CHAPTER FIVE

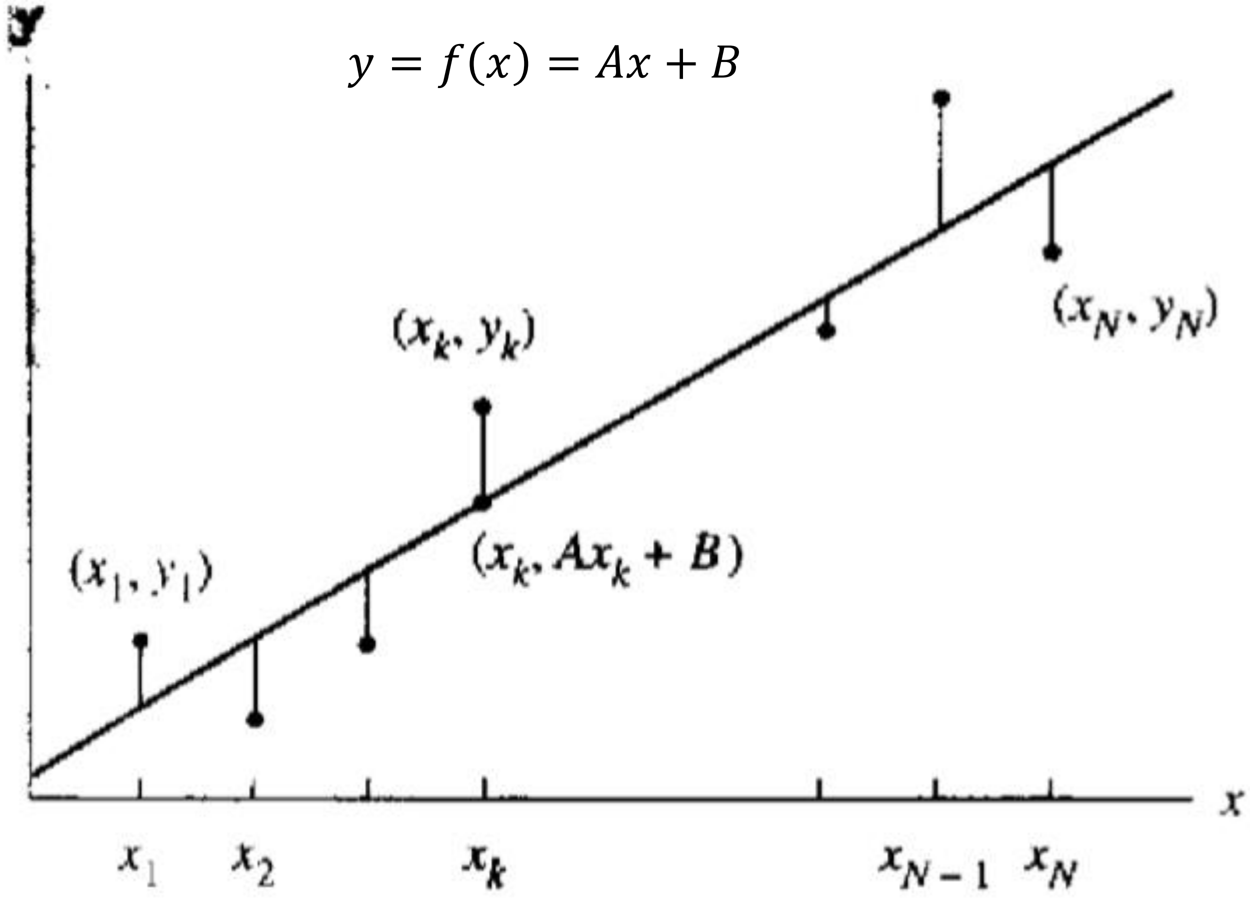
Curve Fitting

Objectives

- Curve Fitting
- Line Fit

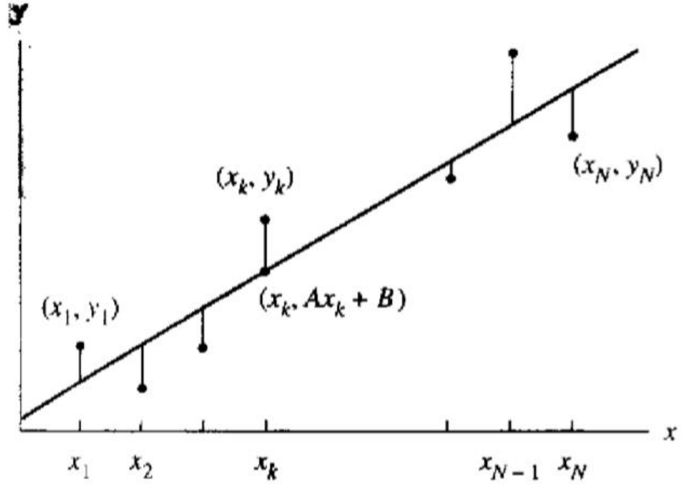
Curve Fitting

- The best curve approximation that goes near (not always through) data points.



[1]

Line Fit



- Find best line that minimize the error between the data points and the line .
- $e_k = f(x_k) - y_k \quad \rightarrow \quad f(x_k) = y_k + e_k$
- $e_k \rightarrow$ Error (deviations or residuals)

- Root–Mean–Square (RMS) error is used to measure how far the curve $y = f(x)$ lies from data.

$$E_{rms} = \left(\frac{1}{N}\sum_{k=1}^{N}(f(x_k) - y_k)^2\right)^{1/2}$$

Line Fit

- $E_{rms} = (\frac{1}{N}\sum_{k=1}^{N}(f(x_k) - y_k)^2)^{1/2}$
- $N(E_{rms}^2) = \boxed{\sum_{k=1}^{N}(f(x_k) - y_k)^2} \longrightarrow$ Sum of the squares of the vertical distances form the point to the line .
- The line that minimizes E_{rms} called least- square line.

$$y = f(x) = A_x + B$$

- Find $A$and B in least square line: $E(A, B) = \sum_{k=1}^{N}(\underbrace{Ax_k + B}_{f(x)} - y_k)^2$

Line Fit

- The minimum value of $E(A,B)$ in determined by setting the partial derivatives $\partial E/\partial A$ and $\partial E/\partial B$ equal to zero and solving there equations for A and B .

$$E(A,B) = \sum_{k=1}^{N}(Ax_k + B - y_k)^2$$

$$\frac{\partial E(A,B)}{\partial A} = \sum_{k=1}^{N} 2\,(Ax_k + B - Y_k)^1(x_k) = 0$$

$$= 2\sum_{k=1}^{N}\left(Ax_k^2 + Bx_k - x_k y_k\right) = 0$$

$$\boxed{(\sum_{k=1}^{N} x_k^2)A + (\sum_{K=1}^{N} x_k)\,B = \sum_{k=1}^{N} x_k y_k} \quad \text{........ } 1$$

Line Fit

$$E(A,B) = \sum_{k=1}^{N}(Ax_k + B - y_k)^2$$

$$\frac{\partial E(A,B)}{\partial B} = \sum_{k=1}^{N} 2\,(Ax_k + B - Y_k)^1(1) = 0$$

$$= 2\sum_{k=1}^{N}(Ax_k + B - y_k) = 0$$

$$\boxed{\left(\sum_{k=1}^{N} x_k\right)A + NB = \sum_{k=1}^{N} y_k} \quad ....2$$

- From equations 1 and 2, find A and B

Line Fit – Example

- Find least–square line far data points: $(-1,10)$ $(0,9)$ $(1,7)$ $(2,5)$ $(3,4)$ $(4,3)$ $(5,0)$ $(6,-1)$ $N = 8$

$$\left(\sum_{k=1}^{8} x_k^2\right)A + \left(\sum_{k=1}^{8} x_k\right) B = \sum_{k=1}^{8} x_k\, y_k \quad \ldots\ldots 1$$

$$\left(\sum_{k=1}^{8} x_k\right) A + 8B = \sum_{k=1}^{8} y_k \quad \ldots.. 2$$

Table 5.2 Obtaining the Coefficients for Normal Equations

x_k	y_k	x_k^2	$x_k y_k$
−1	10	1	−10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	16	12
5	0	25	0
6	−1	36	−6
20	37	92	25

[1]

- From Table 5.2

$\sum x_k = 20 \quad \sum x_k^2 = 92$

$\sum y_k = 37 \quad \sum x_k y_k = 25$

$92A + 20B = 25$

$20A + 8B = 37$

$A = -1.6071429$

$B = 8.6428571$

$y = -1.6071429x + 8.6428571$

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall

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