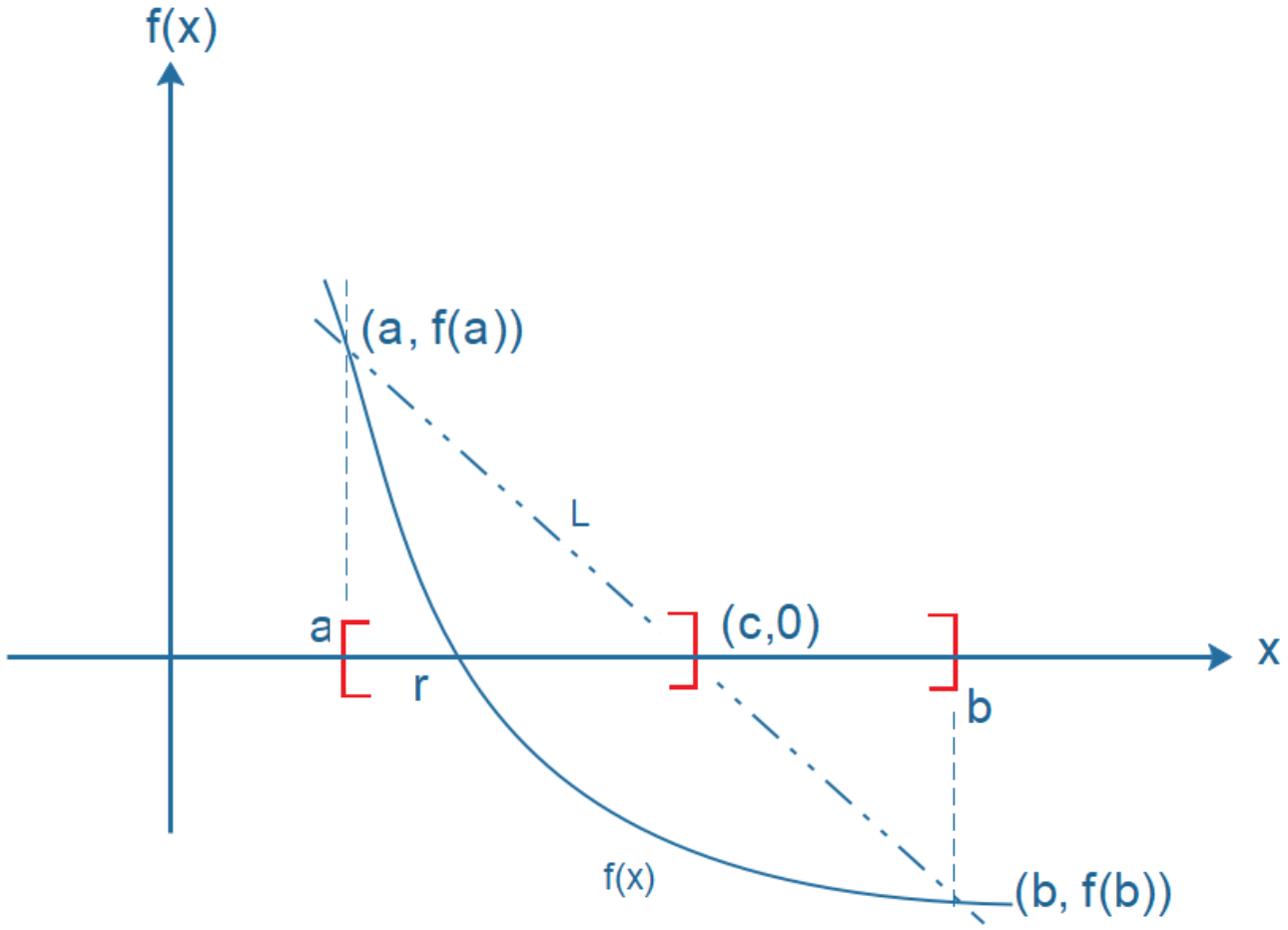
CHAPTER TWO

The Solution of Nonlinear Equations f(x) = 0

Objectives

- False Position Method

False Position Method



False Position Method

- Find the point $(c, 0)$ where the secant line $\boldsymbol{L}$ joining the points $(a, f(a))$ and $(b, f(b))$ crosses the x-axis.

$$m = \frac{f(b)-f(a)}{b-a} \text{ ..... (1)}$$

$$m = \frac{0-f(b)}{c-b} \text{ ..... (2)}$$

Both (1) and (2) are equal equation

- Then,

$$c = b - \frac{f(b)(b-a)}{f(b)-f(a)}$$

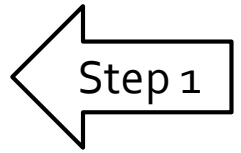
False Position Method

- The same algorithm as Bisection method will be used except for step 2, where the above equation will be used.
- Replacement of curve by straight line gives **false position** of the root.

Example

- The function $f(x) = x \times sin(x) - 1$, is continuous at $[0,2]$.
- Then: $a_0 = 0,\ b_0 = 2$

- $f(a_0) = f(0) = -1,\ f(b_0) = f(2) = 0.818595$ (opposite signs)



- $c_0 = b_0 - \frac{f(b_0)(b_0 - a_0)}{f(b_0) - f(a_0)} = 1.09975017$

- $f(c_0) = -0.02001921$ *(Note: x is in radians)*

Example - continued

- $f(c_0)\, f(b_0) < 0$ → then, root ***r*** lies in the interval $[c_o, b_o]$ Step 3
- Then, $[a_1, b_1] = [c_0, b_0] = [1.09975017, 2]$
- Now, ***start new iteration***:
- $f(a_1) = -0.02001921$, $f(b_1) = f(2) = 0.818595$ *(opposite signs)*
- $c_1 = b_1 - \frac{f(b_1)(b_1 - a_1)}{f(b_1) - f(a_1)} = 1.12124074$
- $f(c_1) = 0.00983461$
- $f(a_1)\, f(c_1) < 0$ → then, root ***r*** lies in the interval $[a_1, c_1]$
- Then, $[a_2, b_2] = [a_1, c_1] = [1.09975017, 1.12124074]$

Example - continued

- The following table show the calculations for 3 iterations.

Table 2.2 False Position Method Solution of $x \sin(x) - 1 = 0$

k	Left end point, a_k	Midpoint, c_k	Right end point, b_k	Function value, $f(c_k)$
0	0.00000000	1.09975017	2.00000000	−0.02001921
1	1.09975017	1.12124074	2.00000000	0.00983461
2	1.09975017	1.11416120	1.12124074	0.00000563
3	1.09975017	1.11415714	1.11416120	0.00000000

[1]

Termination Criterion

- In False Position method, the sequence $\{b_n - a_n\}_{n=0}^{\infty}$ may not go to zero.
- →the interval width $b_n - a_n$ is getting smaller, but it may not go to zero.

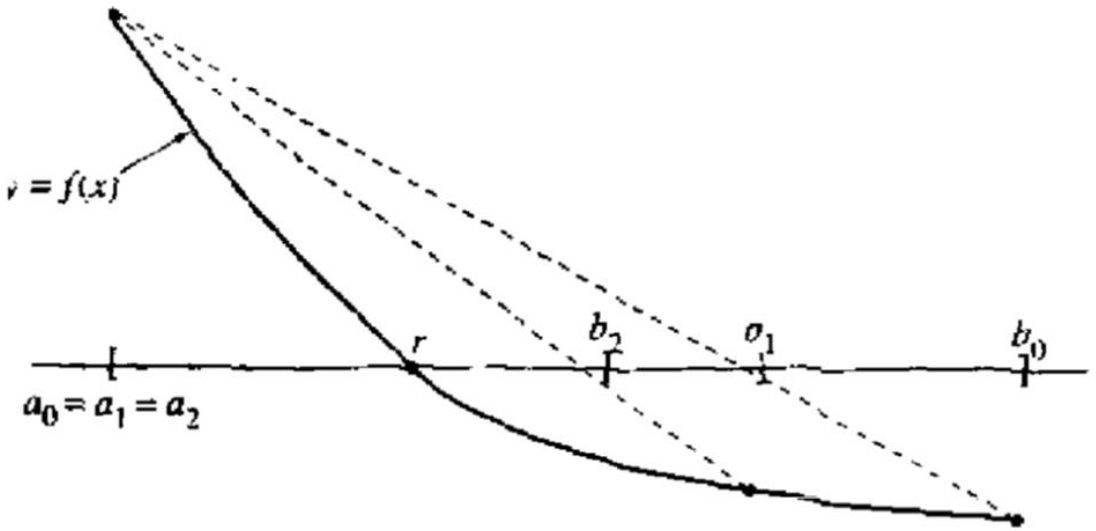


Figure 2.9 The stationary endpoint for the false position method.

[1]

Termination Criterion

- In false position method, to terminate the iterations the following conditions (both or just one of them) should occur.

1- $|f(c_n)| < \epsilon$, where ϵ is the tolerance for $f(c)$

2- $|c_n - c_{n-1}| < \delta$, closeness for consecutive iterations

- Check the following diagram.

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall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