**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**A:** Because the task started after 10 minutes, the average time increased from 45 to 55 minutes for normal distribution:- z = (X-mean)/standard

= (60-55)/8

= 0.625

import scipy.stats as st

1-st.norm.cdf(0.625)= **0.2659855**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**A:** False – with mean = 38 and std dev= 6

P(X<44) = st.norm.cdf(44,38,6) = **84%** which implies that there is only 16% probability that the might be 44+ age

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**A:** True, P(X>30) = 1- st.norm.cdf(30,38,6) = **91%** which implies that there is 9% probability that employees might be of age less than 30 years.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**A:** According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean μ, and the variance σ², and written as .X~N(μ, σ²)

Given X1~ N(μ, σ² ) and X2~N(μ, σ²) are two independent identically distributed random variables.

From the properties of normal random variables,

if X~N(μ 1, σ²1 and Y~N(μ 2, σ²2)are two independent identically distributed random variables then

* the sum of normal random variables is given by

,X+Y~N(μ1+ μ2, σ²1, σ²2)

* and the difference of normal random variables is given by

, X-Y~N(μ1- μ2, σ²1+σ²2)

* When ,Z=aX the product of X is given by

Z~N(aμ1,a² σ²1)

* When , Z=aX+bY the linear combination of X and Y is given by

Z~N(aμ1+b μ2, a² σ²1+ b²σ²2)

Given to find, 2X1

Thus, following the property of multiplication, we get

2X1~N(2μ,2² σ²)  2X1 ~ N(2 μ,4 σ²) and following the property of addition, X1+X2 ~ N(μ+ μ, σ²+ σ²) ~N(2 μ ,2 σ²)

And the difference between the two is given by

2X1-(X1+X2)~N(2 μ -2 μ,2 σ²1+4 σ²2)~N(0,6 σ²)

The mean of 2X1and X1+X2 is same but the var(σ²) of 2X1 is 2 times more than the variance of X1+X2

The difference between the two says that the two given variables are identically and independently distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**A:** p(a<x<b) = 0.99 , mean =100, std= 20

we have to remove area of .005 in each of the left and right tails of Normal Distribution giving as 0.5 and 0.995 percentage Z scores

z score at 99.5 given as => st.norm.ppf(0.995)= 2.5758 z core at 0.005 fiven as =>st.norm.ppf(0.005)= -2.5758

Z = (x - 100)/20 = > x = 20z+100

a = -(20\*2.5758) + 100= 48.484

b = (20\*2.5758)+100= 151.516 hence two values a,b are **48.5, 151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**A:** Total mean profit of the company from 2 divisions 5+7=12 as N(mean,variance) converting it in rupees million 12\*45rs=**540** million rupees

Total Variance is (9+16)=25\*45rs= **1125** million rs

Total Standard Deviation is sqrt(25)\*45rs =**225** million rs 95% prob Rupee Range on annual profit of the company st.norm.interval(0.95,mean=540,std=225)

(**99.008, 980.991**) million rupees

1. Specify the 5th percentile of profit (in Rupees) for the company

**A:** 5th percentile z= st.norm.ppf(0.05)=**-1.6448**

X=mean + Zstd = 540+((-1.6448)\*(225)) X=**169.92**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**A:** Probability of Division 1 making the loss is P(X<0)

st.norm.cdf(0,5,3)= **0.0477**

Probability of Division 2 making loss is P(X<0) st.norm.cdf(0,7,4)=**0.0400**