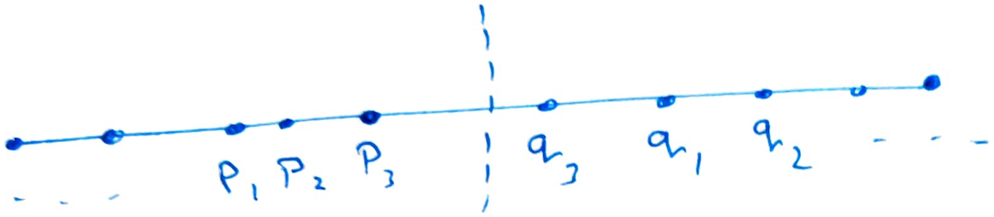
12 Divide & conquer approach

First lets do it on 1D



Median m

Divide the points S into two sets S_1, S_2 by x coordinate so that $p < q$ for all $p \in S_1$ & $q \in S_2$

Let $d_{min} = \min(|p_i - p_j|, |q_i - q_j|)$

So the closest pair is either (p_i, p_j) or (q_i, q_j) or $(p_k\ q_L)$

→ when p_k is rightmost point of S_1 and q_L is the leftmost point of S_2

→ At most one point can lie in the interval $[m - d_{min}, M]$ in S_1 and same is true for S_2

Closest Pair 1 D

Closest-Pair (S)

Input Set of point S

Output ~~indexes~~ index 1 and index 2

if $|S| = 1$, output ∞ or dmin

if $|S| = 2$, output = dmin $|P_2 - P_1|$

and P_1 ~~and t~~ (P_1, P_2)

else

m = median(S).

Divide S into S_1 and S_2 at m.

Dmin1 = Closest-Pair (S_1)

Dmin_2 = Closest. Pair (S_2)

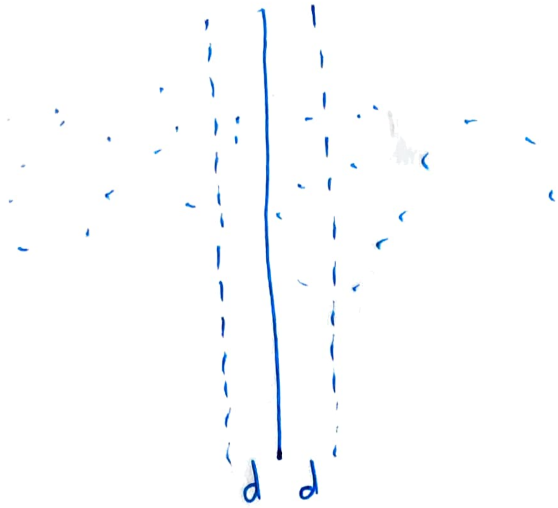
Dmin_3 = Minimum distance ~~at~~ accross the cut (M)

return Dmin = min (Dmin1, Dmin2, Dmi3)

~~M (n)~~

$M(n) = 2M(n/2) + O(n)$

$= O(n \log n)$ as merge sort



1 - Make copies of points sorted by x-coordinates (P_x) and by y-coordinates (P_y)

[$O(n \log n)$ time

2 use Divide & conquer

Base case ?

1 Let Q = Left half of P
R = Right half of P

form Q_x, Q_y, R_x, R_y [Assignment]

2 (p_1, q_1) = closest (Q_x, Q_y)

3 (p_2, q_2) = closest (R_x, R_y)

4 (p_3, q_3) = closest (P_x, P_y)

5 return best of (p_1, q_1), (p_2, q_2), (p_3, q_3)

Key idea!. Only need to bother about split case

1. Let Q = Left half of P

   R = Right half of P

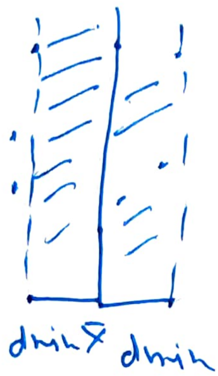
   form Q_x, Q_y, R_x, R_y

2. (p_1, q_1) = Closest pair $(Q_x\ Q_y)$
3. (p_2, q_2) = Closest pair $(R_x\ R_y)$
4. let $d_{min} = \min(d(p_1, q_1), d(p_2, q_2))$
5. $\boxed{(p_3\ q_3) = \text{ClosestSplitPair}(P_x, P_y, d_{min})}$
6. return best of $(d_{min}, d(p_3, q_3))$

we need to do this in $O(n)$ time

closest split pair (P_x, P_y, d_{min})

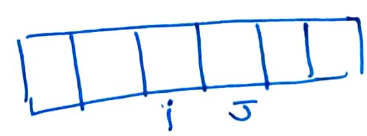
Let $\bar{x}$ = biggest x-coordinate in left of P (O(1))



Let S_y = points of P with x-coordinate in $[\bar{x} - d_{min}, \bar{x} + d_{min}]$, sorted y coordinated

[extract S_y from P_y] (O(n) time)

Initialise best = d_{min}, best pair = NULL



for $i = 1$ to $|S_y| - 7$

for $j = i+1$ to $i+7$

Let $(p, q) = (i^{th}, j^{th})$

if $d(p,q) <$ best

best pair = (p, q)

return best pair

running time O(n)

Correctness claim

Claim : Let $p \in Q$, $q \in R$ be a split pair with $d(p,q) < d_{min}$

(A)- p and q are member of S_y

(b). p and q are at most 7 position apart in S_y.

Proof - (A)

Let $p = (x_1, y_1) \in Q$,

$q = (x_2\ y_2) \in R$, $d(p,q) < d_{min}$

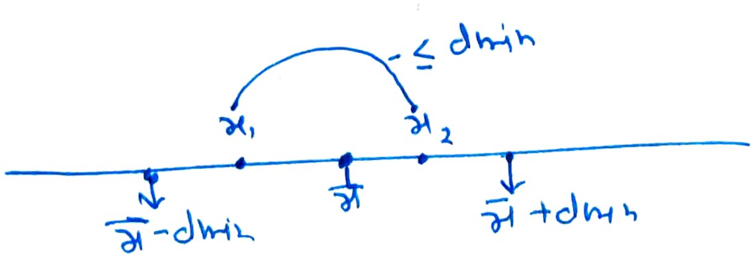
Note :- Since $d(p,q) < d_{min}$

$|x_1 - x_2| < d_{min}$

$|y_1 - y_2| < d_{min}$

p and q are member of S_y - ~~ie $x_1, x_2 \in [\bar{x} - d_{min}$~~

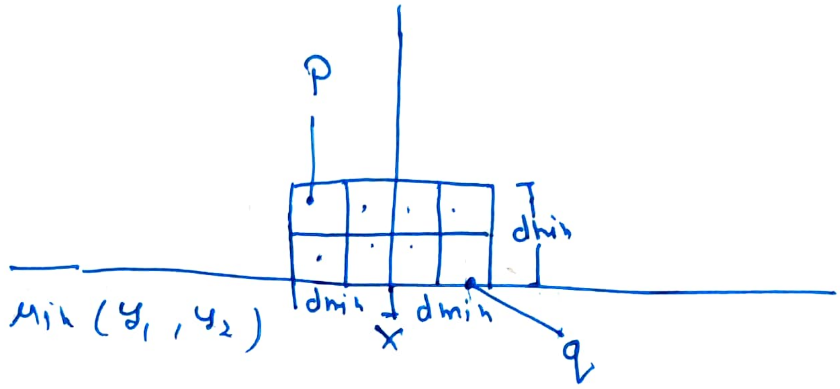
ie $(x_1, x_2) \in [\bar{x} - d_{min}, \bar{x} + d_{min}]$



$p \in Q = x_1 \leq \bar{x}$ and $q \in R$ $x_2 \geq \bar{x}$

Proof - B

p and q are at most 7 position apart in S_y



Each

Lemma: all points of S_y with y-coordinate between those of p and q lie in one one of these 8 boxes.

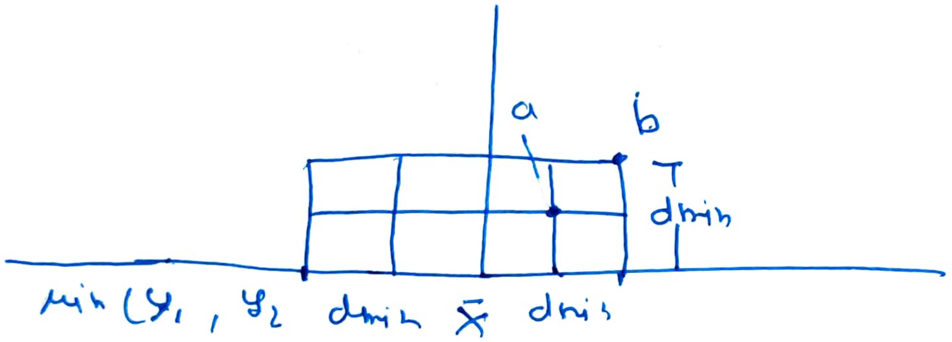
Proof! First, recall y-coordinates of p, q differ by < dmin

second, by definition of S_y all have x-coordinate between $[\bar{x} - d_{min}, \bar{x} + d_{min}]$

Lemma-2:- At most one point of P in each box

Proof by contradiction

suppose a, b lie in the same box.



(i) a, b are either both in Q or both in R

(ii) $d(a,b) \leq \frac{d_{min}}{\sqrt{2}} < d_{min}$

But (i) and (ii) contradict the def of d_{min} as smallest distance between pair of points in Q or R

Remember
No square is shared between 2 halves, its either left or right.