Propositional Logic Homework

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9.1. Write formulas that are logically equivalent to:

9.1a: $p \wedge q$

$$\neg(\neg p \vee \neg q)$$

9.1b: $p \oplus q$

$$\neg(\neg(p \vee q) \vee \neg(\neg p \vee \neg q))$$

9.1c: $p \Leftrightarrow q$

$$\neg(p \vee q) \vee \neg(\neg p \vee \neg q)$$

9.3. Prove the associative laws by comparing truth tables for the two expressions asserted to be equivalent:

9.3a: $(\alpha \vee \beta) \vee \gamma \equiv \alpha \vee (\beta \vee \gamma)$

α	β	γ	$\alpha \vee \beta$	$\beta \vee \gamma$	$(\alpha \vee \beta) \vee \gamma$	$\alpha \vee (\beta \vee \gamma)$
F	F	F	F	F	F	F
F	F	T	F	T	T	T
F	T	F	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	T	T
T	F	T	T	T	T	T
T	T	F	T	T	T	T
T	T	T	T	T	T	T

9.3b: $(\alpha \wedge \beta) \wedge \gamma \equiv \alpha \wedge (\beta \wedge \gamma)$

α	β	γ	$\alpha \wedge \beta$	$\beta \wedge \gamma$	$(\alpha \wedge \beta) \wedge \gamma$	$\alpha \wedge (\beta \wedge \gamma)$
F	F	F	F	F	F	F
F	F	T	F	F	F	F
F	T	F	F	F	F	F
F	T	T	F	T	F	F
T	F	F	F	F	F	F
T	F	T	F	F	F	F
T	T	F	T	F	F	F
T	T	T	T	T	T	T

9.5. Using a truth table, determine whether each of the following compound propositions is satisfiable, a tautology, or unsatisfiable:

9.5a: $p \Rightarrow (p \vee q)$

This proposition is a tautology.

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$
F	F	F	T
F	T	T	T
T	F	T	T
T	T	T	T

9.5b: $\neg(p \Rightarrow (p \vee q))$

Being the negation of a tautology, this proposition is never true—it is unsatisfiable.

p	q	$p \Rightarrow (p \vee q)$	$\neg(p \Rightarrow (p \vee q))$
F	F	T	F
F	T	T	F
T	F	T	F
T	T	T	F

9.5c: $p \Rightarrow (p \Rightarrow q)$

This proposition is satisfiable.

p	q	$p \Rightarrow q$	$p \Rightarrow (p \Rightarrow q)$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

9.7. Prove that $\alpha \equiv \beta$ if and only if $\alpha \Leftrightarrow \beta$ is a tautology.

α	β	$\alpha \Leftrightarrow \beta$	$\alpha \equiv \beta$
F	F	T	T
F	T	F	F
T	F	F	F
T	T	T	T

Since these propositions are the same, then the tautology of $\alpha \Leftrightarrow \beta$ asserts the tautology of $\alpha \equiv \beta$.

9.9. Give real world interpretations of p, q, and r such that $(p \wedge q) \Rightarrow r$ and $p \wedge (q \Rightarrow r)$ mean quite different things, and one is true while the other is false.

p	q	r	$p \wedge q$	$q \Rightarrow r$	$(p \wedge q) \Rightarrow r$	$p \wedge (q \Rightarrow r)$
F	F	F	T	F	T	F
F	F	T	T	F	T	F
F	T	F	T	T	T	F
F	T	T	T	F	T	F
T	F	F	T	F	T	T
T	F	T	T	F	T	T
T	T	F	F	T	F	F
T	T	T	F	F	T	T

- p—cold or not cold
- q—schoolday or not schoolday
- r—brown coat or no brown coat
- $(\mathtt{F} \wedge \mathtt{F}) \Rightarrow \mathtt{T} \equiv \mathtt{T}$—I'm wearing my brown coat, even if it's not a cold schoolday.
- $\mathtt{F} \wedge (\mathtt{F} \Rightarrow \mathtt{T}) \equiv \mathtt{F}$—Although I could wear my brown coat outside of school, it would never happen on a day that wasn't cold.

9.11.:

9.11a: Write $p \Leftrightarrow q$ using $\oplus$ and the constant T.

$$(p \oplus q) \oplus \mathtt{T}$$

p	q	$p \oplus q$	$(p \oplus q) \oplus \mathtt{T}$	$p \Leftrightarrow q$
F	F	F	T	T
F	T	T	F	F
T	F	T	F	F
T	T	F	T	T

9.11b: Show that $\oplus$ and $\Leftrightarrow$ are associative.

$$(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$$

p	q	r	$p \oplus q$	$q \oplus r$	$(p \oplus q) \oplus r$	$p \oplus (q \oplus r)$
F	F	F	F	F	F	F
F	F	T	F	T	T	T
F	T	F	T	T	T	T
F	T	T	T	F	F	F
T	F	F	T	F	T	T
T	F	T	T	T	F	F
T	T	F	F	T	F	F
T	T	T	F	F	T	T

$$(p \Leftrightarrow q) \Leftrightarrow r \equiv p \Leftrightarrow (q \Leftrightarrow r)$$

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	$(p \Leftrightarrow q) \Leftrightarrow r$	$p \Leftrightarrow (q \Leftrightarrow r)$
F	F	F	T	T	F	F
F	F	T	T	F	T	T
F	T	F	F	F	T	T
F	T	T	F	T	F	F
T	F	F	F	T	T	T
T	F	T	F	F	F	F
T	T	F	T	F	F	F
T	T	T	T	T	T	T

9.11c: Prove that $\{\oplus, \Leftrightarrow, \neg, \mathtt{T}, \mathtt{F}\}$ is not a complete set.

$\oplus$ and $\Leftrightarrow$ return 2 true and 2 false values. As a result, they cannot be altered into $\wedge$, which, alongside $\neg$, can represent any function, because $\wedge$ has 3 false values and 1 true value.