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EE267
HW # 1

No
Date

1)

1.1

Object Space $\rightarrow$ $\boxed{\text{Model Transform}}$ $\xrightarrow{\text{World Space}}$ $\boxed{\text{View Transform}}$ $\xrightarrow{\text{Camera / view Space}}$ $\boxed{\text{Projection Transform}}$ $\xrightarrow{\text{Clip Space}}$ $\boxed{\text{Perspective Divide}}$ $\xrightarrow{\text{Normalized Device Space}}$ $\boxed{\text{Viewport Trans-port}}$ $\rightarrow$ Window Space

1.2 (i)

Scaling matrix:

$$\boxed{S = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

Rotation around y axis by 180°

$\pi = 180°$

$$R_y(\pi) = \begin{pmatrix} \cos(\pi) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi) & 0 & \cos(\pi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{R_y(\pi) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

$$S = R_y(\pi)$$

(ii)

• Translation: $t = (1,1,1)^T \rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

• Rotation: around x-axis 90°

$\theta = \pi/2$

$$R_x\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) & 0 \\ 0 & \sin(\frac{\pi}{2}) & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

call this R

To show non-commutative, we must show translate $\rightarrow$ rotate $\neq$ rotate $\rightarrow$ translate

i.e. $TR \neq RT$ TR & RT shown through A & B below

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{A \neq B} \rightarrow$ NOT-COMMUTATIVE

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1.3 $p = (2,1,3)^T$; $n = (\frac{1}{3}, \frac{2}{3}, \frac{2}{3})^T$

i)

Scaling Matrix: $\boxed{S = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$

$\downarrow$

Rotation Matrix: $R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\pi) & -\sin(\pi) & 0 \\ 0 & \sin(\pi) & \cos(\pi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$

$\theta = \pi$

$\downarrow$

Translation Matrix: $\boxed{T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$

represent p as $\begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \end{bmatrix}$

$p_w = TRSp$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \\ -6 \\ 1 \end{pmatrix}$$

$$\boxed{p_w = \begin{pmatrix} 5 \\ -2 \\ -5 \\ 1 \end{pmatrix}}$$

$n_w = (M^{-1})^T \cdot n$ $\quad n = \begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \\ 0 \end{pmatrix}$

① $(S^{-1})^T \cdot n$

only transform upper left 3x3

$$(S^{-1})^T n = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \\ 0 \end{pmatrix} \leftarrow n$$

$$= \begin{pmatrix} 1/6 \\ 2/9 \\ 1/3 \\ 0 \end{pmatrix}$$

②

$$(R^{-1})^T \cdot \begin{pmatrix} 1/6 \\ 2/9 \\ 1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^T \cdot \begin{pmatrix} 1/6 \\ 2/9 \\ 1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/6 \\ -2/9 \\ -1/3 \\ 0 \end{pmatrix}$$

$$(T^{-1})^T \begin{pmatrix} 1/6 \\ -2/9 \\ -1/3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1/6 \\ -2/9 \\ -1/3 \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1/6 \\ -2/9 \\ -1/3 \\ -7/18 \end{pmatrix} = n_w}$$

ii) $e = (0,10,10)$, $c = (0,0,0)$; $u = (0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})^T$; $g = c - e$

$p_w = \begin{pmatrix} 5 \\ -2 \\ 5 \\ 1 \end{pmatrix}$

$$z_c = \frac{\text{eye} - \text{center}}{\lVert \text{eye} - \text{center} \rVert} = \begin{array}{ccc} x & y & z \end{array} \left(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ or } \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$$

$p_v = V \cdot p_w = R \cdot T(-eye)\, p_w$

$$x^c = \frac{up \times z^c}{\lVert up \times z^c \rVert} = \begin{bmatrix} 0 \\ \sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix} \times \begin{bmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ -2 \\ -5 \\ 1 \end{pmatrix}$$

$$y = z^c \times x^c = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$\boxed{p_v = \begin{pmatrix} 5 \\ \frac{3\sqrt{2}}{2} \\ \frac{-27\sqrt{2}}{2} \\ 1 \end{pmatrix}}$$

sanity check:

$e \to \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \to (0,0,0) \to$ camera is centered ✓

$u \to \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \to$ up in y-direction

$g \to (0,-10,-10) \to \begin{pmatrix} 0 \\ 0 \\ -10\sqrt{2} \\ 0 \end{pmatrix}$

transformation is correct by what these points represent in camera space

iii) P, P_{clip}, P_{ndc}

$P_{clip} = P \cdot p_v$ ← proj. matrix

$$P = \begin{pmatrix} \frac{f}{asp.} & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & \frac{-zFar + zNear}{zFar - zNear} & \frac{-2zFar\,zNear}{zFar - zNear} \\ 0 & 0 & -1 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{6}{5} & -\frac{22}{5} \\ 0 & 0 & -1 & 0 \end{pmatrix}}$$

$$P_{clip} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -6/5 & \frac{-22}{5} \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ \frac{3\sqrt{2}}{2} \\ \frac{-27\sqrt{2}}{2} \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 5 \\ \frac{3\sqrt{2}}{2} \\ \frac{81\sqrt{2} - 22}{5} \\ \frac{27\sqrt{2}}{2} \end{pmatrix}}$$

$$P_{ndc} = \begin{pmatrix} 5 / \left(\frac{27\sqrt{2}}{2}\right) \\ \frac{3\sqrt{2}}{2} / \left(\frac{27\sqrt{2}}{2}\right) \\ \left(\frac{81 * \sqrt{2} - 22}{5}\right) / \left(\frac{27\sqrt{2}}{2}\right) \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} \frac{10}{27\sqrt{2}} \\ \frac{1}{9} \\ \frac{6}{5} - \frac{22\sqrt{2}}{135} \\ 1 \end{pmatrix}}$$

iv) $l_w = 200$, $h_w = 200$

$$P_{window} = \begin{pmatrix} X_{wind} \\ Y_{wind} \\ Z_{wind} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{lw}{2}(X_{ndc} + 1) \\ \frac{hw}{2}(Y_{ndc} + 1) \\ \frac{1}{2}(Z_{ndc} + 1) \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 126.19 \\ 111.11 \\ 0.985 \\ 1 \end{pmatrix}}$$

2.3.3

The perspective projection seemed most similar to how I perceive objects (bigger when closer, smaller when farther). It is better in the context of depth. This is more useful in modelling real life and a natural appearance (which might be the goal if trying to replicate human experience in terms of sight). "realistic"

Orthographic projection → no tapering or vanishing points → hard to tell distance if viewing straight on. Not good (probably) for replicating human exper. but ensures that objects appear in true "size" → might be good for more mathematical things like measuring.