CS 763 – Assignment 6

Q1.

1. Let us consider a point in real world $\mathbf{X}$. And the transformation between two cameras is described by rotation matrix $\mathbf{R}$ and a translation vector $\mathbf{t}$ such that $\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{t}$. Assume that the first camera is aligned to world co-ordinate and situated at origin and camera matrix is $\mathbf{K}$. The camera calibration matrix of the second camera is $\mathbf{K}'$.

   (a) What are the expressions for the epipoles, $\mathbf{e}$ and $\mathbf{e}'$ in terms of one or more of the following: $\mathbf{P}$, $\mathbf{P}'$, $\mathbf{R}$, and $\mathbf{t}$, where $\mathbf{P}$ and $\mathbf{P}'$ camera projection matrix.

   (b) An alternate formulation of the fundamental matrix, $\mathbf{F} = [\mathbf{e}']_\times \mathbf{K}'\mathbf{R}\mathbf{K}^{-1}$. Using the results from the earlier parts, show that $\mathbf{e}' \times x' = \mathbf{F}x$ where x' is the image of $\mathbf{X}$ in the second camera coordinate system.

   Note: $[\mathbf{e}']_\times$ denotes skew symmetric.

Solution:

a) Epipoles are the projection of the camera centers on the other image. Therefore:

$$e' = P'O = P' \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$e = PR^{-1}(O - t) = PR^T(-t)$$

where O is the camera center of the first camera and also the origin of the world-coordinate system.

b) On one side we have

$$e' \times x' = l'$$

where l’ is the epipolar line in the second image.

On the other side we can also represent l’ as the projection of the ray Ox on to the second image. The ray can be parametrized as all the solutions to the equation $PX = x$ as:

$$P^{+\cdot} x + \lambda O$$

where $P^{+\cdot}$ is the pseudo-inverse of P.

Picking two points on the ray with $\lambda = 0, \lambda = \infty$ respectively we get $P^{+\cdot}$ and O respectively. The crossproduct of their projection on the 2nd image will again give us the epipolar line l’:

$$l' = P'O \times P'P^{+\cdot} x$$

from a) we learned that $e' = P'O$, we compute $P^{+\cdot} = \begin{bmatrix} K^{-1} \\ 0 \end{bmatrix}$ and from $X' = RX + t$ we derive

$$P' = K'[R|t]$$

so:

$$l' = e' \times K'[R|t] \begin{bmatrix} K^{-1} \\ 0 \end{bmatrix} x = e' \times K'RK^{-1} = [e']_x K'RK^{-1} = Fx$$

Combining these two conclusions we get:

$$e' \times x' = Fx$$