CG1111 Cheat Sheet

Battery and Power

Instantaneous Power

$$p = \frac{dW(t)}{dt} \tag{1}$$

Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \tag{2}$$

$$= \frac{P_{out}}{P_{out} + P_{loss}} \tag{3}$$

Energy Balance Equation

$$\sum E_{in} = \sum E_{out} + \sum E_{stored} + \sum E_{lost} \tag{4}$$

Power

Power is the rate of energy flow.

$$P = \frac{dE}{dt} \tag{5}$$

Power Balance Equation

Differentiating (4) with respect to time,

$$P_{in} = P_{out} + \frac{dE_{stored}}{dt} + P_{loss} \tag{6}$$

Battery Types

1. Primary — non-rechargeable
2. Secondary — rechargeable

Battery Parameters

1. **Open circuit voltage** — voltage across terminals when nothing is connected
2. **Battery capacity** — product of current drawn from battery and time

Mid-point Voltage

It is the approximate operational voltage of the battery and is taken to be the voltage when the battery is 50% charged.

Cycling

This is when the load needs less current for some time and a larger current for another time period. Let I_1 be the lower current and I_2 be the higher current.

If the time period of higher current is less than 65%,

$$I_{discharge} = I_{average} \tag{7}$$

$$= I_1 + (I_2 - I_1)\frac{t_{I_2}}{t_{total}} \tag{8}$$

Else,

$$I_{discharge} = I_2 \tag{9}$$

C-Rate

$$\text{C-rate} = \frac{\text{Discharge Current}}{\text{Battery Capacity}} \tag{10}$$

Depth of Discharge & End of Discharge

If we discharge the battery to 80% of its total capacity, the depth of discharge (DoD) is said to be 80%.

End of discharge voltage V_{eod} is the voltage of the battery at the charge beyond which the operating voltage would drop steeply.

Series and Parallel

Two identical batteries in series would have double the voltage but the same capacity.

Two identical batteries in parallel would have the same voltage but double the capacity.

Battery Design Principles

1. Find mid-point voltage. If C-rate is not given, take it to be 1C.
2. Determine number of batteries to connect in series, n_s.
3. Calculate the power requirement of the system:
   (a) **Parallel** — $P_{in} = \frac{P_{out1}}{\eta_1} + \frac{P_{out2}}{\eta_2} + \cdots$
   (b) **Series** — $P_{in} = \frac{P_{out}}{\eta_1 \eta_2 \cdots}$
4. Find load capacity.
   (a) Calculate battery bank operating voltage V_{BB}.
   $$V_{BB} = n_s \times \text{mid-point voltage} \tag{11}$$
   (b) Calculate load energy.
   $$\text{Load Energy} = P_{in} \times \text{time to run} \tag{12}$$
   (c) Find load capacity.
   $$\text{Load Energy} = V_{BB} \times \text{Load Capacity} \tag{13}$$
5. Estimate battery capacity at DoD.
6. Determine number of parallel branches.
   $$n_p = \frac{\text{Load Capacity}}{\text{Estimated Battery Capacity}} \tag{14}$$

Circuit Basics

Electric Charge

It is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.

$$q_e = -1.602 \times 10^{-19} C$$

Voltage

It is the measure of energy transferred per unit charge when the charge is moved from one point to another (J/C or V).

Active & Passive Elements

If a positive charge enters the negative polarity and exits the positive polarity of an element, (i) the charge gains energy from the element and (ii) the element is a "source" or is said to be active.

If a positive charge enters the positive polarity and exits the negative polarity, the (i) charge loses energy in the element and (ii) the element is said to be a load.

Current

It is the rate of flow electric charges through an element.

$$I = \frac{Q}{t} \tag{15}$$

Resistors & Ohm's Law

It is an empirical relationship that states that the voltage across an ideal resistor is proportional to the current through it. The constant of proportionality is the resistance. It does not hold at very high or very low voltage and current values.

$$V = IR \tag{16}$$

$$R = \rho\frac{L}{A} \tag{17}$$

In series,

$$R_{eq} = R_1 + R_2 + \cdots \tag{18}$$

In parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \tag{19}$$

Kirchoff's Current Laws

At any node in the circuit, $Q_{in} = Q_{out}$ by the conservation of charges. Differentiating with respect to time,

$$I_{in} = I_{out} \text{ at any node} \tag{20}$$

Kirchoff's Voltage Laws

By conservation of power, $P_{supplied} + P_{consumed} = 0$ around any closed loop. Therefore,

$$\sum \text{voltage across elements} = 0 \tag{21}$$

Current Division Principle

If resistors R_1 and R_2 are in parallel,

$$I_1 = \frac{R_2}{R_1 + R_2} \times I \tag{22}$$

Capacitors

$$C = \frac{Q}{V} \tag{23}$$

Capacitance of Plate Capacitor

$$C = \frac{\varepsilon A}{d} \text{where } \varepsilon = \varepsilon_r \varepsilon_0 \tag{24}$$

$$\begin{aligned}
\varepsilon &: \text{Permittivity of dielectric} \\
\varepsilon_r &: \text{Relative permittivity} \\
\varepsilon_0 &: \text{Permittivity of Free Space} \\
A &: \text{Overlap area of conductor} \\
d &: \text{distance between plates}
\end{aligned}$$

Parallel Capacitors

$$C_{eq} = C_1 + C_2 + \cdots \tag{25}$$

$$Q_{total} = C_{eq} \times V \tag{26}$$

Capacitors in Series

From KVL,

$$\begin{aligned}
V &= V_1 + V_2 + \cdots \\
\Rightarrow \frac{Q}{C_{eq}} &= \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots \\
\Rightarrow \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} + \cdots
\end{aligned} \tag{27}$$

The charge across the capacitors must be equal in magnitude because charge stored by a plate must come from the adjacent plate.

I-V Relationship

From 23

$$\begin{aligned}
Q &= CV \\
\frac{dQ}{dt} &= C\frac{dV}{dt} \\
\Rightarrow i &= C\frac{dV}{dt} && (28) \\
\Rightarrow V &= \int \frac{i}{C}\, dt && (29)
\end{aligned}$$

Energy Stored

We note that voltage is a measure of energy transferred (work done) per unit charge.

$$\begin{aligned}
\Rightarrow dw &= v\, dq \\
\Rightarrow W &= \int_0^Q v\, dq \\
&= \int_0^Q \frac{q}{C}\, dq \\
&= \frac{Q^2}{2C} \\
&= \frac{(CV)^2}{2C} \\
W &= \frac{1}{2}CV^2
\end{aligned} \tag{30}$$

Transience

It refers to the time-varying voltages and currents resulting from the addition or removal of a power source to circuits containing energy storage elements.

Capacitor Voltage in RC Circuit

$$v(t) = v(0)e^{-\frac{t}{\tau}} + V_s(1 - e^{-\frac{t}{\tau}}), \tau = RC \tag{31}$$

Inductors

Faraday's Law

$$\begin{aligned}
v &= \frac{d\Phi}{dt} \\
&= \frac{d(Li)}{dt} \\
&= L\frac{di}{dt} && (32) \\
L &= \frac{\varepsilon_r \varepsilon_0 N^2 A}{l} && (33)
\end{aligned}$$

$$\begin{aligned}
\varepsilon_r &: \text{Relative permeability of core material} \\
\varepsilon_0 &: \text{Permeability of free space} \\
N &: \text{Number of turns of coil} \\
A &: \text{Cross sectional area of core} \\
l &: \text{Length of core}
\end{aligned}$$

Parallel Inductors

From 32,

$$\frac{di}{dt} = \frac{V}{L} \tag{34}$$

From KCL,

$$\begin{aligned}
i &= i_1 + i_2 + \cdots \\
\frac{di}{dt} &= \frac{di_1}{dt} + \frac{di_2}{dt} + \cdots \\
\frac{V}{L_{eq}} &= \frac{V}{L_1} + \frac{V}{L_2} + \cdots \\
\frac{1}{L_{eq}} &= \frac{1}{L_1} + \frac{1}{L_2} + \cdots
\end{aligned} \tag{35}$$

Inductors in Series

From KVL,

$$V = V_1 + V_2 + \cdots \tag{36}$$

$$L_{eq}\frac{di}{dt} = \frac{di}{dt}L_1 + \frac{di}{dt}L_2 + \cdots \tag{37}$$

$$L_{eq} = L_1 + L_2 + \cdots \tag{38}$$

Energy Stored

$$
\begin{aligned}
W &= \int_0^T P\,dt \\
&= \int_0^T iL\frac{di}{dt}\,dt \\
&= L\int_0^T i\,di \\
&= \frac{1}{2}LI^2 \qquad (39)
\end{aligned}
$$

Inductor Current in RL Circuit

$$
i(t) = i(0)e^{-\frac{t}{\tau}} + i(\infty)(1-e^{-\frac{t}{\tau}}), \tau = \frac{L}{R} \qquad (40)
$$

AC Circuits

Figure 1: The Target

Phasors

$$
\begin{aligned}
v(t) &= V_m\cos(\omega t+\phi) && (41)\\
&= V_m\angle\phi && (42)\\
&= V_m e^{j\phi} && (43)\\
&= V_m(\cos\phi + j\sin\phi) && (44)
\end{aligned}
$$

Always compare phases using argand diagram or in phasor form.

RMS Voltage

$$
\begin{aligned}
V_{rms} &= \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} && (45)\\
&= \frac{V_{max}}{\sqrt{2}} \text{ for sinusoids} && (46)
\end{aligned}
$$

Impedance of Inductor

Suppose $i(t) = I_m\angle 0$. Then,

$$
\begin{aligned}
v(t) &= L\frac{di}{dt} \\
&= \omega L I_m\angle\frac{\pi}{2} \qquad (47)
\end{aligned}
$$

Now,

$$
\begin{aligned}
Z_L &= \frac{\text{Voltage}}{\text{Current}} \\
&= \frac{\omega L I_m\angle\frac{\pi}{2}}{I_m\angle 0} \\
&= j\omega L = \omega L\angle\frac{\pi}{2} \qquad (48)
\end{aligned}
$$

Impedance of Capacitor

$$
Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} \qquad (49)
$$

PMDC Motors

From mechanics, we have the following equation:

$$
P_{\text{out}} = \tau\times\omega \qquad (50)
$$

Also, note that

$$
\omega = 2\pi\times\frac{\text{RPM}}{60} \qquad (51)
$$

The torque is given by this equation where K_t is the torque constant and I_m is the current through the motor.

$$
\tau = K_t\times I_m \qquad (52)
$$

Since the current-carrying coil of the rotor is spinning in a magnetic field, an opposing emf, or back emf, is created.

$$
E_b = K_e\times\omega \qquad (53)
$$

For PMDC motors, $K_e = K_t$.
Also,

$$
P_{\text{in}} = V_m\times I_m \qquad (54)
$$

and from $P_{\text{out}} = \tau\times\omega$,

$$
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \qquad (55)
$$

Since current through the motor is equal to the current through the internal resistance,

$$
\begin{aligned}
I_m &= \frac{V_m - E_b}{R_m} && (56)\\
&= \frac{V_m}{R_m} - \frac{K_e\omega}{R_m} && (57)
\end{aligned}
$$

Rearranging, we get

$$
\omega = \frac{V_m}{K_e} - \frac{R_m I_m}{K_e} \qquad (58)
$$

Note that $I_m R_m \neq V_m$. V_m is the voltage across the motor while $I_m R_m$ is the voltage across the internal resistor which is given by $V_m - E_b$.

Furthermore, we can deduce the following relationships:

1. $\omega \propto V_m$
2. $\omega \propto \frac{1}{T_{\text{shaft}}}$
3. $T_{\text{shaft}} \propto I_m$.

Operational Amplifiers and Filters

Golden Rules

1. $v_+ = v_-$
2. $i_+ = i_- = 0$

Cut-off Frequency

$$f_c = \frac{1}{2\pi RC} \tag{59}$$

To reduce the power of a particular frequency f by x decibels, a low-pass filter (without amplification) can be constructed with a RC value of

$$RC = \frac{\sqrt{10^{x/10} - 1}}{2\pi f} \tag{60}$$

and the gain is given by

$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \frac{1}{\sqrt{1 + (RC\omega)^2}} \tag{61}$$

Similarly, to build a high-pass filter,

$$RC = \sqrt{\frac{10^{-x/10}}{1 - 10^{-x/10}}}/(2\pi f) \tag{62}$$

and the gain is given by

$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \frac{RC\omega}{\sqrt{1 + (RC\omega)^2}} \tag{63}$$

Power gain

$$P_{\text{gain}}(\text{in decibel}) = 20 \log_{10} \left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| \tag{64}$$

Voltage Ripple Equation

$$\Delta V = I_{\text{load}} \cdot \frac{1}{2 f_s C} \tag{65}$$

$$= \frac{V_{\text{load}}}{R_{\text{load}}} \cdot \frac{1}{2 f_s C} \tag{66}$$

Octaves

Octave	Low (Hz)	High (Hz)
0	16.35	30.87
1	32.70	61.74
2	65.41	123.47
3	130.81	246.94
4	261.63	493.88
5	523.25	987.77
6	1046.50	1975.53
7	2093.00	3951.07
8	4186.01	7902.13

Trigonometric Identities

Degrees	0	30	45	60	90
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-

$$1 + \tan^2 u = \sec^2 u, 1 + \cot^2 u = \csc^2 u$$

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\tan(-x) = -\tan x$$

$$\sin(\frac{\pi}{2} - x) = \cos x \qquad \cos(\frac{\pi}{2} - x) = \sin x$$
$$\tan(\frac{\pi}{2} - x) = \cot x \qquad \cot(\frac{\pi}{2} - x) = \tan x$$
$$\sec(\frac{\pi}{2} - x) = \csc x \qquad \csc(\frac{\pi}{2} - x) = \sec x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin(2x) = 2 \sin x \cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$
$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$
$$\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$$
$$= \frac{\sin x}{1 + \cos x}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x - y) - \cos(x + y)\right]$$
$$\cos x \cos y = \frac{1}{2}\left[\cos(x - y) + \cos(x + y)\right]$$
$$\sin x \cos y = \frac{1}{2}\left[\sin(x + y) + \sin(x - y)\right]$$
$$\tan x \tan y = \frac{\tan x + \tan y}{\cot x + \cot y}$$
$$\tan x \cot y = \frac{\tan x + \cot y}{\cot x + \tan y}$$

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$
$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$
$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$
$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$
$$\tan x \pm \tan y = \frac{\sin(x \pm y)}{\cos x \cos y}$$

$$a \cos\theta \pm b \sin\theta = R\cos(\theta \mp \alpha)$$
$$a \sin\theta \pm b \cos\theta = R\sin(\theta \pm \alpha)$$
$$\alpha = \arctan \frac{b}{a}$$

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