CHAPTER FIVE

Curve Fitting

Data Linearization - Example

- Use data linearization method to find the exponential fit $\boldsymbol{y = Ce^{Ax}}$ for the following five data points: $(0,1.5), (1,2.5), (2,3.5), (3,5.0), (4,7.5)$

- From table:
- $\sum x_k = 10.0$
- $\sum \ln(y_k) = 6.198860$
- $\sum x_k^2 = 30.0$
- $\sum x_k \ln(y_k) = 16.309743$

Table 5.4 **Obtaining Coefficients of the Normal Equations for the Transformed Data Points $\{(X_k, Y_k)\}$**

x_k	y_k	X_k	$Y_k = \ln(y_k)$	X_k^2	$X_k Y_k$
0.0	1.5	0.0	0.405465	0.0	0.000000
1.0	2.5	1.0	0.916291	1.0	0.916291
2.0	3.5	2.0	1.252763	4.0	2.505526
3.0	5.0	3.0	1.609438	9.0	4.828314
4.0	7.5	4.0	2.014903	16.0	8.059612
		10.0 $= \sum X_k$	6.198860 $= \sum Y_k$	30.0 $= \sum X_k^2$	16.309743 $= \sum X_k Y_k$

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Data Linearization – Example (cont'd)

- Using line fit equations:

$$\left(\sum_{k=1}^{N} x_k^2\right)A + \left(\sum_{K=1}^{N} x_k\right) B = \sum_{k=1}^{N} x_k \ln(y_k)$$
$$\left(\sum_{k=1}^{N} x_k\right)A + NB = \sum_{k=1}^{N} \ln(y_k)$$

$$C = e^B$$

- And using the values on previous slides we will have the following:

$$\left.\begin{aligned} 30A + 10B &= 16.309743 \\ 10A + 5B &= 6.198860 \end{aligned}\right\} \quad \begin{aligned} A &= 0.3912023 \\ B &= 0.457367 \end{aligned}$$

- Then, $C = e^{0.457367} = 1.579910$ ➔ $\boxed{y = 1.579910e^{0.3912023x}}$

Transformation for data linearization

$$Y = AX + B$$

Table 5.6 Change of Variable(s) for Data Linearization

Function, $y = f(x)$	Linearized form, $Y = Ax + B$	Change of variable(s) and constants
$y = \frac{A}{x} + B$	$y = A\frac{1}{x} + B$	$X = \frac{1}{x}$, $Y = y$
$y = \frac{D}{x + C}$	$y + \frac{-1}{C}(xy) + \frac{D}{C}$	$X = xy$, $Y = y$ $C = \frac{-1}{A}$, $D = \frac{-B}{A}$
$y = \frac{1}{Ax + B}$	$\frac{1}{y} = Ax + B$	$X = x$, $Y = \frac{1}{y}$
$y = \frac{x}{Ax + B}$	$\frac{1}{y} = A\frac{1}{x} + B$	$X = \frac{1}{x}$, $Y = \frac{1}{y}$
$y = A\ln(x) + B$	$y = A\ln(x) + B$	$X = \ln(x)$, $Y = y$
$y = Ce^{Ax}$	$\ln(y) = Ax + \ln(C)$	$X = x$, $Y = \ln(y)$ $C = e^B$
$y = Cx^A$	$\ln(y) = A\ln(x) + \ln(C)$	$X = \ln(x)$, $Y = \ln(y)$ $C = e^B$
$y = (Ax + B)^{-2}$	$y^{-1/2} = Ax + B$	$X = x$, $Y = y^{-1/2}$
$y = Cxe^{-Dx}$	$\ln\left(\frac{y}{x}\right) = -Dx + \ln(C)$	$X = x$, $Y = \ln\left(\frac{y}{x}\right)$ $C = e^B$, $D = -A$
$y = \frac{L}{1 + Ce^{Ax}}$	$\ln\left(\frac{L}{y} - 1\right) = Ax + \ln(C)$	$X = x$, $Y = \ln\left(\frac{L}{y} - 1\right)$ $C = e^B$ and L is a constant that must be given

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References

- [1] Mathews J. H. and Fink K. D. (1999). Numerical Methods using MATLAB, NJ: Prentice Hall