Computer Organization & Architecture

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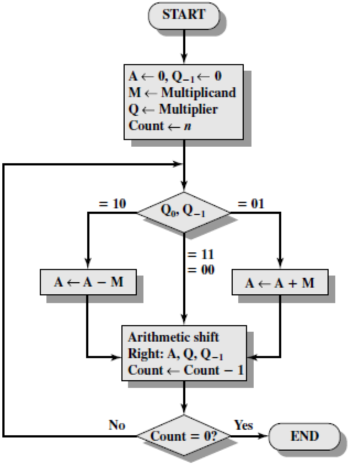
Booth's Multiplication Algorithm



Two major approaches to store real numbers (i.e., numbers with fractional component)

- Fixed Point Notation
- Floating Point Notation

Fixed Point Representation

Fixed-Point Representation:

- In fixed point notation, a fixed number of digits after the decimal point
- Fixed number of bits for integer part and for fractional part.
- a fixed-point number representation consists of three parts:
    - Sign field
    - Integer field
    - Fractional field

Fixed Point Representation

- In all the three schemes, the most-significant bit (msb) is called the sign bit.
  - represent the sign of the integer: 0 for positive integers and 1 for negative integers.
- The magnitude of the number is interpreted differently in different schemes.
- Example: assume number is using 32-bit format which reserve 1 bit for the sign, 15 bits for the integer part and 16 bits for the fractional part.
  - Then, -43.625 is represented as following:

1	000000000101011	1010000000000000
Sign bit	Integer part	Fractional part

Advantages and Disadvantages of Fixed Point Representation

- Advantage: performance
- Disadvantage: relatively limited range of values that they can represent.
  - smallest positive number and largest positive number which can be store in 32-bit representation as given below.

	Sign bit	Integer part	Fractional part
Smallest	0	000000000000000	0000000000000001
Largest	0	111111111111111	1111111111111111

Floating Point Representation

- What is floating point number?
- Need of floating point representation ?
- Floating-point representation:
  - Scientific notation
  - General representation:

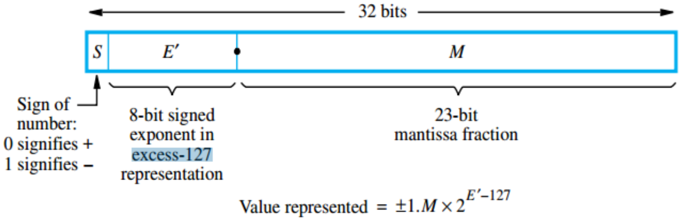
$$\pm M(significand \setminus mantissa) \times B(base)^{E(exponent)}$$

Floating-point Representation in Modern Computers

- IEEE (Institute of Electrical and Electronics Engineers) Standard 754
  1. 32-bit single-precision
  2. 64-bit double-precision.

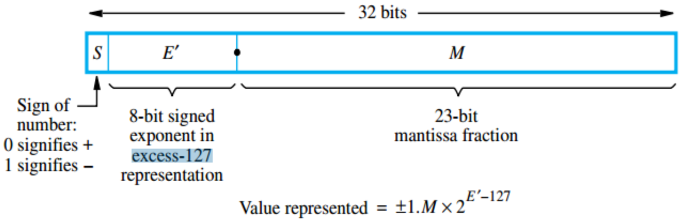
IEEE-754 32-bit Single-Precision Floating-Point Numbers

- In 32-bit single-precision floating-point representation:
  - The most significant bit is the sign bit (S)
  - The next 8 bits, $E^{'}$, represent the signed exponent of the scale factor (with an implied base of 2)
  - The remaining 23 bits, M represents mantissa fraction



(a) Single precision

IEEE (Institute of Electrical and Electronics Engineers) Standard 754



(a) Single precision

- Why the exponent field id $E^{'}$?

Single Precision

- S represents sign bit for the number.
- E represents the signed exponent of the scale factor (with an implied base of 2)
- M represents fractional part of the significant bits called mantissa, always has a leading 1
  $B = 1 \cdot M = 1 \cdot b_{-1} b_{-2} \cdots b_{-23}$
  $V(B) = 1 + b_{-1} \times 2^{-1} + b_{-2} \times 2^{-2} \cdots b_{-23} \times 2^{-23}$
- Normalized number?
- The end values of E are used to represent special values.

Special Values

- The end values 0 and 255 of the excess-127 exponent E are used to represent special values.
  1. When E $= 0$ and M$= 0$: represents zero. If sign bit is 0, then $+0$, else -0.
  2. When E $= 255$ and M $= 0$: the value ∞ is represented, If sign bit is 0, then $+\infty$, else $-\infty$.
  3. The sign bit is still used in these representations, so there are representations for ± 0 and $\pm\infty$
  4. When E $= 0$ and $M \neq 0$: represent denormalized number, then their value is $\pm 0 \cdot M \times 2^{-126}$
  5. When E $= 255$ and $M \neq 0$: Not a Number (NaN).

Exceptions

- A processor must set exception flags if
  1. underflow
  2. overflow
  3. divide by zero
  4. inexact
  5. invalid.

Practice Question: Single Precision

1. Suppose that IEEE-754 32-bit floating-point representation pattern is 0 10000000 110 0000 0000 0000 0000 0000.
2. Suppose that IEEE-754 32-bit floating-point representation pattern is 1 01111110 100 0000 0000 0000 0000 0000.
3. Suppose that IEEE-754 32-bit floating-point representation pattern is 1 00000000 000 0000 0000 0000 0000 0001.
4. Compute the largest and smallest positive numbers that can be represented in the 32-bit normalized form

Work Space

Work Space

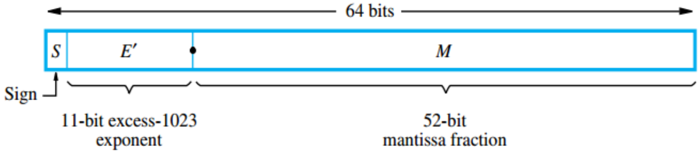
Work Space

Work Space

Work Space

IEEE-754 64-bit Double-Precision Floating-Point Numbers

- Provides more precision and range for floating-point numbers



Value represented $= \pm 1.M \times 2^{E'-1023}$

The number (N) is calculated as follows:

- Normalized form: For $1 \leq E \leq 2046$, $N = (-1)^S \times 1 \cdot F \times 2^{(E-1023)}$
- Denormalized form: For E = 0, $N = (-1)^S \times 0 \cdot F \times 2^{(-1022)}$ These are in the denormalized form
- For E = 2047, N represents special values, such as $\pm\infty$ (infinity), NaN (not a number)

Practice Questions

What about converting to decimal?

- 0011 1110 1010 1000 0000 0000 0000 0000

Arithmetic Operations on Floating-Point Numbers

- Addition and Subtraction Rule
  - Compare the magnitude of the two exponents and make suitable alignment to the number with the smaller magnitude of the exponent.
  - Set the exponent of the result equal to the larger exponent.
  - Perform addition/subtraction on the mantissas and determine the sign of the result.
  - Normalize the resulting value, if necessary.
  - Example: 1.1100×2^4 and 1.100×2^2 ?

Arithmetic Operations on Floating-Point Numbers

1. Multiplication Rule
   - Add the exponents and subtract 127 to maintain the excess-127 representation.
   - Multiply the mantissas and determine the sign of the result.
   - Normalize the resulting value, if necessary
2. Divide Rule
   - Subtract the exponents and add 127 to maintain the excess-127 representation.
   - Divide the mantissas and determine the sign of the result.
   - Normalize the resulting value, if necessary.

Example: $X = 1.000 \times 2^{-2}$ and $Y = -1.010 \times 2^{-1}$?