Lecture XVIII: Games with Incomplete Information II - More Examples

Markus M. Möbius

May 5, 2003

1 Introduction

This lecture gives more examples of games of incomplete information, in particular signalling games.

2 The Lobbying Game

We consider the following model of lobbying.

- Nature chooses whether the lobbyist's industry is headed for Good or Bad times and reveals the state of the world $\{G, B\}$ to the lobbyist.
- The a priori probability of Good times is p.
- The Lobbyist can then send a message to Congress. Following this message, Congress chooses whether or not to enact a subsidy. Let $\{S, N\}$ denote the actions available to Congress. At the end of the period, the state of the world is revealed to Congress.
- A subsidy costs Congress k. It generates a return $r > k$ for Congress if and only if times are Bad. We assume that $(1-p)r < k$.
- The Lobbyist gets a payoff of zero if the subsidy is not passed, a payoff of 1 if the subsidy passes and times are Bad, and a subsidy of 1/2 if the subsidy passes and times are good.

1. **Is there a PBE in which the subsidy passes? Answer: NO**

2. **Now suppose lobbying Congress is costly. In particular, the lobbyist must incur a cost c to be heard. Show that if $c > 1/2$, there is a PBE in which the subsidy passes whenever the state of the world is bad.**

3 Legal Settlements

- There are two players, a plaintiff and a defendant in a civil suit. The plaintiff knows whether or not he will win the case if he goes to trial, but the defendant does not have this information.

- The defendant knows that the plaintiff knows who would win, and the defendant has prior beliefs that there is probability $\frac{1}{3}$ that the plaintiff will win; these prior beliefs are common knowledge.

- If the plaintiff wins, his payoff is 3 and the defendant's payoff is -4; if the plaintiff loses, his payoff is -1 and the defendant's is 0. (This corresponds to the defendant paying cash damages of 3 if the plaintiff wins, and the loser of the case paying court costs of 1.)

- The plaintiff has two possible actions: He can ask for either a low settlement of $m = 1$ or a high settlement of $m = 2$. If the defendant accepts a settlement offer of m, the plaintiff's payoff is m and the defendant's is $-m$. If the defendant rejects the settlement offer, the case goes to court.

1. List all the pure-strategy PBE strategy profiles. For each such profile, specify the beliefs of the defendant as a function of m, and verify that the combination of these beliefs and the profile is in fact a PBE.

2. Explain why the other profiles are not PBE.

4 Corporate Investment

This is a variant of the IPO game. An entrepreneur needs financing to realize a new project and can offer an outside investor an equity stake in his company.

The stake gives the investor a share of the future (next period's) cashflow of the company: the profits from the existing business plus the profits from the project. The profitability of the nw project is known to both investor and entrepreneur. However, only the entrepreneur knows the profitability of the existing company. The investor therefore runs the risk of investing in an unprofitable business. The new project requires investment I and gives payoff $R > (1 + r) I$ in the next period (where r is the interest rate).

- Nature chooses the type of the entrepreneur's firm which can be highly profitable ($\pi = H$) or less profitable ($\pi = L$). The business is not so profitable with probability p.
- The entrepreneur observes π and then offers equity stake s such that $0 \leq s \leq 1$.
- The investor can accept or reject.

5 Monetary Policy

5.1 The Barro-Gordon model (1983)

There are two periods. In the first period firms form expectations about inflation π_e. Their payoff is $-(\pi - \pi_e)^2$. In the second period the Fed sets inflation π. The Fed has objective function:

$$-c\pi^2 - (y - y^*)^2 \tag{1}$$

Actual output y is:

$$y = by^* + d(\pi - \pi^*) \tag{2}$$

where $b < 1$.

5.2 Barro-Gordon with Signalling

Now assume that the Fed can either be weak ($c = W$) or strong ($c = S$) such that $S > W > 0$. The firm and the Fed play the Barro-Gordon game now for two periods: the first period can now be used to signal the resolve of the Fed.