POC (Lec 1) (Concepts)

Basics of communication system

signal = func. of independent variable

communication = exchange of info. in form of signals

when noise added to signal = several frequency components are there in frequency domain

Audible freq range = 20 Hz to 20 kHz
but speech components = 500 Hz vicinity
microphone ⇒ range designed from 500 Hz to 800 Hz

Basic system model

Transmitter — Chanel — Receiver

wired/ wireless

MODULATION = frequency translation

message signal = low frequency signal
weak signal ⇒ cannot be transmitted
over long ~~[illegible]~~ distances

m(t) = low freq
= low energy
= low power

c(t) = high freq signal

$\boxed{E \propto f}$ (Energy ∝ frequency)

superimposing m(t) over c(t) = modulation

After modulation = modified c(t)

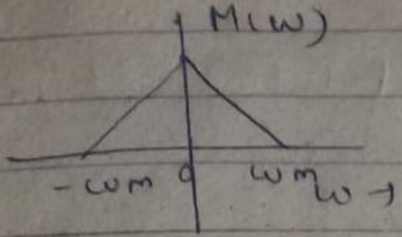
Baseband signal

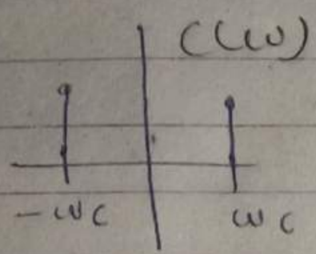
Band pass signal

m(t)
↓ modulation
Band pass signal

$*$ = convolution

MODULATION THEOREM





$m(t)\, c(t)$

$= M(\omega) * C(\omega)$

NEED OF MODULATION

$\downarrow \lambda = \frac{c}{f} \uparrow$

$\downarrow h \propto \frac{\lambda}{4} \downarrow$

91.1, 93.5 MHz = carrier frequency

SOME IMPORTANT TRANSFORM

↳ Fourier transform

HILBERT TRANSFORM

↳ shift the phase of I/P signal by 90°

Hilbert transformer

$\Downarrow$

LTI system

$$\hat{x}(t) = x(t) * \left(\frac{1}{\pi t}\right)$$

$\downarrow$

impulse response of hilbert transformer

$$\hat{X}(\omega) = X(\omega) \cdot (-j\,\mathrm{sgn}(\omega))$$

~~Hilbe~~ Hilbert transform = cos function of sine function

Lec 2 (Concepts)

BAND PASS & LOW PASS SIGNAL REPRESENTATION

Bandpass signal = high freq modulated signal

Baseband signal = low freq = low pass signal
message signal

Any bandpass signal can be represented in terms of complex low frequency signal

low pass equivalent of original band pass signal

high frequency signal = high sampling rates reqd.

ANALYTIC SIGNAL REPRESENTATION

* much simpler to work with complex exponentials of sinusoid signal

sine/cosine ⟶ complex exponential

$$e^{ix} = \cos x + i \sin x$$

* $x(t)$ = real valued signal.

Its complex analytic representation will be:

$$\boxed{x_p(t) = x(t) + j\hat{x}(t)}$$

$\hat{x}(t)$ = Hilbert transform of $x(t)$

pre envelope / complex envelop

Q1) $x(t) = \sin \omega_0 t$

$\hat{x}(t) = -\cos \omega_0 t$

$x_p(t) = \sin \omega_0 t + j(-\cos \omega_0 t)$

$= \cancel{\text{[illegible]}}\ e^{j\omega_0 t}$

~~[illegible]~~ cos ~~[illegible]~~

$$\cancel{-\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2}\right] - j\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]}$$

$$\cancel{= \frac{1}{2}\left[e^{j\omega_0 t} - j e^{j\omega_0 t}\right.}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$\sin(\omega_0 t) \xrightarrow{HT} -\cos(\omega_0 t)$

$\cos(\omega_0 t) \xrightarrow{HT} \sin(\omega_0 t)$ HT = Hilbert Transform

$$x_p(t) = \cancel{\sin(\omega_0 t) + j(-\cos(\omega_0 t))}$$

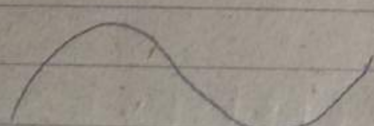
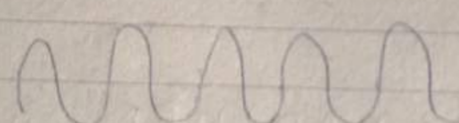
$$= \sin(\omega_0 t) + j(-\cos(\omega_0 t))$$

$$= e^{j\omega_0 t}$$

CONCEPT OF ENVELOPE

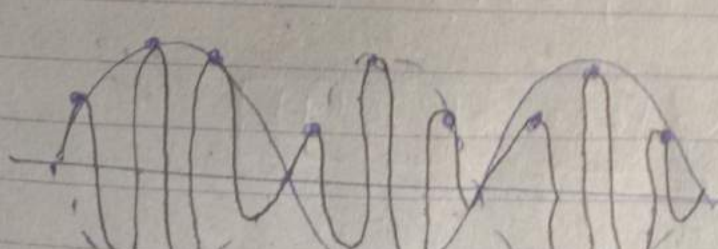
$(\underbrace{m(t)})(\cos(\omega_0 t + \theta))$

message signal × cosine signal (carrier signal)

 × 

$m(t)$ low pass signal

$\cos(\omega_0 t + \theta)$ High pass signal



↳ frequency $= \omega_0$

↳ amplitude = governed by $m(t)$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Amplitude at any time instant "t"

$= m(t)\left[\cos\theta \cos(\omega_0 t) - \sin\theta \sin(\omega_0 t)\right]$

$\text{Amplitude} = \sqrt{m^2(t)\cos^2\theta + m^2(t)\sin^2\theta}$

$\boxed{\text{Amplitude} = |m(t)|}$

$|m(t)|$ = Envelope = Trace of peak amplitude with time

GENERALISED WAY OF REPRESENTING ENVELOPE OF ANY ARBITRARY SIGNAL

$x_p(t) = x(t) + j\hat{x}(t)$

$\boxed{|x_p(t)| = \text{Envelope of } x(t)}$

SPECTRUM OF $x_p(t)$

$x_p(t) = x(t) + j\hat{x}(t)$

$X_p(\omega) = X(\omega) + j\hat{X}(\omega)$

$\hat{X}(\omega) = -j\,\text{sign}(\omega)X(\omega)$

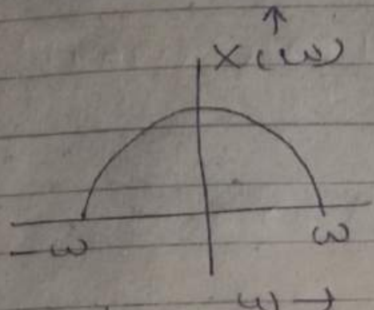
$x_p(\omega) = X(\omega) + X(\omega)\,sgn(\omega)$

$x_p(\omega) = X(\omega)[1 + sgn(\omega)]$

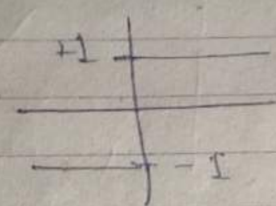
$$x_p(\omega) = \begin{cases} 2X(\omega), & \omega > 0 \\ 0, & \omega < 0 \end{cases}$$

one sided spectrum with positive freq components

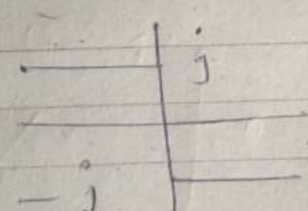
↳ spectrum of $x_p(t)$

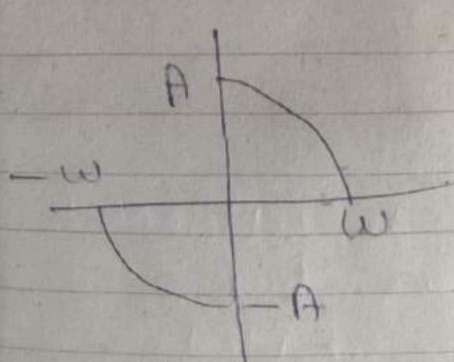


$\hat{X}(\omega) = -j\,sgn(\omega)\,X(\omega)$



|| $-j\,sgn(\omega)$



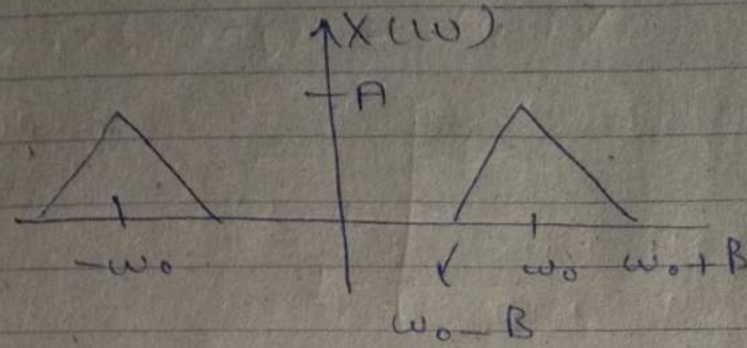


$\Rightarrow$ $2X(\omega),\ \omega > 0$

$0,\ \omega < 0$

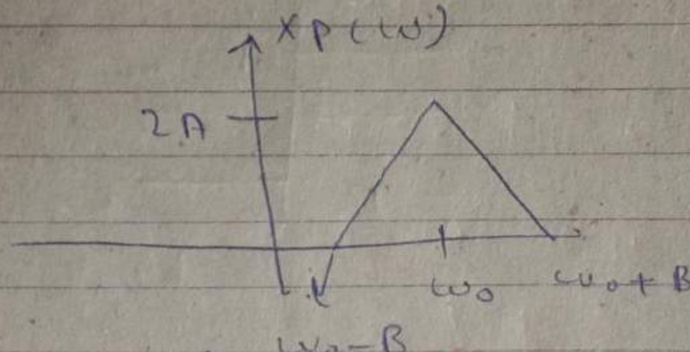
COMPLEX ENVELOPE REPRESENTATION OF BANDPASS SIGNAL

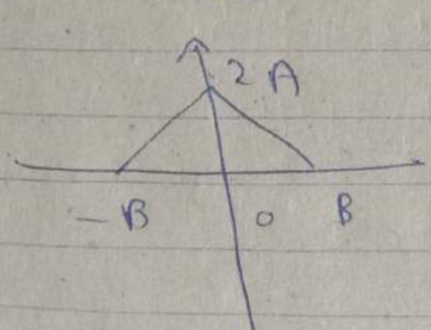
$x(t) \longleftrightarrow X(\omega)$ "Band pass signal"



$\omega_0 >> 0$

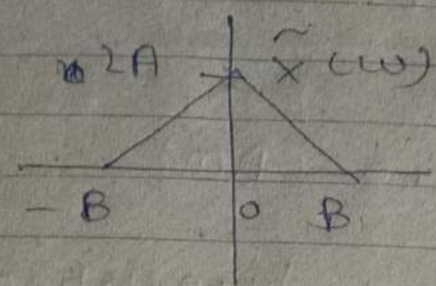
$\Downarrow$





It can be represented as freq shifted version of some low pass signal

Suppose



$\tilde{x}(t) \Rightarrow \tilde{X}(\omega)$

$\Downarrow$

complex valued low pass signal

$$\boxed{x_p(t) = \tilde{x}(t)\, e^{j\omega_0 t}}$$

complex envelope representation

$X_p(\omega) = \tilde{X}(\omega - \omega_0)$

Low pass equivalent of Band pass signal

$x(t) =$ original Band pass signal

$x(t) = Re\{x_p(t)\}$

$\tilde{x}(t) = x_I(t) + j x_Q(t)$

Inphase component

Quadrature phase component

$$\boxed{x(t) = Re\{\tilde{x}(t)\, e^{j\omega_0 t}\}}$$

variation in amplitude varies linearly

Lec 3 (Introduction to DSB-C)

UNIT-2 (LINEAR MODULATION TECHNIQUES)

Amplitude modulation (AM)

* modulation = freq translation

= superimposing a low frequency message signal over high freq carrier signal by varying critical parameters of $c(t)$

like A_c, ω_c or θ_c

AM
- message signal (low freq Base band signal)
- carrier signal (High freq signal)
- modulated signal (High freq Band pass signal)

TYPE OF AM:

1. Conventional AM (DSB-C)
   (Double side band with carrier)

2. DSB-SC (Suppressed carrier)

3. SSB-SC (~~Suppressed~~ single side band - supressed carrier)

4. VSB (Vistigial side band)

1) Conventional AM (DSB-C)

* $m(t)$, $c(t) = A_c \cos(\omega_c t)$

* single tone = message signal has single frequency component (ω_m)

Amplitude of carrier signal varied in accordance to instantaneous value of message signal

$$\boxed{X_{Am}(t) = \left(A_c + m(t)\right)\cos(\omega_c t)}$$

* Amplitude change ↳ eqn ①
* Phase same
* freq same

$$= A_c \cos(\omega_c t) + m(t)\cos(\omega_c t)$$

let $m(t) = A_m \cos(\omega_m t)$ {sinusoid}

$$c(t) = A_c \cos(\omega_c t)$$

so Now, replacing m(t) in ①

$$\boxed{X_{Am}(t) = A_c\left[1 + \frac{A_m}{A_c}\cos(\omega_m t)\right]\cos(\omega_c t)}$$

$$\boxed{\frac{A_m}{A_c} = \mu = \text{modulation index}}$$

✓

gives you the measure of modulation or the extend of modulation

$X_{Am}(t) = $ Amplitude modulated signal

$X_{Am}(t) = \cancel{A_c\cos}\ \cancel{A_c \cos}$

$= A_c\cos(\omega_c t) + A_c \mu \cos(\omega_m t) \cdot \cos(\omega_c t)$

Conventional AM in time domain

$$\boxed{X_{Am}(t) = \underbrace{A_c\cos(\omega_c t)}_{\text{carrier}} + \frac{A_c\mu}{2}\underbrace{\cos(\omega_c+\omega_m)t}_{USB} + \frac{A_c\mu}{2}\underbrace{\cos(\omega_c-\omega_m)t}_{LSB}}$$

USB = Upper side band

LSB = Lower side band

three components

* carrier
* upper side band
* lower side band

To get more useful information, we find the fourier transform of $X_{Am}(t)$, i.e,

$X_{Am}(t) \longleftrightarrow \underbrace{X_{Am}(\omega)}_{\text{we find this}}$

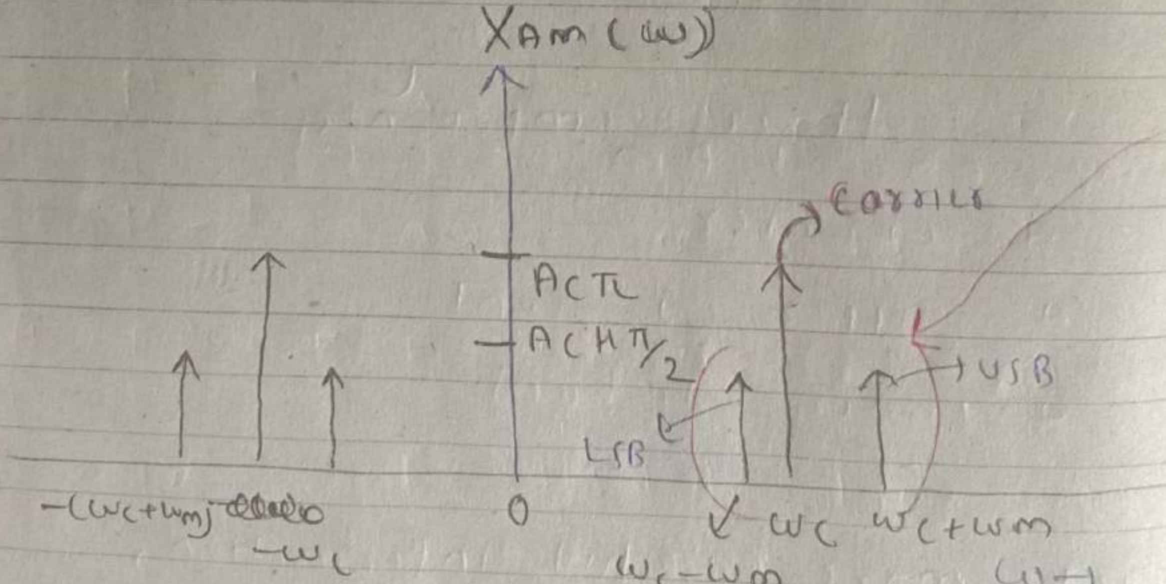
$$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$$

$$X_{AM}(\omega) = \left(A_c\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]\right)$$

$$+\left(\frac{A_c\mu\pi}{2}[\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))]\right)$$

$$+\left(\frac{A_c\mu\pi}{2}[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))]\right)$$

<u>Plotting the spectrum →</u>

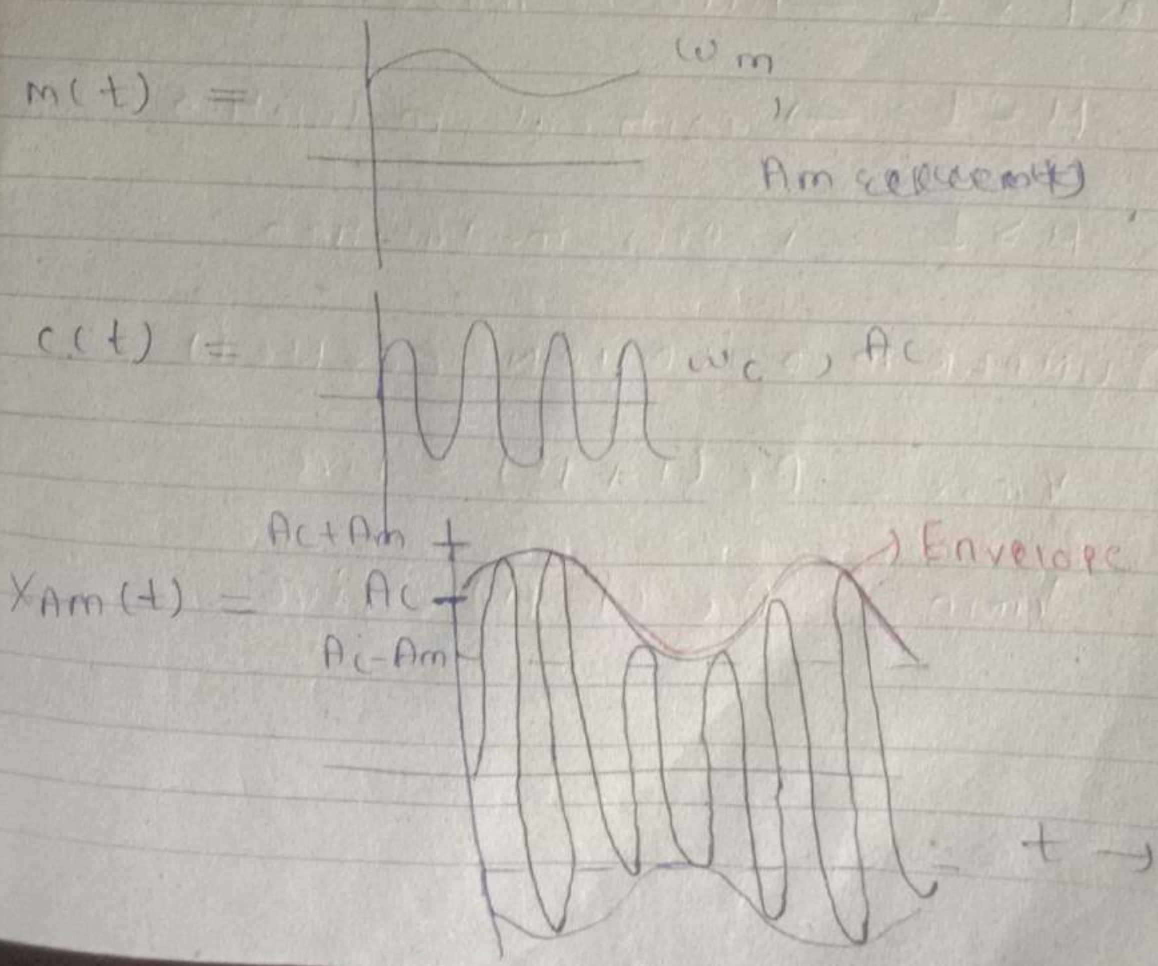


$\mu \in (0, 1)$

(Band of freq. in which major signal energy present)

Bandwidth = $(\omega_c + \omega_m) - (\omega_c - \omega_m)$

$\boxed{\text{Bandwidth} = 2\omega_m}$



V_{min} = minimum amplitude
V_{max} = maximum Amplitude

$$V_{max} = A_c + A_m$$

$$V_{min} = A_c - A_m$$

$$\mu = \frac{A_m}{A_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$V_{max} = A_c + A_m = A_c[1 + \mu]$$

$$V_{min} = A_c - A_m = A_c[1 - \mu]$$

* $0 < \mu < 1$ → under modulation

$\mu = 1$ → critical modulation

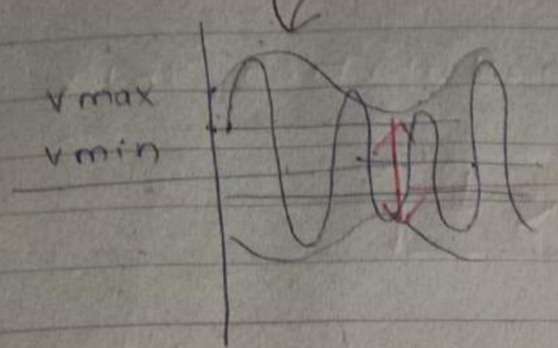
$\mu > 1$ → over modulation

① UNDER MODULATION $(0 < \mu < 1)$

$V_{max} = A_c(1 + \mu)$ = +ive

$V_{min} = A_c(1 - \mu)$ = +ive

positive & negative envelope dont touch each other



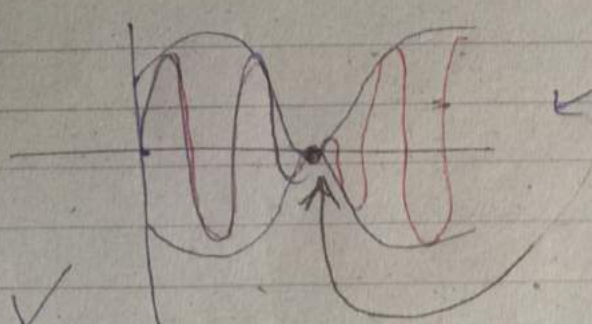
→ Desired

→ some scope of modulation still left

② CRITICAL MODULATION ($M=1$)

$V_{max} = A_c(1+M) = 2A_c$

$V_{min} = A_c(1-M) = 0$



Desired

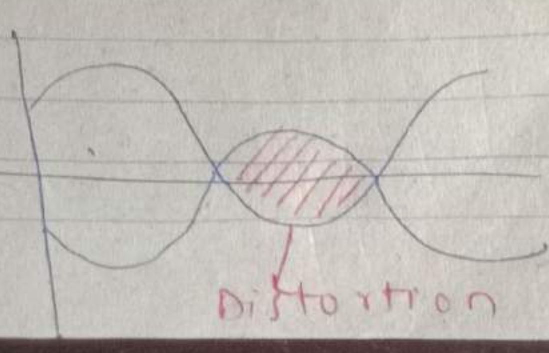
positive & negative envelope touch each other

Desired

③ over modulation ($M>1$)

$V_{max} = A_c(1+M) = +ve$

$V_{min} = A_c(1-M) = -ive$



Undesired

$\mu = 0$ [No modulation]

↳ so we exclude it

∴ Range of modulation index

$$\boxed{0 < \mu \le 1}$$

It can be more than 1, but that is undesired.

Lec 4 (Parameters of DSB-C)

Conventional AM / DSB-C

$$x_{AM}(t) = A_c\cos(\omega_c t) + \frac{A_c\mu}{2}\cos(\omega_m + \omega_c)t + \frac{A_c\mu}{2}\cos(\omega_c - \omega_m)t$$

(1) **Transmitted power ($x_{AM}(t)$):**

$$P_t = P_c + P_{USB} + P_{LSB}$$

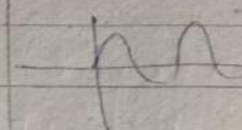
$$= \frac{A_c^2}{2} + \frac{A_c^2\mu^2}{8} + \frac{A_c^2\mu^2}{8}$$

$$P_t = \frac{A_c^2}{2} + \frac{A_c^2\mu^2}{4}$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{P_t = P_c + P_c\frac{\mu^2}{2}}$$



$A_c\cos\theta$

↓

sinusoidal signal

power $= \frac{A^2}{2}$

Lot of wastage since P_c is there since it ~~cannot~~ does not store any useful information

Ex) Let $\mu = 1$ (Critical modulation)

$$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$P_t = \frac{3}{2} P_c$$

$$P_c = \frac{2}{3} P_t = 0.66\, P_t \times 100$$
$$= 66\% \text{ of } P_t$$

wasting 66% of power since carrier does not contain any useful information

(2) Power efficiency:

$$\eta = \frac{(\text{useful power})}{\text{Total power}} \times 100$$

power in side bands

$$= \frac{P_{USB} + P_{LSB}}{P_t} \times 100$$

$$= \frac{\frac{A_c^2 \mu^2}{4}}{\frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right)} = \frac{\mu^2}{2} \times \frac{2}{2 + \mu^2}$$

$$= \frac{\mu^2}{2 + \mu^2} \times 100$$

$$\boxed{\eta = \frac{\mu^2}{2 + \mu^2} \times 100}$$

(3) Bandwidth

$$BW = (\omega_c + \omega_m) - (\omega_c - \omega_m)$$

$$= 2\omega_m$$

(max frequency component in message signal)

(4) modulation index (μ)

$$\mu = \frac{A_m}{A_c}$$

$$\mu \in (0, 1]$$

MULTITONE AM (DSB-C)

more than 1 freq component in message signal

i.e, $m(t) = A_{m_1} \cos(\omega_{m_1} t)$

$$+ A_{m_2} \cos(\omega_{m_2} t)$$

$(A_c + m(t)) \cos(\omega_c t)$

$A_c [1 + \mu \cos(\omega_m t)] \cos(\omega_c t)$

$$X_{AM}(t) = A_c [1 + \mu_1 \cos(\omega_{m_1} t) + \mu_2 \cos(\omega_{m_2} t)] \cos(\omega_c t)$$

Total modulation index:

$$\mu_T = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\mu_1 = \frac{A_{m_1}}{A_c}$$

$$\mu_2 = \frac{A_{m_2}}{A_c}$$

Individual modulation index

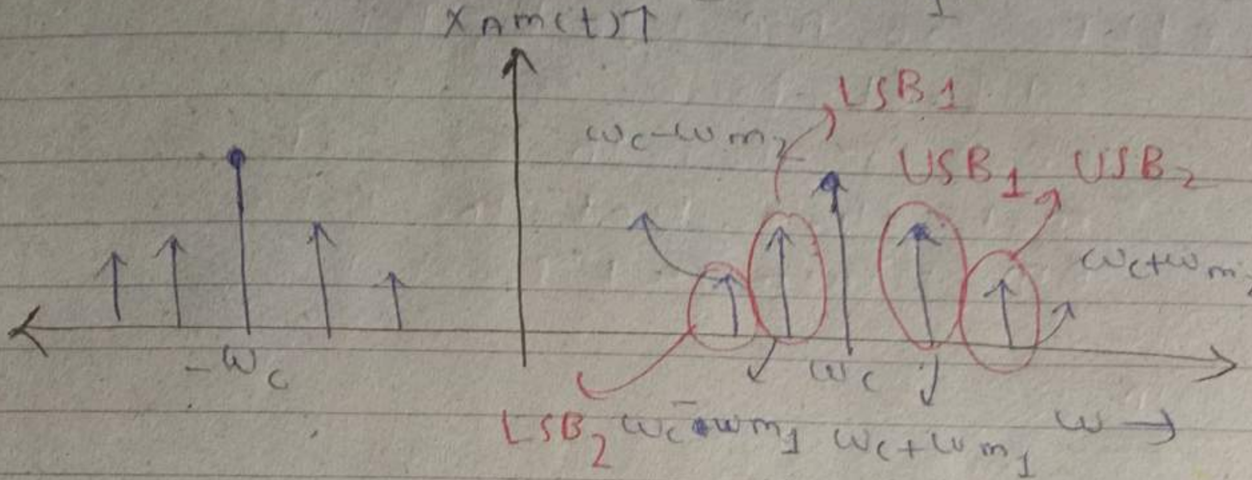
$$P_T = P_c \left[1 + \frac{\mu_T^2}{2}\right]$$

SPECTRUM OF SIGNAL:

↳ instead of single pair of ~~side~~ sidebands, we have 2 pairs of sidebands

Let $A_{m_1} > A_{m_2}$

$\cancel{\omega_{m_1} > \omega}$, $\omega_{m_2} > \omega_{m_1}$



no. of freq component increases	$\Longleftrightarrow$	no. of sideband pairs increases

n. frequency components	=	n sideband pairs

whenever given in question, Am signal & type of Am signal not specified, then assume it to be DSB-C

NUMERICALS ON DSB-C:

Q1) For an Am signal, total sideband power is given by 100 W with 50% of modulation. Find the total Am transmitted power?

Soln:

My Solution:

$$P_{LSB} + P_{USB} = 100\,W = P$$

$$\mu = \frac{50}{100} = \frac{1}{2} = 0.5$$

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right) \qquad P_t = P_c + (P)$$

$$P_c = P_t - P$$

$$P_t = (P_t - P)\left(1 + \frac{\mu^2}{2}\right)$$

$$P_t = (P_t - P)\,1.125$$

$$P_t = 1.125P_t - 1.125P$$

$$P_t(0.125) = 1.125 \times 100$$

$$P_t = 900\,W$$

Sir solution :

$P_{SB} = P_{USB} + P_{LSB} = 100\,W$

$\mu = 0.5$

$P_t = ?$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$

$P_{SB} = P_{USB} + P_{LSB} = \frac{A_c^2 \mu^2}{4}$ — (i)

$P_c = \frac{A_c^2}{2}$ — (ii)

from (i) & (ii),

$P_{SB} = \frac{A_c^2}{2} \cdot \frac{1}{2} \mu^2 = \frac{P_c \mu^2}{2}$

$P_c = \frac{2 \times P_{SB}}{\mu^2} = \frac{2 \times 100}{(0.5)^2} = 800\,W$

$P_t = P_c + P_{SB} = 800\,W + 100\,W$

$= 900\,W$ consuming huge Power which is not read.

MS = My solution
SS = Sir solution

Q2) For an AM each of the SB power is given by 2kW and carrier power is given by 8kW. Find % of modulation?

SS

Soln:

MS:

$P_{LSB} = P_{USB} = 2\,kW$

$P_c = 8\,kW$

$\mu = ?$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$

$P_t = P_c + P_{USB} + P_{LSB}$
$= 8 + 2 + 2$
$= 12\,kW$

$12\,kW = 8\,kW\left(1 + \frac{\mu^2}{2}\right)$

$\frac{\mu^2}{2} = \frac{3}{2} - 1 = \frac{1}{2}$

$\mu^2 = 1$
$\mu = 1$

SS:

$P_{USB} = P_{LSB} = 2kW$

$P_{SB} = 4kW$

$P_c = 8kW$ $\Rightarrow$ again greater than P_{SB}, therefore wastage

$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$

$P_t = \checkmark$
$P_c = \checkmark$
$\mu = 1$

$f = \frac{\omega}{2\pi} \quad T = 1/f \quad \omega = 2\pi f$

Q3) carrier signal, ~~c(t) = 10 cos~~

$c(t) = 10 \cos(2\pi \times 10^6 t)$ and message signal, $m(t) = 4\cos(4\pi \times 10^3 t)$ with 50% of modulation. Antenna resistance is given by 5 Ω

(1) Find all the parameters of AM (BW, P_T, η)

(2) Plot AM spectrum & identify the spectral components

Soln:

$$c(t) = 10\cos(2\pi \times 10^6 t)$$

$$= A_c \cos(2\pi f_c t)$$

$$m(t) = 4\cos(4\pi \times 10^3 t)$$

$$= A_m \cos(2\pi f_m \times t)$$

$A_c = 10\,V$

$f_c = 10^6 = 1\,MHz$

$A_m = 4\,V$

$f_m = 2 \times 10^3\,Hz = 2\,kHz$

Bandwidth of multitone AM signal ?
relation b/w Power, Amplitude & resistance

(1)

$BW = 2\omega m = 2 fm$

$= 2 \times 2k\ HZ$

$= 4 kHZ$

$\mu = \frac{Am}{Ac} = \frac{50}{100} = 0.5$ (given)

$R = 5\Omega$

$\left(P = \frac{V^2}{R} \right)$

$P_c = \frac{Ac^2}{2R} = \frac{100}{2 \times 5}$

$= \frac{100}{10} = 10 W$

$P_t = P_c \left(1 + \frac{\mu^2}{2}\right)$

$= 10 \left(1 + \frac{(0.5)^2}{2}\right)$

$= 11.25 W$

$P_{SB} = P_t - P_c = 11.25 - 10 = 1.25 W$

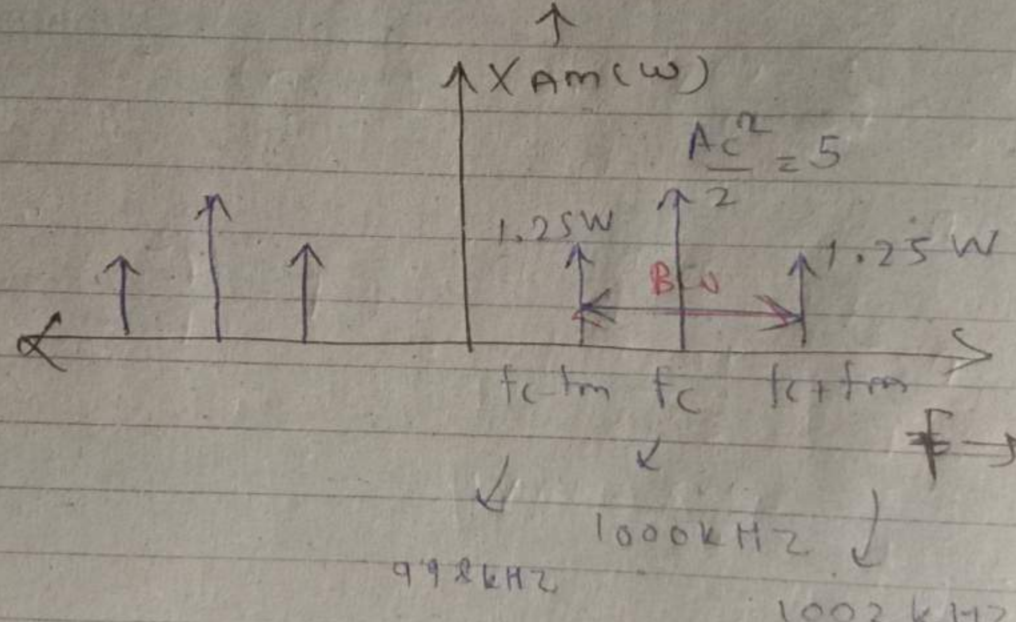
$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = \frac{1.25}{2} = 0.625 W$

$$\eta = \frac{P_{SB}}{P_t} = \frac{\mu^2}{2+\mu^2}$$

$$= \frac{1.25}{11.25} = 0.11 \times 100\%$$

$$= 11\%$$

(2)



$BW = \cancel{100}$ 4 KHZ

$[A_c + m(t)] \cos \omega_c t$ $\qquad$ $A_c[1 + \mu \cos \omega_m t \cos \omega_c t]$

$A_c + A_m \cos \omega_m t \cos \omega_c t$ $\qquad$ $= A_c \cos \omega_c t + A_c \mu$

Recs (Generation of AM/DSB-C)

↳ Square law modulator

↳ Switching modulator

DSB-C.

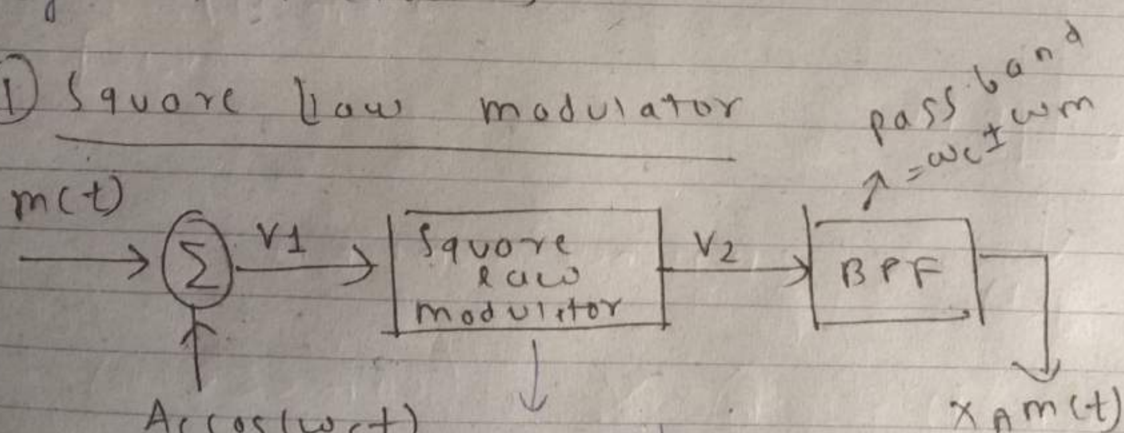
general representation of $x_{Am}(t)$ is:

$$x_{Am}(t) = c(t) + m(t)\,c(t)$$

$$= \underbrace{A_c \cos \omega_c t}_{c(t)} + \underbrace{A_c \mu \cos \omega_m t \cos \omega_c t}_{m(t)\,c(t)}$$

* Scaler components can be removed by amplier circuits

① Square law modulator

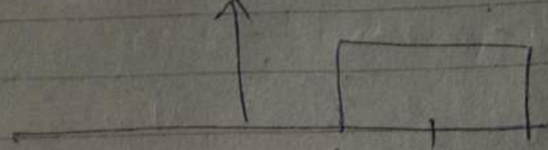


circuit
= combination of discrete elements

X = rejected

✓ = allowed

Frequency characteristics of BPF:



$v_1 = m(t) + A_c \cos(\omega_c t)$

$v_2 = a_1 v_1 + a_2 v_1^2$

$$= a_1 [m(t) + A_c \cos(\omega_c t)] + a_2 [m^2(t) + A_c^2 \cos^2(\omega_c t) + 2m(t) A_c \cos(\omega_c t)]$$

BPF

$y(t)$

frequency component

$a_1 m(t) = \omega_m =$ X $\quad (\omega_m <<< \omega_c)$

$a_1 A_c \cos(\omega_c t) = \omega_c =$ ✓

$a_2 m^2(t) = 2\omega_m$ X $\quad (2\omega_m <<< \omega_c)$

$$\left\{ \begin{array}{l} m(t) = \omega_m \\ m^2(t) = m(t) . m(t) \\ m^2(W) = m(W) \underset{\text{convolution}}{*} m(W) = \omega_m + \omega_m = 2\omega_m \end{array} \right\}$$

Σ = summer

$A_c^2 \cos^2(\omega_c t)$ = ×

$\cos^2(\omega_c t)$

$= \omega_c + \omega_c = 2\omega_c$

$2\omega_c >> \omega_c$

$m(t) A_c \cos(\omega_c t) = \omega_c \pm \omega_m$ ✓

so final expression is:

$$X_{AM}(t) = a_1 A_c \cos(\omega_c t) + a_2 m(t) A_c \cos(\omega_c t)$$

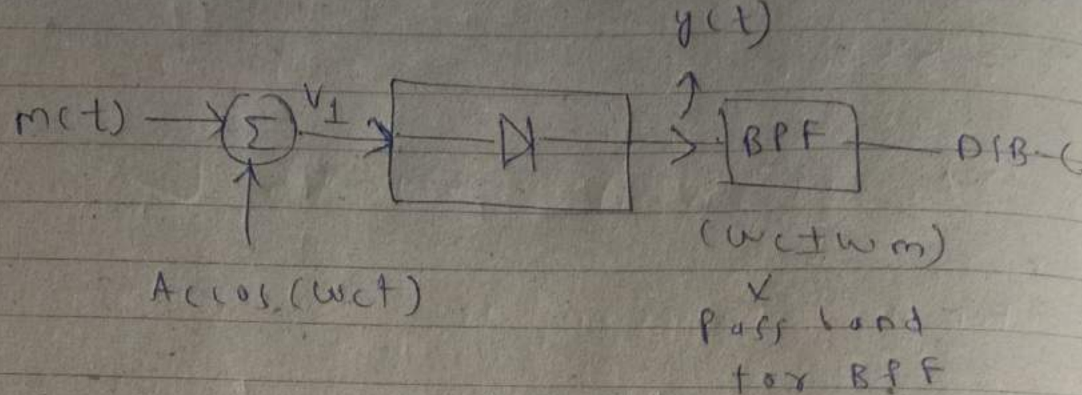
Of form

$$\boxed{X_{AM}(t) = a_1 c(t) + a_2 m(t) c(t)}$$

DSB - C

(2) Switching modulator:

(Hardware implementation simpler)
(simpler to design & cheaper also)



$V_1 = A_c \cos(\omega_c t) + \cancel{A_m}\, m(t)$

Diode → will act as a switch
(switching depends on V_1)

$P_{c(t)} > P_{m(t)}$

$\text{Power}_{c(t)} > \text{Power}_{m(t)}$

$0 < \mu \leq 1$ $\qquad P_{A_c} = \frac{A_c^2}{2}$

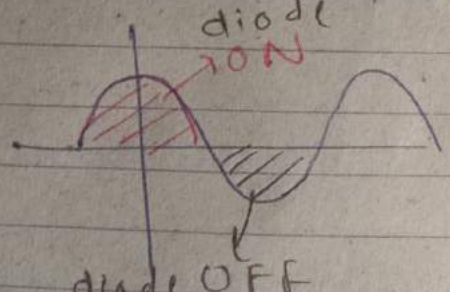
$0 < \frac{A_m}{A_c} \leq 1$ $\qquad P_{A_m} = \frac{A_m^2}{2}$

$A_c \geq A_m$ $\qquad \therefore P_{c(t)} > P_{m(t)}$

* switching of the diode depends on the polarity of the carrier signal because the strength of carrier is more than strength of message signal

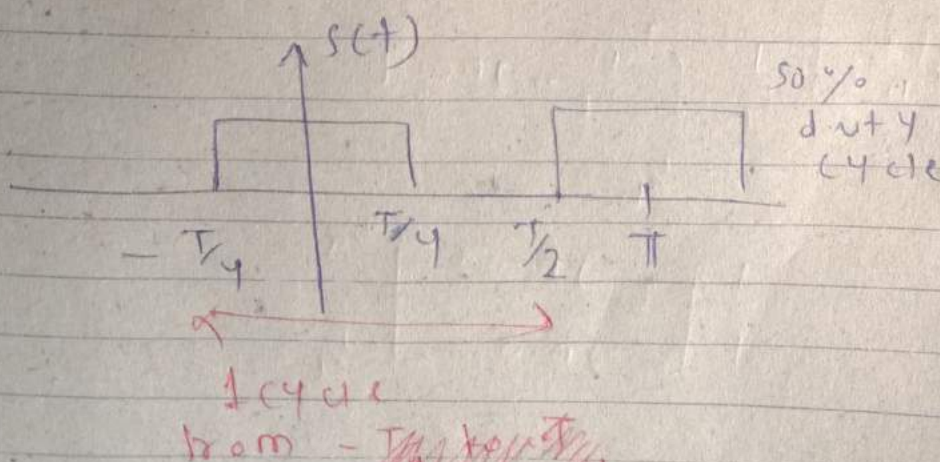
$$y(t) = \left[A_c \cos(\omega_c t) + m(t)\right] s(t)$$

$s(t)$ = switching coefficient
(depends on the carrier, so duty cycle = 50%)



, for 50%, diode ON, rest 50% diode off

Finding the expression for $s(t)$:



Fourier series representation of s(t):

$$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_c t}$$

$$c_n = \frac{1}{T}\int_0^T x(t)\, e^{-j\omega_c t n}\, dt$$

$$c_0 = \frac{1}{T}\int_{-T/4}^{T/4} 1\, dt + \frac{1}{T}\int_{T/4}^{T/2} 0 \cdot dt$$

$$= \frac{1}{T}\int_{-T/4}^{T/4} 1 \cdot dt$$

$$= \frac{1}{T}\cdot\left(\frac{T}{4}+\frac{T}{4}\right) = T/2 \times \frac{1}{T} = \frac{1}{2}$$

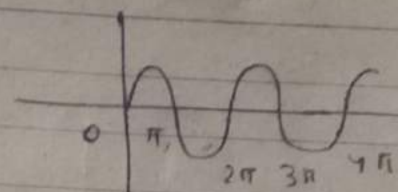
$$c_n = \frac{1}{T}\int_{-T/4}^{T/4} e^{-jn\omega t}$$

$$= \frac{1}{T}\left[\frac{e^{-jn\omega t}}{-jn\omega}\right]_{-T/4}^{T/4}$$

$$\frac{e^{ix}-e^{-ix}}{2i} = \sin x$$

$$C_n = \frac{1}{T}\times\frac{1}{jn\omega}\left[e^{jn\omega T/4} - e^{-jn\omega T/4}\right]$$

$$C_n = \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$



if n is even,

$C_n = 0$

n = odd,

C_1, C_{-1}, C_3, C_{-3} - - -

$$f(t) = \underset{(C_0)}{\frac{1}{2}} + \underset{(C_{-1})}{\frac{\sin(-\pi/2)}{-\pi}} e^{j\omega_c t} + \underset{(C_1)}{\frac{\sin \pi/2}{\pi}} e^{-j\omega_c t}$$

$$+ \frac{\sin(-3\pi/2)}{-3\pi} e^{j3\pi t}$$

$$f(t) = \frac{1}{2} + \frac{1}{\pi}\left[e^{j\omega_c t} + e^{-j\omega_c t} + \frac{1}{3} e^{j\omega_c 3t} - \frac{1}{3} e^{-j3\omega_c t} - -\right]$$

$$= \frac{1}{2} + \frac{1}{\pi}\left[2\cos(\omega_c t) - \frac{1}{3}\cdot 2\cos(3\omega_c t) + - - \right]$$

$$y(t) = \left[m(t) + A_c \cos(\omega_c t)\right] s(t)$$

$$= \left[m(t) + A_c \cos(\omega_c t)\right] \left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_c t) - \frac{2}{3\pi}\cos(3\omega_c t) + \dots\right]$$

$$= \frac{m(t)}{2} + \frac{A_c \cos(\omega_c t)}{2} + \frac{2}{\pi} m(t)\cos(\omega_c t) + \frac{2A_c}{\pi}\cos^2 \omega_c t - \frac{2}{3\pi} m(t)\cos(3\omega_c t) - \frac{2}{3\pi} A_c \cos(\omega_c t)\cos(3\omega_c t) + \dots$$

$\Downarrow$

$\boxed{\text{BPF}}$ $\omega_c \pm \omega_m$

$\downarrow$

?

$X(t) =$

$=$

$\boxed{X_{AM}(t) =}$

$$X(t) = \frac{A_c \cos(\omega_c t)}{2} + \frac{2}{\pi} A_c\, m(t) \cos(\omega_c t)$$

$$= \text{[illegible]}$$

$$\boxed{x_{AM}(t) = \frac{1}{2} c(t) + \frac{2}{\pi} m(t)\, c(t)}$$

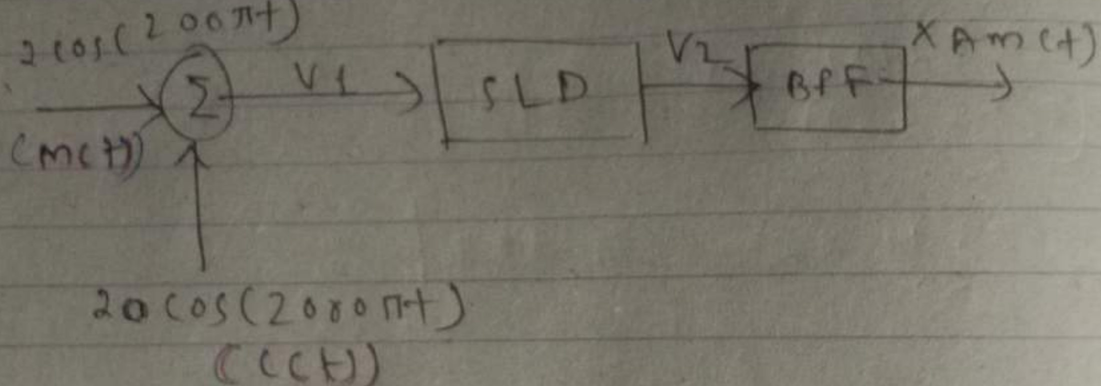
SLD = square law device
BPF = band pass filter

Lec 6 (Questions)

Q1) square law device characterised by $v_2 = v_1 + 0.1 v_1^2$

Passband of BPF is 800–1200 Hz

Find all the parameters of resulting AM signal?



Soln:

MS:

$v_1 = 2\cos(200\pi t) + 20\cos(2000\pi t)$

$v_2 = (2\cos(200\pi t) + 20\cos(2000\pi t)$

$+ 0.1(4\cos^2(200\pi t)$

$+ 400\cos^2(2000\pi t)$

$+ 80\cos(200\pi t)\cos(2000\pi t))$

V_2 — $\boxed{BPF}$ — $x_{Am}(t)$

$$2\pi f t$$

$x_{Am}(t) =$ $2\cos(200\pi t)$ 100 ×

$20\cos(2000\pi t)$ 1000

$0.4\cos^2(200\pi t)$ 200 ×

$40\cos^2(2000\pi t)$ 2000 ×

$8\cos(200\pi t)\cos(2000\pi t)$

$\frac{2200}{2} = 1100$

$$x_{Am}(t) = 20\cos(2000\pi t)$$

$$+ 8\cos(200\pi t)\cos(2000\pi t)$$

$$= 20\cos(2000\pi t)\left[1 + 0.4\cos 200\pi t\right]$$

$$A_c\cos(\omega_c t)\left[1 + \mu \cos\omega_m t\right]$$

$A_c = 20\,V$

$f_c = 1000\,Hz$

$\mu = 0.4$

$f_m = 100\,Hz$

SS:

$v_1 = 2\cos(200\pi t) + 20\cos(2000\pi t)$

$V_2 = v_1 + 0.1 v^2$

100HZ . 1000HZ

$v_2 = (2\cos(200\pi t) + 20\cos(2000\pi t))$

200HZ 2000HZ

$+ 0.1(4\cos^2(200\pi t) + 400\cos^2(2000\pi t) +$

1100HZ

$80\cos(200\pi t)\cos(2000\pi t)]$

$\downarrow$ BPF (800-1200HZ)

$V_2 = 20\cos(2000\pi t) + 8\cos(200\pi t)\cos(2000\pi t)$

$= 20\{1 + 0.4\cos(200\pi t)\}\cos(2000\pi t)$

$= A_c\{1 + \mu\cos(2\pi f_m t)\}\cos(2\pi f_c t)$

$A_c = 20V$ → can be verified ~~from~~ from Block diagram

$\mu = 0.4$

$f_m = 100$ HZ

$f_c = 1000$ HZ

CURRENT RELATION IN AM

$$P_t = P_c \left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{P = i^2 R}$$

$$I_t^2 R = I_c^2 R \left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}}$$

$$\boxed{V_t = V_c \sqrt{1 + \frac{\mu^2}{2}}}$$

$\mu = 0$

Q2) An unmodulated AM transmitter current is given by 5A. Find AM transmitter current with 50% modulation?

Soln: SS:

$$\mu = 0, \quad I_t = I_c \left[1 + \cancel{\frac{\mu^2}{2}}^{\,0}\right]$$

$$I_t = I_c = 5A$$

Carrier signal is constant throughout experiment

$I_c = 5A$

$\mu = \frac{50}{100} = 0.5$

$$I_t = I_c\sqrt{\left[1 + \frac{\mu^2}{2}\right]}$$

$$= 5\sqrt{\left[1 + \frac{(0.5)^2}{2}\right]}$$

$$\boxed{I_t = 5.32\ A}$$

transmitting extra components requires more current, 5.32A > 5A

Q3) An Am transmitter current is given by 10 A with 40% modulation find Am transmitter current with 80% of modulation?

Soln: SS

$I_t = 10A, \ \mu = 0.4$

$$I_t = I_c\sqrt{\left[1 + \frac{\mu^2}{2}\right]}$$

$$10 = I_c\sqrt{\left[1+\frac{(0.4)^2}{2}\right]}$$

$I_c = 9.6\,A$

$\mu = 0.8$, $I_c = 9.6A$, $I_t = ?$

$$I_t = I_c\sqrt{\left[1+\frac{\mu^2}{2}\right]}$$

$$= 9.6\sqrt{\left[1+\frac{(0.8)^2}{2}\right]}$$

$$= 11.05\,A$$

Q4) An unmodulated AM transmitter power is given by 10kW. When the carrier is modulated by single sinusoidal message signal, transmitter power becomes 13.5kW. Find AM transmitter power if the carrier is simultaneously modulated by 2nd message signal with 60% of modulation?

Soln:

Initially unmodulated

$\mu = 0$, $P_t = P_c = 10\,kW$

μ_1 for $m_1(t)$, $P_t = 13.5\,kW$

$$P_t = P_c \left[1 + \frac{\mu_1^2}{2}\right]$$

$$13.5 = 10 \left[1 + \frac{\mu_1^2}{2}\right]$$

$$\mu_1^2 = 0.7$$

$m_2(t)$, $\mu_2 = 0.6$

$$P_t = P_c \left[1 + \frac{\mu_T^2}{2}\right]$$

$$\mu_T = \sqrt{\mu_1^2 + \mu_2^2}$$

$$= \sqrt{0.7 + 0.6^2}$$

$$\boxed{\mu_T \approx 1}$$

$$P_t = P_c \left[1 + \frac{\mu_T^2}{2}\right]$$

$$= 10\left[1 + \frac{1}{2}\right] = 10 \times \frac{3}{2} = 15\,kW$$

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X_{DSB}

X_{DSB}

lec 7 (DSB-SC)

Double sideband suppressed carrier (DSB-SC)

general representation

DSB-C $\Rightarrow$ LSB + USB + Carrier

DSB-SC $\Rightarrow$ LSB + USB + ~~carrier~~

suppressed (not present)

motivation: In DSB-C, carrier power is huge which is wastage of transmitted power

* for $\mu = 1$, $\frac{2}{3}$rd transmitted power consumed by carrier signal

General expression for DSB-SC:

$$x_{DSB-C}(t) = c(t) + m(t)c(t)$$

$$\boxed{x_{DSB-SC}(t) = m(t)c(t)}$$

$$\cos(\omega_c t) = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$$

$$= \frac{1}{2}\left[\pi(\delta(\omega-\omega_c))\right.$$

$$c(t) = A_c \cos(\omega_c t)$$

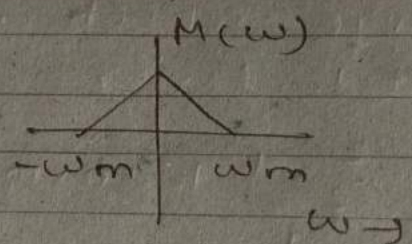
$$x_{DSB-SC}(t) = m(t)\, A_c \cos(\omega_c t)$$

$$\Updownarrow \text{FT}$$

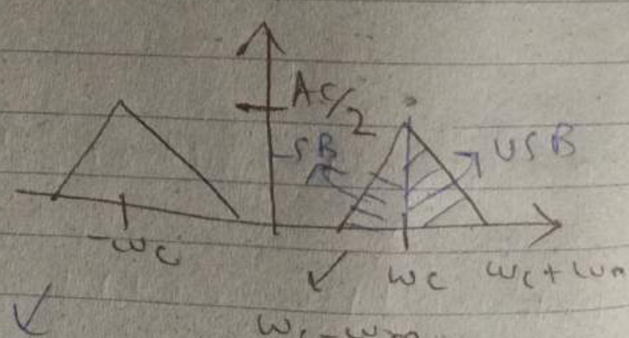
$$X_{DSB-C}(\omega) = M(\omega) * \frac{A_c}{2}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

$$\boxed{X_{DSB-C}(\omega) = \frac{A_c}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]}$$

let $m(t) =$



$X_{DSB-SC}(t) =$



no carrier signal present, only USB & LSB

BW = Band of frequency in which max. energy components present

$$\boxed{BW_{DSB-SC} = 2\omega_m}$$

↳ ~~same~~ same as DSB-C

↳ difference in transmission efficiency

→ <u>single-tone DSB-SC</u>

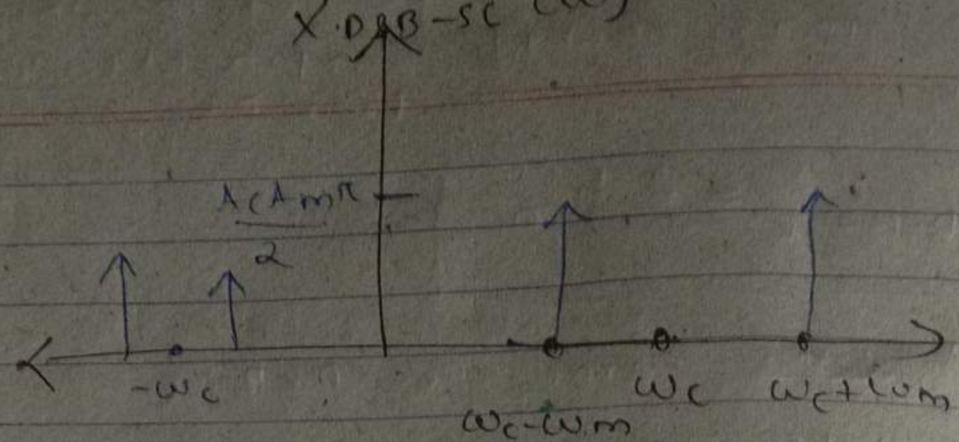
$$m(t) = A_m \cos(\omega_m t)$$
$$c(t) = A_c \cos(\omega_c t)$$

$$X_{DSB-SC}(t) = A_c A_m \cos(\omega_m t)\cos(\omega_c t)$$

$$= \underbrace{\frac{A_c A_m}{2}\cos(\omega_c - \omega_m)t}_{LSB} + \underbrace{\frac{A_c A_m}{2}\cos(\omega_c + \omega_m)t}_{USB}$$

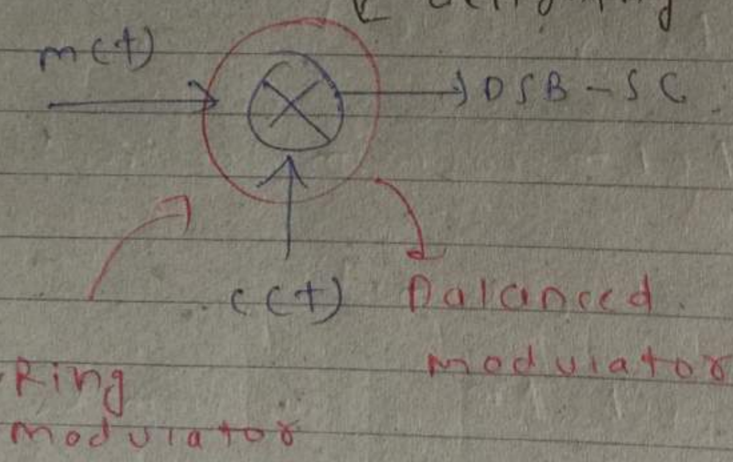
$$X_{DSB-SC}(\omega) = \frac{A_c A_m \pi}{2}\left[\delta(\omega - (\omega_c - \omega_m)) + \delta(\omega + (\omega_c - \omega_m))\right]$$

$$+ \frac{A_c A_m}{2}\pi\left[\delta(\omega - (\omega_m + \omega_c)) + \delta(\omega + (\omega_m + \omega_c))\right]$$



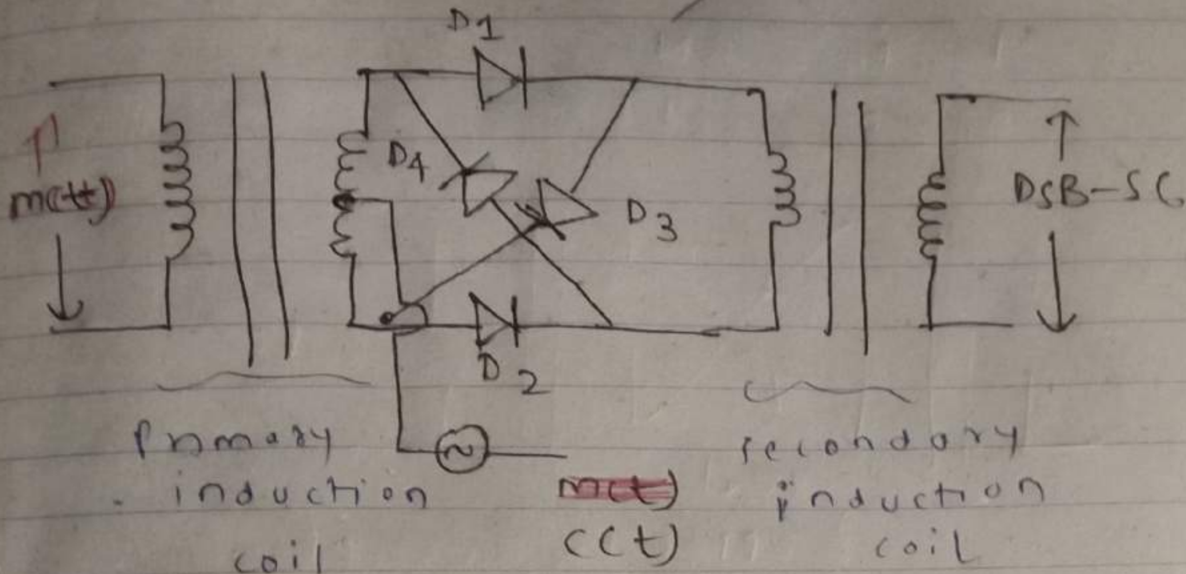
GENERATION OF DSB-SC SIGNAL

↳ Ring modulator
↳ Balanced modulator
(commonly used)
← designing not easy



RING MODULATOR

diodes form a ring



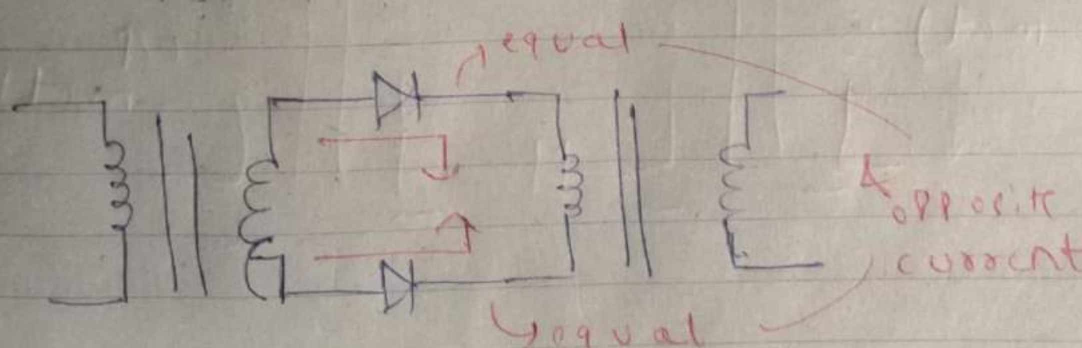
CASE 1: when only c(t) is applied

$m(t) = 0$

(i) +ive cycle:

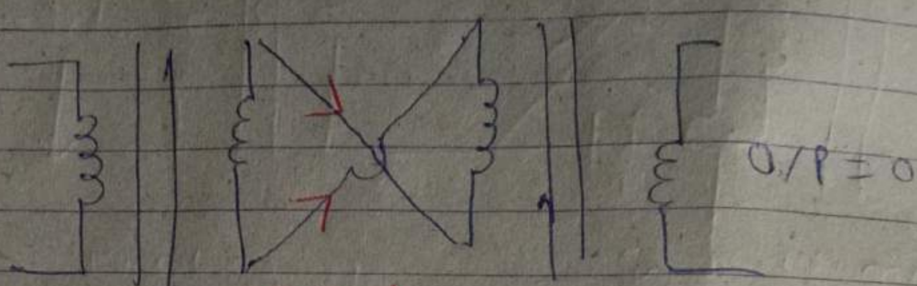
D_1, D_2 = ON (forward biased)

D_3, D_4 = OFF (Reverse biased)



(ii) -ive cycle:

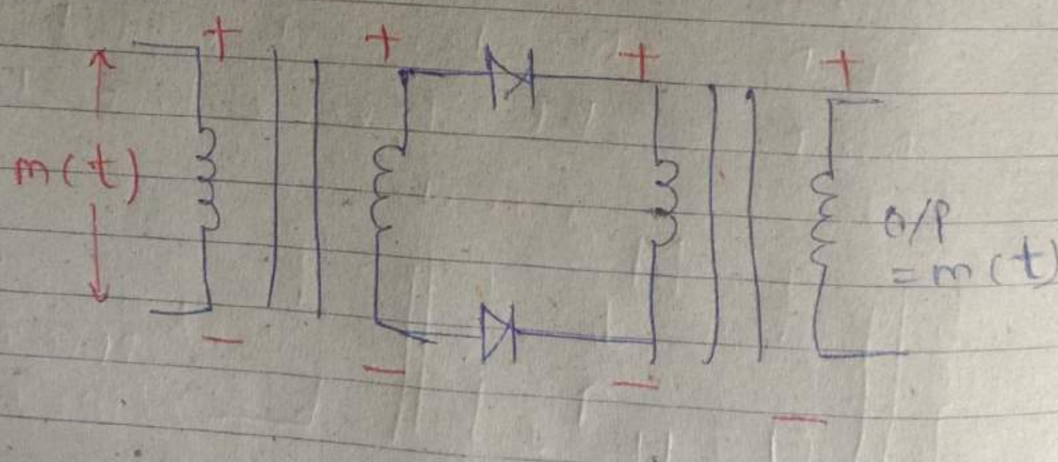
$D_3, D_4 = ON$
$D_1, D_2 = OFF$



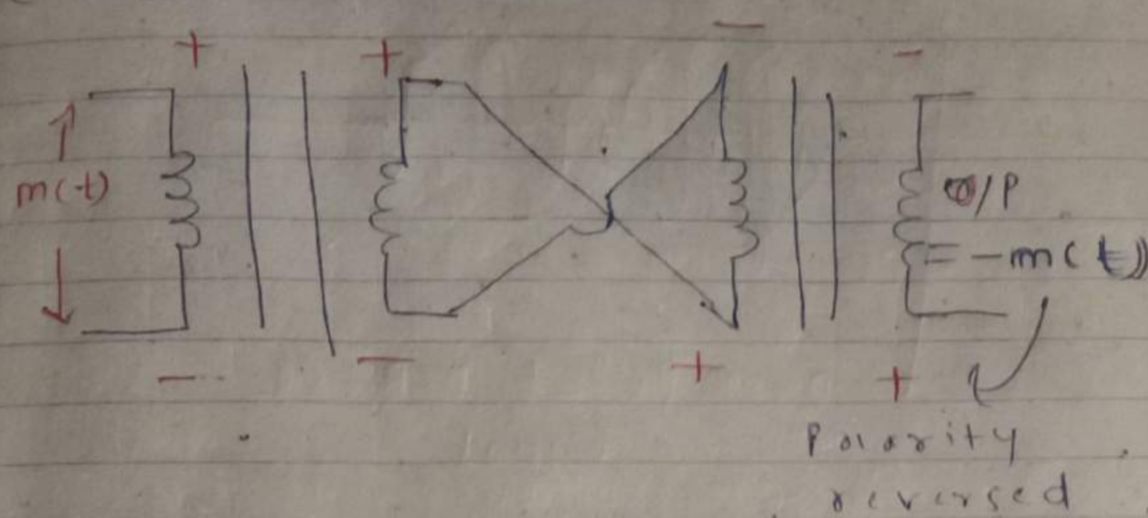
equal
& opposite current

CASE 2: When m(t) is also applied

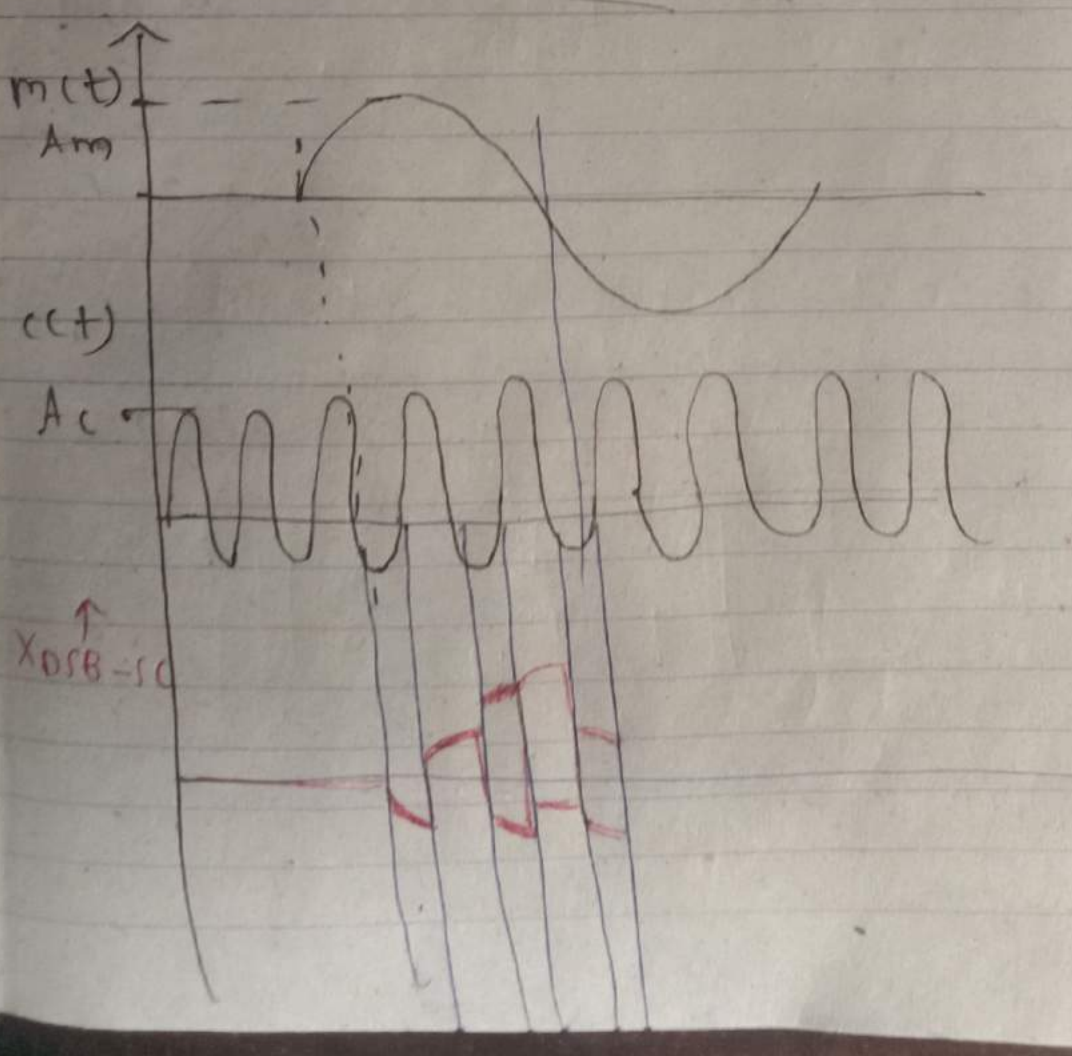
(i) +ive cycle of c(t)

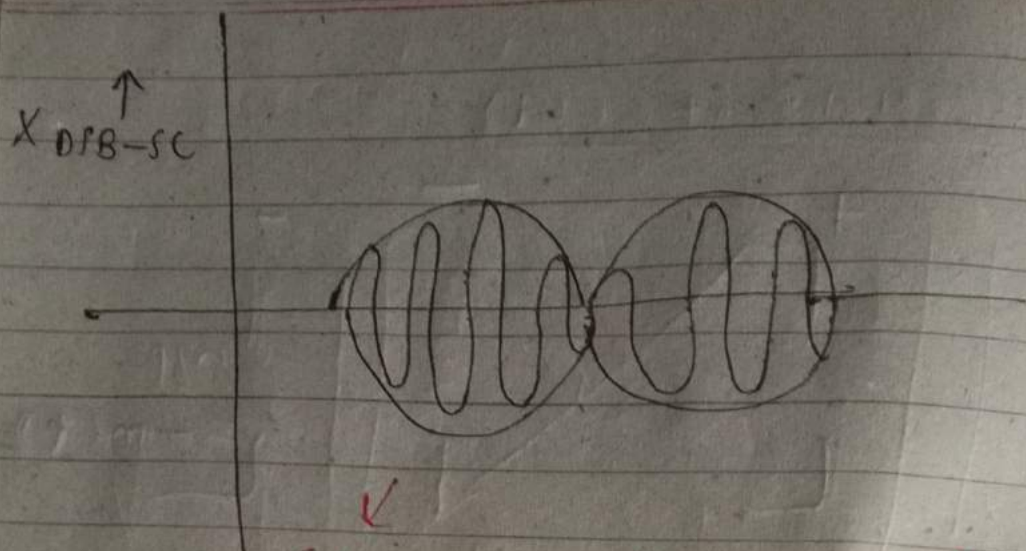


(ii) -ive cycle of c(t)



GRAPHICAL ANALYSIS:





Envelope of DSB-SC
signal

L(8) (Balanced mod. & SSB-SC)

DSB-SC

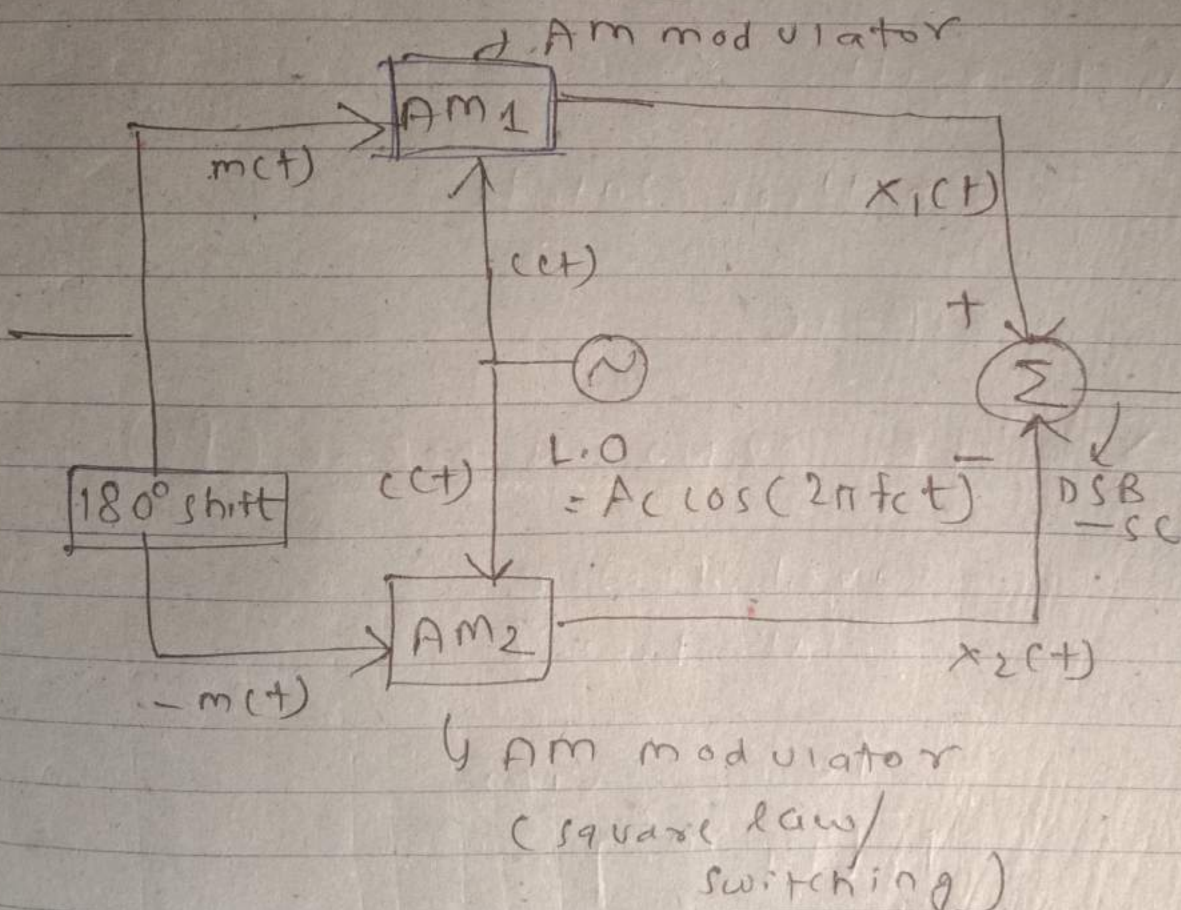
$= c(t) + m(t)\,c(t) = \text{DSB-C}$

$= \cancel{c(t)} + m(t)\,c(t)$

↳ DSB-SC

$x_{DSB-SC}(t) = m(t)\,A_c \cos(\omega_c t)$

Balanced modulator



System model

For AM_1

$I/P = m(t)$ & $c(t)$

For AM_2

$I/P =$ ~~[illegible]~~ $-m(t)$ & $c(t)$

$$x_1(t) = (A_c + m(t)) \cos(2\pi f_c t)$$

$$x_2(t) = (A_c - m(t)) \cos(2\pi f_c t)$$

$$x_1(t) - x_2(t)$$

$$= 2\, m(t) \cos(2\pi f_c t)$$

DSB-SC

$\Rightarrow$ POWER SAVING IN DSB-SC (PS)

$$= \frac{\text{Saved Power}}{\text{Total Power}}$$

$$= \frac{P_c}{P_c\left[1 + \frac{m^2}{2}\right]} \times 100$$

$$= \frac{2}{2 + m^2} \times 100\%$$

If (r

PS

MULT

m(t)

c(t)

X_{DSB-SC}

If critical modulation $(m = 1)$

$$PS = \frac{2}{2+1} \times 100$$

$$= \frac{200}{3} = 66.6\ \%$$ power saved

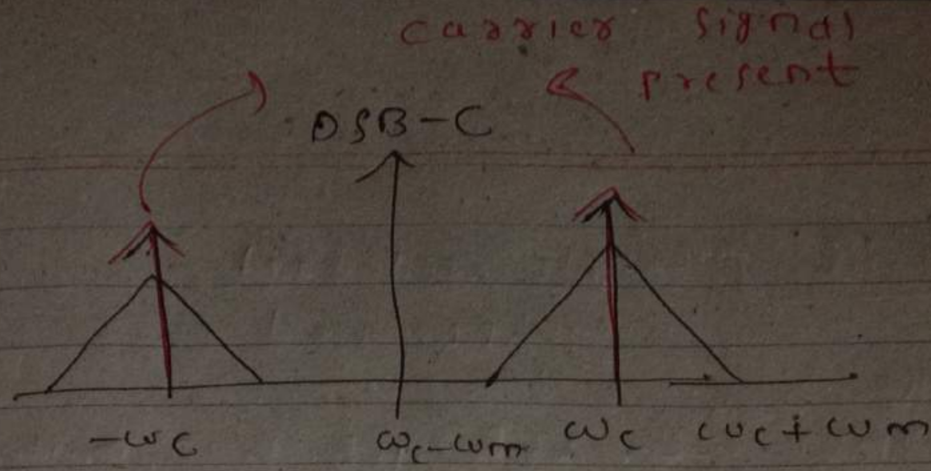
MULTITONE DSB-SC

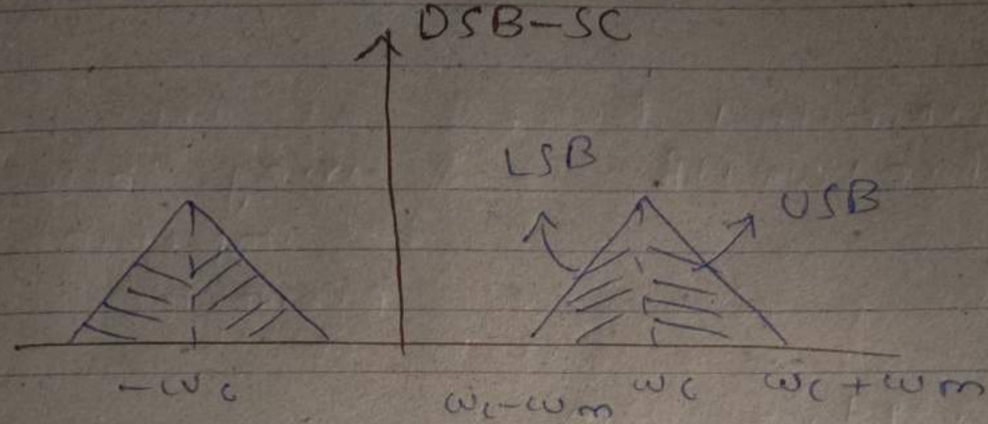
$$m(t) = A_{m_1} \cos(\omega_{m_1} t) + A_{m_2} \cos(\omega_{m_2} t)$$

$$c(t) = A_c \cos(\omega_c t)$$

$$x_{DSB-SC}(t) = \frac{A_c A_{m_1}}{2} \cos(\omega_c + \omega_{m_1}) t + \frac{A_c A_{m_1}}{2} \cos(\omega_c - \omega_{m_1}) t + \frac{A_c A_{m_2}}{2} \cos(\omega_c + \omega_{m_2}) t + \frac{A_c A_{m_2}}{2} \cos(\omega_c - \omega_{m_2}) t$$

DSB-C	DSB-SC
(1) P_t is high	P_t is lower
$P_t = P_c\left(1 + \frac{m^2}{2}\right)$	$P_t = \frac{P_c m^2}{2}$
(2) $BW = 2\omega_m$	$BW = 2\omega_m$
(3)	although power saved but bandwidth reqd = more that leads to SSB-SC





* Both sidebands are identical in nature, replica of each other.

* same mathematical expression, carry same information

* knowledge of one sideband = we can generate another sideband

Principle behind SSB-SC

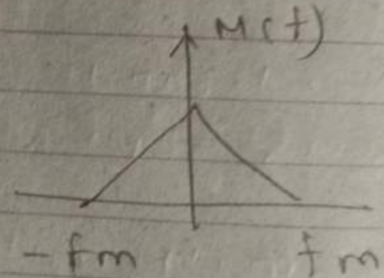
SSB-SC (Single sideband - suppressed carrier)

either ~~[illegible]~~ LSB/USB transmitted ~~[illegible]~~

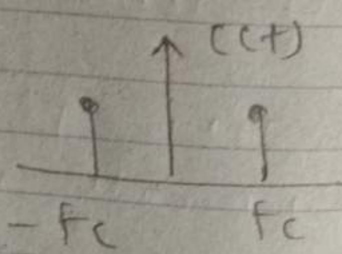
Transmitted power } we are saving these two important parameters
Bandwidth

BW reqd. = ω_m

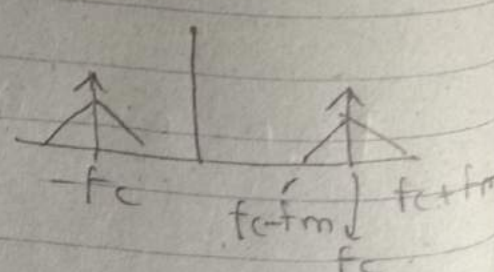
$m(t) \longleftrightarrow$ Spectrum



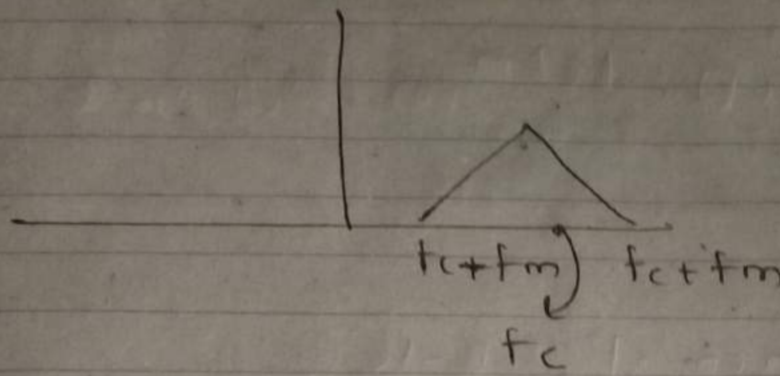
$c(t)$
$(A_c \cos(\omega_c t))$ $\longleftrightarrow$



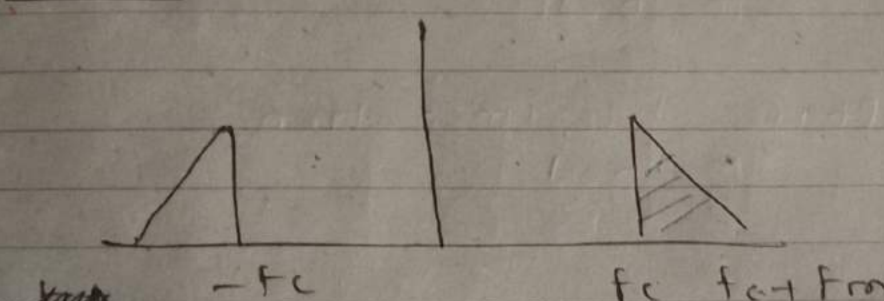
DSB-C $\longleftrightarrow$



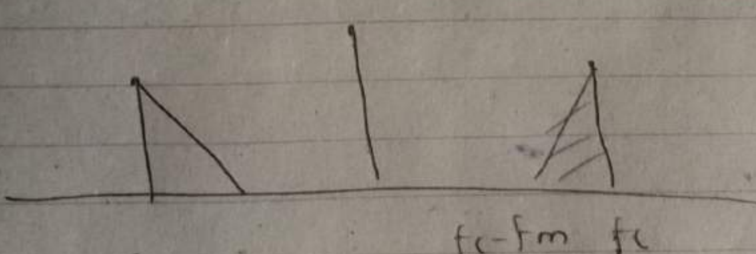
DSB-SC



SSB-SC



OR $\boxed{\begin{aligned} B(\omega) &= f_c + f_m - f_c \\ &= f_m \end{aligned}}$



Therefore,

$$\boxed{x_{SSB-SC}(t) = \frac{A_c A_m}{2} \cos(\omega_c \pm \omega_m) t}$$

Generation of SSB-SC

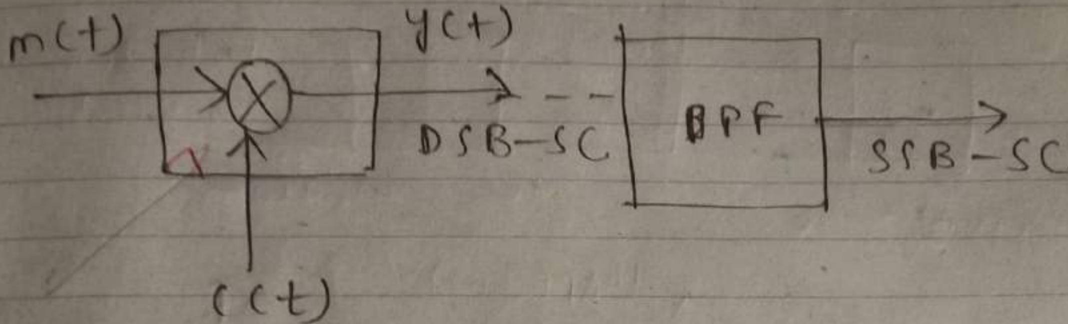
↳ frequency discrimination method

↳ Phase discrimination method

(Generation of SSB-SC)

① Frequency discrimination method

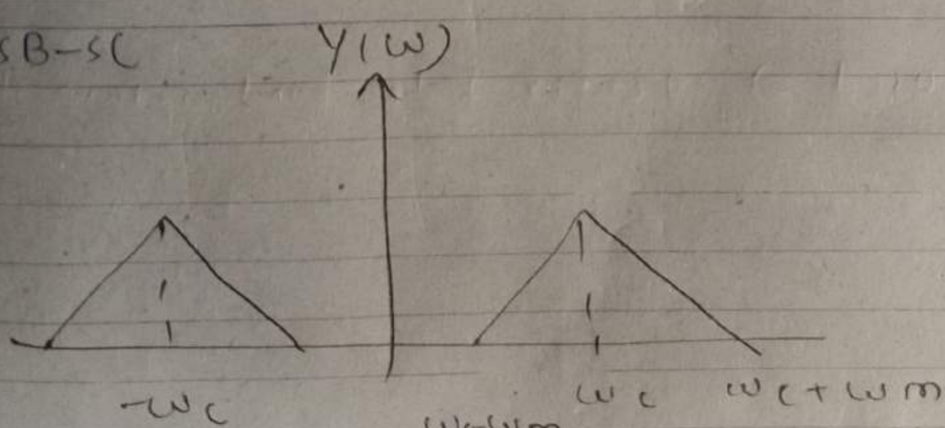
↳ based on frequency components



multiplier
(either Balanced modulator OR ring modulator)

$y(t) \longleftrightarrow Y(\omega)$

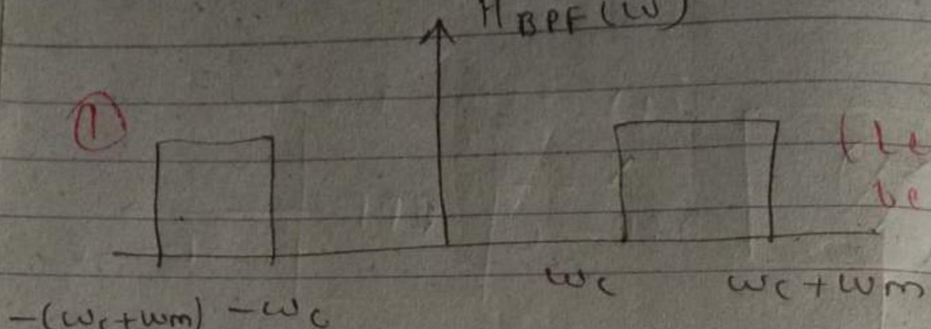
DSB-SC



$H_{BPF}(\omega)$ = frequency response of Band Pass signal

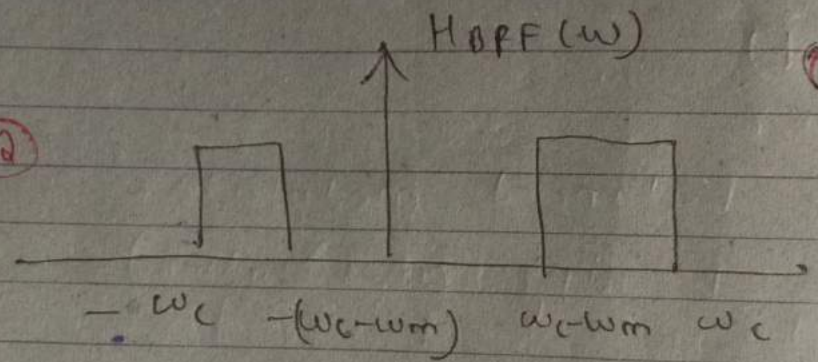
Band-Pass filter:

Any one of the following Band pass filter

1. 

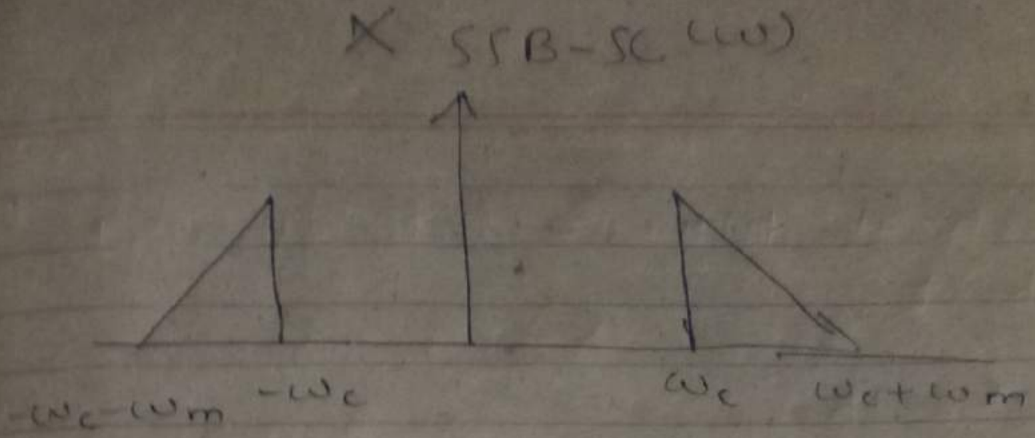
(Let this be used)

OR

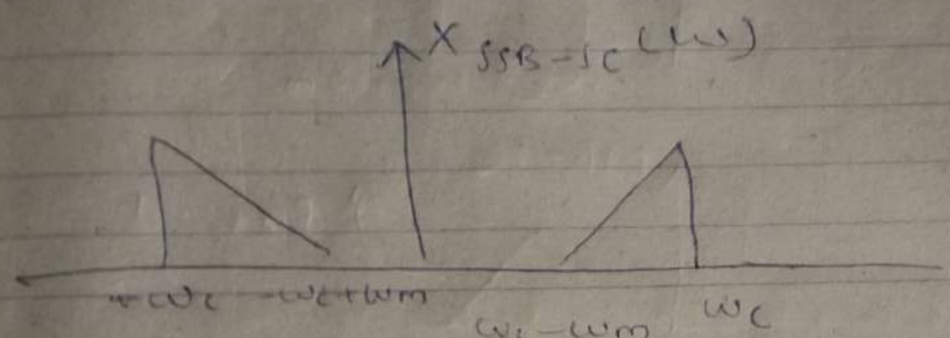
2. 

(i) used ⟹ upper side band retained

(ii) used ⟹ lower side band retained



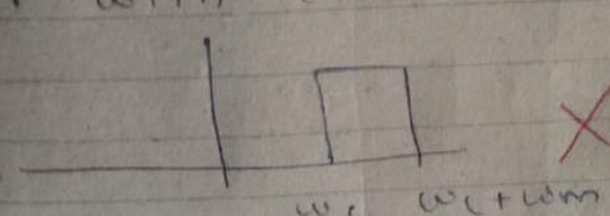
OR



DISADVANTAGE :

Ideal filters practically impossible

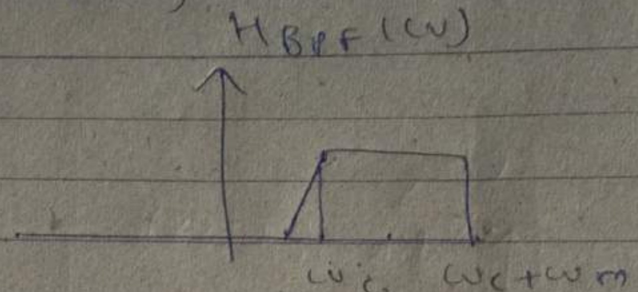
BPF with characteristics

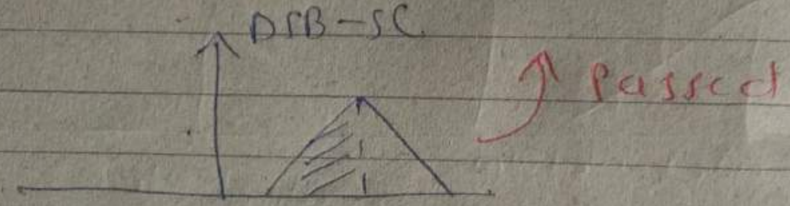


not possible

Practical filters = slope associated with it

like this,





USB passed along with some unwanted part of LSB also passed $\Rightarrow$ leads to distortion of signal

Convolution in time domain
= multiplication in frequency domain

HILBERT TRANSFORM

↳ phase diff. of $\pi/2$ in signal
↳ rotates by 90° the signal I/P

$x(t) \longrightarrow \boxed{HT} \longrightarrow \hat{x}(t)$

$$x(t) \xrightarrow[\text{shift}]{90^\circ \text{ phase}} \hat{x}(t)$$

$$\boxed{\hat{x}(t) = x(t) * \frac{1}{\pi t}}$$

impulse response of hilbert transformer

$$\hat{x}(t) = \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t-\tau)}\, d\tau$$

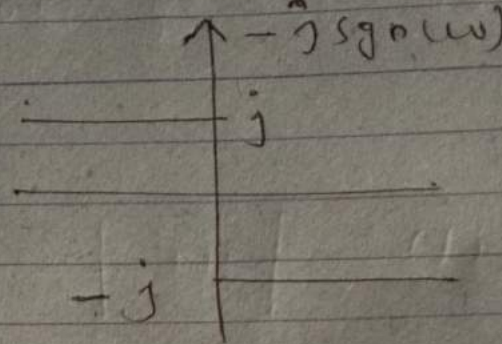
$$\boxed{\hat{X}(\omega) = \cancel{x}\; X(\omega)\,(-j\,\mathrm{sgn}(\omega))}$$

frequency response of hilbert transformer

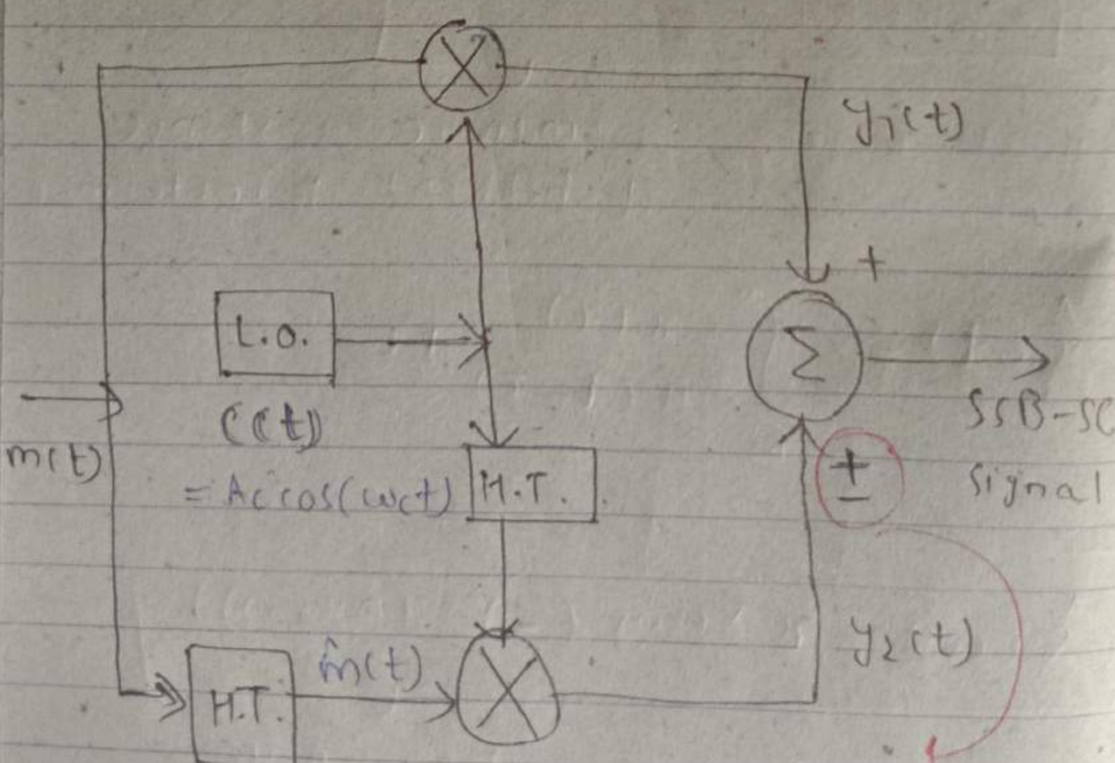
* $\cos(\omega_c t) \xleftrightarrow{HT} \sin(\omega_c t)$

* $\sin(\omega_c t) \xleftrightarrow{HT} -\cos(\omega_c t)$

$-j\,\mathrm{sgn}(\omega) \Longrightarrow$



(2) PHASE DISCRIMINATION METHOD



either

$y_1 - y_2$ OR $y_1 + y_2$

L.O. = Local oscillator generates the carrier signal

$$y_1(t) = A_c\, m(t) \cos(\omega_c t)$$

$$\longleftrightarrow \frac{A_c}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]$$

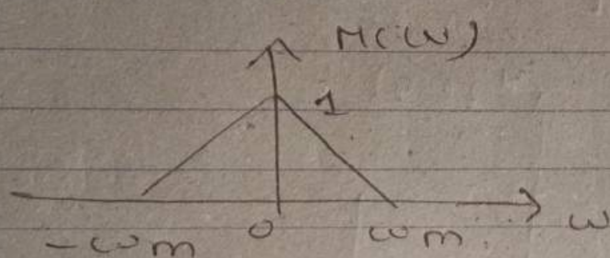
$\hookrightarrow Y_1(\omega)$

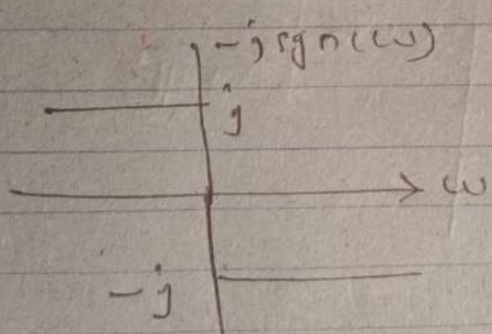
$$y_2(t) = A_c\, \hat{m}(t) \sin(\omega_c t)$$

$$\longleftrightarrow \frac{A_c}{2j}\left[\hat{M}(\omega-\omega_c) \mp \hat{M}(\omega+\omega_c)\right]$$

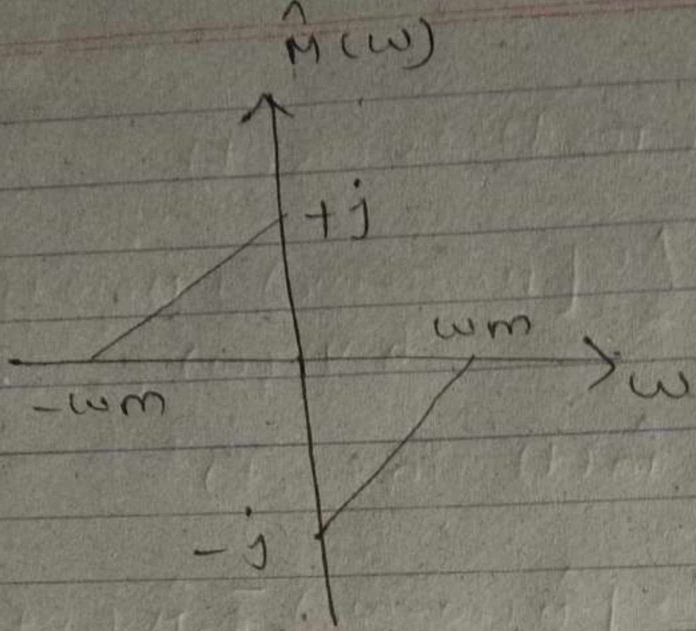
$\hookrightarrow Y_2(\omega)$

Let $m(\omega)$ be:

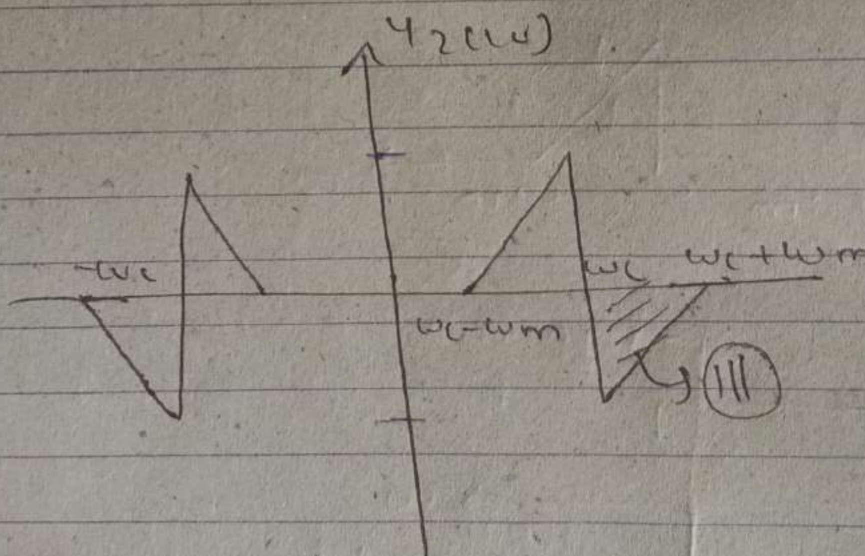




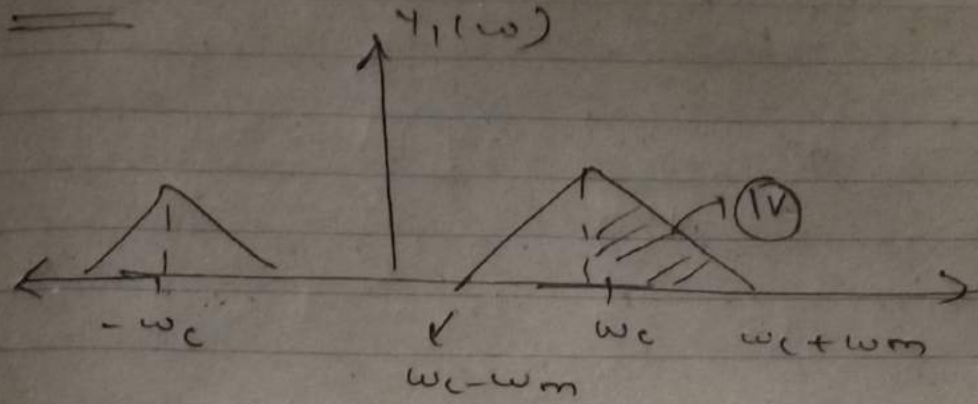
$$\hat{M}(\omega) = M(\omega)\left(-j\,\mathrm{sgn}(\omega)\right)$$



$$Y_2(\omega) = \frac{A_C}{2j}\left[\hat{M}(\omega-\omega_c) - \hat{M}(\omega+\omega_c)\right]$$



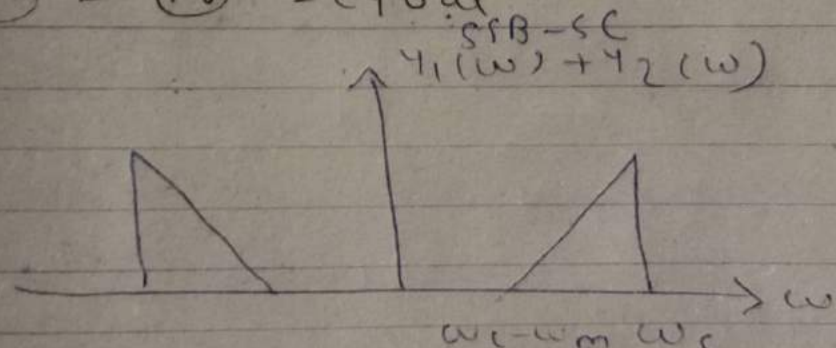
$Y_1(\omega)$



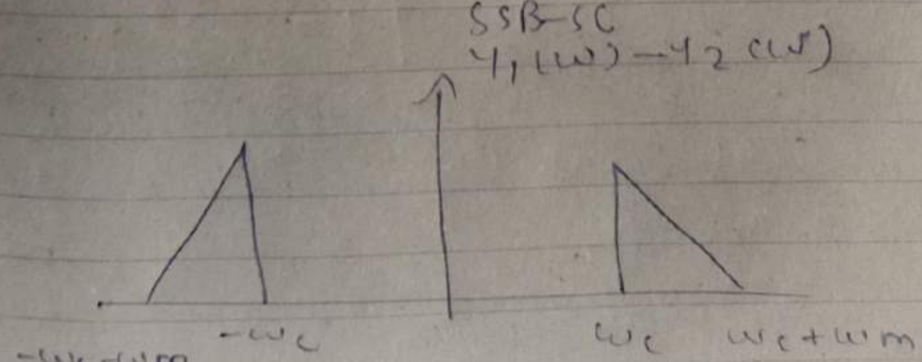
Final steps: either $Y_1(\omega)+Y_2(\omega)$

or $Y_1(\omega)-Y_2(\omega)$

(III) & (IV) = equal



only lower side bands



only upper side bands