Vanishing points and Horizon

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Q5. a) A line of 3D points is represented as $\boldsymbol{X}(\boldsymbol{\omega}) = \boldsymbol{A} + \omega\boldsymbol{D}$**.** Where **'D'** is the direction vector and **'A'** is a point in 3D. Using x = c **X**/Z(from similar triangles).

The vanishing point its image is: $\alpha = \lim\limits_{\omega \to \pm\infty} x(\boldsymbol{\omega})$ = c($\boldsymbol{A} + \omega\boldsymbol{D}$)/($A_Z + \omega D_Z$) = c **D**/$D_Z$ = c $(\frac{Dx}{Dz}, \frac{Dy}{Dz}, 1)^T$

Here we can see that if we draw another line with point B lying on it and having direction **D,** we will also have the same equation as above since it is independent of **'A'.**

Therefore, we can say that parallel lines in R^3 have intersection point called vanishing point. Hence proved.

- From similar triangles: $x/X = c/Z$
- $(X, Y, Z)^T \longmapsto (cX/Z,\ cY/Z)$

b) Let direction vector corresponding to three sets of parallel lines be **D1,D2** and **D3**. As these lines lie on a plane we can say that **(D1 X D2) . D3** = 0 (Scalar Triple Product). ----- i)

As we know that line passing through two points in homogeneous coordinates is represented as
l = y1 X y2 and for a point 'y3' to pass through line 'l', l.y3 = 0.

Therefore, for three points(y1,y2,y3) to be collinear: (y1 X y2).y3 = 0 ----- ii)

Here three vanishing points are (Dx1,Dy1,Dz1); (Dx2,Dy2,Dz2); (Dx3,Dy3,Dz3).

From equation i) and ii) we can say that three vanishing points are collinear. Hence Proved