Exercise 8.1, Problem 36

Mohit Singh

***Abstract*—This document provides the solution to the problem no. 36 given in the exercise 8.1. The problem is based on the congruence rules in triangles. The figures are provided using python and LaTeXcodes.**

This documentation can be downloaded from

svn co https://github.com/mohit−singh−9/Summer −2020/tree/master/geometry/triangles.git

1 PROBLEM No. 36

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of triangle PQR. Show that:
a) $\triangle ABM \cong \triangle PQN$
b) $\triangle ABC \cong \triangle PQR$

2 CONSTRUCTION

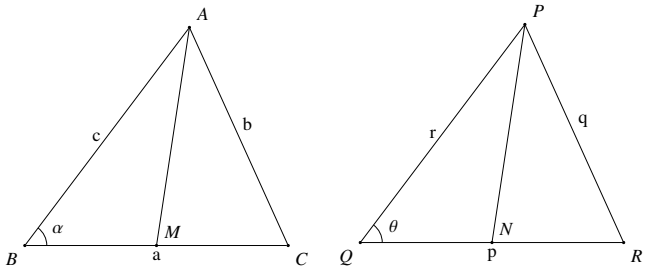

Fig. 2.0: $\triangle ABC$ and $\triangle PQR$ by Latex-Tikz

2.1. Since it is given that the triangles ABC and PQR have two sides and a median length equal, we have considered the following inputs for constructing the figures:

Parameter	Value
a , p	6
b , q	4
c , r	5

TABLE 2.1: To construct △ ABC and △ PQR

2.2. The coordinates of the various points of triangle ABC in Fig. **??** are:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{2.2.1}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \tag{2.2.2}$$

$\because$ **M** is the midpoint of BC,

$$\mathbf{M} = \frac{\mathbf{B}+\mathbf{C}}{2} = \begin{pmatrix} a/2 \\ 0 \end{pmatrix}, \tag{2.2.3}$$

Using the coordinates of **B** , **C** and input parameters, we can find the coordinates of vertex **A**. The derived values are listed in Table. **??**

Derived Values	
	Coordinates (x,y)
M	(3,0)
A	(3.75, 6.25)

TABLE 2.2: To construct △ ABC

2.3. Using the similar construction steps as triangle ABC, we can construct triangle PQR because they both have the same input parameters.

2.4. To get the python code for Fig **??**, download it from

codes/triangle_python.py

and the equivalent latex-tikz code for Fig. **??** from

figs/triangle.tex

The above latex code can be compiled as a standalone document as

figs/triangle_fig.tex

3 SOLUTION

a)
In $\triangle ABM$ and $\triangle PQN$

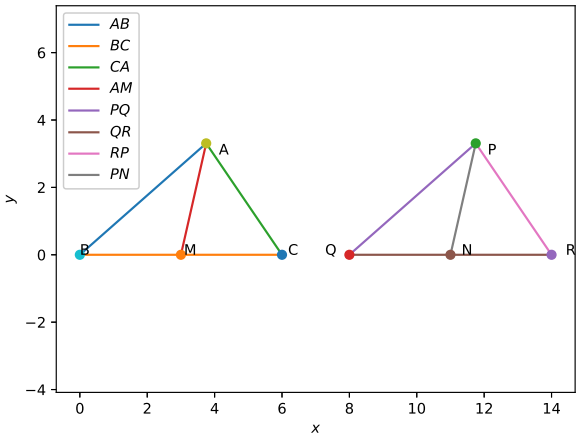

Fig. 2.4: $\triangle ABC$ and $\triangle PQR$ using Python

AB = PQ (Given)
AM = PN (Given)
Since M and N are midpoints and BC = QR ,
BM = QR
:. By SSS congruence rule , $\triangle ABM \cong \triangle PQN$

This implies that $\angle ABM = \angle PQN$ i.e $\alpha = \theta$

b)
Now in $\triangle ABC$ and $\triangle PQR$
AB = PQ (Given)
$\alpha = \theta$
BC = QR (Given)
:. By SAS congruence rule , $\triangle ABC \cong \triangle PQR$