S.M.M.R(A, B)

n = A.rows

let C be a new $n \times n$ matrix

if $n == 1$

$c_{11} = a_{11} \cdot b_{11}$

else Partition A, B, and C as discussed.

C_{11} = S.M.M.R(A_{11}, B_{11}) + S.M.M.R(A_{12}, B_{21})

C_{12} = S.M.M.R(A_{11}, B_{12}) + S.M.M.R(A_{12}, B_{22})

C_{21} = S.M.M.R(A_{21}, B_{11}) + S.M.M.R(A_{22}, B_{21})

C_{22} = S.M.M.R(A_{21}, B_{12}) + S.M.M.R(A_{22}, B_{22})

return C

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \qquad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

Strassen M.M

$P_1 = A_{11} \cdot B_{12} - A_{11} B_{22}$

$P_2 = A_{11} \cdot B_{22} + A_{12} \cdot B_{22}$

$P_3 = A_{21} \cdot B_{11} + A_{22} \cdot B_{11}$

$P_4 = A_{22} \cdot B_{21} - A_{22} \cdot B_{11}$

$P_5 = A_{11} \cdot B_{11} + A_{11} \cdot B_{22} + A_{22} \cdot B_{11} + A_{22} \cdot B_{22}$

$P_6 = A_{12} \cdot B_{21} + A_{12} \cdot B_{22} - A_{22} \cdot B_{21} - A_{22} \cdot B_{22}$

$P_7 = A_{11} \cdot B_{11} + A_{11} \cdot B_{12} - A_{21} \cdot B_{11} - A_{21} \cdot B_{12}$

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create 10 Matrices as follows.

$S_1 = B_{12} - B_{22}$, $S_2 = A_{11} + A_{22}$, $S_3 = A_{21} + A_{22}$

$S_4 = B_{21} - B_{11}$, $S_5 = A_{11} + A_{22}$, $S_6 = B_{11} + B_{22}$

$S_7 = A_{12} - A_{22}$, $S_8 = B_{21} + B_{22}$, $S_9 = A_{11} - A_{21}$

$S_{10} = B_{11} + B_{12}$

$P_1 = A_{11} \cdot S_1$, $P_2 = S_2 \cdot B_{22}$, $P_3 = S_3 \cdot B_{11}$

$P_4 = A_{22} \cdot S_4$, $P_5 = S_5 \cdot S_6$, $P_6 = S_7 \cdot S_8$

$P_7 = S_9 \cdot S_{10}$

(9)

$$C_{11} = P_5 - P_4 - P_2 + P_6$$

$$C_{12} = P_1 + P_2$$

$$C_{21} = P_3 + P_4$$

$$C_{22} = P_5 + P_1 - P_3 - P_7$$

$T(n) = 7(n/2)$ for $n > 1$, $T(1) = 1$

Let $n = 2^k$

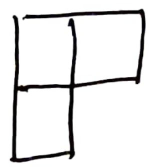
$$T(2^k) = 7\cancel{T}(2^{k-1})$$
$$= 7.[7\cancel{T}(2^{k-2}) = 7^2 . T(2^{k-2})$$
$$= 7^i\, T(2^{k-i})$$
$$\vdots$$
$$7^k\, T(2^{k-k})$$
$$= 7^k$$
$$= 7^{\log_2 n}$$
$$= n^{\log_2 7}$$
$$\boxed{\approx n^{2.807}}$$

$$\left[a^{\log_b c} = c^{\log_b a} \right]$$

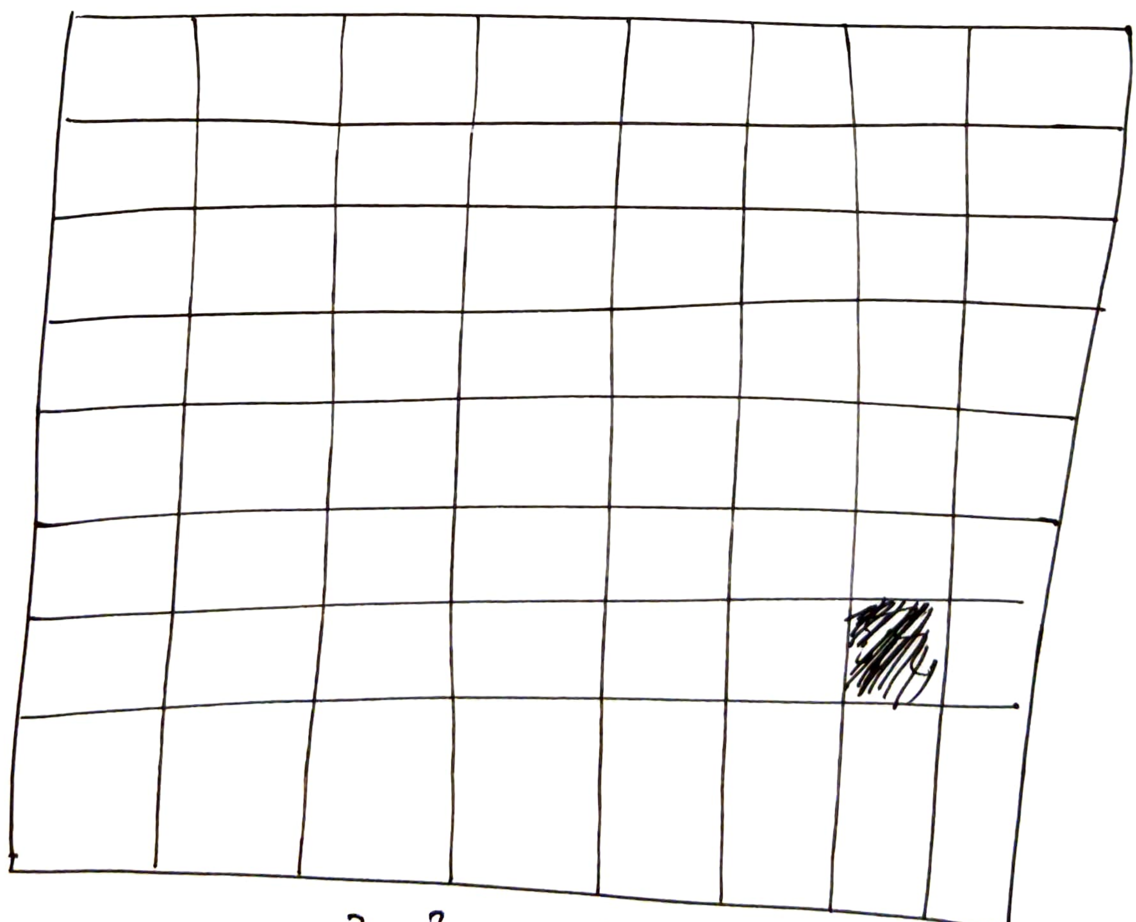
Tromino tiling

You are given a $2^n \times 2^n$ board with one missing square

→ You must cover all squares except the missing one exactly using right trominoes



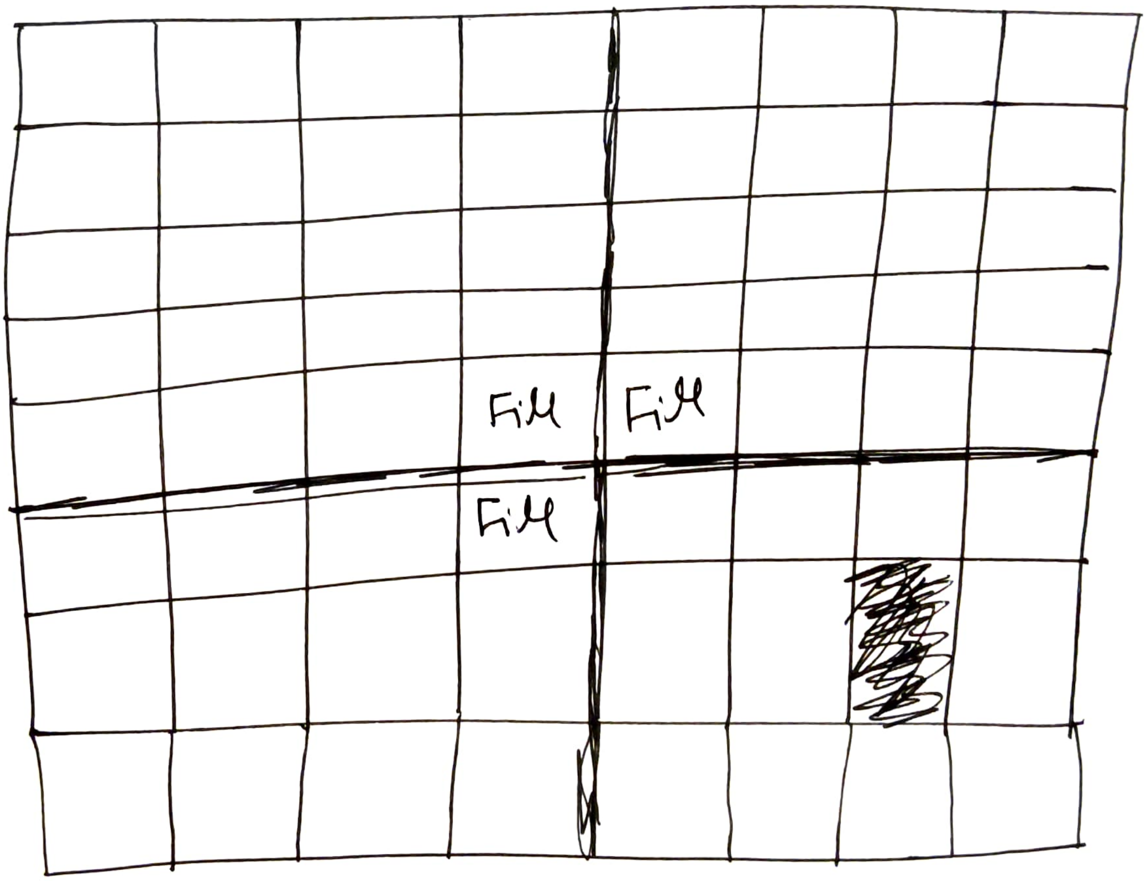
→ The trominoes must not overlap



$2^3 \times 2^3$ board

Step 1 - Divide the $2^n \times 2^n$ board into 4 disjoint $2^{n-1} \times 2^{n-1}$ sub board.

2 - Place a tromino at the center so that it fully cover one square from each of the three sub boards, and misses the fourth sub board completely (Problem Reduced)



3 - Solve each smaller subproblem recursively using the same technique

Algo → TrominoTile (n, location)

Input - $2^n \times 2^n$ Board with a defected square

Output - A tiled board

if $n = 0$

Tile the board and return

else

Divide the board $2^{n-1} \times 2^{n-1}$ sub board.

Place the tile at the center such that one square of each subboard is filled by the tromino except the sub board with defect.

Let l_1, l_2, l_3 and l_4 are the location of defect in each tile.

TrominoTile $\left(\frac{n}{2}, l_1\right)$

TrominoTile $(n/2, l_2)$

TrominoTile $(n/2, l_3)$

Trominotile $(n/2, l_4)$

return tiled board

$$T(n) = 4T(n/2) + c$$