CHAPTER SEVEN

Numerical Integration

Objectives

- Numerical Integration

Numerical Integration

- $\int_a^b f(x)dx = Q[f] + E[f]$
- Suppose that $a = x_0 < x_1 < \ ... < x_M = b$ → Sample points

- $Q[f] = \sum_{k=0}^{M} w_k f(x_k) = w_0 f(x_0) + w_1 f(x_1) + \cdots + w_M f(x_M)$
- $E[f] = kf^{(n+1)}(c)$, k is constant

- Where:
- $Q[f]$: Numerical Integration (Quadrature form)
- $E[f]$: Truncation Error for integration
- $\{x_k\}_{k=0}^{M}$: Quadrature Nodes
- $\{w_k\}_{k=0}^{M}$: Weights

The goal is to approximate the definite integral of $f(x)$ over the interval $[a, b]$ by evaluating $f(x)$ at a finite number of sample points

Newton-Cotes Quadrature Formulas

- When $Q[f]$ derived from polynomial approximates $f(x)$ over $[a, b]$, resulting formula called **Newton-Cotes quadrature formula**
- $P_M(x)$: polynomial of degree $\leq M$ pass through $M + 1$ equally spaced points $\{x_k , f(x_k)\}_{k=0}^{M}$
- $P_M(x)$: approximates $f(x)$ → integral of $P_M(x)$ approximates integral of $f(x)$

Newton-Cotes Quadrature Formulas

- For $M = 4$: $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$

$\int_{x_0}^{x_1} f(x)dx \approx \int_{x_0}^{x_1} P_1(x)dx = \frac{h}{2}(f_0 + f_1)$ → Trapezoidal Rule

$\int_{x_0}^{x_2} f(x)dx \approx \int_{x_0}^{x_2} P_2(x)dx = \frac{h}{3}(f_0 + 4f_1 + f_2)$ → Simpson's Rule

$\int_{x_0}^{x_3} f(x)dx \approx \int_{x_0}^{x_3} P_3(x)dx = \frac{3h}{8}(f_0 + 3f_1 + 3f_2 + f_3)$ → Simpson's $^3/_8$Rule

$\int_{x_0}^{x_4} f(x)dx \approx \int_{x_0}^{x_4} P_4(x)dx = \frac{2h}{45}(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4)$ → Boole's Rule

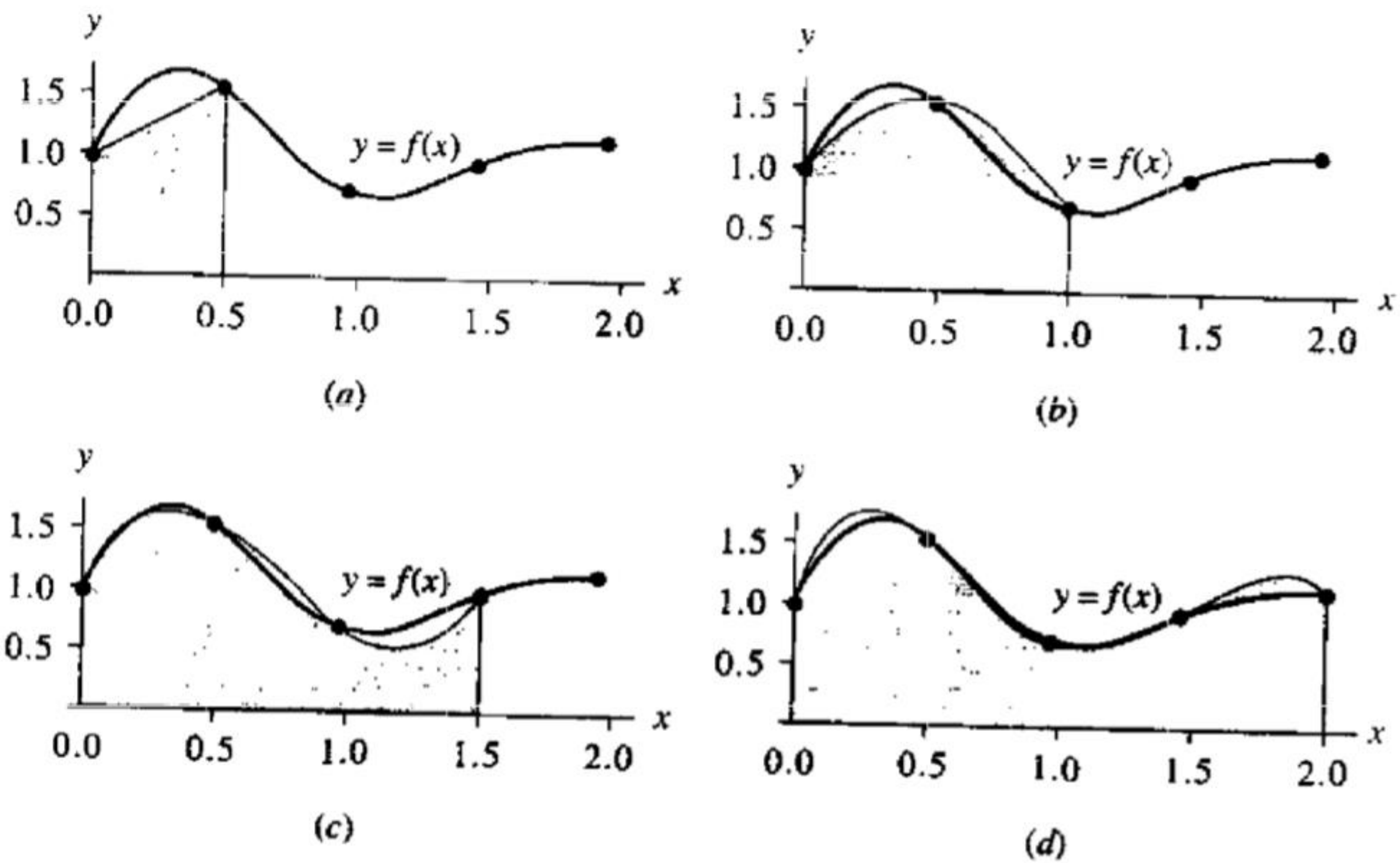

Figure 7.2 (a) The trapezoidal rule integrates $y = P_1(x)$ over $[x_0, x_1] = [0.0, 0.5]$. (b) Simpson's rule integrates $y = P_2(x)$ over $[x_0, x_1] = [0.0, 1.0]$. (c) Simpson's $\frac{3}{8}$ rule integrates $y = P_3(x)$ over $[x_0, x_3] = [0.0, 1.5]$. (d) Boole's rule integrates $y = P_4(x)$ over $[x_0, x_4] = [0.0, 2.0]$. [1]

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall

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