Fundamentals of Computer Vision - Spring 1397 - Final Exam	Instructor: B. Nasihatkon	دانشگاه صنعتی خواجه نصیرالدین طوسی K. N. TOOSI UNIVERSITY OF TECHNOLOGY
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Question 1 - Harris Corner Detection (20 points)

Consider the following image:

I

0	0	1	4	9
1	0	5	7	11
1	4	9	12	16
3	8	11	14	16
8	10	15	16	20

d/dx

-1	0	1

d/dy

-1
0
1

Compute the Harris matrix

$$H = \sum_{(x,y)\in W} \begin{bmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y(x,y)^2 \end{bmatrix}$$

for the 3 by 3 highlighted window. In the above formula $I_x = dI/dx$, $I_y = dI/dy$, and W is the window highlighted in the image.

A) First, compute the derivatives using the differentiation kernels shown above. No normalization (division by 2) is needed. (5 points).

$I_x = dI/dx$

X	X	X	X	X
X	**4**	**7**	**6**	X
X	**8**	**8**	**7**	X
X	**8**	**6**	**5**	X
X	X	X	X	X

$I_y = dI/dy$

X	X	X	X	X
X	**4**	**8**	**8**	X
X	**8**	**6**	**7**	X
X	**6**	**6**	**4**	X
X	X	X	X	X

B) Now compute the Harris Matrix based on the derivative matrices. (10 points).

$$H = \sum_{(x,y)\in W} \begin{bmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y(x,y)^2 \end{bmatrix}$$

$$\sum_{(x,y)\in W} I_x(x,y)^2 = 4^2 + 7^2 + 6^2 + 8^2 + 8^2 + 7^2 + 8^2 + 6^2 + 5^2 = 403$$

$$\sum_{(x,y)\in W} I_y(x,y)^2 = 4^2 + 8^2 + 8^2 + 8^2 + 6^2 + 7^2 + 6^2 + 6^2 + 4^2 = 381$$

$$\sum_{(x,y)\in W} I_x(x,y)\, I_y(x,y) = 4 * 4 + 7 * 8 + 6 * 8 + 8 * 8 + 8 * 6 + 7 * 7 + 8 * 6 + 6 * 6 + 5 * 4 = 385$$

$$H = \begin{matrix} [403 & 385] \\ [385 & 381] \end{matrix}$$

C) Compute the Harris cornerness score $C = det(H) - k\ trace(H)^2$ for $k = 0.04$. What do we have here? A corner? An edge? Or a flat area? Why? (5 points)

$$C = det(H) - K\ trace(H)^2 = 5318 - 0.04 * (784)^2 = -19268.24$$

A negative Harris score indicates an edge.

Question 2 - Scale Space / SIFT Detection (20 points)

The matrices in the left column are the output of applying Gaussian filters with different bandwidths for a single octave in the SIFT detection algorithm. There is a **single** sift keypoint in the scale space. Your job is to find it. (This is **before** removing the edges and low contrast points, and sub-pixel tuning). Report the x, y and scale of the key point. As a reference, for the highlighted cell at scale=2, the scale-space location is (x=3, y=1, scale=2).

To find the keypoint, you first need to build the Difference of Gaussian Images in the scale-space. The key points are found at the locations of extrema in scale-space as explained in the class. Fill in the Difference of Gaussian values and locate the key point. Why is this a SIFT key point?

Hint: The keypoints do not exist in the in the first and last scale.

The keypoint is located at
x=1, y=1, scale=3, since

Compared to neighbours at scale 2
-3 > 1,2,0,1,1,0,1,1,1

Compared to neighbours at scale 4
-3 > 2,5,0,4,-2,8,0,8,-2

Compared to neighbours in the same scale (scale 3)
-3 > 2,3,0,1,1,2,0,1

Gaussian Filtered Images

25	22	20	17
25	28	19	17
20	19	19	17
15	15	15	15

25	20	20	16
25	30	19	16
19	17	19	16
13	13	14	14

24	18	20	14
25	32	19	15
18	16	19	14
12	12	13	13

22	15	20	12
24	35	18	14
16	16	18	13
11	11	11	12

20	10	20	8
20	37	10	8
16	8	20	10
12	10	9	10

Difference of Gaussian Images

scale = 1

0	**2**	**0**	**1**
0	**-2**	**0**	**1**
1	**2**	**0**	**1**
2	**2**	**1**	**1**

scale = 2

1	**2**	**0**	**2**
0	**-2**	**0**	**1**
1	**1**	**0**	**2**
1	**1**	**1**	**1**

scale = 3

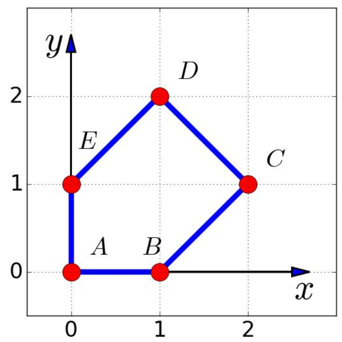
2	**3**	**0**	**2**
1	**-3**	**1**	**1**
2	**0**	**1**	**1**
1	**1**	**2**	**1**

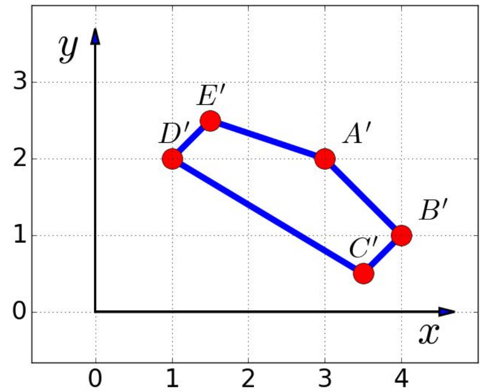
scale = 4

2	**5**	**0**	**4**
4	**-2**	**8**	**6**
0	**8**	**-2**	**3**
-1	**1**	**2**	**2**

Question 3 - Image Transformations (25 points)

A) The image below on the left has undergone an **affine** transformation $y = Mx + t$ to create the image on the right. The locations of the transformed points A', B' and D' are marked in the transformed image. Calculate the affine transformation (the 2 by 2 matrix M, and the vector t) from the point correspondences (A, A'), (B, B'), and (D, D'). (10 points)





$$\mathtt{M}A + t = A' \Rightarrow \mathtt{M}\begin{pmatrix}0\\0\end{pmatrix} + t = \begin{pmatrix}3\\2\end{pmatrix} \Rightarrow t = \begin{pmatrix}3\\2\end{pmatrix}$$

Let

$$\mathtt{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}.$$

Then

$$\mathtt{M}B + t = B' \Rightarrow \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix} \Rightarrow \begin{pmatrix}m_{11}\\m_{21}\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}$$

$$\mathtt{M}D + t = D' \Rightarrow \begin{bmatrix} 1 & m_{12} \\ -1 & m_{22} \end{bmatrix}\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$$
$$\Rightarrow \begin{pmatrix}1+2m_{12}\\-1+2m_{22}\end{pmatrix} = \begin{pmatrix}-2\\0\end{pmatrix} \Rightarrow \begin{pmatrix}m_{12}\\m_{22}\end{pmatrix} = \begin{pmatrix}-3/2\\1/2\end{pmatrix}.$$

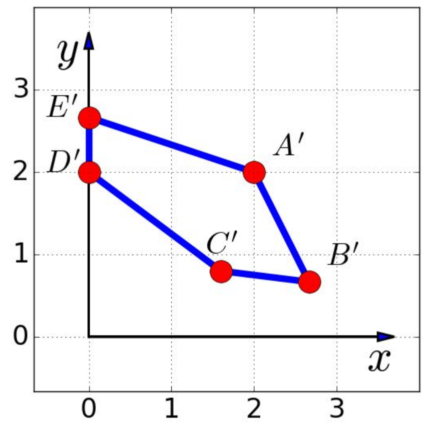
Thus

$$\mathtt{M} = \begin{bmatrix} 1 & -1.5 \\ -1 & 0.5 \end{bmatrix}, \quad t = \begin{pmatrix}3\\2\end{pmatrix}.$$

B) Compute the coordinates of C' and E'. (5 points)

$$C' = \mathtt{M}C + t = \begin{bmatrix} 1 & -1.5 \\ -1 & 0.5 \end{bmatrix}\begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}3.5\\0.5\end{pmatrix}.$$
$$E' = \mathtt{M}E + t = \begin{bmatrix} 1 & -1.5 \\ -1 & 0.5 \end{bmatrix}\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}1.5\\2.5\end{pmatrix}.$$

C) Apply the following homography transformation to the input image of part A (the image on the left). Derive the corresponding transformed points A', B', C', D', E', and draw the output image (10 points).



$$\begin{bmatrix} 2 & -2 & 2 \\ -1 & 2 & 2 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$

$$\mathtt{H} = \begin{bmatrix} 2 & -2 & 2 \\ -1 & 2 & 2 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$

$$\mathtt{H}\begin{pmatrix} A \\ 1 \end{pmatrix} = \begin{bmatrix} 2 & -2 & 2 \\ -1 & 2 & 2 \\ 0.5 & 0.5 & 1 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \Rightarrow A' = \begin{pmatrix} 2/1 \\ 2/1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathtt{H}\begin{pmatrix} B \\ 1 \end{pmatrix} = \begin{bmatrix} 2 & -2 & 2 \\ -1 & 2 & 2 \\ 0.5 & 0.5 & 1 \end{bmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 1.5 \end{pmatrix} \Rightarrow B' = \begin{pmatrix} 4/1.5 \\ 1/1.5 \end{pmatrix} = \begin{pmatrix} 2+2/3 \\ 2/3 \end{pmatrix}$$

$$\mathtt{H}\begin{pmatrix} C \\ 1 \end{pmatrix} = \begin{bmatrix} 2 & -2 & 2 \\ -1 & 2 & 2 \\ 0.5 & 0.5 & 1 \end{bmatrix}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 2.5 \end{pmatrix} \Rightarrow C' = \begin{pmatrix} 4/2.5 \\ 2/2.5 \end{pmatrix} = \begin{pmatrix} 1.6 \\ 0.8 \end{pmatrix}$$

$$\mathtt{H}\begin{pmatrix} D \\ 1 \end{pmatrix} = \begin{bmatrix} 2 & -2 & 2 \\ -1 & 2 & 2 \\ 0.5 & 0.5 & 1 \end{bmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 2.5 \end{pmatrix} \Rightarrow D' = \begin{pmatrix} 0/2.5 \\ 5/2.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\mathtt{H}\begin{pmatrix} E \\ 1 \end{pmatrix} = \begin{bmatrix} 2 & -2 & 2 \\ -1 & 2 & 2 \\ 0.5 & 0.5 & 1 \end{bmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1.5 \end{pmatrix} \Rightarrow E' = \begin{pmatrix} 0/1.5 \\ 4/1.5 \end{pmatrix} = \begin{pmatrix} 0 \\ 2+2/3 \end{pmatrix}$$

Question 4 - RANSAC (15 points)

We want to do panorama using homographies for stitching images. We find a number of matches between the key points of two consecutive images, out of which at most 40 per cent are outliers. We run the RANSAC algorithm for 100 iterations. Derive a lower-bound on the probability of detecting the outliers (and hence correctly estimating the homography).

Here is the relation between the probability of getting at least one sample with all inliers (p), the minimum number of point matches to compute the transformation (s), and the proportion of outliers (e):

$$(1-p) = (1-(1-e)^s)^N$$

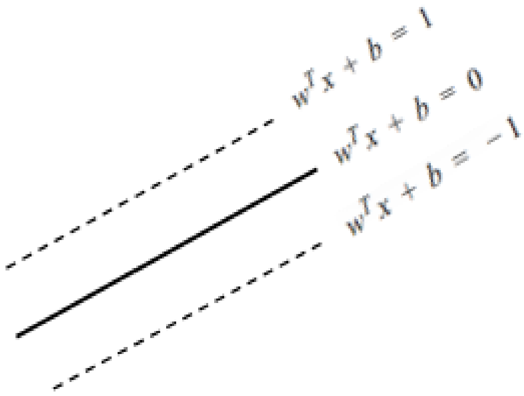
Homography $\Rightarrow s = 4$

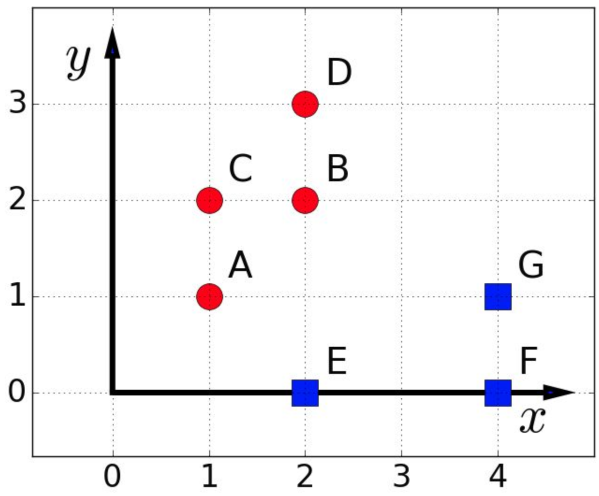
$$e \leq 0.40 \Rightarrow 1-e \geq 0.6 \Rightarrow (1-e)^4 \geq 0.6^4 \Rightarrow 1-(1-e)^4 \leq 1-0.6^4$$

$$1-p = (1-(1-e)^4)^{100} \leq (1-0.6^4)^{100} \Rightarrow p \geq 1-(1-0.6^4)^{100} \approx 1-9.37 * 10^{-7}$$

Question 5 - SVM (20 points)

We intend to train a 2-class SVM on data points below. The data is linearly separable. Your task is to determine the support vectors, and compute the optimal **w** and **b** for the SVM classifier. Hint: find all potential sets of support vectors, for each of them, compute **w** and **b,** and choose the one with the widest margin).





The support vectors from the circle class must be a subset of {A,B}, otherwise the two classes are not separated by a line.

The support vectors from the square class must be a subset of {E,G}, otherwise the two classes are not separated by a line.

There are only two possibilities:
Support Vectors = {A,B, E}, and
Support Vectors = {A,E,G};
For the other couple of cases {A,B,G} and {B,E,G} the data is not linearly separated.

Let $w = [u, v]^T$
First case: Support Vectors = {A,B, E}

$w^T A + b = 1 \Rightarrow u*1 + v*1 + b = 1 \quad \Rightarrow u + v + b = 1 \quad (I)$

$w^T B + b = 1 \Rightarrow u*2 + v*2 + b = 1 \quad \Rightarrow 2u + 2v + b = 1 \quad (II)$

$w^T E + b = 1 \Rightarrow u*2 + v*0 + b = -1 \quad \Rightarrow 2u + b = -1 \quad (III)$

$(II) - (III) \quad \Rightarrow 2v = 2 \Rightarrow v = 1$

$2 * (I) - (II) \quad \Rightarrow b = 1$

$(III) \quad \Rightarrow 2u + b = -1 \Rightarrow 2u = -2 \Rightarrow u = -1$

$w = [u, v]^T = [-1, 1]^T \Rightarrow \| w \| = \sqrt{2} \Rightarrow$ Magin(1) = $2 / \|w\| = \sqrt{2}$

Second case: Support Vectors = {A, E, G}

$w^T A + b = 1 \Rightarrow u*1 + v*1 + b = 1 \quad \Rightarrow u + v + b = 1 \quad (I)$

$w^T E + b = 1 \Rightarrow u*2 + v*0 + b = -1 \quad \Rightarrow 2u + b = -1 \quad (II)$

$w^T G + b = 1 \Rightarrow u*4 + v*1 + b = -1 \quad \Rightarrow 4u + v + b = -1 \quad (III)$

$(III) - (I) \quad \Rightarrow 3u = -2 \Rightarrow u = -2/3$

$(II) \quad \Rightarrow 2 * -2/3 + b = -1 \quad \Rightarrow b = 1/3$

$(I) \quad \Rightarrow v = 1 - u - b \Rightarrow v = 4/3$

$w = [u, v]^T = [-2/3, 4/3]^T \Rightarrow \| w \| = 2\sqrt{5}/3,$

$\Rightarrow$ Magin(2) = $2 / \|w\| = 3\sqrt{5} / 5 < \sqrt{2} = Margin(1)$

Margin(1) > Margin(2) => Support Vectors = {A,B, E}, $w = [-1,\ 1]^T,\ b = 1$