Lecture IV: Nash Equilibrium II - Multiple Equilibria

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Readings for this class: Osborne and Rubinstein, Chapter 2.1-2.3

1 Multiple Equilibria I - Coordination

Lots of games have multiple Nash equilibria. In this case the problem arises how to select between different equilibria.

1.1 New-York Game

Look at this simple coordination game:

	E	C
E	1,1	0,0
C	0,0	1,1

This game has two Nash equilibria - (E,E) and (C,C). In both cases no player can profitably deviate. (E,C) and (C,E) cannot be NE because both players would have an incentive to deviate.

1.2 Voting Game

Three players simultaneously cast ballots for one of three alternatives A,B or C. If a majority chooses any policy that policy is implemented. If the votes split 1-1-1 we assume that the status quo A is retained. Suppose the preferences are:

$$u_1(A) > u_1(B) > u_1(C)$$
$$u_2(B) > u_2(C) > u_2(A)$$
$$u_3(C) > u_3(A) > u_3(B)$$

Claim 1 *The game has several Nash equilibria including* (A, A, A)*,* (B, B, B)*,* (C, C, C)*,*(A, B, A)*, and* (A, C, C)*.*

Informal Proof: In the first three cases no single player can change the outcome. Therefore there is no profitable deviation. In the last two equilibria each of the two A and two C players, respectively, is pivotal but still would not deviate because it would lead to a less desirable result.

1.3 Focal Points

In the New York game there is no sense in which one of the two equilibria is 'better' than the other one.

For certain games Schelling's (1961) concept of a tipping point can be a useful way to select between different Nash equilibria. A focal point is a NE which stands out from the set of NE - in games which are played frequently social norms can develop. In one-shot games strategies which 'stand out' are frequently played. In both cases, players can coordinate by using knowledge and information which is not part of the formal description of our game.

An example of a social norm is the fact that Americans drive on the right hand side of the road. Consider the following game. Tom and Jerry drive in two cars on a two lane road and in opposite directions. They can drive on the right or on the left, but if they mis-coordinate they cause a traffic crash. The game can be represented as follows:

	R	L
R	1,1	0,0
L	0,0	1,1

We expect both drivers to choose (R,R) which is the social norm in this game.

Next, let's conduct a class experiment.

Class Experiment 1 *You have to coordinate on what of the following four actions - coordinating with your partner gives you a joint payoff of 1 Dollar. Otherwise you both get zero Dollars. The actions are* $\{Fiat95, Fiat97, Saab98, Fiat98\}$.

We played the game with four pairs of students - three pairs coordinated on SAAB98, one pair did not coordinate.

This experiment is meant to illustrate that a strategy which looks quite distinct from the set of other available strategies (here, Fiats) can be a focal point in a one-shot game (when no social norm can guide us).

2 Multiple Equilibria II - Battle of the Sexes

The payoffs in the Battle of the Sexes are assumed to be Dollars.

	F	O
F	2,1	0,0
O	0,0	1,2

(F,F) and (O,O) are both Nash equilibria of the game. The Battle of the Sexes is an interesting coordination game because players are not indifferent on which strategy to coordinate. Men want to watch Football, while Women want to go to the Opera.

Class Experiment 2 *You are playing the battle of the sexes. You are player 2. Player 1 will make his choice first but you will not know what that move was until you make your own. What will you play?*

Last year: We divided the class up into men and women. 18 out of 25 men (i.e. 72 percent) chose the action which in case of coordination would give them the higher payoff. In contrast, only 6 out of 11 women did the same. These results replicate similar experiments by Rubinstein at Princeton and Tel Aviv University. Men are simply more aggressive creatures...

When we aggregate up we found that 24 out of 36 people (66 percent) play the preferred strategy in BoS.

Because there is an element of conflict in the BoS players use the framing of the game in order to infer the strategies of their opponent. In the following experiments the underlying game is always the above BoS. However, in each case the results differ significantly from the basic experiment we just conducted. This tells us that players signal their intention to each other, and that the normal strategic form does not capture this belief formation process.

Class Experiment 3 *You are player 1 in the Battle of the sexes. Player 2 makes the first move and chooses an action. You cannot observe her action until you have chosen your own action. Which action will you choose.*

Last year: Now a significantly higher number of students (17 instead of 12) choose the less desirable action (O). Note, that the game is still the same simultaneous move game as before. However, players seem to believe that player 1 has an advantage by moving first, and they are more likely to 'cave in'.

Class Experiment 4 *You are player 1 in the Battle of the sexes. Before actually playing the game, your opponent (player 2) had an opportunity to make an announcement. Her announcement was "I will play O". You could not make a counter-announcement. What will you play ?*

Now 35 out of 36 students chose the less desirable action. The announcement seems to strengthen beliefs that the other player will choose O.

This kind of communication is called *cheap talk* because this type of message is costless to the sender. For exactly this reason, it should not matter for the analysis of the game. To see that, simply expand the strategy set of player 2. Instead of strategies F and O she now has 4 strategies - Ff, Fo, Of, Oo - where strategy Ff means that player 2 plays F and announces to play f, while Of means that player 2 announces O and plays f. Clearly, the strategies Of and Oo yield exactly the same payoffs when played against any strategy of player 1. Therefore, the game has exactly the same NE as before. However, the announcement seems to have successfully signalled to player 1 that player 2 will choose her preferred strategy.

Class Experiment 5 *Two players are playing the Battle of the Sexes. You are player 1. Before actually playing the game, player 2 (the wife) had an opportunity to make a short announcement. Player 2 choose to remain silent. What is your prediction about the outcome of the game?*

Less than 12 people choose the less desirable action in this case. Apparently, silence is interpreted as weakness.

3 Multiple Equilibria III - Coordination and Risk Dominance

The following symmetric coordination game is given.

	A	B
A	9,9	−15,8
B	8,−15	7,7

Class Experiment 6 *Ask class how many would choose strategy A in this coordination game.*

Observations:

1. This game has the two Nash equilibria, namely (A,A) and (B,B). Coordinating on A Pareto dominates coordination on B. Unlike the New York and the Battle of the Sexes game, one of equilibria is clearly 'better' for both players. We might therefore be tempted to regard (A,A) as the more likely equilibrium.

2. However, lots of people typically choose strategy B in most experimental settings. Playing A seems too 'risky' for many players.

3. Harsanyi-Selten developed the notion of risk-dominance. Assume that you don't know much about the other player and assign 50-50 probability to him playing A or B. Then playing A gives you utility -3 in expectation while playing B gives you 7.5. Therefore, B *risk-dominates* A.

4 Interpretations of NE

IDSDS is a constructive algorithm to predict play and does not assume that players know the strategy choices of other players. In contrast, in a Nash equilibrium players have precise beliefs about the play of other players, and these beliefs are self-fulfilling. However, where do these beliefs come from?

There are several interpretations:

1. **Play Prescription:** Some outside party proposes a prescription of how to play the game. This prescription is stable, i.e. no player has an incentive to deviate from if she thinks the other players follow that prescription.

2. **Preplay communication:** There is a preplay phase in which players can communicate and agree on how to play the game. These agreements are self-enforcing.

3. **Rational Introspection:** A NE seems a reasonable way to play a game because my beliefs of what other players do are consistent with

them being rational. This is a good explanation for explaining NE in games with a unique NE. However, it is less compelling for games with multiple NE.

4. **Focal Point:** Social norms, or some distinctive characteristic can induce players to prefer certain strategies over others.

5. **Learning** Agents learn other players' strategies by playing the same game many time over.

6. **Evolution:** Agents are programmed to play a certain strategy and are randomly matched against each other. Assume that agents do not play NE initially. Occasionally 'mutations' are born, i.e. players who deviate from the majority play. If this deviation is profitable, these agents will 'multiply' at a faster rate than other agents and eventually take over. Under certain conditions, this system converges to a state where all agents play a Nash equilibrium, and mutating agents cannot benefit from deviation anymore.

Remark 1 *Each of these interpretations makes different assumptions about the knowledge of players. For a play prescription it is sufficient that every player is rational, and simply trusts the outside party. For rational introspection it has to be common knowledge that players are rational. For evolution players do not even have to be rational.*

Remark 2 *Some interpretations have less problems in dealing with multiplicity of equilibria. If we believe that NE arises because an outside party prescribes play for both players, then we don't have to worry about multiplicity - as long as the outside party suggests some Nash equilibrium, players have no reason to deviate. Rational introspection is much more problematic: each player can rationalize any of the multiple equilibria and therefore has no clear way to choose amongst them.*