Exercise 8.5, Problem 16

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Abstract—**This document provides the solution to the problem no. 16 given in the exercise 8.5. The figures are constructed using python and LaTeXcodes.**

This documentation can be downloaded from

```
svn co https://github.com/mohit−singh−9/Summer
    −2020/tree/master/geometry/circle.git
```

1 PROBLEM No. 16

AB is a diameter of the circle, ***CD*** is a chord equal to the radius of the circle. ***AC*** and ***BD*** when extended intersect at point ***E*** . Prove that $\angle AEB = 60°$.

2 CONSTRUCTION

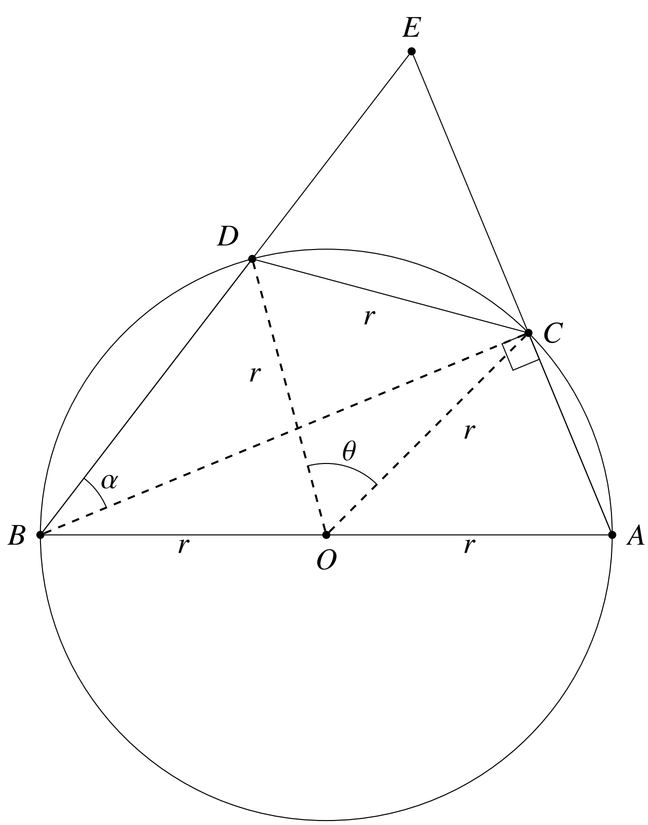

Fig. 2.0: Using Latex-Tikz

2.1. The following inputs were taken for constructing the figure:

Parameter	Value
Radius (r)	2
Centre (O)	(0,0)

TABLE 2.1: Input Table for construction

2.2. The coordinates of **A** and **B** such that **AB** is a diameter as in Fig. 2.0 are:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \tag{2.2.1}$$

$$\mathbf{B} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.2.2}$$

2.3. Let **C** be a point on circle such that its coordinates are:

$$\mathbf{C} = \begin{pmatrix} 1.414 \\ 1.414 \end{pmatrix} \tag{2.3.1}$$

Now **D** should be a point on the circle such that **CD** = 2 units (radius of the circle). Let,

$$\mathbf{D} = \begin{pmatrix} x' \\ y' \end{pmatrix} \tag{2.3.2}$$

So the coordinates of **D** will satisfy the following equations:

$$(1.414 - x')^2 + (1.414 - y')^2 = 4 \tag{2.3.3}$$

$$x'^2 + y'^2 = 4 \tag{2.3.4}$$

Eqn. 2.3.3 is the distance formula between two cordinates. Eqn. 2.3.4 is the equation of the circle with radius 2 units and centre at (0,0). On solving these two euations, you get the value of x' and y'. So ,

$$\mathbf{D} = \begin{pmatrix} -0.518 \\ 1.932 \end{pmatrix} \tag{2.3.5}$$

2.4. The point **E** is obtained by the intersection of the extended lines of **AC** and **BD**.
As the coordinates of points **A**, **B**, **C** and **D** are now known, you can find the line equation

of **AC** and **BD**.
By equating the line equations of **AC** and **BD**, you get the intersection point i.e. point **E**.

$$1.414x + 0.586y - 2.828 = 0 \tag{2.4.1}$$
$$1.932x - 1.482y + 3.864 = 0 \tag{2.4.2}$$

Eqn. 2.4.1 is the line equation of **AC** and Eqn. 2.4.2 is the line equation of **BD**.
By solving these two equations, you get the coordinates of **E**.

$$\mathbf{E} = \begin{pmatrix} 0.597 \\ 3.385 \end{pmatrix}, \tag{2.4.3}$$

2.5. The derived values are listed in Table. 2.5

Derived Values	
	Coordinates (x,y)
A	(2,0)
B	(-2, 0)
C	(1.414, 1.414)
D	(-0.518, 1.932)
E	(0.597, 3.385)

TABLE 2.5: To construct △ ABC

2.6. For solving the problem, join **OC**, **OD** and **BC**.
2.7. To get the python code for Fig 2.7, download it from

codes/circle.py

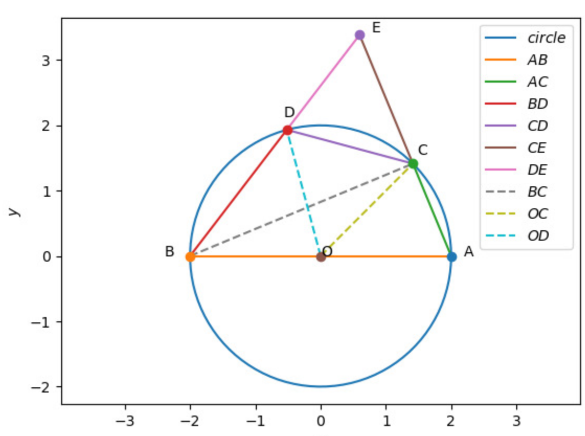

Fig. 2.7: Constructed using Python

and the equivalent latex-tikz code for Fig. 2.0 from

figs/circle.tex

The above latex code can be compiled as a standalone document as

figs/circle_fig.tex

3 SOLUTION

From Fig. **??**
In $\triangle OCD$
OD = OC = r (Radius of circle)
CD = r (Given)

$\therefore$ $\triangle OCD$ is an equilateral triangle ,
So $\angle COD = \theta = 60°$

In $\triangle CBD$
$\angle CBD = \alpha = \frac{1}{2}.\angle COD = \frac{1}{2}$. 60°= 30°
Using the Theorem : Angle subtended by chord at the centre of circle is twice the angle subtended by it at any other point on the circle

In $\triangle BCA$
$\angle BCA = 90°$[Angle subtended by a diameter at any point on circle is 90°]
So, $\angle BCE = 90°$

Now in $\triangle EBC$
$\angle BCE + \alpha + \angle BEC = 180°$[Triangle sum property]
90°+ 30°+ $\angle BEC = 180°$
$\angle BEC = 60°$

$\therefore \angle AEB = 60°$
Hence proved.