CHAPTER FIVE

Curve Fitting

Objectives

- Data Linearization

Data Linearization

- Non-Linear-Least-Square method for $y = Ce^{Ax}$

$$E(A,C) = \sum_{k=1}^{N}(Ce^{Ax_k} - y_k)^2$$

$$\frac{\partial E(A,C)}{\partial A} = 2\sum_{k=1}^{N}(Ce^{Ax_k} - y_k)^1 (Cx_k e^{Ax_k}) = 0$$

$$\boxed{\mathrm{C}\sum_{k=1}^{N} x_k e^{2Ax_k} - \sum_{k=1}^{N} x_k y_k e^{Ax_k} = 0 \quad \ldots\ldots (1)}$$

$$\frac{\partial E(A,C)}{\partial C} = 2\sum_{k=1}^{N}(Ce^{Ax_k} - y_k)^1 (e^{Ax_k}) = 0$$

$$\boxed{\mathrm{C}\sum_{k=1}^{N} e^{2Ax_k} - \sum_{k=1}^{N} y_k e^{Ax_k} = 0 \quad \ldots\ldots (2)}$$

Data Linearization

- $C\sum_{k=1}^{N} x_k e^{2Ax_k} - \sum_{k=1}^{N} x_k y_k e^{Ax_k} = 0$ …… (1)
- $C\sum_{k=1}^{N} e^{2Ax_k} - \sum_{k=1}^{N} y_k e^{Ax_k} = 0$ …… (2)

- Two equations with two unknown variables (Non-linear)
- Time-consuming computations
- Non-linear method to solve the system

- Solution: Use data linearization method

Data Linearization Method

- Non-linear curves: $y = Ce^{Ax}$ or $y = Cx^{A}$ → time-consuming computations.
- Solution is to use **Transformation for Linearization**.

$$y = Ce^{Ax}$$

$$\underbrace{\ln(y)}_{Y} = \underbrace{Ax}_{AX} + \underbrace{\ln(C)}_{B}$$

$$Y = AX + B \quad \Rightarrow (X_i, Y_i) \rightarrow (x_i, \ln(y_i))$$

- Now, use least-square line.
- Once we solve the linear-fit, we can transform back for our least square fit of $y = Ce^{Ax}$.

$$\left(\sum_{k=1}^{N} X_k^2\right)A + \left(\sum_{K=1}^{N} X_k\right)B = \sum_{k=1}^{N} X_k Y_k$$
$$\left(\sum_{k=1}^{N} X_k\right)A + NB = \sum_{k=1}^{N} Y_k$$

OR

$$\left(\sum_{k=1}^{N} x_k^2\right)A + \left(\sum_{K=1}^{N} x_k\right)B = \sum_{k=1}^{N} x_k \ln(y_k)$$
$$\left(\sum_{k=1}^{N} x_k\right)A + NB = \sum_{k=1}^{N} \ln(y_k)$$

$$C = e^{B}$$