Why this lecture is important?

17 May 2023 07:16

$\boxed{17}$ → $\boxed{\text{top5}}$

Bias Var tradeoff — know
↳ understand X

→ Interview → 8 mins

→ 1% students →

→ Super useful → to all the ML algorithms → Deep learning

→ Linear rg
↓
Stats

$\boxed{\text{2hr}}$ → understand

What are we going to study?

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Bias Var tradeoff

- → Bias
- → Variance
- → Overfitting
- → underfitting
- → tradeoff
- → Bias Var decomposition

The Hidden Truth

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1000 students

x_1 cgpa	x_2 iq	lpa	
8	80	8	100
9	90	9	
.	.	.	

1000

$\rightarrow$ model $\rightarrow$ predict $\rightarrow$ cgpa | iq $\rightarrow$ lpa

X y $\rightarrow$ $\boxed{Y = f(X)}$

x_1 x_2

$\boxed{\text{cgpa} \mid \text{iq}}$ $\rightarrow$ lpa

for all students

sample $\rightarrow$ pop predict

$Y = \boxed{f(X)} \rightarrow$ pop

$\hat{Y} = f'(X)$ $\rightarrow$ reducible error

$\boxed{f(X) - f'(X)}$

reduce

$f'(X) \rightarrow f(X)$

$\boxed{Y = f(X) + \boxed{\text{error}}}$

Irreducible

ml model

irreducible error

$Y - \hat{Y}$

Bias variance tradeoff

Reducible error

$\boxed{\text{Reducible error} = \text{Bias}^2 + \text{Var}}$

Bias variance decomposition $\rightarrow$ Regularization

Bias Variance Tradeoff

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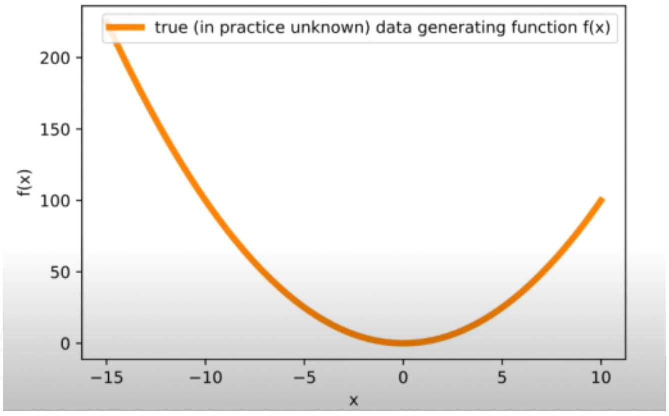
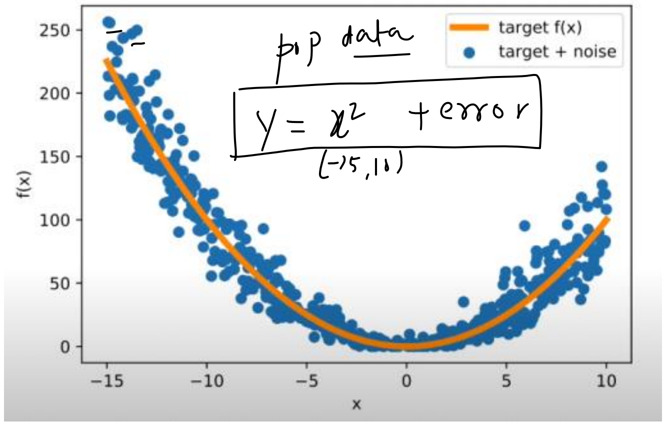
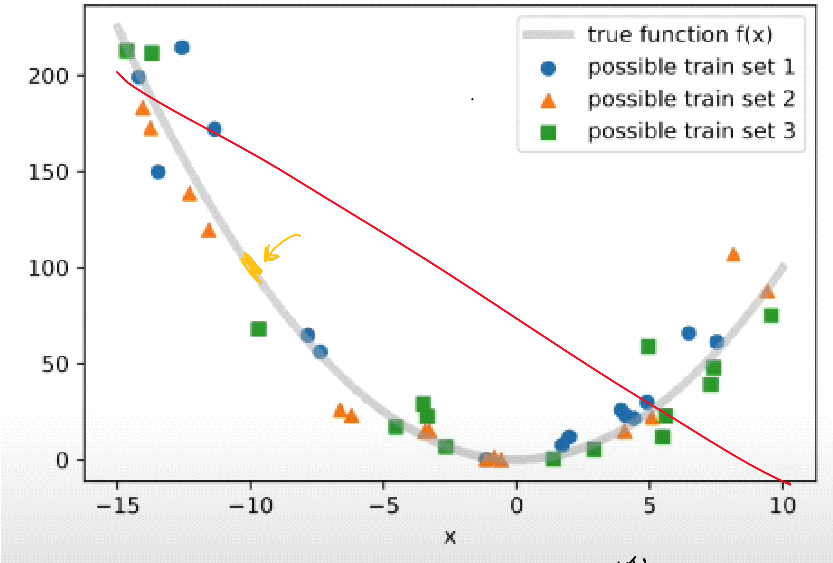
$y = f(x) = x^2 \; [-15, 10]$ $\boxed{y = x^2}$

$y = x^2 + error$ pop $\boxed{1000 \text{ points}}$



3 samples random





● → 1

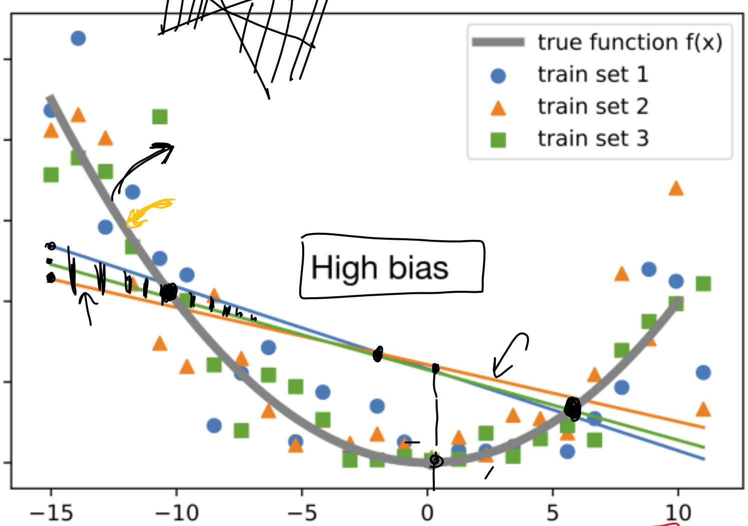
▲ → 2

■ → 3

→ 3 student

linear model

reg



high bias → $\boxed{\text{underfitting}}$

$\boxed{\text{Bias}}$ → the inability of a ML model to fit the training data

Variance

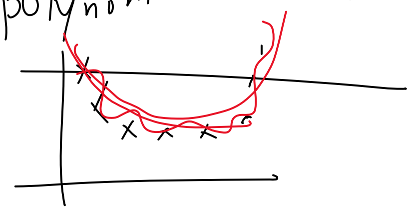
high bias ✓ $\boxed{\text{Low variance}}$

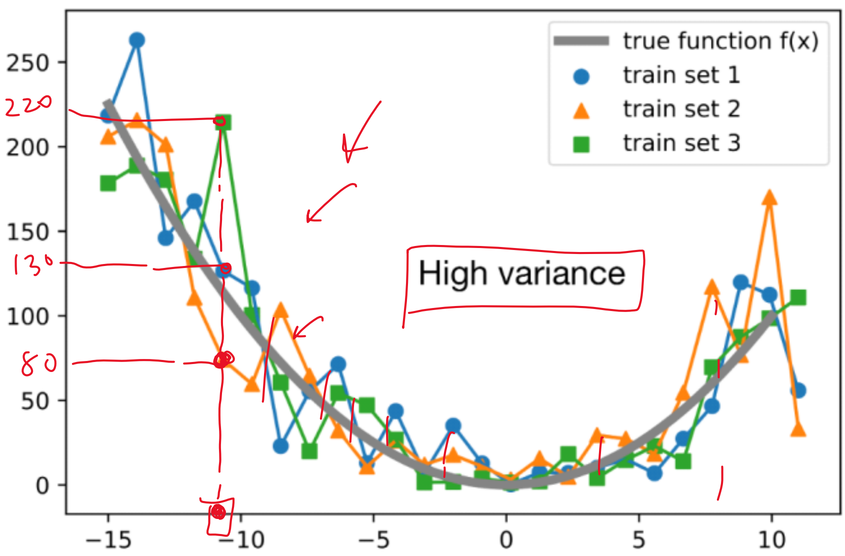
low bias

ml model predict when the training data is chan

polynomial

degree = 3





polynomial high degree

Overfitting

high bias

high var

low bias ✓

→ low bias high variance

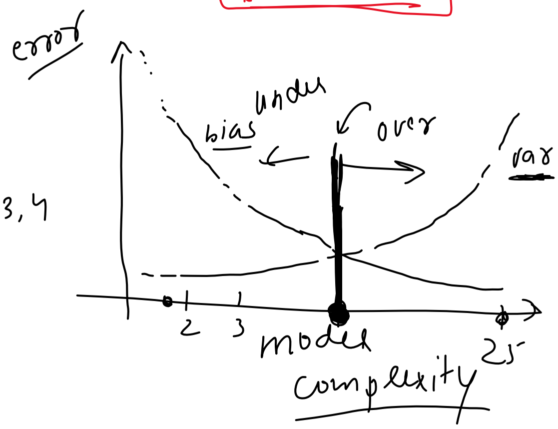
high var → overfitting

low bias low variance → bias ↓ var ↑ var ↓ bias ↑

The "trade-off" in bias-variance trade-off refers to the fact that minimizing bias will usually increase variance and vice versa.

{ bias variance }

poly → degree 2, 3, 4



Some questions

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1. How would you define bias and variance mathematically?
2. How is bias and variance related to overfitting and underfitting mathematically?
3. Why is there a tradeoff between bias and variance mathematically?

Expected Value and Variance

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pop mean $\boxed{\text{Prob}}$ ← expected value → pop mean

Expected value represents the average outcome of a random variable over a large number of trials or experiments. → pop mean

In a simple sense, the expected value of a random variable is the long-term average value of repetitions of the experiment it represents. For example, the expected value of rolling a six-sided die is 3.5 because, over many rolls, we would expect to average about 3.5.

$\boxed{\text{1 Lac roll}}$

1, 2, 3, 6, 4, 5

deterministic

$\boxed{\text{mean}}$ = ?

$\boxed{1/6}$ $\boxed{3.5}$

X = rolling a die 1Lac

$$E[X] = x_1 p(x_1) + x_2 p(x_2) + \dots + x_n p(x_n)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$X \to x_1 \to 1$, $x_2 \to 2$, $x_3 \to 3$, x_n, $x_6 \to 6$

$$= \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

overall avg mean

$\boxed{4.1}$

$\boxed{5\ 3\ 4\ 5\ 3\ 5}$ → mean

$$\frac{5+3+4+5+3+5}{6} = \frac{25}{6}$$

5

$$\frac{3(5) + 2(3) + 1(4)}{6}$$

$$= \frac{3}{6}(5) + \frac{2}{6}(3) + \frac{1}{6}(4)$$

mean → expected value

$$= x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3)$$

$E[X]$

discrete random var (X)

$$E[X] = \sum_{i=1}^{n} x_i p(x_i)$$

→ expected value → $\boxed{\text{pop mean}}$ $E[X]$

continuous random var

$$E[X] = \int x_i f(x_i)\, dx$$

$\boxed{Var(X)}$ → var of pop   pop var of X

$$\boxed{Var(X) = E[X^2] - (E[X])^2}$$

sample

sample

$$var = \sum \frac{(x_i - \bar{X})^2}{n} = E\left[(X - E[X])^2\right] = Var(x)$$

avg

$E[X]$ = number

constant

$\frac{1}{n}$

$E[X]$

$$Var(X) = E\left[(X - E[X])^2\right]$$

$$= E\left[X^2 + (E[X])^2 - 2XE[X]\right]$$

$$= E[X^2] + E\left[(E[X])^2\right] - E\left[2XE[X]\right]$$

$$= E[X^2] + E\left[(E[X])^2\right] - E[2]\,E[X]\,E[E[X]] \quad E[E[X]]$$

$$= E[X^2] + E\left[(E[X])^2\right] - 2E[X]E[X]$$

$$= E[X^2] + E\left[(E[X])^2\right] - 2(E[X])^2$$

$$= E[X^2] + (E[X])^2 - 2(E[X])^2$$

$$= E[X^2] - (E[X])^2$$

$E[X+Y] = E[X] + E[Y]$

$E[XY] = E[X]E[Y]$ given X and Y are independent

$E[\text{constant}] =$ constant

$$\boxed{Var(X) = E[X^2] - (E[X])^2}$$

$E[X]$

discrete

continuous

age

22

22

feature

$[X]$

$$Var(X) = E[X^2] - (E[X])^2$$

$$= E\left[(X - E[X])^2\right]$$

mean

$E[X]$

pop

$$\boxed{var = \sum \frac{(x_i - \bar{x})^2}{n}}$$

mean $\boxed{\frac{\sum}{n}} \rightarrow E$

mean

pop

bq

$$= E\left[(X - E[X])^2\right]$$

Bias ?

sample

$E[X]$

Bias ?
Var ?

→ mathemati' $\underline{E[x]}$

What exactly are Bias and Variance Mathematically?

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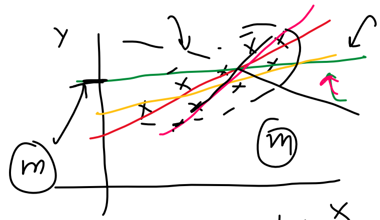
Bias

In the context of machine learning and statistics, bias refers to the systematic error that a model introduces because it cannot capture the true relationship in the data. It represents the difference between the expected prediction of our model and the correct value which we are trying to predict. More bias leads to underfitting, where the model does not fit the training data well.

Variance

In the context of machine learning and statistics, variance refers to the amount by which the prediction of our model will change if we used a different training data set. In other words, it measures how much the predictions for a given point vary between different realizations of the model.



100 samples

↓

LR → m

true function

$f(x)$ → $\boxed{f'(x)}$ $f'(x)$

$\boxed{E[f'(x)]}$ $E[m]$

$\boxed{\text{Bias} = E[f'(x)] - f(x)}$

$f(x)$ (+) 0

$f(x) - f(x) = 0$

$\boxed{\text{bias} = 0}$

unbiased predictor

$\boxed{E[m]} - m$

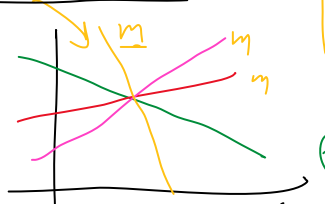
m

bias

$m - m = 0$

$\boxed{\text{bias} = 0}$

m



param

$b_0 \; b_1 \; b_2 \; \dots \; b_n$

bias(b)

bias(b_0)

bias(b_1)

$\rightarrow \text{Bias}(f'(x)) = E[f'(x)] - f(x)$

+ve -ve pop

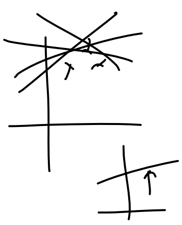
high bias ↓ underfitting

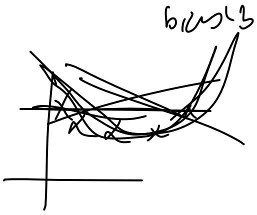
$f(x) \Rightarrow \boxed{y = x^2 + 5}$

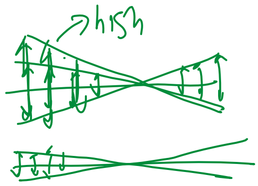
$f'(x)$

$f(x) = x^2 + 5$

$x + 5$







$$\boxed{\text{bias}(f'(x)) = E[f'(x)] - f(x)}$$

train → 90%

test → 80%

$$\boxed{\text{Var}(f'(x)) = E\left[\left(f'(x) - E[f'(x)]\right)^2\right]}$$

← high → data change change

overfitting

$E[f'(x)]$

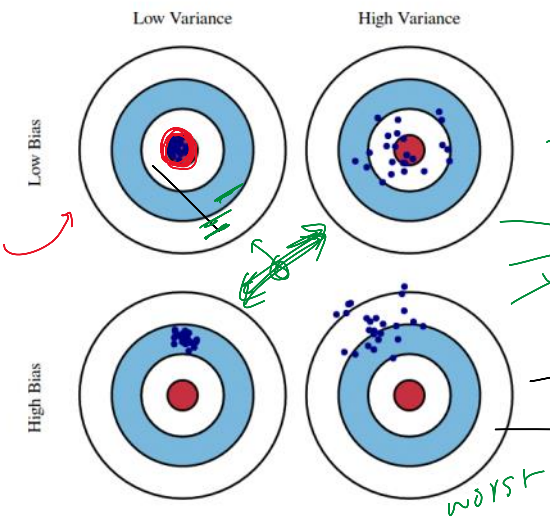
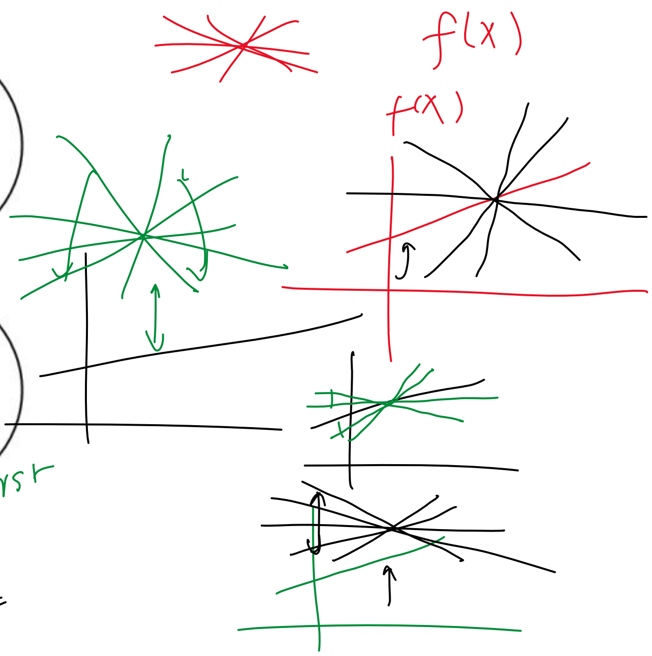


Fig. 1 Graphical illustration of bias and variance.



$\text{var} = \dfrac{\sum (x_i - \bar{x})^2}{n}$

$y = f(x) + \text{error}$ mse $\hat{\theta}$ θ

var =

$y = f(x) + \text{error}$ mse   n $\hat{\theta}$ θ

$\hat{y} = f'(x)$

$$\begin{cases} Bias(f'(x)) = E[f'(x)] - f(x) \\ Var(f'(x)) = E[(f'(x) - E[f'(x)])^2] \end{cases}$$

$\boxed{\text{Bias Variance Decomposition}}$ $E[(\hat{\theta} - E[\hat{\theta}])^2]$

$Bias = E[\hat{\theta}] - \theta$

Bias Variance Decomposition

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Bias-variance decomposition is a way of analysing a learning algorithm's expected generalization error with respect to a particular problem by expressing it as the sum of three very different quantities: bias, variance, and irreducible error.

1. Bias: This is the error from erroneous assumptions in the learning algorithm. High bias can cause an algorithm to miss the relevant relations between features and target outputs (underfitting).
2. Variance: This is the error from sensitivity to small fluctuations in the training set. High variance can cause an algorithm to model the random noise in the training data, rather than the intended outputs (overfitting).
3. Irreducible Error: This is the noise term. This part of the error is due to the inherent noise in the problem itself, and can't be reduced by any model.

Loss → mse

y → actual

$\hat{y}$ → predicted

$$\boxed{\frac{\sum (y_i - \hat{y}_i)^2}{n}}$$

Loss = bias + variance + irreducible

derive

$$Loss = \underbrace{[bias^2 + variance]}_{\text{reducible (reducible)}} + \underbrace{[var(\epsilon)]}_{\text{irreducible noise}}$$

cgpa | iq | lpa | pred

cgpa	iq	lpa	pred	
—	—	8	9	→ 1
—	—	8	8.1	— 0.1
—	—	7	6.9	
—	—	9	10.1	

reducible, irreducible

lr → irreducible

$$\sum_{i=1}^{n} (y - \hat{y})^2 \leftarrow \text{minimize mse}$$

$$y = \beta_0 + \beta_1 cgpa + \beta_2 iq$$

$\boxed{\sigma^2}$

θ

irreducible error

$\hookrightarrow$ mean $= 0$, var $= \sigma^2$

Derivation

$$mse = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n} = E[(y - \hat{y})^2]$$

$(a+b)^2$

$$(\underbrace{\theta - \hat{\theta}}_{a} + \underbrace{\epsilon}_{b})^2$$

$$y = f(x) + \epsilon = \theta + \epsilon$$

$$E[(\underbrace{\theta + \epsilon - \hat{\theta}}_{a})^2]$$

$$\hat{y} = f'(x) = \hat{\theta}$$

$\theta - \hat{\theta} + \epsilon$

ϵ eplison irreducible

$$= E[\underbrace{(\theta - \hat{\theta})^2}_{a^2} + \underbrace{\epsilon^2}_{b^2} + 2\epsilon(\theta - \hat{\theta})]$$

$E[x+y] = E[x] + E[y]$

$$= E[(\theta - \hat{\theta})^2] + E[\epsilon^2] + E[2\epsilon(\theta - \hat{\theta})]$$

$$= E[(\theta - \hat{\theta})^2] + E[\epsilon^2] + \underbrace{E[2]\,E[\epsilon]\,E[\theta - \hat{\theta}]}$$

$\rightarrow E[\epsilon] = 0$

$E[xy]$

$E[x]E[y]$

$$E[(\theta - \hat{\theta})^2] + E[\epsilon^2] \leftarrow$$

$E[XY]$

$E[X]E[Y]$

$$\boxed{mse} = \underbrace{E[(\theta-\hat{\theta})^2]} + \underbrace{E[\epsilon^2]}$$

$$\underbrace{E[\epsilon^2]} \rightarrow \quad var(\epsilon) = \sigma^2 = E[(\epsilon - E[\epsilon])^2]$$

$E[\epsilon^2] = var(\epsilon) \quad = E[(\epsilon - 0)^2] = E[\epsilon^2]$

$$mse = \underbrace{E[(\theta-\hat{\theta})^2]} + \underbrace{\boxed{var(\epsilon)}}_{\text{irreducible}}$$

$$E[(\theta-\hat{\theta})^2] = E[(\overbrace{\theta - E[\hat{\theta}]}^{a} + \overbrace{E[\hat{\theta}] - \hat{\theta}}^{b})^2]$$

$$E\left[(\theta - E[\hat{\theta}])^2 + \underbrace{(E[\hat{\theta}] - \hat{\theta})^2}_{b^2} + 2(\theta - E[\hat{\theta}])(E[\hat{\theta}] - \hat{\theta})\right]$$

$E[XY] = E[X]E[Y]$

$$\underbrace{E[(\theta - E[\hat{\theta}])^2]} + \underbrace{E[(E[\hat{\theta}] - \hat{\theta})^2]} + \boxed{E[2(\theta - E[\hat{\theta}])(E[\hat{\theta}] - \hat{\theta})]} = 0$$

$E[2]\, E[(\theta - E[\hat{\theta}])]\, E[(E[\hat{\theta}] - \hat{\theta})]$

$2(\theta - E[\hat{\theta}])\{E[E[\hat{\theta}]] - E[\hat{\theta}]\}$

0

$E[\hat{\theta}] - E[\hat{\theta}]$

$\theta - E[\hat{\theta}]$

$$\underbrace{E[(\theta - E[\hat{\theta}])^2]} + \underbrace{E[(E[\hat{\theta}] - \hat{\theta})^2]}_{var}$$

$$\underbrace{(\theta - E[\hat{\theta}])^2}$$

$$\rightarrow (\theta - E[\hat{\theta}])^2$$

$$(Bias)^2 + var$$

$y = \theta + \text{error}$

$$mse = \underbrace{(bias)^2 + \boxed{variance}}_{\text{reducible error}} + \underset{var(\epsilon)}{\overset{\text{noise}}{\text{irreducible}}}$$

Bias variance decomposition

$mse = \text{reducible} + \text{irreducible}$

$\rightarrow bias + var$

cgpa	iq	lpa	0.1
→ 8	80	8	8.1 → 0.05
→ 7	70	7	6.9 0.1

0.05

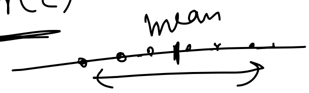
0.04

$var(\epsilon)$

→ bias + var

→ 7 70 7 — 6.1 0.04

Var(ε)



Diagram

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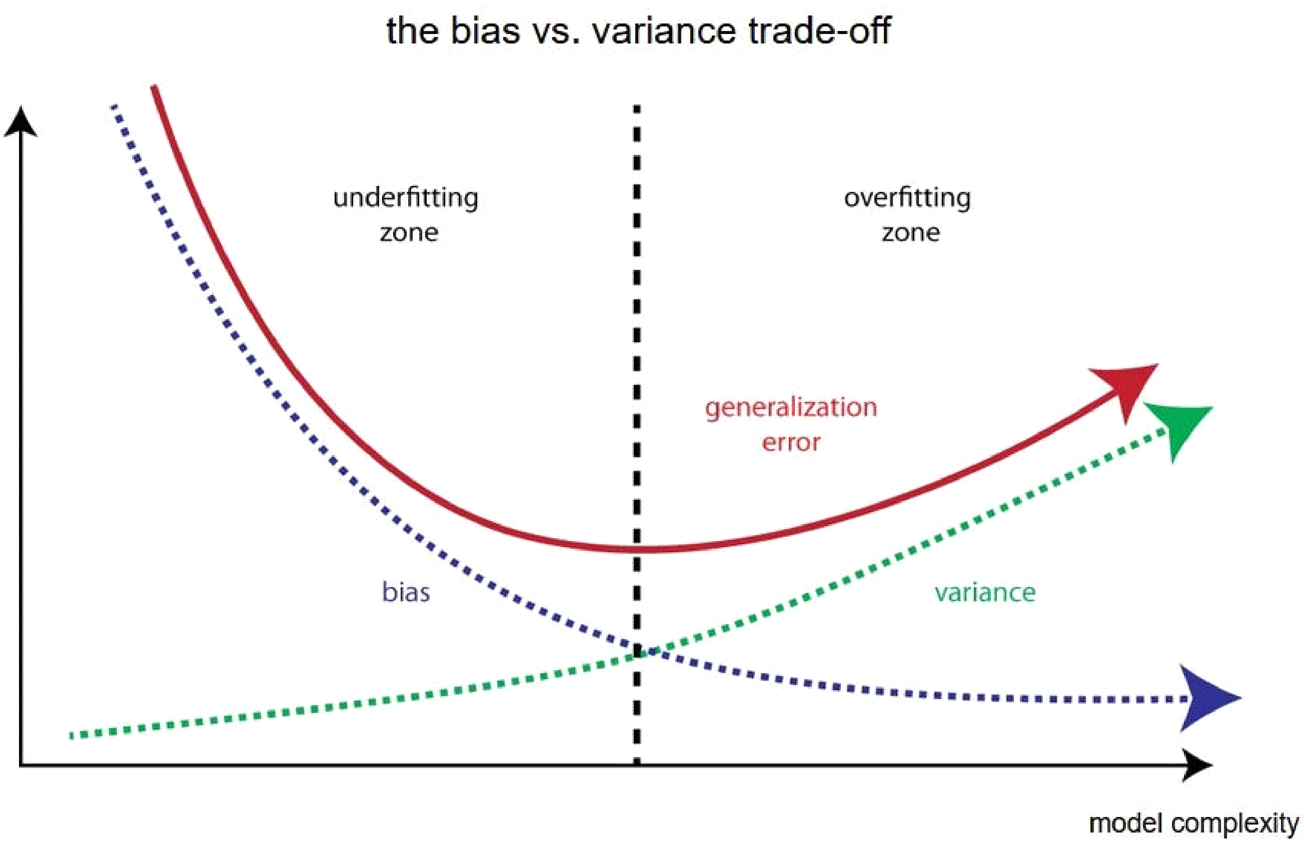

Analogy

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irreducible

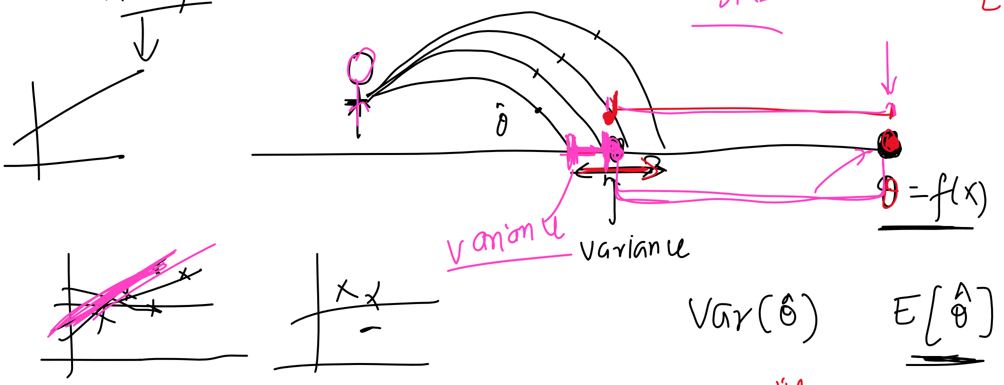
$$\underline{mse} = (\underline{bias})^2 + \underline{variance} + \boxed{\underbrace{\text{variance}(\epsilon)}} \neq 0 \qquad y = f(x)$$

multiple shots

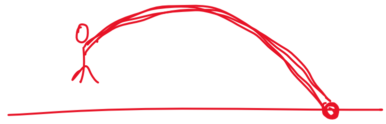
bias

$E[\hat{\theta}] - \theta = \text{bias}$

var on avg mean



$\hat{\theta} = f(x)$

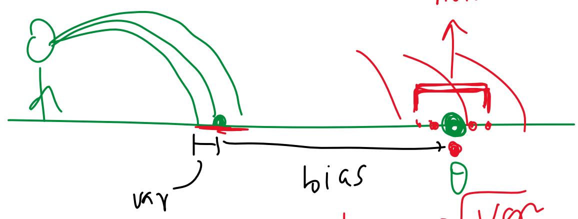


$\text{Var}(\hat{\theta})$ $\quad E[\hat{\theta}]$

noise

$\text{var}(\epsilon) = 0$

$\boxed{\text{loss} = 0}$ ←

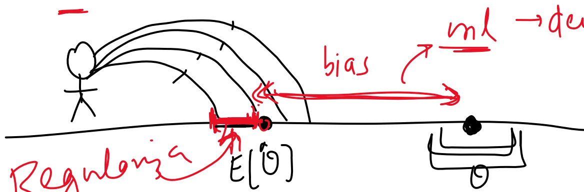


bias + 2√var

std

→ $(y - \hat{y})^2 = (\text{bias})^2 + \text{var}$

ml → decision $(y - \hat{y})$

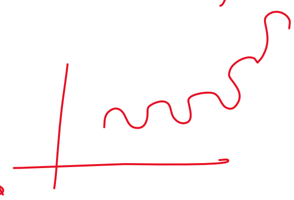


Regularization

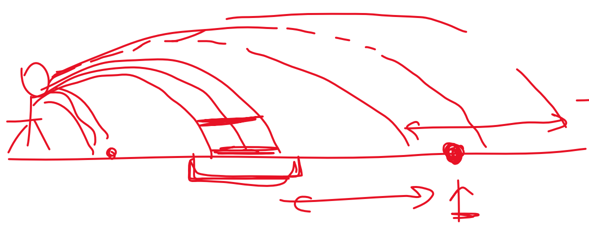
Bias variance

bias

polynomial degree



loss → bias, var, target



→ $\boxed{\text{Relugi}}$

↓

reduce variance

↳ reduce overfitting

Code Example

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bias vari dec

bias vari

$\text{loss} = \underbrace{(\text{bias})^2}_{\uparrow} + \underbrace{\text{var}}_{\uparrow} + \underbrace{\text{some error}}_{\uparrow}$

$(\text{bias})^2 + (\text{var}) \leftarrow$ technique

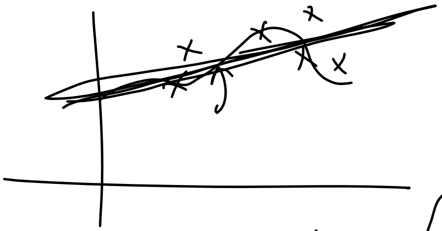
↓

oversiting

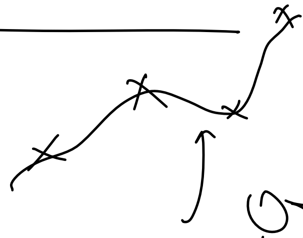
↓

Regularization

complex models



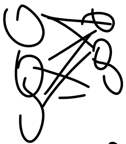
decision



linear → poly

decision →

neur



100 day of ML

Ridge

Lasso

Elastic

Intro to reg

2 session intro to reg

7 videos → Monday

↓

3 hrs → Doubt clear

Smoth

SMOTE → imbal.