CHAPTER SIX

Numerical Differentiation

Objectives

- Differentiation of Newton Polynomial

Differentiation of Newton Polynomial

General Method

- Generates central , backward and forward difference formulas using Newton Polynomials.
- Newton Polynomial $P(t)$ of degree $N = 2$ that approximates $f(t)$ using point t_0, t_1, and t_2 is:

$$P(t) = a_0 + a_1(t - t_0) + a_2(t - t_0)(t - t_1)$$

- Note: on Newton Polynomials, points do not need to be equally spaced

Differentiation of Newton Polynomial

- General Method $P(t) = a_0 + a_1(t - t_0) + a_2(t - t_0)(t - t_1)$
- From Chapter 4:

$$a_0 = f(t_0)$$

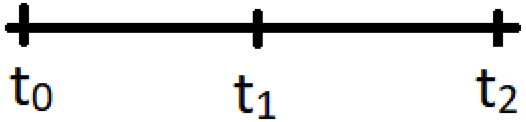
$$a_1 = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

$$a_2 = \frac{\frac{f(t_2) - f(t_1)}{t_2 - t_1} - \frac{f(t_1) - f(t_0)}{t_1 - t_0}}{t_2 - t_0}$$

- Then, $P'(t) = a_1 + a_2((t - t_0) + (t - t_1))$

$$\rightarrow \quad P'(t_0) = a_1 + a_2\big((t_0 - t_1)\big) \approx f'(t_0)$$

Case (1) Forward-Difference:



$$t_0 = x$$
$$t_1 = x + h$$
$$t_2 = x + 2h$$

$$a_1 = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

$$a_2 = \frac{\frac{f(t_2)-f(t_1)}{t_2-t_1} - \frac{f(t_1)-f(t_0)}{t_1-t_0}}{t_2-t_0}$$

$$a_1 = \frac{f(x+h)-f(x)}{h}$$

$$a_2 = \frac{\frac{f(x+2h)-f(x+h)}{h} - \frac{f(x+h)-f(x)}{h}}{2h} = \frac{f(x)-2f(x+h)+f(x+2h)}{2h^2}$$

$$P'(t_0) = a_1 + a_2\big((t_0 - t_1)\big)$$

$$P'(x) = a_1 + a_2\Big(\big(x - (x + h)\big)\Big) = a_1 - a_2 h$$

Case (1) Forward-Difference:

$$P'(x) = a_1 + a_2\left(\left(x - (x+h)\right)\right)$$

$$= a_1 - a_2 h$$

$$= \frac{f(x+h) - f(x)}{h} - \frac{f(x) - 2f(x+h) + f(x+2h)}{2h^2} h$$

$$= \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$

$$= \frac{-3f_0 + 4f_1 - f_2}{2h}$$ → Second-order forward-difference formula for $f'(x)$

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall

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