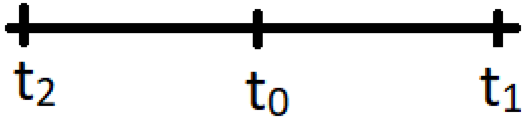
CHAPTER SIX

Numerical Differentiation

Case (2) Central-Difference:



$$t_0 = x$$
$$t_1 = x + h$$
$$t_2 = x - h$$

$$a_1 = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

$$a_2 = \frac{\frac{f(t_2)-f(t_1)}{t_2-t_1} - \frac{f(t_1)-f(t_0)}{t_1-t_0}}{t_2-t_0}$$

$$a_1 = \frac{f(x+h)-f(x)}{h}$$

$$a_2 = \frac{\frac{f(x-h)-f(x+h)}{-2h} - \frac{f(x+h)-f(x)}{h}}{-h} = \frac{f(x+h)-2f(x)+f(x-h)}{2h^2}$$

$$P'(t_0) = a_1 + a_2\big((t_0 - t_1)\big)$$

$$P'(x) = a_1 + a_2\Big(\big(x - (x+h)\big)\Big) = a_1 - a_2 h$$

Case (2) Central-Difference:

$$P'(x) = a_1 + a_2\left(\left(x - (x+h)\right)\right)$$

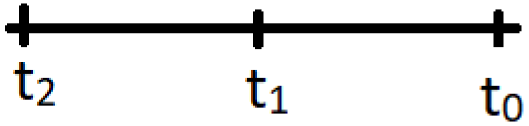
$$= a_1 - a_2 h$$

$$= \frac{f(x+h)-f(x)}{h} - \frac{f(x+h)-2f(x)+f(x-h)}{2h^2}h$$

$$= \frac{f(x+h) - f(x-h)}{2h}$$

$$= \frac{f_1 - f_{-1}}{2h}$$ → Second-order central-difference formula for $f'(x)$

Case (3) Backward-Difference:



$$t_0 = x$$
$$t_1 = x - h$$
$$t_2 = x - 2h$$

$$a_1 = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

$$a_2 = \frac{\frac{f(t_2)-f(t_1)}{t_2-t_1} - \frac{f(t_1)-f(t_0)}{t_1-t_0}}{t_2-t_0}$$

$$a_1 = \frac{f(x-h)-f(x)}{-h} = \frac{f(x)-f(x-h)}{h}$$

$$a_2 = \frac{\frac{f(x-2h)-f(x-h)}{-h} - \frac{f(x-h)-f(x)}{-h}}{-2h} = \frac{f(x)-2f(x-h)+f(x-2h)}{2h^2}$$

$$P'(t_0) = a_1 + a_2\big((t_0 - t_1)\big)$$

$$P'(x) = a_1 + a_2\Big(\big(x - (x - h)\big)\Big) = a_1 + a_2 h$$

Case (3) Backward-Difference:

$$P'(x) = a_1 + a_2\Big(\big(x - (x-h)\big)\Big)$$

$$= a_1 + a_2 h$$

$$= \frac{f(x) - f(x-h)}{h} + \frac{f(x) - 2f(x-h) + f(x-2h)}{2h^2}\, h$$

$$= \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$

$$= \frac{3f_0 - 4f_{-1} + f_{-2}}{2h}$$ → Second-order backward-difference formula for $f'(x)$

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall

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