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Summary

The aim of this project is to identify the right level of settings (or dimensions) to maximize the flight time of a Paper Helicopter. The physical dimensions considered in designing the helicopter are wing length, wing width, body length, body width, middle body length and fold at the tip. The

-

Burman Design we designed the initial orthogonal array OA (12, 2^{11}

Effect Sparsity

For doing so we follow the following steps:

Step I: Implementing the Plackett-Burman Design to make the required Orthogonal Array OA (12, 2^{11}) which we would use to make the paper helicopters.
Step II: Constructing the required paper helicopters using the runs in the design matrix using the

Step III: Randomizing the order of runs to get random and more reliable analysis.
(**Randomization** means randomly assigning (by chance) the runs to the experimental group. This helps us in creating homogeneous treatment groups eliminating potentials bias and judgements in the process.)
Step IV: Replicating the results to get better approximations. Here, we replicated each run thrice. (**Replication** is a process to repeating the run to reproduce the results and averaging the output at the end.)
Step V:
Further, we averaged the data and calculated its variance which we used to calculate the Main Effects and 2 Factor Interaction Effects using which the significant factors were identified using Half Normal Plots.
Step VI: The Stepwise Regression Method suggested in the Hamada Wu Effect Sparsity Analysis was adopted and the equations for both location and dispersion were constructed.
Step VII: Using the location and dispersion equations, we identified the optimal level settings for the significant factors to maximize the flight time of our paper helicopter.
Step VIII: A new model was constructed using our optimal model (optimal level settings for the significant factors) and the above results were tested.

Employing the above steps, we successfully increased the flight time of our paper helicopter to 3.08 seconds from our previous average of 2.4136 seconds. This value is for our optimal model constructed in Step VIII.

The aim of this project was to apply the practical knowledge about the methods learnt in the

identifying the right settings for its various parameters. In this case, we identified the right levels for the dimensions of a paper helicopter to maximize its flight time.

Abstract

This project is aimed to improve the design of a paper helicopter to maximize its flight time by identifying the significant factors and setting them at their optimal levels. We employed Plackett-Burman Design and Hamada Wu Effect Sparsity Analysis to achieve this setting of the significant factors of the paper helicopter. Using Randomization and Replication, we ensured the random nature of our experiment and improved its approximation.

Important Keywords: Design of Experiments (DoE), Orthogonal Array (OA), Plackett-Burman Design, Hamada Wu Effect Sparsity Analysis, Effect Heredity Principle, Main Effects, 2 Factorial Effects, Half Normal Plot, Significant Factors, Location, Dispersion, Regression.

Introduction

Each experiment has many factors affecting its outcome. Design of Experiments (DoE) helps us to identify the significant factors broadly affecting the outcome of the experiment and set them at the optimal level and thus, improving our products efficiency and performance. Here, using Plackett Burman Design we constructed our design matrix using 12 runs. The parameters and their levels available to us at the beginning of the experiment are as follows:

Table 1: Levels & Factors of Paper Helicopter

Factors	**Symbol**	**Dimensions**	
		- level	**+ level**
Wing Length	l	3 inches	4.5 inches
Wing Width	w	1.8 inches	2.4 inches
Body Length	L	3 inches	4.5 inches
Body Width	W	1.25 inches	2 inches
Middle Body Length	d	1 inch	1.5 inches
Fold at Tip	F	NO	YES

The parameters mentioned in the above table apply to our paper helicopter design as follows:

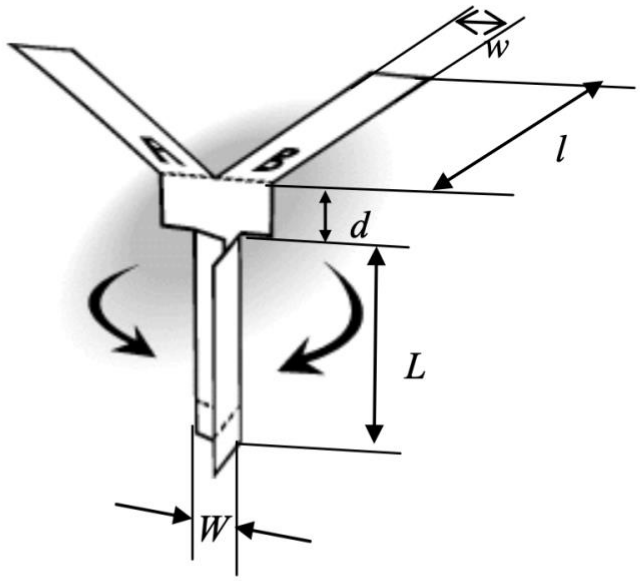

Figure 1: Paper Helicopter Diagram

Using the above table and design, we constructed 12 helicopters, record their flight times, perform randomization & replication, and identify the significant factors & their optimal levels.

Method & Design Analysis

For maximizing the flight time, we identify the optimal design of our paper helicopter, the steps for doing so which were previously stated in Executive Summary are elaborated below:

Design Table:

We construct an Orthogonal Array OA (12, 2^{11}) by applying Plackett Burman Design.

Orthogonal Array:

An **orthogonal array** OA (N, $s1^{m_1}$ ... s^{m}, t) of strength t is an N×m matrix, m = m_1 + ... + m , in which m_i columns have s_i

possible combinations of symbols appear equally often in the matrix. *(From, Slide Number 4, Unit 7: Orthogonal Array Experiments and Response Surface Methodology Slideset)*

The two reason for using Orthogonal Arrays are as follows:

Run Size Economy: Instead of using a uniform factor design, we use mixed factor designs to reduce the number of runs and thus, make our experiment more economically viable.

Flexibility: Many Orthogonal Arrays exist for various flexible combinations of different Factor Levels. Here, the design is more flexible, and we make use of 12 runs in our design experiment.

re table w.r.t the

first row. Our OA (12, 2^{11}) design will have the following format:

Table 2: 12 runs for Design Matrix

$N = 12.$

Run	1	2	3	4	5	6	7	8	9	10	11
1	+	+	−	+	+	+	−	−	−	+	−
2	−	+	+	−	+	+	+	−	−	−	+
3	+	−	+	+	−	+	+	+	−	−	−
4	−	+	−	+	+	−	+	+	+	−	−
5	−	−	+	−	+	+	−	+	+	+	−
6	−	−	−	+	−	+	+	−	+	+	+
7	+	−	−	−	+	−	+	+	−	+	+
8	+	+	−	−	−	+	−	+	+	−	+
9	+	+	+	−	−	−	+	−	+	+	−
10	−	+	+	+	−	−	−	+	−	+	+
11	+	−	+	+	+	−	−	−	+	−	+
12	−	−	−	−	−	−	−	−	−	−	−

experimental data with mean flight time (Ybar), variance (s^2), and logarithmic value of variance (lnssq) are as follows:

Table 3: Experimental Data

Run	A	B	C	D	E	F	7	8	9	10	11	Y1	Y2	Y3	Ybar	s^2	lnssq
1	1	1	-1	1	1	1	-1	-1	-1	1	-1	2.81	2.78	2.98	2.85666667	0.01163333	-4.4538807
2	-1	1	1	-1	1	1	1	-1	-1	-1	1	2.41	1.91	1.86	2.06	0.0925	-2.3805466
3	1	-1	1	1	-1	1	1	1	-1	-1	-1	2.1	2.45	2.65	2.4	0.0775	-2.5574773
4	-1	1	-1	1	1	-1	1	1	1	-1	-1	2.1	2.23	2.54	2.29	0.0511	-2.9739708
5	-1	-1	1	-1	1	1	-1	1	1	1	-1	1.83	2.02	2.26	2.03666667	0.04643333	-3.0697377
6	-1	-1	-1	1	-1	1	1	-1	1	1	1	2.03	2.28	2.28	2.19666667	0.02083333	-3.871201
7	1	-1	-1	-1	1	-1	1	1	-1	1	1	2.3	2.63	2.16	2.36333333	0.05823333	-2.8432974
8	1	1	-1	-1	-1	1	-1	1	1	-1	1	3.13	3.31	3.3	3.24666667	0.01023333	-4.5821049
9	1	1	1	-1	-1	-1	1	-1	1	1	-1	2.93	2.83	2.46	2.74	0.0613	-2.7919754
10	-1	1	1	1	-1	-1	-1	1	-1	1	1	2.3	2.2	2.13	2.21	0.0073	-4.9198809
11	1	-1	1	1	1	-1	-1	-1	1	-1	1	2.15	2.2	2.33	2.22666667	0.00863333	-4.7521246
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	2.38	2.1	2.53	2.33666667	0.04763333	-3.0442225

Helicopter Construction

Now, we construct helicopters as per the design matrix and record flight times thrice. The helicopters constructed by us are shown below:

Figure 2: 12 Paper Helicopters

After recording the flight times, we used it to find the average flight time for each run and the variance of the flight times urther analysis by Effect Sparsity Principle suggested by the Hamada Wu Method.

Traditional Interpretation on Main Factors

The analysis for Location and Dispersion of Main Effects are as follows:

Table 4: Parameters for Half Normal Plots

Name (Main)	Main Effect Ybar (Main)	Main Effect ln (s^2) (Main)
A	0.45055556	-0.2868835
B	0.30722222	-0.3273832
C	-0.2694444	0.21615577
D	-0.1005556	-0.8027751
E	-0.2161111	0.21555072
F	0.105	0.06842054
7	-0.1438889	1.2339138
8	0.02166667	0.05791365
9	0.085	-0.3069682
10	-0.0261111	-0.2765877
11	-0.0594444	-0.7429818

Based on the Main Effects, we can get the significant factors:

<u>Half Normal Plot for Location:</u>

```
halfnorm(`Main Effect Ybar (Main)`, nlab = 11, labs = as.character(`Name
(Main)`), ylab = "Sorted Data")
```

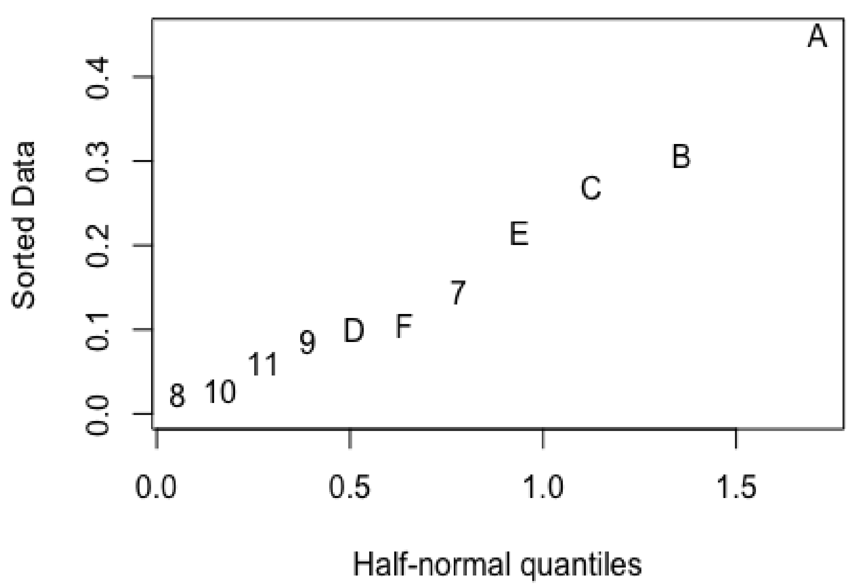

Figure 3: Location Half Normal Plot

From the above plot, we can conclude that the Factor A is the most significant factor.

<u>Half Normal Plot for Dispersion:</u>

```
halfnorm(`Main Effect ln (s^2) (Main)`, nlab = 11, labs = as.character(`Name (Main)`), ylab = "Sorted Data")
```

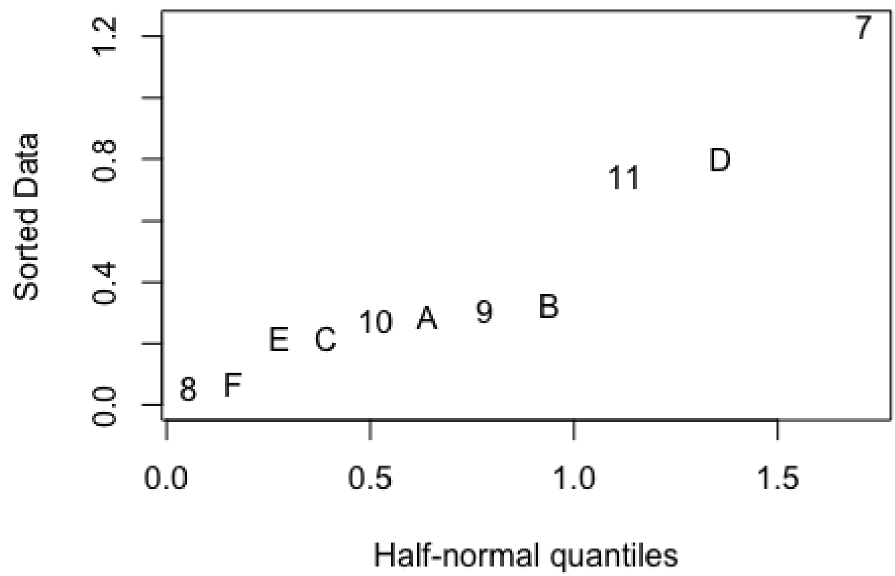

Figure 4: Dispersion Half Normal Plot

From the above plot, we can conclude that the factors 7, 11, and D are the most significant but since, 7 and 11 are assumed, we would only consider factor D to be a significant one.

From the above analysis (Page 8 and Page 9), we have the Location and Dispersion equations as follows:

<u>Location Equation:</u>

Y-hat = 2.41361111 + 0.22527778 X_A ← Equation (I)

<u>Dispersion Equation:</u>

Z-hat = ln (s-hat)^2 = -3.52003499 0.40139755 X_D ← Equation (II)

Design Analysis: Strategy & Execution

Since, methods like complex aliasing, and partial aliasing are computationally very demanding and expensive, hence we would use a different method to analyze our data (Hamada Wu Method).

Hamada Wu Effect Sparsity Method:

Here, we would limit our model to 2 factor interactions to take advantage of Complex Aliasing by applying Effect Sparsity Principle to our Complex Aliasing Procedure.

The steps used in Hamada Wu method to find the best model are as follows:

Step I: For each Factor X, apply Stepwise Regression to a model involving X and all the 2 factor interactions involving X.
Step II: Follow Step I for each factor involved in our experiment.
Step III: Use Stepwise Regression on a model which involves all the Main Effects in the experiment, and all the 2 Factor Interactions which are found to be significant in Steps I, and II.
Step IV: Use Effect Heredity Principle, to eliminate the 2 factor interactions which are found (Effect Heredity Principle favors the exclusion of higher order interaction effects which do not have at least one of their respective parent lower order effects in the model.)
Step V: Again, apply Stepwise Regression to the remaining model and check if any of the

Step VI: Follow Steps IV, and V till no such 2 factor interactions can be found which do have a parent Main Effect as significant.
Step VII: The model identified in at the end of Step VI is the final required model.

Hamada Wu Analysis for Location Parameters:

The table below contains all the Main Effects, the 2 Factor Interactions and the mean flight time of all the replications for each run:

Table 5: Experimental Data for Location

Run	A	B	C	D	E	F	7	8	9	10	11	AB	AC	AD	AE	AF	BC	BD	BE	BF	CD	CE	CF	DE	DF	EF	Ybar
1	1	1	-1	1	1	1	-1	-1	-1	1	-1	1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	1	1	2.85666667
2	-1	1	1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1	2.06
3	1	-1	1	1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	2.4
4	-1	1	-1	1	1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	-1	-1	2.29
5	-1	-1	1	-1	1	1	-1	1	1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	2.03666667
6	-1	-1	-1	1	-1	1	1	-1	1	1	1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	2.19666667
7	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	2.36333333
8	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	-1	1	-1	-1	3.24666667
9	1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	1	1	1	2.74
10	-1	1	1	1	-1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	2.21
11	1	-1	1	1	1	-1	-1	-1	1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	2.22666667
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2.33666667

```
library(readxl)
Effects <- read_excel("/Users/mohitdeepakchhaparia/Documents/ISEN 616/Project
/Effects.xlsx")

## New names:
## * `` -> ...34
## * `` -> ...35
## * `` -> ...39
## * `` -> ...43
## * `` -> ...44
## * ...

attach(Effects)

## The following object is masked from package:base:
##
##     F

library(olsrr)

## Registered S3 methods overwritten by 'car':
##   method                          from
##   influence.merMod                lme4
##   cooks.distance.influence.merMod lme4
##   dfbeta.influence.merMod         lme4
##   dfbetas.influence.merMod        lme4

##
## Attaching package: 'olsrr'

## The following object is masked from 'package:faraway':
##
##     hsb

## The following object is masked from 'package:datasets':
##
##     rivers

fit.A <- lm(Ybar ~ A + AB + AC + AD + AE + AF, data = Effects)
ols_step_both_p(fit.A, pent = 0.05)

##
##                              Stepwise Selection Summary
## --------------------------------------------------------------------------
---------
##                      Added/                  Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## --------------------------------------------------------------------------
---------
##    1       A        addition       0.432       0.375    30.4670    7.5927
0.2833
```

```
## -------------------------------------------------------------------------------
---------

fit.B <- lm(Ybar ~ B + AB + BC + BD + BE + BF, data = Effects)
ols_step_both_p(fit.B, pent = 0.05)

##
##                              Stepwise Selection Summary
## -------------------------------------------------------------------------------
---------
##                      Added/                       Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## -------------------------------------------------------------------------------
---------
##    1       AB       addition       0.205       0.126    2.8700    11.6169
0.3350
## -------------------------------------------------------------------------------
---------

fit.C <- lm(Ybar ~ C + AC + BC + CD + CE + CF, data = Effects)
ols_step_both_p(fit.C, pent = 0.05)

##
##                              Stepwise Selection Summary
## -------------------------------------------------------------------------------
---------
##                      Added/                       Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## -------------------------------------------------------------------------------
---------
##    1       CF       addition       0.234       0.157    0.3580    11.1738
0.3288
## -------------------------------------------------------------------------------
---------

fit.D <- lm(Ybar ~ D + AD + BD + CD + DE + DF, data = Effects)
ols_step_both_p(fit.D, pent = 0.05)

##
##                              Stepwise Selection Summary
## -------------------------------------------------------------------------------
---------
##                      Added/                       Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## -------------------------------------------------------------------------------
---------
##    1       DE       addition       0.349       0.284    -0.8790    9.2252
0.3032
```

```
## ----------------------------------------------------------------------------
---------

fit.E <- lm(Ybar ~ E + AE + BE + CE + DE + EF, data = Effects)
ols_step_both_p(fit.E, pent = 0.05)

## 
##                              Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                         Added/                    Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ----------------------------------------------------------------------------
---------
##    1       DE       addition       0.349       0.284    -0.8710    9.2252
0.3032
## ----------------------------------------------------------------------------
---------

fit.F <- lm(Ybar ~ F + AF + BF + CF + DF + EF, data = Effects)
ols_step_both_p(fit.F, pent = 0.05)

## 
##                              Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                         Added/                    Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ----------------------------------------------------------------------------
---------
##    1       CF       addition       0.234       0.157    0.9610    11.1738
0.3288
## ----------------------------------------------------------------------------
---------
```

Based on the above analysis, we found that the 2 Factor Interactions AB, CF, and DE are significant. Using these three 2 Factor Interactions and the 6 Main Effects, we will apply stepwise regression as follows:

```
newfit <- lm(Ybar ~ A + B + C + D + E + F + AB + CF + DE, data = Effects)
ols_step_both_p(newfit, pent = 0.065)

## 
##                               Stepwise Selection Summary
## ----------------------------------------------------------------------------
----------
##                         Added/                    Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
```

```
## ----------------------------------------------------------------------------
----------
##    1        A        addition       0.432       0.375    29.9760     7.5927
0.2833
##    2        AB       addition       0.637       0.556    18.2810     4.2256
0.2387
##    3        B        addition       0.837       0.776     6.8780    -3.4094
0.1695
## ----------------------------------------------------------------------------
----------
```

paper helicopter), and Interaction Factor AB are significant.

Hamada Wu Analysis for Dispersion Parameters:

The table below contains all the Main Effects, the 2 Factor Interactions and the variance and logarithmic variance for flight time of all the replications for each run:

Table 6: Experimental Data for Dispersion

Run	A	B	C	D	E	F	7	8	9	10	11
1	1	1	-1	1	1	1	-1	-1	-1	1	-1
2	-1	1	1	-1	1	1	1	-1	-1	-1	1
3	1	-1	1	1	-1	1	1	1	-1	-1	-1
4	-1	1	-1	1	1	-1	1	1	1	-1	-1
5	-1	-1	1	-1	1	1	-1	1	1	1	-1
6	-1	-1	-1	1	-1	1	1	-1	1	1	1
7	1	-1	-1	-1	1	-1	1	1	-1	1	1
8	1	1	-1	-1	-1	1	-1	1	1	-1	1
9	1	1	1	-1	-1	-1	1	-1	1	1	-1
10	-1	1	1	1	-1	-1	-1	1	-1	1	1
11	1	-1	1	1	1	-1	-1	-1	1	-1	1
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Run	AB	AC	AD	AE	AF	BC	BD	BE	BF	CD	CE	CF	DE	DF	EF	s^2	lnssq
1	1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	1	1	0.01163333	-4.4538807
2	-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1	0.0925	-2.3805466
3	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	0.0775	-2.5574773
4	-1	1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	-1	-1	0.0511	-2.9739708
5	1	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	0.04643333	-3.0697377
6	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	0.02083333	-3.871201
7	-1	-1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	0.05823333	-2.8432974
8	1	-1	-1	-1	1	-1	-1	-1	1	1	1	-1	1	-1	-1	0.01023333	-4.5821049
9	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	1	1	1	0.0613	-2.7919754
10	-1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	0.0073	-4.9198809
11	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	0.00863333	-4.7521246
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.04763333	-3.0442225

```
fit.A <- lm(lnssq  ~ A + AB + AC + AD + AE + AF, data = Effects)
ols_step_both_p(fit.A, pent = 0.05)

##
##                               Stepwise Selection Summary
## ----------------------------------------------------------------------------
----------
##                      Added/                       Adj.
## Step    Variable    Removed     R-Square     R-Square     C(p)        AIC
RMSE
## ----------------------------------------------------------------------------
----------
##    1        AE       addition       0.267       0.194    -1.3390    33.5763
0.8363
## ----------------------------------------------------------------------------
----------
```

```
fit.B <- lm(lnssq ~ B + AB + BC + BD + BE + BF, data = Effects)
ols_step_both_p(fit.B, pent = 0.05)

##
##                               Stepwise Selection Summary
## -----------------------------------------------------------------------------
----------
##                        Added/                  Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## -----------------------------------------------------------------------------
----------
##    1       BE       addition       0.118       0.030    -2.0680    35.7946
0.9173
## -----------------------------------------------------------------------------
----------

fit.C <- lm(lnssq ~ C + AC + BC + CD + CE + CF, data = Effects)
ols_step_both_p(fit.C, pent = 0.05)

##
##                              Stepwise Selection Summary
## -----------------------------------------------------------------------------
---------
##                        Added/                  Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## -----------------------------------------------------------------------------
---------
##    1       CF       addition       0.631       0.594    3.9430    25.3296
0.5931
## -----------------------------------------------------------------------------
---------

fit.D <- lm(lnssq ~ D + AD + BD + CD + DE + DF, data = Effects)
ols_step_both_p(fit.D, pent = 0.05)

##
##                               Stepwise Selection Summary
## -----------------------------------------------------------------------------
----------
##                        Added/                  Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## -----------------------------------------------------------------------------
----------
##    1       D        addition       0.203       0.123    -0.3620    34.5860
0.8722
## -----------------------------------------------------------------------------
----------
```

```
fit.E <- lm(lnssq ~ E + AE + BE + CE + DE + EF, data = Effects)
ols_step_both_p(fit.E, pent = 0.05)

##
##                               Stepwise Selection Summary
## ---------------------------------------------------------------------------
----------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
----------
##    1       AE       addition       0.267       0.194    -0.7400    33.5763
0.8363
## ---------------------------------------------------------------------------
----------
```

```
fit.F <- lm(lnssq ~ F + AF + BF + CF + DF + EF, data = Effects)
ols_step_both_p(fit.F, pent = 0.05)

##
##                              Stepwise Selection Summary
## ---------------------------------------------------------------------------
---------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ---------------------------------------------------------------------------
---------
##    1       CF       addition       0.631       0.594    2.5450    25.3296
0.5931
## ---------------------------------------------------------------------------
---------
```

From the above analysis, we conclude that the three 2 Factor Interactions AE, BE, and CF are significant. Using these three 2 Factor Interactions and the 6 Main Effects, we will apply stepwise regression as follows:

```
newfit <- lm(lnssq ~ A + B + C + D + E + F + AE + BE + CF, data = Effects)
ols_step_both_p(newfit, pent = 0.065)

##
##                               Stepwise Selection Summary
## ---------------------------------------------------------------------------
----------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
----------
##    1       CF       addition       0.631       0.594    -2.8830    25.3296
```

```
0.5931
## ---------------------------------------------------------------------------
----------
```

From the above stepwise regression, the 2 Factor Interaction CF is found significant but by Effect Heredity Principle, we remove all the 2 Factor Interactions for which none of the the 2 Factor Interaction CF.

```
newfit <- lm(lnssq ~ A + B + C + D + E + F + AE + BE, data = Effects)
ols_step_both_p(newfit, pent = 0.065)

##
##                              Stepwise Selection Summary
## ---------------------------------------------------------------------------
---------
##                        Added/                    Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
---------
##    1       AE       addition       0.267       0.194    5.3240    33.5763
0.8363
## ---------------------------------------------------------------------------
---------
```

From the above stepwise regression, the 2 Factor Interaction AE is found significant. But by Effect Heredity Principle, we remove all the 2 Factor Interactions for which none of the the 2 Factor Interaction AE, and CF (from the previous step).

```
newfit <- lm(lnssq ~ A + B + C + D + E + F + BE, data = Effects)
ols_step_both_p(newfit, pent = 0.065)

##
##                              Stepwise Selection Summary
## ---------------------------------------------------------------------------
---------
##                        Added/                    Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
---------
##    1       D        addition       0.203       0.123    2.1840    34.5860
0.8722
##    2       BE       addition       0.477       0.361    0.6810    31.5274
0.7447
## ---------------------------------------------------------------------------
---------
```

From the above stepwise regression, the 2 Factor Interaction BE, and the Main Effect D are found significant. But by Effect Heredity Principle, we remove all the 2 Factor Interactions
the above step after
removing the 2 Factor Interaction BE, AE, and CF (from the previous step).

```
newfit <- lm(lnssq ~ A + B + C + D + E + F, data = Effects)
ols_step_both_p(newfit, pent = 0.065)

##
##                              Stepwise Selection Summary
## ---------------------------------------------------------------------------
----------
##                        Added/                    Adj.
## Step    Variable     Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
----------
##    1       D        addition       0.203       0.123    -2.3610    34.5860
0.8722
## ---------------------------------------------------------------------------
----------
```

The final significant factor for the dispersion model is only D which cannot be further changed.

From the above analysis (Page 10 to Page 18), we have the Location and Dispersion equations as follows:

Location Equation:

Y-hat = 2.41361111 + 0.22527778 X_A + 0.15361111 X_B + 0.15527778 X_{AB} ← Equation (III)

Dispersion Equation:

Z-hat = ln (s-hat)^2 = -3.52003499 0.40139755 X_D ← Equation (IV)

factors in our Location and Dispersion Equations.

For minimizing the dispersion and in turn, minimizing the variance, we would fix the (which is the Body Width of the Paper Helicopter) at the higher level which is 2 inches.

For maximizing the flight time, we would fix the significant factors A and B at a (+) level, that

(which is the Wing Width of the Paper Helicopter) at the higher level which are 4.5 inches, and 2.4 inches respectively.

From the above equations, we get the optimal design where the levels for various significant factors are as follows:

A (+)

B (+)

D (+)

Thus, for increasing the flight time of our Paper Helicopter the significant parameters should having the following dimensions:

Wing Length (l) 4.5 inches

Wing Width (w) 2.4 inches

Body Width (W) 2 inches

Using the above-mentioned set of level for the significant factors we get the theoretical value of Variance as 0.0198127 and the theoretical value of Y-hat as 2.95 seconds.

Validation of Optimal Design

We need to construct our Paper Helicopter again by fixing the levels of the three significant factors mentioned above. From trial and error, we can fix the levels of the other factors which . Thus, the final set of dimensions for our Optimal Helicopter are as follows:

Table 7: Levels & Factors for Optimal Paper Helicopter

Factors	**Symbol**	**Level Setting**	**Optimal Dimension Setting**
Wing Length	l	+	4.5 inches
Wing Width	w	+	2.4 inches
Body Length	L	-	3 inches
Body Width	W	+	2 inches
Middle Body Length	d	-	1 inch
Fold at Tip	F	+	YES

Hence, by constructing a new Paper Helicopter as per me the above mentioned dimensions, we get an increase in our flight time from an average of the experiment of 2.4136 seconds to the flight time of our optimal model of 3.08 seconds (which is an increase of 27.61% increase in the flight time). The image of the constructed Paper Helicopter as per the Optimal Model is as follows:

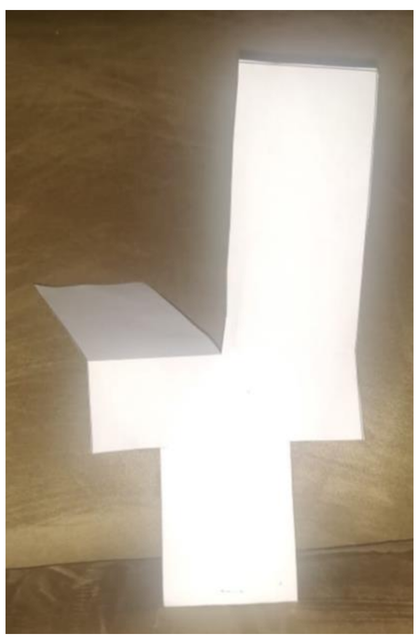

Figure 5: Optimal Paper Helicopter

Results, Conclusions & Recommendations

The design was constructed using Plackett Burman Method and for 12 runs the Orthogonal Array was constructed. With the help of this design, we constructed our Paper Helicopters and recorded their flight times.

Using Half nificant

n and factor D

By comparison, we can see that the Half

icant.

We set the levels of these significant factors to achieve the required optimization (or maximization) in the flight time of our Paper Helicopter. The dimensions of the Optimal Model are shown in Table 7 in Validation of Optimal Design on Page number 20.

There is an increase in the flight time of our optimal model compared to the average flight time of all the models in this experiment by 27.61%. Thus, it can be concluded

Design and Analysis of

Appendix

<u>Definitions & Descriptions:</u>