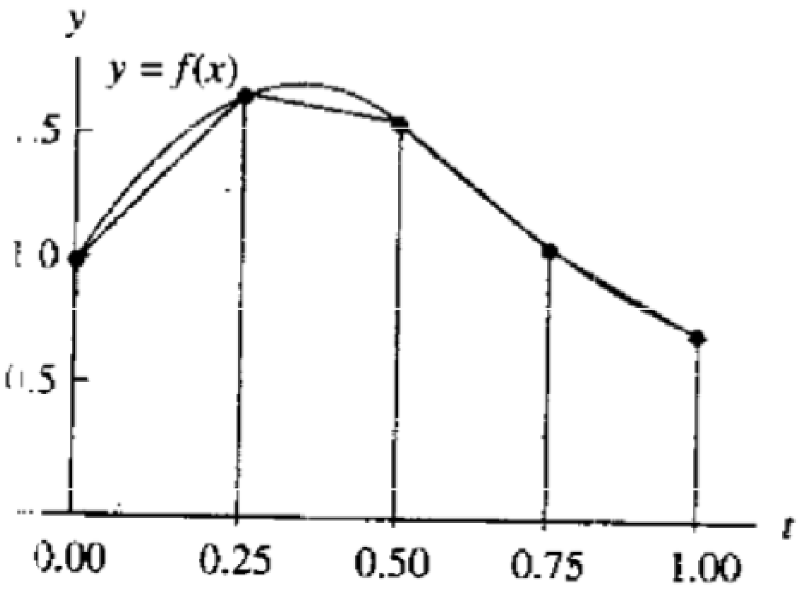
CHAPTER SEVEN

Numerical Integration

Objectives

- Composite Rules

Composite Trapezoidal Rule



[1]

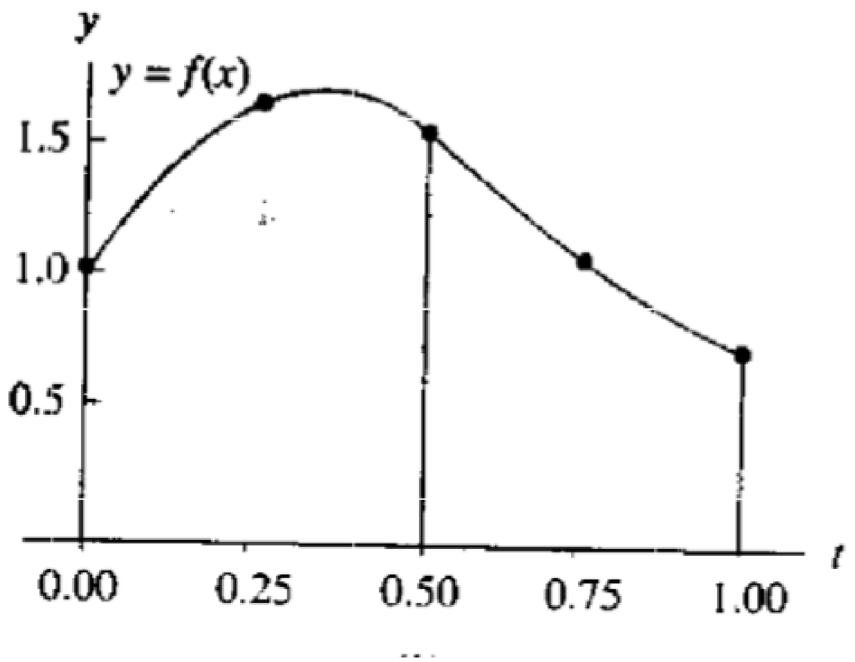
- $\int_{x_0}^{x_4} f(x)dx = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx + \int_{x_3}^{x_4} f(x)dx$
- $\int_{x_0}^{x_4} f(x)dx = \frac{h}{2}(f_0 + f_1) + \frac{h}{2}(f_1 + f_2) + \frac{h}{2}(f_2 + f_3) + \frac{h}{2}(f_3 + f_4)$
- $\int_{x_0}^{x_4} f(x)dx = \frac{h}{2}(f_0 + 2f_1 + 2f_2 + 2f_3 + f_4)$

Composite Trapezoidal Rule

$\int_a^b f(x)dx \rightarrow \quad a = x_0 < x_1 < \ \ldots < x_M = b$

- Apply Trapezoidal rule on M subintervals $[x_k, x_{k+1}]$ of width $h = \frac{b-a}{M}$ using equally spaced nodes. Where $x_k = a + kh$ for $k = 0, 1, 2, 3, \ldots, M$
- $\int_a^b f(x)dx = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx + \cdots + \int_{x_{M-1}}^{x_M} f(x)dx$
- $\int_{x_0}^{x_M} f(x)dx = \frac{h}{2}(f_0 + f_1) + \frac{h}{2}(f_1 + f_2) + \frac{h}{2}(f_2 + f_3) + \cdots + \frac{h}{2}(f_{M-1} + f_M)$
- $\int_{x_0}^{x_M} f(x)dx = \frac{h}{2}(f_0 + 2f_1 + 2f_2 + 2f_3 + \cdots + 2f_{M-2} + 2f_{M-1} + f_M)$
- $\int_a^b f(x)dx = \frac{h}{2}\big(f(a) + f(b)\big) + h\sum_{k=1}^{M-1} f_k$

Composite Simpson's Rule



[1]

- $\int_{x_0}^{x_4} f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx$
- $\int_{x_0}^{x_4} f(x)dx = \frac{h}{3}(f_0 + 4f_1 + f_2) + \frac{h}{3}(f_2 + 4f_3 + f_4)$
- $\int_{x_0}^{x_4} f(x)dx = \frac{h}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + f_4)$

Composite Simpson's Rule

$\int_a^b f(x)dx \;\rightarrow\quad a = x_0 < x_1 < \ \ldots < x_{2M} = b$

- Apply Simpson's rule on $2M$ subintervals $[x_k, x_{k+2}]$ of width $h = \frac{b-a}{2M}$ using equally spaced nodes. Where $x_k = a + kh$ for $k = 0, 1, 2, 3, \ldots, 2M$
- $\int_a^b f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \cdots + \int_{x_{2M-2}}^{x_{2M}} f(x)dx$
- $\int_{x_0}^{x_{2M}} f(x)dx = \frac{h}{3}(f_0 + 4f_1 + f_2) + \frac{h}{3}(f_2 + 4f_3 + f_4) + \cdots + \frac{h}{2}(f_{2M-2} + 4f_{2M-1} + f_{2M})$
- $\int_a^b f(x)dx = \frac{h}{3}\big(f(a) + f(b)\big) + \frac{2h}{3}\sum_{k=1}^{M-1} f_{2k} + \frac{4h}{3}\sum_{k=1}^{M} f_{2k-1}$

Composite Rules - Example

- Having $f(x) = 1 + e^{-x}\sin(4x)$ and the interval $[0,1]$
- Apply composite Trapezoidal and Simpson's rules with 5 sample points.

$M = \#of\ points\ - 1 \rightarrow h = \frac{b-a}{M} = \frac{1-0}{4} = \frac{1}{4}$

True value of interval $= 1.30825$

- Trapezoidal Rule

$$\int_0^1 f(x)dx = \frac{1/4}{2}\left(f(0) + 2f\left(\frac{1}{4}\right) + 2f\left(\frac{2}{4}\right) + 2f\left(\frac{3}{4}\right) + f(1)\right) = 1.28358$$

- Simpson's Rule

$$\int_0^1 f(x)dx = \frac{1/4}{3}\left(f(0) + 4f\left(\frac{1}{4}\right) + 2f\left(\frac{2}{4}\right) + 4f\left(\frac{3}{4}\right) + f(1)\right) = 1.30938$$

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall

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