ME2 Computing- Coursework summary

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Words: 976/1000

A) What physics are you trying to model and analyse? (Describe clearly, in words, what physical phenomenon you wish to analyse)

The temperature distribution of wine placed inside a wine cooler of 15°C was investigated over time. The initial temperature of the wine was 25°C and the time taken for all the wine to reach the ideal range of serving temperatures (15°C - 16 °C (Wine and Barrels Co, 2020)) was investigated. It was assumed that the thermal diffusivity of the wine remained constant at α = 0.392 mm^2/s (Hlavac, et al., 2016) and that there was no internal heat generation.

The wine bottle was modelled as two rectangles, a large rectangle to represent the body of the bottle and a small rectangle to represent the neck as in Figure 1. The boundary conditions were set such that the empty space surrounding the wine bottle remains constant at the cooler temperature (see Section D).

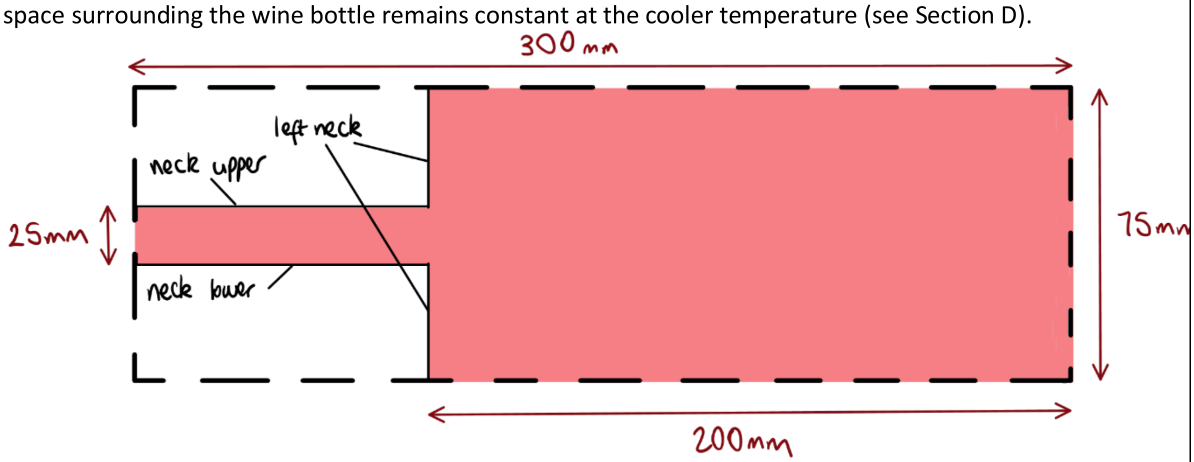

Figure 1: Control volume representation

The dimensions of the bottle were found using standards (Wine Racks, 2017).

B) What PDE are you trying to solve, associated with the Physics described in A? (write the PDE)

Heat diffusion equation:

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t} \tag{1}$$

C) Boundary value and/or initial values for my specific problem: (be CONSISTENT with what you wrote in A)

Initial conditions
The initial temperature inside the bottle was 25ºC:

$$\left.\frac{\partial T}{\partial t}\right|_{t=0} = 25\ °\mathrm{C} \tag{2}$$

The implementation of the initial conditions are shown in Figure 2 below.

```python
# Initial Conditions & Implementation
    T_initial = 25
    T[:, :, 0] = T_initial
```

Figure 2: Implementation of the initial conditions in the code

Boundary conditions

The surfaces of the wine bottle were assumed to be 15°C (T_∞) once it entered the cooler.

Therefore, the boundary conditions at x = 0mm, x = 300mm, y = 0mm and y = 75mm were set at 15°C:

$$T|_{x=0} = 15\text{ °C} \tag{3}$$

$$T|_{x=300} = 15\text{ °C} \tag{4}$$

$$T|_{y=0} = 15\text{ °C} \tag{5}$$

$$T|_{y=75} = 15\text{ °C} \tag{6}$$

Certain areas inside the control volume were part of the cooler instead of the wine bottle, therefore were assumed to have a constant temperature of 15°. Both conditions below must be satisfied to describe this area.

$$T|_{x=a,\ y=b} = 15\text{ °C} \begin{cases} 200 < a < 300 \\ 0 < b < 25 \text{ or } 50 < b < 75 \end{cases} \tag{7}$$

The implementation of the boundary conditions is shown in Figure 3.

```python
# Boundary Conditions & Implementation
    T_cooler = 15
    left_bc = T_cooler #(x = 0)
    right_bc = T_cooler #(x = length)
    top_bc = T_cooler #(y = out_diam)
    bottom_bc = T_cooler #(y=0)

    T[0,:,:] = left_bc #(x = 0)
    T[-1,:,:] = right_bc #(x = length)
    T[:, 0, :] = top_bc #(y=out_diam)
    T[:, -1, :] = bottom_bc #(y=0)

    ## Define indexes required for neck boundaries
    neck_lower_index = int((0.5 * (y_final - in_diam)) / h) # neck bottom y value
    neck_upper_index = int((0.5 * (y_final + in_diam)) / h) # neck top y value
    left_neck_index = int(body_length / h) # neck and body boundary x value

    ## Implement BC to cooler region within control volume
    for i in range(left_neck_index, x_intervals + 1):
        for j in range(0,y_intervals + 1):
            if (j > neck_upper_index) or (j < neck_lower_index):
                T[i,j,:] = T_cooler # if in the cooler region apply BC
```

Figure 3: Implementation of Boundary Conditions in code

D) What numerical method are you going to deploy and why? (Describe, in words, which method you intend to apply and why you have chosen it as opposed to other alternatives)

The finite difference method (FDM) was chosen. FDMs solve differential equations by approximating them with difference equations, making this is a simple method. It is efficient in regards to memory handling, as it requires minimal recall of previous objects compared to other methods. As well as simplicity, the finite difference method was chosen as it can easily model irregular shapes, like the wine bottle.

The FDM is an explicit method with an O(N) time. An implicit method would require matrix inversion and therefore has $O(N^2)$ time. This is an order higher than the proposed solution, therefore would require more operations than necessary.

E) I am going to discretise my PDE as the following (show the steps from continuous to discrete equation and boundary/initial conditions:

The PDE was discretised as shown below, with step size h constant for the x and y domain.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

$$\frac{1}{k}\left[T^{t+1}_{i,j} - T^{t}_{i,j}\right] = \frac{\alpha}{h_1^2}\left[T^{t}_{i+1,j} + T^{t}_{i-1,j} - 2T_{i,j}\right] + \frac{\alpha}{h_2^2}\left[T^{t}_{i+1,j} + T^{t}_{i-1,j} - 2T_{i,j}\right]$$

assuming $h_1 = h_2 = h$

$$T^{t+1}_{i,j} = \frac{k\alpha}{h^2}\left[T^{t}_{i+1,j} + T^{t}_{i-1,j} + T^{t}_{i,j+1} + T^{t}_{i,j-1}\right] + T^{t}_{i,j}\left[1 - \frac{4k\alpha}{h^2}\right]$$

Let $r = \frac{k\alpha}{h^2}$

$$T^{t+1}_{i,j} = r\left[T^{t}_{i+1,j} + T^{t}_{i-1,j} + T^{t}_{i,j+1} + T^{t}_{i,j-1}\right] + T^{t}_{i,j}\left[1 - 4r\right]$$

Figure 4: Derivation of PDE discretisation

F) Plot the numerical results comprehensively and discuss them (discuss how the results describe the physics and comment on any discrepancies or unexpected behaviours). Use multiple types of visual graphs. Present and discuss any outcomes of the grid analysis, as requested in Task 8, too.

The graphing techniques employed to investigate the distribution of the temperature of the wine at various time points.

At t = 0s:

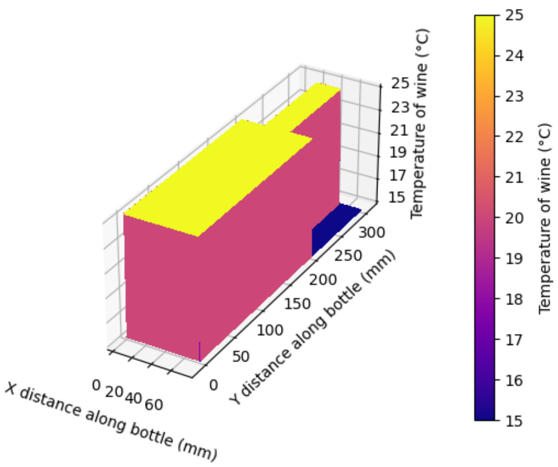

Figure 5: 3D Surface Plot at t=0s

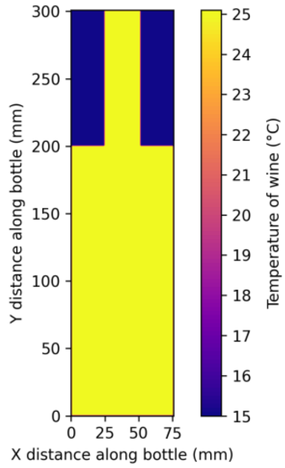

Figure 6: Contour plot of temperature at t=0s

Figure 5 shows a 3D plot of the temperature of each point in the control volume. The boundary conditions (Equations 3-8) can be seen implemented with the clear bottle shape, arguably more clear in Figure 6.

There is an angle between the boundary and wine in the 3D plot, due to the discretisation of the domain. A smaller step size (h) would reduce the angle.

At t=10s & 100s:
The contour plot in Figure 7 shows that the temperature near the boundaries have changed from the initial conditions after the wine bottle has been in the cooler for 10 seconds. Figure 8 shows that at 100 seconds, more of the wine close to the boundary has decreased in temperature, but the temperature at the centre is still close to the initial condition of 25 °C.

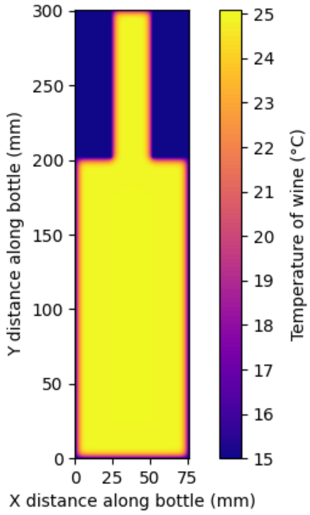

Figure 7: Contour plot of temperature distribution at t=10s

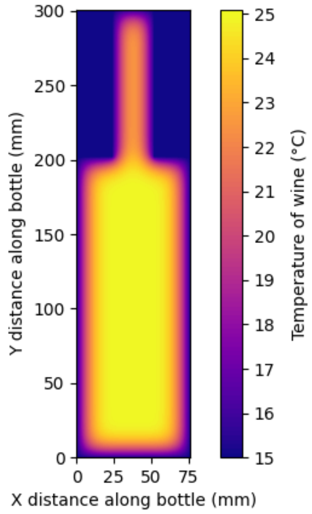

Figure 8: Contour plot of temperature distribution at t=100s

Figure 9 and Figure 10 help illustrate that the main body of the wine bottle has higher temperatures than that of the neck and that this disparity grows with time.

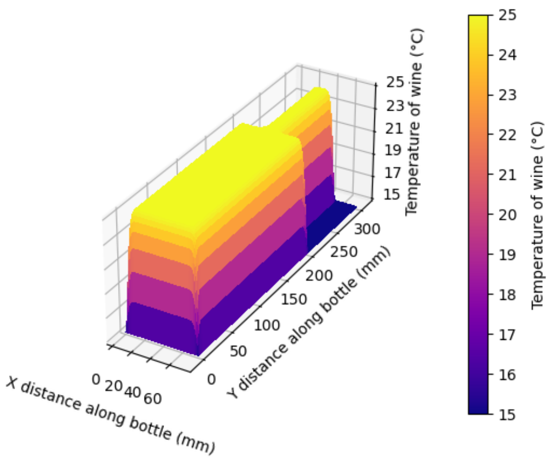

Figure 9: 3D surface plot of temperature distribution at t=10s

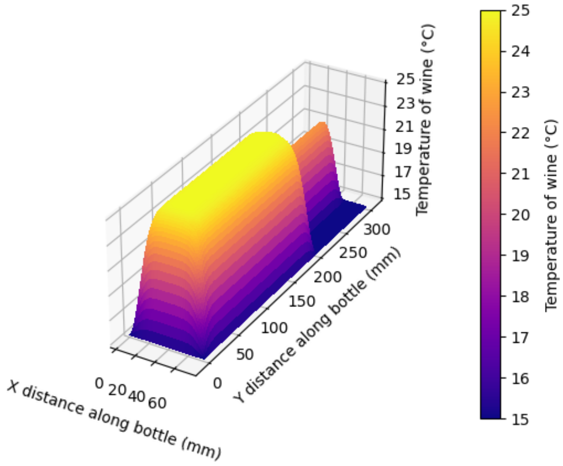

Figure 10: 3D surface plot of temperature distribution at t=100s

At t = 3600s:
The time for the wine to reach the acceptable temperature range was found to be 3589 seconds, using the maximum temperature within each time interval. A 3D surface plot (Figure 11) and contour plot (Figure 12) show the temperature distribution and prove that the wine is within the acceptable region.

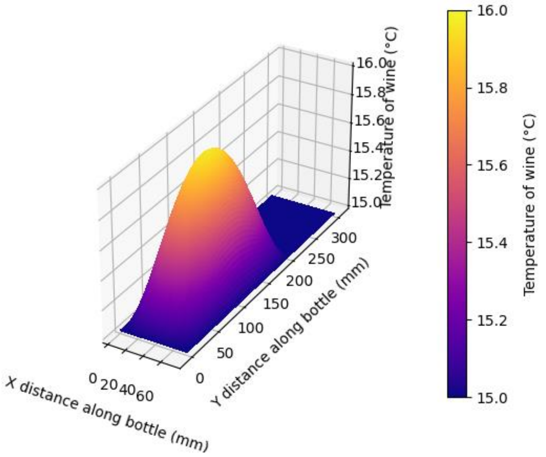

Figure 11: 3D surface plot of temperature distribution at t=3600s

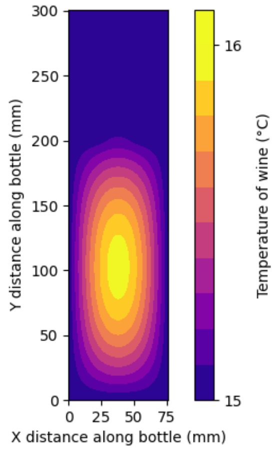

Figure 12: Contour plot of temperature distribution at t=3600s

In Figure 12, it is seen that the temperature of the neck has already reached the initial conditions, while the body has greater variations in temperature.

The maximum temperature occurs at the centre of the large rectangle (main body). A line graph of this maximum temperature was plotted with respect to time (Figure 13). As expected, the temperature of the centre point does not change to begin with. After sufficient time, the surrounding points feel the effect of the boundary conditions and the temperature changes at a rate similar to exponential decay.

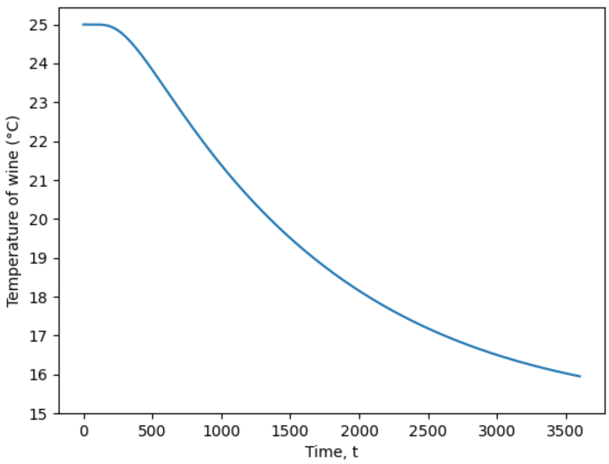

Figure 13: Temperature distribution at midpoint of body over time

The value of 3589 seconds (about 1 hour) to seems plausible as placing a wine bottle with the initial conditions would take around this time to cool to the appropriate range.

G) Other remarks (limits of the model, convergence problems, possible alternative approaches, anything you find relevant and important to mention):

An explicit method was used to find the temperature distribution of the bottle. A key disadvantage of this is that there is a range of convergence shown in Equation 8 below.

$$r = \frac{k\alpha}{h^2} \leq 0.25 \quad (88)$$

A small h value was chosen for high accuracy, however this constrained k to very small values resulting in lower computational efficiency as the code had to be run many times. If convergence doesn't occur, the program prints that the convergence will not occur and no other code is run.

There are limitations of the model due to how the shape of the bottle was simplified. Firstly, the curved neck of the bottle was neglected and instead modelled as a straight line. An alternative, more accurate approach would be to use part of sine wave to model the curve. Another simplification of the shape is the straight edge of the base of the bottle. Wine bottles typically have a curved bottom edge which could have been modelled using a quadratic curve. These alterations would result in a more accurate model of the bottle, however are complicated to implement and would have a minimal effect on the result.

Though improvements could be made to the model, the results are plausible and appear to be a good approximation for the temperature distribution.

References

Hlavac, P., Božiková, M., Hlaváčová, Z. & Kardjilova, K., 2016. Changes in selected wine physical properties. *Research in Agricultural Engineering,* 62(3), pp. 147-153.
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