lec 12

## Am receivers

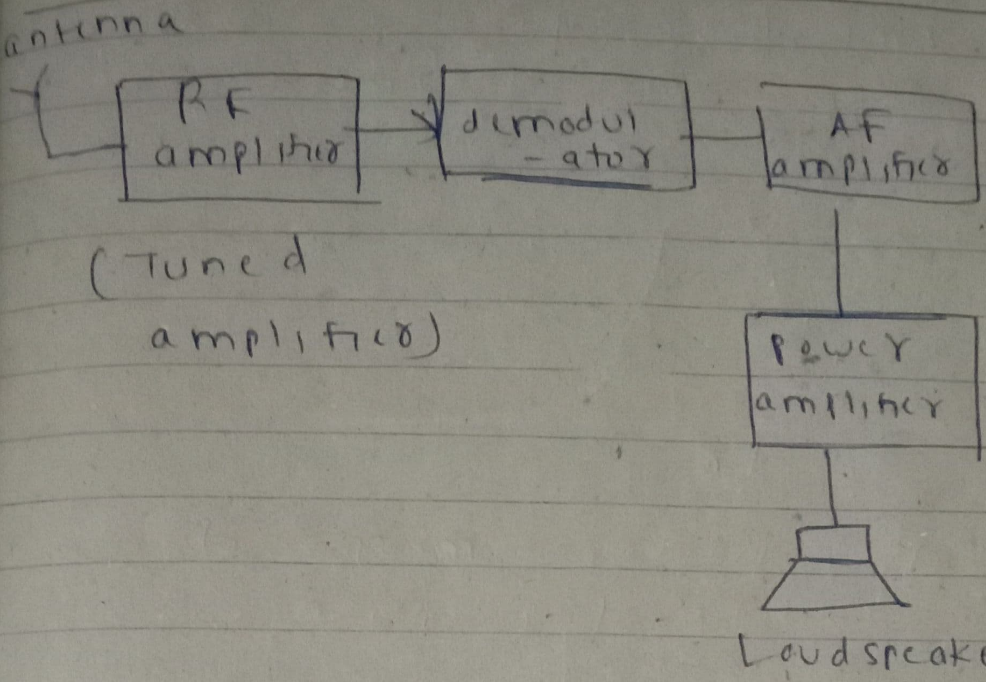
1. Tuned radio frequency receiver (TRF)
2. Super heterodyne receiver

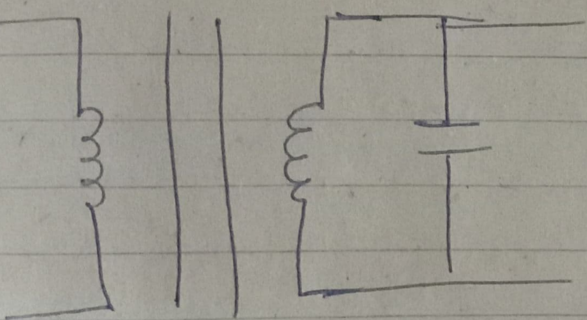
Two properties satisfied by Am receivers are:

1. selectivity = ability to select desired signal from
2. Sensitivity

extracting the information correctly from the signal received

# Tuned radio frequency receiver



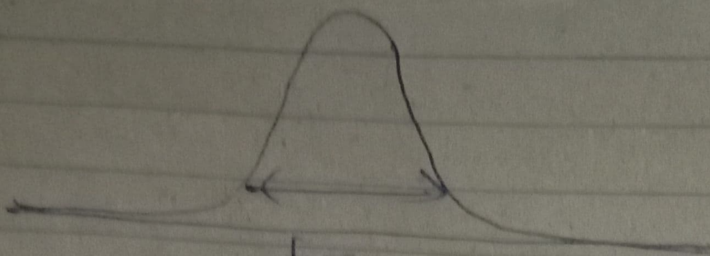
= LC tank circuit for tuning

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

resonating frequency

(~~Take this~~ allows only single frequency, reject other frequency components)

Range of frequency which the tuned amplifier passes without attenuation = BW of tuned amplifier




Q = Quality factor = measure of quality of reception

$$\boxed{Q = \frac{f_r}{B.W.}}$$

B.W. ↑, Q ↓

B.W ↓, Q ↑

Q ↑, selectivity boosted ↑

Quality factor gives the sharpness to the gain frequency characteristics.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{f_r}{BW}$$

$$Q = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$$

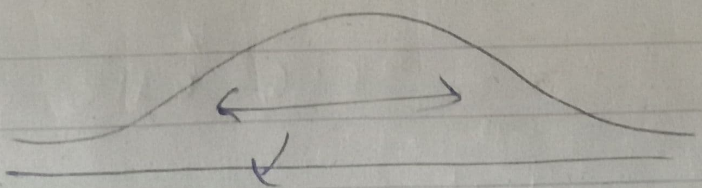
## Problem with TRF

1. Poor selectivity

case 1 :

Ex ) Assume receiver is tuned to 600 ~~H~~ KHZ = fs

BW = 10 KHZ



BW = 10 K

fs = 600 K = fr

$$Q = \frac{f_r}{BW} = \frac{600K}{10K} = 60$$

resonant frequency
= central frequency

Case 2: Reciver is tuned to 1000KHZ frequency

$f_s = f_r$

$BW = 10K$

$$Q = \frac{f_r}{BW} = \frac{1000KHZ}{10K}$$

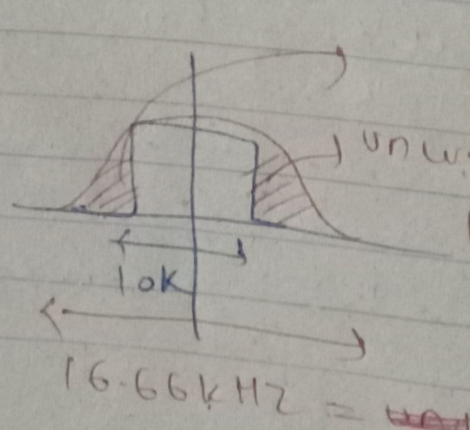
$$\boxed{Q = 100}$$

it is difficult to achieve this

For $Q = 60$, $f_r = 1000K$,

$$BW = \frac{f_r}{Q} = \frac{1000}{60} = 16.66\,KHZ$$

Now analysing, actual $BW = 10K$
but obtained $BW = 16.66K\,HZ$, we are getting unwanted signals




unwanted, so poor selectivity

## superheterodyne receiver

↳ to overcome poor selectivity of TRF

→ use MIXER (down converter)
it converts input frequency $f_i$ downconverted to 455KHZ

$f_i \rightarrow$ mixer $\rightarrow$ 455KHZ

Intermediate frequency (IF)

→ Tuned amplifier has to be tuned at about 455KHZ

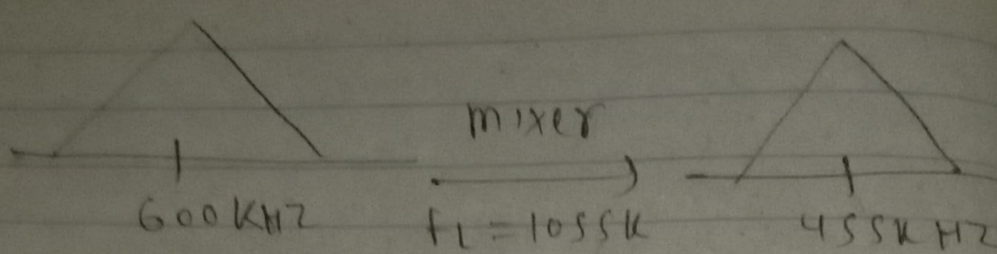
$$Q = \frac{455k}{10k} = 45.5$$

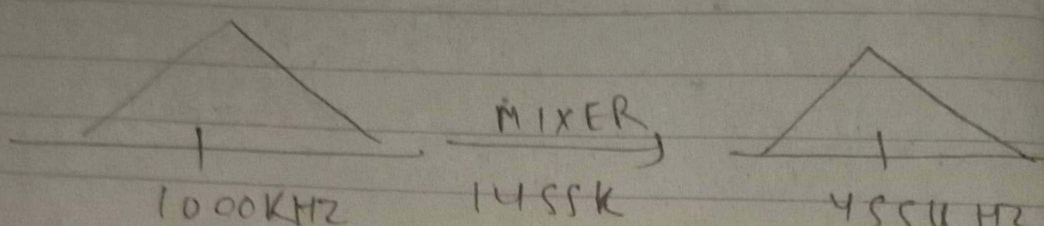
$$\boxed{IF = f_L - f_s}$$

$f_L$ ↳ frequency of local oscillator

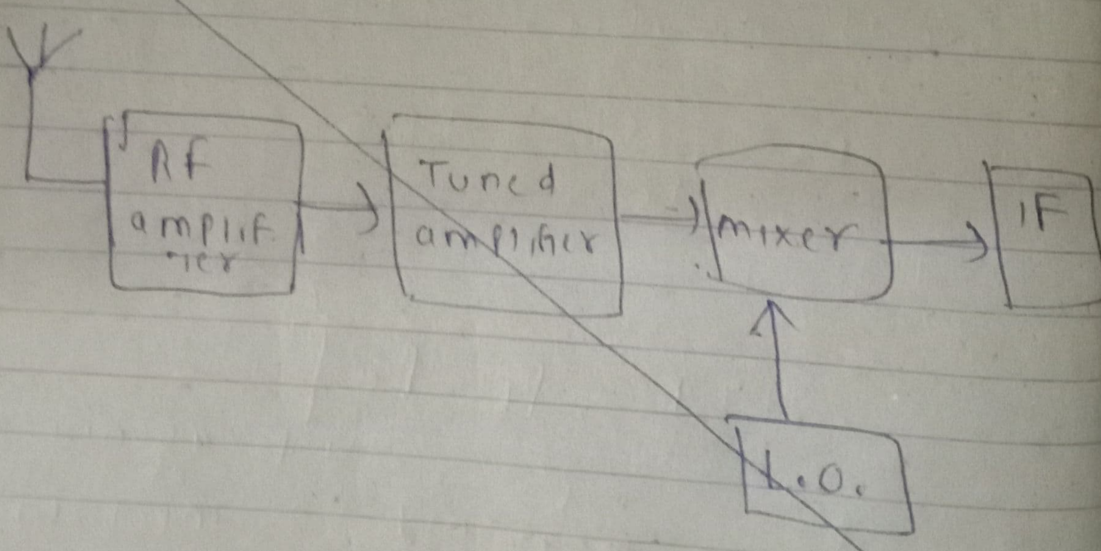
$f_s$ ↳ frequency of input signal

→ does not change with input signal frequency

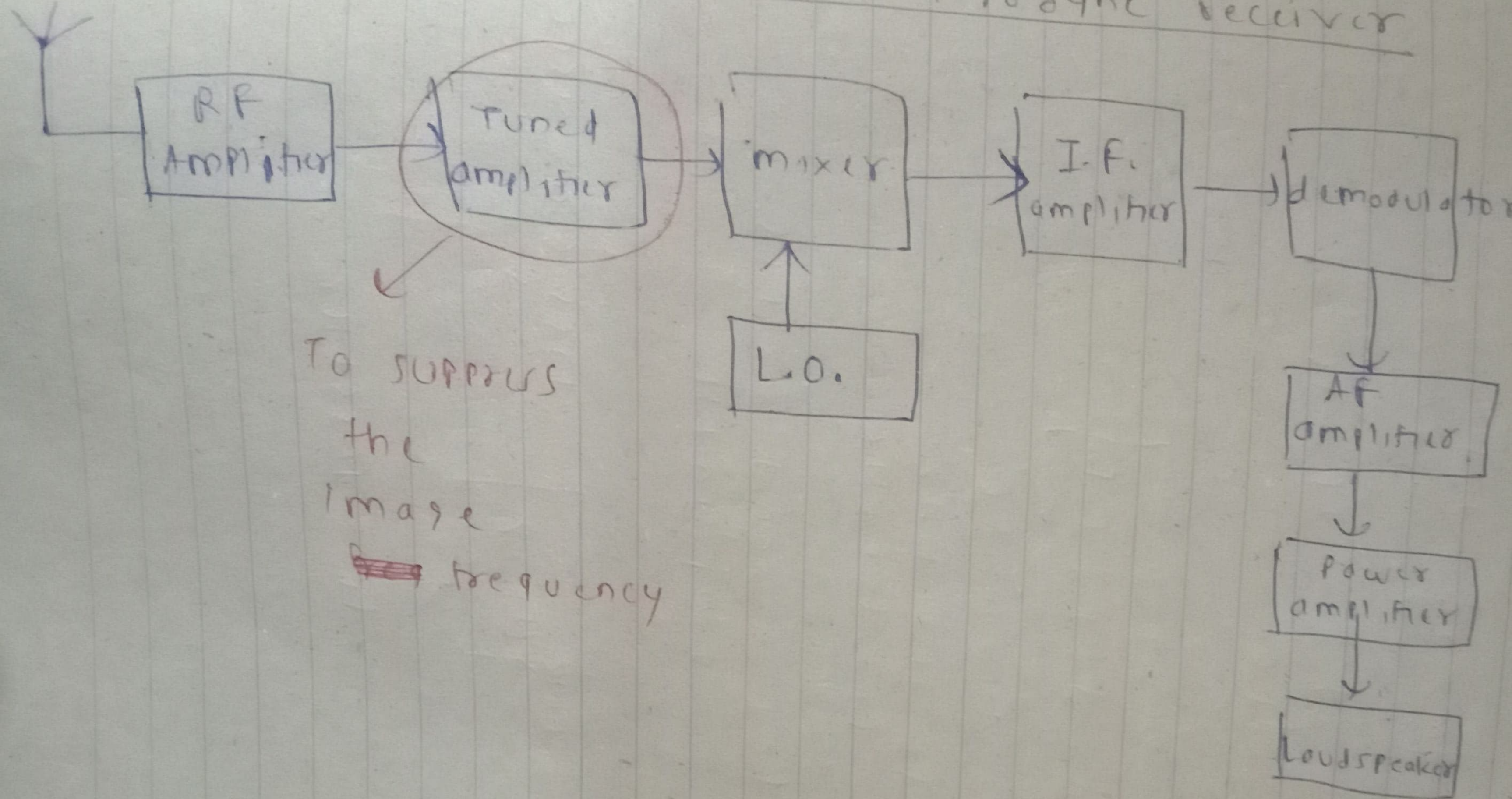


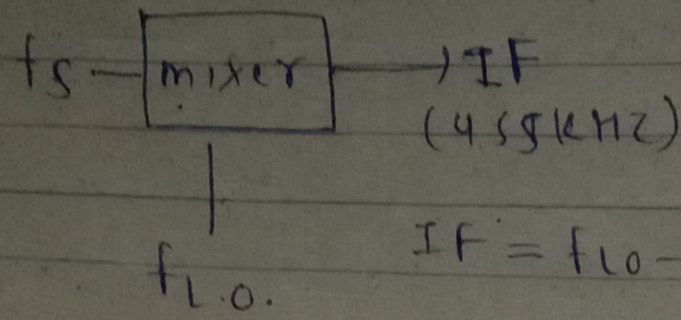

Block diagram of superheterdyne receiver



Block diagram of superheterodyne receiver



# IMAGE FREQUENCY

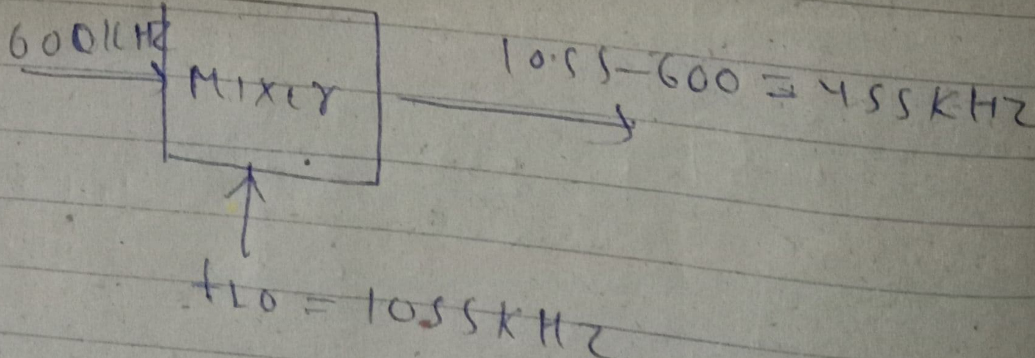



$IF = f_{LO} - f_s$

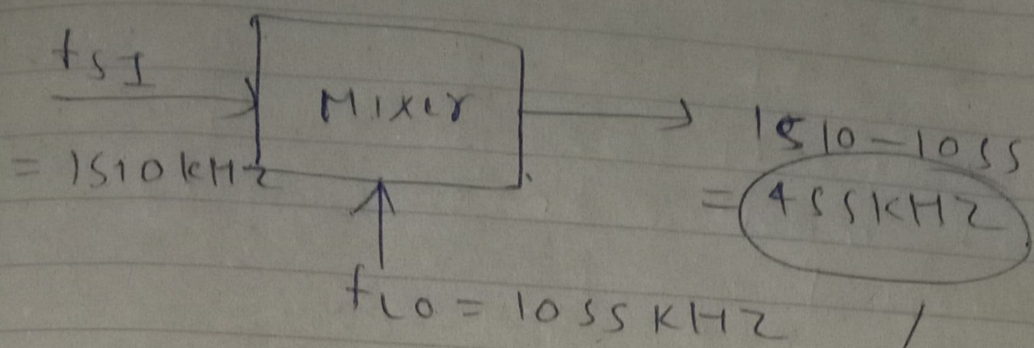
$$\boxed{f_{sI} = f_s + 2\,I.F.}$$

Ex) $f_s = 600\ KHZ$
$I.F. = 455\ KHZ$

$f_{sI} = 600 + 2 \times 455$
$= 1510\ KHZ$



If we give $f_{SI}$ to mixer




again I.F.,

hence $f_{SI}$ also passed

drawback of superheterodyne receiver

To overcome above drawback, use:

Tuned amplifier = low frequency amplifier which only passes input signal frequency and not the image signal frequency

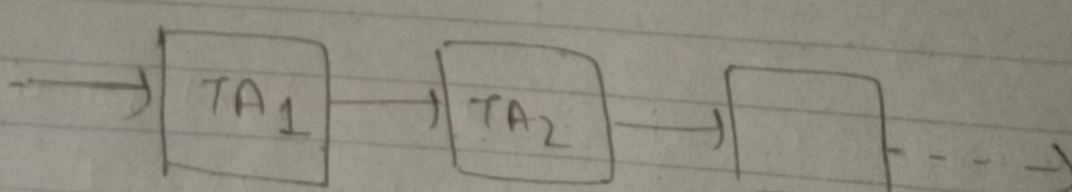
## IMAGE REJECTION RATIO (IRR)

$\alpha$

It is effectiveness of the tuned amplifier in suppressing the image frequency

$$\boxed{\alpha = \sqrt{1+\rho^2 Q^2}}$$

$\rho = \frac{f_{SI}}{f_S} - \frac{f_S}{f_{SI}}$, $Q$ = Quality factor




$\alpha_1$ $\alpha_2$ $\alpha_3$

$$\boxed{\alpha_{total} = \alpha_1 \alpha_2 \alpha_3 \cdots}$$

Q1) A receiver is tuned to 500 KHZ, Local oscillator frequency is given by 1050 KHZ

find (i) IF

(ii) $f_{SI}$

(iii) IRR

Q = 50 ?

Soln: $f_s = 500$ KHZ

$f_L = 1050$ KHZ

$I.F. = f_L - f_s = 1050 - 500$

$= 550$ KHZ

$f_{SI} = f_s + 2IF$

$= 500 + 2 \times 550$

$= 1600$ KHZ

$\alpha = \sqrt{1 + \rho^2 Q^2}$

$$\rho = \frac{f_{SI}}{f_s} - \frac{f_s}{f_{SI}}$$

$$= \frac{1600}{500} - \frac{500}{1600}$$

$= 2.8$

$$\alpha = \sqrt{1 + p^2 \omega^2}$$

$$= \sqrt{1 + (2.8)^2 \times 50^2}$$

$$= \sqrt{19601}$$

$$\boxed{\alpha = 144.3}$$

~~EC13~~ (UNIT 1 – Random var. & Process)

Signals – Deterministic
– Random

**Deterministic signals** ⇒ reproduced exactly by repeated experiments

* represented by exact ~~mathem~~ mathematical expressions ⇒ we can know the outcome at any instant

For Ex) Unit step signal

**Random signals** ⇒ cannot be ~~rep~~ ~~reproduced~~ repeated in exact manner, cannot be represented by exact mathematical expressions.

For Ex) Noise in any ~~system~~ signals

Random signals

↳ can be described in terms of the statistical properties
avg. power, probability density function, cdf

## RANDOM VARIABLE

It is a variable whose possible values are the numerical outcomes of random phenomenon.

* Experiment ⇒ Events (Possible outcomes) ⇒ Probability of Events

* Sample point = Each possible outcome

* Sample space = combinely all sample points

Ex.)

Throw of dice

$S = 1, 2, 3 - -$

$\{S\} = \{1, 2, 3, 4, 5, 6\}$

R.V.

| discrete R.V. | continuous R.V |
| --- | --- |
| can take only finite / countable no. of discrete values | can take infinite number if distinct values |

## Cummulative distribution function (CDF)

"$F_X(x)$"

$$\boxed{F_X(x) = P(X \le x)}$$

R.V. = X

### Properties of C.D.F:

1. It is bounded b/w 0 & 1.
2. $F_X(x)$ is non-decreasing function

$$F_X(x_1) \le F_X(x_2) \text{ if } x_2 \ge x_1$$

3. $F_X(-\infty) = 0$ (Initial value)

   $F_X(\infty) = 1$ (Final value)

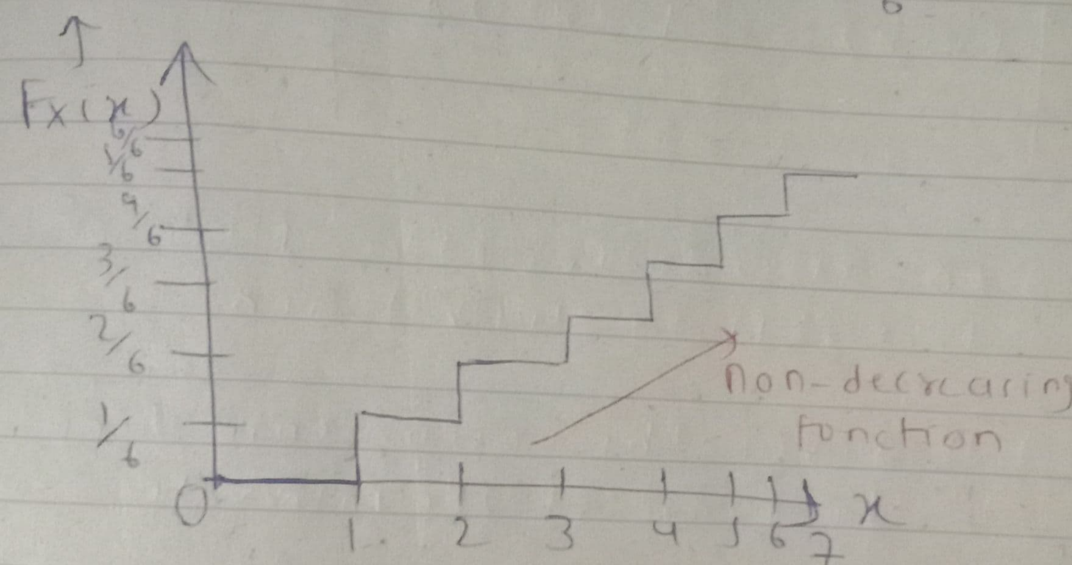
Ex 1) Throw of a dice:

Soln:

$X$ = throw of dice

$X = \{1, 2, 3, 4, 5, 6\}$

$P(1) = P(2) = \ldots P(6) = 1/6$




$F_X(0.5) = P(X \le 0.5) = 0$

$F_X(1) = P(X \le 1) = 1/6$

$F_X(1.5) = P(X \le 1.5) = 1/6$

$F_X(2) = P(X \le 2) = P(X=1) + P(X=2)$
$= 2/6$

derivative of C.D.F.

## Probability density function (PDF)

$f_X(x)$

$$\boxed{f_X(x) = \frac{d}{dx} F_X(x)}$$

$$\boxed{F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx}$$

⇒ An alternative descriptive of probability distribution

⇒ It gives the occurrence density in the range of the intervals

$x_1 \leq X \leq x_2$

$$\boxed{\begin{aligned} P(x_1 \leq X \leq x_2) &= P(X \leq x_2) - P(X \leq x_1) \\ &= F_X(x_2) - F_X(x_1) \\ &= \int_{x_1}^{x_2} f_X(x)\, dx \end{aligned}}$$

Prope

① $f_X$

its

is

② $\int_{-\infty}$

③ $F_X($

Properties of PDF:

① $f_X(x) \geq 0$ for all '$x$'

Since $F_X(x)$ is non-decreasing, its ~~PDF is not~~ derivative (PDF) is non-negative

② $$\boxed{\int_{-\infty}^{\infty} f_X(x)\, dx = 1}$$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = F_X(\infty) - F_X(-\infty)$$
$$= 1 - 0 = 1$$

③ $$F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$$

Q1) R.V. X has p.d.f

$$f_X(x) = a e^{-b|x|}$$

$$-\infty < x < \infty$$

a = 3 ?

find (i) relation b/w a & b

(ii) CDF of RV X

(iii) Prob. that X lies between -1 & 2 ?

Soln: $f_X(x) = a e^{-b|x|}$

$$= \begin{cases} a e^{bx}, & -\infty < x < 0 \\ a e^{-bx}, & 0 < x < \infty \end{cases}$$

(i) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$ (Property)

$$= \int_{-\infty}^{0} a e^{bx} + \int_{0}^{\infty} a e^{-bx} = 1$$

$$\frac{a}{b} e^{bx} \Big|_{-\infty}^{0} + \left(\frac{a}{-b}\right) e^{-bx} \Big|_{0}^{\infty} = 1$$

$$= \frac{a}{b}\left[e^{0} - e^{-\infty}\right] - \frac{a}{b}\left[e^{-\infty} - e^{0}\right] = 1$$

$$= \frac{a}{b}(1) - \frac{a}{b}(0-1) = 1$$

$$\frac{2a}{b} = 1$$

$$2a = b$$

$$\boxed{b = 2a}$$

$a = 3, \quad b = 6$

(ii) C.D.F. of RV '$x$'

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$$

Case 1:

$x < 0$

$$F_X(x) = \int_{-\infty}^{x} a e^{bx}\, dx$$

$$= \frac{a}{b}\left[e^{bx}\right]_{-\infty}^{x} = \frac{a}{b}\left[e^{bx}\right]$$

$$= \frac{1}{2} e^{bx}, \quad x < 0$$

Case 2 : $x > 0$

$$F_X(x) = \int_{-\infty}^{0} f_X(x)\,dx + \int_{0}^{x} a e^{-bx}\,dx \quad \text{(iii)}$$

$$= \frac{1}{2} e^{bx}\Big|_{-\infty}^{0} + \left(\frac{a}{-b}\right) e^{-bx}\Big|_{0}^{x}$$

$$= \cancel{\frac{1}{2}(1) - 0 - \frac{1}{2}\left[e^{-bx}\right]}$$

$$= \frac{1}{2} - \frac{1}{2}\left[e^{-bx} - 1\right]$$

$$= \frac{1}{2}\left[1 - e^{-bx} + 1\right]$$

$$= \frac{1}{2}\left[2 - e^{-bx}\right], \; x > 0$$

$$= 1 - \frac{1}{2} e^{-bx}, \; x > 0$$

$$F_X(x) = \begin{cases} \frac{1}{2} e^{bx} & x < 0 \\ 1 - \frac{1}{2} e^{-bx}, & x > 0 \end{cases}$$

(iii) $P(-1 \le X \le 2)$

$$= \cancel{F_X(2)} \int_{-1}^{2} f_X(x)\,dx$$

$$= \int_{-1}^{0} a e^{bx}\,dx + \int_{0}^{2} a e^{-bx}\,dx$$

$$= \frac{a}{b}\left[e^{bx}\right]_{-1}^{0} - \frac{a}{b}\left[e^{-bx}\right]_{0}^{2}$$

$$= \frac{1}{2}\left[1 - e^{-6}\right] - \frac{1}{2}\left[e^{-12} - 1\right]$$

$$\frac{1}{2}\left[1 - e^{-6} - e^{-12} + 1\right]$$

$$\frac{1}{2}\left[2 - e^{-6} - e^{-12}\right]$$

$$= 1 - \frac{1}{2} e^{-6}\left[1 + e^{-6}\right]$$

Ans