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TECHNICAL NOTE

The Ornstein–Uhlenbeck third-order Gaussian process (OUGP) applied directly to the un-resampled heart rate variability (HRV) tachogram for detrending and low-pass filtering

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Abstract The heart rate variability signal derived from the ECG is a beat-to-beat record of RR-intervals and is, as a time series, irregularly sampled. It is common engineering practice to resample this record, typically at 4 Hz, onto a regular time axis for conventional analysis using IIR and FIR filters, and power spectral estimators, in the time and frequency domain, respectively. However, such interpolative resampling introduces noise into the signal and the information quality is compromised. Here, the Ornstein–Uhlenbeck third-order band-pass filter is presented which operates on data sampled at arbitrary time and preserves fidelity. The algorithm is available as open source code for MATLAB® (MathWorks™ Inc.) and supported by an interactive website at http://clinengnhs.liv.ac.uk/OUGP.htm.

Keywords HRV · ECG · EKG · RR-intervals · PSD · Ornstein–Uhlenbeck · Spectral analysis · Bandpass filter

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1 Introduction

The heart rate variability signal (HRV tachogram) derived from the ECG is a beat-to-beat record of RR-intervals and is, as a time series, irregularly sampled. It is common engineering practice to resample this record, typically at 4 Hz, onto a regular time axis for conventional analysis using IIR and FIR filters, and power spectral estimators (PSD's), in the time and frequency domain, respectively. However, such interpolative resampling introduces noise into the signal and the information quality is compromised [2, 9].

The information content of the HRV tachogram is limited at high frequencies by Shannon–Nyquist criterion. For both regularly and irregularly sampled data, the upper limiting frequency is simply half the mean sampling rate (the Nyquist frequency). In real life HRV tachograms, which are irregularly sampled and subject to noise (e.g. ectopic beats and recording artefacts), a robust estimate, such as the reciprocal of the median sampling interval, is appropriate. The low frequency information limit is determined by the assumed stationarity. Formal definitions of stationarity can be found elsewhere. The assumption of stationarity is an axiomatic requirement in the estimation of the power spectrum. The Task Force [11] recommendation is that stationarity beyond 20 min should not be assumed, although this limit is quite arbitrary. Hence, the common practice is to detrend the HRV tachogram, either by identifying and removing by subtraction a low frequency content trend component or by high-pass filtering. Trend component identification methods using fixed low-order polynomials [10, 13] and adaptive high-order polynomials [16] are readily applied to the raw irregularly sampled data but with poorly described cut-off frequencies. Reported methods using high-pass filtering, necessarily resampled onto a regular time axis, include Butterworth-response IIR's [7], locally applied FIR's [1] and more recently, a Gaussian process-based smoothing priors (GPSP) approach [14, 15, 18]. The latter method was extended in [3] (ext-GPSP), to handle non-uniformly sampled data, resulting in a time-varying IIR filter with a second order amplitude response.

2 Methods

2.1 The Ornstein–Uhlenbeck third-order Gaussian process (OUGP) filter

Consider a Gaussian process $x(t)$ with the exponential covariance function:

$$k(t_i, t_j) = \exp(-\gamma |t_i - t_j|). \tag{1}$$

This stationary covariance function describes the Ornstein–Uhlenbeck process which was originally introduced as the model of the velocity of a particle undergoing Brownian motion. The process is mean-square (MS) continuous:

$$\lim_{\varepsilon \to 0} \mathrm{E}\left[(x(t+\varepsilon) - x(t))^2\right] = 0 \tag{1a}$$

but not MS differentiable:

$$\lim_{\varepsilon \to 0} \mathrm{E}\left[\left(\frac{x(t+\varepsilon) - x(t)}{\varepsilon} - \dot{x}(t)\right)^2\right] \neq 0 \tag{1b}$$

The properties above are expressed in terms of expectations.

In the following, the basic theory of the OUGP filter is outlined: for details, the Reader is referred to [17].

Let us consider the Gram matrix K, obtained by evaluating the covariance function at a (not necessarily uniform) sequence of ordered times $t_1 < t_2 < \cdots < t_n$. At first sight, the matrix K does not seem a good candidate for a low-pass filter, because the discontinuity in the slope on the diagonal [due to property in Eq. (1b)] introduces potentially significant high-frequency leakage. A fundamental modification, however, is to define γ as a complex number, so we can consider the filter as the real part of the result. It is then possible to show [17] that there exists a unique exponential filter with the following properties:

- The derivative of the impulse response is continuous;
- The frequency response is unity at 0 and flat up to the third derivative;
- The frequency response falls off at 24 dB per octave when $f > f_c$, where f_c is the cut-off frequency at -3 dB (frequencies normalised w.r.t. Nyquist).

Defining values for γ as γ_l and γ_h, for low-pass and high-pass filters, respectively, with $i = \mathrm{sqrt}(-1)$,

$$\begin{aligned} \gamma_l &\equiv \sqrt{2}\pi(\sqrt{2}-1)^{-1/4}(1+i)f_c \\ \gamma_h &\equiv \sqrt{2}\pi(\sqrt{2}-1)^{1/4}(1+i)f_c \end{aligned} \tag{2}$$

scales the respective 3 dB points of the filter to the cut-off frequency f_c.

Let us define for

$j = 1 \ldots n-1$, where, as previously defined, γ stands for γ_l or γ_h:

$$\begin{aligned} w_j &\equiv \gamma(t_{j+1} - t_j) \\ r_j &\equiv \exp(-w_j) \\ e_j &\equiv (r_j^{-1} - r_j)^{-1}. \end{aligned}$$

It can then be proved that the inverse of the covariance matrix K is tridiagonal with the following entries:

$$T_{ij} = \begin{cases} 1 + r_1 e_1 & i = j = 1 \\ -e_i & 1 \le i = j-1 \le n-1 \\ 1 + r_i e_i + r_{i-1} e_{i-1} & 1 < i = j < n \\ -e_j & 1 \le j = i-1 \le n-1 \\ 1 + r_{n-1} e_{n-1} & i = j = n \\ 0 & \text{otherwise.} \end{cases}$$

Note that to evaluate and store the T matrix, it is not necessary to store and calculate the entries of the K matrix. This is crucial for large data sets, since the space complexity of the algorithm is linear in the sample size rather than quadratic or higher ordered.

Hence, filtering a sequence s_j produces an output:

$$u_i = \sum_j K_{ij} s_j, \quad i = 1 \ldots n \tag{3}$$

which can be more efficiently evaluated in terms of the solution of a sparse tridiagonal system involving the matrix T as:

$$\sum_j T_{ij} u_j = s_i.$$

It is known [8] that tridiagonal systems can be solved in linear time, requiring only $8n$ arithmetic operations, so the above operation is actually faster than for a regular matrix–vector product.

Given a set of measurements y_j at the times t_j, $j = 1 \ldots n$, the above equation specialises as:

$$\sum_j T_{ij} u_j = \begin{cases} (y_1 - y_2)/2w_1 & i = 1 \\ (y_i - y_{i+1})/2w_i + (y_i - y_{i-1})/2w_{i-1} & i = 2, \ldots, n-1 \\ (y_n - y_{n-1})/2w_{n-1} & i = n \end{cases}.$$

It should be noted that the data have been transformed by a piecewise-linear quadrature formula. For a rationale of the choice of this transform, see [17].

The high-pass (H) or low-pass (L) filtered data can finally be obtained by the following [$\Re(\cdot)$ denotes the real part]:

$$\mathrm{Hy} = \Re(\mathrm{u}), \quad \mathrm{Ly} = \mathrm{y} - \Re(\mathrm{u}) \tag{4}$$

Equations (3) and (4) describe the operation of a non-causal filter; in particular, the matrix K can be seen as its impulse response, though the data are further processed by a piecewise-linear quadrature. It can also be shown that the filter has zero phase, since its frequency response is a real, even and positive function of frequency in the pass band [14].

It is useful to compare this method with the ext-GPSP as proposed in [3]. Whereas the former directly specifies a covariance function, the latter specifies an inverse covariance function, which is somewhat less intuitive and the associated Gaussian process prior is also improper [14]. Table 1 explicitly compares the theoretical characteristics of previous ext-GPSP with the present OUGP filter. The comparative stop band roll-off performance is illustrated in Fig. 1.

3 Results and discussion

3.1 Frequency response

A frequency-of-interest (FoI) vector was defined as:

$$\mathrm{FoI} = \left[0.0005\ldots0.003,\ 0.003\ldots0.01,\ 0.04\ldots0.15,\ 0.15\ldots0.40\right]\ \mathrm{Hz},$$

corresponding to the standard frequency ranges [11] ultra-low (ULF), very-low (VLF), low-frequency (LF) and high-frequency (HF), and additionally the range ultra-low-star (ULF*):

$$\mathrm{FoI}* = [0.002\ldots0.01]\ \mathrm{Hz}.$$

In 5 series of 1,000 realisations, the mean amplitude responses to white Gaussian noise (amplitudes), projected onto an irregular time axis formed as the running cumulative sum of amplitudes (as in an HRV tachogram) [2, 9], were determined using the OUGP filter in band-pass configuration (see website resources [6]). The performance is illustrated in Fig. 2 and the agreement between theory and its practical realization were shown in Table 2.

Table 1 Comparison of previous extended Gaussian process smoothing priors filter (ext-GPSP) [3] and the present Ornstein–Uhlenbeck Gaussian process filter (OUGP)

Feature	Filter model	
	ext-GPSP	OUGP
Filter order	Second	Third
Definition terms	Inverse covariance function	Explicit covariance function
Gaussian process dynamics	Non-stationary	Stationary
Definition of Gaussian process prior	Improper, i.e., it cannot be integrated (but the posterior expectation is well-defined nonetheless)	Proper probability density function

3.2 Detrending and low-pass filtering

A 30 min synthetic HRV tachogram with a median frequency of 1 Hz was realised from the sum of four sinusoids

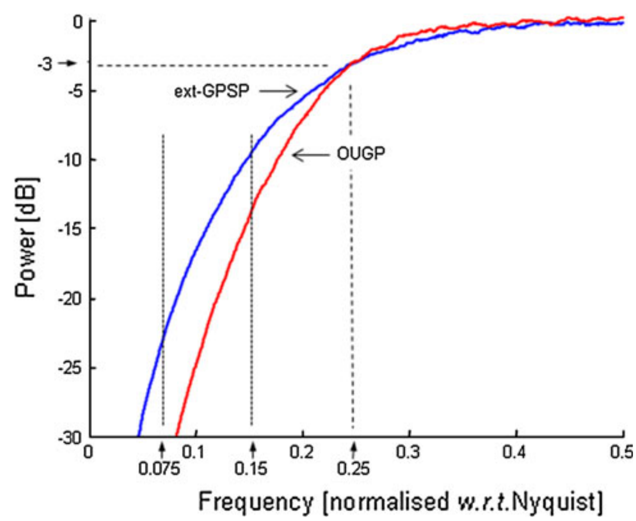


Fig. 1 Comparison of ext-GPSP and OUGP stop-band roll off in high-pass configuration with a normalised Fc of 0.25 in the octave [0.075 … 0.15] (average of 1,000 Monte Carlo realisations)

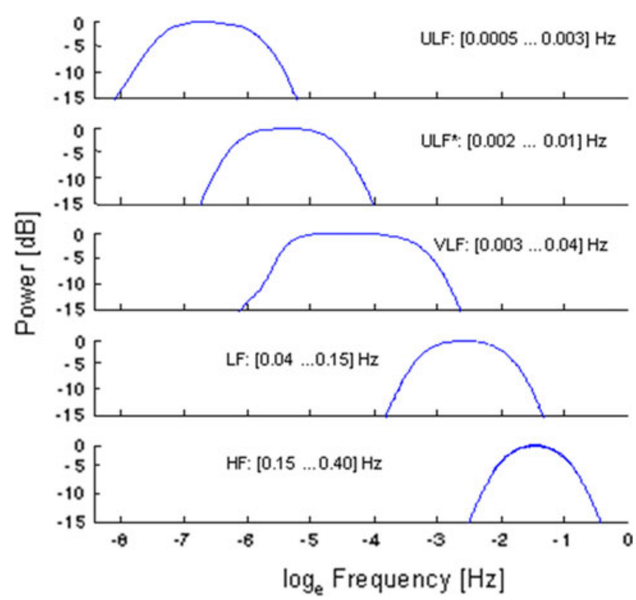


Fig. 2 Power band-pass characteristics achievable using the OUGP band-pass filter as estimated from Monte Carlo analysis of Gaussian random noise tachogram (pass-band gain set to 1)

of Gaussian-distributed centre frequency and zero phase, viz:

$$\mathrm{HRV}(t) = \sum_{i \in \{\mathrm{ulf,vlf,lf,hf}\}} G_i \sin(2\pi f_{i,\sigma_i} t) \tag{4}$$

where G is the amplitude gain, f and σ are centre frequency and standard deviation (Hz).

The sinusoidal components were defined as in Table 3.

A non-stationary feature was added as a 4 dB (w.r.t. total sinusoidal power) Brownian component constructed as the cumulative sum of a random Gaussian series.

Table 2 Comparison of theoretical and simulated −3 dB bandwidths

Frequency band	Design (Hz)	Realisation (Hz)
ULF	[0.00050 … 0.00300]	[0.00054 … 0.00300]
VLF	[0.00200 … 0.01000]	[0.00205 … 0.00990]
LF	[0.00300 … 0.04000]	[0.00305 … 0.03990]
HF	[0.15000 … 0.40000]	[0.14000 … 0.40660]

Table 3 Components of synthetic HRV tachogram

Frequency band	Amplitude gain, G	Centre frequency, f (Hz)	STD frequency, σ (Hz)
ULF	2	0.002	0.15
VLF	1	0.025	0.15
LF	1	0.01	0.5
HF	1	0.25	0.5

This tachogram was band-pass filtered with OUGP to achieve detrending (high-pass response) and low-pass band-limiting at 0.003 and 0.4 Hz −3 dB points, respectively. The time domain and frequency domain (as the Lomb Scargle PSD [2, 9]) performance is shown in Fig. 3. The decomposition in the time domain by a series band-pass operations corresponding to the VLF, LF and HF frequency bands is shown in Fig. 4.

3.3 GPOU interactive web pages

The internet-accessible GPOU website provides an interactive GUI in which the User can apply the GPOU band-pass filter to any one of six HRV data sets:

Synthetic 3 peak ~[0.045, 0.12, 0.25] Hz
Above with addition of 4 dB Brownian noise (see Performance)
Gaussian white noise
Normal subject at heart rate ~0.85 Hz
Premature baby at heart rate ~2.5 Hz
Textbook [12]: LF at ~0.1 (SD 0.010) Hz and HF at ~0.25 (SD 0.015) Hz

Bandwidths are user-selectable (at −3 dB points):

High-pass (detrending): {0.001, 0.002, 0.005, [0.001 … 0.1]} Hz
Low-pass: {0.0125, 0.025, [0.3…1.0]} Hz.

Results are displayed both as the time series decomposition or as the Lomb Scargle PSD. The MATLAB

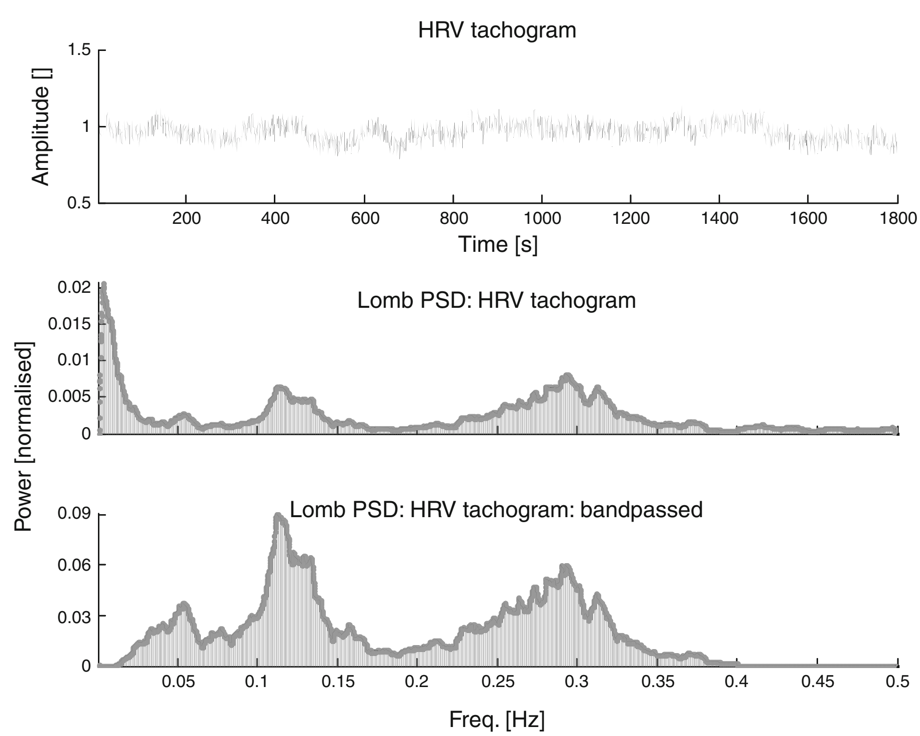
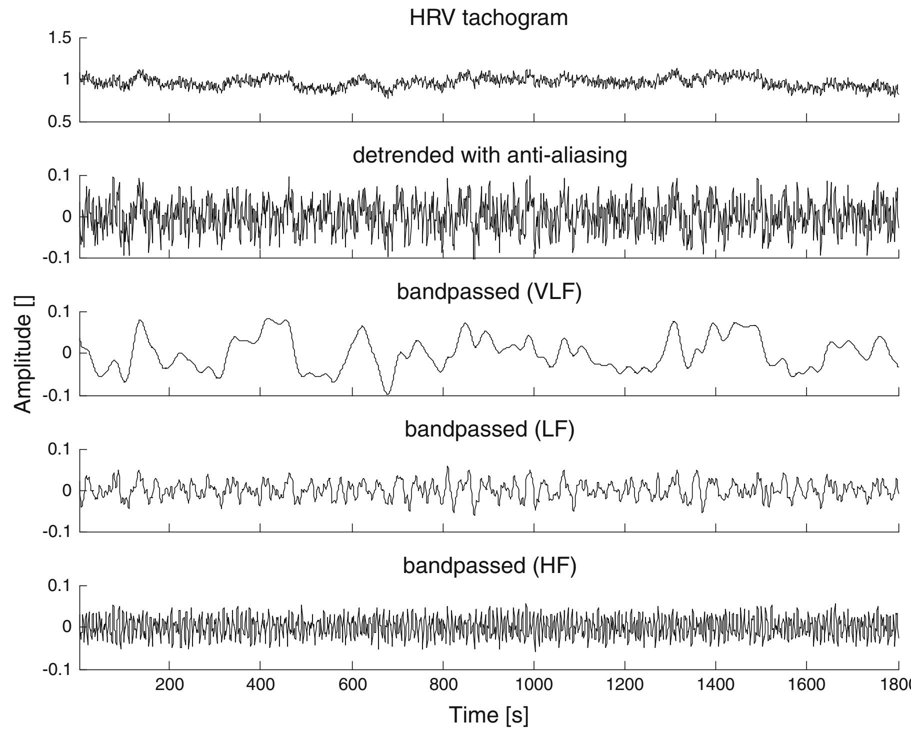


Fig. 3 Artificial HRV tachogram as a time series (*top pane*) with its Lomb Scargle PSD (*middle pane*). The PSD after OUGP band-pass filtering [0.003 … 0.35] Hz for detrending and low-pass filtering (*bottom pane*). Note: the 0.35 Hz low-pass effect reduces the amplitude of the local peak around 0.29 Hz: this is consistent with the implicit third-order response

(version $\geq$ 2006b) code fully commented in HTML for the GPOU filter (gpsmooth_2.m) and the supporting code for the optimized Lomb Scargle (fLSPw.m) are downloadable. A guide is given to the rationale for setting the bandwidth for the exemplar datasets. An example session is given as Fig. 5.

Fig. 4 Decomposition of the artificial tachogram by a series of OUGP band-pass filters



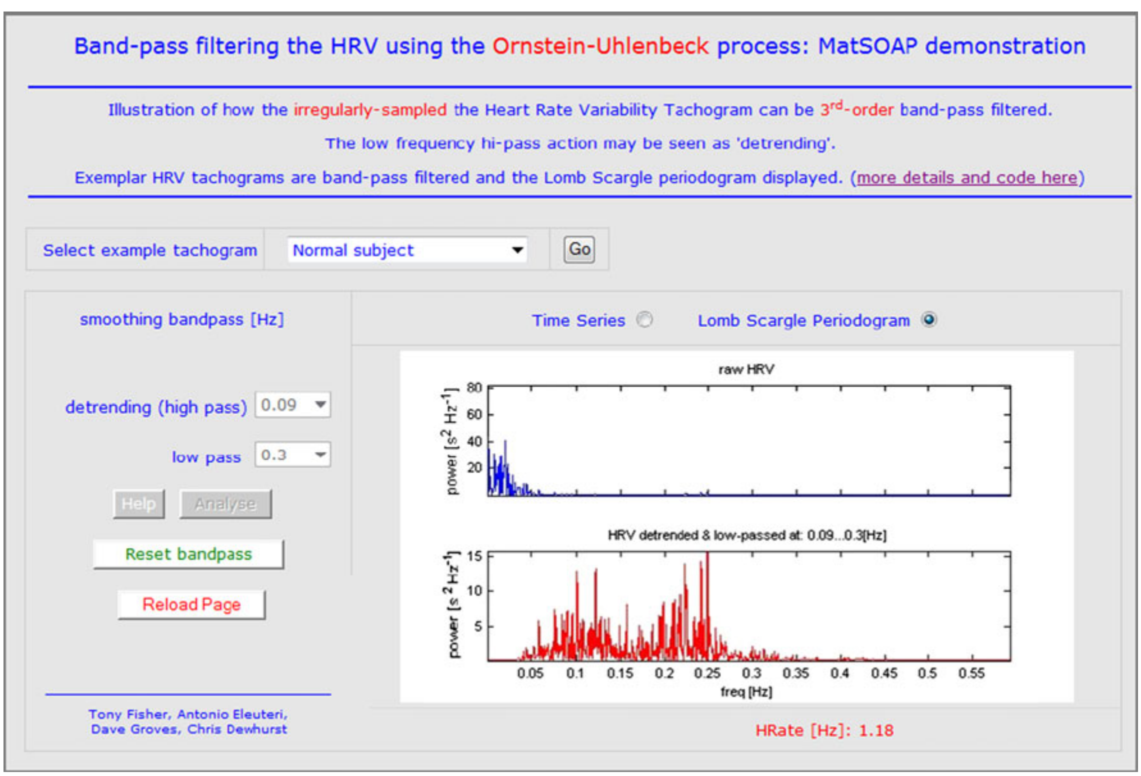


Fig. 5 Internet-accessible interactive demonstration of the OUGP band-pass filter. The User has selected the 'Normal subject' data set, detrended at 0.09 Hz, low-pass filtered at 0.3 Hz and enabled the Lomb Scargle PSD display

The web pages require JavaScript to be enabled and are optimized for the Microsoft Internet Explorer® but run adequately in Firefox® and Safari® under Microsoft Windows® and Linux operating systems. The GPOU and its associated applications execute on a MatSOAP© server which provides access to automation instances of MATLAB over the internet using the Simple Object Access Protocol (SOAP) [5]. Details of MatSOAP can be obtained from the authors [4].

A comparison of GPOU with the extended Gaussian-process smoothing model (ext-GPSP) [3] can be made interactively by referring to http://clinengnhs.liv.ac.uk/links.htm.

4 Conclusion

The OUGP filter is efficiently implemented in MATLAB. As a time domain band-pass filter, it exhibits a predictable and stable third-order zero-phase frequency response with explicit -3 dB points. It can be applied to both regularly and irregularly spaced data without the requirement for resampling. The latter property is suitable to analysis of the HRV tachogram, either as a pre-processing operation prior to PSD estimation by, for example, the Lomb Scargle method (with detrending and low-pass filtering), or directly in the time domain as a series of band-pass filters. The open source code and an interactive demonstration webpage with five exemplar HRV tachograms are maintained at http://clinengnhs.liv.ac.uk/links1.htm [6].

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