EE267 HW5

1.1)

$v = (v_x, v_y, v_z)^T, \|v\| = 1 ; \theta$

$q = (q_0, q_1, q_2, q_3)$

$$\left.\begin{aligned} q_0 &= \cos(\theta/2) \\ q_1 &= v_x \sin(\theta/2)\, i \\ q_2 &= v_y \sin(\theta/2)\, j \\ q_3 &= v_z \sin(\theta/2)\, k \end{aligned}\right\} \quad \|q\| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}$$

$$= \sqrt{\cos^2(\tfrac{\theta}{2}) + v_x^2\sin^2(\tfrac{\theta}{2}) + v_y^2\sin^2(\tfrac{\theta}{2}) + v_z^2\sin^2(\tfrac{\theta}{2})}$$

$$\|q\| = \sqrt{\cos^2(\tfrac{\theta}{2}) + \sin^2(\tfrac{\theta}{2})\,(\cancel{v_x^2 + v_y^2 + v_z^2})^{\,1}} \qquad \|v\| = \sqrt{v_x^2 + v_y^2 + v_z^2} = 1$$

$$v_x^2 + v_y^2 + v_z^2 = 1$$

$$= \sqrt{\cos^2(\theta/2) + \sin^2(\theta/2)}$$

$$\boxed{\|q\| = 1}$$

$f = 1\,Hz \quad \theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon = \omega(t) + \_\_$

1.2) $\omega^{(i)} = (\omega_x^{(i)}, \omega_y^{(i)}, \omega_z^{(i)})^T \quad i = 1,2,3,4$

(i) rotation axis, amt of rotation

$\omega_1: \begin{bmatrix} \frac{\pi}{2} \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ rotation around x axis by 90°

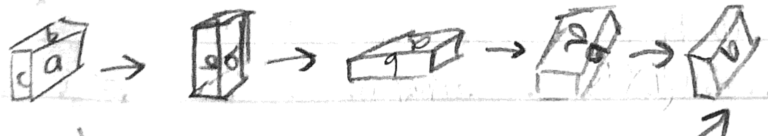
$w_2: \begin{bmatrix} 0 \\ 0 \\ -\frac{\pi}{2} \end{bmatrix} \Rightarrow$ rotation around z-axis by −90°

$w_3 = \begin{bmatrix} 0 \\ -\frac{\pi}{2} \\ 0 \end{bmatrix} \Rightarrow$ rotation around y-axis by −90°

$w_4 = \begin{bmatrix} 0 \\ 0 \\ \pi/2 \end{bmatrix} \Rightarrow$ rotation around z-axis by 90°

ii) Upside down

↳ final relative to original: rotation around x-axis by 180°



180°/−180° rotation around x

iii) $\Delta t = 1/f = 1s$; $\theta^{0} = (0,0,0)^T$; $f = 1\,Hz$

$t = 1s$:

$$\theta^{(1)} = \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}^{1} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} \pi/2 \\ 0 \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} \frac{\pi}{2} \\ 0 \\ 0 \end{pmatrix}} = \theta^{(1)}$$

$t = 2s$:

$$\theta^{(2)} = \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}^{2} = \begin{pmatrix} \frac{\pi}{2} \\ 0 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 0 \\ -\frac{\pi}{2} \end{pmatrix} = \boxed{\begin{pmatrix} \frac{\pi}{2} \\ 0 \\ -\frac{\pi}{2} \end{pmatrix} = \theta^{(2)}}$$

$t = 3s$:

$$\theta^{(3)} = \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}^{3} = \begin{pmatrix} \frac{\pi}{2} \\ 0 \\ -\frac{\pi}{2} \end{pmatrix} + 1 \begin{pmatrix} 0 \\ -\frac{\pi}{2} \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} \frac{\pi}{2} \\ -\frac{\pi}{2} \\ -\frac{\pi}{2} \end{pmatrix} = \theta^{(3)}}$$

$t = 4s$:

$$\theta^{(4)} = \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}^{4} = \begin{pmatrix} \frac{\pi}{2} \\ -\frac{\pi}{2} \\ -\frac{\pi}{2} \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 0 \\ \frac{\pi}{2} \end{pmatrix} = \boxed{\begin{pmatrix} \frac{\pi}{2} \\ -\frac{\pi}{2} \\ 0 \end{pmatrix} = \theta^{(4)}}$$

$R =$

$\theta_x = -90^\circ \quad \theta_y = 0^\circ \quad \theta_z = 0^\circ$

$$R^{(1)} = R_z(0)\,R_x\left(-\tfrac{\pi}{2}\right)R_y(0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$R^{(2)} = R_z\left(\tfrac{\pi}{2}\right)R_x\left(-\tfrac{\pi}{2}\right)R_y(0) = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

$$R^{(3)} = R_z\left(\tfrac{\pi}{2}\right)R_x\left(-\tfrac{\pi}{2}\right)R_y\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$R_{ii} = R_z(0)\,R_y(0)\,R_x(+180) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$R^{(4)} = R_z(0)\,R_x\left(-\tfrac{\pi}{2}\right)R_y\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

iv) For euler angles, gimbal lock exists and can present an issue. If the pitch (rot around x-axis) is $\pm 90^\circ$, this can happen (w/ this order of rotations). This means that rotations in yaw, roll produce the same results. This can make the rotations/movements unexpected. We could try changing the order (though will still be susceptible to gimbal lock in other ways).

No.
Date

2.1.1: GYR_BIAS: 0.87667 -0.46089 0.86697
ACC_BIAS: 0.516 0.269 9.630

If these values were perfect, the acceleration bias should be 0, 0, 9.81, where 9.81 refers to +g. The gyroscope bias should be 0, 0, 0

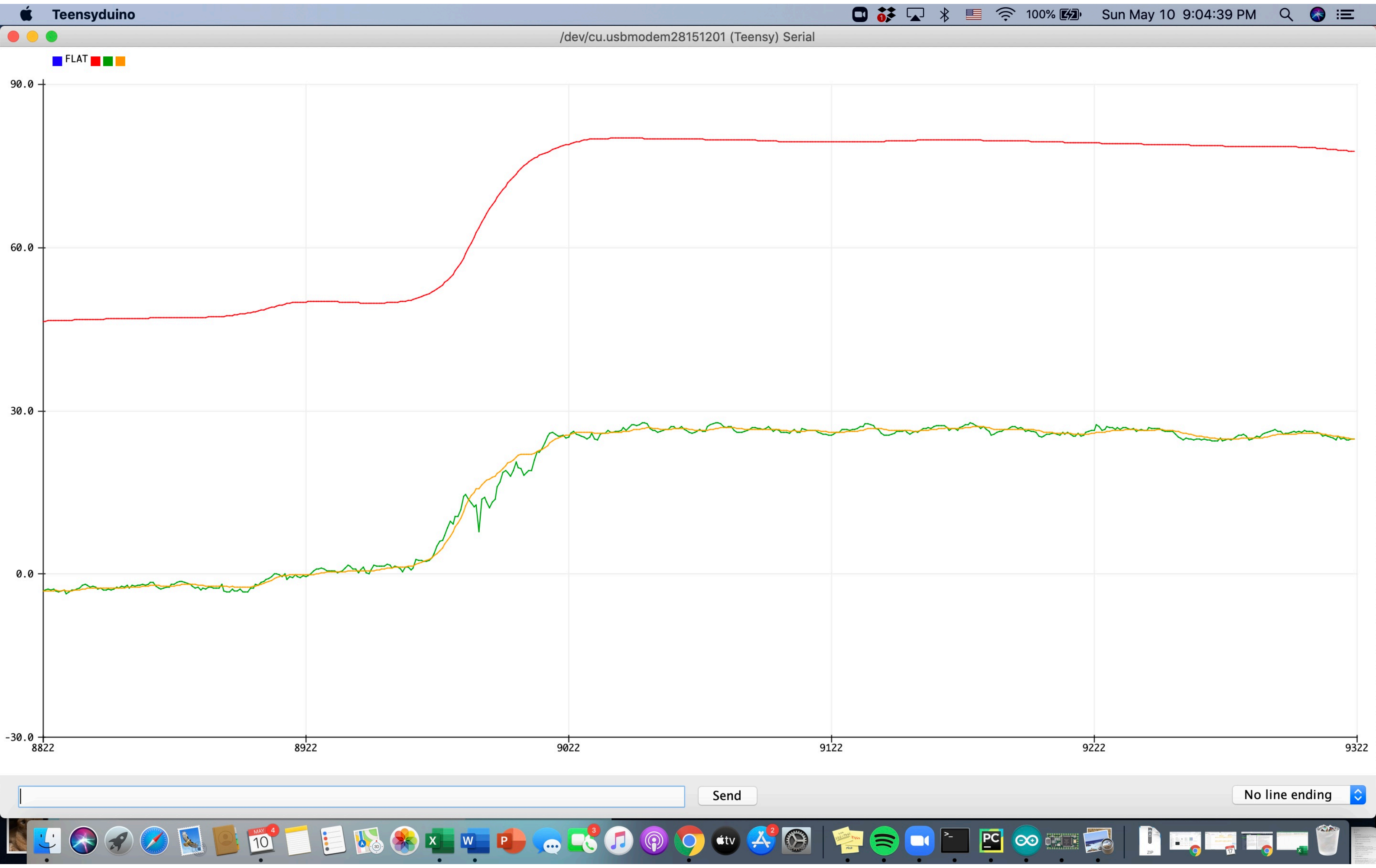
2.1.2: GYR_VAR: 0.01044 0.00945 0.00796
ACC_VAR: 0.000 0.000 0.001

2.2.4 In my case, the gyro appeared as red, the accel. appeared as green, and combined appeared as orange (confirmed by trying $\alpha = 1.0$, $\alpha = 0.0$ and verifying graphs. The acc. plot tends to be noisier (not smooth). The gyro one looks much smoother but with a noticeable offset. The complementary/combined plot ~~is~~ has values similar to that of the accelerometer in terms of the ranges of values (centered around acc.) but has the shape/smoothness closer to that of the gyro graph.

→ Different α values changes the "weights" of each of the gyro/acc. graphs/values. For example, with $\alpha = 1.0$, angles of complimentary is equivalent to just gyro, $\alpha = 0.0$ produces combined graph equal to acc. and anywhere in between is a mix of the two to varying degrees.

2.4.4) $\alpha = 0.9$ ⇒ complimentary very closely mimics motion of the gyroscope quaternion, though the extent of rotation is not ~~the~~ exactly the same. Motion is very smooth, no shakiness.
Can definitely still see the influence of accel. especially during tilt, like angle (true during all rotations)

$\alpha = 0.0$
Complementary only mimics accel. when tilting up/down.



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2.5.3

Doesn't seem to be a huge difference between the two. My main focus is more drawn to the general shakiness of the posts (due to the motion of the HMD) that is making me dizzy. Seems to be more or less consistent between the two versions.