Lecture XVI: Static Games of Incomplete Information II

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April 24, 2003

Readings for this class: Gibbons Chapter 3

1 Purification

People are often uncomfortable with mixed equilibria - why should agents who are indifferent between strategies mix precisely at the right rate to make the other player indifferent between the strategies in her support?

Harsanyi (1973) developed the idea of 'purifying' mixed strategies. The idea is best explained with an example. Consider the standard BOS game below:

	F	O
F	2,1	0,0
O	0,0	1,2

The game has two pure Nash equilibria (F,F) and (O,O) and a mixed one $(\frac{2}{3}F + \frac{1}{3}O, \frac{1}{3}F + \frac{2}{3}O)$.

Harsanyi pointed out that even if players know each other reasonably well the above game is an idealization. In reality none of the two players can be absolutely sure about the other's payoffs. Instead, he suggested real-world payoffs are more likely to look as follows:

	F	O
F	2+a,1	0,0
O	0,0	1,2+b

Each player payoffs are slightly disturbed - assume that both a and b are distributed uniformly over $[0, \epsilon]$ for ϵ small. Each player knows his own type but not the type of the other player.

We now have a static game of incomplete information. As before an informed guess might tell us that an equilibrium has some nice monotonicity properties. In particular, we guess that player 1 chooses F whenever $a > \alpha$ and player 2 chooses O when $b > \beta$.

Note, that we are looking for equilibria in which each player plays a pure strategy.

In this case, the probability that player 1 will player action F is $p = \frac{\epsilon-\alpha}{\epsilon}$ and the probability that player 2 will do so is $q = \frac{\beta}{\epsilon}$. The marginal agent has to be indifferent between choosing F and O in both cases. This gives us the following equilibrium conditions:

$$\begin{aligned}(2+\epsilon\alpha)q &= (1-q) \\ p &= (2+\epsilon\beta)(1-p)\end{aligned}$$

This gives us a quadratic equation in α and β. The important point is, however, that as $\epsilon \to 0$ the probabilities p and q approach precisely the probabilities of the mixed equilibrium.

In other words, the BNE of the perturbed game in which each type of agent plays a pure strategy 'looks' just like the mixed strategy of the original game.

Harsanyi showed (1) that each perturbation gives rise to an equilibrium which looks like a mixed equilibrium, and (2) that each mixed equilibrium can be approximated by a sequence of BNE as constructed above.