lec 1 - NP

→ lecnotes 1

Pg-9 – Nuclear Physics history

& Pg-10

→ accelerator = accelerates particles to higher energy

→ LHC's = large ~~hydrogen~~ hadron collider

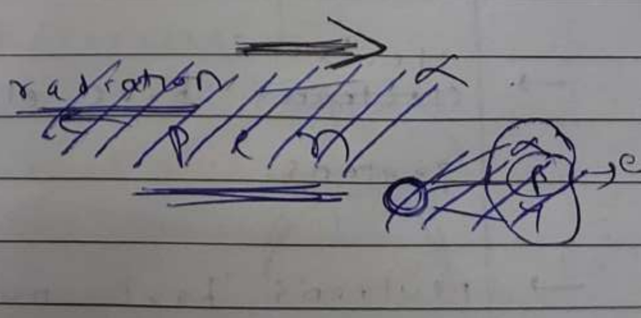
→ nuclear energy more than energy to knock off electrons.

→ electrons are small & light
nucleus = dense & solid

→ electrons & protons = fermions

→ earlier it was found that N-14 (nitrogen) nucleus has 14 protons & $7e^-$ and $7e^-$ in outermost shells, but this gives $= 14 + 7 = 21$ fermions & spin of $\frac{1}{2}$
but with discovery of neutrons
⇒ Nitrogen nucleus have 7 protons & 7 neutrons, total 14 fermions (no overall unpaired) but 1 unpaired proton & 1 unpaired neutron ⇒ $\frac{1}{2} + \frac{1}{2} = 1$

~~N-14~~



& reactions

→ properties of ~~atom~~ elements = due to electrons

properties of elements = due to nucleus existence

→ Isotopes = same no of protons but different no. of neutrons

→ $^{A}_{Z}X$ A = mass no. (Total no. of nucleons)

protons + neutrons

neutrons = A − Z

$E = mc^2$

J

→ 12 AMU = mass of 1 atom of carbon

1 AMU = $\frac{1}{12}$th mass of carbon atom

1 AMU = 1.66×10^{-27} kg

1 u ≈ 931.49 MeV (energy equivalent)

1.66×10^{-27}

→ $\boxed{\text{energy in (eV)} = \frac{\text{energy (Joules)}}{\text{charge of electron}}}$

→ ~~electrons~~ electrons have electromagnetic interaction with protons.

→ neutrons have nuclear interactions

→ protons have both nuclear & electromagnetic

scattering — neutron = mass distr.
scattering — electron = charge distr.

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⇒ mass distribution = distribution of both protons & neutrons

⇒ charge distribution = distribution of only protons

→ R = nuclear radius

$$R = R_0 A^{1/3}$$

$$R_0 = 1.2 \times 10^{-15} \text{ m}$$
$$= 1.2 \text{ fm}$$
or
fermi

→ density of nucleus remains almost constant

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⇒ At points where density falls to 0.05 nucleons/fm^3

those are considered as radius.

→ Density of nucleons inside the nucleus remains almost constant.

$$\rho = \frac{m}{V} \qquad m \propto A, \quad V \propto A$$ therefore A cancels

density of all nuclei that exist are almost constant.

Camlin

$|\vec{S}| = \sqrt{s(s+1)}\,\hbar$ $\quad \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\frac{1}{2}\right)} \quad \frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{3}{2}\right)} \quad \frac{\sqrt{3}}{4}$

↳ spin

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⟹ Spin & magnetic moment of nucleus

Spin angular momentum

$s = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar$

spin angular momentum

* even-even nuclei ⟹ $\boxed{\text{spin} = 0}$

* magnetic moment of nucleus are not very important property of nucleus.

* Spin angular ~~magnetic~~ moment & magnetic moment are in same direction for proton and in different direction for neutron.

* Spin ~~Spin~~ angular momentum = intrinsic property of the particles themselves (neutrons and protons)

* orbital angular momentum = due to the motion of neutrons and protons about each other

⇒ Nuclear Binding energy

* mass of nucleus is less than mass of its constituents

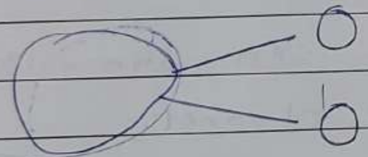
* more B.E. ⇒ more stable nucleus ⇒ more energy reqd. to break nucleus

* B.E. is higher for heavy nucleus

* BE/~~nucleon~~ = Total B.E./ Total no. of nucleons

* Fission — source of nuclear power.

daughter



A ≅ 50–100

⇓

~~therefore~~ release energy since they have higher B.E./nucleon

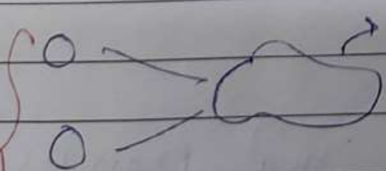
A = 235

⇓

less B.E./nucleon

~~release~~ excess B.E./nucleon release

* Fusion



high B.E./nucleon, therefore excess B.E./nucleon is released as energy.

less B.E/nucleon

on rising part of curve.

Nuclear physics

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Nuclear units & conversions

mass no $\rightarrow$ $^{A}_{Z}X$
atomic no $\rightarrow$

$A =$ no. of nucleons $(N+Z)$
$Z =$ no. of protons = no. of e^- in neutral atoms
$N =$ no. of neutrons

$1 \text{ amu} = 1.66 \times 10^{-27} \text{ Kg}$
$= 1/12$ of mass of $^{12}_{6}C$ atom
$= 931.5 \text{ MeV}$

$m(^{1}_{1}H) = 1.007825 u$

$$\text{Energy in eV} = \frac{\text{energy in Joules}}{1.6 \times 10^{-19} C}$$

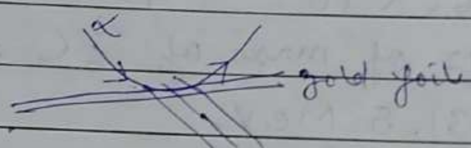
$m_p = 1.007276 u$
$m_n = 1.008669 u$
$m_e = 9 \times 10^{-31} \text{ Kg}$

Properties of nucleus

1) Size
2) Spin, magnetic moment
3) Binding Energy

Size

- Rutherford scattering expt



distance of closest approach ~ 10^{-14} m

nuclear size < 10^{-14} m

fast neutron & electron scattering expts. told nuclear size

Radius of hydrogen nucleus

radius → $\boxed{R = R_0 A^{1/3}}$

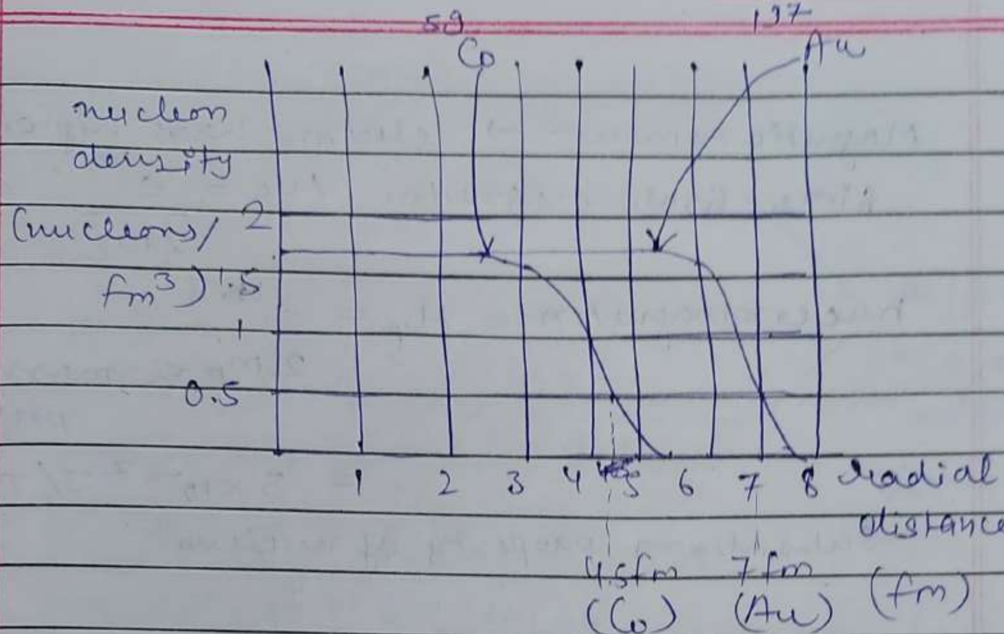
volume of nucleus $\propto A$

$R_0 = 1.2 \times 10^{-15}$ m

$= 1.2$ fm

$1 \text{ fm} = 10^{-15}$ m

Radius is where nucleon density falls to < 0.5 nucleons/fm^3



Spin, magnetic moment

nucleons = fermions = spin 1/2 particles

Angular momentum
- spin - rotation of earth (intrinsic)
- orbital - revolution of earth

fermions — 1/2 integral spin (1/2, 3/2) — odd A — $\frac{1}{2} - \frac{1}{2} + \frac{1}{2}$, $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

boson — integral (0, 1) — Even A

h - plank's constant

Magnetic moment → electrons have higher

since Bohr magneton $\mu_e = \frac{e\hbar}{2m_e} = 9 \times 10^{-24}$ J/T

nuclear magneton $= \mu_p = \frac{e\hbar}{2m_p}$ ← mass of proton

$\simeq 5 \times 10^{-27}$ J/T

- Subsidiary property of nucleus

m_n = neutron mass
m_N = Nuclear mass

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Binding Energy of nucleus

$m({}^2_1H) = 2.014102\,u$
Deuterium

$m({}^1_1H) = 1.007825\,u$
$m_n = 1.008665\,u$
$2.01649\,u$

difference $= 0.002388\,u$
missing mass / mass defect

Energy equivalent $= 0.002388 \times 931.5$
$= 2.224\,MeV$

Binding Energy = Energy equivalent of mass defect

Experimentally, 2.22 MeV energy required to break ${}^2_1H$ nucleus to its constituted particles

= energy reqd. to break a nucleus into its constituted particles
= energy released when constituted particles come together to form nucleus

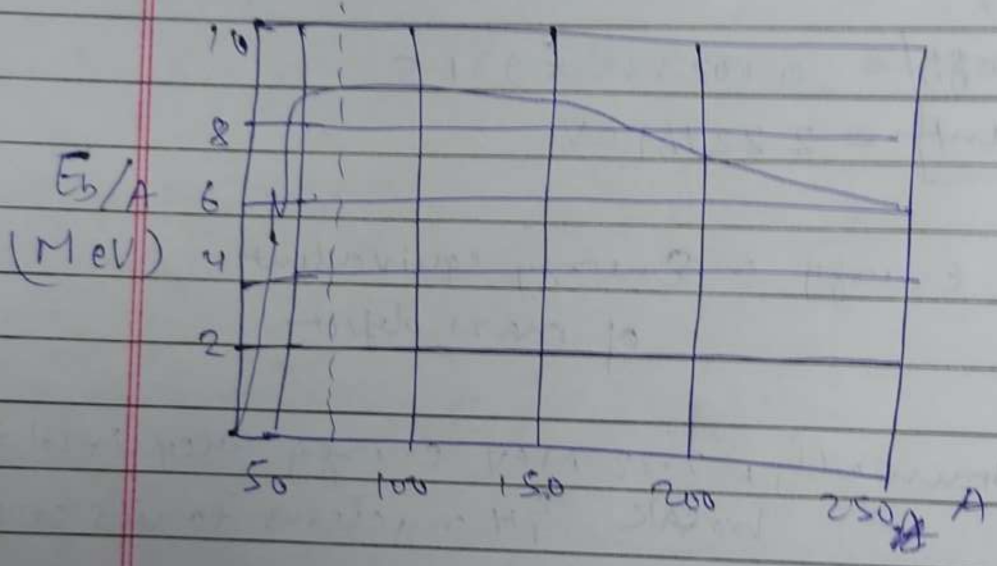
Stability $\propto$ binding energy per nucleon

$$BE = \left(Z\, m({}^1_1H) + N\, m_n - m_N({}^A_ZX)\right) \times 931\ MeV$$
or m_p

$E_b = 2.22$ MeV for ${}^{2}_{1}H$
$= 1640$ MeV for ${}^{209}_{83}Bi$

$E_b/A = 1.11$ MeV for ${}^{2}_{1}H$
$= 7.8$ MeV for ${}^{209}_{83}Bi$

$^{56}Fe \rightarrow$ most stable nucleus



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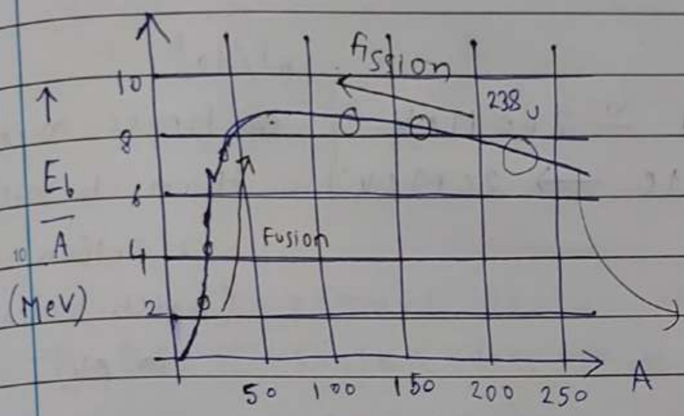
⇒ **Binding energy** → measure of stability of nucleus

$$E_b = \left(Z\,m(^1_1H) + N\,m_n - m(^A_Z X)\right) \times 931.5 \text{ MeV}$$



more binding energy per nucleon
⇒ more stable nucleus

less stable heavy nuclei

$$m(^{42}_{20}Ca) = 41.958619 \text{ u}$$

$$m(^{41}_{20}Ca) = 40.962278 \text{ u}$$

$$m_n = 1.008665 \text{ u}$$

1) **Fission** = Daughter nuclei have higher E_b/A than parent nuclei resulting in release of excess E_b/A

↳ exothermic/exergic (heat is released)

2) **Fusion** = Two lighter nuclei join to form heavy nuclei

$$R = R_0 A^{1/3} \quad (A = 1 \text{ to } 250)$$

$\frac{\text{mass}}{\text{volume}} \leftarrow$ nuclear density $\simeq$ constant

mass $\propto$ A

* <u>Fusion</u> = ~~P~~ daughter nuclei has higher E_b/A than parent nuclei resulting in release of excess E_b/A (exothermic/ exergic)

* 1 Fission $^{235}U \simeq 200\,MeV$
1 fusion $H \to He \longrightarrow 20\,MeV$ } $10^7/10^8$ times more than chemical reaction (which releases in eV)

Q1) Calculate binding energy of neutron and proton separately in $^{42}_{20}Ca$?

<u>Soln:</u> $^{(42)}_{20}Ca$ (→ proton+neutron; ↙ proton) $\xrightarrow{-\text{neutron}}$ $^{41}_{20}Ca$ + neutron

$m(^{42}_{20}Ca) = 41.958619\,u$

$m(^{41}_{20}Ca) = 40.962278\,u$

$m_{\text{neutron}} = 1.008665\,u$

$$\begin{array}{r} 40.962278 \\ 1.008665 \\ \hline 41.970943 \end{array}$$

diff in masses = ~~6~~ $41.970943 - 41.958619$
=
$= 0.012324\,u$

same Z diff. A ⇒ Isotopes

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energy reqd. to take away neutron from nucleus

Binding energy of neutron = 0.012324 u × 931.5
= 11.48 MeV

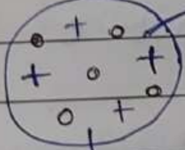
* Calculate B.E. of proton ?

10.27 MeV

$^{42}_{20}Ca \rightarrow ^{41}_{19}K$ + proton

* easier to remove proton than neutron
→ (B.E. of proton less than B.E. of neutron)
→ (outward pushing force on the proton, so easier to remove)

⇒ NUCLEAR FORCES = 2 forces exist in nucleus



→ electric/coulomb repulsion between protons

→ Attraction

→ 4 forces exist in nature:

1) Gravity
2) electromagnetic

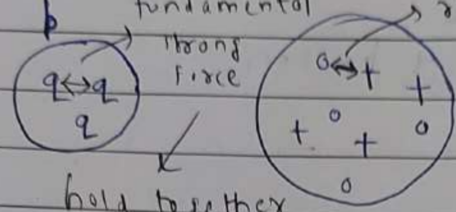
long range

3) strong
4) weak

short range

There are 3 quarks inside proton

* There are 2 kinds of strong forces:
  - fundamental strong force
  - residual strong force



fundamental strong force → hold together quarks

residual strong force (hold together proton & neutron)

⇒ RESIDUAL STRONG FORCE OR NUCLEON-NUCLEON FORCE

Properties
1) Attractive (Strongly than coulomb repulsion)
2) Short range
3) Repulsive at very short distance
4) Acts on n and p not e
5) Charge independent
6) Spin dependent
7) Potential
8) Force carrying particle

* If strong force is 1, Coulomb force is 10^{-2}
If strong force is 1, gravity force is 10^{-37}

gravity force → important at macroscopic level

Repulsive = positive represent
attractive = negative represent

nucleons
↳ protons + neutrons

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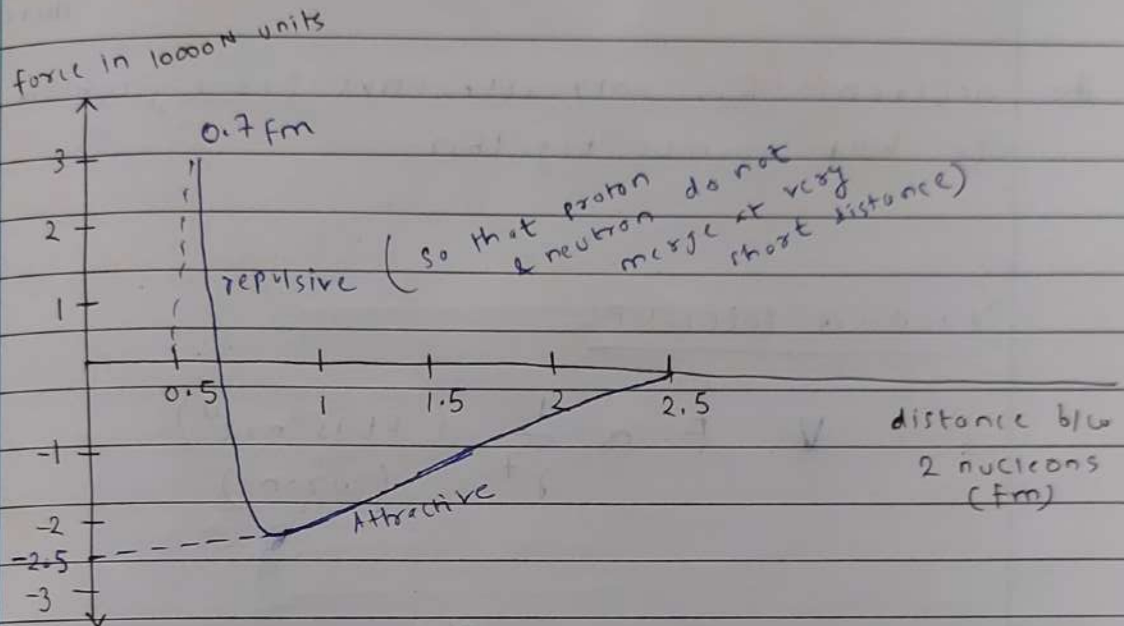
If not short range ? ⇒ Acting in range of fm (femtometres)

Acts in same way on proton and neutron ⇒ Charge independent



Repulsive at short dist < 0.7 fm

spin dependent ⇒ slightly more for parallel spins than anti-parallel spins

→ reason for repulsion at short dist.

PAULI'S EXCLUSION PRINCIPLE

↳ 2 identical fermions can't be in the same state

→ therefore two protons & two neutrons cannot come close together

protons & neutrons have ~~same~~ u and d quarks, therefore quarks are identical fermions and hence they cannot come together close according to Pauli exclusion principle

Camlin

Attractive nucleon force must be fine tuned.

* If nucleon-nucleon force 1% less attractive, then stable nuclei cannot be formed.

If 1% more attractive, we would have nuclei without neutrons., clumps of protons would be there

* neutrons have only attractive force, which helps to hold nucleus together.

YUKAWA POTENTIAL

$$F \propto \frac{1}{r^7}$$ (It is "not") (wrong)

Actually, $F \propto \dfrac{e^{-\alpha r}}{r^2}$

Yukawa

ϕ remains same if force carrying particles are massless & don't decay

If force carrying particles are massive it decays as $\phi = \phi_0 e^{-\mu_0 r}$

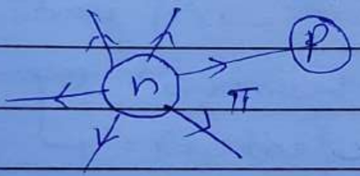
function of its mass

Force $F \propto \frac{e^{-\mu_0 r}}{r^2}$

Potential $V \propto \frac{e^{-\mu_0 r}}{r}$

→ massive particle $\simeq 200\, m_e$ or $\geq 200\, m_e$

π mesons or pions



Massive force carrying particles carry away mass

Heisenberg's uncertainity

$\Delta x \Delta p \simeq \hbar$

$\Delta E \Delta t \simeq \hbar$

energy = mass uncertainity is allowed for a short time

$m_\pi c^2 \Delta t \simeq \hbar$

$\Delta t \simeq \frac{r}{c}$ → range of strong force $\simeq 2\,fm$

c → speed of pion (actually $< c$)

$m_\pi \simeq \frac{\hbar}{rc} \simeq \frac{(10^{-34}\,Js)}{(2\times10^{-15}\,m)\times(3\times10^{8}\,m/s)} \simeq 2\times10^{-28}\,kg$

$\simeq 200\, m_e$

boson ← $m_\pi = 270\, m_e$

Integral spin

FCP — boson — 0, 1] spin

IP — fermion — $1/2$, $3/2$

Non-integral

fermion can't change to boson or vice-versa

force carrying particle has to be integral spin

$1/2$

Nuclear Models

Liquid drop model →

→ Bethe- Weizsacher semi-empirical mass formula (BE)

~~experimental~~

Postulates

i) Tends to sphere, surface tension

ii) Nearest - neighbors interactions

iii) Uniform density

Terms

1) Volume energy

2) Surface energy

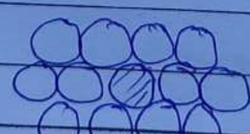
3) Coulomb energy

Volume Energy

Let energy of one nucleon-nucleon bond be U

BE of 1 nucleon due to 1 bond $= \frac{U}{2}$

In least volume, each nuclon has 12 nucleons around it.



BE of each nucleon due to all nucleons around it

$$= 12 \frac{U}{2} = 6U$$

If all nucleons are inside nucleus,

$$BE = 6AU$$

$$\boxed{E_v = \alpha_v A}$$

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$$\underline{E_b = \alpha_v A}$$
↳ considering only volume energy (neglecting others)

<u>Surface Energy</u>

- Surface nucleons have less nearest neighbors
⇒ BE is reduced due to some nucleons being on surface.

Reduced by a factor proportional to surface area of nucleus.

$E_S \propto 4\pi R^2 \Rightarrow E_S \propto A^{2/3}$

$$\boxed{E_b = \alpha_v A - \alpha_S A^{2/3}}$$

<u>Coulomb energy</u>

Coulomb PE between 2 protons $= \frac{e^2}{4\pi\varepsilon_0 r} \propto \frac{1}{r}$

" " b/w any pair of protons $\propto \frac{Z(Z-1)}{|r|_{avg}}$

average distance between any 2 protons ↗
$|r_{avg}| \propto R$

$$E_c = \alpha_c \frac{Z(Z-1)}{R} = \alpha_c \frac{Z(Z-1)}{A^{1/3}}$$

$$\boxed{E_b = \alpha_v A - \alpha_S A^{2/3} - \alpha_c \frac{Z(Z-1)}{A^{1/3}}}$$

$\left.\begin{array}{l}\alpha_v\\ \alpha_s\\ \alpha_c\end{array}\right\}$ experimental

$\left.\begin{array}{l}A\\ Z\end{array}\right\}$ Theoritical

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BW BE Formula

empirical

$$E_b = \alpha_v A - \alpha_s A^{2/3} - \alpha_c Z(Z-1)/A^{1/3}$$

Binding energy

~~repul~~ reduction due to Coulomb repulsion

* $\alpha_v, \alpha_s, \alpha_c$ found by fitting the numerical values of known Binding energy.

→ **Wapstra fitting**

$$\left.\begin{array}{l}\alpha_v = 14.1 \text{ MeV} \\ \alpha_s = 13 \text{ MeV}\end{array}\right\} \rightarrow$$

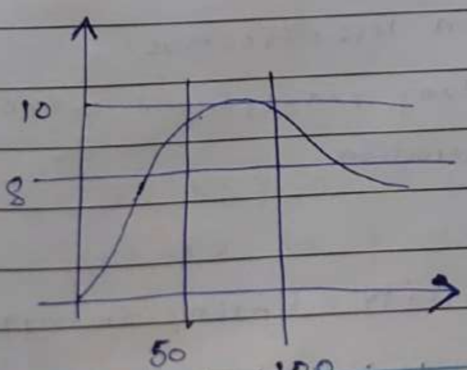
large due to it being ~~because~~ weak force

$\alpha_c = 0.595$ MeV

→ **Assymetric energy**

⇒ **Pairing energy**

* Plotting the three terms $(\alpha_v, \alpha_s, \alpha_c)$ and then summing them gives the following E_b/A curve:



proton-proton (coulomb) ~~coulomb~~ repulsion = long range
nucleon-nucleon = short range

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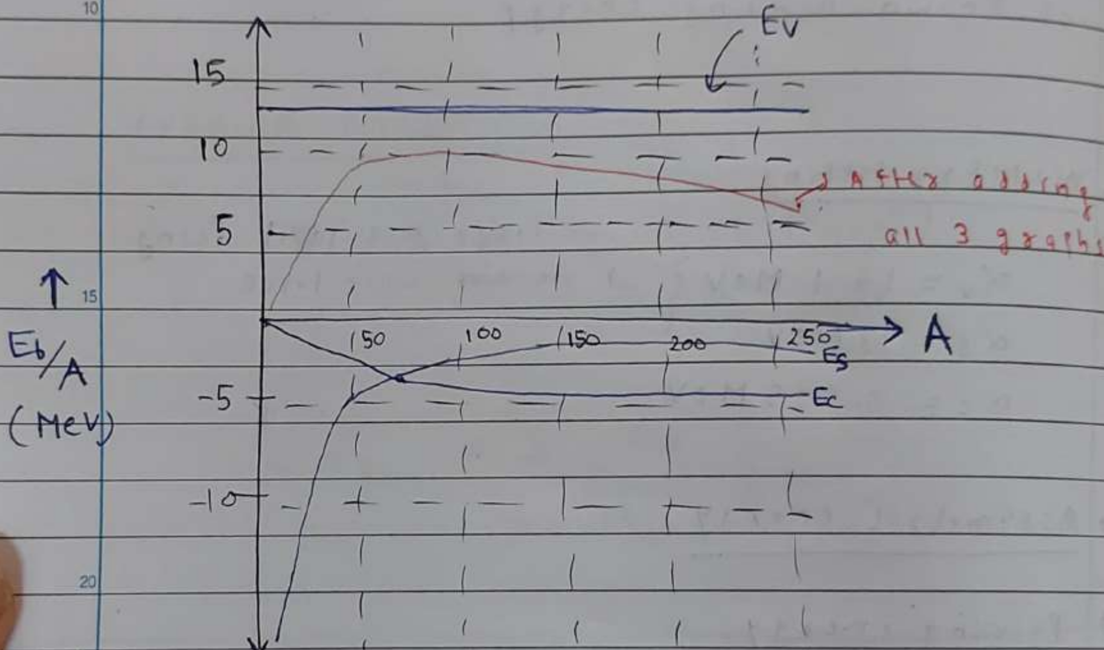
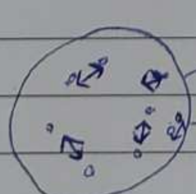
→ <u>Dividing formula by A:</u>

$$\frac{E_b}{A} = \underbrace{\alpha_v}_{\text{Volume energy } (E_v)} - \underbrace{\frac{\alpha_s}{A^{1/3}}}_{\text{Surface energy } (E_s)} - \underbrace{\frac{\alpha_c Z(Z-1)}{A^{4/3}}}_{\text{coulomb energy } (E_c)}$$



* Es term more important
  smaller spheres = surface more imp. than volume

* For higher values of A, Ec more important



→ nucleon-nucleon less effective
→ proton-proton long range, hence more coulomb repulsion

* good measure of stability = binding energy/nucleon

same A, diff. N & Z = isobars

protons & neutrons arranged in shells inside nucleus

Assymetry energy = For given A, assymetric nucleus is less stable (less binding energy)

Ex:

$A = 16$

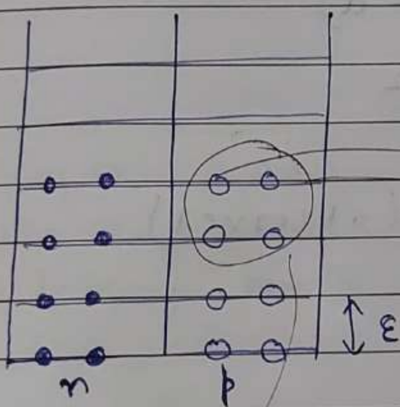
${}^{16}_{8}X$ ${}^{16}_{4}Y$

$Z = 8$ $Z = 4$ $(N - Z = 8)$

$N = 8$ $N = 12$

$(N - Z = 0)$

${}^{16}_{8}X$

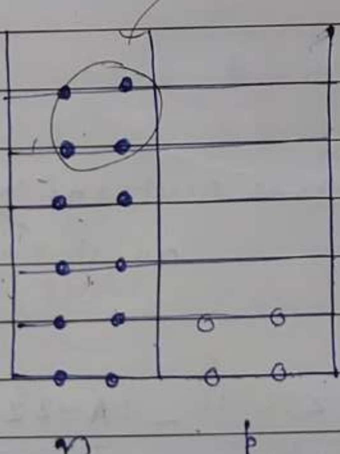


2ε

* Gaps b/w energy levels = constant

* only 2 protons allowed in a state, not more than 2 due to pauli exclusion principle

${}^{16}_{4}Y$



* $\frac{N-Z}{2} = 8/2 = 4$

no. of extra neutrons in ${}^{16}_{4}Y$ than ${}^{16}_{8}X$

* Extra energy of 1 extra neutron in ${}^{16}_{4}Y$ compared to p in ${}^{16}_{8}X$ is 2ε.

* $4 \cdot \varepsilon/2$

* $\left(\frac{N-Z}{2}\right) \cdot \frac{\varepsilon}{2} \Rightarrow$ extra energy of 1 extra neutron.

* extra energy of $\left(\frac{N-Z}{2}\right)$ neutrons

$$\propto \varepsilon (N-Z)^2$$

* Assymetry energy (E_a)

$$E_a \propto \varepsilon \, (N-Z)^2$$

$$\Rightarrow \quad \varepsilon \propto 1/A \qquad \text{(observed)}$$

$$\Rightarrow \quad E_a \propto \frac{(N-Z)^2}{A}$$

$$N + Z = A$$

$$\Rightarrow \quad E_a \propto \frac{(A-2Z)^2}{A}$$

(this term comes when no. of neutrons > no. of protons)

$$\therefore \; E_b = \alpha_v A - \alpha_s A^{2/3} - \alpha_c \frac{Z(Z-1)}{A^{1/3}} - \underbrace{\frac{(A-2Z)^2}{A} \cdot \alpha_a}_{E_a}$$

$$\boxed{\begin{aligned} \alpha_a &= 19 \text{ MeV} \\ \alpha_p &= 33.5 \text{ MeV} \end{aligned}}$$

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* The fifth term is purely experimental.

$$\boxed{E_b = \alpha_v A - \alpha_s A^{2/3} - \alpha_c \frac{Z(Z-1)}{A^{1/3}} - \alpha_a \frac{(A-2Z)^2}{A} (\pm, 0) \frac{\alpha_p}{A^{3/4}}}$$

pairing energy.

more stable

even-even nucleus : + (even N, even Z)

odd-odd nucleus : − (odd N, odd Z)

less stable

odd-even nucleus : 0 (N, Z OR Z, N)

"zero"

even-even = adds the pairing energy

odd-odd nucleus = subtracts the pairing energy

odd-even ~~nu~~ = adds 0

Q1) Nucleus = $^{64}_{30}Zn$

(i) Find BE using initial formula

(ii) Find BE using the above new expression

Soln: $E_b^{th} = 559.1$ MeV $< 0.5\%$

$E_b^{BWMF} = 561.7$ MeV

more accurate

$A = 64$

$Z = 30$

$N = 34$

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* **Uses of above formulae:**

(1) Find Z stable for given A

$$\frac{dE_b}{dZ} = 0 \longrightarrow Z_{stable}$$

$$\boxed{Z_{stable} = \frac{\alpha_c A^{-1/3} + 4\alpha_A}{2\alpha_c A^{-1/3} + 8\alpha_A A^{-1}}} = \text{Derive?}$$

if $35.8 \to 36$ (Find the nearest value of Z)
$35.3 \to 35$

(2) $$\boxed{\left(\frac{N}{Z}\right)_{stable} \simeq 1 + \frac{\alpha_c}{2\alpha_A} A^{2/3}}$$ $\frac{\alpha_c}{\alpha_A} \approx 1/60$ OR $1/80$

For ~~large~~ smaller values of A, it is almost 1.
For ~~very~~ large values of A, it becomes greater than 1

(3) Find most stable nucleus

Find A for maximum E_b/A

So, $\frac{d(E_b/A)}{dA} = 0$ (gives value for A)

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$A \simeq 63$ (copper ~~approx~~.)

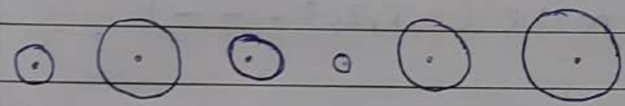
For this A, E_b/A would be maximum

VIBRATION SPECTRA of NUCLEI (excited)

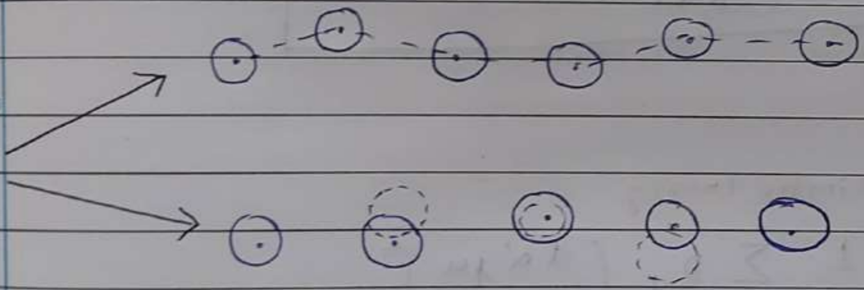
Different vibration modes are:

modes. time (t) →

d=0 Breathing

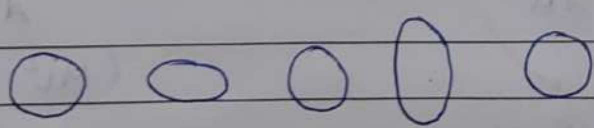


d=1

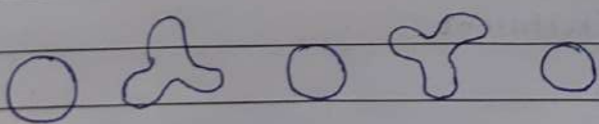


(protons may vibrate, neutrons may remain at same place)

d=2



d=3



ML^2T^{-2}

Y = dimensionless

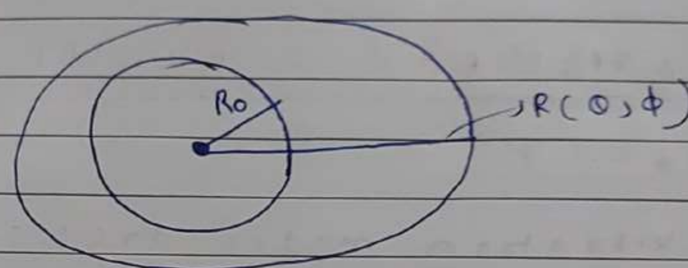
α = dimensionless

Spherical harmonics

$$R(\theta,\phi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha_{\lambda,\mu}(t)\, Y_{\lambda,\mu}(\theta,\phi)\right)$$

Shape parameters,

collective coordinates



λ = mode (0, 1, 2, 3 - - -)

μ = take values from $-\lambda$ to λ
($2\lambda+1$ values)

Kinetic energy

* $$T = \frac{1}{2}\sum_{\lambda\mu} D_\lambda \left(\frac{d\alpha_{\lambda\mu}}{dt}\right)^2$$

collective mass parameter

Potential energy

* $$V = \frac{1}{2}\sum_{\lambda\mu} C_\lambda\, \alpha_{\lambda\mu}^2$$

$$D_\lambda = \frac{\rho R_0^5}{\lambda}, \quad \rho = \text{density}$$

(ML^2 dimension)

Stiffness

Coefficient

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* $H_\lambda = \frac{1}{2} \sum_{\mu} D_\lambda \left(\frac{d\alpha_{\lambda\mu}}{dt} \right)^2 + \frac{1}{2} \sum_{\mu} C_\lambda \alpha_{\lambda\mu}^2$

Total energy
(must be constant)

* if we do, $\frac{dH_\lambda}{dt} = 0$

$$\Rightarrow D_\lambda \frac{d^2\alpha_{\lambda\mu}}{dt^2} + C_\lambda \alpha_{\lambda\mu} = 0$$

of form $\left(\frac{d^2x}{dt^2} + \omega^2 x = 0 \right)$

$$\omega_\lambda = \sqrt{\frac{C_\lambda}{D_\lambda}}$$

(Frequency)

C.S = cross section

lec 6 - NP (25th Jan, 2023)

NUCLEAR SHELL MODEL

PROPERTIES OF MORE STABLE NUCLEI →

1) Greater B.E. than predicted by B.W. Formula
2) Abundance = ~~stable nuclei are~~ found more in nature
3) High neutron separation energy
   = takes lot of energy to separate neutron from nucleus
4) Low n capture C.S.
   ↳ low probability of catching another neutron by nucleus.
5) Zero quadrupole moments
   ↳ nuclei are exactly spherical

N or Z or both = magic no.s

* Magic nos. = 2, 8, 20, 28, 50, 82, 128

B.E. from BW & actual B.E. are ~~quit~~ different at large)

	E_b (BW)	Actual E_b
$^{56}_{28}Ni$	478 MeV	484 MeV
$^{132}_{50}Sn$	1084 MeV	1110 MeV

* Not exactly spherical = ~~not~~ less stable
(~~high~~ non-zero quadrupole moment) ↑

octet rule - outermost shell cannot contain more than 8 e^-

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ATOMIC SHELL MODEL

Principal quantum no. (n)

shells			
1	k	$\longrightarrow$ s	2
2	L	$\longrightarrow$ s, p	8
3	m	$\longrightarrow$ s, p, d	18
4	n		
5	o		

azimuthal quantum no. (l) → orbital angular momentum

→ $2(2l+1)$ available states per orbital

0	0 — s
1	1 — p
2	2 — d
	3 — F
	4 — g

l	no. of available states	states per ~~orbitals~~
0	S → 2	$2(0+1)=2$
1	p → 6	$2(2+1)=6$
2	d → 10	
3	F → 14	
4	g → 18	

orbitals

* He = 2
  Ne = 10
  Ar = 18 } magic numbers for atomic shell model

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* energy and angular moment are quantized (can have discrete values)
↳ labelled by quantum nos.

* principal quantum nos in Atomic shell model ⟶ energy levels

* orbitals ⟶ orbital angular momentum levels

* For each l, m can take $(2m+1)$ magnetic quantum ~~nos.~~ values from $-l$ to l

* Each $2l+1$ angular momentum states accomodate $2e^-$, with different spins., therefore multiplicity of 2 from Pauli exclusion Principle

$2(2l+1)$

* The electrons also have spin angular momentum

* NUCLEAR SHELL MODEL $(n l_j)$ → remember

principal quantum no.

"spin-orbit coupling is important for nucleus"
↳ (20 times more than for atom)

$$\boxed{\vec{J} = \vec{L} + \vec{S}}$$

Total angular momentum

orbital angular momentum

spin angular momentum

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~~Short~~ For J For L For S

quantum no : j	l	s

$\boxed{j = l + s}$

l ↳ orbital quantum no.

s ↳ spin quantum no.

* "n" is no longer important
↳ energy level quantum no.

* Here, l and j are important

nlj

$j = l + s$ $\qquad s = \pm \frac{1}{2}$

For each l, two j's, $j = l \pm \frac{1}{2}$

How many j states?
↳ $(2j+1)$ available states for each j

$\Longrightarrow$

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increasing magnitude of energy

without LS coupling	with LS coupling	no. of available states per level $(2j+1)$	no. of available states per shell	Total no. of available states (add. up from below)
1g	$1g_{9/2}$	10	22	50
2p	$2p_{1/2}$	2		
	$1f_{5/2}$	6		
	$2p_{3/2}$	4		
1f	$1f_{7/2}$	8	8	28
2s	$1d_{3/2}$	4	12	20
1d	$2s_{1/2}$	2		
	$1d_{5/2}$	6		
1p	$1p_{1/2}$	2	6	8 $(2+6)$
	$1p_{3/2}$	4		
1s	$1s_{1/2}$	$2(1/2)+1 = 2$	2	2

	l
s	0
p	1
d	2
f	3
g	4

no $1s_{1/2}$
quantum no.
are always +ive

2 neutrons / protons
fill up first shell

* Independent and identical shell structure for both neutrons & protons

* $nl_j \Rightarrow j$ = angular momentum

$$\boxed{j = l + s}$$

* For every l, 2 values of j, $j = l \pm 1/2$

* Protons & neutrons have separate shell structure

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lec 7 - NP (30th Jan, 2023)

* Angular momentum in Q.M. (Part-2)

$$\boxed{\text{operator (state)} = \text{value (state)}}$$

all observables
are operators
in Q.M.

$H\psi = E\psi$ → value of energy

Total
energy operator

$H\psi_1 = E_1\psi_1$
$H\psi_2 = E_2\psi_2$

H acting on different ψ_i, different $H_i\psi_i$

→ eigen values

$$\boxed{\begin{aligned} J^2\psi &= j(j+1)\hbar^2\psi \\ L^2\psi &= l(l+1)\hbar^2\psi \\ S^2\psi &= s(s+1)\hbar^2\psi \end{aligned}}$$ → eigen value equations

* $\vec{J} = \vec{L} + \vec{S}$ (spin-orbit-coupling in vector form)

$$J^2 = L^2 + S^2 + 2\bar{L}\cdot\bar{S}$$

LS coupling (Spin orbit coupling)
Potential is proportional
to this.

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spin-orbit coupling potential

$\Rightarrow V_{LS} \propto \bar{L}\cdot\bar{S}$

$\Rightarrow V_{LS} \propto J^2 - L^2 - S^2$

$\Rightarrow \langle V_{LS} \rangle \propto j(j+1) - l(l+1) - s(s+1)$

value of V_{LS} denoted by $\langle \rangle$

* $E_{ls}^{+} \propto (l+\frac{1}{2})(l+\frac{3}{2}) - l(l+1) - \frac{3}{4}$

$(j = l + \frac{1}{2})$

energy of spin-orbit coupling for $j = l + \frac{1}{2}$

constant of proportionality

$\therefore \boxed{E_{ls}^{+} = V_0 l}$

* $E_{ls}^{-} \propto (l+\frac{1}{2})(l-\frac{1}{2}) - l^2 - l - \frac{3}{4}$

$(j = l - \frac{1}{2})$

$\boxed{E_{ls}^{-} = -V(l+1)}$

$\Rightarrow$ corrections to energy with spin orbit coupling

LS coupling = spin-orbit coupling

$E_0 > 0$

* Energy without LS coupling = $-E_0$

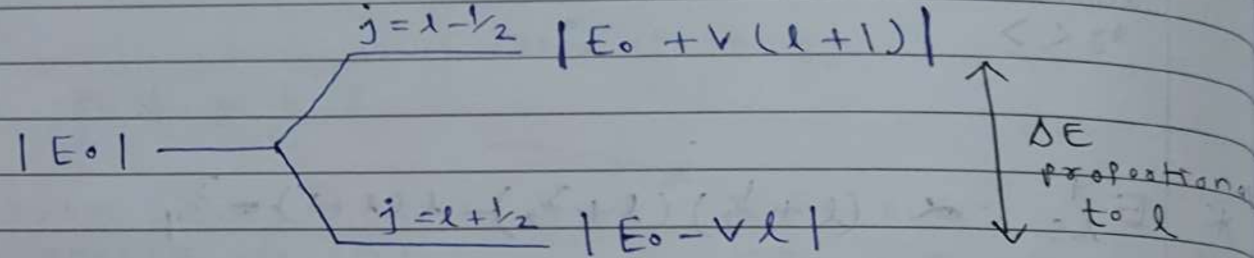
* Energy with LS coupling :

less magnitude

$$\boxed{\begin{aligned} E(j = l + 1/2) &= -E_0 + Vl \\ E(j = l - 1/2) &= -E_0 - V(l+1) \end{aligned}}$$

more magnitude

∴



* $$\left.\begin{aligned} \Delta E &= E_{ls^+} - E_{ls^-} \\ \Delta E &= V(2l+1) \end{aligned}\right\}$$

explains why for $(l=0)$ lower $1p_{3/2}$ & $1p_{1/2}$ gap is less

for $(l=1)$ $1d_{5/2}$ & $1d_{3/2}$ gap is more

& for $l = 2$ ($1f_{7/2}$ & $1f_{5/2}$), gap is even more

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difference b/w levels b/w 2 isotopes = difference b/w B.E. of last neutron b/w two ~~elements~~ isotopes

Q1) Find energy separation between $1p_{1/2}$ & $1d_{5/2}$ levels using B.E. and shell model

given $^{15}_{8}O = 111.96$ MeV

$^{16}_{8}O = 127.62$ MeV

$^{17}_{8}O = 131.76$ MeV

Soln. Protons same, neutrons differ

neutrons ~~neutrons~~ in $^{15}_{8}O = 7$

shell structure $= 1s_{1/2} - 2,\ 1p_{3/2} - 4,\ 1p_{1/2} - 1$

$^{16}_{8}O = 1s_{1/2} - 2,\ 1p_{3/2} - 4,\ 1p_{1/2} - 2$

$^{17}_{8}O = 1s_{1/2} - 2,\ 1p_{3/2} - 4,\ 1p_{1/2} - 2,\ 1d_{5/2} - 1$

B.E. of 8th neutron in $^{16}_{8}O = 127.62 - 111.96$ MeV
$= 15.66$ MeV

more stable, therefore B.E. more than $^{17}_{8}O$

B.E. of 9th neutron in $^{17}_{8}O = 131.76 - 127.62$ MeV
$= 4.14$ MeV

Difference b/w $1p_{1/2}$ and $1d_{5/2}$ levels $= 15.66 - 4.14$
$= 11.52$ MeV

Ans.

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we want to find total j of nucleus

SPIN-PARITY ASSIGNMENT OF NUCLEUS USING SHELL MODEL

j μ

* Parity: behavior of ψ (wave function of nucleus) under reversal of spatial coordinates (reflection)

$$\psi(-\bar{r}_1, -\bar{r}_2 - - - -\bar{r}_A) = \mu\,\psi(\bar{r}_1, \bar{r}_2 - - - \bar{r}_x)$$

$\bar{r}_n$ = space coordinates of n^{th} nucleon w.r.t centre of nucleus

* $\mu = \pm 1$

+1 = even-parity
−1 = odd-parity

* $\boxed{\mu = (-1)^l}$ (orbital angular momentum)

~~therefore~~ we just to find j and l

A	N	Z	
even	even-even nucleus : $j = 0,\ l = 0,\ \mu = +1$		
	N	Z	
odd	odd - even nucleus : j of unpaired nucleon, l of unpaired nucleon		
	N Z	Z N	
even	~~even~~ odd - odd nucleus : ~~will depend on j_1 & j_2~~		

l_1 & l_2 of 2 unpaired nucleons

$j = |j_1 - j_2|$ to $|j_1 + j_2|$ in steps of 1

$\mu = (-1)^{l_1 + l_2}$

Q2) $^{47}_{22}Ti$ (odd-even nucleus)

Soln: $N = 25$
$P = 22$

25^{th} neutron will be ~~unpaired~~ unpaired, and it will go into $1f_{7/2}$ (nlj)

s - 0
p - 1
d - 2
f - 3

$\therefore \boxed{\begin{matrix} j = 7/2 \\ l = 3 \end{matrix}}$

$\therefore \mu = (-1)^3 = -1$

$\boxed{j^{\mu} = (7/2)^{-1}}$

Q3) $^{6}_{3}Li$ $\longrightarrow$ $\boxed{j^{\mu} = (0, 1, 2, 3)^{+1}}$

Soln: 3N & 3P

3P in level $= 1p_{3/2}$ $\quad j_1 = 3/2,\ l_1 = 1$

3N in level $= 1p_{3/2}$ $\quad j_2 = 3/2,\ l_2 = 1$

$\mu = (-1)^{1+1} = 1$

$j = (3/2 - 3/2)$ to $(3/2 + 3/2)$

$= 0, 1, 2, 3$

stable nuclei = spherical nuclei

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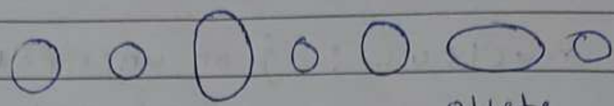
lec 8 – NP (1st Feb, 2023)

Rotational spectra of Nuclei



These shapes (which are not spherical) give rotational spectra

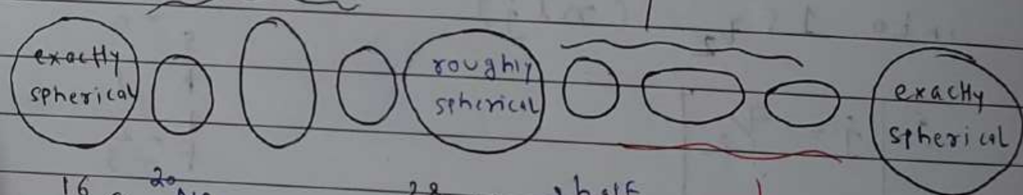
Closed shell nuclei = N & Z shells are filled

~~elongated towards poles~~

A →

Prolate shape

~~oblete~~ oblate



$^{16}_{8}O$ — 0 Quadrupole moment

$^{20}_{10}Ne$ — +ve quadrupole moment

$^{28}_{14}Si$ → half filled ~~third~~ shell → ~0 // quadrupole moment

−ve quadrupole moment

$^{40}_{20}Ca$ — 0 quadrupole moment

gives rotational spectra not necessarily no effect on rotation on spherical nucleus since it spherical symmetric

* Highly non-spherical (prolate/oblate) nuclei gives rotational spectra because they are not spherically symmetric.

* nuclei with $A = 150$ to 180 and $A = 220$ to 250 give observable rotational spectra without excitation

* Lighter non-spherical nuclei give rotational spectra ~~with~~ when excited.

* RADIOACTIVE DECAY = α, β, γ decay

"Becquerel, Curie, Rutherford"

→ $\alpha = {}_{2}^{4}He^{2+}$ (Helium nuclei)
heavy, least penetrating

→ $\beta = e^-$
lighter, more penetrating

→ γ = energy released in terms of photons
massless, most penetrating

* Radioactive decay is spontaneous (which happens by itself)

* → requires no input of energy

* → happens to make an unstable nucleus stable

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$\rightarrow$ ${}^{A}_{Z}X^{*} \rightarrow$ excited nucleus

$\rightarrow$ ${}^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + \gamma$

ground state nucleus

gamma

$\Rightarrow$ REASONS FOR RADIOACTIVE DECAY:

1) Attractive force is short range and not strong enough to hold only protons due to repulsive force, so neutrons are required.

2) For larger nuclei, the repulsive ~~coulomb~~ force becomes more important since the attractive force is localized. $\Rightarrow$ more neutrons are required for large A, to hold nucleus together.

$$\left(\frac{N}{Z}\right)_{\text{stable}} \simeq 1 + \underbrace{\left(\frac{\alpha_C}{2\alpha_A}\right)A^{2/3}}_{\approx 10^{-2}}$$

neutron-proton ratio for maximum stability for given A

$\Downarrow$

This is small, so A must be large for stable nucleus

becomes non-negligible for $A > 20$

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$A < 20 \Rightarrow$ almost equal protons & neutrons
$A = 20 - 209 \Rightarrow$ stable with more neutrons than protons
$A > 209 \Rightarrow$ unstable

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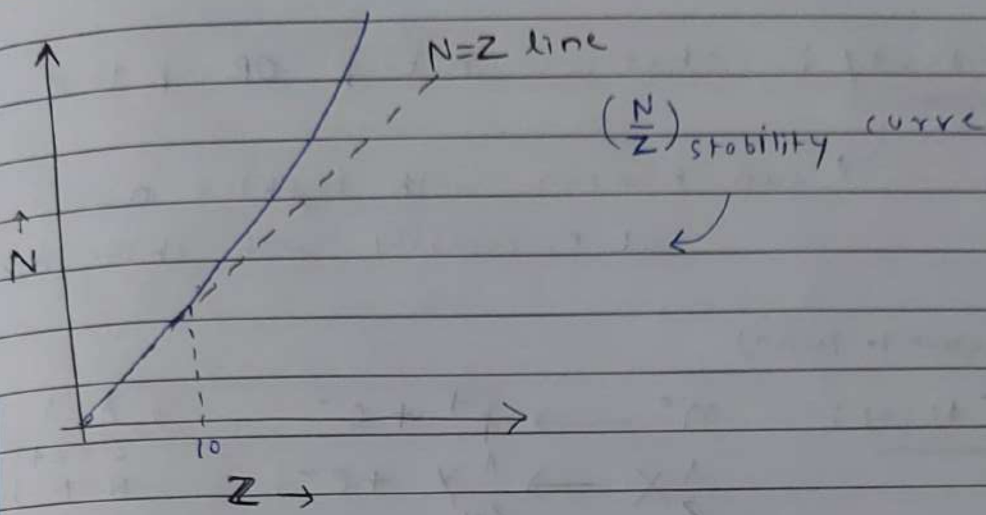

3) $^{209}_{83}Bi$ is the heaviest stable nucleus., beyond which nucleus is unstable no matter how many neutrons are there, and these nuclei with $A > 209$ are radioactive nuclei.

They emit alpha particle ~~beta particle~~ to become stable nuclei

* α-decay: → to become lighter nuclei with $A < 209$

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

$A = A - 4$
$Z = Z - 2$
$N = N - 2$

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α = heavy to lighter
γ = excited to ground

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* **β-decay :** changes n to p OR p to n
  ↳ can happen with lighter nuclei also not necessarily with those A > 209

(neutron to proton)

→ **β^- decay:** $n^0 \longrightarrow p^+ + e^-$

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^-$$ → A = A, Z = Z+1, N = N−1

→ **β^+ decay:** $p^+ \longrightarrow n^0 + e^+$

(proton to neutron)

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+$$ → Z = Z−1, N = N+1

→ to change wrong $\left(\frac{N}{Z}\right)$ ratio to come to correct (N/Z) ratio.

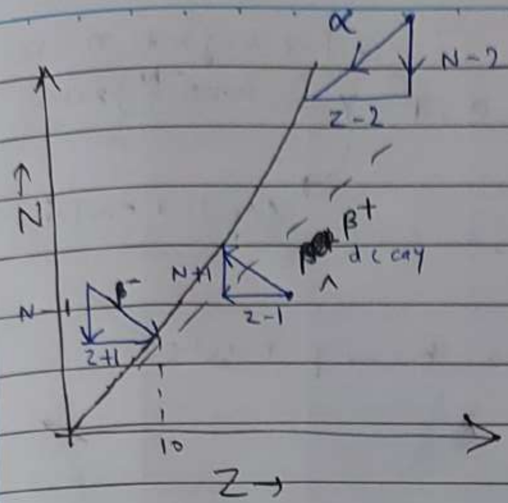
OR

to change $\left(\frac{N}{Z}\right)$ ratio to $\left(\frac{N}{Z}\right)_{stable}$

→ **e^- capture :** (when more protons) $p^+ + e^- \longrightarrow n^0$

$${}^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}Y$$ → Z = Z−1, N = N+1

Protons in nucleus capture electrons in innermost orbit


α
N-2
Z-2
N↑
β+
decay
N+1
β-
N
Z-1
Z+1
10
Z→

lec 9 - NP (7th Feb, 2023)

* **Radioactive decay** = α, β, γ

$\beta \rightarrow$ For light nuclei for stable $\left(\frac{N}{Z}\right)$ ratio

$\gamma \rightarrow$ excited nuclei

$$\underbrace{{}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He}_{}$$

α-decay happens for heavy nuclei

* decay constant (λ), Half life ($T_{1/2}$), mean lifetime, activity

$$N = N_0 \, e^{-\lambda t}$$

$\lambda \rightarrow$ decay constant

$N \rightarrow$ no. of nuclei at time "t"

$N_0 \rightarrow$ no. of nuclei at time "t = 0"

Proof:

λ = Probability time for 1 ~~nuclei~~ nucleus to decay

Probability for 1 nuclei to decay in time
$dt = \lambda \, dt$

No. of nuclei decaying in time $dt = N \lambda \, dt$
(N = no. of nuclei at a given time)

$\Rightarrow dN = -N \lambda \, dt$ (since dN ~~in~~ is a decrease)

If N_0 = Initial no. of nuclei at $t = 0$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t \mu \qquad (N < N_0)$$

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$\Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t$

Taking anti-log of both sides:

$$\Rightarrow \boxed{N = N_0 e^{-\lambda t}}$$

→ **Half life ($T_{1/2}$)** = time in which N_0 falls to $N_0/2$.

rivation:

$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$ (at $t = T_{1/2}$, $N = N_0/2$)

$\Rightarrow 2 = e^{\lambda T_{1/2}}$

$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

can be measured

can't be measured

→ **Mean life ($\bar{T}$)** ⇒ $\boxed{\bar{T} = 1/\lambda}$

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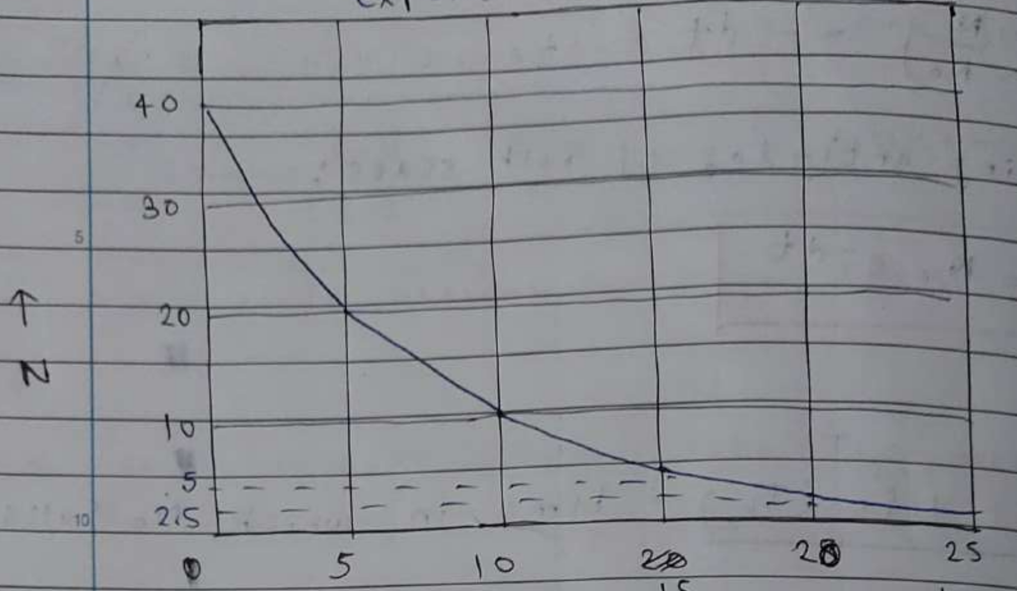
exponential decay



$T_{1/2} = 5$ hours

Q1) How long does it take for 60% of a sample to decay with $T_{1/2} = 3.82$ days?

Soln:
$$N = N_0 e^{-\frac{0.693t}{T_{1/2}}}$$

$$\Rightarrow \ln\left(\frac{N}{N_0}\right) = -\frac{0.693t}{T_{1/2}}$$

N = "how much is left"

$$\Rightarrow \ln\left(\frac{N_0}{N}\right) = \frac{0.693t}{T_{1/2}}$$

60% decayed so, 40% left

$$\Rightarrow t = \frac{T_{1/2}}{0.693}\ln\left(\frac{N_0}{N}\right)$$

$N = 0.4N_0$

$$\Rightarrow N = 0.4N_0$$

$$\Rightarrow \boxed{t = 5.05 \text{ days}}$$

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$N = N_0 e^{-\lambda t}$ $\quad R = \lambda N$

$R = R_0 e^{-\lambda t}$ $\quad T_{1/2} = \frac{0.693}{\lambda}$ } Formulae

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* Activity of sample R = rate of decay

$$= -\frac{dN}{dt}$$

(−ve since R should be positive)

$$\Rightarrow R = N_0 \left(-\frac{d}{dt} e^{-\lambda t}\right)$$

$$\boxed{R = N_0 \lambda e^{-\lambda t}}$$

$$\Rightarrow R_0 = \lambda N_0 \text{ at } t = 0 \Rightarrow \boxed{R = R_0 e^{-\lambda t}}$$

$$\boxed{R = \lambda N}$$

RADIOACTIVE SERIES

all these parent nuclei goes α-decay till they reach a stable daughter nuclei with $A \le 209$

* RADIOACTIVE SERIES

	$4n$	$4n+1$	$4n+2$	$4n+3$
Series	Thorium	Neptunium	Uranium	Actinium
Parent	$^{232}_{90}Th$ ↓	$^{237}_{93}Np$ ↓	$^{238}_{92}U$ ↓	$^{235}_{92}U$ ↓
Stable end product ($A \le 209$)	$^{208}_{82}Pb$	$^{209}_{83}Bi$	$^{206}_{82}Pb$	$^{207}_{82}Pb$

$4n$ $\quad ^{232}Th \longrightarrow 228 \longrightarrow 224$

$4n+1$ $\quad 237 \longrightarrow 233 \longrightarrow 229$

$4n+2$ $\quad 238 \longrightarrow 234 \longrightarrow 230 \longrightarrow 226$

$4n+3$ $\quad 235 \longrightarrow 231 \longrightarrow 227 \longrightarrow 223$

no nuclei in ~~comon~~ common

* 4-series are not related to each other with multiple of 4

not stable end product since Z is messed up, so it will go β-decay

* $^{232}_{90} \xrightarrow{\alpha} {}^{228}_{88} \xrightarrow{\alpha} {}^{224}_{86} \xrightarrow{\alpha} {}^{220}_{84} \xrightarrow{\alpha} {}^{216}_{82} \xrightarrow{\alpha} {}^{212}_{80} \xrightarrow{\alpha} {}^{208}_{78}$

It will undergo β-decay from now-on so as to correct the (N/Z) ratio.

* Sample of million Th nuclei ⇒ becomes mixture of all radioactive nuclei in that nuclei → some nuclei have decayed, some have not

This kind of sample finally reaches radioactive equilibrium (which means activity becomes equal of all nuclei in the series)

Ex: Let sample = A, B, C, D

$\Rightarrow R_A = R_B = R_C = R_D$

Their activities will be equal, important formulae

$$\Rightarrow \boxed{\lambda_A N_A = \lambda_B N_B = \lambda_C N_C = \lambda_D N_D \text{-----}}$$

$$\Rightarrow \frac{0.693 N_A}{T_{A_{1/2}}} = \frac{0.693 N_B}{T_{B_{1/2}}} \text{-----}$$

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* WHY α-decay ?

$$Q = (\underbrace{\Sigma m_i}_{\text{masses of initial nuclei}} - \underbrace{\Sigma m_f}_{\text{masses of final nuclei}}) \times 931.5 \text{ MeV}$$

actually nucleor masses

Q-value / disintegration energy
(energy released or absorbed in a nuclear reaction)

* For α-decay, Q is: $\left({}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He\right)$

$$Q_\alpha = \left(m\left({}^{A}_{Z}X\right) - m\left({}^{A-4}_{Z-2}Y\right) - m\left({}^{4}_{2}He\right)\right) \times 931.5 \text{ MeV}$$

radioactive decay is valid (it will happen)
radioactive decay is "energetically feasible"

* Q → +ve (if energy released)
Q → -ive (if energy absorbed)
→ not "energetically feasible" to be a radioactive decay (radioactive decay will not happen)

* $${}^{210}_{84}Po \longrightarrow {}^{206}_{82}Pb + \alpha$$

→ α is the smallest particle that heavy nucleus can lose with +ve Q-value

$m\left({}^{210}_{84}Po\right) = 209.9829 \text{ u}$

$m\left({}^{206}_{82}Pb\right) = 205.9745 \text{ u}$

$m_\alpha = 4.0026 \text{ u}$

$KE_\alpha = 5.3$ MeV
(Q value $\simeq KE_\alpha$ but not $= KE_\alpha$)

Soln: $Q\text{-value} = (209.9829 - 205.9745 - 4.0026) \times 931.5$

$= 5.4$ MeV (+ve) → this reaction can happen → energetically feasible

* $^{210}_{84}Po \longrightarrow {}^{209}_{83}Bi + p$ → not energetically feasible channel for Po to lose ~~become~~ weight

$m(^{209}_{83}Bi) = 208.9804$ u

$m(p) = 1.0073$ u

Soln: $Q\text{-value} = (209.9829 - 208.9804 - 1.0073) \times 931.5$

$= -4.4$ MeV (−ve)

→ not energetically feasible (so this reaction can't happen)

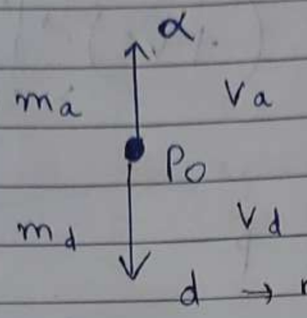
NOTE:

1) Q should be in nuclear masses, but for α-decay atomic masses give negligible error.

why?

(since masses of e^- negligible ~~in~~ w.r.t. mass of α-particle)

* <u>α-decay</u> $\Rightarrow$ parent at rest, daughter and α are moving



d → moving at lesser magnitude since it is heavy

From conservation of momentum:

$$\Rightarrow \quad m_\alpha v_\alpha = m_d v_d$$

$$\frac{m_d}{m_\alpha} = \frac{A-4}{4}$$

$$\Rightarrow \quad v_d = \frac{m_\alpha}{m_d} v_\alpha$$

$$\left({}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He \right)$$

$$= \frac{4}{A-4} v_\alpha$$

$$\Rightarrow \quad KE_\alpha = \frac{1}{2} m_\alpha v_\alpha^2$$

&

$$Q = KE_\alpha + KE_d = \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_d v_d^2$$
(↳ distributed b/w nucleus & daughter)

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$$\Rightarrow Q = \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_\alpha \left(\frac{A-4}{4}\right)\left(\frac{4}{A-4}\right)^2 v_\alpha^2$$

$$\Rightarrow Q = KE_\alpha \left[1 + \frac{4}{A-4}\right] = KE_\alpha \frac{A}{A-4}$$

$$\Rightarrow Q = \frac{A}{A-4} KE_\alpha$$

$$\Rightarrow \boxed{KE_\alpha = \frac{A-4}{A} Q}$$

very close but slightly different (since A is very large, $\frac{A-4}{A}$ is slightly less than 1)

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midterm
(Till ~~MT~~.) $V_d << V_\alpha$

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Lec 10 - NP (14th Feb, 2023)

BETA DECAY

To reach a stable $\left(\frac{N}{Z}\right)$ ratio & can happen in lighter / heavy nuclei

$\beta^- \Rightarrow n \longrightarrow p^+ + e^- \quad \left({}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^-\right)$

$\beta^+ \Rightarrow p^+ \longrightarrow n + e^+ \quad \left({}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+\right)$

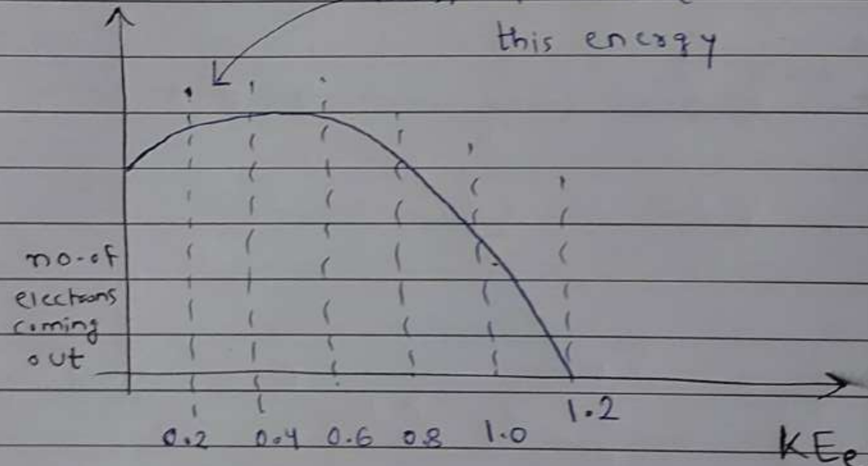
e^- capture $\Rightarrow p^+ + e^- \longrightarrow n \quad \left({}^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}Y\right)$

All e^- coming out will have this energy

* ${}^{210}_{83}Bi \longrightarrow {}^{210}_{84}Po + e^- \Rightarrow \boxed{Q = 1.17\ MeV}$

most of the e^- coming out with this energy

Energy spectrum of β-decay



If we apply conservation of linear momentum here:

$$m_d v_d = m_e v_e$$

$$m_d >> m_e$$

$$V_d << V_e$$

$$\boxed{KE_d << KE_e}$$

$\boxed{Q \simeq KE_e}$

(this should happen by conservation of energy), but doesn't happen

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Problem 1

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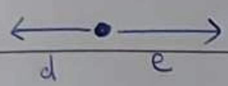
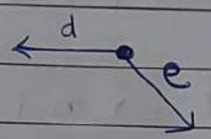
missing energy ? $\Rightarrow$ violation of conservation of energy

* $KE_{max} \simeq Q$-value

problem 2 $\Rightarrow$ Daughter nucleus and electron don't have equal and opposite momenta.



this should happen but it not



$\boxed{\text{Violation of conservation of linear momentum}}$

Problem 3 $\Rightarrow$

$$n \longrightarrow p + e$$

spin $\frac{1}{2}$ spin $\frac{1}{2}$ spin $\frac{1}{2}$

Half integral spin   integral spin } This not allowed.

$\boxed{\text{Violation of conservation of Angular momentum?}}$

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1953 ← ν = uncharged & unreactive

* How to solve above problems ?

↳ 1930 ⟹ Fermi ~~said~~ postulate another particle coming out

& Pauli named it ⟹ neutrino (little one)

β^- : $n \rightarrow p + e^- + \bar{\nu}$

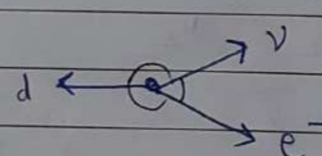
β^+ : $p^+ \rightarrow n + e^+ + \nu$

e^- capture : $p^+ + e^- \longrightarrow n + \nu$

* Problem 1 ⟹ "ν carrying away the missing energy."

$\therefore Q = KE_e + KE_\nu$

* Problem 2 ⟹



"ν momentum is balancing the total linear momentum"

* Problem 3 ⟹ $n \Rightarrow p + e + \nu$

assuming this is fermion (spin-$\frac{1}{2}$)

If ν is Fermion, it adds its spin-$\frac{1}{2}$ to RHS, so RHS also become half-integral

messengers of far-away things
early things (Big-Bang)....
providing signatures
of far away things

* properties of neutrino(ν):

1) small
2) almost massless
3) chargeless
4) fermions
5) unreactive
6) $\nu : p = 10^9 : 1$

means they are hard to detect

↳ neutrinos are found everywhere

7) messengers of far-away galaxies & times like Big-Bang.

---

* Q-value of β decay? (For calculation, forget the neutrino)

$$\Rightarrow {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^-$$

$$Q_{\beta^-} = \big(\underbrace{m_N({}^{A}_{Z}X) - m_N({}^{A}_{Z+1}Y)}_{\text{nuclear masses}} - m_e\big) \times 931.5$$

$$\underbrace{m_N({}^{A}_{Z}X)}_{\text{nuclear mass}} = \underbrace{m({}^{A}_{Z}X)}_{\text{Atomic mass}} - Zm_e$$

$$m_N({}^{A}_{Z+1}Y) = m({}^{A}_{Z+1}Y) - (Z+1)m_e$$

$$\therefore\ Q = (m({}^{A}_{Z}X) - Zm_e - m({}^{A}_{Z+1}Y) + (Z+1)m_e - m_e) \times 931.5$$

$$\boxed{Q_{\beta^-} = m({}^{A}_{Z}X) - m({}^{A}_{Z+1}Y)}$$ → more easily to be +ve } more happen

* Both β^+ & e^- capture reduces protons & hence e^- capture more likely to happen

$\Rightarrow$ $${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+$$

$$Q_{\beta^+} = (m_N({}^{A}_{Z}X) - m_N({}^{A}_{Z-1}Y) - m_e)$$

$$m_N({}^{A}_{Z-1}Y) = m({}^{A}_{Z-1}Y) - (Z-1)m_e$$

$$Q_{\beta^+} = m({}^{A}_{Z}X) - Zm_e - m({}^{A}_{Z-1}Y) + (Z-1)m_e - m_e$$

$$\boxed{Q_{\beta^+} = m({}^{A}_{Z}X) - m({}^{A}_{Z-1}Y) - 2m_e}$$

harder to be +ve due to this

(less likely to happen)

$\Rightarrow$ $${}^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}Y$$

$$Q_{e^-\text{-capture}} = m_N({}^{A}_{Z}X) + m_e - m_N({}^{A}_{Z-1}Y)$$

$$\boxed{Q_{e^-\text{-capture}} = m({}^{A}_{Z}X) - m({}^{A}_{Z-1}Y)}$$

more likely to be +ve

(more likely to happen than β^+ decay) energetically feasible $\Rightarrow$ does not require energy but will emit energy

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* what about free "n" OR free "p" decay ?

free $Q_{\beta^-} \Rightarrow (m_n - m_p - m_e) \times 931.5$ MeV (+ve)

free $Q_{\beta^+} \Rightarrow (m_p - m_n - m_e) \times 931.5$ MeV (−ve)

$m_n = 1.008665$ u

$m_p = 1.007276$ u

$m_e = 0.000548$ u

neutron decay happens $\Rightarrow$ (+ve) Q-value

proton decay impossible $\Rightarrow$ (−ve) Q-value
↳ (proton is stable & doesn't decay)

lifetime of $\approx$ 15 mins.

γ radiation $\nearrow d \sim nm$
$\searrow \nu \sim 10^{17}$ Hz (freq)

γ-DECAY

* release of energy by excited nuclei ⇒ observation requires excited nucleus state to last for a while

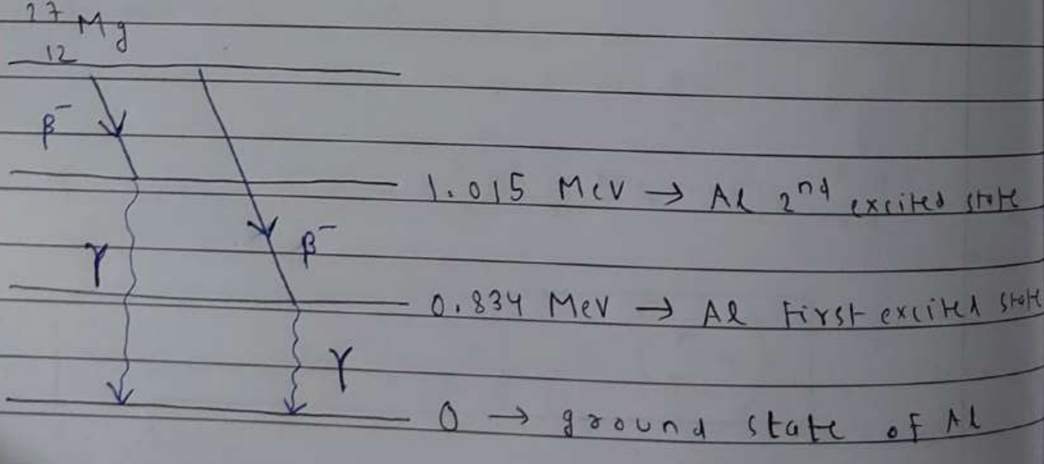
* $^{87}_{38}Sr^{(*)} \longrightarrow {}^{87}_{38}Sr + \gamma \quad (T_{1/2} \simeq 2.8 \text{ hours})$

excited ← (on $^{87}_{38}Sr^{(*)}$); ground state ← (on $^{87}_{38}Sr$)

* γ-decay can follow β-decay

$$^{27}_{12}Mg \longrightarrow {}^{27}_{13}Al^{*} + e^{-} + \bar{\nu}$$

$$^{27}_{13}Al^{*} \longrightarrow {}^{27}_{13}Al + \gamma$$



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→ Another way for excited to ground

* Internal conversion ⇒ nucleus ~~gets~~ gives energy to atomic e^{-1}'s which becomes excited and then come back to ground state causing atomic spectra.

(AM)
lec 11 - NP (14th March, 2023)

* Types of nuclear reactions :

1) Scattering — elastic / inelastic   ~~X,~~ $X(a, a)X$ (incoming particle: a; outgoing particle: a)

2) Capture (incoming particle captured by nucleus, outgoing energy in form of γ) $X(a,\gamma)Y$

3) Transfer ⇒ $X(a, b)Y$

4) Decay ⇒ α, β, γ

5) Fission ⇒ 1 nucleus = 2 nuclei

6) Fusion ⇒ 2 nuclei = 1 nucleus

* $X(a, b)Y \Rightarrow X + a = Y + b$
(X, Y: nucleus; a, b: particle)

* Conservation laws followed in nuclear reactions

1) Nucleon number (A is conserved)

2) Charge

3) Momentum — Linear / Angular (Spin & ~~angular~~ orbital angular momentum both need to be conserved)

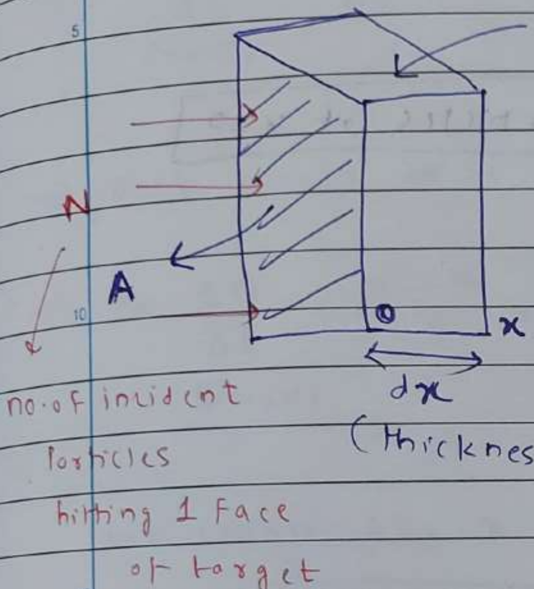
4) Mass & Energy (including mass-energy equivalence)
↳ Total amount of mass-energy conserved.

σ depends on nature of incoming particle & parent nucleus
It also depends on energy of incoming particle

* <u>Cross-section (σ)</u> = measure of likelihood of a reaction written as an effective area.

target (with parent nuclei)

$$\boxed{\text{no. density } (n) = \frac{\text{no. of atoms}}{\text{volume}}}$$



HOW DOES N BEHAVE AS FUNCTION of x (distance moved through target)?

⇒ dN = no. of particles reacting after moving thru thickness dx of target.
= decrease in no. of incoming particles

⇒ if σ = effective area offered by 1 nucleus to incoming beam.

⇒ $nAdx$ = Total no. of nuclei in target

⇒ aggregate σ = Total effective area of the target for incoming beam
$= \sigma n A dx$

⇒ $\frac{dN}{N} = \frac{\sigma n A dx}{A} = \frac{\text{effective area}}{\text{Actual area}}$

dN → no. of particles reacting
N → no. of incident particles

σ different for diff. materials.

<u>Writing as negative:</u>

$$\Rightarrow \frac{dN}{N} = -\sigma n \, dx$$

$\boxed{N_0 \text{ is incident no. of particles at } x=0}$

$\Rightarrow$ Integrating both sides

$$\Rightarrow \int_{N_0}^{N} \frac{dN}{N} = -\int_0^x \sigma n \, dx$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\sigma n \int_0^x dx$$

why it can be taken outside ?

(Uniform density target with same nuclei throughout, uniform incident beam with particular energy)
(same)

$$\Rightarrow \boxed{N = N_0 \, \underbrace{e^{-\sigma n x}}_{\text{Probability factor}}}$$

distance moved by particles through the target

Probability factor

n = no. density of atoms or nuclei in target
N_0 = initial no. of incident particles
N = no. of incident particles after distance x
σ = cross-section (area)
x = dist. through target

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how much of beam goes away in time interval Δt

REACTION RATE

$$\Rightarrow \underset{\text{(small)}}{\frac{\Delta N}{\Delta t}} = \frac{N_0 - N}{\Delta t}$$

$n' =$ no. of atoms or nuclei in target

$\phi =$ flux $=$ no. of incoming particles / area × time

$\Rightarrow$ Dist. moved is x in time Δt

$$\Rightarrow \underset{\text{(small)}}{\frac{\Delta N}{\Delta t}} = \frac{N_0 - N}{\Delta t} = \frac{N_0(1 - e^{-n\sigma x})}{\Delta t}$$

for small Δt, x is also small & $\boxed{n\sigma x \ll 1}$

$$\boxed{e^{-n\sigma x} = 1 - n\sigma x} + \underbrace{O(N^2)}_{\text{can be neglected since } n\sigma x \text{ is small}} - - - -$$

only this much is taken

$$\Rightarrow \frac{\Delta N}{\Delta t} = \frac{N_0}{\Delta t}(1 - 1 + n\sigma x)$$

$$\Rightarrow \frac{\Delta N}{\Delta t} = \frac{N_0 (n\sigma x)}{\Delta t}$$

usually known. (measured qty.)

Incoming flux of particles $(\phi) = \frac{N_0}{\text{Area} \times \text{time}} = \frac{N_0}{A \cdot \Delta t}$

$\Rightarrow \left(\frac{N_0}{\Delta t} = A\phi\right)$ {to remove Δt factor from above equation}

$$\Rightarrow \frac{\Delta N}{\Delta t} = A\sigma x n \phi$$

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KE_{CM} = KE in centre of mass frame

⇒ "n" is no. density

$n = \frac{n'}{Ax}$ ← no. of nuclei in target

to remove factor 'x' from previous page

⇒ $\boxed{\frac{\Delta N}{\Delta t} = \sigma n' \phi}$

"reaction rate"

RELATION B/W Q & K.E. OF INCOMING PARTICLE

* What KE_{lab} will make reaction happen?

* Q-value, if -ve, gives $K.E._{CM}$ required for reaction to happen.

KE_{CM} has to be $\geq |Q|$

this much energy reqd. for reaction to happen.

$\boxed{\sigma \text{ is calculated in barns} = 10^{-28}\, m^2 = 100\, fm^2}$

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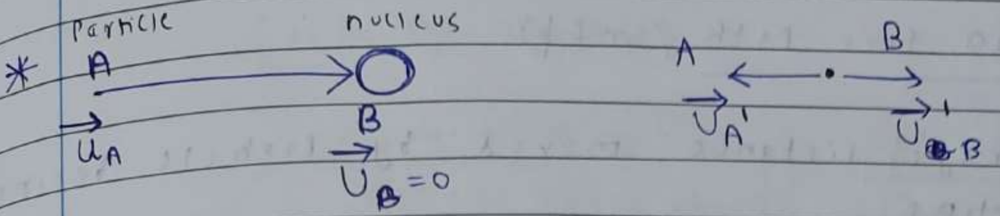
lab frame

C.O.M frame



$$\vec{u}_A' = \frac{m_B \vec{u}_B}{m_A + m_B} \qquad \vec{u}_B' = -\frac{m_A \vec{u}_A}{m_A + m_B}$$

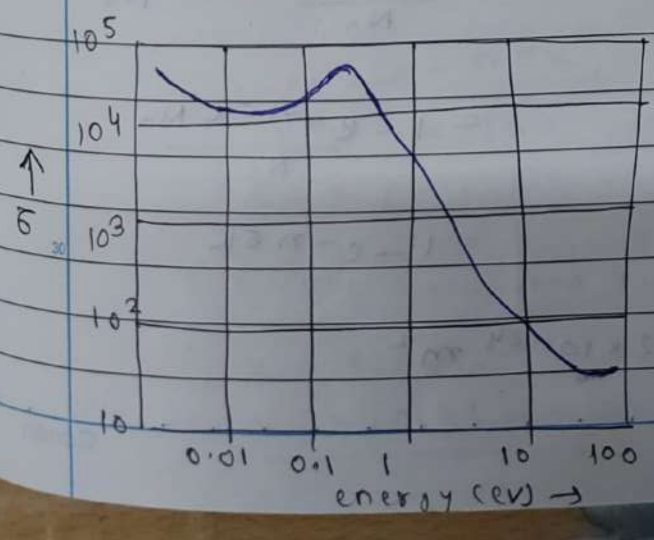
$$KE_{lab} = \frac{1}{2} m_A v_A^2$$

$$KE_{COM} = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 = \frac{m_B}{m_A + m_B}\left(\frac{1}{2} m_A v_A^2\right)$$

$$\boxed{KE_{COM} = \frac{m_B}{m_A + m_B} KE_{lab}}$$

energy that particle must carry if Q-value is < 0

* $^{113}_{46}Ca + n \longrightarrow {}^{114}_{48}Ca + \gamma$



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(AM)

lec 12 – NP (17th March, 2023)

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* **Mean free path (mfp)**

Average distance moved by particle before reacting

$$\boxed{mfp = \frac{\int_0^\infty x e^{-n\sigma x}\,dx}{\int_0^\infty e^{-n\sigma x}\,dx} = \frac{1}{n\sigma}}$$

Q1) $^{113}Cd + n \longrightarrow {}^{114}Cd$ $\quad \sigma = 2\times10^4$ barns $(2\times10^4 b)$

mean atomic mass of natural Cd = 112u

density = 8.64 g/cm^3

^{113}Cd is 12% of natural cadmium

$(1u = 1.66\times10^{-27}$ kg$)$

Find (i) Fraction of incident beam of n absorbed by 0.1 mm thick cadmium sample

Soln: No. of n absorbed $= N_0 - N$

Fraction of n absorbed $= \frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0}$

$$= 1 - \frac{e^{-n\sigma x} N_0}{N_0}$$

$$= 1 - e^{-n\sigma x}$$

$\sigma = 2\times10^4$ b $= 2\times10^{-24}$ m^2

$x = 10^{-4}$ m

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$\boxed{n}$ = no. density

$$= \boxed{\frac{\text{density}}{\text{mass of 1 atom}}} = \frac{8.64 \times 10^3 \ kg/m^3}{112 \times 1.66 \times 10^{-27} \ kg}$$

"This is of every type of cadmium but we want only ^{113}Cd"

therefore

$$n_{actual} = \frac{12}{100} \times \left(\frac{8.64 \times 10^3 \ kg/m^3}{112 \times 1.66 \times 10^{-27} \ kg} \right)$$

$$= 5.58 \times 10^{27} \text{ atoms}/m^3$$

$$n\sigma = 1.12 \times 10^4 \ m^{-1}$$

$$\boxed{\text{Fraction}} = 1 - e^{-n\sigma x} = \boxed{0.67}$$

(ii) Thickness of Cadmium needed to absorb 99% of incident neutrons

Soln:

$$N = N_0 e^{-n\sigma x}$$

99% absorbed $\Rightarrow$ 1% left

$$N = 0.01 N_0$$

$$\frac{N}{N_0} = 0.01$$

$$0.01 = e^{-n\sigma x}$$

$$e^{n\sigma x} = 100$$

$$n\sigma x = \ln 100$$

Put value calculated previously

$$\boxed{\begin{aligned} x &= 4.1 \times 10^{-4} m \\ &= 0.41 \ mm \end{aligned}}$$

if we have around 0.5mm, then 100% may be absorbed

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ΔN = no. of reactions happening in time Δt

(iii) mean free path

Soln: $\boxed{m f p} = \frac{1}{n\sigma} = \boxed{0.0893 \text{ mm}}$

Q2) Natural gold has only ^{197}Au
σ for: $^{197}Au + n \longrightarrow {}^{198}Au$ is 99 b

How long should a ~~10 mm~~ 10 mg of gold foil be exposed to flux of 2×10^4 neutrons/$m^2 s$ in order for the sample to have 2.48×10^{12} 198 Au atoms ?

Soln: $\Delta t = \frac{\Delta N}{\phi n' \sigma}$ $\quad \sigma = 99 \text{ b}$
$\phi = 2 \times 10^4 \ m^{-2} s^{-1}$

$$\boxed{n' = \frac{\text{mass of target in kg}}{\text{mass of 1 atom in kg}}}$$

mass of target in kg → 10 mg
mass of 1 atom in kg → $197 \times 1.66 \times 10^{-27}$ kg

$n' = 3.06 \times 10^{19}$ atoms

ΔN = no. of 198 Au after Δt time
= no. of reactions in Δt time
$= 2.48 \times 10^{12}$

$$\boxed{\Delta t = 6 \text{ min } 49 \text{ sec.}}$$

β decay = Parity is not conserved

* TO STUDY A REACTION FOR CONSERVATION OF ANGULAR MOMENTUM & PARITY

$(l_{rel})^{\pi_{rel}} = ?$ $\quad (l_{rel})^{\pi_{rel}} = ?$ $\Rightarrow$ these we need to find in this topic ↓

$${}^{7}_{3}Li + {}^{1}_{1}H \longrightarrow {}^{4}_{2}He + {}^{4}_{2}He$$

$j^{\pi} \quad j^{\pi} \quad j^{\pi} \quad j^{\pi}$

$Q = 17.35$ MeV

"We want to know j & π of individual nuclei"

	j^{π}
${}^{4}_{2}He$	$=(0)^{+}$
${}^{4}_{2}He$	$=(0)^{+}$
${}^{7}_{3}Li =$	$(3/2)^{-}$
${}^{1}_{1}H =$	$(1/2)^{+}$

$\boxed{\text{even-even} \Rightarrow \pi = +1}$

$1p \begin{cases} 1p_{1/2} \\ 1p_{3/2} \end{cases}$

$1s - 1s_{1/2}$

$P = 3$
$N = 4$

* $\boxed{(\pi_{rel}) \text{ relative motion parity} = (-1)^{l_{rel}}}$

→ Parity is also conserved in this reaction

"Parity is conserved in some reactions only (will be given in exam)"

→ Parity is multiplicative

* We want to calculate $(l_{rel})^{\pi_{rel}}$ on both sides

* RHS is 2 identical bosons → wavefunction has to be symmetric

→ +ve parity μ^{rel} (+1)

it means RHS ⇒ l_{rel} has to be even
⇒ l_{rel} can be 0, 2, 4 - - -

$\boxed{RHS = (0, 2, 4 - - -)^{+}} \rightarrow (l_{rel})^{\mu_{rel}}$

$\boxed{\text{Total } j^{\mu} = (0, 2, 4 - - -)^{+} \text{ on RHS}}$

* μ_{rel} on LHS = −ve (so as total becomes +ve)

$\mu_{rel} \times (-) \times (+)$
on LHS. | For ${}^{7}_{3}Li$ | For ${}^{1}_{1}H$

⇒ $\mu_{rel} = (-1)^{l_{rel}} = (-1)$ 1, 3, 5 - - -

⇒ l_{rel} can have odd values = $(1, 3, 5 - - -)^{-}$

⇒ We choose 1^{-} on LHS since higher l_{rel} doesn't allow nuclei to come close enough to react

Reason.
if higher values, particles move faster past each other & do not spend time with another to react

"Angular momentum is additive"

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$\boxed{\text{Total } j^H \text{ on LHS}} = (\cancel{0}, 1, 2, \cancel{3} \dots) +$ not allowed, since only 1 & 2 are on RHS & we need conservation

$\left\{ \begin{array}{c} 1 + 3/2 - 1/2 \\ 1 - 3/2 + 1/2 \\ -1 + 3/2 + 1/2 \\ \dots \\ 1 + 3/2 + 1/2 \end{array} \right\}$ angular momentum is additive and since they can be parallel/antiparallel, can be added or subtracted.

of $(L_{rel})^{H_2 el}$ of L_1 of H

(AM)

lec 13 - NP (21st March, 2023)

Nuclear reactions

energy = few eV to <1 MeV (slow neutrons)

Time reqd. for incident particle to be in target ~ 10^{-15} s

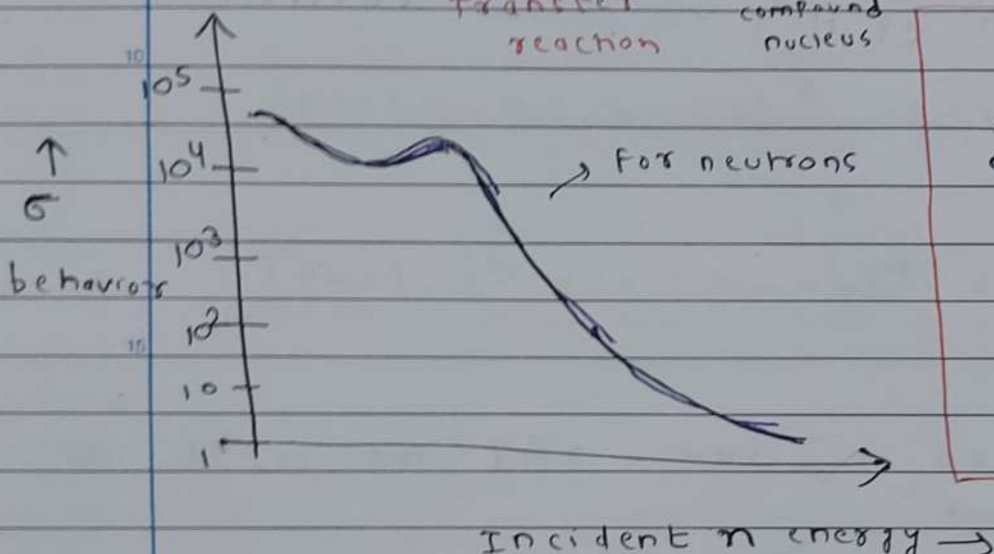
Compound nucleus reactions

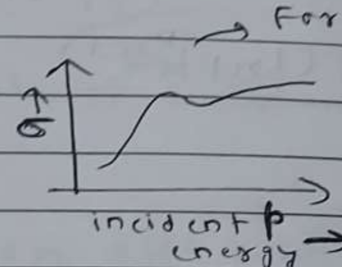
Capture ~~reaction~~ reaction: $^{107}Ag + n \longrightarrow {}^{108}Ag + \gamma^*$

~~$^{107}Ag + n \rightarrow {}^{108}Ag^*$~~
~~$^{108}Ag^* \rightarrow {}^{108}Ag + \gamma$~~

$\boxed{X + a \longrightarrow Y + b^* \qquad X + a \longrightarrow C^* \longrightarrow Y + b}$

Transfer reaction — compound nucleus





(~~light~~ proton cross-section increases with increasing K.E. since proton repelled by nucleus & that much min. energy reqd to overcome repulsion

* Why decreasing? = because n has to spend some time in target for reaction to happen.

* ~~W~~ Why peak? = More energetic (faster) n pass thru in quickly for reaction to happen

→ Faster n is less likely to have the reaction

→ σ decreases with increasing n energy.

So High energy of proton required to reach the target.

$$* \quad {}^{107}Ag + n \longrightarrow {}^{108}Ag^*$$ compound nucleus

$${}^{108}Ag^* \longrightarrow {}^{108}Ag + \gamma$$

* compound nucleus loses memory of ~~formation~~ Formation.

formation channel

$$X + a \longrightarrow C^*$$

$$C^* \longrightarrow Y + b$$ decay channel

$$* \quad \left.\begin{array}{l} {}^{1}_{0}n + {}^{13}_{7}N \\ {}^{1}_{1}H + {}^{13}_{6}C \\ {}^{2}_{1}H + {}^{12}_{6}C \\ {}^{3}_{1}H + {}^{11}_{6}C \\ {}^{3}_{2}He + {}^{11}_{5}B \end{array}\right\} \longrightarrow {}^{14}_{7}N^* \longrightarrow \left\{\begin{array}{l} {}^{13}_{7}N + {}^{1}_{0}n \\ {}^{13}_{6}C + {}^{1}_{1}H \\ {}^{12}_{6}C + {}^{2}_{1}H \\ {}^{10}_{5}B + {}^{4}_{2}He \end{array}\right.$$

imp. Decay channel depends on properties & excitation state of compound nucleus, not on formation channel.

* C.S. for formation of compound nucleus

$$\sigma = A, \; B$$

$$\sigma_a = A \left[\frac{B}{(E - E_R)^2 + C} \right]$$

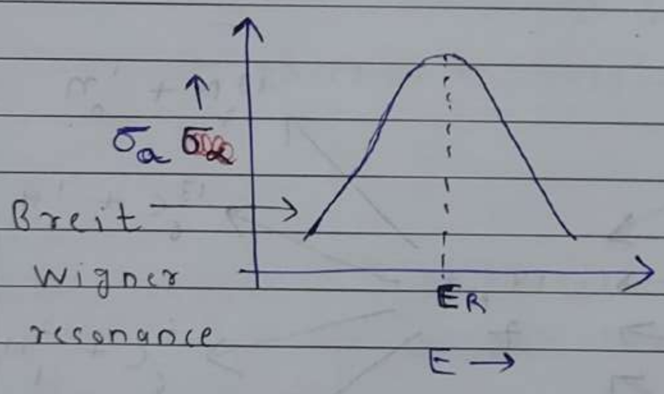
σ_a — cross section of formation channel

E — K.E. of incident particle

C — constant energy

A, B, C — independent of energy

$\Downarrow$ graph



E_R = energy of compound nucleus (state)

* C.S. for reaction to happen is a function of function σ_a → some function of σ_a

$$\sigma_{ab} = F(\sigma_a)$$

σ_{ab} — cross section for reaction

10% to 15% are direct reactions

↑

* Direct reactions → Fast n react only with surface of nuclei within 10^{-21} s ~~to directly~~ give end particles.

~~[illegible]~~

$x + a \longrightarrow y + b$ } these reactions can also happen

* Slow n ~ few eV to < 1 MeV spend 10^{-15} s time

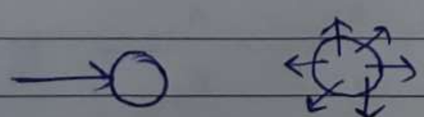
* Fast n > 1 MeV, spend 10^{-21} s time

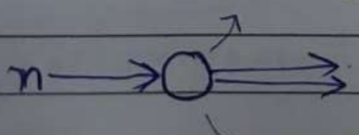
Features of Direct reactions

1) ~~Mono~~ Monotonic dependence of σ on neutron energy
   → no peaks in graph →  → not allowed

2) Highly energetic output particles
   → due to ~~highly~~ energetic input particles

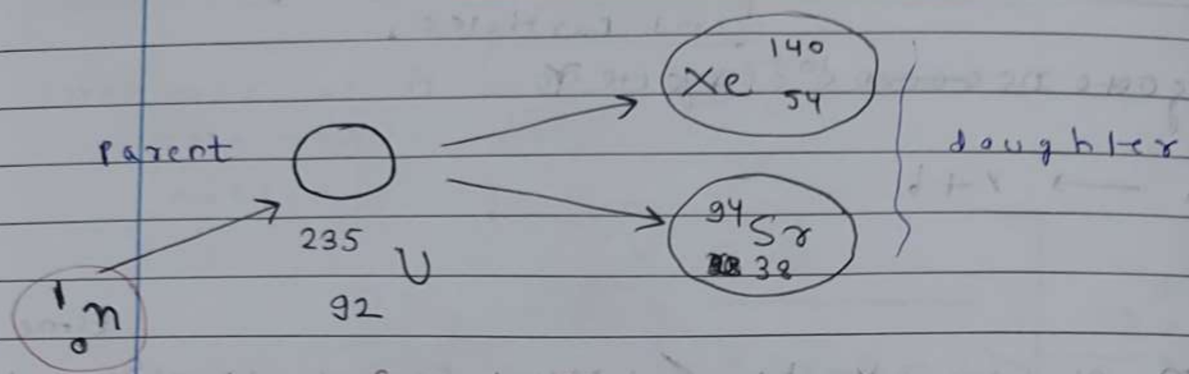
3) Forward peaking of output particles spectrum.
   → O/P particles tend to go in same direction as that of I/P particles.

C.N.R.  O/P particles in all directions

D.R.  O/P particles in same direction as I/P particles

this also happen but mostly in same direction.

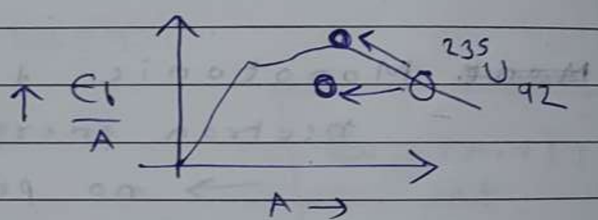
NUCLEAR FISSION



Incoming beam of neutrons

* exothermic / exergic reaction because E_b/A of parent is lower than of daughters.

* excess E_b/A of daughter is released.



* 99.9% = make it happen
  0.1% = spontaneously

* nuclear weapons & artificial way of generating energy

* releases 200 MeV of energy (Huge source of Power)

* Fission is compound nucleus reaction

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has to be sufficiently excited to vibrate ~~violently~~ violently & break up

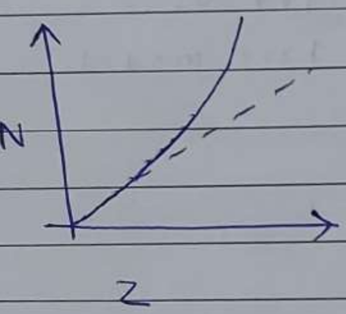
$$2\left({}^{1}_{0}n\right)$$

* $${}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{236}_{92}U^{*} \longrightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + \underbrace{2\,{}^{1}_{0}n}_{}$$

2 neutrons released.

* no. of released ~~neutrons~~ n > no. of incident n

because $\left(\frac{N}{Z}\right)_{stable}$ ratio is higher for heavier nuclei



* $$\underbrace{{}^{1}_{0}n + {}^{A+B+C}_{Y+Z}X \longrightarrow {}^{A+B+C+1}_{Y+Z}X^{*}}_{} \longrightarrow {}^{A}_{Y}X' + {}^{B}_{Z}X'' + (C+1)\,{}^{1}_{0}n$$

"C+1" neutrons

"general expression for nuclear fission"

* artificially induced fission (99%)

* $$\underbrace{{}^{A+B+C}_{Y+Z}X \longrightarrow {}^{A}_{Y}X' + {}^{B}_{Z}X'' + C\,{}^{1}_{0}n}_{\text{spontaneous fission (rare)}}$$

(AM)

lec 14 – NP (24th March, 2023)

* **Nuclear fission** → fair probability of happening

$${}^{A+B+C}_{Y+Z}X + {}^{1}_{0}n \longrightarrow {}^{A+B+C+1}_{Y+Z}X^{*} \longrightarrow {}^{A}_{Y}X' + {}^{B}_{Z}X'' + (C+1)\,{}^{1}_{0}n$$

⇒ Needs energy but exothermic

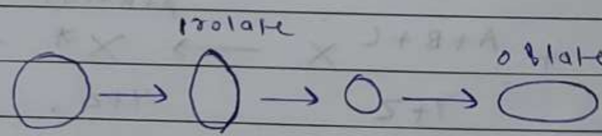
* Fission chain reaction
↳ makes ~~it~~ fission useful for us

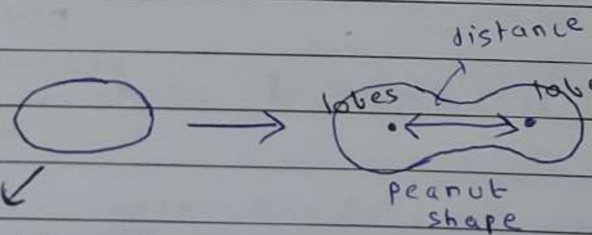
* Fission as per Liquid drop model

* ^{235}U ~~and~~ vs. ^{238}U

* **Liquid drop (LD)**

LD → excited Nucleus →





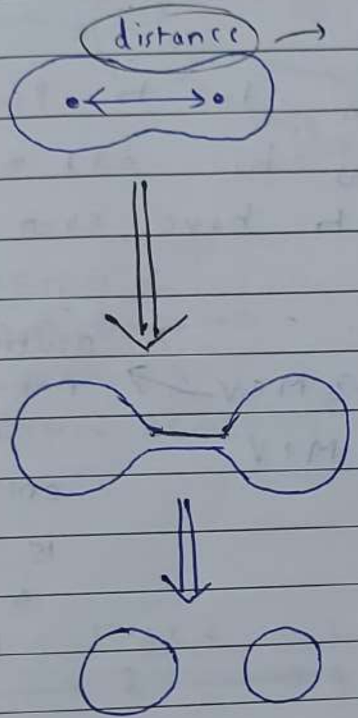
if vibrate violently enough, then nuclei may become peanut shape

form 2 lobes separated by distance.

almost Impossible to create 8 MeV incoming neutrons

2 forces exist in nuclei:

* N-N short range attractive force
* Repulsive coulomb force



→ if > critical distance, attractive force loses its efficiency and only repulsive force acts

"dumbell shape" (only repulsive force and no attractive force)

2 nuclei

$KE_{\text{incoming neutrons}} + BE_{\text{incoming neutrons}}$

* About 7-8 MeV energy is reqd. to form lobes

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* $BE_n = BE_{in} = \underline{BE \text{ incoming neutrons}}$

higher means it wants another neutron.

* ^{235}U vs. ^{238}U

⟹ ^{235}U has higher B.E. (~~in~~) incoming ~~neutrons~~ &

^{238}U has lower BE_{in} due to Pauli's exclusion principle, ^{235}U has odd no. of neutrons and wants to have even no. of neutrons

⟹ BE_n for $^{235}U \simeq 7-8$ MeV → neutrons does not need to carry K.E. only its B.E. is enough to make fission happen

BE_n for $^{238}U \simeq 6$ MeV → neutrons need to have ~2 MeV K.E. to make fission happen

⟹ hard to make fission happen in ~~238U~~ ^{238}U

⟹ ^{238}U more likely to capture the incoming neutron (even of ~ 1 MeV).

⟹ "Fissile nuclear fuel" ⟹ ^{235}U, fuel which makes fission easily happen

⟹ "Fissionable" ⟹ ^{238}U

○ = nuclei • = neutrons

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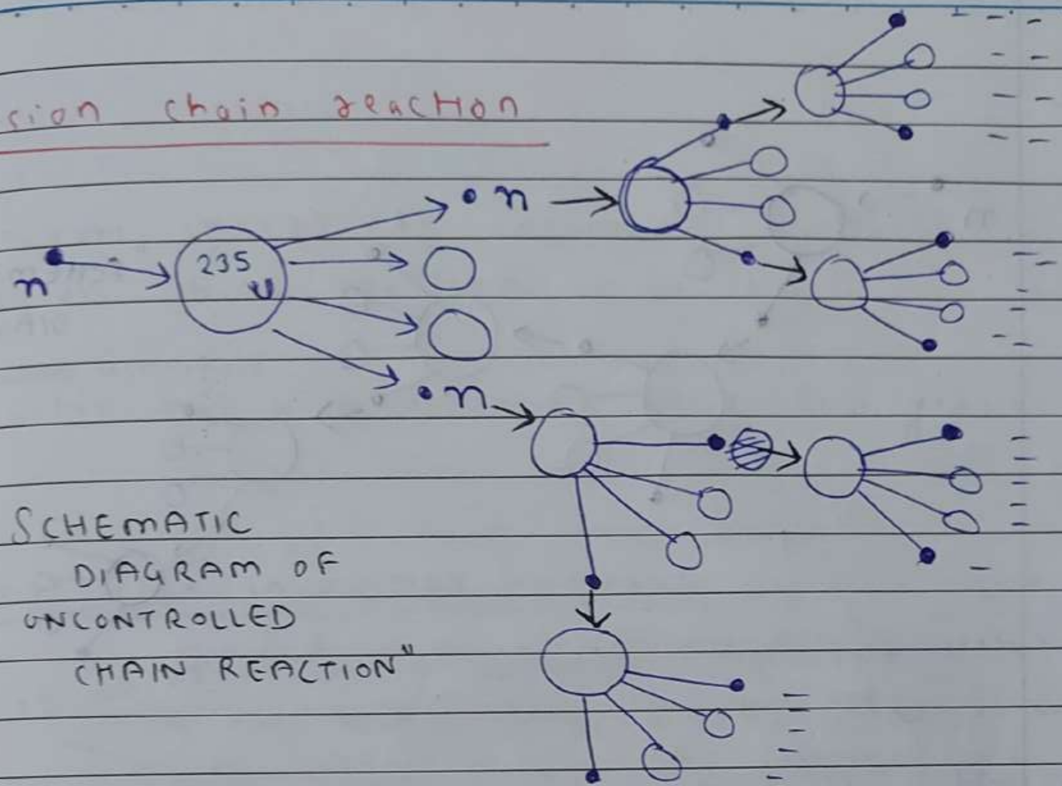
* Fission chain reaction



"SCHEMATIC DIAGRAM OF UNCONTROLLED CHAIN REACTION"

Stage 1	stage 2	stage 3	stage 4	stage 5

* 1 ⟶ 2 ⟶ 4 ⟶ 8 ⟶ 16 - - - -

exponentionally increasing fission reactions if both neutrons used for fission in each stage

* "Super critical or uncontrolled chain reaction"

→ more than 1 neutron from one fission is used in the next fission

→ release uncontrolled amount of energy (explosion)

→ Used in nuclear weapon / bombs

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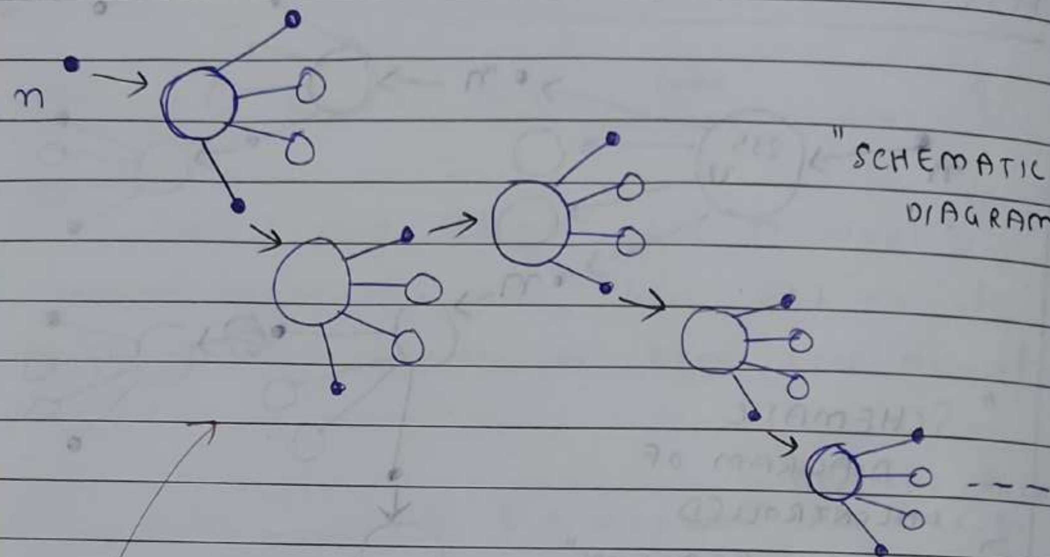

"SCHEMATIC DIAGRAM"

* $\boxed{\text{Critical or controlled chain reaction}}$

→ exactly 1 neutron from one fission is used in the next fission

→ controlled release of energy

→ ~~Nucle~~ used in nuclear reactors

* $\boxed{\text{Sub-critical chain reaction}}$ ⟹ less than 1 neutron from 1 fission is used in the next fission

→ chain reaction dies out with time

→ not useful to us

Natural U ⇒ 99.3% ^{238}U
0.7% ^{235}U

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Nuclear reactors

⇒ Generate energy by controlled (critical) fission chain reaction.

~~Generally~~ Generally
⇒ Reactor has a lifetime of a few years.

* Things reqd. in nuclear reactors

1) Fuel ⇒ uranium → we want ^{235}U but Natural U has 99.3% ^{238}U & 0.7% ^{235}U so we use processes such as diffusion etc. to get enriched U with 97% ^{238}U and 3% ^{235}U

→ uses uranium oxide (UO_2)

2) ~~Control rods~~

3) ~~Coolant~~

4) ~~Moderator~~

2) Control rods
⇒ "Cd" rods with large σ of neutron capture, which is used to keep chain reaction in critical phase

Cadmium

⇒ can be pushed in and out of core of reactor which has fuel.
⇒ Have a sensor → if too much energy released, pushed in
→ if less energy released, pushed out

⇒ maintains the reaction in critical phase (controlled phase)

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O/P of nuclear reactor

3) Coolant

⇒ material (usually H_2O) which carries away the generated energy for its use, use can be like run turbine etc.

4) Moderator

⇒ slows down neutrons so that they don't get captured by ^{238}U and instead have fission with ^{235}U

to overcome problem of capture by ^{238}U

⇒ slows down neutrons to few eV

⇒ we want atoms that collide with neutrons & slow them

⇒ max. KE transfer happens if 2 particles are of same mass.

⇒ Hydrogen has same mass as neutrons, so H_2O is an efficient moderator.

HE = Heat energy   EE = electrical energy.

ME = mech. energy

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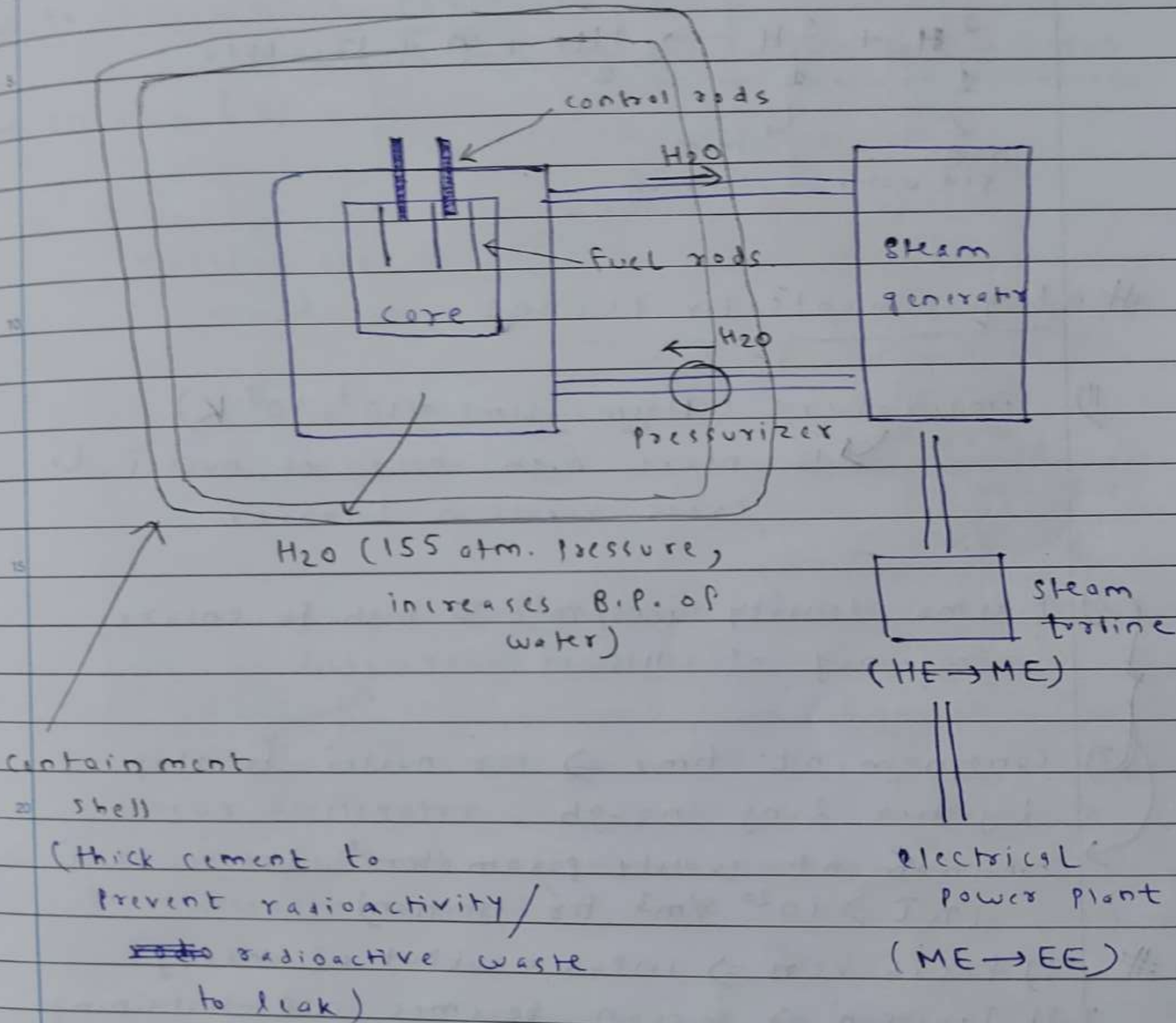
* <u>SCHEMATIC DIAGRAM OF NUCLEAR REACTOR</u>



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lec-15 (31st March)

Breeder reactor (No moderator)

$^{238}_{92}U + n \longrightarrow {}^{239}_{92}U + \nu$ → half life of 5 min

$^{239}_{92}U \longrightarrow {}^{239}_{93}Np + e^- + \nu$ → half life of 5 hours

$^{239}_{93}Np \longrightarrow {}^{239}_{94}Pu + e^- + \nu$ → unstable

Reactor or Breeder of nuclear fuel $^{239}_{94}Pu$

* Here 1 reactor cycle = $\boxed{3 \text{ years}}$

* More ^{239}Pu generates than ^{235}U

* ^{238}U is being used, no enriching required

* No moderator

* New fuel is generated = ^{239}Pu

* Problems

(1) Operational costs are high since coolant is different from water.
↳ D_2O (Heavy water) is used, so that there are no H to slow down neutrons!

(2) ^{239}Pu is fuel for nuclear weapons since more fissile than ^{235}U ⟹ Security risk

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NUCLEAR FUSION

* lighter nuclei join to form heavier nuclei releasing energy.

* Energy in stars, creation of elements

* Fusion of hydrogen happens in $\boxed{\text{core of stars}}$

* Req. of Fusion

1) $\boxed{\text{High temp.} \sim 10^7\,K}$ $\Rightarrow$ so that lighter nuclei have enough energy to approach close enough to overcome repulsion.

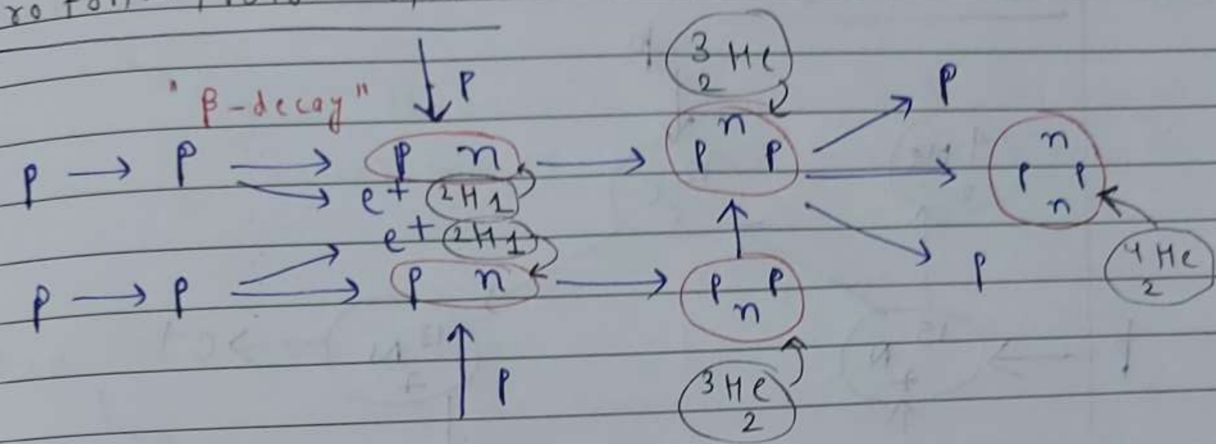
2) $\boxed{\text{High Density}}$ $\Rightarrow$ To ensure high frequency of collisions.

3) $\boxed{\text{Large reacting mass}}$ $\Rightarrow$ To give nuclei enough time to have fusion.

* Hydrogen fusion $\sim 10^7 k$ temperature
Sun core temp. $\sim 1.5 \times 10^7\,K$

* Sun is an intermediate star, young/middle aged star (4.6 billion years)
Sun $\sim$ 70% Hydrogen, 28% Helium, 2% other elements.

* Proton-Proton cycle



$$2 \times \left({}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu \right)$$

$$2 \times \left({}^2_1H + {}^1_1H \rightarrow {}^3_2He \right)$$

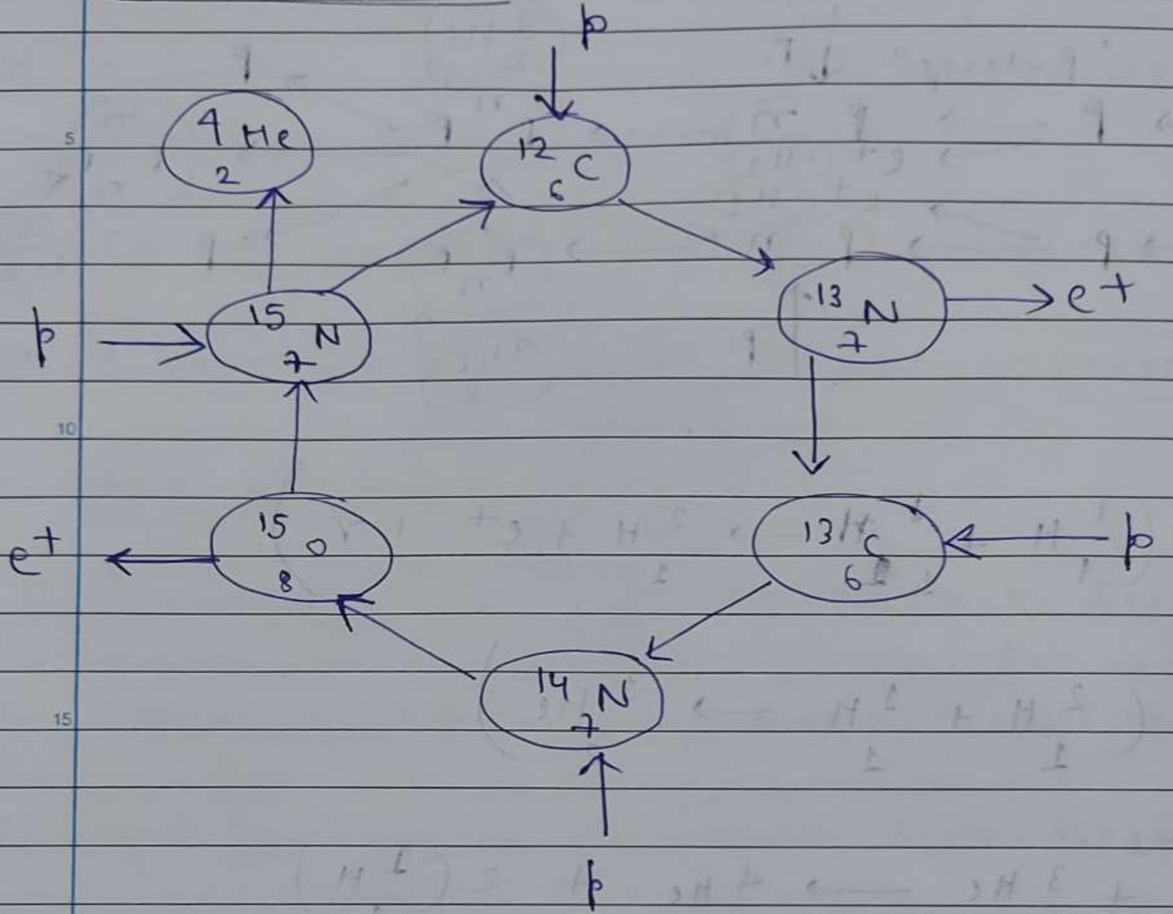
$${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2\left({}^1_1H\right)$$

* **I/P** = 4p

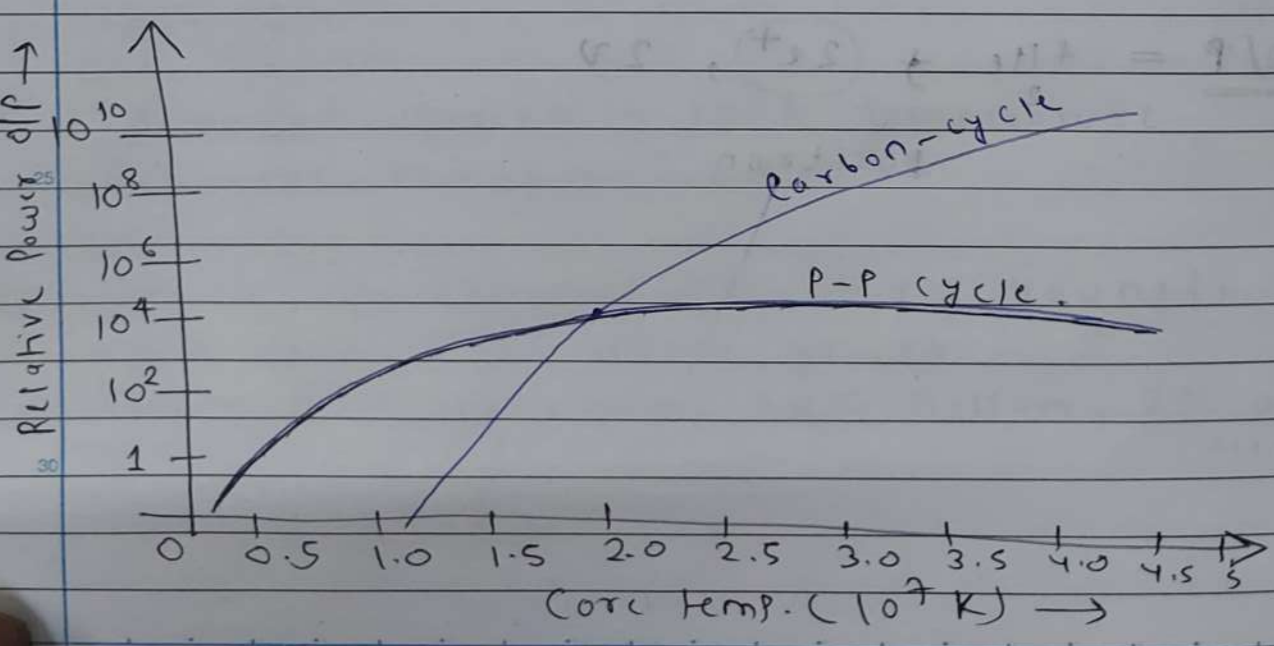
* **O/P** = ${}^4_2He$ + $2e^+$, 2ν

positron

* Carbon cycle



* Relative Power o/p vs. core temp.:



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* For stars of size of sun (5 billion years)
  → when Hydrogen used up ⇒ red giant ⇒ white dwarf ⇒ dead star

* For massive stars
  → nova & supernova

---

* Formation of elements = higher core temp. $\sim 10^8$ K

* Triple α reaction

$$ {}^{4}_{2}He + {}^{4}_{2}He \longrightarrow {}^{8}_{4}Be $$
$$ {}^{4}_{2}He + {}^{8}_{4}Be \longrightarrow {}^{12}_{6}C $$

"can happen is sun!"

* $$ {}^{12}_{6}C + {}^{4}_{2}He \longrightarrow {}^{16}_{8}O $$
$$ {}^{12}_{6}C + {}^{12}_{6}C \longrightarrow {}^{24}_{12}Mg $$
$$ {}^{12}_{6}C + {}^{12}_{6}C \longrightarrow {}^{20}_{10}Ne + {}^{4}_{2}He $$

~~Sun~~ * Stars > 10x sun's mass
* temp. $\sim 10^8$ K

* Fusion forms elements upto $^{56}_{28}Fe$

* For $A > 56$, neutron capture followed by β-decay creates new elements.

$56 \leq A \leq 209 \rightarrow$ highest stable element

* $209 < A \leq 260 \Rightarrow$ Formed by "n-capture" in supernova explosion

↙

High n-density is there

* We are made up of stardust scattered from nova / supernova explosion.

(AM)

Lec 16 – NP (11th April, 2023)

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Nuclear fusion reactors → possible?
↳ experimentally not, theoretically yes

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + n + 17.6\,MeV$$

$^{2}_{1}H$ from sea water; $^{3}_{1}H$ from lithium

Requirements for fusion:

1) Temperature (plasma temp $\sim 10^{7} - 10^{8}$ K)
→ to ensure high energy of nuclei to cross repulsion barrier

2) Plasma density (ions/m^3) ⇒ high to ensure frequency of collisions

3) Confinement time ⇒ for nuclei to stay together long enough

→ "confinement quality parameters"
($n\tau > 10^{20}$ s/m^3 for ignition)

1) Breakeven ⇒ input = output of energy
2) Ignition ⇒ fusion becomes self-sustaining

We want to achieve these in fusion reactors

$\boxed{\text{ITER}}$ = International thermonuclear ~~exptal~~ ~~exptal~~ experimental reaction (35 countries)

$\boxed{\text{tokamak}}$ = To confine plasma using magnetic field.

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PARTICLE ACCLERATORS :

⇒ to accelerate particles upto 10's of GeV
↳ to probe much more inside the nucleus (go to smallest particle)

→ energy $(E) = \frac{hc}{\lambda}$ ← de Broglie wavelength

nuclear size $\approx 10^{-15}$ m
$h = 4.135 \times 10^{-15}$ eVs

↳ putting these values, we get
$E = 1000$ MeV
$= 1$ GeV

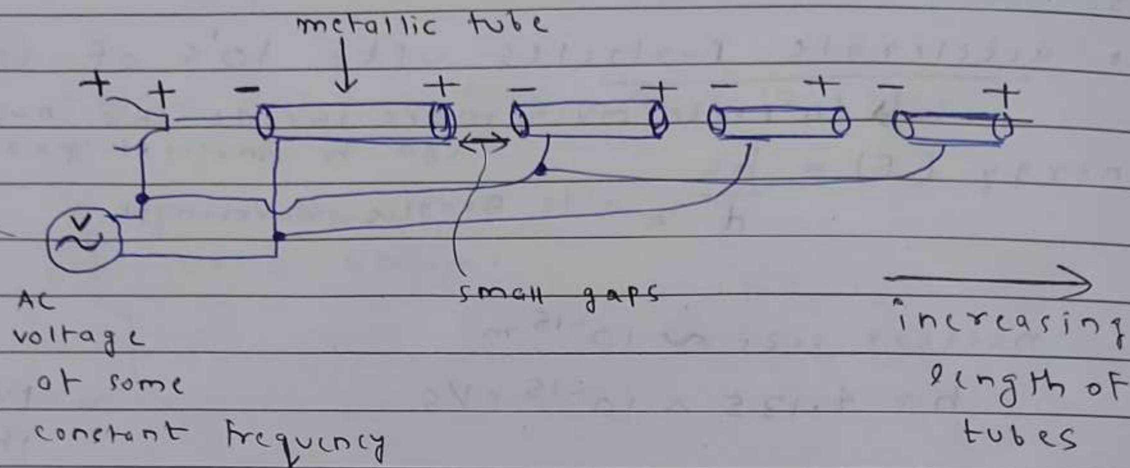
Accelerator

Linear accelerator
⇒ uses electric field to accelerate charged particles

Cyclotron
uses electric & magnetic field to accelerate charged particles

AC voltage = V

* Linear Accelerator (LINAC)



* After constant intervals of time, gaps change polarities

* increasing velocity ⇒ requires increasing length of tubes to pass thru in the same time.

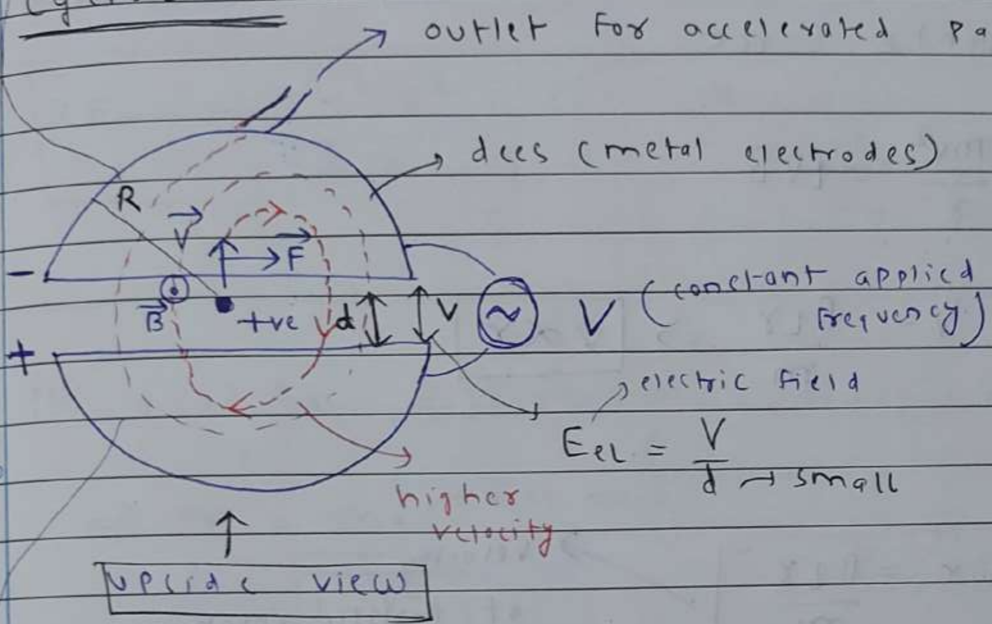
* SLAC ~ 3 kms long ~ tens of GeV

* Frequency of V has to match with frequency of particle reaching the gap.

Problems
1) only electric field is used ⇒ high voltage required
2) long lengths of tube needed ⇒ many kms needed

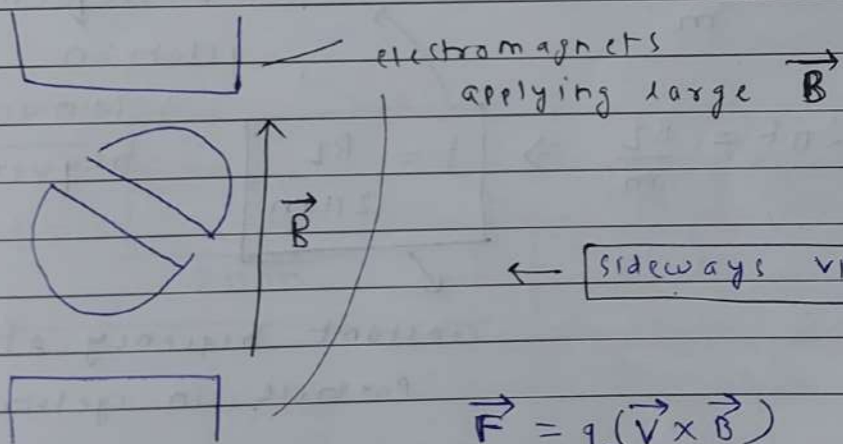
radius of dees.

* Cyclotron



$$E_{el} = \frac{V}{d} \rightarrow \text{small}$$

→ By the time, particle reaches this "dee", it changes its polarity to -ve



← sideways view

$$\vec{F} = q(\vec{v} \times \vec{B})$$

makes particle move in a semicircular path & come back to the gap.

* Spiral motion = radius increases with increasing velocity

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* centripetal force is due to magnetic lorentz force

$$\frac{mv^2}{r} = qvB$$

$$V = \frac{Bqr}{m} \Rightarrow \boxed{v \propto r}$$

* $\boxed{v = \omega r = \frac{Bqr}{m}}$ → velocity of particle rotating in cyclotron

$$\Rightarrow \omega = \frac{Bq}{m}$$

$$\Rightarrow 2\pi f = \frac{Bq}{m} \Rightarrow \boxed{f = \frac{Bq}{2\pi m}}$$

natural frequency

cyclotron resonance frequency

constant frequency of particle in cyclotron

* f should be = applied AC voltage frequency for cyclotron to work

* $\boxed{V_{max} = \frac{BqR}{m}}$ → velocity is max., when particle reaches boundary ($r = R$)

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* $KE_{max} = \frac{1}{2} m v_{max}^2$

$$\boxed{KE_{max} = \frac{B^2 q^2 R^2}{2m}}$$

if mass less, KE more
⇒ electrons will have energy in cyclotron than protons

* if $v > 0.3c$ (relativistic velocity)

$$\Rightarrow m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

m_0 → rest mass

m → relativistic mass

* For relativistic particle:

$$\boxed{f = \frac{\sqrt{1 - v^2/c^2}\ Bq}{2\pi m_0}}$$

{This is varying, applied AC voltage} frequency is constant, then how will cyclotron work?

For cyclotron to work, f = applied AC voltage freq.

To solve this

Cyclotron

Synchrotron (Freq. of applied AC voltage is constantly decreased to match with f at all times.)

fine tuning

Isochronous cyclotron (B is varied to match f with applied voltage frequency)

B is increased at same rate as $\sqrt{1-v^2/c^2}$ decreases to keep f constant

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* Neutral particle acceleration

1) Neutron is already energetic when produces

2) Accelerate a charged particle then make it neutral

↳ accelerate p, add e^- when accelerated $\longrightarrow$ accelerated neutral particle of high energy

(AM)

Lec-17 NP (18th April, 2023)

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* **particle detectors** ⟹ charged particle ionization track

(1) Bubble & cloud chambers ⟹ supersaturated vapour / superheated liquid → pressure drop leads to condensation or bubble formation on ions ⟹ track is visible.

(2) Scintillation detectors ⟹ ionized particles causes scintillation (light emission) in organic compounds ($C_{15}H_{11}NO$) → visible track, photodetector ⟹ electric signal

(3) Spark chambers ⟹ Grid of oppositely charged wires → ions causes sparks → visible track → output electric signal

(4) semiconductor detectors ⟹ Reverse based diode → ions causes e^--hole pairs → increase in reverse bias current

(5) Counters ⟹ Ionized particles cause electric pulses in anode → No. of particles detected → (GM counters)

(6) Cerenkov detectors ⟹ charged particle with velocity > velocity of light in a medium causes Cerenkov radiation → visible track → come with angle $\cos\theta = c/nV$ ⟹ energy info. of particle

refractive index

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$\boxed{\cos\theta = \text{conical angle}}$

* Charged particle → ○ → e^-, positive ion

* <u>Details regarding particle detectors:</u>

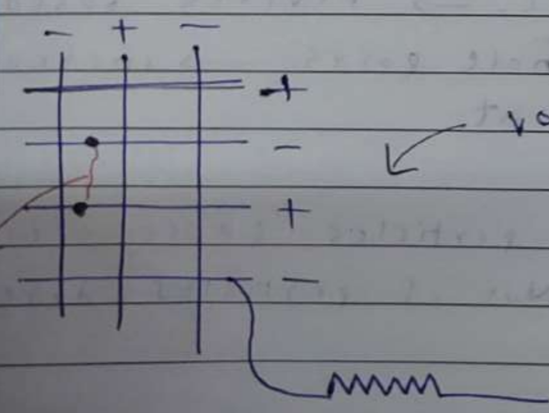
(1) First particle detector
Bubble chamber = liquid used
Gas chamber = vapour used

Bubbles ⇒ overall liquid ⇒ bubbles of air formed on ionization track

Cloud chamber ⇒ track of condensation of ions
Bubble chamber ⇒ track of bubbles on ions

(2) Photodetector ⇒ converts light energy to electric energy.

(3)



Spark

voltage applied on grids, spark b/w wires when particles get attached to wires of opposite charges.

(connected to external circuit which gives O/P electric pulse)

V_{light} in medium $= \frac{c}{n}$; $c \to 3\times10^8$ m/s ; $n \to$ refractive index

(4) depends on e^--hole pairs
↳ increase in reverse bias current
⇒ particle has passed through it

(5) Tells the frequency of the particles passing through it.

(6) "Carenkov radiation on the path of the particle"
"Does not need ionization"
"v = velocity of the particle"
"radiation generated in form of cones"

GM counter.

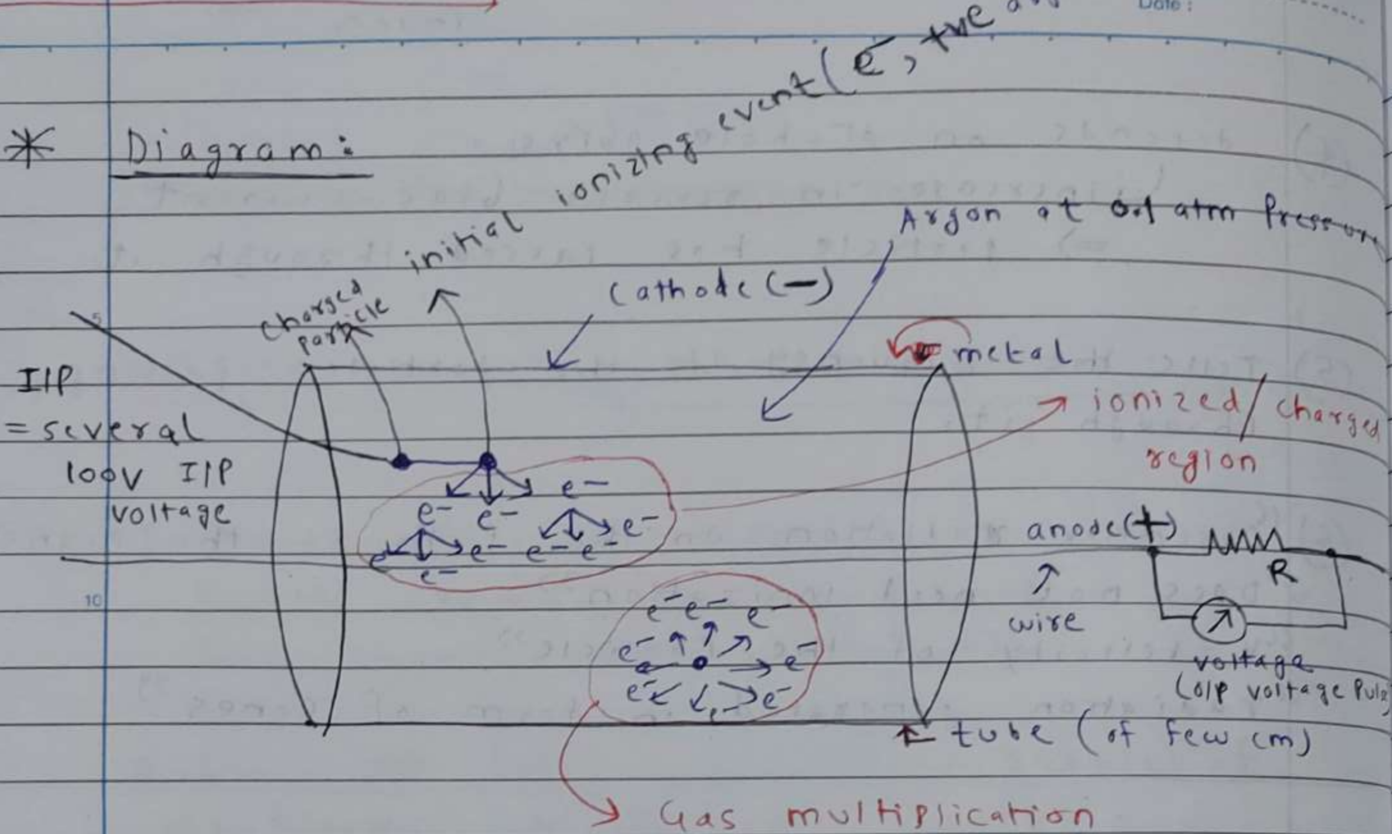
* produces electric pulses in reaction to charged particles passing through it.

* Electric pulse generated for every charged particle.

⟹

UV Photon ~ 10^{15} Hz

* Diagram:



* ($1e^- \rightarrow 2e^- \rightarrow$ — — — —) millions of e^- produced when e^- move towards anode, this is called
"Cascading effect"
OR
"Townsend Avalanche"
($10^6 \sim 10^8$ ions produced)

* Along with e^-, ultraviolet photons are produced (these are energetic) and cause further ionization).

these can move parallel to anode, up-down or in any direction & cause further ionization of argon atoms leading to e^-

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* "Gas-multiplication / Avalanche multiplication" due to photons.

↳ $10^9 \sim 10^{10}$ ions, entire gas ionized

* Propagation of avalanche $\sim 2.4\,cm/\mu s$
$\Rightarrow$ few μs needed for whole gas to be ionized.

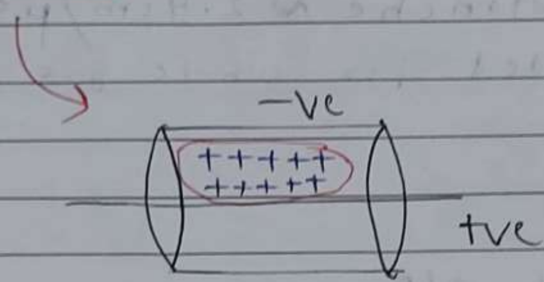
* O/P = large current O/P
I/P = several 100V I/P

* electrons are much lighter and therefore free to move and cause avalanche.

$\Longrightarrow$

* WHY DOES O/P PULSE END ?

⟹ +ve argon ions move towards cathode. They don't reach cathode with the time of avalanche and create a positive space charge b/w cathode and anode.



* This "positive space charge" terminates the pulse by reducing ionization and avalanches.

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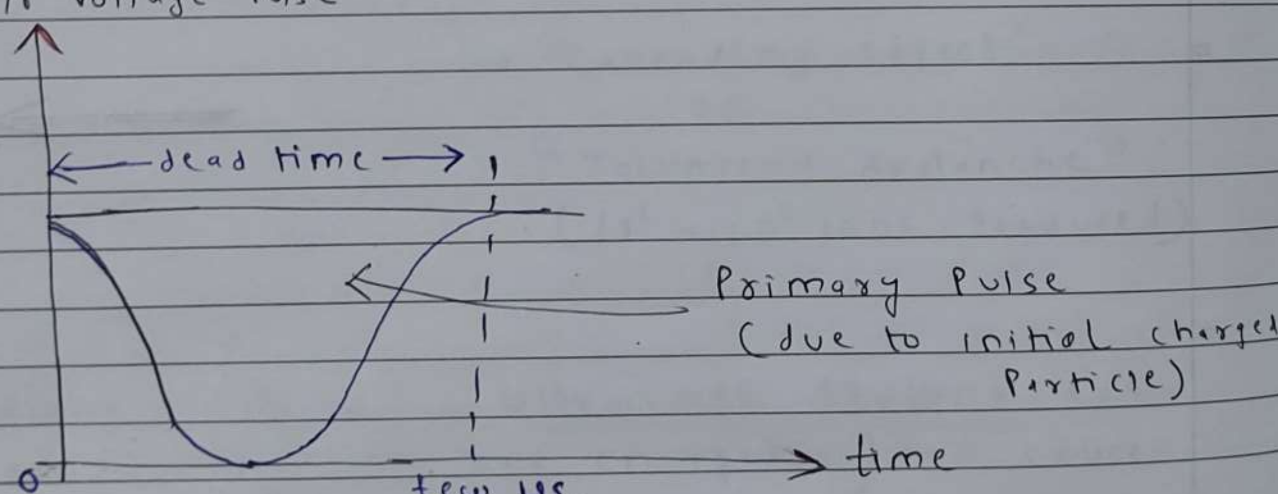
* O/P voltage Pulse



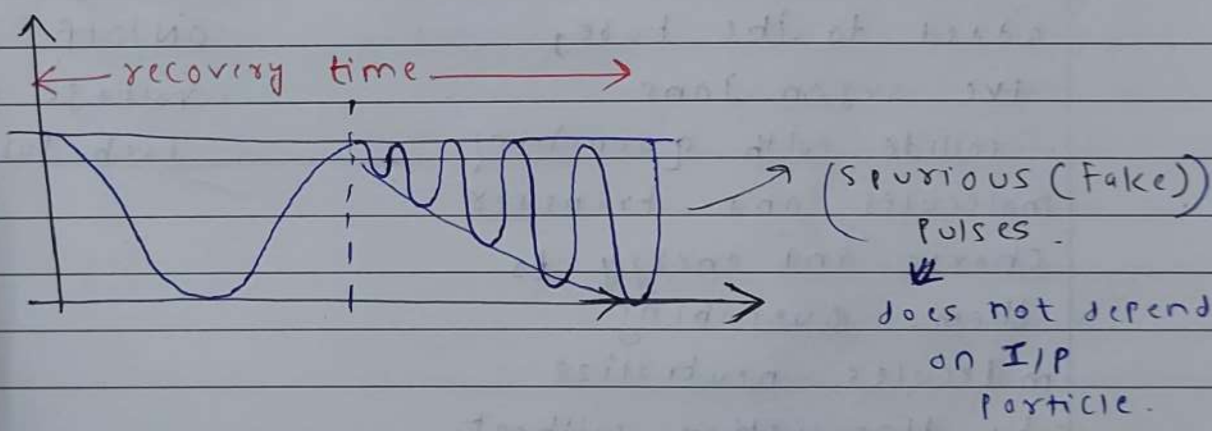
* $\boxed{\text{dead time}}$ = amount of time taken to generate 1 pulse and terminate 1 pulse.

* Next particle in GM counter should come after the dead time.

* PROBLEMS OF GM COUNTER:

(1) Can't identify the type of particle or its energy ⇒ because O/P pulse depends on tube geometry and I/P voltage and not on type or energy of particle.

(2)



* Recovery time = of tube during which it cannot count a new incoming particle.

* Reason for Spurious Pulses:

→ When +ve argon ions reach the cathode and get neutralized.

When +ve argon ions reach cathode, they become neutral by reacting with cathode, and generates UV photons and these photons generate "Fake pulses".

→ "We want to stop this!"

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* WE DON'T WANT +ve IONS TO REACH CATHODE

⇓

Quenching

Internal/self Quenching

Halogen or butane is added to the tube, +ve argon ions collide with quenching molecules and transfer charge and energy to them. Quenching molecules neutralize by dissociation without releasing UV photons.

Electronic quenching

Rapidly switch ON/OFF I/P voltage after each pulse.

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* WHY ARGON AT LOW PRESSURE ?

$$\boxed{\text{energy of } e^- = \frac{\text{Applied electric } \cancel{\text{force}} \text{ field}}{\text{Gas pressure } (P)}}$$

⇒ If P is high, energy of e^- too low to cause ionization.

⇒ If P is too low, frequency of collisions is not enough to cause ionization.

⇒ So we need optimal pressure $\sim \boxed{0.1 \text{ atm.}}$