CHAPTER TWO

The Solution of Nonlinear Equations $f(x) = 0$

Objectives

- Secant Method
- Comparison of Speed of Convergence

Secant Method

- Here, there is no need to evaluate $f'(x)$
- Converges almost as fast as Newton's method → $R = 1.618$
  - Newton's → $R = 2$

- Needs two initial points $(p_0, f(p_0))$ and $(p_1, f(p_1))$ near the point $(p, 0)$, where p is the root of the function.

Secant Method

- $m = \frac{f(p_1) - f(p_0)}{p_1 - p_0}$
- Also, $m = \frac{0 - f(p_1)}{p_2 - p_1}$
- Then,
- $p_2 = g(p_1, p_0)$
- $\quad = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)}$

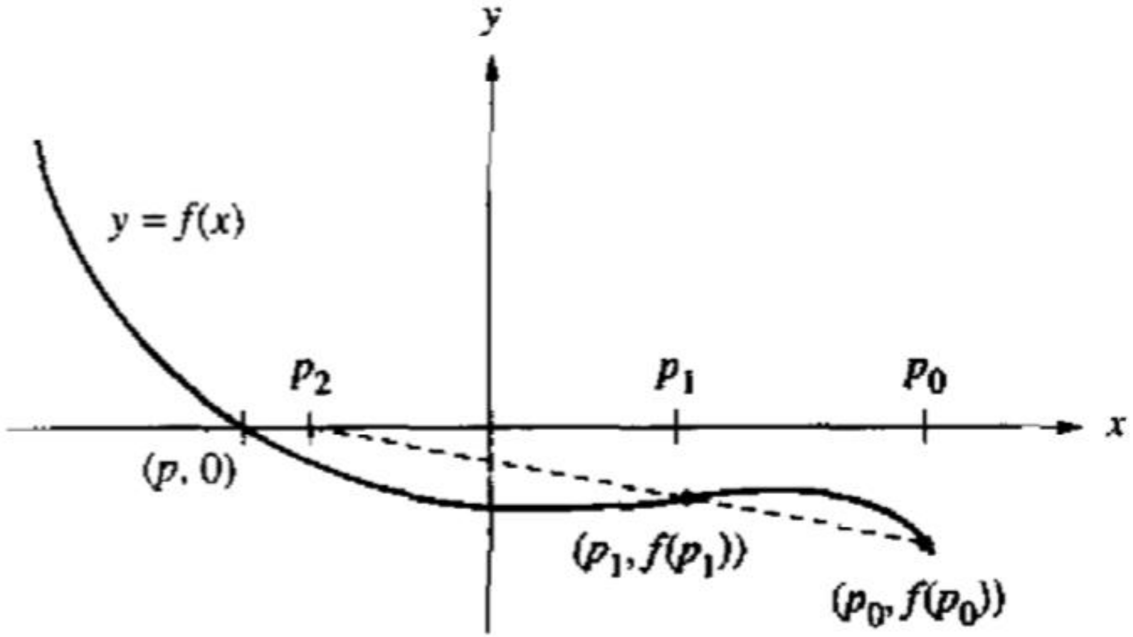

Figure 2.16 The geometric construction of p_2 for the secant method. [1]

Secant Method

- On general,

$$p_{k+1} = g(p_k, p_{k-1}) = p_k - \frac{f(p_k)(p_k - p_{k-1})}{f(p_k) - f(p_{k-1})}$$

- Convergence rate for simple root:

$$|e_{k+1}| \approx |e_k|^{1.618} \left|\frac{f''(p)}{2f'(p)}\right|^{0.618}$$

→ This relation only for simple root

Example – Simple Root

- Let $f(x) = x^3 - 3x + 2 = (x+2)(x-1)^2$.
- Start with $p_0 = -2.6, p_1 = -2.4$ to find the root $p = -2$ (Simple Root)

- $p_{k+1} = g(p_k, p_{k-1}) = p_k - \frac{(p_k^3 - 3p_k + 2)(p_k - p_{k-1})}{p_k^3 - p_{k-1}^3 - 3p_k + 3p_{k-1}}$
- $p_2 = g(p_1, p_0) = -2.106598985$
- $p_3 = g(p_2, p_1) = -2.022641412$

....

- $p_7 = g(p_6, p_5) = -2.000$

Newton's → 4 iterations

Example – Simple Root (cont'd)

Table 2.7 Convergence of the Secant Method at a Simple Root

k	p_k	$p_{k+1} - p_k$	$E_k = p - p_k$	$\frac{\lvert E_{k+1}\rvert}{\lvert E_k\rvert^{1.618}}$
0	−2.600000000	0.200000000	0.600000000	0.914152831
1	−2.400000000	0.293401015	0.400000000	0.469497765
2	−2.106598985	0.083957573	0.106598985	0.847290012
3	−2.022641412	0.021130314	0.022641412	0.693608922
4	−2.001511098	0.001488561	0.001511098	0.825841116
5	−2.000022537	0.000022515	0.000022537	0.727100987
6	−2.000000022	0.000000022	0.000000022	
7	−2.000000000	0.000000000	0.000000000	

[1]

Example – Simple Root (cont'd)

- Now let us check the relation $|e_{k+1}| \approx |e_k|^{1.618} \left|\frac{f''(p)}{2f'(p)}\right|^{0.618}$

- $e_{k+1} = e_5 = |p - p_5| = 0.000022537$
- $e_k = e_4 = |p - p_4|^{1.618} = 0.001511098^{1.618} = 0.000027296$

- $A = \left|\frac{f''(-2)}{f'(-2)}\right|^{0.618} = (^2/_3)^{0.618} = 0.778351205$

- $|p - p_5| = A|p - p_4|^{1.618} = 0.000021246 \approx |p - p_5|$

Comparison of Speed of Convergence

Method	**Special Consideration**	**Relation between successive error terms**
Bisection		$\|e_{k+1}\| \cong \frac{1}{2}\|e_k\|$
Secant	Multiple Root	$\|e_{k+1}\| \cong A\|e_k\|$
Newton's	Multiple Root	$\|e_{k+1}\| \cong A\|e_k\|$
Secant	Simple Root	$\|e_{k+1}\| \cong A\|e_k\|^{1.618}$
Newton's	Simple Root	$\|e_{k+1}\| \cong A\|e_k\|^2$

- Note that the value of A is different for each method

References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall

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