

Q.2

Let K_1 and K_2 be two camera matrices. We know that the fundamental matrix corresponding to these camera matrices is of the following form:

$$F = S_b R \quad (1)$$

where S_b is the matrix

$$S_b = \begin{pmatrix} 0 & b_y & -b_z \\ -b_y & 0 & b_x \\ b_z & -b_x & 0 \end{pmatrix} \quad (2)$$

Assume that $K_1 = [I|0]$ and $K_2 = [\mathbf{R}|\mathbf{b}]$, where \mathbf{R} is a 3×3 (non-singular) matrix. Prove that the last column of K_2 , denoted by \mathbf{b} , is one of the epipoles.

Solution:

$$S_b = \begin{pmatrix} 0 & b_y & -b_z \\ -b_y & 0 & b_x \\ b_z & -b_x & 0 \end{pmatrix}$$

We know that the two epipoles of any stereo system can be expressed as:

$$F\mathbf{e}' = 0 ; F = S_b R \quad \text{and} \quad F\mathbf{e} = 0 ; F^T = R^T S_b^T$$

Since S_b is skew-symmetric, we have $S_b^T = -S_b$.

Thus, we can simply plug in \mathbf{b} for \mathbf{e} and \mathbf{e}' :

$$\begin{aligned} F^T \mathbf{b} &= R^T S_b^T \mathbf{b} \\ &= -R^T S_b \mathbf{b} \\ &= -\mathbf{A} \mathbf{0} \end{aligned}$$

Since $S_b \mathbf{b} = 0$, \mathbf{b} must clearly be the epipole. Hence proved