

## Assignment 53


① `int a=0, b=0;`  $\rightarrow$  constant  
`for (i=0; i<N; i++)`  
`{`  
`a = a + rand();`  
`}`  $\rightarrow (n+1)$

`for (j=0; j<M; j++)`  
`{`  
`b = b + rand();`  
`}`  $\rightarrow (M+1)$

time complexity  $\Rightarrow O(n)$

space complexity  $\Rightarrow 6$  (total 6 variables)

② `int a=0;`  $\rightarrow$  constant  
`for (i=0; i<N; i++)  $\rightarrow n$`   
`{`  
`for (j=N; j>1; j--)  $n^2$`   
`{`  
`a = a + i+j`  
`}`  
`}`  $\left. \vphantom{\begin{matrix} \text{for (j=N; j>1; j--) } \\ \text{for (i=0; i<N; i++) } \end{matrix}} \right\} n^2$

 time complexity =  $O(n^2)$

③ int i, j, k = 0; ——— constant

for (i =  $\frac{n}{2}$ ; i <= n; i++)  $\rightarrow \frac{n}{2} \approx n$

{  
for (j = 2; j <= n; j += 2)  $\rightarrow n \log n$

{  
k = k +  $\frac{n}{2}$ ;

}

}

time complexity  $\rightarrow O(\underline{n \log n})$

④ void fun(int n)

{

for (int i = 0; i <=  $\frac{n}{2}$ ; i++)  $\rightarrow \frac{n}{2} \approx n$

for (int j = 1; j +  $\frac{n}{2}$  <= n; j++)  $\approx \sqrt{n}$

for (int k = 1; k <= n; k += 2)  $\rightarrow \log n$

cout << "Prabek Jain";

}

time complexity  $\Rightarrow O(\underline{n^{\frac{3}{2}} \log n})$

$$\underline{5.} \quad T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$$

$$a=3, b=3, k=1, p=0, b^k=3$$

$$\frac{a}{b^k} = \frac{3}{3}$$

$$T(n) = O(n^{\log_3 3} \log n)$$

$$\boxed{T(n) = O(n \log n)}$$

$$\frac{\log 3}{\log 3} = 1$$

$$\underline{6.} \quad T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$$

$$a=6, b=3, k=2, p=1, b^k = 3^2 = 9$$

$$b^k > a$$

$$9 > 6$$

$$\boxed{T(n) = O(n^2 \log n)}$$

$$\underline{7.} \quad T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

$$a=4, b=2, k=1, p=-1, b^k = 2^1 = 2$$

$$a > b^k$$

$$4 > 2$$

$$\therefore T(n) = O(n^{\log_2 4} \log^{-1+1} n) \Rightarrow T(n) = O(n^2)$$

$$\frac{\log 4}{\log 2} = \frac{\log 2^2}{\log 2} = 2$$

$$(8) \quad T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$$

$$a = 64, b = 8, k = 2, p = 1, b^k = 8^2 = 64$$

$$a = b^k$$

$$64 = 64$$

$$\therefore T(n) = O(n^{\log_8 64} \log^{(1+1)} n)$$

$$T(n) = O(n^2 \log^2 n)$$

$$\Rightarrow \frac{\log 64}{\log 8}$$

$$\Rightarrow \frac{\log 8^2}{\log 8}$$

$$\Rightarrow \frac{2 \log 8}{\log 8}$$

$$\Rightarrow 2$$

$$(9) \quad T(n) = 7T\left(\frac{n}{3}\right) + n^2$$

$$a = 7, b = 3, k = 2, p = 0, b^k = 3^2 = 9$$

$$b^k > a$$

$$9 > 7$$

$$\therefore T(n) = O(n^2 \log^0 n)$$

$$T(n) = O(n^2)$$



$$(10) \quad T(n) = 4T\left(\frac{n}{2}\right) + \log n$$

$$a=4, \quad b=2, \quad k=0, \quad p=1, \quad b^k = 2^0 = 1$$

$$b^k < a$$

$$1 < 4$$

$$\therefore T(n) = O(n^{\log_2 4})$$

$$\therefore \boxed{T(n) = O(n^2)}$$

$$\frac{\log 4}{\log 2} \Rightarrow \frac{2 \log 2}{\log 2}$$