lec 14

Quantization noise power

$(Q_e)_{max} = \frac{\Delta}{2}$

↳ undeterministic, so a RV

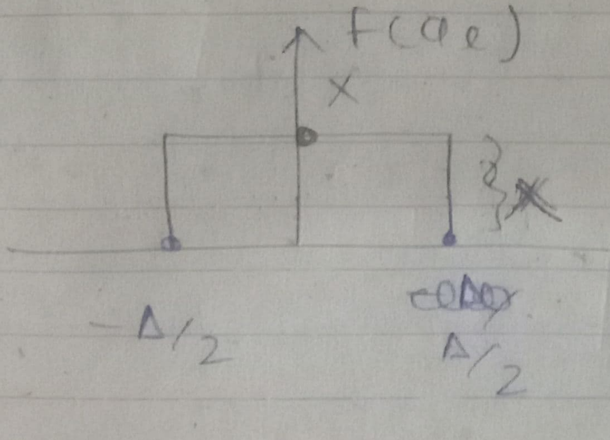
N_q = power (Q_e)

= mean square (Q_e)

$= E(Q_e^2)$

$= \int_{-\infty}^{\infty} Q_e^2 \, f_{Q_e}(Q_e) \, dQ_e$

Assume Q_e satisfied uniform probability density fn



Area (PDF) = 1

$x \cdot \Delta = 1$

$\boxed{x = 1/\Delta}$ = pdf = $f_{Qe}(Qe)$

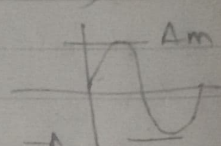
$$N_q = \int_{-\Delta/2}^{\Delta/2} Qe^2 \times \frac{1}{\Delta}\, dQe$$

$$= \frac{1}{\Delta} \left.\frac{Qe^3}{3}\right|_{-\Delta/2}^{\Delta/2}$$

$$\boxed{N_q = \frac{\Delta^2}{12}}$$; Δ = step size

Let $m(t) = A_m \cos(2\pi f_m t)$

$$\Delta = \frac{A_m + A_m}{L}$$



$$\boxed{\Delta = \frac{2A_m}{2^n}}$$

$$N_q = \frac{4A_m^2}{(2^n)^2} \times \frac{1}{12}$$

$$\boxed{N_q = \frac{1}{3}\frac{A_m^2}{2^{2n}}}$$

SIGNAL TO QUANTIZATION NOISE RATIO

$$SQNR = \frac{\text{Signal power}}{\text{Quantization noise power}}$$

$$= \frac{A_m^2/2}{\frac{1}{3}\frac{A_m^2}{2^{2n}}}$$

$$\boxed{SQNR = \frac{3}{2}2^{2n}}$$ ← should be as high as possible

$n\uparrow$, $L\uparrow$, $\Delta\downarrow$, $Qe\downarrow$, $N_q\downarrow$

$SQNR\uparrow$

$n\uparrow$, $BW\uparrow$

$$\boxed{BW = \frac{R_b}{2} = \frac{nf_s}{2}}$$

$$\boxed{SQNR = \frac{3}{2} 2^{2n}}$$

$n \to n+k$,

SQNR inc. by 2^{2k} times

$$SQNR_{dB} = 10\log_{10}\left(\frac{3}{2} 2^{2n}\right)$$

$$= 10\log\left(\frac{3}{2}\right) + 10\log_{10} 2^{2n}$$

$$\boxed{SQNR_{dB} = 1.8 + 6.02n}$$

Q1) $m(t) = 8 \sin(8\pi \times 10^3 t)$ is transmitted through PCM system. Sampling rate is 50% higher than nyquist rate & min SQNR should be 22 dB. Find

1. Transmission BW
2. $SQNR_{dB}$

(ii) $(SQNR)_{dB} = 1.8 + 6n$

$= 1.8 + 6 \times 4$

$= 25 \, dB$

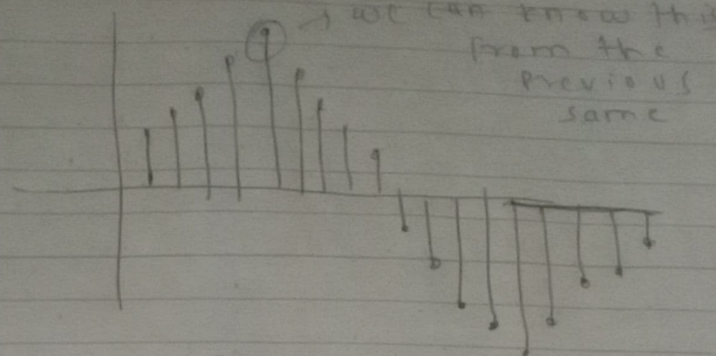
DPCM (differential ~~pulse code~~ PCM)

$(Q_e)_{max} = \frac{\Delta}{2} \Rightarrow \Delta = \frac{V_{max} - V_{min}}{2^n}$

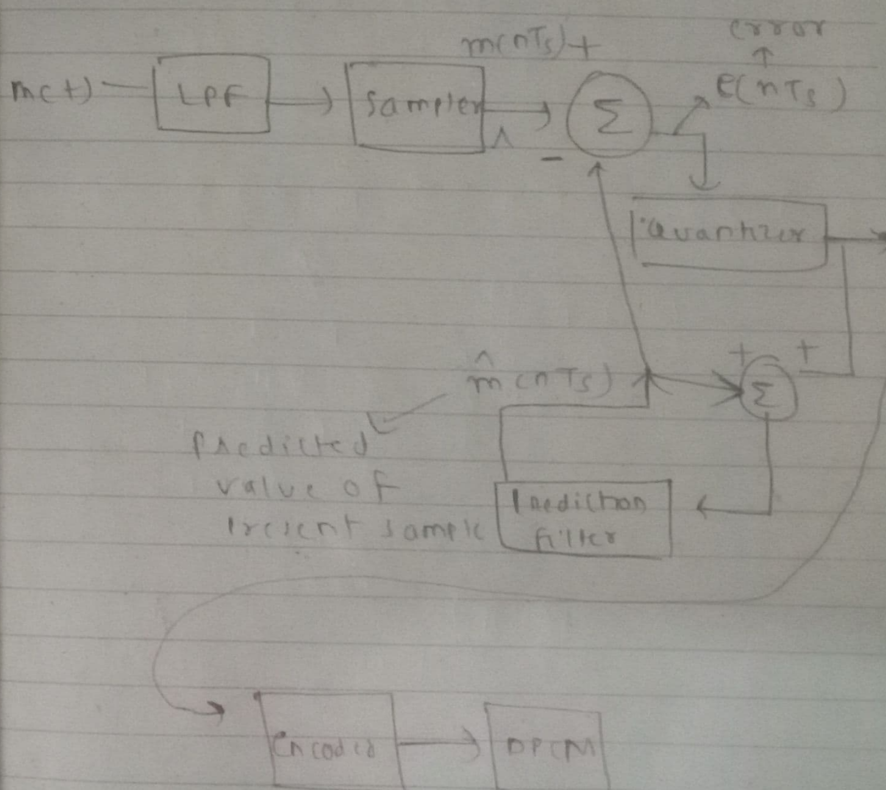
to decrease Δ, $V_{max} - V_{min} \downarrow$

dynamic range $\downarrow$

In DPCM, aim is to reduce the dynamic range



more the samples are nearer, more nearer their amplitude



In PCM, original samples were quantized, Dynamic range more

In DPCM, error b/w the sample & predicted value of sample and now this error is quantized, Dynamic range less

Prediction filter ⇒ By analyzing the past behaviour of the signal, it predicts the present sample value

$$e(nT_s) = m(nT_s) - \hat{m}(nT_s) \quad -(1)$$

Present sample value

Predicted value of present sample

$e(nT_s)$ = prediction ~~error~~ error

~~Present~~s all quantized values stored in prediction filter and by analyzing the previous quantized values signal prediction value place & present sample is detected

$$\boxed{q(nT_s) = e(nT_s) - e_q(nT_s)}$$

quantization error

This is DPCM transmitter

Prediction filter = $\hat{m}(nT_s) + e_q(nT_s)$

I/P

from eqn (2)

$$e_q(nT_s) = e(nT_s) - q(nT_s)$$

$$= \hat{m}(nT_s) + e(nT_s) - q(nT_s)$$

error

$$= m(nT_s) - q(nT_s) \quad \text{(Using eqn (1))}$$

original sample value

$$\boxed{QV = SV - QE}$$

quantization error

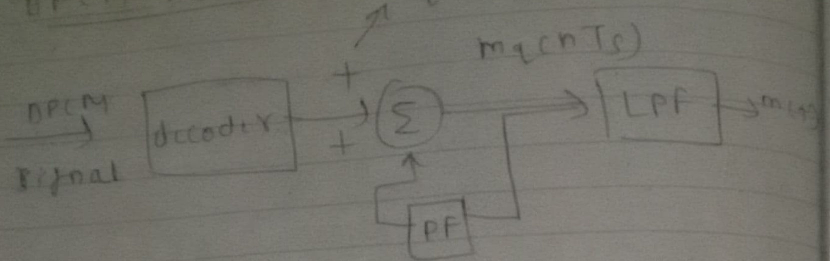
Quantized value

Sampled value

$$= m_q(nT_s)$$

Quantized value of $m(nT_s)$

DPCM receiver



$$LPF_{o/p} = \underbrace{e_q(nT_s)}_{\text{quantized value}} + \underbrace{\hat{m}(nT_s)}_{\text{predicted value}}$$

$= m_q(nT_s)$ → Quantized signal

⇓

$\boxed{\text{LPF}}$

⇓

$m(t)$

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DELTA MODULATION

* Special case of DPCM
* 1 bit DPCM
* Prediction error less
* 2 Quantization level are required & 1 bit encoding
* Transmission BW very less

$$BW = \frac{n f_s}{2}$$

If $n = 1$, "BW very less"

$n = 1$, levels$(L) = 2$

↳ these 2 levels can be represented by $+\Delta$

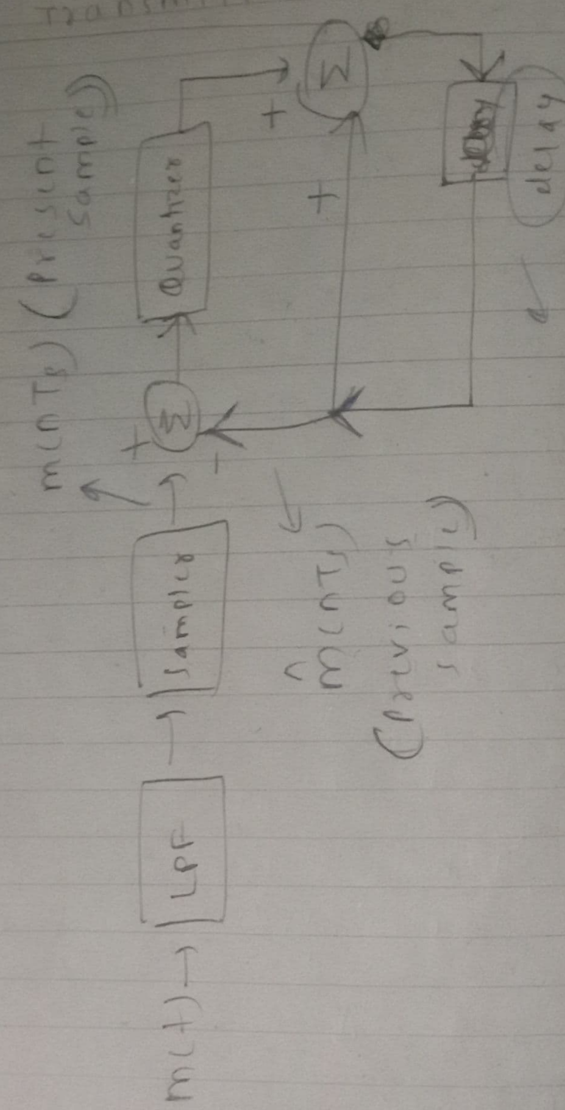
$-\Delta$

Δ = step size

$\Delta \approx 1$ encoded

$-\Delta \approx 0$ encoded

DM Transmitter



in DPCM, there was prediction filter

$e(nT_s) =$ "very less"

present sample — previous sample

$= m(nT_s) - \hat{m}(nT_s)$

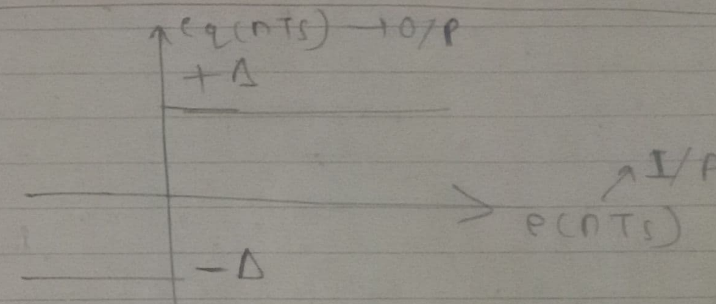
Very small

"results in small dynamic range"

therefore only 2 levels required

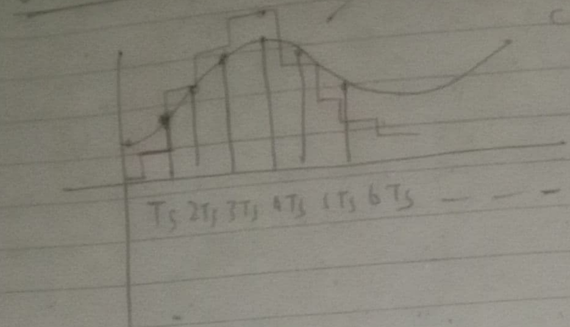
& 1 bit.

CHARACTERISTICS OF QUANTIZER



WORKING

we achieve stair case



$e(nT_s) = +ive = +\Delta$ (mapped to 1)

OR

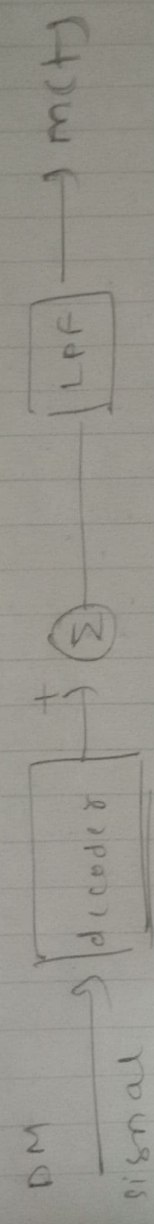
$-ive = -\Delta$ (mapped to 0)

DM receiver

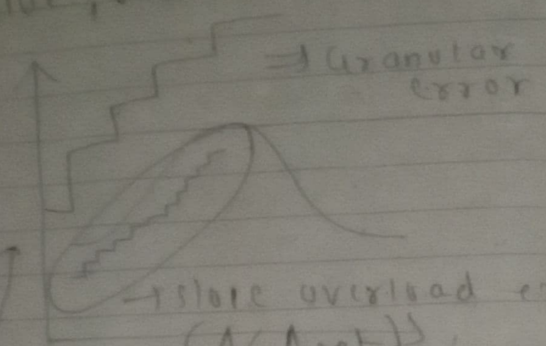
similar to DPCM receiver, instead of prediction filter, use "delay block"

In DM, sampling freq very very high (oversampling), diff between the present & previous sample very low.

two levels of quantization required



Δ should have an optimal value, otherwise issues:



if $\Delta < \Delta_{optimal}$ distortion and DM signal may not be able to trace the original message signal.

If ($\Delta > \Delta_{optimal}$), distortions in receiving end

ADAPTIVE DELTA MODULATION

(to overcome previous discussed errors)

* step size changes continuously according to change of message signal

→ $e(nT_s) = +ive\ (+\Delta)$

$e(nT_s) = -ive\ (-\Delta)$

DM is limited for only transmission of signal which are having constant rate of change

Adaptive DM ⇒ no slope overload error & granular error

$$\boxed{m(t) = A_m \cos(2\pi f_m t)}$$

$$\boxed{\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m}$$

$$\boxed{\frac{\Delta}{T_s} < 2\pi f_m A_m}$$

condition for slope overload error

$$\boxed{\frac{\Delta}{T_s} > 2\pi f_m A_m}$$

condition for granular error to occur

Q1) continuous signal of $8 \sin(8\pi \times 10^3 t)$ is passed through DM whose pulse rate = 4000 pulses / sec. Find Δ_{opt} ?

Soln $f_s = 4000$ pulses / sec

$m(t) = 8 \sin(8\pi \times 10^3 t)$

$A_m = 8$, $f_m = 4$ kHz

$$\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m$$

in DM, $\boxed{\text{pulse rate} = \text{sampling rate}}$

$n = 1$

$R_b = n f_s = f_s$

$f_s = 1/T_s = 4000$ samples / sec

$\Delta_{opt} \times 4000 = 2\pi \times 4 \times K \times 8$

$$\boxed{\Delta_{opt} = 16\pi \text{ Volts}}$$