CHAPTER FIVE

Curve Fitting

Objectives

- Polynomial Fitting

Polynomial Fitting

- For any polynomial:

$$f(x) = c_1 + c_2x + c_3x^2 + \ldots + c_{m+1}x^m$$

- Find Least square Parabola: $f(x) = Ax^2 + Bx + C$

$$E(A,B,C) = \sum_{k=1}^{N}(Ax_k^2 + Bx_k + C - y_k)^2$$

Polynomial Fitting

$$E(A,B,C) = \sum_{k=1}^{N}(Ax_k^2 + Bx_k + C - y_k)^2$$

$$\frac{\partial E}{\partial A} = 2\sum_{k=1}^{N}(Ax_k^2 + Bx_k + C - y_k)^1(x_k^2) = 0$$

$$\frac{\partial E}{\partial B} = 2\sum_{k=1}^{N}(Ax_k^2 + Bx_k + C - y_k)^1(x_k) = 0$$

$$\frac{\partial E}{\partial C} = 2\sum_{k=1}^{N}(Ax_k^2 + Bx_k + C - y_k)^1(1) = 0$$

$$(\sum x_k^4)A + (\sum x_k^3)B + (\sum x_k^2)C = \sum y_k x_k^2$$

$$(\sum x_k^3)A + (\sum x_k^2)B + (\sum x_k)C = \sum y_k x_k$$

$$(\sum x_k^2)A + (\sum x_k)B + NC = \sum y_k$$

Polynomial Fitting - Example

- Find the least-squares Parabola for the following four data points: $(-3,3), (0,1), (2,1), (4,3)$

Table 5.7 Obtaining the Coefficients for the Least-Squares Parabola of Example 5.6

x_k	y_k	x_k^2	x_k^3	x_k^4	$x_k y_k$	$x_k^2 y_k$
−3	3	9	−27	81	−9	27
0	1	0	0	0	0	0
2	1	4	8	16	2	4
4	3	16	64	256	12	48
3	8	29	45	353	5	79

[1]

Polynomial Fitting – Example (cont'd)

- From table:

- $\sum x_k = 3$ $\qquad \sum y_k = 8$
- $\sum x_k^2 = 29$ $\qquad \sum x_k^3 = 45$
- $\sum x_k^4 = 353$ $\qquad \sum y_k x_k = 5$
- $\sum y_k x_k^2 = 79$

$$(\sum x_k^4)A + (\sum x_k^3)B + (\sum x_k^2)C = \sum y_k x_k^2$$
$$(\sum x_k^3)A + (\sum x_k^2)B + (\sum x_k)C = \sum y_k x_k$$
$$(\sum x_k^2)A + (\sum x_k)B + NC = \sum y_k$$

- $A = \frac{585}{3278}$ $\qquad B = \frac{-631}{3278}$ $\qquad C = \frac{1394}{1639}$

- $\boxed{y = 0.178462x^2 - 0.192495x + 0.850519}$

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall

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