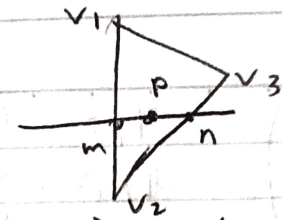

1.1) Intensity at p(x,y)

$v_1 \to I_1$, $v_2 \to I_2$, $v_3 \to I_3$; $m(x_m, y_m)$, $n(x_n, y_n)$

$$I_m = \left(\frac{y_m - y_2}{y_1 - y_2}\right) I_1 + \left(\frac{y_1 - y_m}{y_1 - y_2}\right) I_2$$

$$I_n = \left(\frac{y_n - y_3}{y_1 - y_3}\right) I_1 + \left(\frac{y_1 - y_n}{y_1 - y_3}\right) I_3$$

$$I_p = \left(\frac{x - x_m}{x_n - x_m}\right) I_n + \left(\frac{x_n - x}{x_n - x_m}\right) I_m$$

$$\boxed{I_p = \left(\frac{x - x_m}{x_n - x_m}\right)\left[\left(\frac{y_n - y_3}{y_1 - y_3}\right) I_1 + \left(\frac{y_1 - y_n}{y_1 - y_3}\right) I_3\right] + \left(\frac{x_n - x}{x_n - x_m}\right)\left[\left(\frac{y_m - y_2}{y_1 - y_2}\right) I_1 + \left(\frac{y_1 - y_m}{y_1 - y_2}\right) I_2\right]}$$

1.2). vertex coordinates in window space: $V_1 = (1.5, 4)$; $V_2 = (0.5, 1)$; $V_3 = (4.5, 0.5)$

- depth values in window space w/ range $[0,1]$: $z_1 = 0.7$, $z_2 = 0.5$; $z_3 = 0.3$
- normals in view space $n_1 = \left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)^T$ $n_2 = \left(-\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)^T$ $n_3 = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)^T$

(1) 3D coord. in view space: $a = 1$; fovy $= 90°$; zNear $= 2$, zFar $= 22$

$$\begin{pmatrix} x_{ndc} \\ y_{ndc} \\ z_{ndc} \end{pmatrix} = \begin{pmatrix} \frac{2x_{win}}{w} - 1 \\ \frac{2y_{win}}{h} - 1 \\ (2z_{win}) - 1 \end{pmatrix}$$

$h = 6$, $w = 6$

$$V_{1,ndc} = \begin{pmatrix} -1/2 \\ \cancel{+}1/3 \\ 2/5 \end{pmatrix} ; \quad V_{2,ndc} = \begin{pmatrix} -5/6 \\ -2/3 \\ 0 \end{pmatrix}$$

$$V_{3,ndc} = \begin{pmatrix} 1/2 \\ -5/6 \\ -2/5 \end{pmatrix}$$

$z_{1,ndc} = 2/5$

$z_{2,ndc} = 0$

$z_{3,ndc} = -1/3$

Date

$$\begin{pmatrix} X_{ndc} \\ Y_{ndc} \\ Z_{ndc} \\ 1 \end{pmatrix} = \begin{pmatrix} X_{clip}/W_{clip} \\ Y_{clip}/W_{clip} \\ Z_{clip}/W_{clip} \\ W_{clip}/W_{clip} \end{pmatrix}$$

$T_1 = -6/5$ $\quad Z_{1,ndc} = 2/5$

$T_2 = -22/5$ $\quad Z_{2,ndc} = 0$

$E_1 = -1$ $\quad Z_{3,ndc} = -2/5$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{6}{5} & -\frac{22}{5} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$W_{clip} = \frac{T_2}{Z_{ndc,i} - \frac{T_1}{E_1}} \Rightarrow \boxed{\begin{aligned} W_{clip,1} &= 11/2 \\ W_{clip_2} &= 11/3 \\ W_{clip3} &= 11/4 \end{aligned}}$$

so $X_{clip} = X_{ndc} W_{clip}$

$Y_{clip} = Y_{ndc} W_{clip}$

$Z_{clip} = Z_{ndc} W_{clip}$

$$V_{1,ndc} = \begin{pmatrix} -1/2 \\ 1/3 \\ 2/5 \end{pmatrix} \quad V_{2,ndc} = \begin{pmatrix} -5/6 \\ -2/3 \\ 0 \end{pmatrix}, \quad V_{3,ndc} = \begin{pmatrix} 1/2 \\ -5/6 \\ -2/5 \end{pmatrix}$$

$$V_{1,clip} = \begin{pmatrix} -11/4 \\ 11/6 \\ 11/5 \\ 11/2 \end{pmatrix} \quad V_{2,clip} = \begin{pmatrix} -55/18 \\ -22/9 \\ 0 \\ 11/3 \end{pmatrix} \quad V_{3,clip} = \begin{pmatrix} 11/8 \\ -55/24 \\ -11/10 \\ 11/4 \end{pmatrix}$$

$$P_{view} = V_{view} = P^{-1} \cdot P_{clip} \text{ ie. } (M_{proj})^{-1} \cdot V_{i,clip}. \quad M_{proj}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -\frac{5}{22} & \frac{3}{11} \end{pmatrix}$$

$$V_{1,view} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -\frac{5}{22} & 3/11 \end{pmatrix} \cdot \begin{pmatrix} -11/4 \\ 11/6 \\ 11/5 \\ 11/2 \end{pmatrix} = \begin{pmatrix} -11/4 \\ +11/6 \\ -11/2 \\ 1 \end{pmatrix}$$

$V_{2,view} \Rightarrow Z_{view} = -W_{clip} = -11/3$

$$X_{view} = \frac{X_{clip} - M_{02} Z_{view}}{M_{00}} = \frac{-55/18 - 0}{1}$$

$$Y_{view} = \frac{Y_{clip} - M_{12} Z_{view}}{M_{11}} = \frac{-22/9 - 0}{1}$$

$$\begin{pmatrix} -55/18 \\ -22/9 \\ -11/3 \\ 1 \end{pmatrix}$$

$$V_{3,view} = \quad X_{view} = \frac{X_{clip} - \cancel{M_{02} Z_{view}}^{\,0}}{\cancel{M_{00}}\ 1} = 11/8$$

$$Y_{view} = \frac{Y_{clip} - M_{12} Z_{view}^{\,0}}{\cancel{M_{11}}\ 1} = -55/24$$

$$\begin{pmatrix} 11/8 \\ -55/24 \\ -11/4 \\ 1 \end{pmatrix}$$

$Z_{view} = -W_{clip} = -11/4$

$$\boxed{V_{1,view} = \begin{pmatrix} -11/4 \\ 11/6 \\ -11/2 \\ 1 \end{pmatrix} \quad V_{2,view} = \begin{pmatrix} -55/18 \\ -22/9 \\ -11/3 \\ 1 \end{pmatrix} \quad V_{3,view} = \begin{pmatrix} 11/8 \\ -55/24 \\ -11/4 \\ 1 \end{pmatrix}}$$

light source:

ii) $l = (5,5,5)$; $I_l = (10,10,10)^T$

diff. vec. $m_1 = (1,0,0)^T$, $m_2\ (0,1,0)^T$, $m_3 = (0,0,1)^T$

Phong lighting

↓

per fragment

→ neglect specular, ambient component

→ square distance fallout → $(K_c = K_l = 0,\ K_q = 1)$

Phong Lighting Diffuse : $m^{diff} \cdot l^{diff} \cdot \max(L \cdot N, 0)$

N = normalized surf. normal

L = normalized vector pointing toward light source

* drop the 1 (w)

V1

$$V_{1,view} = \begin{pmatrix} -11/4 \\ 11/6 \\ -11/2 \end{pmatrix} \quad L_1 = \frac{l - V_{1,view}}{\|l - V_{1,view}\|} = \left(\frac{93}{\sqrt{25969}}, \frac{38}{\sqrt{25969}}, \frac{126}{\sqrt{25969}}\right)^T = L_1$$

$\frac{\sqrt{25969}}{12}$

$$N_1 = \begin{pmatrix} -2/3 \\ 2/3 \\ 1/3 \end{pmatrix} \quad L_1 \cdot N = \begin{pmatrix} 93/\sqrt{25969} \\ 38/\sqrt{25969} \\ 126/\sqrt{25969} \end{pmatrix} \cdot \begin{pmatrix} -2/3 \\ 2/3 \\ 1/3 \end{pmatrix} = \boxed{\frac{116}{3\sqrt{25969}} > 0}$$

$$\underset{\text{attenuation}}{A} = \frac{1}{\cancel{K_c} + \cancel{K_l d} + K_q d^2} = \frac{1}{d^2} = \frac{1}{\|l - v_{view}\|^2} = \boxed{\frac{144}{25969} = A}$$

light source = 1

scalar, float

$$\text{color}_{v_1} = A\left(m_1 \cdot I_l \cdot \max(L \cdot N, 0)\right)$$

$$= \frac{144}{25969} \cdot \frac{116}{3\sqrt{25969}} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix}$$

$$\boxed{\text{Color}_{v_1} \approx \begin{pmatrix} 0.001835 \\ 0 \\ 0 \end{pmatrix}}$$

$V_{2,view} = \begin{pmatrix} -53/16 \\ -22/9 \\ -11/3 \end{pmatrix}$

No.

Date

(V2) $color_{v_2} = a(m_2 \cdot I_e \cdot \max(L \overset{\text{dot p}}{\cdot} N, 0))$

$a = \frac{1}{d^2} = \frac{1}{\|l - V_{2,view}\|^2} = \boxed{0.007913}$

$N_2 = \begin{pmatrix} -2/3 \\ -2/3 \\ 1/3 \end{pmatrix}$; $L_2 = \frac{l - V_{2,view}}{\|l - V_{2,view}\|} = \begin{pmatrix} 0.576 \\ 0.533 \\ 0.620 \end{pmatrix}$

$L_2 \cdot N_2 = -0.5325 \rightarrow \boxed{\max(L_2 \cdot N_2, 0) = 0}$

$m_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$; $I_L = \begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix}$

$m_2 I_L = \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix}$

$color_{v_2} = a(m_2 I_e \max(\underbrace{\cancel{L_2 \cdot N_2}}_{0}, 0)) = \boxed{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = color_{v_2}}$

(V3) $a = \frac{1}{d^2} = \frac{1}{\|l - V_{3,view}\|} = \boxed{0.007913}$

$V_{3,view} = \begin{pmatrix} 11/8 \\ -55/24 \\ -11/4 \end{pmatrix}$; $N_3 = \begin{pmatrix} 2/3 \\ -2/3 \\ 1/3 \end{pmatrix}$; $L_3 = \begin{pmatrix} 0.3225 \\ 0.6486 \\ 0.6894 \end{pmatrix}$

$\boxed{L_3 \cdot N_3 = 0.012355}$

$Color_{v3} = a(\underbrace{m_3 I_e}_{\begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}} \max(L_2 \cdot N_2, 0)) = \boxed{\begin{pmatrix} 0 \\ 0 \\ 0.0009767 \end{pmatrix} = Color_{v3}}$

iii) Yes, it depends on the # of fragments inside, since there are always 3 vertices. If there are <3 fragments, then Phong will be faster (less computation).

2.1.1.3th the addition of an $K_q = 0.001$ term definitely made the Gourand diffuse pot seem more dim overall. There is less observable lighting differences btwn different areas of the pot.

2.2.2.1
The lighting on the Phong is definitely smoother when you shine the light and move it around. Less "lines" appear. A glossier look? Not sure exactly how to describe it.

Phong

+ Better quality color, light appears more naturally/realistically

- More computation, slower calculations

Gourand

+ faster, generally less computation (less vertices than fragments)

- weird effects w/ specular reflection (not as natural looking)