CG2023 Cheat Sheet

Complex Numbers

$$z + z* = 2Re\{z\}$$
$$z - z* = 2Im\{z\}$$
$$sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$
$$cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

Energy of a signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Power of a signal

$$P = \lim_{\tau \to \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} |x(t)|^2 \, dt$$

Convolution

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) \, d\tau$$

Graphically, this is the sliding sum of the signal x multiplied with the reflected signal h.

Fourier Series Analysis

$$c_k = \frac{1}{T_p} \int_{T_p} x_p(t) e^{-j2\pi kt/T_p} \, dt, \forall k \in \mathbb{Z}$$

Let $a_k = \frac{c_{-k}+c_k}{2}$ and $b_k = \frac{c_{-k}-c_k}{j2}$. Then,

$$x_p(t) = a_0 + 2\sum_{k=1}^{\infty} [a_k cos(2\pi kt/T_p) + b_k sin(2\pi kt/T_p)]$$

Alternatively,

$$a_k = \frac{1}{T_p} \int_{T_p} x_p(t) cos(2\pi kt/T_p) \, dt; k \geq 0$$
$$b_k = \frac{1}{T_p} \int_{T_p} x_p(t) sin(2\pi kt/T_p) \, dt; k > 0$$

Fourier Series Synthesis

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kt/T_p}$$

Forward Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt$$

$$\text{Note that } c_k = \frac{1}{T_p} X(\frac{k}{T_p})$$

The fourier series coefficients can thus be obtained by performing Fourier transform on one period of the periodic signal and sampling the resulting transform with scaling.

$$\Im(Ae^{-\alpha t}u(t)) = \frac{A}{\alpha + j2\pi f}, \alpha > 0$$

Inverse Fourier Transform

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} \, dt$$

Dirichlet Conditions for Existence of Fourier Transforms

1. Signal has finite number of maxima and minima in any finite interval
2. Signal has finite number of discontinuities in any finite interval
3. $\int_{-\infty}^{\infty} |x(t)| \, dt < \infty$

Properties of Fourier Transforms

1. **Linearity**
   $\alpha x_1(t) + \beta x_2(t) \longleftrightarrow \alpha X_1(f) + \beta X_2(f)$
2. **Time Scaling**
   $x(\beta t) \longleftrightarrow \frac{1}{|\beta|} X(\frac{f}{\beta})$
3. **Duality**
   $X(t) \longleftrightarrow x(-f)$
4. **Time Shifting**
   $x(t - t_0) \longleftrightarrow X(f) e^{-j2\pi f t_0}$
5. **Frequency Shifting/Modulation**
   $x(t) e^{j2\pi f_0 t} \longleftrightarrow X(f - f_0)$
6. **Differentiation in t-domain**
   $\frac{d}{dt} x(t) \longleftrightarrow j2\pi f X(f)$
7. **Integration in t-domain**
   $\int_{-\infty}^{t} x(\tau) \, d\tau \longleftrightarrow \frac{1}{j2\pi f} X(f) + \frac{1}{2} X(0) \delta(f)$
8. **Convolution in t-domain**
   $x_1(t) * x_2(t) \longleftrightarrow X_1(f) X_2(f)$
9. **Multiplication in t-domain**
   $x_1(t) x_2(t) \longleftrightarrow X_1(f) * X_2(f)$

Spectral Properties of Real Signals

If $x(t)$ **is real**

$$X^*(f) = X(-f) \qquad c_k^* = c_{-k}$$
$$|X(f)| = |X(-f)| \qquad |c_k| = |c_{-k}|$$
$$\angle X(-f) = -\angle X(f) \qquad \angle c_k = -\angle c_{-k}$$

If $x(t)$ **is real and even**

$$X^*(f) = X(f) \qquad c_k^* = c_k$$
$$X(f) = X(-f) \qquad c_k = c_{-k}$$

If $x(t)$ **is odd and even**

$$X^*(f) = -X(f) \qquad c_k^* = -c_k$$
$$X(-f) = -X(f) \qquad c_k = -c_{-k}$$

Properties of Dirac Delta

1. **Symmetry**
   $\delta(t) = \delta(-t)$
2. **Sampling**
   $x(t)\delta(t - \lambda) = x(\lambda)\delta(t - \lambda)$
3. **Sifting**
   $\int_{-\infty}^{\infty} x(t)\delta(t - \lambda) \, dt = x(\lambda) \int_{-\infty}^{\infty} \delta(t - \lambda) \, dt = x(\lambda)$
4. **Replication**

$$\begin{aligned} x(t) * \delta(t - t_0) &= \int_{-\infty}^{\infty} x(\tau)\delta(t - t_0 - \tau) \, d\tau \\ &= \int_{-\infty}^{\infty} x(\tau)\delta(\tau - (t - t_0)) \, d\tau \\ &= x(t - t_0) \end{aligned}$$

5. **White Spectrum**

$$\begin{aligned} \Delta(f) &= \Im\{\delta(t)\} \\ &= \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} \, dt \\ &= 1 \end{aligned}$$

Fourier Transform of Periodic Signal

$$X_p(f) = \sum_{k=-\infty}^{\infty} c_k \delta(f - \frac{k}{T_p})$$

The c_k are first obtained by computing fourier series coefficients.

Rayleigh Energy Theorem

$$\begin{aligned} E &= \int_{-\infty}^{\infty} |x(t)|^2 \, dt \\ &= \int_{-\infty}^{\infty} |X(f)|^2 \, df \end{aligned}$$

Energy Spectral Density

$$E_x(f) = |X(f)|^2 \text{ Joules/Hz}$$

1. $E_x(f)$ is a real function of f.
2. $E_x(f) \geq 0 \forall f$.
3. $E_x(f)$ is even if $x(t)$ is real.

Parseval Power Theorem

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt = \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{1}{2T} |X_T(f)|^2\, df$$
$$= \sum_{k=-\infty}^{\infty} |c_k|^2$$

where $X_T(t) = x(t) rect(\frac{t}{2T})$.

Power Spectral Density

$$P_x(f) = \lim_{T\to\infty} \frac{1}{2T} |X_T(f)|^2 \text{ Watt/Hz}$$
$$= \sum_{k=-\infty}^{\infty} |c_k|^2 \delta \left(f - \frac{k}{T_p} \right)$$

1. $P_x(f)$ is a real function of f.
2. $P_x(f) \geq 0 \forall f$.
3. $P_x(f)$ is even if $x(t)$ is real.

Lowpass Signal

$$|X(f)| = 0 \text{ when } |f| > B$$

Bandpass signal

$$|X(f)| = 0 \text{ when } ||f| - f_c| > \frac{B}{2}$$

3dB Bandwidth

$$\frac{|X(f)|}{|X(0)|} = \frac{1}{\sqrt{2}}$$
$$\frac{E_x(f)}{E_x(0)} = \frac{1}{2}$$

1st-Null Bandwidth

$$|X(f)| = 0 \text{ first occurs }, f > 0$$

η% Energy Containment Bandwidth

Smallest bandwidth that contains at least η% of the total signal energy.

$$E = \int_{-\infty}^{\infty} E_x(f)\, df$$

η% Power Containment Bandwidth

Smallest bandwidth that contains at least η% of the average signal power.

$$P = \int_{-\infty}^{\infty} P_x(f)\, df$$

Minhaj's Sabotage

$$\text{Let } Q(f) = \Im(q(t))$$
$$\Im\left(q\left(\frac{t-\alpha}{\beta} \right) \right) = |\beta| Q(f\beta) e^{-j2\pi\alpha f}$$

Tippinyu's Tips

$$\text{Your Grade} = \Im(sinc(t))$$

Compiled by **Mohideen Imran Khan**.
Template from http://wch.github.io/latexsheet/