Scalar Feedback based Joint Time-Frequency Precoder interpolation for MIMO-OFDM Systems

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***Abstract*—In MIMO-OFDM systems, knowledge of the precoding matrix enables effective resource allocation for high rates and low BER. However, feeding back precoders for all subcarriers introduces large overheads. Past work has shown that the orthonormal structure of the precoder enables effective parameterization and quantization using Givens rotations and Householder transformations. Also, the use of temporal and frequency correlations can further reduce the feedback requirements. We present a framework that utilizes adaptive feedback with joint time-frequency prediction for precoder construction at the transmitter. Simulations reveal that the proposed method achieves lower BERs for various channel profiles compared to other approaches.**

I. Introduction

The use of multiple antennas at the transmitter and receiver can significantly enhance the performance of wireless systems. Moreover, in these multiple-input multiple-output (MIMO) wireless systems, the transmitter can allocate resources more efficiently if the channel state information (CSI) is made available at the transmitter, both to improve data rates and to reduce BER [1]. To this end, it is necessary to efficiently feedback the CSI from the receiver to the transmitter. In wireless systems that use orthogonal frequency division multiplexing (OFDM), the feedback needs to be provided to the transmitter across all subcarriers, and this places a large feedback demand. However, since the CSI varies gradually in both time and frequency, effective techniques to reduce feedback overheads exist, using frequency domain interpolation temporal prediction. In particular, knowledge of the precoding matrices (that are typically the orthonormal right singular vectors of the channel matrix) is essential to achieve link capacity, as well as to parallelize the channel. In this paper, we focus on the problem of tracking the precoders of a MIMO-OFDM system across time and frequency for efficient CSI feedback by tracking a parametrized version of the matrices.

The problem of effective quantization and feedback of precoders for MIMO-OFDM systems has attracted significant attention in recent years. Many approaches involve codebook design over the Grassmann manifold [2–4], the Stiefel manifold [5, 6] and the Flag manifold [7], wherein the manifold structure of the precoders is used to obtain tangents for predictive quantization and interpolation across time and frequency. While these methods have been shown to be very effective for precoder feedback, they involve operations over higher dimensional manifolds. In particular, optimization and interpolation is complicated, especially for manifolds where geodesic paths for interpolation may be difficult to obtain, especially for joint time-frequency predictive coding. An alternate approach is to parametrize the precoding matrices using scalar parameters. The space of unitary matrices parameterized into independent scalar parameters that correspond to Givens rotations and Householder transformations of the unitary precoding matrices [8, 9]. These approaches have been combined with adaptive delta modulation effectively track precoding matrices both in the time [8] and frequency domains [9], albeit separately.

In this paper, we propose a method for exploiting the joint time-frequency correlation of the precoder's scalar parameters on the transmitter using CSI feedback. Typical approaches to predictive quantization and interpolation do not exploit the joint time-frequency correlation to enhance the precoder estimate at the transmitter. However, exploiting the correlations jointly not only reduces the feedback requirement, but also enhances the precoder reconstruction accuracy significantly. Further, the independence of the scalar parameters reduces the problem to one that involves interpolation of separate scalar valued functions, as opposed to operations over tangent spaces in manifolds [6]. Simulations reveal that the proposed method matches the performance of manifold based approaches with a much lower implementation complexity and fewer number of bits for quantization.

The rest of this paper is organized as follows: Section II describes the system model and the precoder feedback and interpolation techniques. Section III compares the BER and achievable rates for our proposed methods with other approaches. Finally, Section IV concludes with discussions on future work.

II. System Model

For a MIMO-OFDM system with N_T transmitter and N_R receiver antennas, the channel for the i-th subcarrier at time instant t can be modelled as a flat-fading MIMO channel. This flat-fading MIMO channel can be modelled as

$$\mathbf{y}_{i,t} = \mathbf{H}_{i,t}\tilde{\mathbf{V}}_{i,t}\mathbf{x}_{i,t} + \eta_{\mathbf{i},\mathbf{t}} \tag{1}$$

where $\mathbf{H}_{i,t} \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix, $\mathbf{x}_{i,t} \in \mathbb{C}^{N_d}$ is the input data vector, $\mathbf{y}_{i,t} \in \mathbb{C}^{N_R}$ is the output vector, $\tilde{\mathbf{V}}_{i,t} \in \mathbb{C}^{N_T \times N_R}$ is the unitary precoder, and η is the complex additive white Gaussian noise distributed as $\mathcal{CN}(0, I_{N_R})$. We consider the case where $N_R < N_T$, and the data vector size $N_d = N_R$. The "thin" SVD (that keeps only those right and left singular vectors that correspond to non-zero singular values) of $\mathbf{H}$ is

given by $\mathbf{H}_{i,t} = \mathbf{U}_{i,t}\Sigma_{i,t}\mathbf{V}_{i,t}^H$, where $\mathbf{U}_{i,t} \in \mathbb{C}^{N_R \times N_R}$, $\Sigma_{i,t} \in \mathbb{C}^{N_R \times N_R}$ and $\mathbf{V}_{i,t} \in \mathbb{C}^{N_T \times N_R}$. $\mathbf{U}_{i,t}$ and $\mathbf{V}_{i,t}$ are matrices whose columns are orthonormal, and $\Sigma_{i,t}$ contains the singular values $\sigma_1 \geq \ldots \geq \sigma_{N_R} > 0$ of $\mathbf{H}_{i,t}$. The entries of $\mathbf{H}_{i,t}$ are assumed to be distributed i.i.d.
$mathcalCN(0,1)$. To obtain a precoder at the transmitter, we need to quantize and feed back information about $\mathbf{V}_{i,t}$ for each subcarrier. However, the frequency coherence and temporal correlations would reduce the overall feedback requirement, as discussed in subsequent sections.

A. Scalar Parametrization

The degrees of freedom in the $\tilde{\mathbf{V}}_{i,t}$ matrices is smaller than the number of real entries in the matrix because of the geometrical structure between the columns of the matrix [8]. The orthonormal matrix $\tilde{\mathbf{V}}_{i,t}$ with N_T rows and N_R columns can be decomposed as follows [8] (we suppress the (i,t) subscripts in the decomposition matrices)

$$\tilde{\mathbf{V}}_{i,t} = \left[\prod_{k=1}^{N_R} \mathbf{D}_k\left(\phi_{k,k},\ldots,\phi_{k,N_R}\right) \prod_{l=1}^{N_T-k} \mathbf{G}_{N_T-l,N_T-l+1}\left(\theta_{k,l}\right)\right]\tilde{\mathbf{I}} \tag{2}$$

where $\mathbf{D}_k$ is a diagonal matrix defined as $\mathbf{D}_k\left(\phi_{k,k},\ldots,\phi_{k,N_T}\right) = \operatorname{diag}\left(\mathbf{1}_{k-1}, e^{j\phi_{k,k}},\ldots,e^{j\phi_{k,N_T}}\right)$, $\mathbf{1}_{k-1}$ represents $k-1$ ones, and the $N_R \times N_T$ matrix $\tilde{\mathbf{I}} = \left[\mathbf{I}_{N_R}, \mathbf{0}_{N_T,N_T-N_R}\right]^T$. Here, $\mathbf{0}_{N_T,N_T-N_R}$ is an $N_T \times (N_T - N_R)$ matrix all of whose elements are zero. Finally, we also have

$$\mathbf{G}_{m-1,m}(\theta) = \begin{bmatrix} \mathbf{I}_{m-2} & & & \\ & \cos\theta & -\sin\theta & \\ & \sin\theta & \cos\theta & \\ & & & \mathbf{I}_{t-m} \end{bmatrix} \tag{3}$$

Thus, for a 4×2 orthogonal matrix $\mathbf{V}_{i,t}$, we have

$$\begin{aligned}\mathbf{V}_{i,t} = {} & \mathbf{D}_1(\phi_{1,1},\ldots,\phi_{1,4})\mathbf{G}_{3,4}(\theta_{1,1})\mathbf{G}_{2,3}(\theta_{1,2})\mathbf{G}_{1,2}(\theta_{1,3})\times \\ & \mathbf{D}_2(\phi_{2,2},\phi_{2,3},\phi_{2,4})\mathbf{G}_{3,4}(\theta_{2,1})\mathbf{G}_{2,3}(\theta_{2,2})\tilde{\mathbf{I}}\end{aligned}$$

Here, ϕ_ks are referred to as phases, and their collection for the i-th subcarrier at time instant t is captured in a vector denoted by $\Phi_{i,t}$. θ_ls as called rotation angles, and their collection is captured in a vector referred to as $\Theta_{i,t}$. We note that $\Phi_{i,t}$ and $\Theta_{i,t}$ represent the phase and rotation angles for the i-th subcarrier at time instant t, as represented in Fig. 1. It is known that $\phi_k \in (-\pi,\pi]$ is uniformly distributed between the two extremes [8], while θ_l is distributed as [8]

$$2l(\sin\theta_l)^{2l-1}\cos\theta_l,\ \ \theta_l \in \left[0,\frac{\pi}{2}\right),\ l = 1,2,\ldots,N_T-1 \tag{4}$$

In addition, ϕ_k and θ_l are statistically independent, making this representation useful for tracking them separately across time, as discussed below. The total number of parameters obtained from the decomposition of a complex orthogonal matrix $N_T \times N_R$ is $N_R(2N_T - N_R)$. The number of ϕ_k parameters is $N_R(2N_T - N_R - 1)/2$ while the number of θ parameters is $N_R(2N_T - N_R + 1)/2$. Thus, $\Phi_{i,t}$ is a vector of length $N_R(2N_T - N_R - 1)/2$, while $\Theta_{i,t}$ has length $N_R(2N_T - N_R + 1)/2$.

The quantization approach described above is related to the channel state information feedback present in several wireless standards, including 802.11ac [10], although adaptive quantization and feedback are not discussed. We show in the further sections that the use of adaptive approaches can greatly enhance performance with only very minor modifications to the system.

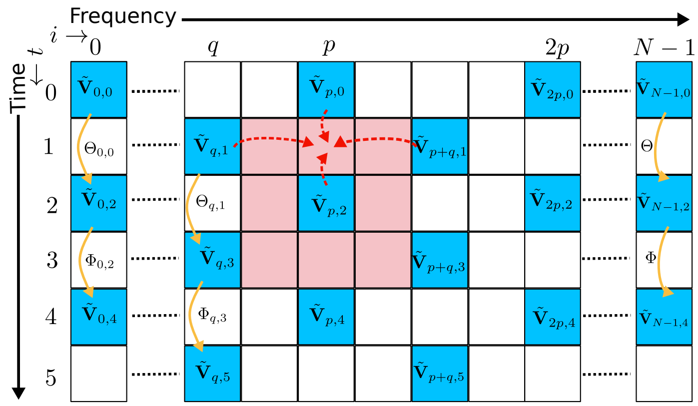

Fig. 1: Subcarriers which need to be quantized. In the figure, we see that the subcarrier indicated by red arrows has a precoder whose parameters are determined jointly using the neighbouring parameters, both in time and frequency.

B. Differential quantization and channel tracking

For channels that vary slowly with time, adaptive differential quantization is an effective method for tracking parameters over time using very few bits (often just one bit per parameter). For example, while tracking a slowly varying discrete random process x_n at time n, the one bit can be used to determine the direction in which to move in order to track the parameter. This can be denoted by $\beta_n = \operatorname{sign}(x_n - \hat{x}_{n-1})$, where x_n is the unquantized value and $\hat{x}_{n-1}$ is the unquantized (accurate) value. Further, the step size for the move can be adapted based on the speed of channel variation as well the accuracy needed. If Δ_n is the step size, we define our adaptive quantizer as

$$\hat{x}_n = \hat{x}_{n-1} + \beta_n\Delta_n \text{ where } \Delta_n = \begin{cases} M\Delta_{n-1}, & \text{if } \beta_n = \beta_{n-1} \\ \Delta_{n-1}/M, & \text{if } \beta_n \neq \beta_{n-1}. \end{cases} \tag{5}$$

where M is the increment used during the positive and negative moves. We initialize Δ_1 as $\Delta_1 = |x_2 - \hat{x}_1|$. We note that this is performed separately for each of the quantized parameters.

To quantize the subcarriers efficiently and exploit the joint time-frequency interpolation at the transmitter, we select the subcarriers for quantization in an alternating fashion as shown in Fig. 1. The figure shows the channel evolution of an N subcarrier MIMO-OFDM system with the subcarriers that are quantized. Every p-th subcarrier starting with 0 is quantized for even time instances, where, p is the gap between two quantized subcarrier indices. For odd time instances, subcarriers at the position $pk + q$ for $k = 0, 1, \ldots, \frac{N-1}{p}$ with $q = \frac{p}{2}$.

For the adaptive channel tracking scheme, the arrows in Fig. 1 show how previous quantized values along with the 1-bit enhancement are used to find the new quantized value. Since we use a single bit adaptation, the amount of feedback required is 1 bit for each scalar parameter that is being tracked. On average, this can be reduced by half if we transmit only Θ values for one-time instance and only Φ values for the next time instant in an alternating fashion (the untransmitted value is assumed to be the same as that at the previous instant). In this case, the average number of bits required to quantize a channel matrix will be $N_T(2N_T - N_R)/2$.

One issue that is associated with the tracking the ϕ_i parameters is that they may change abruptly between $-\pi$ and π due to jumps of 2π. We have avoided this by unwrapping the phases to facilitate continuous tracking (i.e., we avoid jumps in phase of magnitude close to 2π).

C. Joint Time Frequency Interpolation

After obtaining the β values (using the single bit feedback) at the transmitter, we can construct the precoder at subcarrier indices pk or $pk + q$, in keeping with the same notation as the previous section. To construct the rest of the subcarriers, we will use precoder parameters from both the past and future and time instances to do the interpolation (using backtracking to ensure causality). In Fig. 1, each parameter is interpolated using the neighbouring subcarriers using bilinear interpolation (equivalent to linear interpolation in two dimensions). These interpolated values and then used to reconstruct the precoding matrices all together. Thus, the approach here is able to exploit the time and frequency correlation jointly to enhance performance.

One reason why a linear predictive quantization is preferred is because the underlying parameters can generally be thought to emerge from an autoregressive (AR) process. For such processes, it is well known that linear prediction based on past samples is optimal. Even though the Θ and Φ parameters do not manifest as AR processes, for small changes, a linear approximation works sufficiently well, as described in [8], although that consideration was limited to tracking the temporal evolution of the parameters.

III. Simulation and Discussion

We now present simulations of the scalar feedback based joint time-frequency predictive quantization outlined in the previous section. In particular, we analyze the performance of the proposed quantization and interpolation method for time-varying MIMO-OFDM systems. The channel is simulated using the typical Jakes Model for for the COST207 channel power delay profiles [11, 12]. The BER achieved by the proposed predictive quantization based precoder is compared against the ideal precoder for 100 channel evolutions in each run and averaged. Simulations are performed for normalised Doppler values of $f_d T_s = 3.5 \times 10^{-6}$ (corresponding to a velocity of 36 km/h for $f_c = 1$ GHz) with 1024 subcarriers, and with $f_d T_s = \times 1.05 \times 10^{-5}$ (velocity of 108 km/h) with 64 subcarriers. The sampling time period T_s was 5×10^{-8} s. For both the situations, we considered $N_T = 4$ transmit antennas and and $N_R = 2$ receive antennas. Therefore, the total number of parameters that determine the precoder is $N_R(2N_T - N_R) = 12$, with five θ_ls and and seven ϕ_is. We assume that the spacing between fed back subcarrier indices p to be 33 and 9 for the fast and slow channel profiles respectively (i.e. for the slower channels with 1024 subcarriers, the subcarriers indices fed back would be $0, 33, 66, \dots 1023$ and for the faster channel, with 64 subcarriers, $0, 9, 18, \dots 63$ would be fed back).

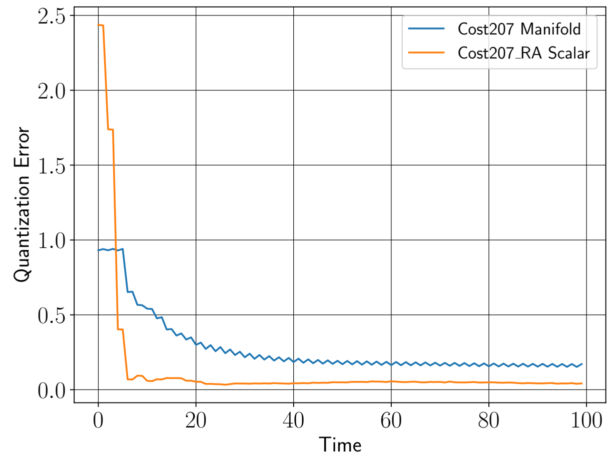

Fig. 2: A comparison of the 4×2 precoder quantization error vs. time instant for the proposed approach (labelled "TODO") and direct quantization on the Stiefel manifold for the Vehicular channel profile.

For the first and the second simulation time instances, since the precoder is not available at the transmitter, we use 2 bits for initializing each parameter for effective representation of the precoder. Therefore, the number of bits for each subcarrier $= 10 \times 2 = 20$ (we note that $\phi_0 = \phi_4 = 0$ due to the non-uniqueness of the SVD). Quantization is performed as given in Section II. We note that this is not significant, since, amortized over long durations, this number becomes small. Subsequent quantization takes place in the time-domain using one bit for each parameter. Since the bit budget is small, if the channel varies sufficiently slowly, either the θ_l or the ϕ_i parameters are fed back alternately, while retaining the unsent parameters as being to their previous values, as shown in Fig. 1. Here, Θ is the collection of all θ_ls and Φ is the collection of all ϕ_is. This brings down the bit budget to an average of 5 bits(the length of Θ) and 7 bits (the length of Φ) over alternate feedback instances, thereby resulting in an average feedback budget of 5 bits.

To enable adaptive tracking of the channel parameters, we start with an initialization of the step size for each parameter, as discussed in Equation 5. The value of M was chosen to be 1.4, and can be adjusted appropriately to track the temporal evolution of parameters for different Doppler frequencies.

Fig. 2, shows the rate at which error diminishes with time for the proposed approach and direct quantization on the Stiefel manifold using codebooks described in [6]. We see that, since the adaptation of independent scalar parameters causes the error to decay rapidly, the error is lower, and the convergence is faster.

Fig. 3 compares the BER obtained using the proposed quantization method with that obtained using exact precoders. We can observe that the performance of the proposed method is very close to the performance with exact precoders. This can be attributed to the fact that even the 1-bit update to the scalar parameters is able to approximate the precoder very accurately. Since the channel variation across subcarriers is gradual,

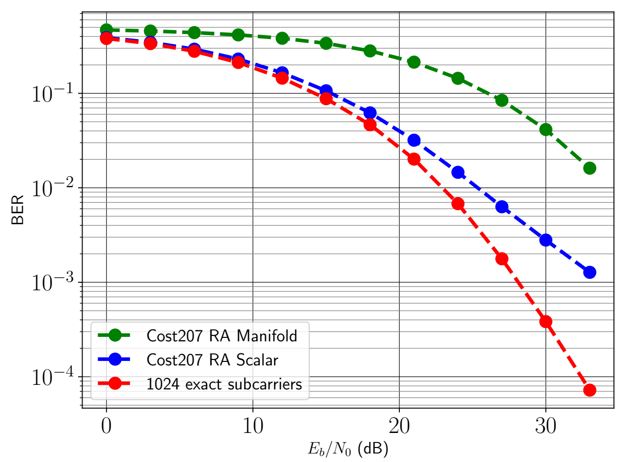

Fig. 3: BER vs. SNR for QPSK transmission over the 4×2 1024 subcarrier MIMO-OFDM system, with the ITU Cost207-RA channel profile for $f_d T_s = 3.41 \times 10^{-6}$, corresponding to a velocity of 37.8 km/h.

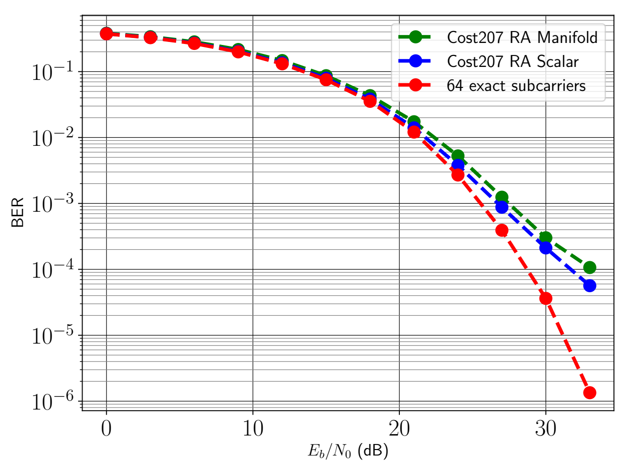

Fig. 4: BER vs. SNR for QPSK transmission over the 4×2 64 subcarrier MIMO-OFDM system, with the ITU Cost207-RA channel profile for $f_d T_s = 1.56 \times 10^{-5}$, corresponding to a velocity of 172.8 km/h.

owing to the limited delay spread of the channel profile, linear interpolation and prediction yields good performance. Moreover, this method outperforms the Stiefel manifold based approach due to the fact that the adaptation is much more accurate for scalar parameters than the tangent space based adaptation used over the Stiefel manifold.

Fig. 4 shows a similar comparison for a similar situation with a higher speed. The faster variation of the channel taps with time necessitates the use of a smaller number of subcarriers. The smaller number of subcarriers implies that the predictive quantization and interpolation needs to be performed across a larger frequency range, thereby potentially reducing the accuracy. Even in this case, we observe that the scalar parameter-based approach outperforms manifold based quantization by about 5 dB.

IV. Conclusion

We have presented a predictive quantization based precoder reconstruction of a MIMO-OFDM system that employs scalar quantization based feedback of precoder parameters. By quantizing and adapting the Givens rotation and Householder transformation parameters of the precoder, the receiver can send minimal feedback to the transmitter and yet enable effective precoder adaptation. Exploiting the temporal and frequency correlations jointly enhances precoder accuracy significantly. Simulations reveal that the proposed approach provides faster convergence and smaller errors in the precoder when compared to approaches that involve direct quantization on manifold tangent spaces. Future work would involve tuning the adaptation rate of the scalar parameters and further reductions of quantization redundancy to reduce the feedback rates.

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