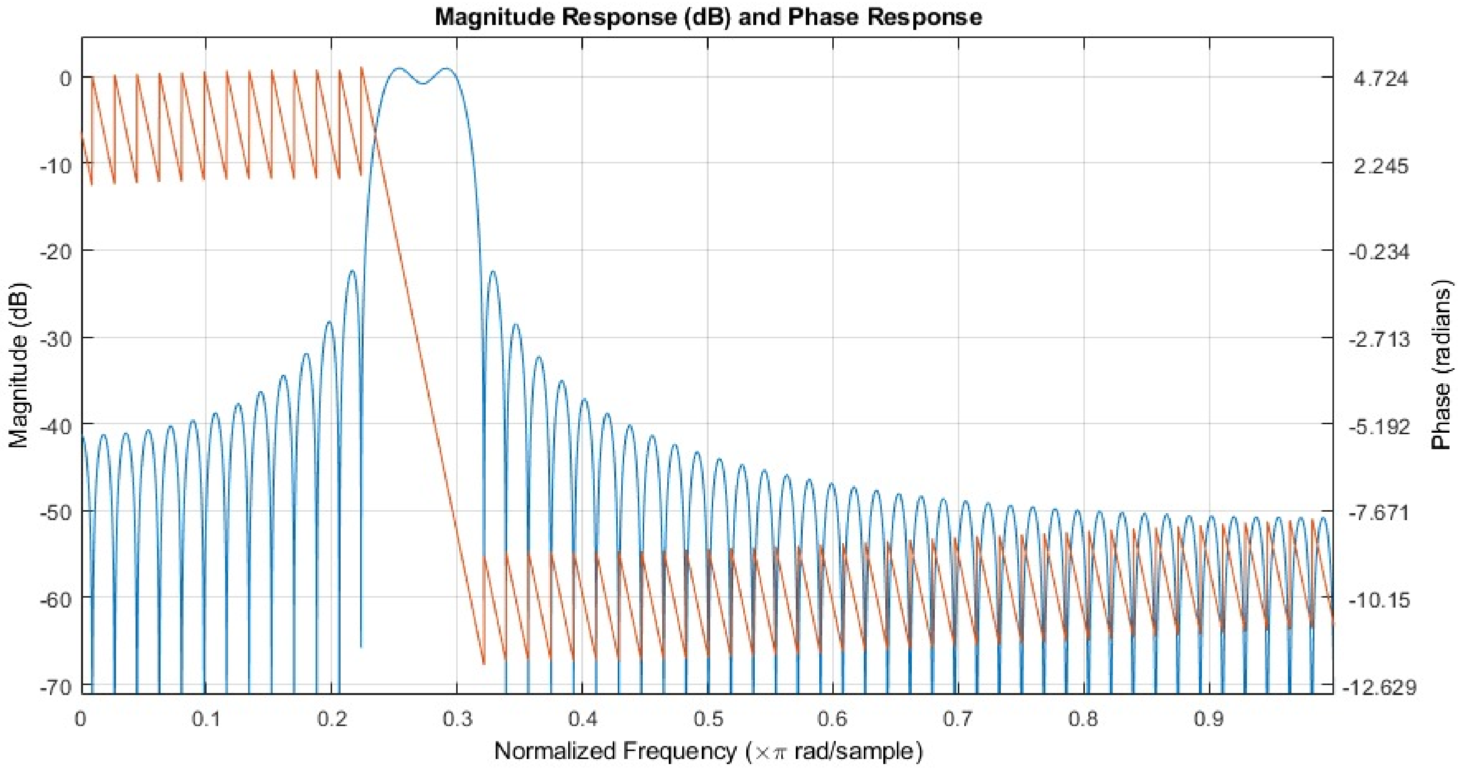

Figure 9: Attenuation and Phase Response

The magnitude response plot shows that the constraints on tolerances are met with the phase plot being linear as expected for a FIR filter.

4.2 BandStop Filter (Chebyschev)

4.2.1 IIR Design

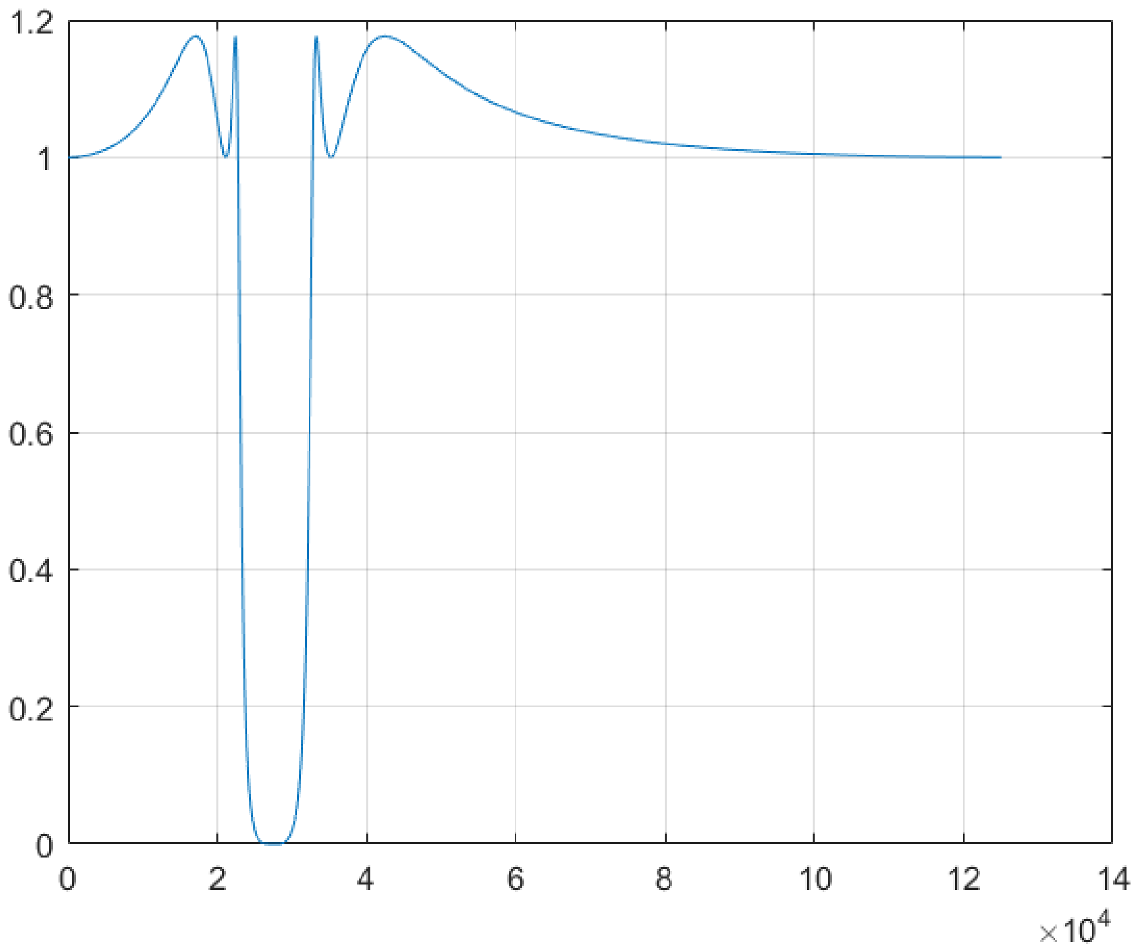

Figure 10: Magnitude Response

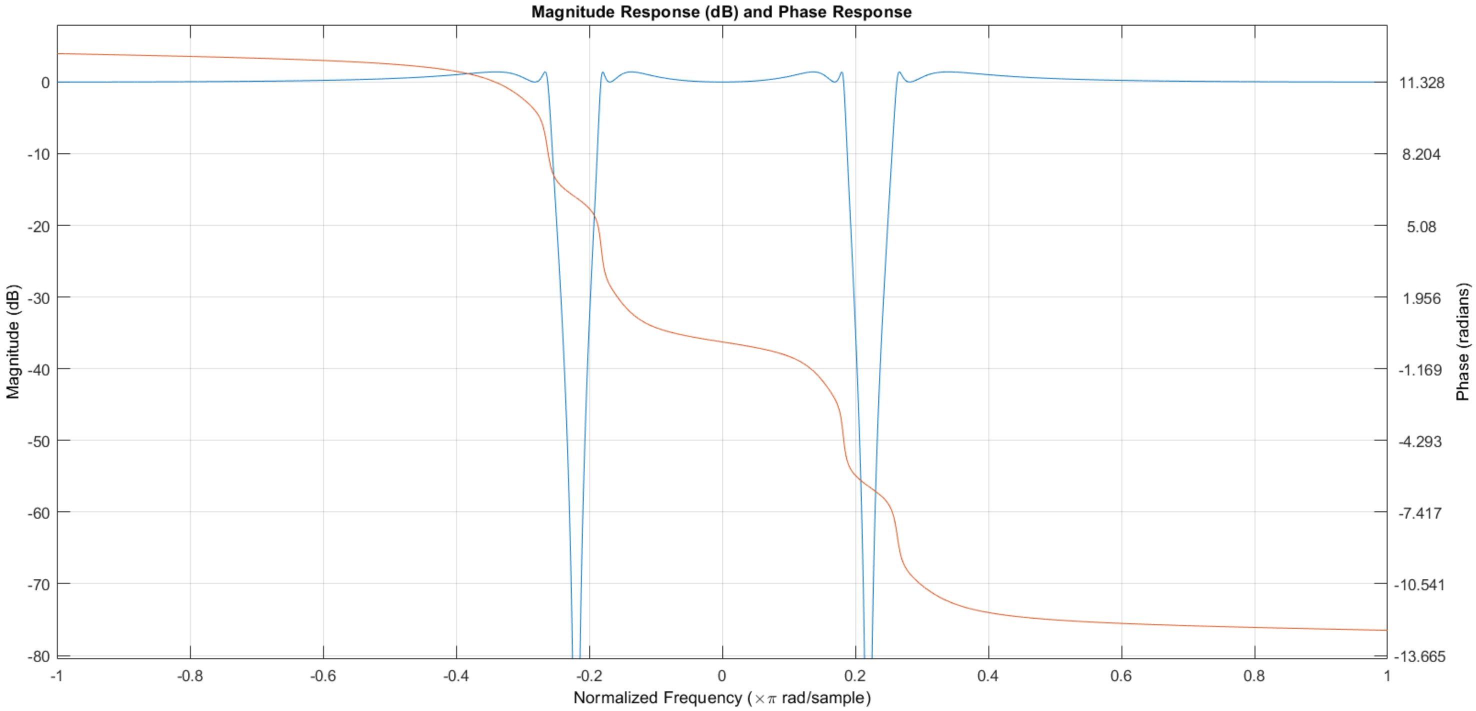

Figure 11: Attenuation and Phase Response

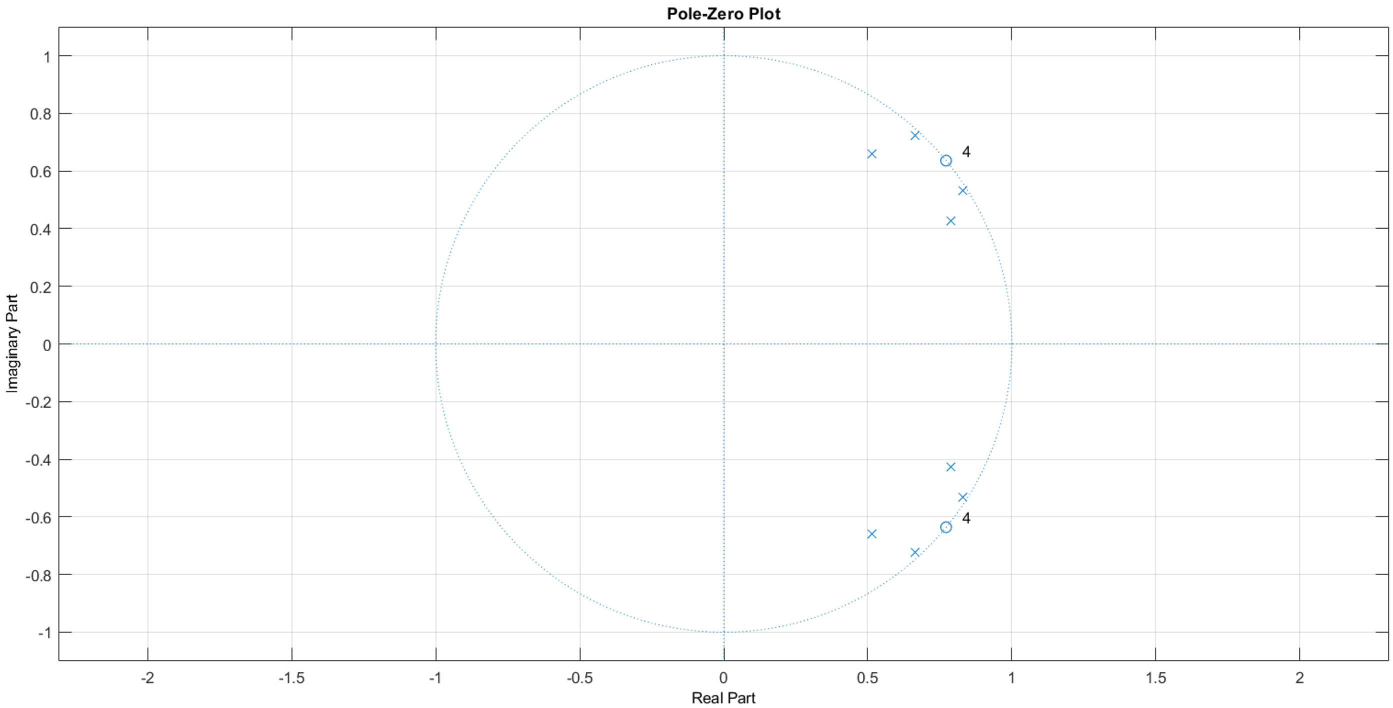

Figure 12: Pole/Zero Plot

The Magnitude plot justifies that the constraints on tolerances are met, the phase plot is non-linear as expected. The pole-zero plot shows all poles(CROSSES) are within the unit circle, hence the filter transfer function is stable.

4.2.2 FIR Design

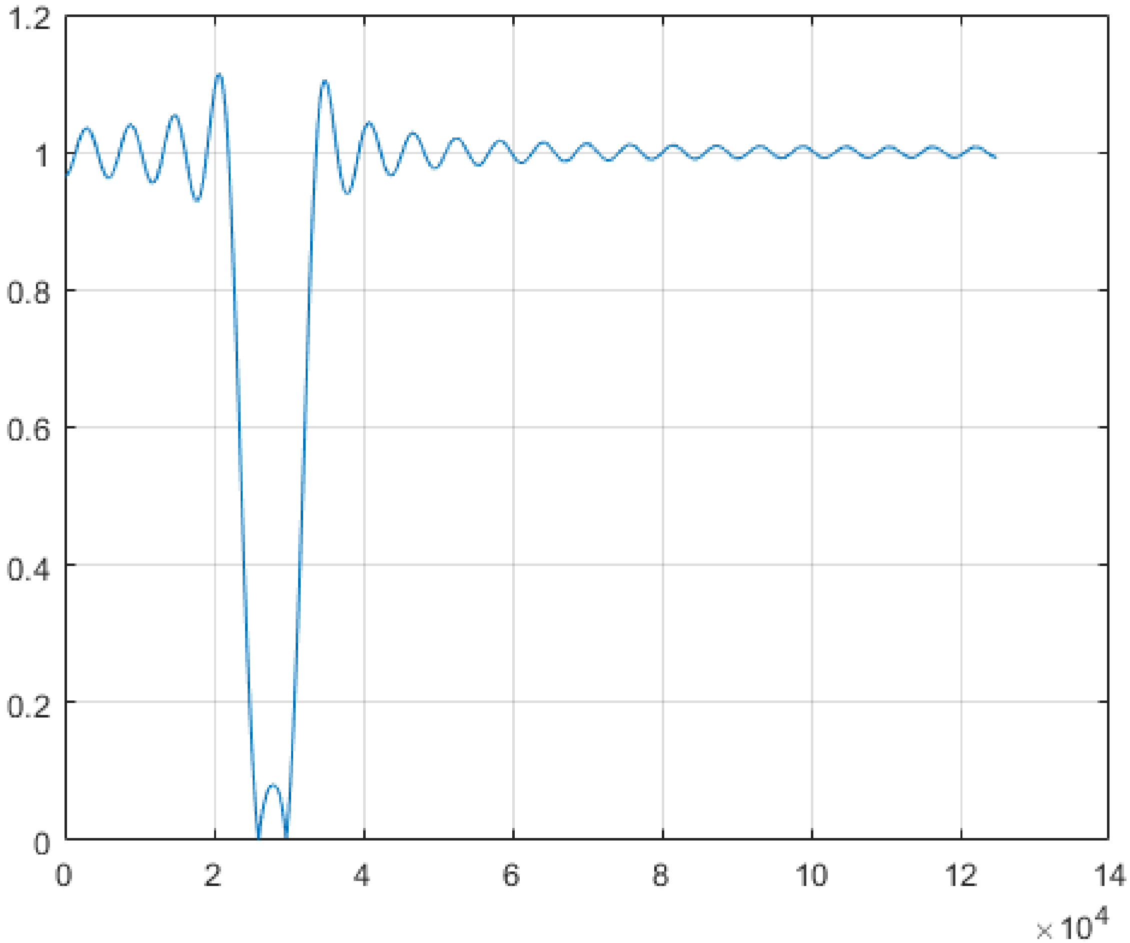

Figure 13: Magnitude Response

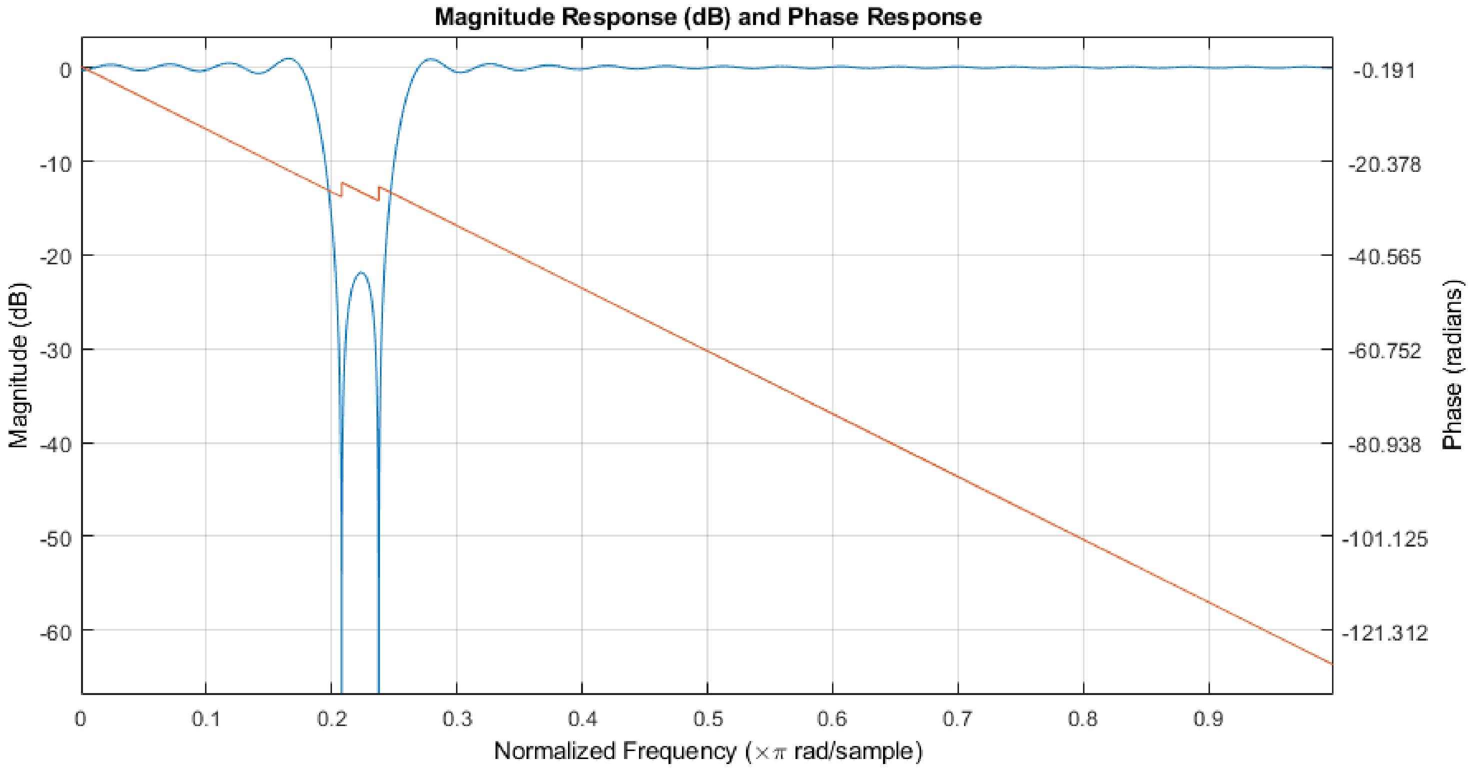

Figure 14: Attenuation Response

The magnitude response plot shows that the constraints on tolerances are met, with the phase plot being linear as expected for a FIR filter.