The LNMIIT, Jaipur
Electronics and Communication Department
Principles of Communication (ECE-XXX)

Subject Code: ECE-XXX	Course Title: **Principles of Communication**	Total Contact Hours: **40**	L: 0	T: 0	P: 0	C: 3
Pre-requisite: **Signals and Systems**		Year: **2nd**	Semester: **Even**			

*** L → Lectures, T → Tutorials, P → Projects C → Credit*

Learning Objective:

This course deals with the basic principles of analog communication techniques. The emphasis is on discussions of both linear and non-linear modulation techniques like amplitude modulation, frequency modulation, etc. In the first term (of about 10 hours) the analytical background needed for studying these methods is prepared. The second term and part of the third term is spent on discussions of the various angle modulation techniques. Understand the process of digitizing the analog signals and digital transmission of analog signals is discussed in the remaining part of the semester.

Course outcomes (COs):

On completion of this course, the students will have the ability:		Bloom's Level
CO-1	**Describe** random variables and processes. **Apply** basic mathematical tools and analytical background in communication systems.	2,3
CO-2	**Outline** and **analyze** the working principles of different linear modulation techniques (AM and its different versions) and associated demodulators	1,4
CO-3	**Outline** and **analyze** the working principles of different non-linear modulation techniques (FM and PM) and associated demodulators	1,4
CO-4	**Determine** and **critically analyze** the performance of different modulation techniques in terms of power utilization, bandwidth requirement, complexity of modulator and demodulator circuits, etc.	3,4
CO-5	**Explain** digital transmission of analog signals and **outline** the process of digitizing the analog signals.	1,2
CO-6	**List** and **illustrate** types of pulse modulation schemes	1,3

Course Topics	Lecture Hours	
UNIT – I (Introduction to Random variable and Processes)		
1.1 Random variables, probability density function, cumulative distribution function, mean, auto-correlation function, cross-correlation function, power spectral density.	02	06
1.2 Transformation of random variables, probability distributions, central limit theorem.	02	
1.3 Random process, classification of random processes, transmission of random process through a linear system.	02	

UNIT – II (Linear Modulation Techniques)		
2.1 Modulation (single tone & multi tone), need of modulation, pre-envelope and complex envelope, representation of band-pass signals, hilbert transform, fourier transforms of some important functions.	**02**	**14**
2.2 Classification of AM techniques, conventional AM technique (DSB-C), generation of DSB-C signals (square law modulator, switching modulator), detection of DSB-C signals (envelope detector), double side-band suppressed carrier (DSB-SC), generation of DSB-SC signals (balanced modulator, ring modulator), synchronous detection of DSB-SC signals.	**05**	
2.3 Single side band (SSB) technique, generation of SSB signals (frequency discrimination method, phase discrimination method), synchronous detection of SSB signals, vestigial side band (VSB) technique.	**05**	
2.4 Receivers : Tuned radio frequency receiver, superhetrodyne receiver, Image frequency.	**02**	
UNIT – III (Non-linear Modulation Techniques)		
3.1 Frequency Modulation (FM) technique, narrowband FM, wideband FM, carson's rule, direct and indirect method to generate FM.	**03**	**10**
3.2 Demodulators for FM: balanced slope detector, ratio detector, foster-seeley discriminator, phase locked loop, application of PLL and VCO in modulating and demodulating the signals, phase modulation (generation and detection).	**05**	
3.3 Pre-emphasis, De-emphasis, Frequency division multiplexing	**02**	
UNIT – IV (Digital Representation of Analog Signals)		
4.1 Introduction, Why Digitize Analog Sources?, The Sampling process, Pulse Amplitude Modulation, Time Division Multiplexing, Pulse width modulation, Pulse-Position Modulation, Generation of PPM Waves, Detection of PPM Waves.	**04**	**10**
4.2 The Quantization Process, Quantization Noise, Pulse– Code Modulation: Sampling, Quantization, Encoding, Regeneration.	**04**	
4.3 Differential PCM, Delta Modulation, Adaptive Delta Modulation (ADM)	**02**	

Text Books:

1. *Principles of Communication Systems*, Herbert Taub, Donald L. Schilling, and Gautam Saha, McGraw Hill, New York, 4th Ed., 2013.
2. *Modern Digital and Analog Communication Systems*, B. P. Lathi, Oxford University Press, 3rd Ed.
3. *Communication Systems*, Simon Haykin, John Wiley Publications, 4th Ed.

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Reference Books:

1. *Communication Systems*, A. Bruce Carlson and Paul B. Crilly, McGraw Hill, New York, 5th Ed., 2011.

Additional Resources (NPTEL, MIT Video Lectures, Web resources etc.): NA

Evaluation Methods:	
Item	Weightage
Quiz 1	15
Quiz 2	15
Mid-term Examination	30
End-term Examination	40

Please note, as per the notice circulated in the ECE department on 5th march 2018 students having attendance less than 60% will not be allowed to sit in the final examination.

CO and PO Correlation Matrix

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO 1	3	2	3	1					2	1		3	3	1	
CO 2	3	3	1	1					2	1		3	3	1	
CO 3	3	3	1	1					2	1		3	3	1	
CO 4	3	3		3					2	1		3	3	1	
CO 5	3	2	1						2	1		3	3	2	
CO 6	3	1	1						2	1		3	3	2	

Last Updated On: 18-11-2020

Updated By: Dr. Nikhil Sharma

Approved By:

BASIC'S OF COMMUNICATION SYSTEM

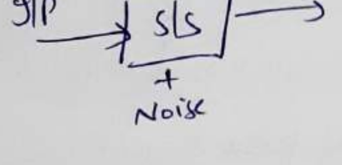
Signal :- Signal is a function of independent variable that contains some information.

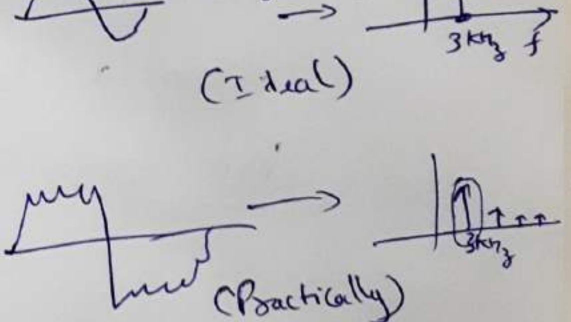
Communication :- It is the process of Exchanging information in the form of signals.

Time domain & Freq. domain Analysis of Signal

- Advantages of Freq. domain Analysis
  - → (1) System designing becomes Easy (Hardware Implementation).
  - → (2) Analysis of Signal propagating through a System is Easy.
  - → (3) Easier to implement necessary mathematical operations.
  - → (4) Can predict the Stability of system.

Eg:

(1) 



Freq. domain analysis helps in filtering process.

(2) Audible freq. range :- 20Hz - 20KHz

→ Humans speak in range of 500Hz, so microphone tuned to 200 - 800Hz.

Transmitter → Channel (wired, wireless) → Receiver

ak

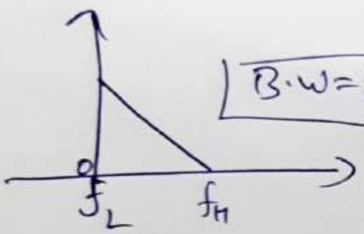
- Message can be Analog or Digital.
- **Transmitter** :- **Transducer** + **Signal Pre-processing** + **Modulation**
- Message / information signal has low frequency and is a weak signal cannot be transmitted to a longer distance.
- **MODULATION** :- Process of superimposing the information of a baseband modulating signal (message) on a high freq carrier signal by altering its characteristics (Amp., freq., Phase).

$$c(t) = A_c \cos(\omega_c t + \theta)$$

→ Modulation is basically frequency translation from a low frequency baseband signal to a high frequency band pass signal.

Baseband Signal

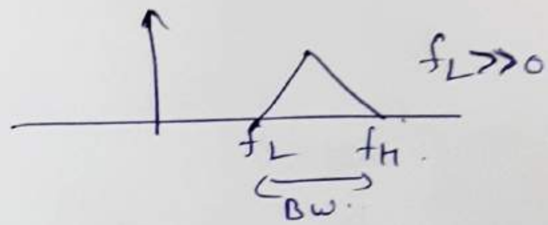
- A low frequency signal having lower cutoff freq. either '0' or very close to zero.



$\boxed{B.W = f_H - f_L}$

Band Pass Signal

- It represents a high freq. Signal.
- It Exists in a range of frequencies
- Lower Cutoff freq. is much greater than zero.



NARROW BAND SIGNAL

$f_H \gg B$

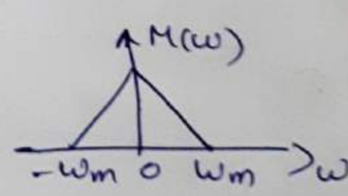
$\frac{f_H}{B} \gg 1$

WIDE BAND SIGNAL

$\frac{f_H}{B} \approx 1$

⟹ MODULATION THEOREM

$m(t) \rightarrow$ message signal

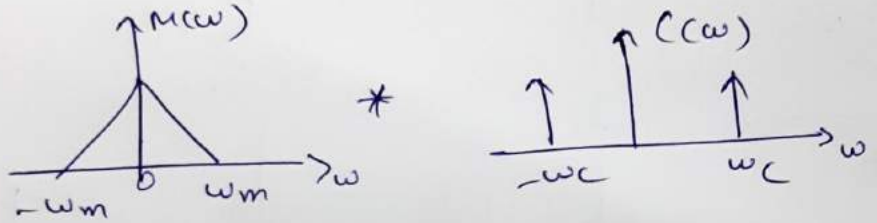


$C(t) \rightarrow$ Carrier Signal $= A_c \cos \omega_c t$

$$m(t) \cdot c(t) \longleftrightarrow M(\omega) * C(\omega)$$

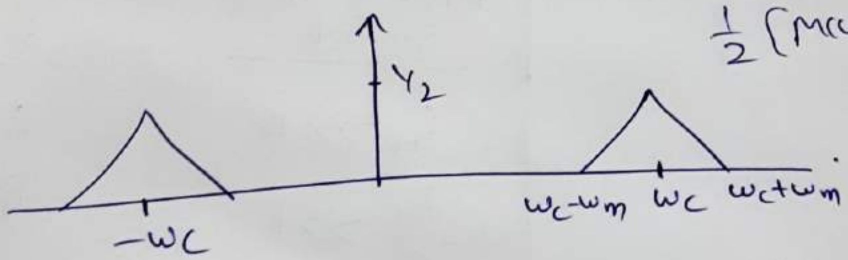
$$M(\omega) * \frac{1}{2}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

$$\frac{1}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]$$



$\Downarrow$

$$\frac{1}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]$$

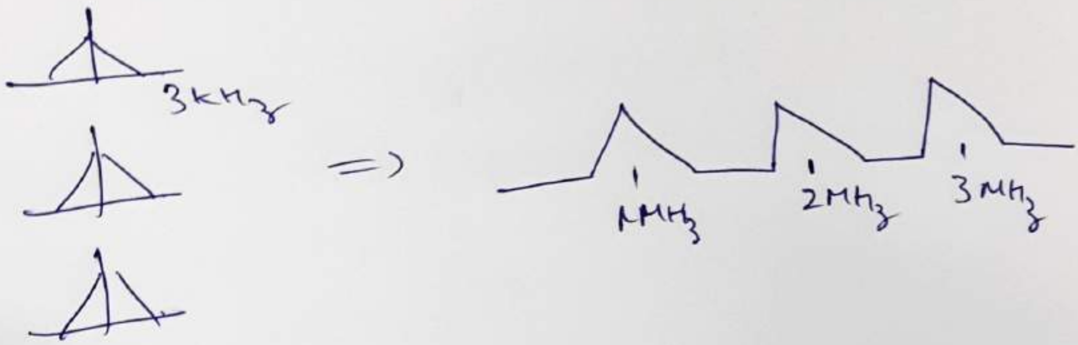


$\Longrightarrow$ NEED OF MODULATION

① Height of Antenna :- $\left\{ \lambda = \frac{c}{f} \right.$, $h \propto$

Eg. $f = 15\,KHz$, $h \approx 5\,Km$ | $f = 15\,MHz$, $h \approx 5\,m$

(2) Avoids mixing of Signal

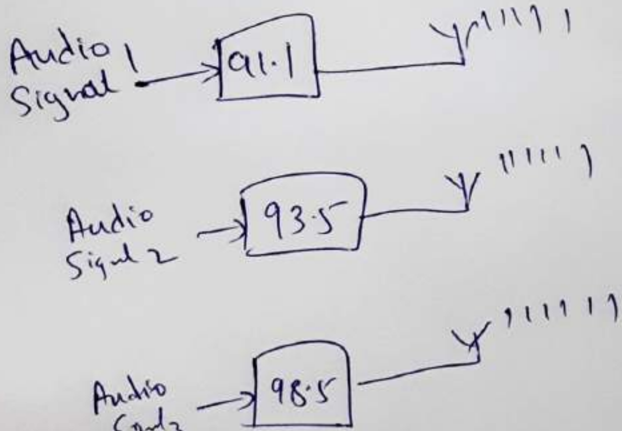


(3) Increase Range of Communication.

(4) Improves quality of Reception

$\left(P \propto \frac{1}{\lambda^2}\right)$, $(f \propto \text{Energy})$

(5) Allow's multiplexing



Some Important Transforms

$$\boxed{\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \\ x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega \end{aligned}}$$

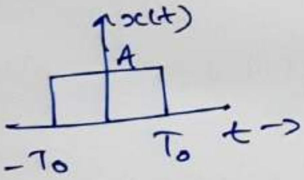
1) $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

2) $e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+\omega^2}$

3) $\delta(t) \longleftrightarrow 1$

4) $1 \longleftrightarrow 2\pi\,\delta(\omega)$

5)



$x(t) = A\,\mathrm{rect}\left(\frac{t}{2T_0}\right) \longleftrightarrow 2AT_0\,\mathrm{Sinc}\left(\frac{\omega T_0}{\pi}\right)$

6) $\cos\omega_0 t \longleftrightarrow \pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$.

$\longleftrightarrow \frac{1}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$.

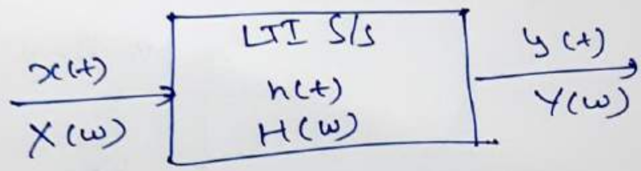
7) $\sin\omega_0 t \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$

$\frac{1}{2j}\left[\delta(f-f_0) - \delta(f+f_0)\right]$.

8) $\operatorname{Sgn}(t) \longleftrightarrow \dfrac{2}{j\omega}$.

9) $u(t) \longleftrightarrow \pi\,\delta(\omega) + \dfrac{1}{j\omega}$.

⟹ Signal Transmission through LTI S/S.

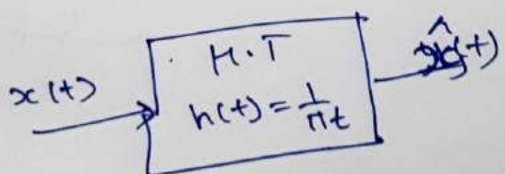


$$\boxed{\begin{aligned} y(t) &= x(t) * h(t) \\ Y(\omega) &= X(\omega)\cdot H(\omega). \end{aligned}}$$

$h(t) \rightarrow$ Impulse Response

⟹ HILBERT Transform.

- Provides a phase shift of 90°



$$\boxed{\begin{aligned} \hat{x}(t) &= x(t) * \frac{1}{\pi t} \\ \hat{X}(\omega) &= X(\omega)\,(-j\,\operatorname{sgn}(\omega)) \end{aligned}}$$

Q Find the F.T. of $g(t) = \frac{1}{\pi t}$?

Sol^n

$$\text{Sgn}(t) \longleftrightarrow \frac{2}{j\omega}$$

$$\frac{j}{2\pi}\,\text{Sgn}(t) \longleftrightarrow \frac{1}{\pi\omega}$$

$$x(t) \longleftrightarrow X(\omega)$$

$$\frac{1}{\pi t} \longleftrightarrow 2\pi\, x(-\omega)$$

$$\longleftrightarrow \cancel{2\pi}\, \frac{j}{\cancel{2\pi}}\, \text{Sgn}(-\omega)$$

$$\boxed{\frac{1}{\pi t} \longleftrightarrow -j\,\text{Sgn}(\omega)}$$

Q $g(t) = A\,\text{Sinc}\left(\frac{t}{2\pi}\right)$, $G(\omega) = ?$

$$\Rightarrow \quad A\,\text{rect}\left(\frac{t}{2T_0}\right) \longleftrightarrow 2AT_0\,\text{Sinc}\left(\frac{\omega T_0}{\pi}\right)$$

Substituting $2T_0 = 1$

$$A\,\text{rect}(t) \longleftrightarrow A\,\text{Sinc}\left(\frac{\omega}{2\pi}\right)$$

$$x(t) \longleftrightarrow X(\omega)$$

$$A\,\text{Sinc}\left(\frac{t}{2\pi}\right) \longleftrightarrow 2\pi\, x(-\omega) = 2\pi A\,\text{rect}(-\omega)$$

$$= 2\pi A\,\text{rect}(\omega).$$

POC (Lec 1) (Concepts)

Basics of communication system

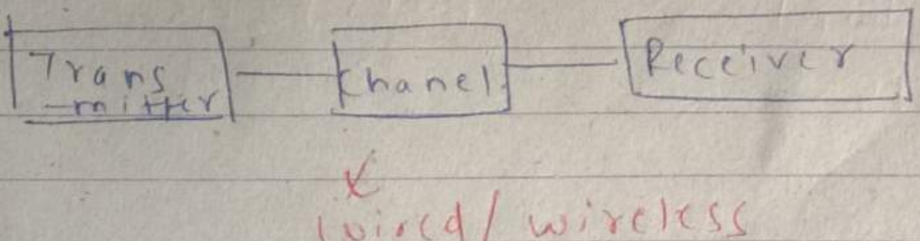
signal = func. of independent variable

communication = exchange of info. in form of signals

when noise added to signal = several frequency components are there in frequency domain

Audible freq range = 20 Hz to 20 kHz
but speech components = 500 Hz vicinity
microphone $\Rightarrow$ range designed from 500 Hz to 800 Hz

Basic system model



MODULATION = frequency translation

message signal = low frequency signal
weak signal ⇒ cannot be transmitted
over long ~~[illegible]~~ distances

m(t) = low freq
= low energy
= low power

c(t) = high freq signal

$\boxed{E \propto f}$ (Energy ∝ frequency)

superimposing m(t) over c(t) = modulation

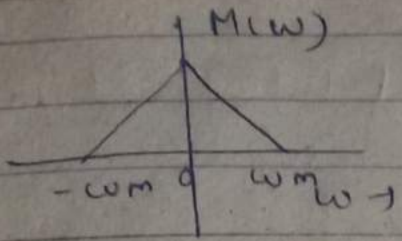
After modulation = modified c(t)

Baseband signal — Band pass signal

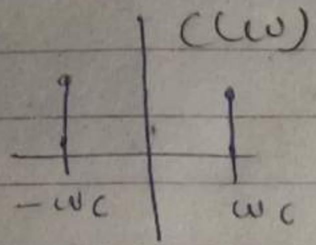
m(t)
↓ modulation
Band pass signal

$*$ = convolution

MODULATION THEOREM



$$m(t)\,c(t) = M(\omega) * C(\omega)$$



NEED OF MODULATION

$\downarrow \lambda = \frac{c}{f} \uparrow$

$\downarrow h \propto \frac{\lambda}{4} \downarrow$

91.1, 93.5 MHz = carrier frequency

SOME IMPORTANT TRANSFORM

↳ fourier transform

HILBERT TRANSFORM

↳ shift the phase of I/P signal by 90°

Hilbert transformer

$\Downarrow$

LTI system

$$\hat{x}(t) = x(t) * \left(\frac{1}{\pi t}\right)$$

$\Downarrow$

impulse response of hilbert transformer

$$\hat{X}(\omega) = X(\omega) \cdot (-j\,\mathrm{sgn}(\omega))$$

~~Hilb~~ Hilbert transform = cos function of sine function

Lec 2 (Concepts)

BAND PASS & LOW PASS SIGNAL REPRESENTATION

Bandpass signal = high freq modulated signal

Baseband signal = low freq = low pass signal
(message signal)

Any bandpass signal can be represented in terms of complex low frequency signal

↳ low pass equivalent of original band pass signal

high frequency signal = high sampling rates reqd.

ANALYTIC SIGNAL REPRESENTATION

* much simpler to work with complex exponentials of sinusoid signal

sine/cosine ⟶ complex exponential

$e^{ix} = \cos x + i \sin x$

* $x(t)$ = real valued signal.

Its complex analytic representation will be:

$$\boxed{x_p(t) = x(t) + j\hat{x}(t)}$$

$\hat{x}(t)$ = Hilbert transform of $x(t)$

pre envelope / complex envelop

(Q1) $x(t) = \sin \omega_0 t$

$\hat{x}(t) = -\cos \omega_0 t$

$x_p(t) = \sin \omega_0 t + j(-\cos \omega_0 t)$

$= e^{j\omega_0 t}$

$$\cancel{-\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2}\right] - j\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]}$$

$$\cancel{= \frac{1}{2}\left[e^{j\omega_0 t} - j e^{j\omega_0 t}\right.}$$

$e^{i\theta} = \cos\theta + i \sin\theta$

$\sin(\omega_0 t) \xrightarrow{HT} -\cos(\omega_0 t)$

$\cos(\omega_0 t) \xrightarrow{HT} \sin(\omega_0 t)$

HT = Hilbert Transform

$x_p(t) = \cancel{\sin(\omega_0 t) + j(-\cos(\omega_0 t))}$

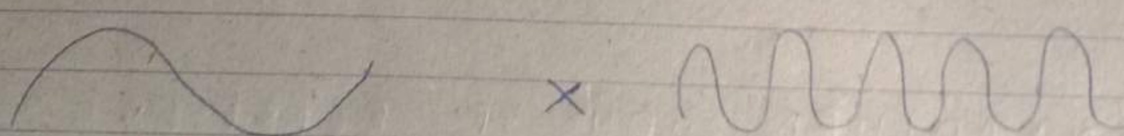
$= \sin(\omega_0 t) + j(-\cos(\omega_0 t))$

$= e^{j\omega_0 t}$

CONCEPT OF ENVELOPE

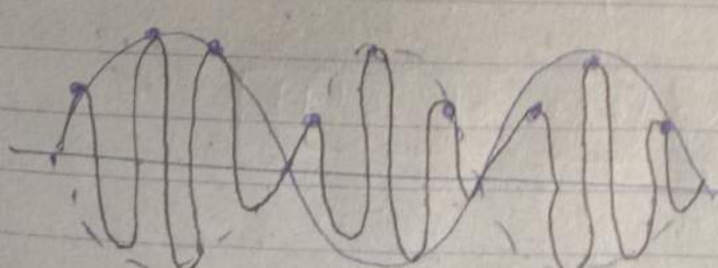
$(\underbrace{m(t)})(\cos(\omega_0 t + \theta))$

message signal × cosine signal (carrier signal)



$m(t)$ — Low pass signal

$\cos(\omega_0 t + \theta)$ — High pass signal



↳ frequency $= \omega_0$

↳ amplitude = governed by $m(t)$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Amplitude at any time instant "t"

$= m(t)\left[\cos\theta \cos(\omega_0 t) - \sin\theta \sin(\omega_0 t)\right]$

Amplitude $= \sqrt{m^2(t)\cos^2\theta + m^2(t)\sin^2\theta}$

$\boxed{\text{Amplitude} = |m(t)|}$

$|m(t)|$ = Envelope = Trace of peak amplitude with time

GENERALISED WAY OF REPRESENTING ENVELOPE OF ANY ARBITRARY SIGNAL

$x_p(t) = x(t) + j\hat{x}(t)$

$\boxed{|x_p(t)| = \text{Envelope of } x(t)}$

SPECTRUM OF $x_p(t)$

$x_p(t) = x(t) + j\hat{x}(t)$

$X_p(\omega) = X(\omega) + j\hat{X}(\omega)$

$\hat{X}(\omega) = -j\,\text{sign}(\omega)X(\omega)$

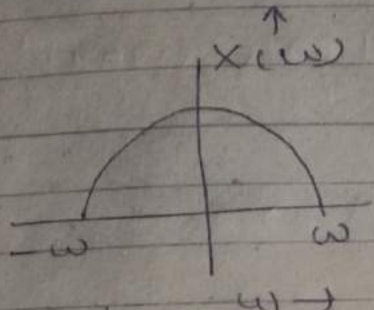
$x_p(\omega) = X(\omega) + X(\omega)\,sgn(\omega)$

$x_p(\omega) = X(\omega)[1 + sgn(\omega)]$

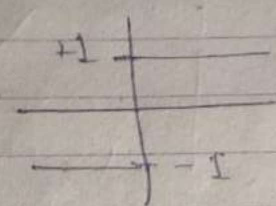
$$x_p(\omega) = \begin{cases} 2X(\omega), & \omega > 0 \\ 0, & \omega < 0 \end{cases}$$

one sided spectrum with positive freq components

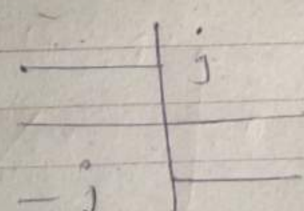
↳ spectrum of ~~[illegible]~~ $x_p(t)$

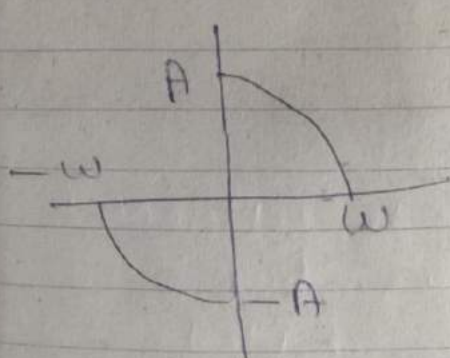


$\hat{X}(\omega) = -j\,sgn(\omega)\,X(\omega)$



|| $-j\,sgn(\omega)$



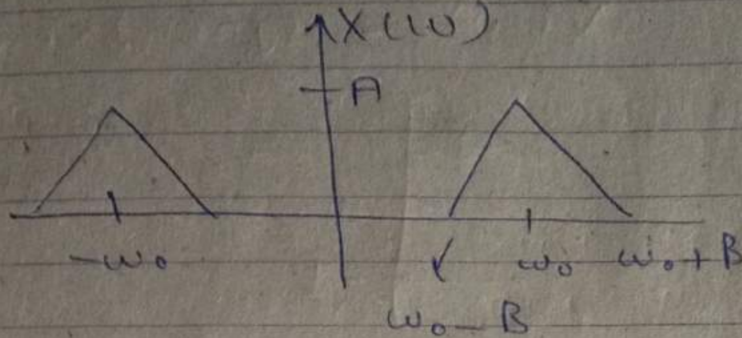


$\Rightarrow 2X(\omega),\ \omega > 0$

$0,\ \omega < 0$

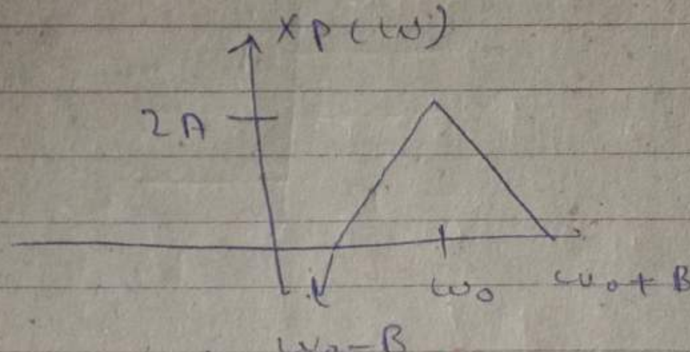
COMPLEX ENVELOPE REPRESENTATION ~~REAL~~ OF BANDPASS SIGNAL

$x(t) \longleftrightarrow X(\omega)$ "Band pass signal"

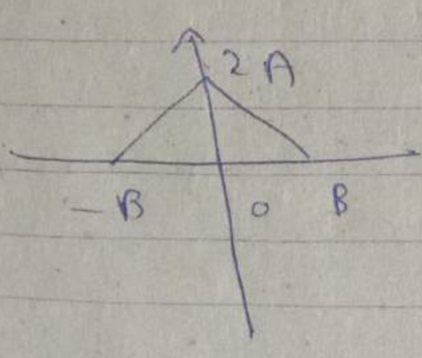


$\omega_0 >> 0$

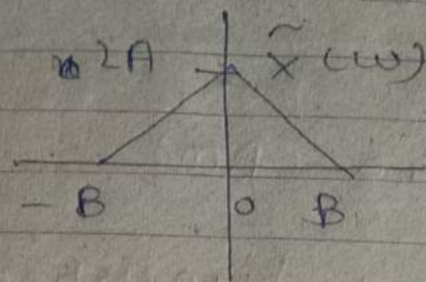
$\Downarrow$



It can be represented as freq shifted version of some low pass signal



Suppose



$\tilde{x}(t) \Rightarrow \tilde{X}(\omega)$

$\Downarrow$

complex valued low pass signal

$$\boxed{x_p(t) = \tilde{x}(t) e^{j\omega_0 t}}$$

complex envelope representation

$X_p(\omega) = \tilde{X}(\omega - \omega_0)$

Low pass equivalent of Band pass signal

$x(t)$ = original Band pass signal

$x(t) = Re\{x_p(t)\}$

$\tilde{x}(t) = x_I(t) + j x_Q(t)$

Inphase component

Quadrature phase component

$$\boxed{x(t) = Re\{\tilde{x}(t) e^{j\omega_0 t}\}}$$

variation in amplitude varies linearly

Lec 3 (Introduction to DSB-C)

UNIT-2 (LINEAR MODULATION TECHNIQUES)

Amplitude modulation (AM)

* modulation = freq translation

= superimposing a low frequency message signal over high freq carrier signal by varying critical parameters of c(t)

like A_c, ω_c or θ_c

AM

- message signal (low freq Base band signal)
- carrier signal (High freq signal)
- modulated signal (High freq Band pass signal)

TYPE OF AM:

1. Conventional AM (DSB-C)
   (Double side band with carrier)

2. DSB-SC (suppressed carrier)

3. SSB-SC (~~suppressed~~ single side band - supressed carrier)

4. VSB (vestigial side band)

1) Conventional AM (DSB-C)

* $m(t)$, $c(t) = A_c \cos(\omega_c t)$

* single tone = message signal has single frequency component (ω_m)

Amplitude of carrier signal varied in accordance to instanstaneous value of message signal

$$\boxed{x_{AM}(t) = \left(A_c + m(t)\right)\cos(\omega_c t)}$$

* Amplitude change ↳ eqn (1)
* Phase same
* freq same

$$= A_c \cos(\omega_c t) + m(t)\cos(\omega_c t)$$

let $m(t) = A_m \cos(\omega_m t)$ {sinusoid}

$$c(t) = A_c \cos(\omega_c t)$$

so Now, replacing $m(t)$ in (1)

$$\boxed{x_{AM}(t) = A_c\left[1 + \frac{A_m}{A_c}\cos(\omega_m t)\right]\cos(\omega_c t)}$$

$$\boxed{\frac{A_m}{A_c} = \mu = \text{modulation index}}$$

✓

gives you the measure of modulation or the extend of modulation

$X_{Am}(t)$ = Amplitude modulated signal

$X_{Am}(t) = \cancel{A_c \ldots}$

$= A_c \cos(\omega_c t) + A_c \mu \cos(\omega_m t) \cos(\omega_c t)$

Conventional AM in time domain

$$\boxed{X_{Am}(t) = \underbrace{A_c \cos(\omega_c t)}_{\text{carrier}} + \frac{A_c \mu}{2} \underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \frac{A_c \mu}{2} \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}}}$$

USB = Upper side band

LSB = Lower side band

three components

* carrier
* upper side band
* lower side band

To get more useful information, we find the fourier transform of $X_{Am}(t)$ i.e,

$$X_{Am}(t) \longleftrightarrow \underbrace{X_{Am}(\omega)}_{\text{we find this}}$$

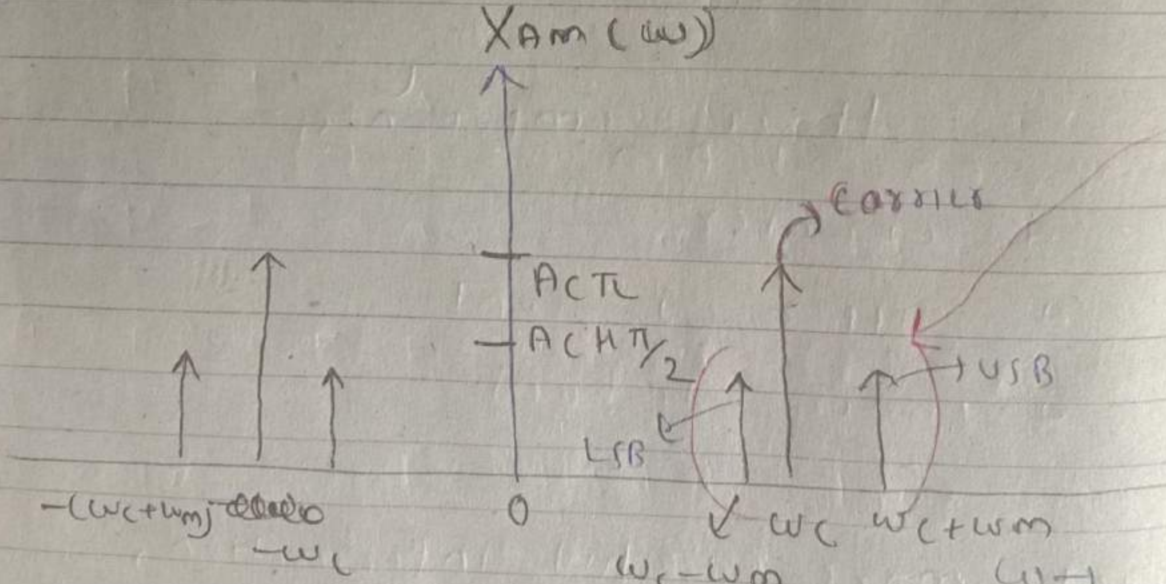
$$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)]$$

$$X_{AM}(\omega) = \left(A_c \pi[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)]\right)$$

$$+ \left(\frac{A_c \mu \pi}{2}[\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))]\right)$$

$$+ \left(\frac{A_c \mu \pi}{2}[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))]\right)$$

Plotting the spectrum →



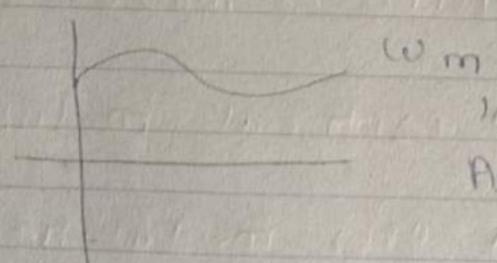
$\mu \in (0,1)$

(Band of freq. in which major signal energy present)

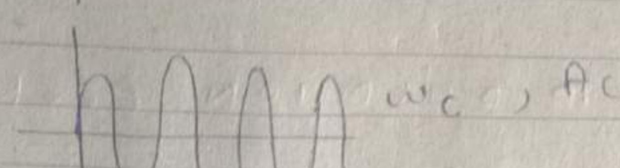
Bandwidth $= (\omega_c + \omega_m) - (\omega_c - \omega_m)$

$\boxed{\text{Bandwidth} = 2\omega_m}$

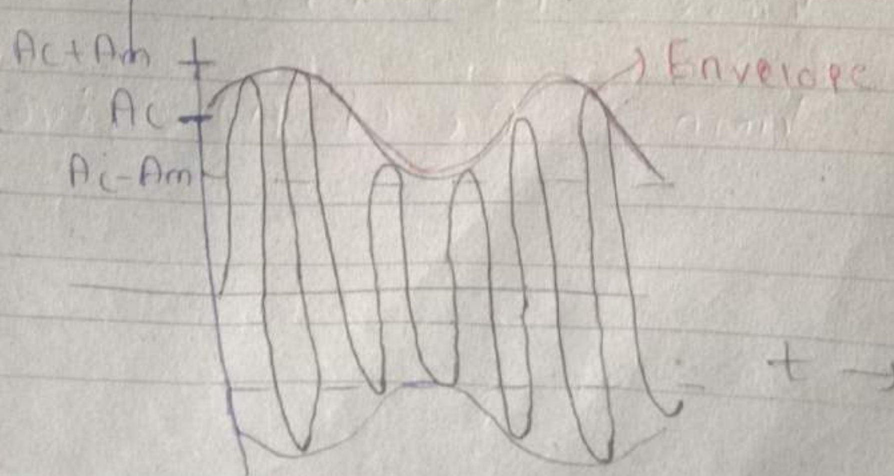
$m(t) =$



$c(t) =$



$x_{Am}(t) =$



V_{min} = minimum amplitude
V_{max} = maximum Amplitude

$$V_{max} = A_c + A_m$$

$$V_{min} = A_c - A_m$$

$$\mu = \frac{A_m}{A_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$V_{max} = A_c + A_m = A_c[1 + \mu]$$

$$V_{min} = A_c - A_m = A_c[1 - \mu]$$

* $0 < \mu < 1$ → under modulation

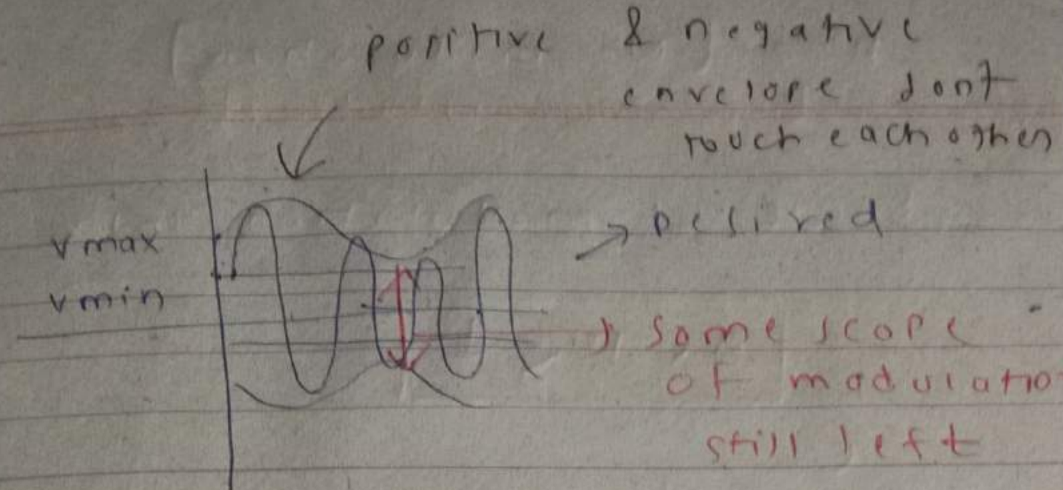
$\mu = 1$ → critical modulation

$\mu > 1$ → over modulation

① UNDER MODULATION $(0 < \mu < 1)$

$V_{max} = A_c(1 + \mu) = +ive$

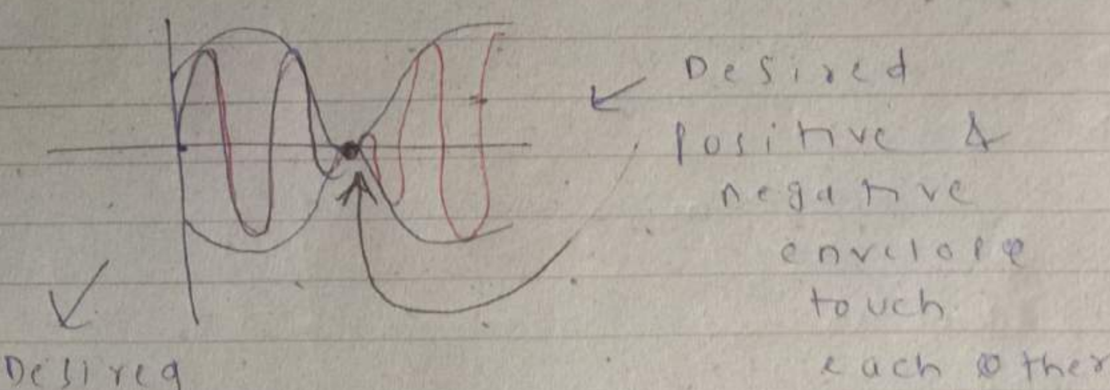
$V_{min} = A_c(1 - \mu) = +ive$



② CRITICAL MODULATION ($\mu = 1$)

$V_{max} = A_c(1+\mu) = 2A_c$

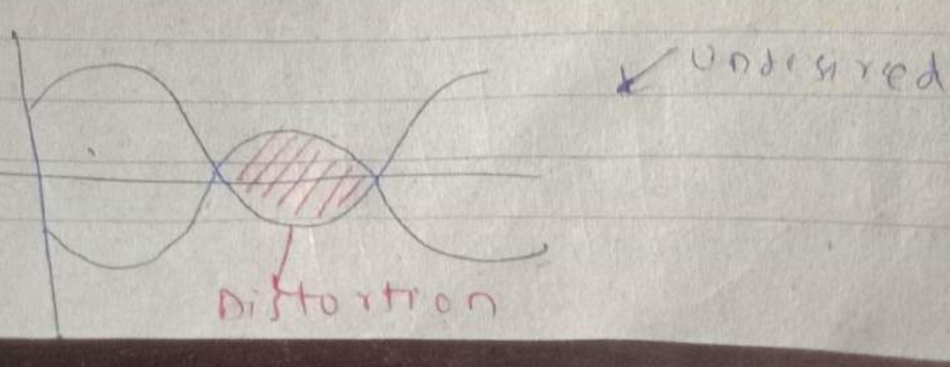
$V_{min} = A_c(1-\mu) = 0$



③ over modulation ($\mu > 1$)

$V_{max} = A_c(1+\mu) = +ve$

$V_{min} = A_c(1-\mu) = -ive$



$\mu = 0$ [No modulation]
↳ so we exclude it

∴ Range of modulation index

$$\boxed{0 < \mu \leq 1}$$

It can be more than 1, but that is undesired.

Lec 4 (Parameters of DSB-C)

Conventional AM / DSB-C

$$x_{AM}(t) = A_c \cos(\omega_c t) + \frac{A_c \mu}{2} \cos(\omega_m + \omega_c)t + \frac{A_c \mu}{2} \cos(\omega_c - \omega_m)t$$

(1) **Transmitted power ($x_{AM}(t)$):**

$$P_t = P_c + P_{USB} + P_{LSB}$$

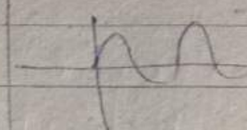
$$= \frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{8} + \frac{A_c^2 \mu^2}{8}$$

$$P_t = \frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{4}$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{P_t = P_c + P_c \frac{\mu^2}{2}}$$



$A_c \cos\theta$

sinusoidal signal

power $= \frac{A^2}{2}$

Lot of wastage since P_c is there since it ~~cannot~~ does not store any useful information

Ex) Let $\mu = 1$ (Critical modulation)

$$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$P_t = \frac{3}{2} P_c$$

$$P_c = \frac{2}{3} P_t = 0.66\, P_t \times 100$$
$$= 66\% \text{ of } P_t$$

wasting 66% of power since carrier does not contain any useful information

(2) Power efficiency:

$$\eta = \frac{\text{(useful power)}}{\text{Total power}} \times 100$$

→ power in side bands

$$= \frac{P_{USB} + P_{LSB}}{P_t} \times 100$$

$$= \frac{\frac{A_c^2 \mu^2}{4}}{\frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right)} = \frac{\mu^2}{2} \times \frac{2}{2 + \mu^2}$$

$$= \frac{\mu^2}{2 + \mu^2} \times 100$$

$$\boxed{\eta = \frac{\mu^2}{2 + \mu^2} \times 100}$$

(3) B

(4) m

MU

in

i.e,

(3) Bandwidth

$$BW = (\omega_c + \omega_m) - (\omega_c - \omega_m)$$

$$= 2\omega_m$$

↳ max frequency component in message signal

(4) modulation index (μ)

$$\mu = \frac{A_m}{A_c}$$

$$\mu \in (0, 1]$$

MULTITONE AM (DSB-C)

↳ more than 1 freq component in message signal

i.e, $m(t) = A_{m_1}\cos(\omega_{m_1}t)$

$$+ A_{m_2}\cos(\omega_{m_2}t)$$

$(A_c + m(t)) \cos(\omega_c t)$

$A_c [1 + \mu \cos(\omega_m t)] \cos(\omega_c t)$

$$X_{AM}(t) = A_c [1 + \mu_1 \cos(\omega_{m_1} t) + \mu_2 \cos(\omega_{m_2} t)] \cos(\omega_c t)$$

Total modulation index:

$$\mu_T = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\mu_1 = \frac{A_{m_1}}{A_c}$$

$$\mu_2 = \frac{A_{m_2}}{A_c}$$

Individual modulation index

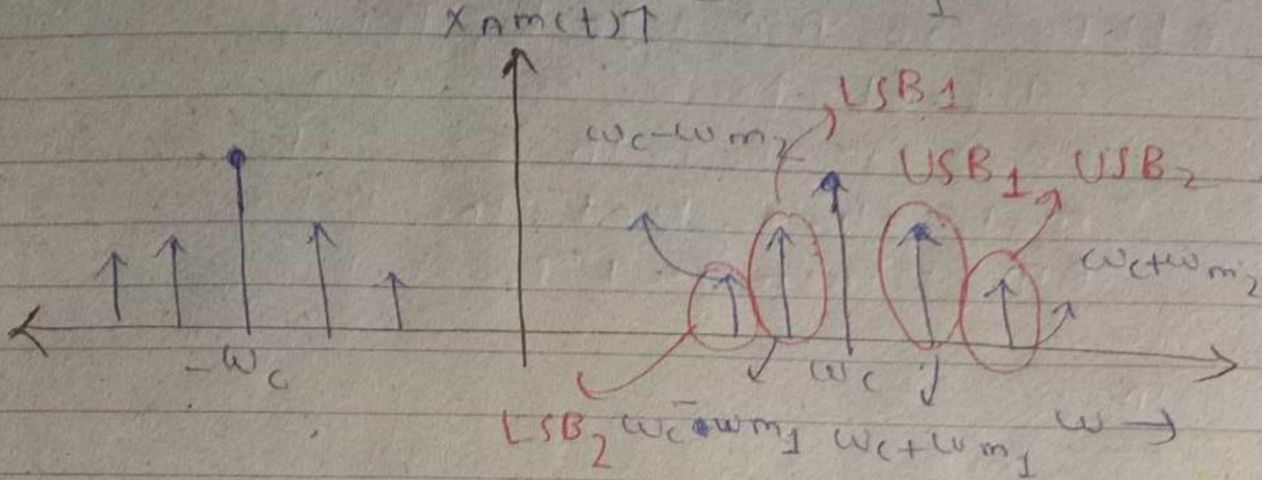
$$P_T = P_c \left[1 + \frac{\mu_T^2}{2}\right]$$

SPECTRUM OF SIGNAL:

↳ instead of single pair of ~~side~~ sidebands, we have 2 pairs of sidebands

Let $A_{m_1} > A_{m_2}$

$\cancel{\omega_{m_1} > \omega}$, $\omega_{m_2} > \omega_{m_1}$



$\boxed{\text{no. of freq component increases} \iff \text{no. of sideband pairs increases}}$

$\boxed{n \text{ frequency components} = n \text{ sideband pairs}}$

whenever given in question, Am signal & type of Am signal not specified, then assume it to be DSB-C

NUMERICALS ON DSB-C :

Q1) For an Am signal, total sideband power is given by ~~100~~ 100 W with 50% of modulation. Find the total Am transmitted power ?

Soln:

My Solution:

$P_{LSB} + P_{USB} = 100W = P$

$\mu = \frac{50}{100} = \frac{1}{2} = 0.5$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$ $\quad P_t = P_c + (P)$

$P_c = P_t - P$

$P_t = (P_t - P)\left(1 + \frac{\mu^2}{2}\right)$

$P_t = (P_t - P)$ ~~(...)~~ 1.125

$P_t = 1.125P_t - 1.125P$

$P_t(0.125) = 1.125 \times 100$

$P_t = 900W$

Sir solution :

$P_{SB} = P_{USB} + P_{LSB} = 100\ W$

$\mu = 0.5$

$P_t = ?$

$P_t = P_c \left(1 + \frac{\mu^2}{2}\right)$

$P_{SB} = P_{USB} + P_{LSB} = \frac{A_c^2 \mu^2}{4}$ — (I)

$P_c = \frac{A_c^2}{2}$ — (II)

from (I) & (II),

$P_{SB} = \frac{A_c^2}{2} \cdot \frac{1}{2} \mu^2 = \frac{P_c \mu^2}{2}$

$P_c = \frac{2 \times P_{SB}}{\mu^2} = \frac{2 \times 100}{(0.5)^2} = 800\ W$

$P_t = P_c + P_{SB} = 800\ W + 100\ W$

$= 900\ W$ consuming huge Power which is not reqd.

MS = My solution

SS = Sir solution

Q2) For an AM each of the SB power is given by 2kW and carrier power is given by 8kW. Find % of modulation?

Soln:

MS:

$P_{LSB} = P_{USB} = 2kW$

$P_c = 8kW$

$\mu = ?$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$

$P_t = P_c + P_{USB} + P_{LSB}$

$= 8 + 2 + 2$

$= 12kW$

$12kW = 8kW\left(1 + \frac{\mu^2}{2}\right)$

$\frac{\mu^2}{2} = \frac{3}{2} - 1 = \frac{1}{2}$

$\mu^2 = 1$

$\mu = 1$

SS:

$P_{USB} = P_{LSB} = 2\,kW$

$P_{SB} = 4\,kW$

$P_c = 8\,kW$ $\Rightarrow$ again greater than P_{SB}, therefore wastage

$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$

$P_t = \checkmark$

$P_c = \checkmark$

$\mu = 1$

$f = \frac{\omega}{2\pi}$ $T = 1/f$ $\omega = 2\pi f$

Q3) carrier signal, ~~c(t) = 10 cos~~

$c(t) = 10 \cos(2\pi \times 10^6 t)$ and

message signal, $m(t) = 4\cos(4\pi \times 10^3 t)$

with 50% of modulation. Antenna

resistance is given by 5 Ω

(1) Find all the parameters of

Am (BW, P_T, η)

(2) Plot Am spectrum & identify

the spectral components

Soln:

$c(t) = 10\cos(2\pi \times 10^6 t)$

$= A_c \cos(2\pi f_c t)$

$m(t) = 4\cos(4\pi \times 10^3 t)$

$= A_m \cos(2\pi f_m \times t)$

$A_c = 10\,V$

$f_c = 10^6 = 1\,MHz$

$A_m = 4\,V$

$f_m = 2\times 10^3\,Hz = 2\,kHz$

Bandwidth of multitone AM signal ?
relation b/w Power , Amplitude & resistance

(1)

$BW = 2\omega m = 2 fm$

$= 2 \times 2 kHZ$

$= 4 kHZ$

$\mu = \frac{Am}{Ac} = \frac{50}{100} = 0.5$ (given)

$R = 5\Omega$

$(P = \frac{V^2}{R})$

$P_c = \frac{Ac^2}{2R} = \frac{100}{2 \times 5}$

$= \frac{100}{10} = 10 W$

$P_t = P_c (1 + \frac{\mu^2}{2})$

$= 10 (1 + \frac{(0.5)^2}{2})$

$= 11.25 W$

$P_{SB} = P_t - P_c = 11.25 - 10 = 1.25 W$

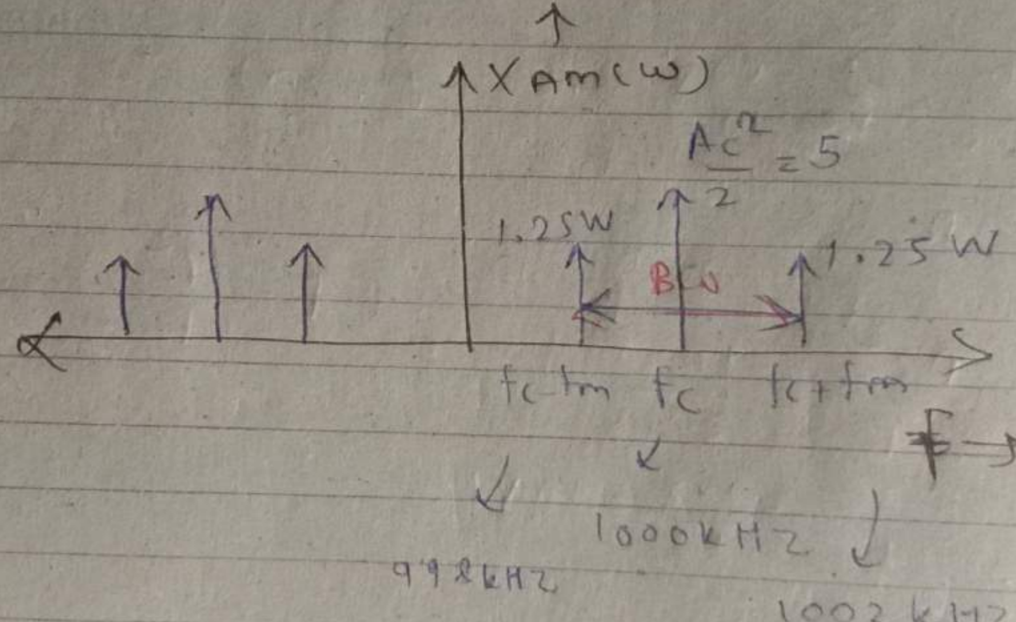
$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = \frac{1.25}{2} = 0.625 W$

$$\eta = \frac{P_{SB}}{P_t} = \frac{\mu^2}{2+\mu^2}$$

$$= \frac{1.25}{11.25} = 0.11 \times 100\%$$

$$= 11\%$$

(2)



BW = 4 kHz

$[A_c + m(t)] \cos \omega_c t$ $\qquad A_c[1 + \mu \cos \omega_m t \cos \omega_c t]$

$A_c + A_m \cos \omega_m t \cos \omega_c t$ $\qquad = A_c \cos \omega_c t + A_c \mu$

Recs (Generation of AM/DSB-C)

- ↳ Square law modulator
- ↳ switching modulator

<u>DSB-C.</u>

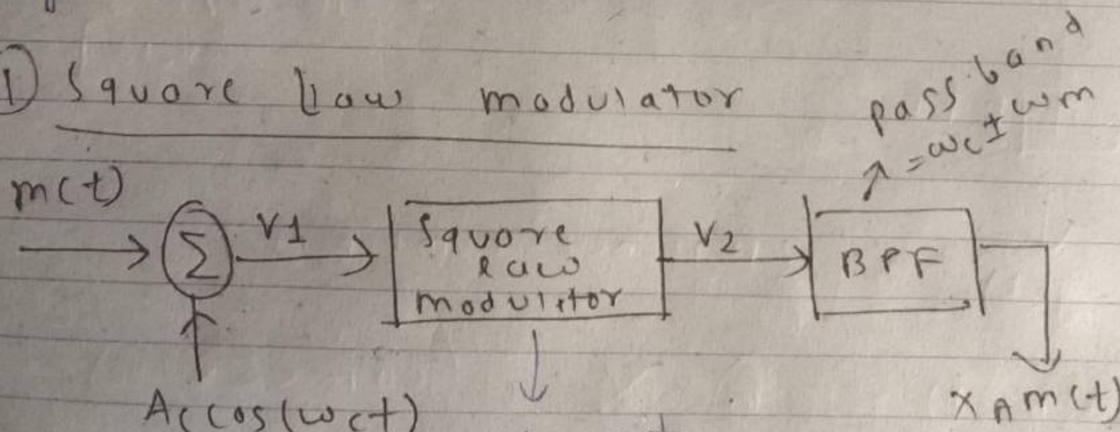
general representation of $x_{Am}(t)$ is:

$$x_{Am}(t) = c(t) + m(t)\,c(t)$$

$$= \underbrace{A_c \cos \omega_c t}_{c(t)} + \underbrace{A_c \mu \cos \omega_m t \cos \omega_c t}_{m(t)\,c(t)}$$

* Scaler components can be removed by amplier circuits

① <u>Square law modulator</u>

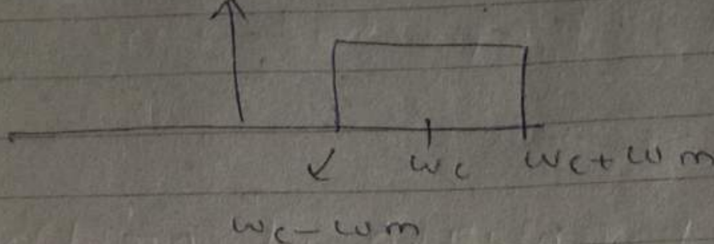


circuit = combination of discrete elements

X = rejected

✓ = allowed

Frequency characteristics of BPF:



$v_1 = m(t) + A_c \cos(\omega_c t)$

$v_2 = a_1 v_1 + a_2 v_1^2$

$$= a_1 [m(t) + A_c \cos(\omega_c t)] + a_2 [m^2(t) + A_c^2 \cos^2(\omega_c t) + 2m(t) A_c \cos(\omega_c t)]$$

BPF

$y(t)$

frequency component

$a_1 m(t) = \omega_m = $ X $\quad (\omega_m <<< \omega_c)$

$a_1 A_c \cos(\omega_c t) = \omega_c = $ ✓

$a_2 m^2(t) = 2\omega_m$ X $\quad (2\omega_m <<< \omega_c)$

$$\left\{ \begin{array}{l} m(t) = \omega_m \\ m^2(t) = m(t) \cdot m(t) \\ m^2(W) = \underbrace{m(W) * m(W)}_{\text{convolution}} = \omega_m + \omega_m = 2\omega_m \end{array} \right\}$$

Σ = summer

$A_c^2 \cos^2(\omega_c t) = \times$

$\cos^2(\omega_c t)$

$= \omega_c + \omega_c = 2\omega_c$

$2\omega_c >> \omega_c$

$m(t) A_c \cos(\omega_c t) = \omega_c \pm \omega_m$

so final expression is:

$$X_{AM}(t) = a_1 A_c \cos(\omega_c t) + a_2 m(t) A_c \cos(\omega_c t)$$

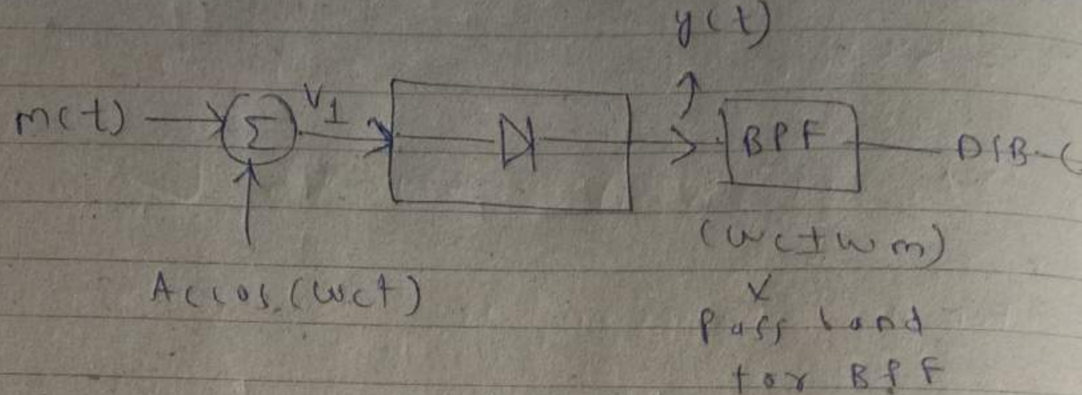
of form

$$\boxed{X_{AM}(t) = a_1 c(t) + a_2 m(t) c(t)}$$

DSB-C

② <u>Switching modulator:</u>

(Hardware implementation simpler)
(simpler to design & cheaper also)



$V_1 = A_c \cos(w_c t) + \cancel{A_m}\ m(t)$

Diode → will act as a switch
(switching depends on V_1)

$P_{c(t)} > P_{m(t)}$

$Power_{c(t)} > Power_{m(t)}$

$0 < \mu \leq 1$ $\quad P_{A_c} = \frac{A_c^2}{2}$

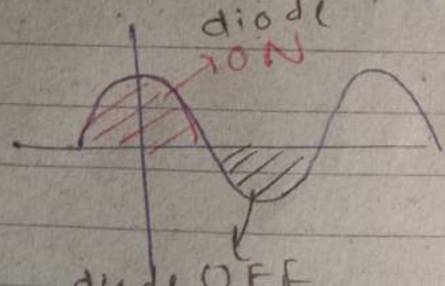
$0 < \frac{A_m}{A_c} \leq 1$ $\quad P_{A_m} = \frac{A_m^2}{2}$

$A_c \geq A_m$ $\quad \therefore P_{c(t)} > P_{m(t)}$

* switching of the diode depends on the polarity of the carrier signal because the strength of carrier is more than strength of message signal

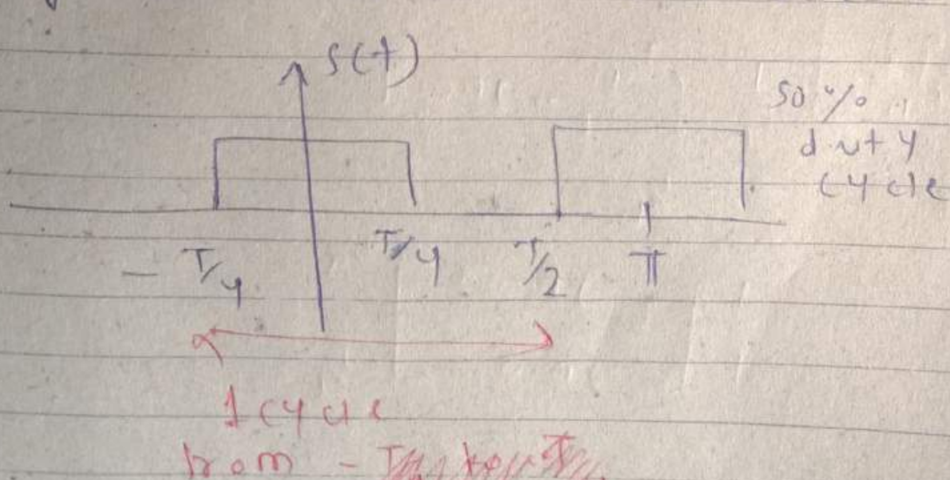
$$y(t) = \left[A_c \cos(\omega_c t) + m(t)\right] s(t)$$

$s(t)$ = switching coefficient
(depends on the carrier, so duty cycle = 50%)



, for 50%, diode ON, rest 50% diode off

Finding the expression for $s(t)$:



Fourier series representation of s(t):

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_c t}$$

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-j\omega_c t n}\, dt$$

$$C_0 = \frac{1}{T}\int_{-T/4}^{T/4} 1\, dt + \frac{1}{T}\int_{T/4}^{T/2} 0 \cdot dt$$

$$= \frac{1}{T}\int_{-T/4}^{T/4} 1 \cdot dt$$

$$= \frac{1}{T}\cdot\left(\frac{T}{4} + \frac{T}{4}\right) = T/2 \times \frac{1}{T} = \frac{1}{2}$$

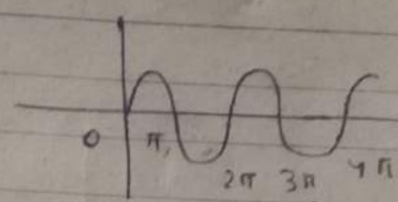
$$C_n = \frac{1}{T}\int_{-T/4}^{T/4} e^{-jn\omega t}$$

$$= \frac{1}{T}\left[\frac{e^{-jn\omega t}}{-jn\omega}\right]_{-T/4}^{T/4}$$

$$\frac{e^{ix}-e^{-ix}}{2j} = \sin x$$

$$c_n = \frac{1}{T} \times \frac{1}{jn\omega}\left[e^{jn\omega T/4} - e^{-jn\omega T/4}\right]$$

$$c_n = \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$



if n is even,

$c_n = 0$

$n =$ odd,

$c_1, c_{-1}, c_3, c_{-3} \ldots$

$$s(t) = \frac{1}{2} + \frac{\sin(-\pi/2)}{-\pi} e^{j\omega_c t} + \frac{\sin \pi/2}{\pi} e^{-j\omega_c t}$$

(c_0) $\quad (c_{-1})$ $\quad (c_1)$

$$+ \frac{\sin(-3\pi/2)}{-3\pi} e^{j3\pi t}$$

$$s(t) = \frac{1}{2} + \frac{1}{\pi}\left[e^{j\omega_c t} + e^{-j\omega_c t} + \frac{1}{3} e^{j\omega_c 3t} - \frac{1}{3} e^{-j3\omega_c t} \ldots\right]$$

$$= \frac{1}{2} + \frac{1}{\pi}\left[2\cos(\omega_c t) - \frac{1}{3} \cdot 2\cos(3\omega_c t) + \ldots\right]$$

$$y(t) = \left[m(t) + A_c \cos(\omega_c t)\right] s(t)$$

$$= \left[m(t) + A_c \cos(\omega_c t)\right] \left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_c t) - \frac{2}{3\pi}\cos(3\omega_c t) + \dots\right]$$

$$= \frac{m(t)}{2} + \frac{A_c \cos(\omega_c t)}{2} + \frac{2}{\pi} m(t)\cos(\omega_c t) + \frac{2A_c}{\pi}\cos^2 \omega_c t - \frac{2}{3\pi} m(t)\cos(3\omega_c t) - \frac{2}{3\pi} A_c \cos(\omega_c t)\cos(3\omega_c t) + \dots$$

$\Downarrow$

$\boxed{\text{BPF}}$ $\omega_c \pm \omega_m$

$\downarrow$

?

$X(t) =$

$=$

$\boxed{X_{AM}(t) =}$

$$x(t) = \frac{A_c \cos(\omega_c t)}{2} + \frac{2}{\pi} m(t) A_c \cos(\omega_c t)$$

= [illegible]

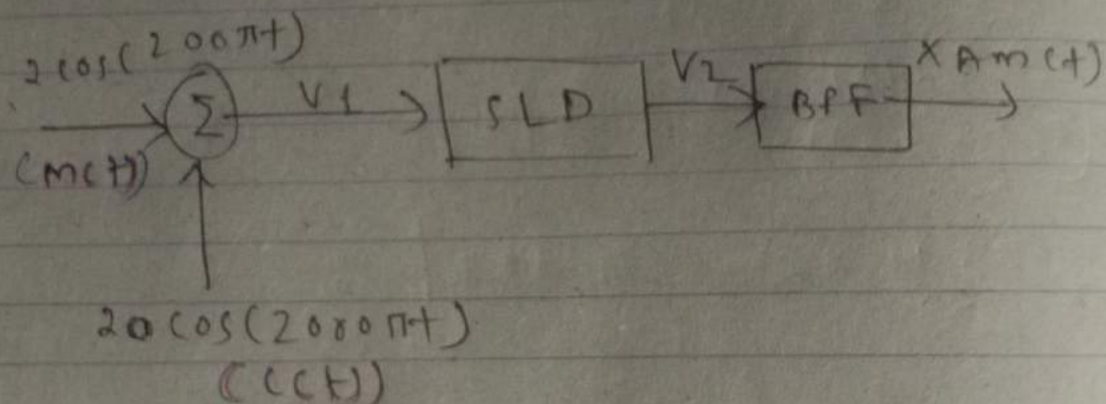
$$\boxed{x_{AM}(t) = \frac{1}{2} c(t) + \frac{2}{\pi} m(t)\, c(t)}$$

SLD = square law device
BPF = band pass filter

Lec 6 (Questions)

Q1) square law device characterised by $v_2 = v_1 + 0.1 v_1^2$

Pass band of BPF is 800–1200 Hz. Find all the parameters of resulting AM signal?



Soln:

MS:

$$v_1 = 2\cos(200\pi t) + 20\cos(2000\pi t)$$

$$v_2 = (2\cos(200\pi t) + 20\cos(2000\pi t)) + 0.1\,(4\cos^2(200\pi t) + 400\cos^2(2000\pi t) + 80\cos(200\pi t)\cos(2000\pi t))$$

$V_2 \longrightarrow \boxed{BPF} \longrightarrow X_{Am}(t)$

$$2\pi f t$$

$$X_{Am}(t) = 2\cos(200\pi t) \quad 100$$

$$20\cos(2000\pi t) \quad 1000$$

$$0.4\cos^2(200\pi t) \quad 200$$

$$40\cos^2(2000\pi t) \quad \cancel{1000}\ 2000$$

$$8\cos(200\pi t)\cos(2000\pi t)$$

$$\frac{2200}{2} = 1100$$

$$X_{Am}(t) = 20\cos(2000\pi t)$$

$$+ 8\cos(200\pi t)\cos(2000\pi t)$$

$$= 20\cos(2000\pi t)\left[1 + 0.4\cos 200\pi t\right]$$

$$A_c\cos(\omega_c t)\left[1 + \mu \cos\omega_m t\right]$$

$A_c = 20\ V$

$f_c = 1000\ HZ$

$\mu = 0.4$

$f_m = 100\ HZ$

SS:

$v_1 = 2\cos(200\pi t) + 20\cos(2000\pi t)$

$V_2 = v_1 + 0.1v^2$

100HZ · 1000HZ

$v_2 = (2\cos(200\pi t) + 20\cos(2000\pi t))$

200HZ 2000HZ

$+0.1(4\cos^2(200\pi t) + 400\cos^2(2000\pi t) +$

1100HZ

$80\cos(200\pi t)\cos(2000\pi t)]$

$\downarrow$ BPF (800-1200HZ)

$V_2 = 20\cos(2000\pi t) + 8\cos(200\pi t)\cos(2000\pi t)$

$= 20\{1 + 0.4\cos(200\pi t)\}\cos(2000\pi t)$

$= A_c\{1 + \mu\cos(2\pi f_m t)\}\cos(2\pi f_c t)$

$A_c = 20V$ → can be verified from Block diagram

$\mu = 0.4$

$f_m = 100$ HZ

$f_c = 1000$ HZ

CURRE

Pt

Q2) An

cur

Am

50%

Soln:

CURRENT RELATION IN AM

$$P_t = P_c \left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{P = i^2 R}$$

$$I_t^2 R = I_c^2 R \left[1 + \frac{\mu^2}{2}\right]$$

$$\boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}}$$

$$\boxed{V_t = V_c \sqrt{1 + \frac{\mu^2}{2}}}$$

Q2) An unmodulated ($\mu = 0$) AM transmitter current is given by 5A. Find AM transmitter current with 50% modulation?

Soln: SS:

$$\mu = 0, \quad I_t = I_c \left[1 + \cancel{\frac{\mu^2}{2}}^{\,0}\right]$$

$$I_t = I_c = 5A$$

Carrier signal is constant throughout experiment

$I_c = 5A$

$\mu = \frac{50}{100} = 0.5$

$$I_t = I_c\sqrt{\left[1+\frac{\mu^2}{2}\right]}$$

$$= 5\sqrt{\left[1+\frac{(0.5)^2}{2}\right]}$$

$$\boxed{I_t = 5.32\,A}$$

transmitting extra components requires more current, $5.32A > 5A$

Q3) An AM transmitter current is given by 10 A with 40% modulation find AM transmitter current with 80% of modulation?

Soln: SS

$I_t = 10A, \ \mu = 0.4$

$$I_t = I_c\sqrt{\left[1+\frac{\mu^2}{2}\right]}$$

$$10 = I_c \sqrt{\left[1 + \frac{(0.4)^2}{2}\right]}$$

$$I_c = 9.6\,A$$

$\mu = 0.8$, $I_c = 9.6\,A$, $I_t = ?$

$$I_t = I_c \sqrt{\left[1 + \frac{\mu^2}{2}\right]}$$

$$= 9.6 \sqrt{\left[1 + \frac{(0.8)^2}{2}\right]}$$

$$= 11.05\,A$$

Q4) An unmodulated AM transmitter power is given by 10kW. When the carrier is modulated by single sinusoidal message signal, transmitter power becomes 13.5kW. Find AM transmitter power if the carrier is simultaneously modulated by 2nd message signal with 60% of modulation?

Soln:

Initially unmodulated

$\mu = 0$, $P_t = P_c = 10\,kW$

μ_1 for $m_1(t)$, $P_t = 13.5\,kW$

$$P_t = P_c \left[1 + \frac{\mu_1^2}{2}\right]$$

$$13.5 = 10 \left[1 + \frac{\mu_1^2}{2}\right]$$

$$\mu_1^2 = 0.7$$

$m_2(t)$, $\mu_2 = 0.6$

$$P_t = P_c \left[1 + \frac{\mu_T^2}{2}\right]$$

$$\mu_T = \sqrt{\mu_1^2 + \mu_2^2}$$

$$= \sqrt{0.7 + 0.6^2}$$

$$\boxed{\mu_T \approx 1}$$

$$P_t = P_c \left[1 + \frac{\mu_T^2}{2}\right]$$

$$= 10\left[1 + \frac{1}{2}\right] = 10 \times \frac{3}{2} = 15\,kW$$

elec 7

DOU

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DSB

DSB

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* for

Gener

X_{DSB}

X_{DSB}

lec 7 (DSB-SC)

Double sideband suppressed carrier (DSB-SC)

general representation

DSB-C ⇒ LSB + USB + carrier

DSB-SC ⇒ LSB + USB + ~~carrier~~

suppressed (not present)

motivation: In DSB-C, carrier power is huge which is wastage of transmitted power

* for $\mu = 1$, $\frac{2}{3}$rd transmitted power consumed by carrier signal

General expression for DSB-SC:

$$x_{DSB-C}(t) = c(t) + m(t)\,c(t)$$

$$\boxed{x_{DSB-SC}(t) = m(t)\,c(t)}$$

$$\cos(\omega_c t) = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$$

$$= \frac{1}{2}\left[\pi(\delta(\omega-\omega_c)\right)$$

$$c(t) = A_c \cos(\omega_c t)$$

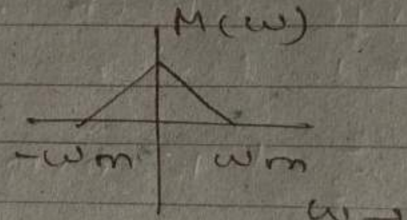
$$x_{DSB-SC}(t) = m(t) A_c \cos(\omega_c t)$$

$$\Updownarrow FT$$

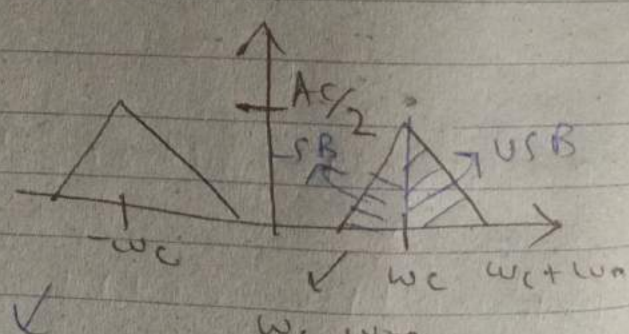
$$X_{DSB-C}(\omega) = M(\omega) * \frac{A_c}{2}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

$$\boxed{X_{DSB-C}(\omega) = \frac{A_c}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]}$$

let $m(t)$ =



$X_{DSB-SC}(t)$ =



no carrier signal present, only USB & LSB

BW = Band of frequency in which max. energy components present

$$\boxed{BW_{DSB-SC} = 2\omega_m}$$

↳ ~~same~~ same as DSB-C

↳ difference in transmission efficiency

→ <u>Single-tone DSB-SC</u>

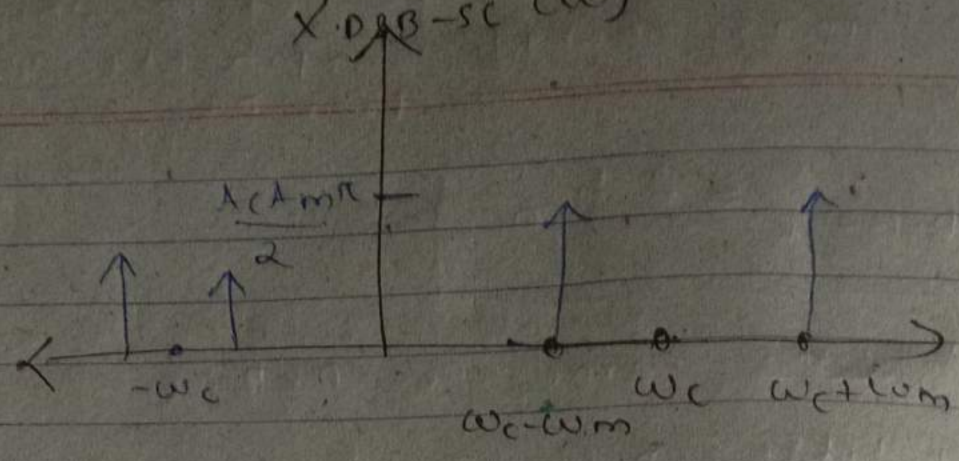
$$m(t) = A_m \cos(\omega_m t)$$
$$c(t) = A_c \cos(\omega_c t)$$

$$X_{DSB-SC}(t) = A_c A_m \cos(\omega_m t)\cos(\omega_c t)$$

$$= \underbrace{\frac{A_c A_m}{2}\cos(\omega_c - \omega_m)t}_{LSB} + \underbrace{\frac{A_c A_m}{2}\cos(\omega_c + \omega_m)t}_{USB}$$

$$X_{DSB-SC}(\omega) = \frac{A_c A_m \pi}{2}\left[\delta(\omega - (\omega_c - \omega_m)) + \delta(\omega + (\omega_c - \omega_m))\right]$$

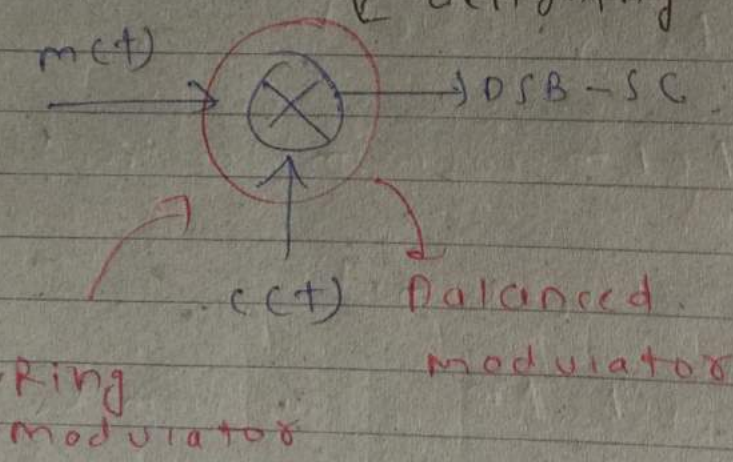
$$+ \frac{A_c A_m}{2}\pi\left[\delta(\omega - (\omega_m + \omega_c)) + \delta(\omega + (\omega_m + \omega_c))\right]$$



GENERATION OF DSB-SC SIGNAL

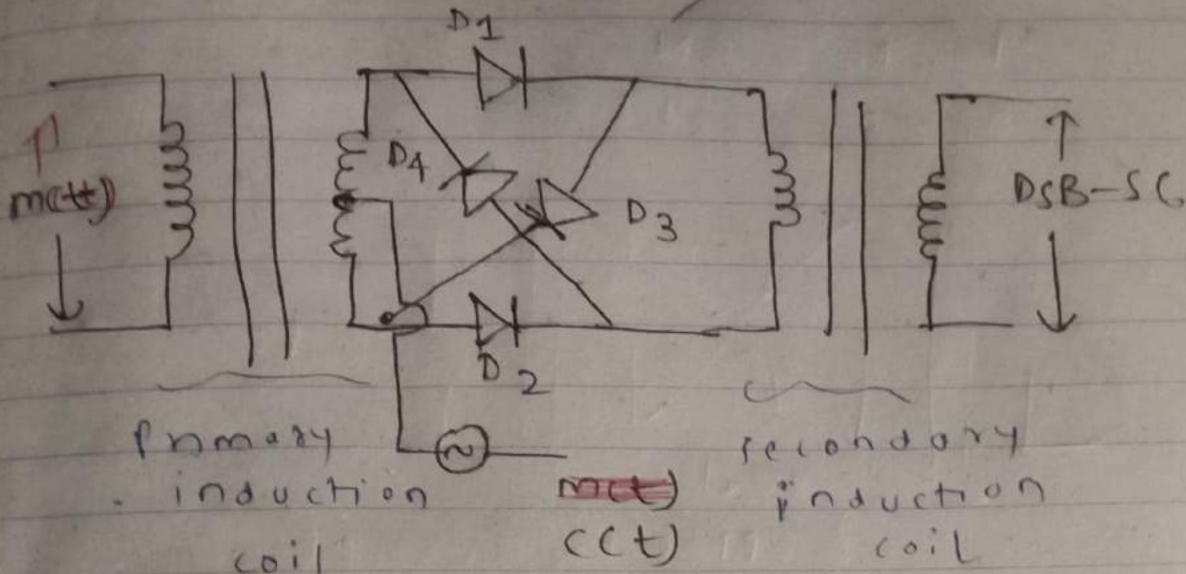
↳ Ring modulator

↳ Balanced modulator
(commonly used)
designing not easy



RING MODULATOR

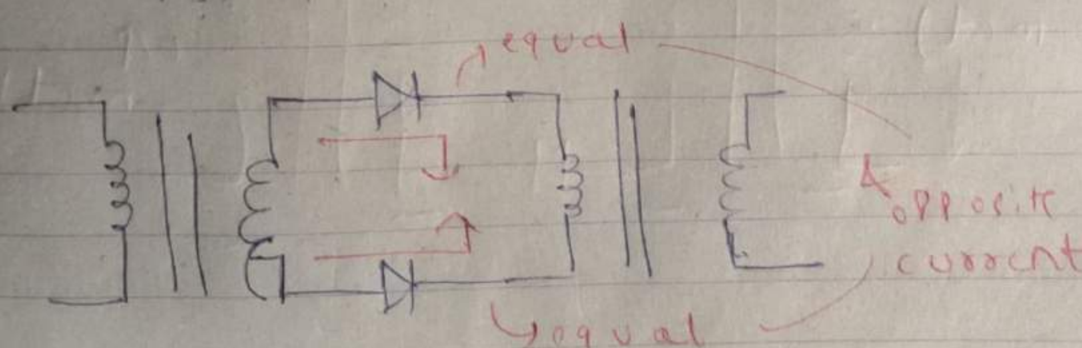
diodes form a ring



CASE 1: when only c(t) is applied
$m(t) = 0$

(i) +ive cycle:

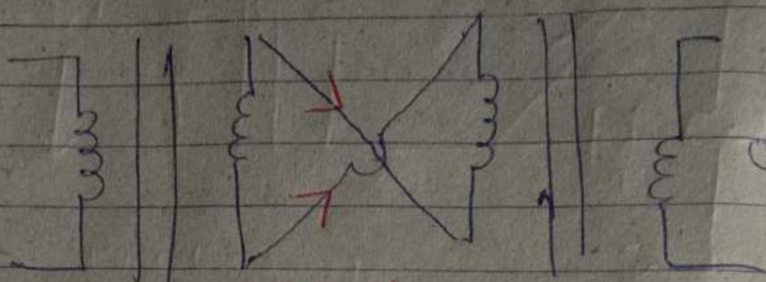
$D_1, D_2 = $ ON (forward biased)
$D_3, D_4 = $ OFF (Reverse biased)



(ii) -ive cycle:

D_3, D_4 = ON

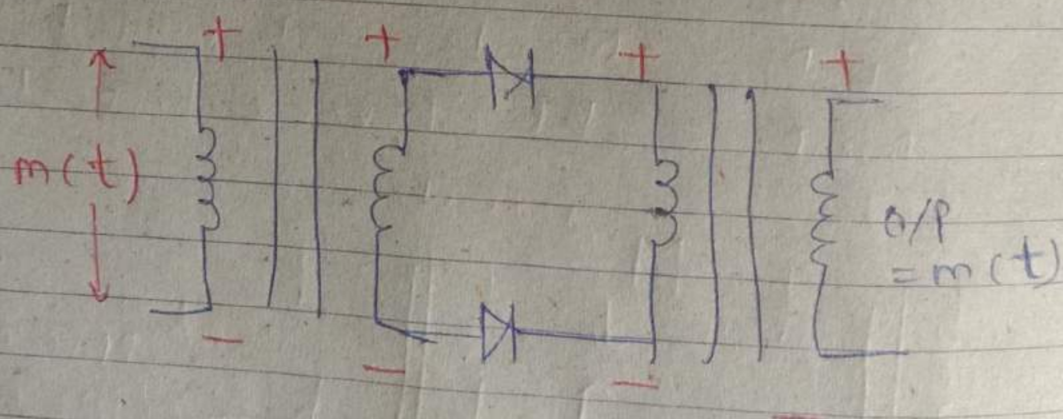
D_1, D_2 = OFF



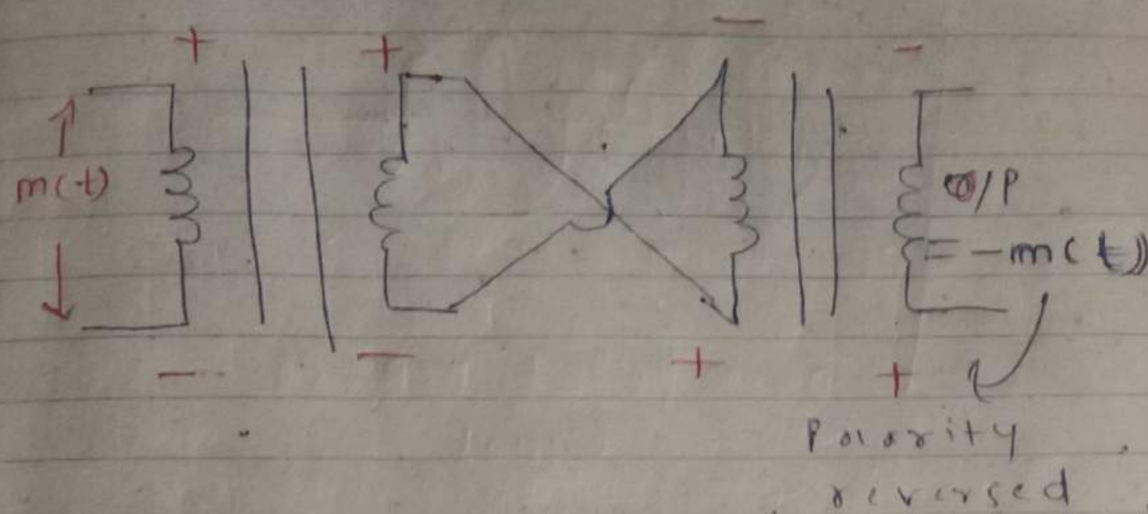
equal
& opposite current

CASE 2: when m(t) is also applied

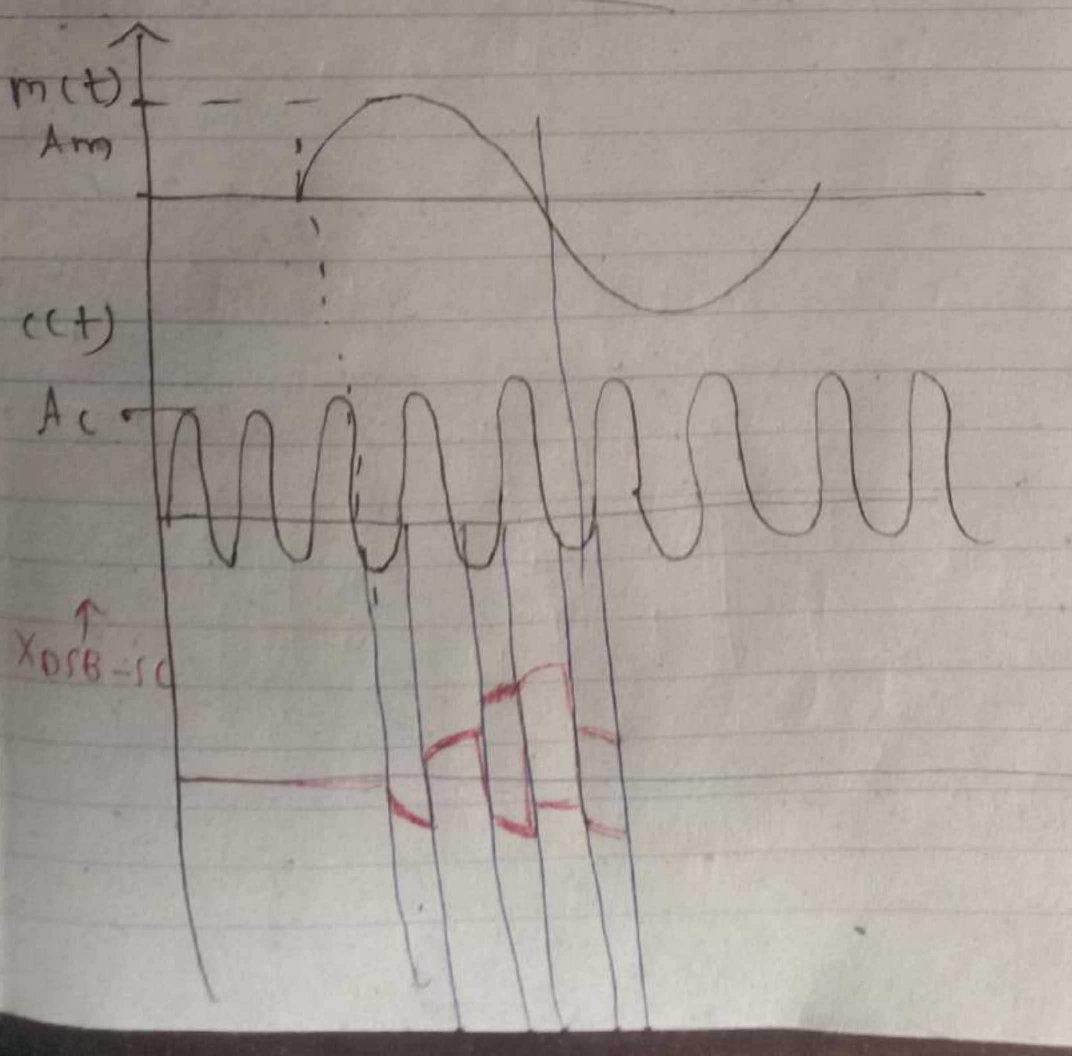
(i) +ive cycle of c(t)

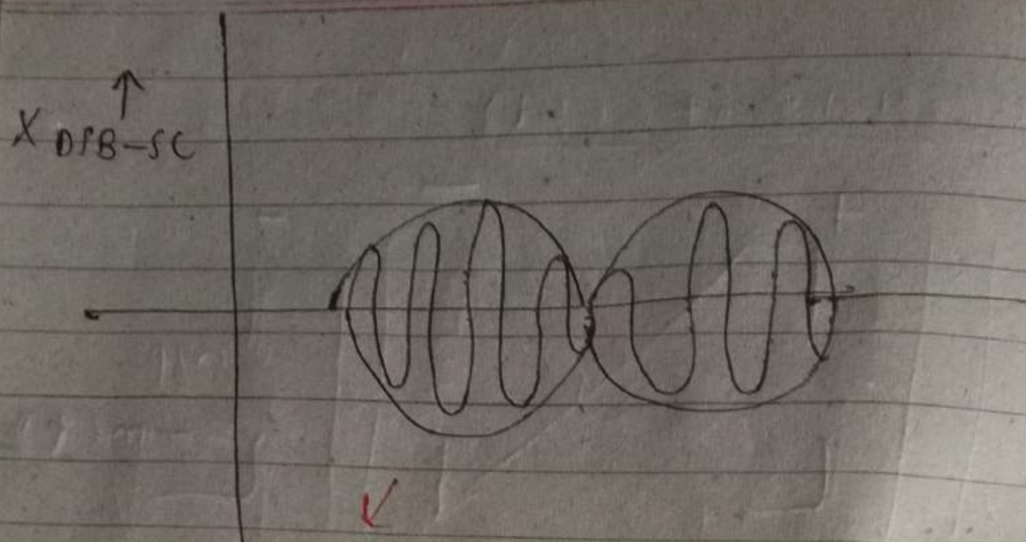


(ii) -ive cycle of c(t)



GRAPHICAL ANALYSIS:





Envelope of DSB-SC signal

L(8) (Balanced mod. & SSB-SC)

DSB-SC

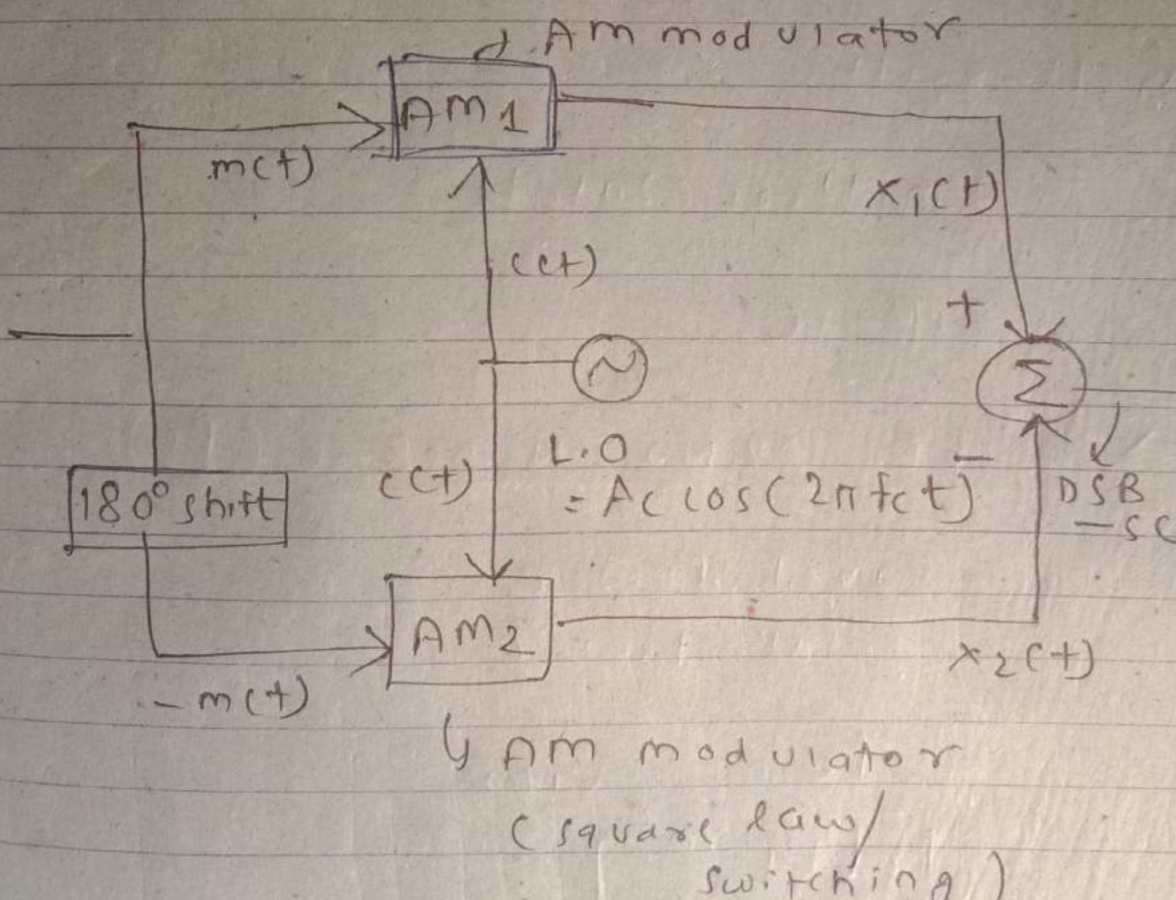
$= c(t) + m(t)\,c(t) = \text{DSB-C}$

$= \cancel{c(t)} + m(t)\,c(t)$

↳ DSB-SC

$x_{DSB-SC}(t) = m(t) A_c \cos(\omega_c t)$

Balanced modulator



↗ system model

For AM_1

$I/P = m(t)$ & $c(t)$

For AM_2

$I/P = -m(t)$ & $c(t)$

$$x_1(t) = (A_c + m(t))\cos(2\pi f_c t)$$

$$x_2(t) = (A_c - m(t))\cos(2\pi f_c t)$$

$$x_1(t) - x_2(t)$$

$$= \underbrace{2m(t)\cos(2\pi f_c t)}$$

DSB-SC

⇒ POWER SAVING IN DSB-SC (PS)

$$= \frac{\text{Saved Power}}{\text{Total Power}}$$

$$= \frac{P_c}{P_c\left[1 + \frac{m^2}{2}\right]} \times 100$$

$$= \frac{2}{2 + m^2} \times 100\%$$

If critical modulation $(m = 1)$

$$PS = \frac{2}{2+1} \times 100$$

$$= \frac{200}{3} = 66.6\,\%$$ Power saved

MULTITONE DSB-SC

$$m(t) = A_{m_1} \cos(\omega_{m_1} t) + A_{m_2} \cos(\omega_{m_2} t)$$

$$c(t) = A_c \cos(\omega_c t)$$

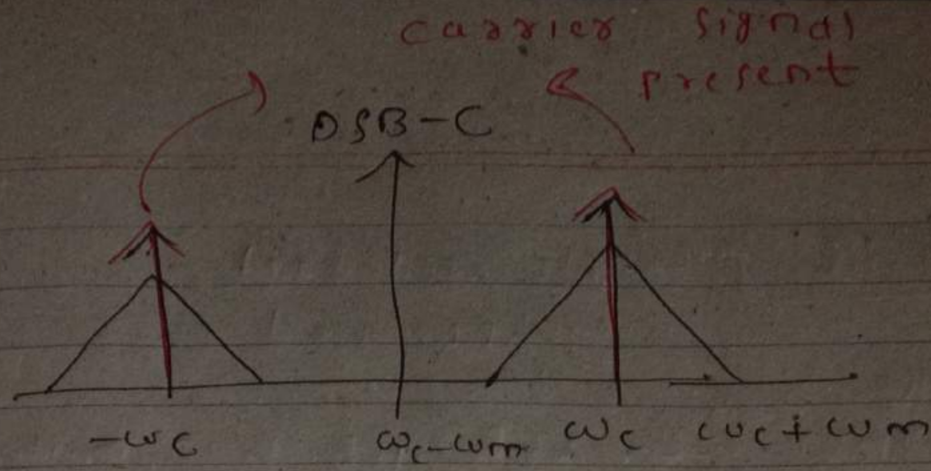
$$x_{DSB-SC}(t) = \frac{A_c A_{m_1}}{2} \cos(\omega_c + \omega_{m_1}) t$$

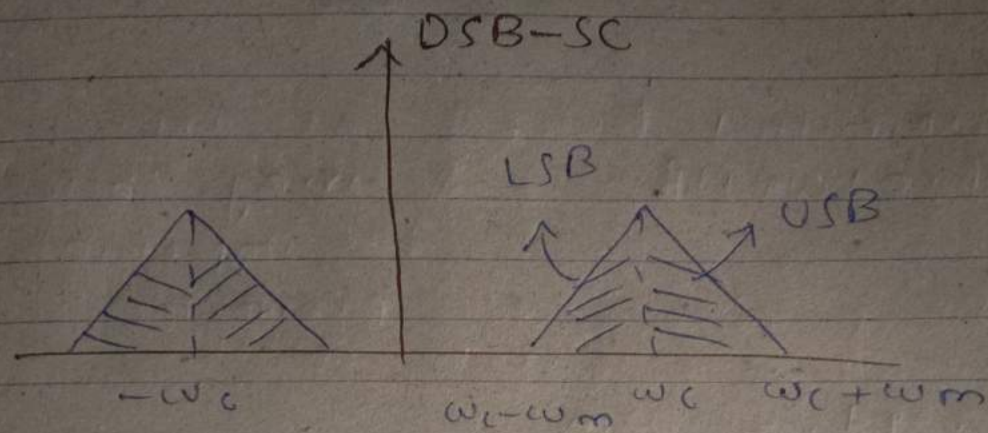
$$+ \frac{A_c A_{m_1}}{2} \cos(\omega_c - \omega_{m_1}) t$$

$$+ \frac{A_c A_{m_2}}{2} \cos(\omega_c + \omega_{m_2}) t$$

$$+ \frac{A_c A_{m_2}}{2} \cos(\omega_c - \omega_{m_2}) t$$

DSB-C	DSB-SC
(1) P_t is high	P_t is lower
$P_t = P_c\left(1+\frac{m^2}{2}\right)$	$P_t = \frac{P_c m^2}{2}$
(2) $BW = 2\omega_m$	$BW = 2\omega_m$
(3)	although Power saved but bandwidth reqd. more that leads to SSB-SC





* Both sidebands are identical in nature, replica of each other.

* same mathematical expression, carry same information

* knowledge of one sideband = we can generate another sideband

Principle behind SSB-SC

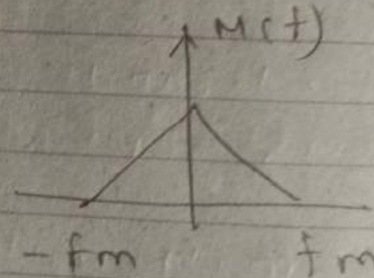
SSB-SC (Single sideband - suppressed carrier)

either ~~two~~ LSB / USB transmitted

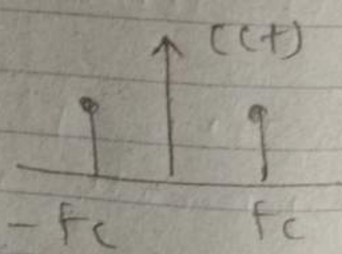
Transmitted power } we are
Bandwidth } saving these two important parameters

BW reqd. = ω_m

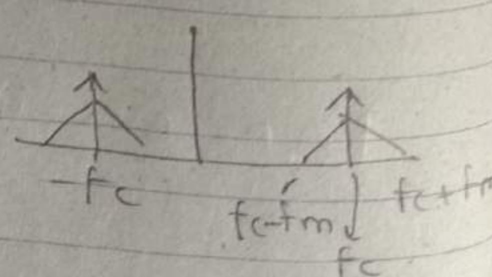
$m(t) \longleftrightarrow$ spectrum



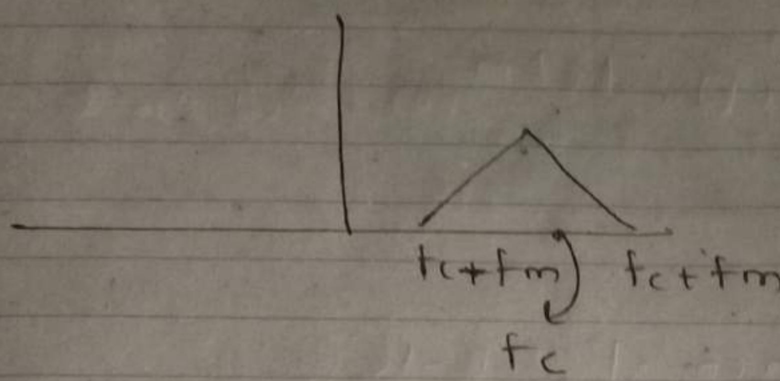
c(t)
($A_c \cos(\omega_c t)$) $\longleftrightarrow$



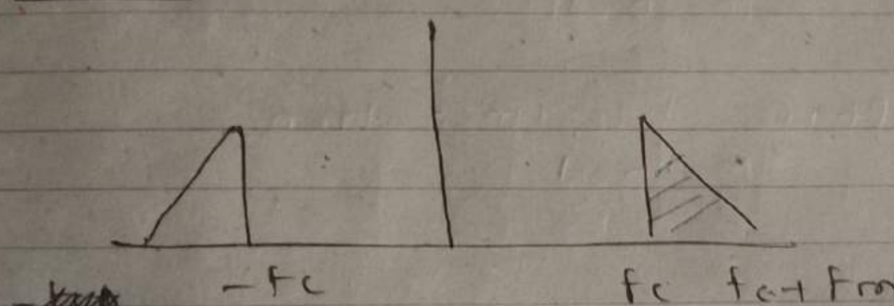
DSB-C $\longleftrightarrow$



DSB-SC

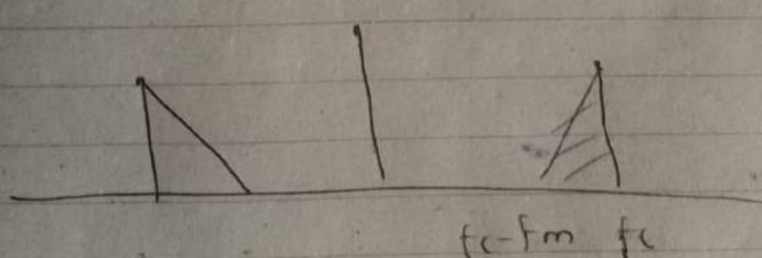


SSB-SC



OR

$$\boxed{\begin{aligned} B.W &= f_c + f_m - f_c \\ &= f_m \end{aligned}}$$



Therefore,

$$\boxed{x_{SSB-SC}(t) = \frac{A_c A_m}{2} \cos(\omega_c \pm \omega_m) t}$$

Generation of SSB-SC

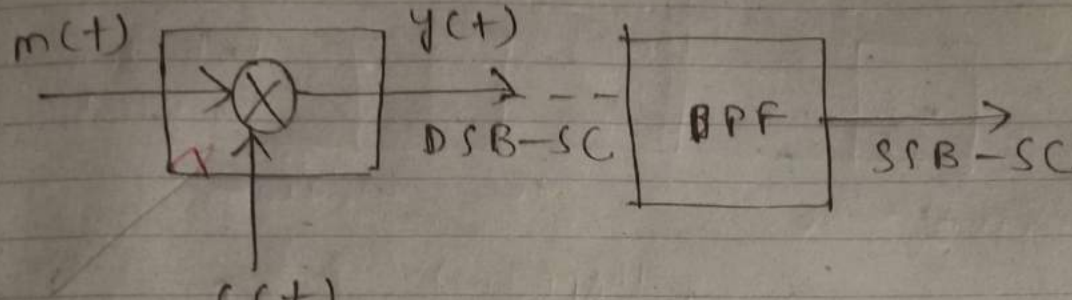
↳ frequency discrimination method

↳ Phase discrimination method

(Generation of SSB-SC)

① Frequency discrimination method

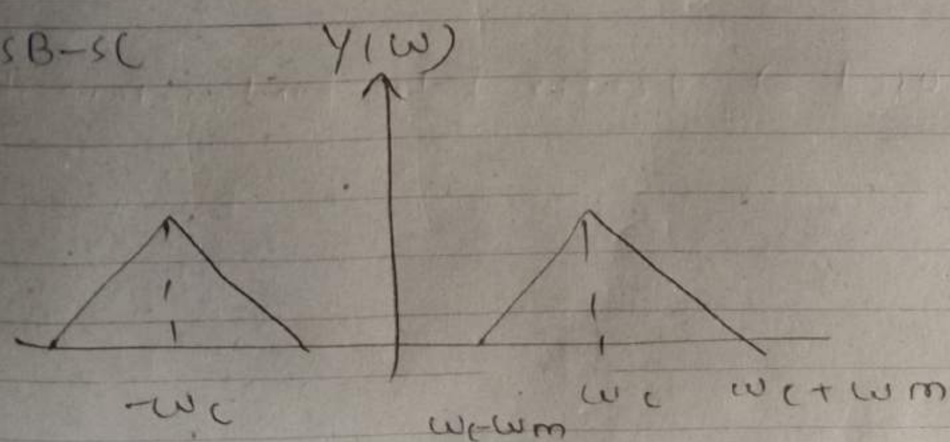
↳ based on frequency components



multiplier
(either Balanced modulator OR ring modulator)

$y(t) \longleftrightarrow Y(\omega)$

DSB-SC

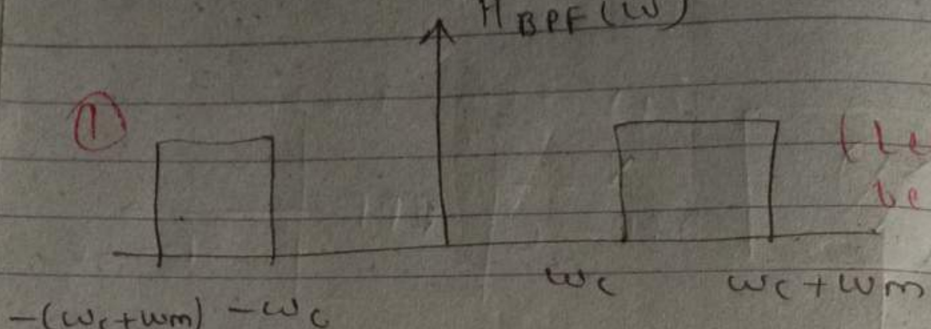


$H_{BPF}(\omega)$ = frequency response of Band Pass signal

Band-Pass filter:

Any one of the following Band Pass filter

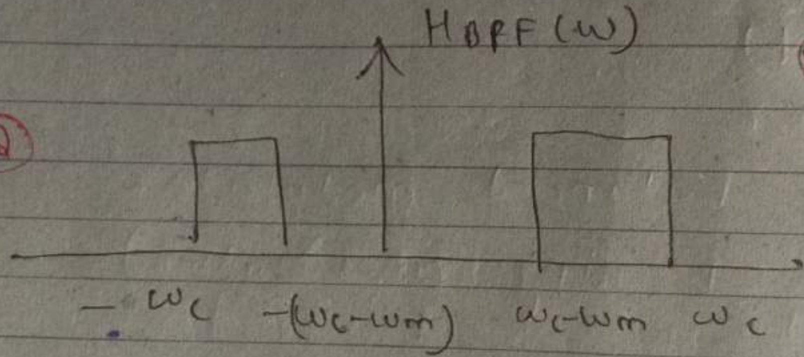
(1)



(Let this be used)

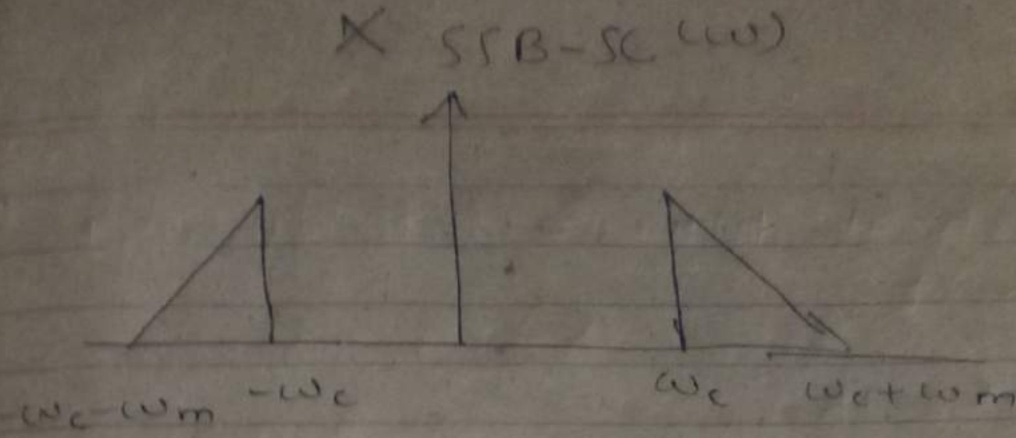
OR

(2)

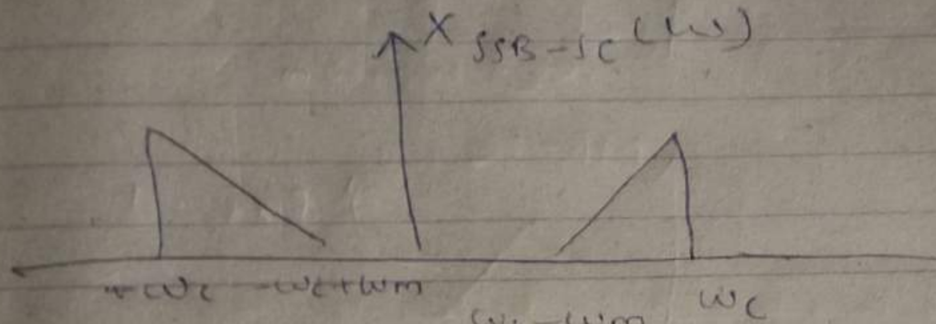


(i) used ⇒ upper side band retained

(ii) used ⇒ lower side band retained



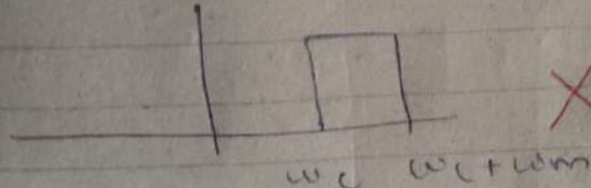
OR



DISADVANTAGE :

Ideal filters practically impossible

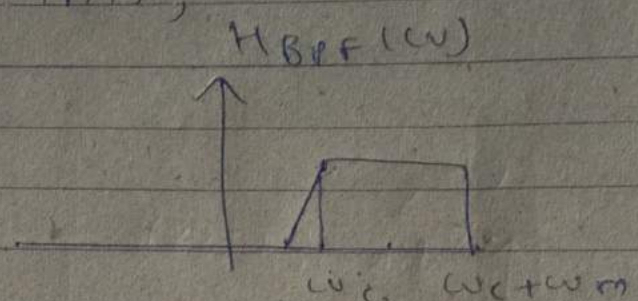
BPF with characteristics

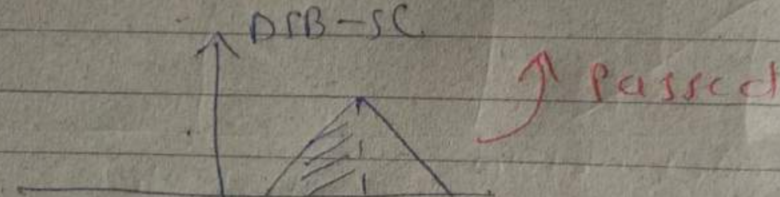


✗ not possible

Practical filters = slope associated with it

like this,





USB passed along with some unwanted part of LSB also passed $\Rightarrow$ leads to distortion of signal

Convolution in time domain
= multiplication in frequency domain

HILBERT TRANSFORM

↳ phase diff. of $\pi/2$ in signal
↳ rotates by 90° the signal I/P

$$x(t) \longrightarrow \boxed{HT} \longrightarrow \hat{x}(t)$$

$$x(t) \xrightarrow[\text{shift}]{90^\circ\text{ phase}} \hat{x}(t)$$

$$\boxed{\hat{x}(t) = x(t) * \frac{1}{\pi t}}$$

impulse response of hilbert transformer

$$\hat{x}(t) = \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t-\tau)}\, d\tau$$

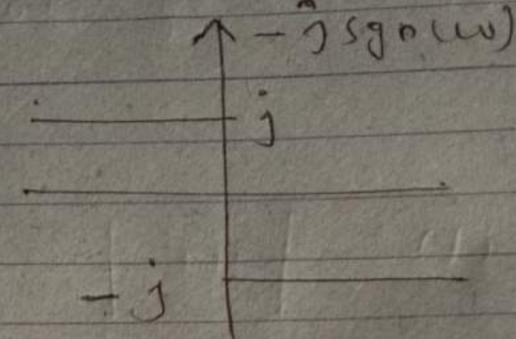
$$\boxed{\hat{X}(\omega) = X(\omega)\,(-j\,\mathrm{sgn}(\omega))}$$

frequency response of hilbert transformer

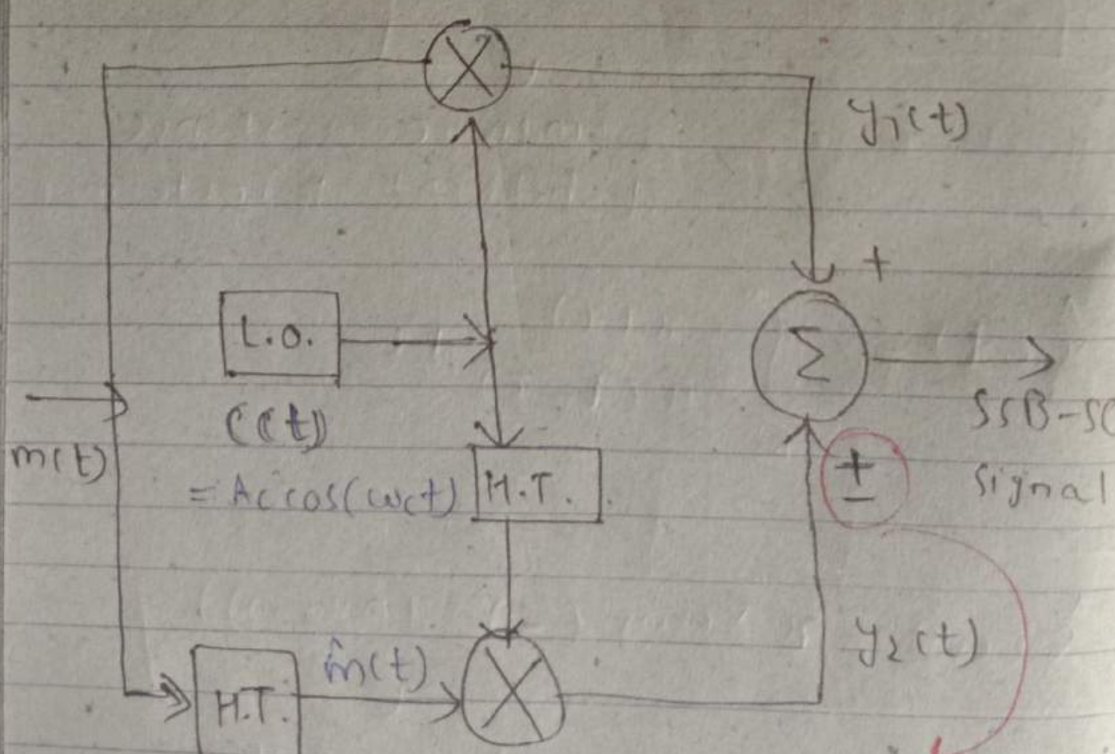
* $\cos(\omega_c t) \xleftrightarrow{HT} \sin(\omega_c t)$

* $\sin(\omega_c t) \xleftrightarrow{HT} -\cos(\omega_c t)$

$-j\,\mathrm{sgn}(\omega) \Longrightarrow$



(2) PHASE DISCRIMINATION METHOD



either
$y_1 - y_2$ OR
$y_1 + y_2$

L.O. = Local oscillator generates the carrier signal

$$y_1(t) = A_c\, m(t) \cos(\omega_c t)$$

$$\leftrightarrow \frac{A_c}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]$$

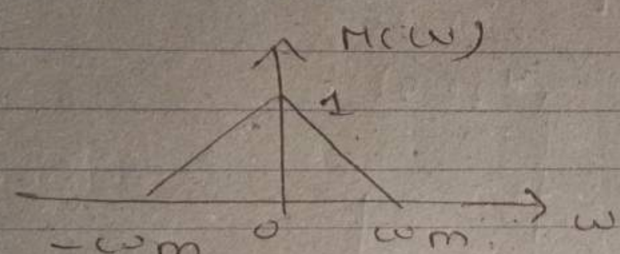
↳ $Y_1(\omega)$

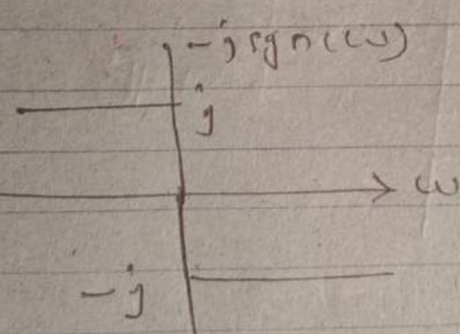
$$y_2(t) = A_c\, \hat{m}(t) \sin(\omega_c t)$$

$$\leftrightarrow \frac{A_c}{2j}\left[\hat{M}(\omega-\omega_c) - \hat{M}(\omega+\omega_c)\right]$$

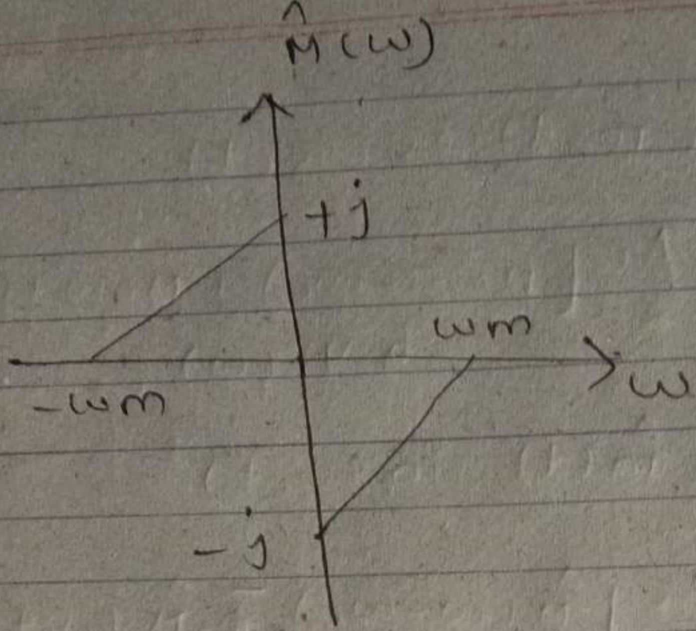
↳ $Y_2(\omega)$

Let m(ω) be:

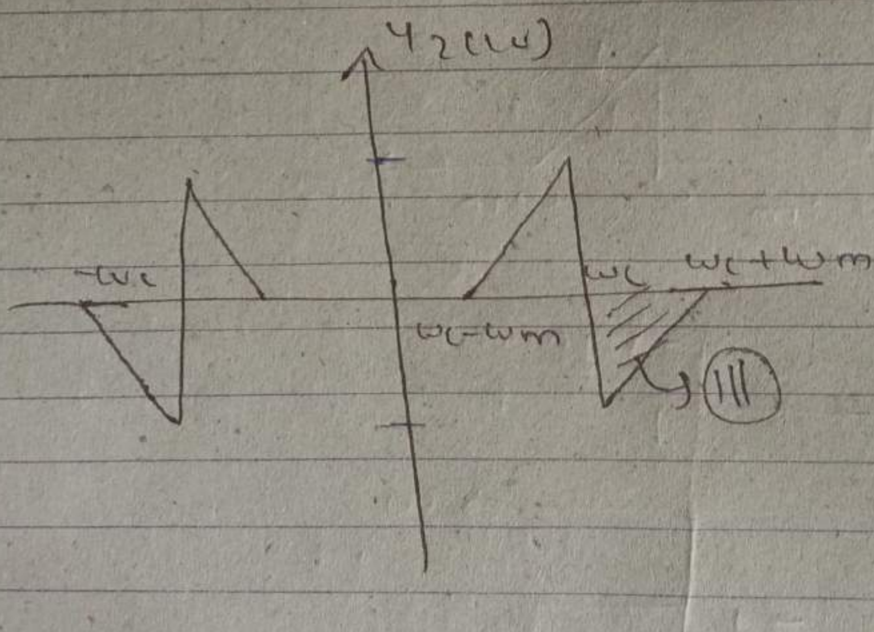




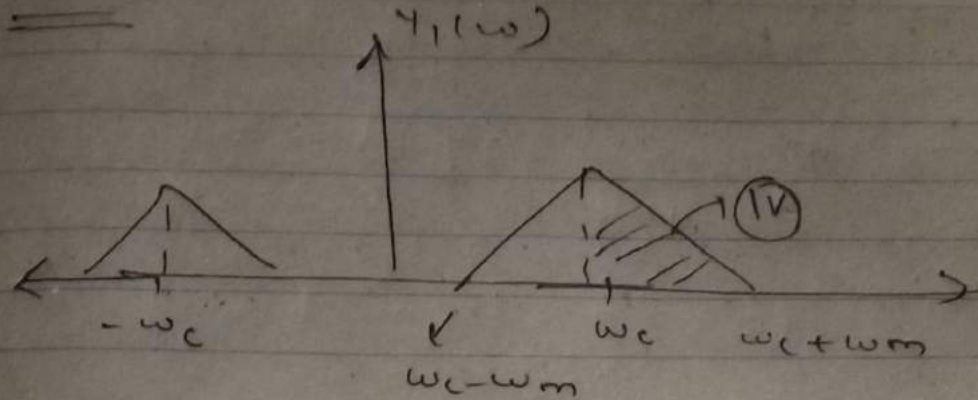
$$\hat{M}(\omega) = M(\omega)\,(-j\,\mathrm{sgn}(\omega))$$



$$Y_2(\omega) = \frac{A_c}{2j}\left[\hat{M}(\omega-\omega_c) - \hat{M}(\omega+\omega_c)\right]$$



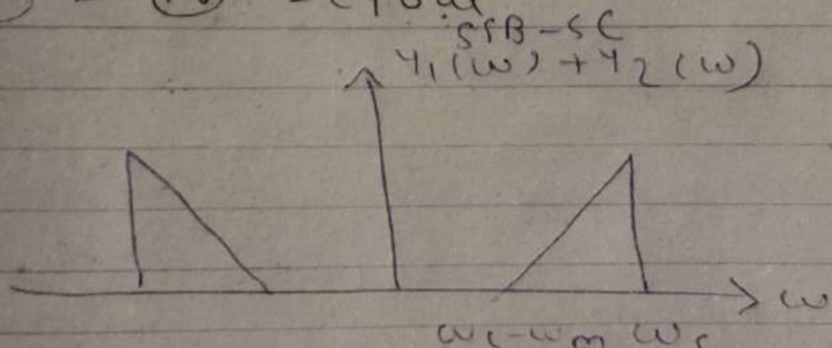
$Y_1(\omega)$



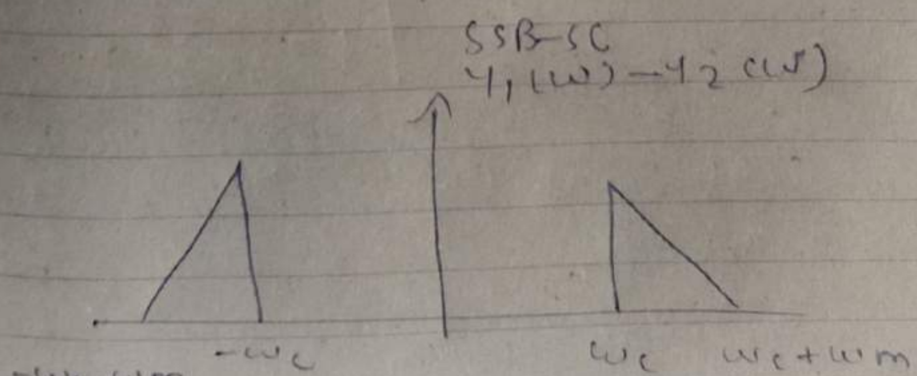
Final steps: either $Y_1(\omega)+Y_2(\omega)$

or $Y_1(\omega)-Y_2(\omega)$

III & IV = equal



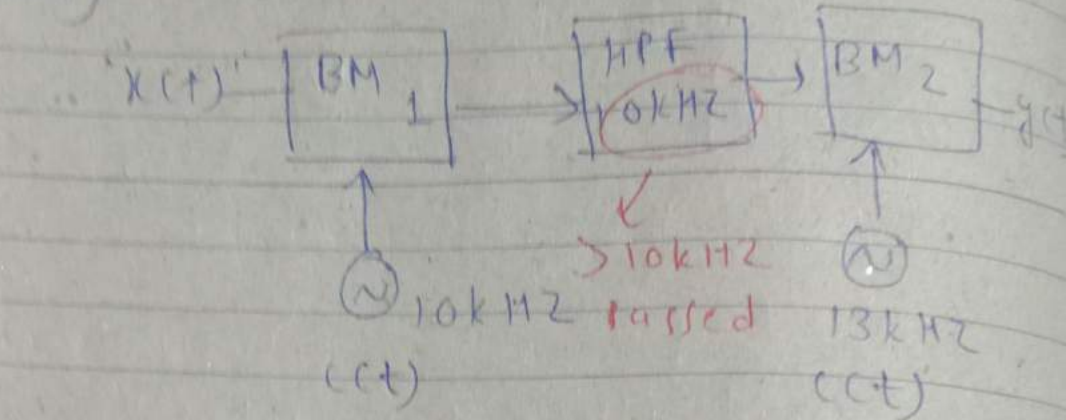
only lower side bands



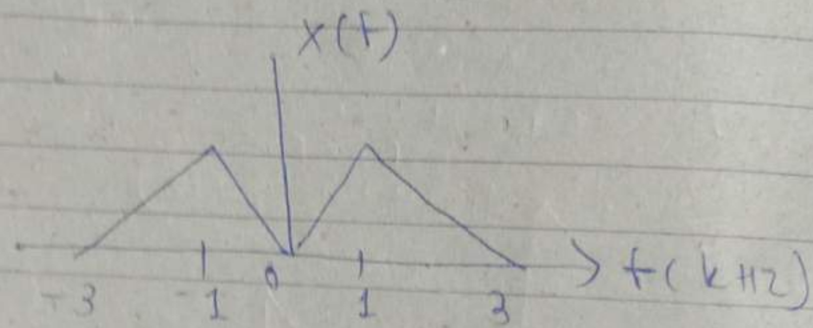
only upper side bands

Lec 10 (Demodulation Techniques)

Q1)



Spectrum of x(t) is:



Find out the positive frequencies where Y(f) has peaks?

Soln:

$y(t) \longleftrightarrow Y(f)$

$X_f = \left.\begin{matrix} 1\,kHz \\ -1\,kHz \end{matrix}\right\}$ Peaks

Balanced modulator $= x(t)\,c(t)$

$\omega_c \pm \omega_m \quad -\omega_c \pm \omega_m$

O/P of $BM_1 = x(t)\, c(t)$
$$\underset{10\,kHz}{\checkmark}$$

= frequencies added & subtracted

$$f \begin{cases} \cancel{10 \pm 1kHz}\; 10\,kHz \pm 1\,kHz \\ 10\,kHz \pm (-1\,kHz) \end{cases}$$

→ peaks

= 11 kHz; 9 kHz
9 kHz, 11 kHz

⇓ HPF (>10 kHz)

11 kHz passed

O/P of $BM_2 = Y(f) = 13\,kHz \pm 11\,kHz$

= 24 kHz
OR
2 kHz

Peaks

Ans)

T_x = Transmitter
R_x = Receiver

Amplitude modulation

↳ DSB-C (conventional AM)

$$\Downarrow$$

$$x_{AM}(t) = c(t) + m(t)c(t)$$

↳ DSB-SC

↳ only 2SB $(m(t)c(t))$

↳ SSB-SC = only 1SB, carrier suppressed

$$T_x \xrightarrow{x_{AM}(t)} R_x$$

↑ Demodulation takes place here

DEMODULATION/DETECTION of AM SIGNALS

(1) Envelope detection (Asynchronous / non-coherent detector)

(2) Synchronous / coherent detection

(3) Square law detector

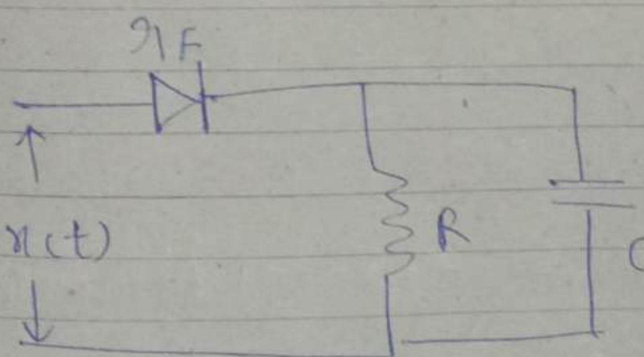
ENVELOPE DETECTION

→ used for conventional AM type of signal

→ non-coherent / Asynchronous
(does not require carrier signal at the receiver for detection)

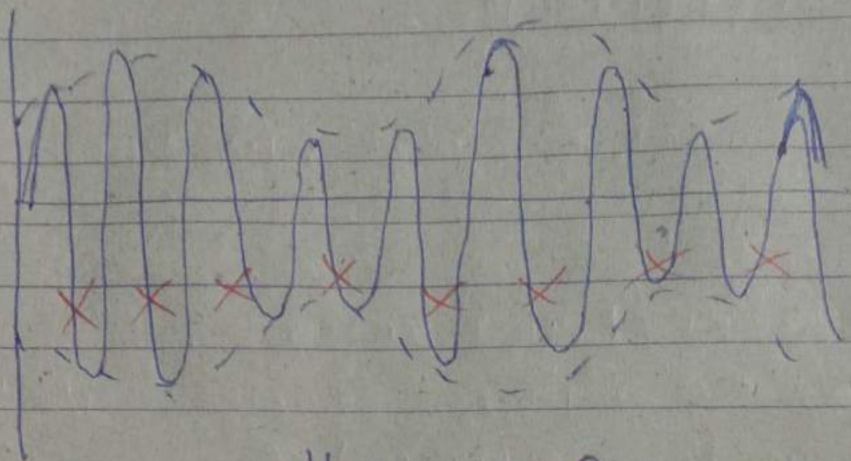
Since carrier signal already present in $x_{AM}(t)$

CIRCUIT:

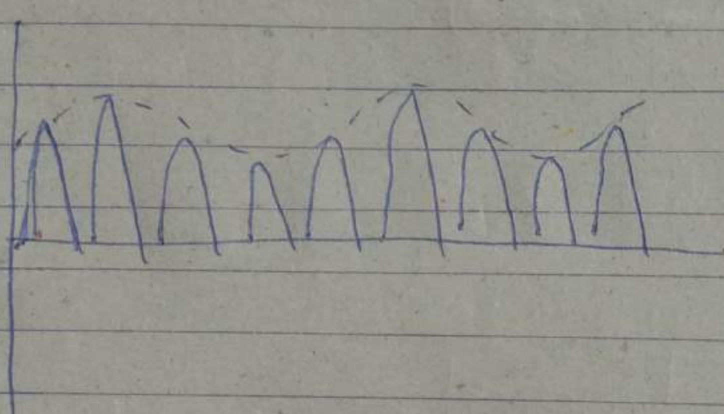


Envelope detector

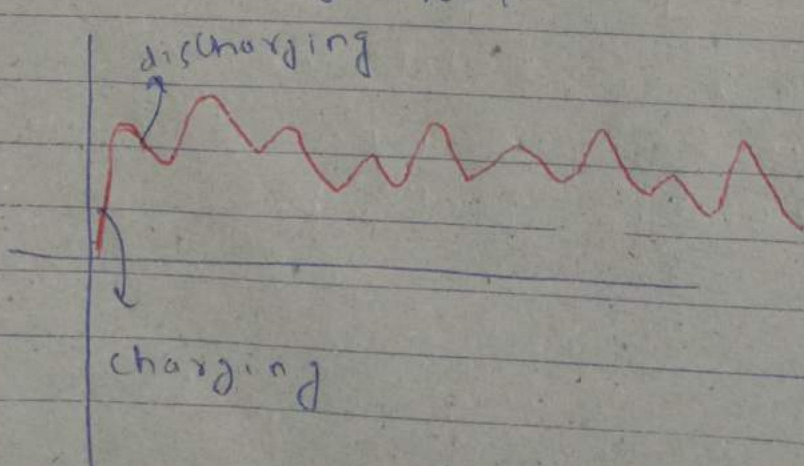
* Simple & cheap detector
* consists of unexpensive discrete elements like diode, R, C



diode (all negative polarities removed)

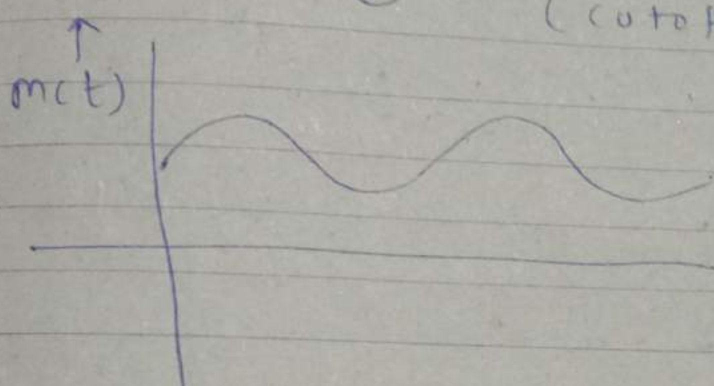


RC = charging / discharging circuit



charging/discharging = leads to distorted envelope consisting of high frequency components

$\Downarrow$ LPF
(cutoff = ω_m)



CHARGING TIME CONSTANT

r_f = forward bias resistance

Charging through route Diode $\rightarrow$ C

$\tau_{charging} = r_f C$

$$\boxed{r_f C \ll \frac{1}{f_c}}$$

DISCHARGING TIME CONSTANT

$$\tau_{discharge} = R \times C$$

discharging through path = $C \to R$

This cannot be too low / too high to properly detect the envelope.

It should be moderate

$$RC \gg \frac{1}{f_c}$$

$$RC \ll \frac{1}{f_m}$$

$$\boxed{\frac{1}{f_c} \ll RC \ll \frac{1}{f_m}}$$

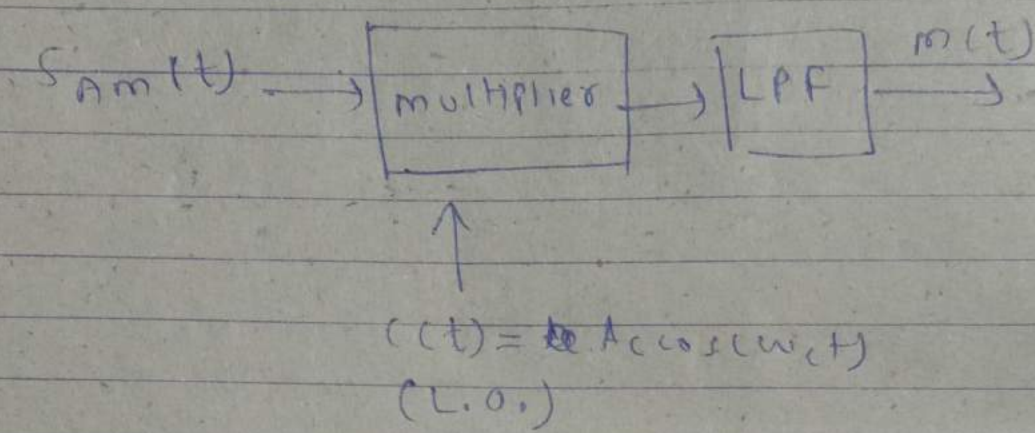
If not this, then discharge very quickly, may lead to distorted envelope

* DSB-C / conventional Am used for ~~brea~~ broad casting since detection technique very cheap

* ~~reuer~~

Lec11 (Demodulation techniques)

SYNCHRONOUS DETECTION
(locally generated carrier signal reqd.)



* $c(t)$ should be perfectly synchronized in phase & frequency to the $c(t)$ at the transmitter end

* generation of carrier signal
  = complex task

ANALYSIS

Case 1: Perfect synchronization of $c(t)$

$$S_{Am}(t) = A_c \cos(w_c t) + A_c m(t) \cos(w_c t)$$

$\cos(\omega_c t) = 2\cos^2(\omega_c t) - 1$

$2\cos(\omega_c t) =$

$S_{AM}(t) \times c(t) = \left[A_c \cos(\omega_c t) + A_c m(t) \cos(\omega_c t)\right]$

$A_c \cos \omega_c t$

$= A_c^2 \cos^2(\omega_c t) + A_c^2 m(t) \cos^2(\omega_c t)$

$= \frac{A_c^2}{2}\left[1 + \cos(2\omega_c t)\right] + \frac{A_c^2 m(t)}{2}(1 + \cos(2\omega_c t))$

$\downarrow$ LPF (ω_m)

$\frac{A_c^2}{2}\cos(2\omega_c t)$ ✗

$\frac{A_c^2 m(t)}{2}$ ✓

$\frac{A_c^2 m(t)}{2}\cos(2\omega_c t)$ ✗

$\frac{A_c^2 m(t)}{2}$ ⇒ required signal = $m(t)$
we are getting scaled version of $m(t)$, scaling factor can be removed.

L.O. $c(t)$ = Locally generated carrier signal

Case 2:

L.O. $c(t) = A_c \cos(\omega_c t + \phi)$

$S_{AM}(t) \times A_c \cos(\omega_c t + \phi)$

$= (A_c \cos(\omega_c t + \phi) \cancel{m(t)} + A_c\, m(t) \cos(\omega_c t + \phi))$

$A_c \cos(\omega_c t + \phi)$

$\Downarrow$ LPF

$\left(\frac{A_c^2}{2} m(t)\right) \cos\phi$ → distortion in demodulated signal ~~varies~~ varied in accordance to ϕ

scaled factor of $m(t)$

if $\phi = 90^\circ$,

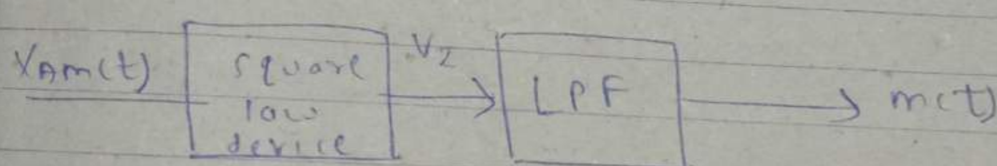
$\cos 90^\circ = 0$, hence total loss of signal.

This condition is called <u>Quadrature null effect</u>

Quadrature → orthogonal phase

null effect → total loss of $m(t)$

SQUARE LAW DETECTOR



$$V_2 = a_1 x_{Am}(t) + a_2 [x_{Am}(t)]^2$$

$\Downarrow$ LPF

↳ eliminates all high frequency components & keeps lower frequency

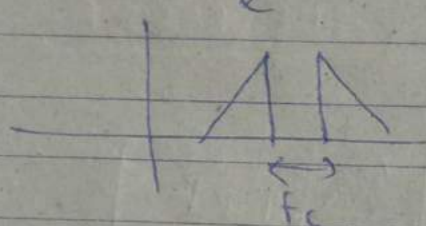
$$\boxed{LPF_{out} = \frac{a_2^2 A_c^2}{2} m(t)}$$

VSB = vestigial side band modulation

SSB-SC = best in case of Power efficiency & ~~Band~~ Transmission efficiency

→ demodulation circuit is complex.

→ limited to voice-signal



sufficient guard band between USB & LSB, so easier to detect

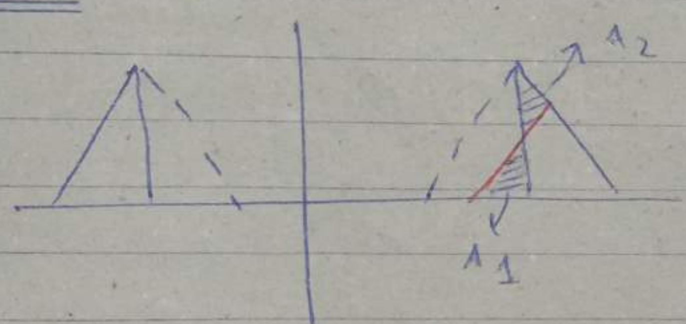
VSB = variation of SSB-SC

VSB

(1) B.W requirement is almost same as the bandwidth of SSB-SC and also allows video signal transmission

* freq. range = 4.5 MHZ
* no guard band between sidebands

(2) VSB

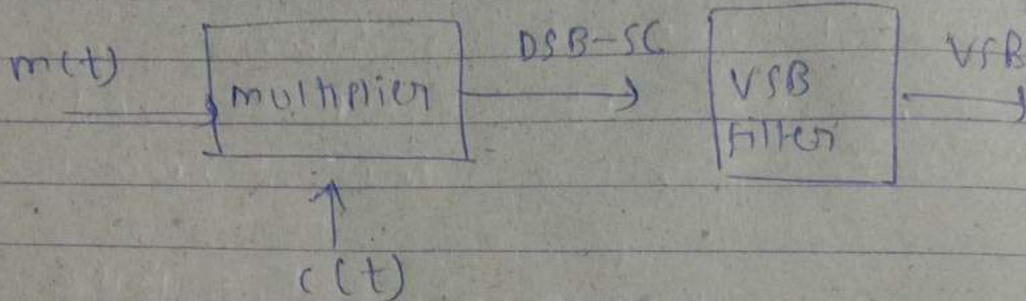


area A_1 = area A_2

* small part of vestige unwanted ~~rear~~ sideband added & equal amount of desired sideband subtracted. so no loss of information.

* helps us to use practical filtering process.

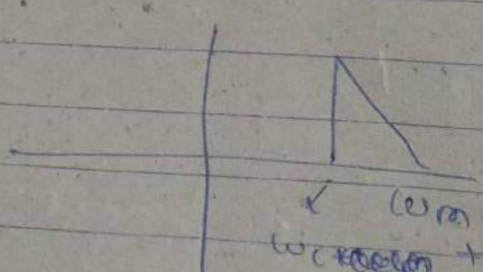
GENERATION OF VSB



Bandwidth requirement

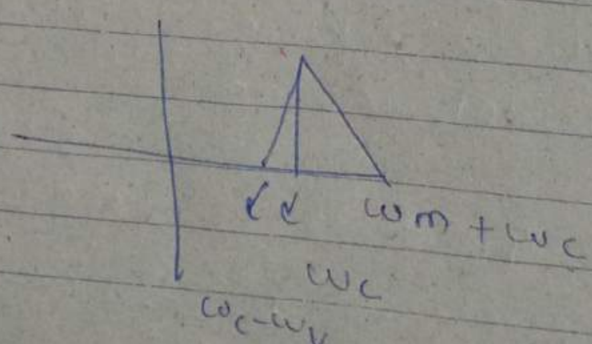
DSB-C & DSB-SC $(2\omega_m)$ > VSB > SSB-SC (ω_m)

but slight difference



$BW = \omega_m$

$= SSB-SC$



$\boxed{BW = \omega_m + \omega_v \quad \omega_v = \text{small}}$

Power requirement

DSB-C > DSB-SC > (VSB &
SSB-SC)

Power
= same

∴ VSB-SC = efficient technique

lec 12

Am receivers

(1) Tuned radio frequency receiver (TRF)

(2) super hetero dyne receiver

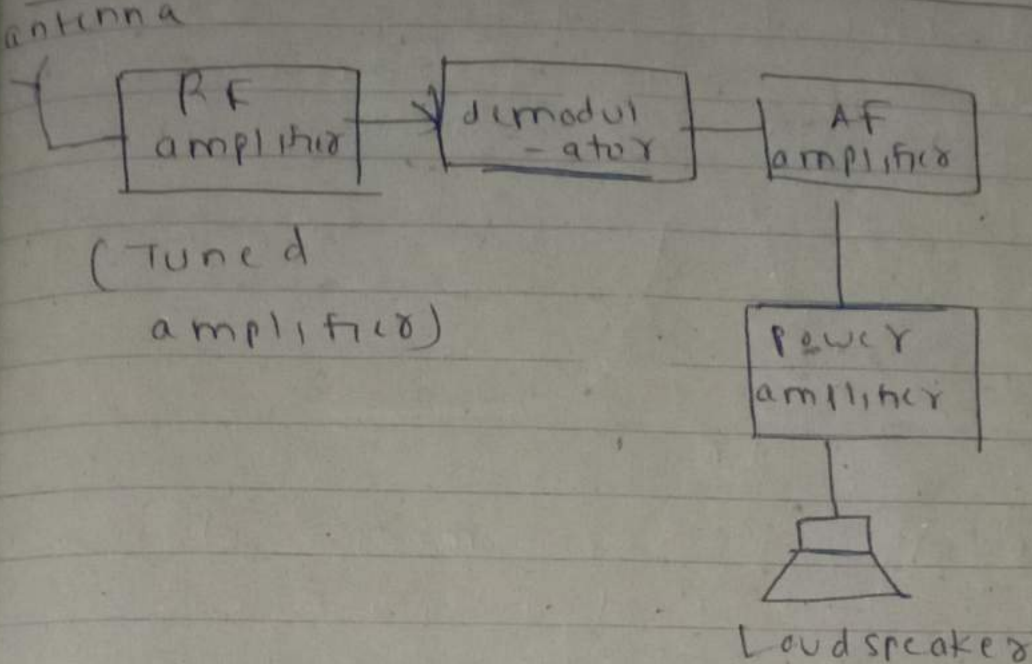
Two properties satisfied by Am receivers are:

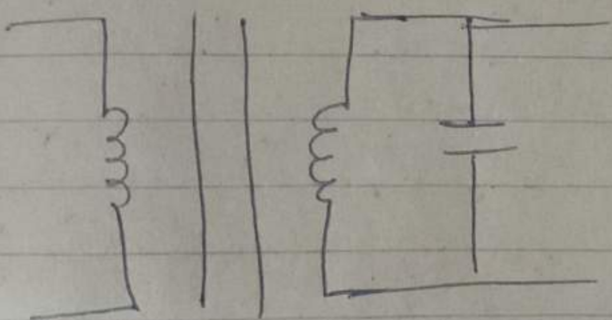
(1) selectivity = ability to select desired signal from

(2) Sensitivity

extracting the information correctly from the signal received

Tuned radio frequency receiver





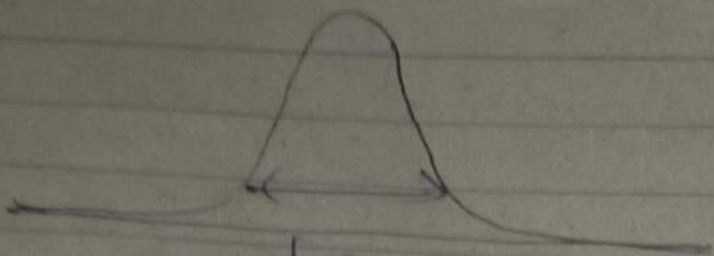
= LC tank circuit for tuning

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

resonating frequency

(~~Take this~~ . allows only single frequency, reject other frequency components)

Range of frequency which the tuned amplifier passes without attenuation = BW of tuned amplifier



f_r

Q = Quality factor = measure of quality of reception

$$\boxed{Q = \frac{f_r}{B.W.}}$$

B.W. ↑, Q ↓

B.W ↓, Q ↑

Q ↑, selectivity boosted ↑

Quality factor gives the sharpness to the gain frequency characteristics.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{f_r}{BW}$$

$$Q = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$$

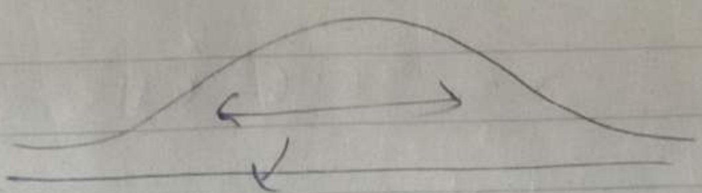
Problem with TRF

(1) Poor selectivity

case 1:

Ex) Assume receiver is tuned to 600 KHZ = Fs

$BW = 10\,KHZ$



$BW = 10K$

$F_S = 600K = F_r$

$$Q = \frac{F_r}{BW} = \frac{600K}{10K} = 60$$

resonant frequency
= central frequency

<u>Case 2:</u> Reciver is tuned to
<u>1000kHz</u> frequency

$f_s = f_r$

$BW = 10k$

$$Q = \frac{f_r}{BW} = \frac{1000kHz}{10k}$$

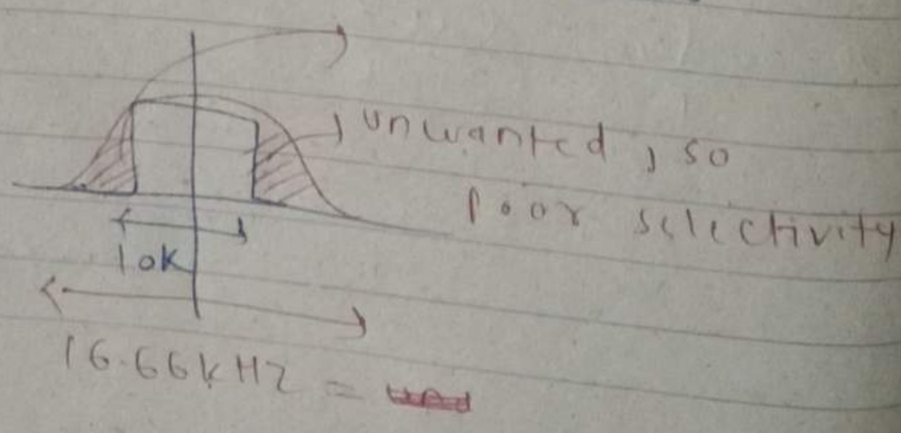
$$\boxed{Q = 100}$$

it is difficult to achieve this

For $Q = 60$, $f_r = 1000k$,

$$BW = \frac{f_r}{Q} = \frac{1000}{60} = 16.66\,kHz$$

Now analysing, actual $BW = 10k$
but obtained $BW = 16.66kHz$, we are getting unwanted signals



superheterodyne receiver

↳ to overcome poor selectivity of TRF

→ use MIXER (down converter)
it converts input frequency
f_i downconverted to 455KHZ

$$f_i \longrightarrow \boxed{\text{mixer}} \longrightarrow 455\text{KHZ}$$

Intermediate
frequency (IF)

→ Tuned amplifier has to be
tuned at about 455KHZ

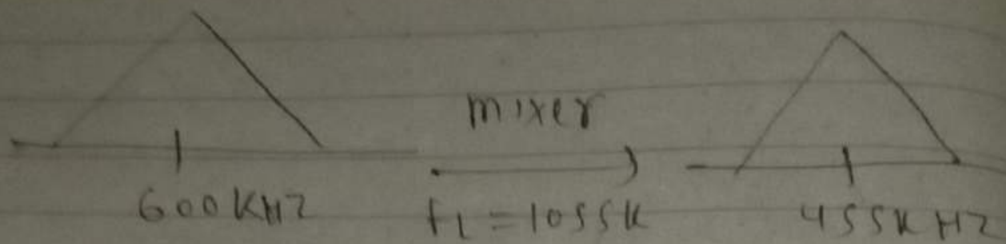
$$Q = \frac{455k}{10k} = 45.5$$

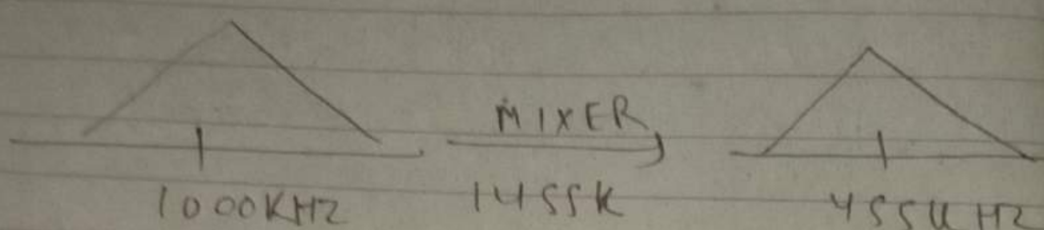
$$\boxed{IF = f_L - f_s}$$

f_s ↳ frequency of input signal

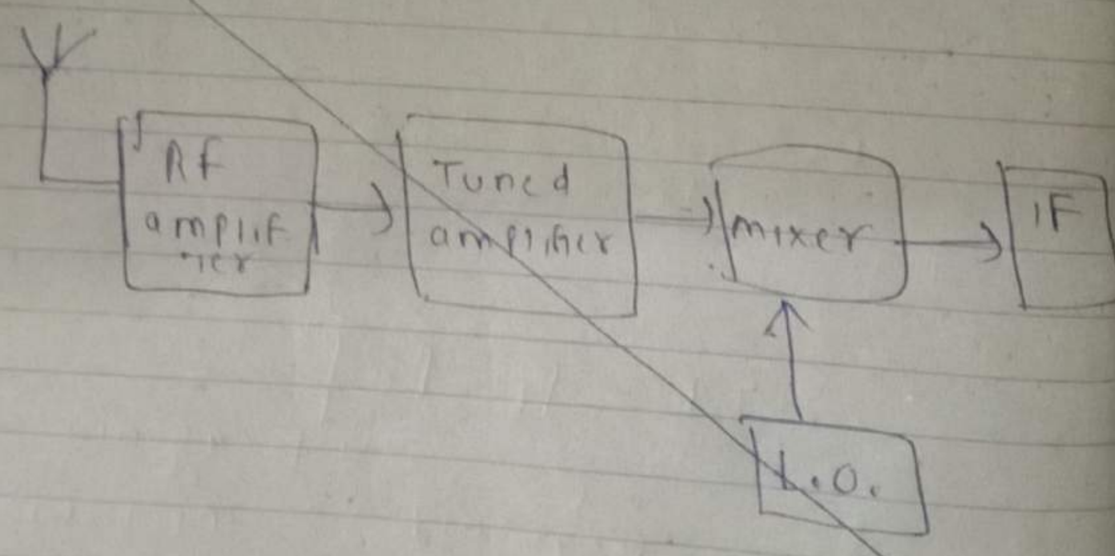
f_L → frequency of local oscillator

→ does not change with
input signal
frequency





~~Block diagram of superheterdyne receiver~~



Block diagram of superheterodyne receiver

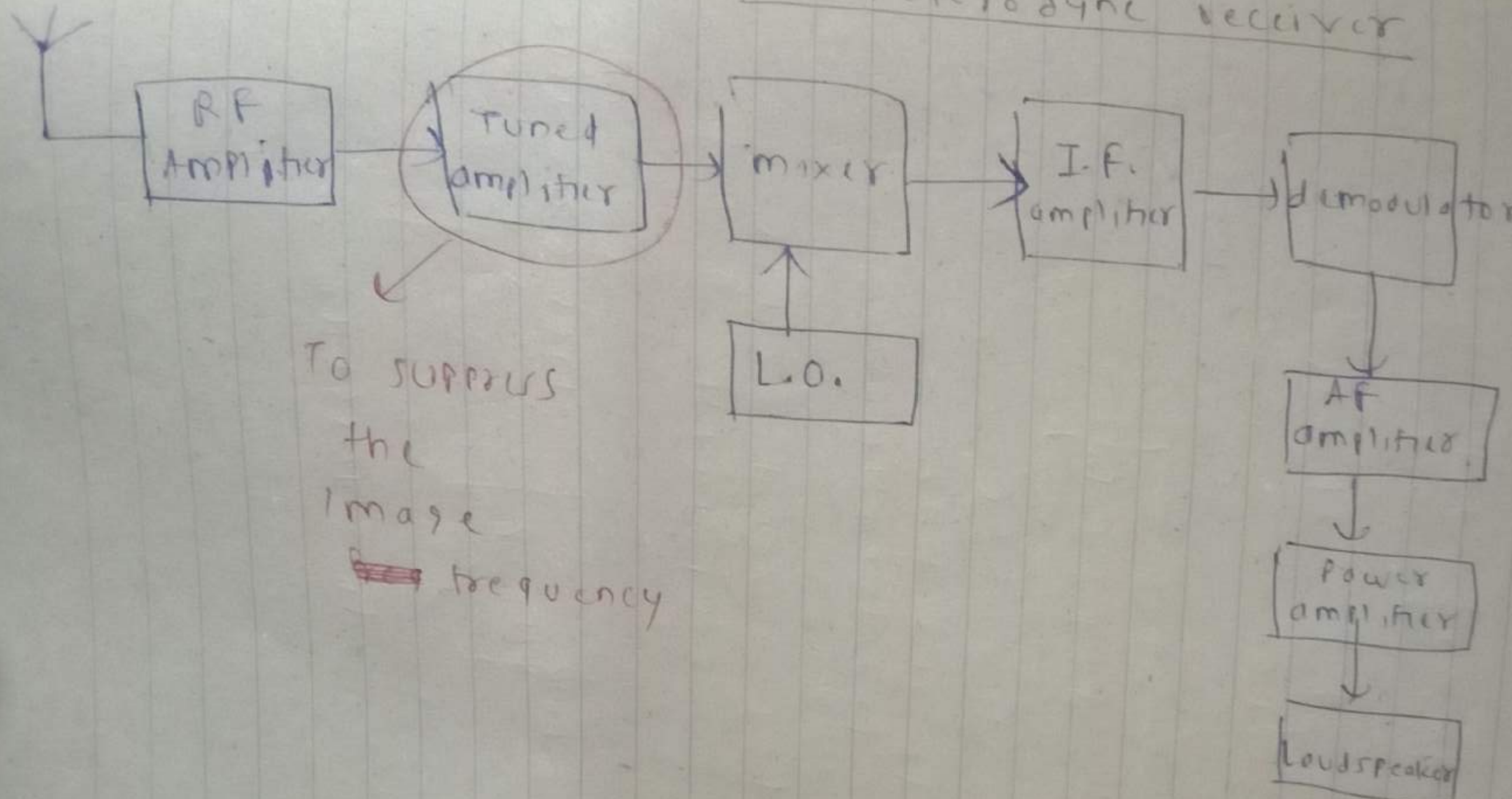
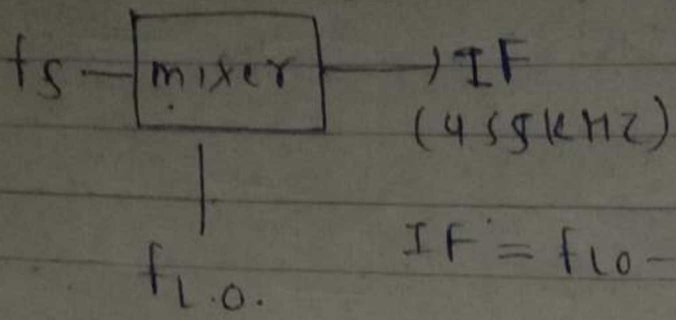


IMAGE FREQUENCY

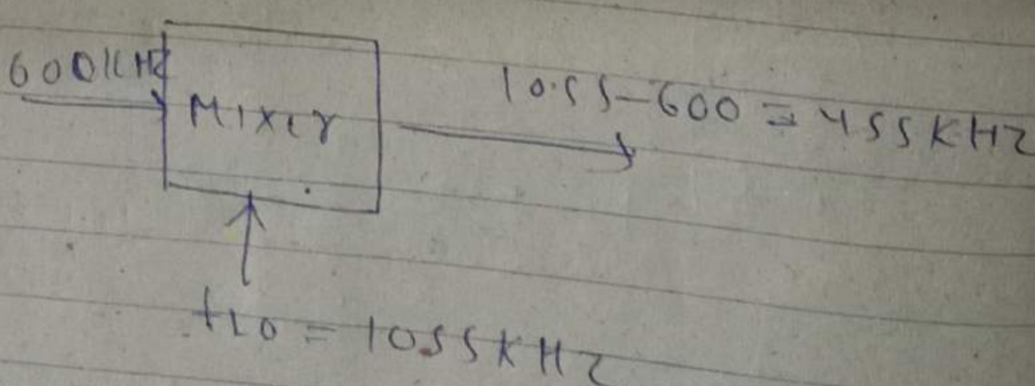


$$IF = f_{LO} - f_s$$

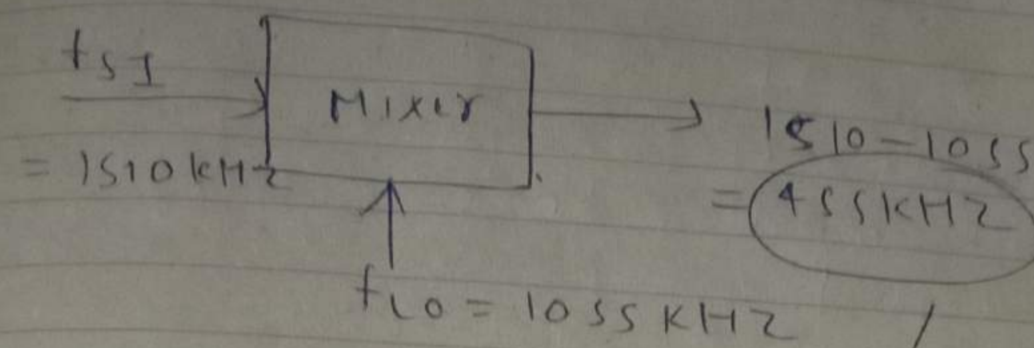
$$\boxed{f_{SI} = f_S + 2\,I.F.}$$

Ex) $f_S = 600\ KHZ$
$I.F. = 455\ KHZ$

$$f_{SI} = 600 + 2 \times 455$$
$$= 1510\ KHZ$$



If we give f_{SI} to mixer



again I.F.,

hence f_{SI} also passed

drawback of superheterodyne receiver

To overcome above drawback, use:

Tuned amplifier = low frequency amplifier which only passes input signal frequency and not the image signal frequency

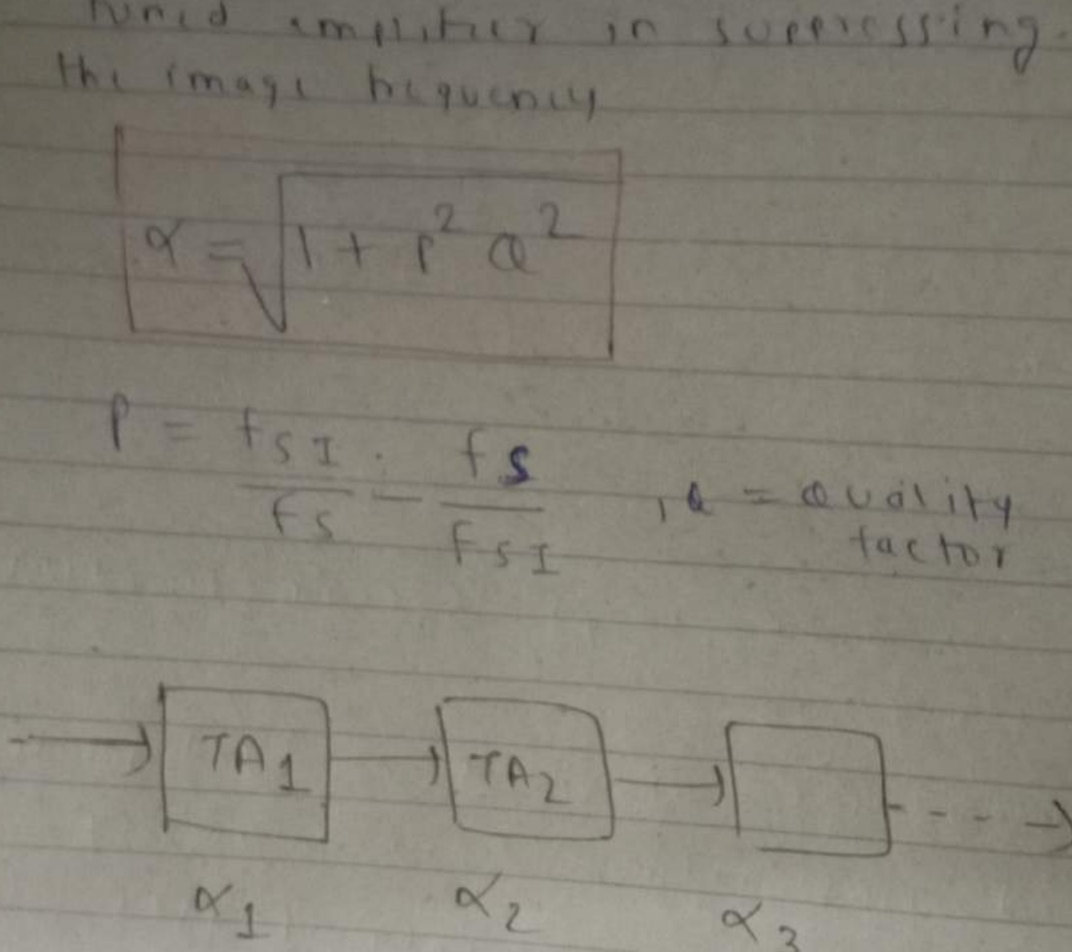
IMAGE REJECTION RATIO (IRR)

α

It is effectiveness of the tuned amplifier in suppressing the image frequency

$$\boxed{\alpha = \sqrt{1 + \rho^2 Q^2}}$$

$$\rho = \frac{f_{SI}}{f_S} - \frac{f_S}{f_{SI}} \quad , Q = \text{Quality factor}$$



$$\boxed{\alpha_{total} = \alpha_1 \alpha_2 \alpha_3 \ldots}$$

Q1) A receiver is tuned to 500 KHZ, Local oscillator frequency is given by 1050 KHZ

find (i) IF

(ii) f_{SI}

(iii) IRR

$Q = 50$?

soln: $f_s = 500$ KHZ

$f_L = 1050$ KHZ

$I.F. = f_L - f_s = 1050 - 500$

$= 550$ KHZ

$f_{SI} = f_s + 2IF$

$= 500 + 2 \times 550$

$= 1600$ KHZ

$\alpha = \sqrt{1 + \rho^2 Q^2}$

$\rho = \frac{f_{SI}}{f_s} - \frac{f_s}{f_{SI}}$

$= \frac{1600}{500} - \frac{500}{1600}$

$= 2.8$

$$\alpha = \sqrt{1+\rho^2 a^2}$$

$$= \sqrt{1+(2.8)^2 \times 50^2}$$

$$= \sqrt{19601}$$

$$\boxed{\alpha = 144.3}$$

EC13 (UNIT-1 — Random var. & Process)

Signals — Deterministic
— Random

Deterministic signals ⇒ reproduced exactly by repeated experiments

* represented by exact ~~mathem~~ mathematical expressions ⇒ we can know the outcome at any instant

For Ex) Unit step signal

Random signals ⇒ cannot be ~~repe~~ ~~reproduced~~ repeated in exact manner, cannot be represented by exact mathematical expressions.

For Ex) Noise in any ~~system~~ signals

Random Signals

↳ can be described in terms of the statistical properties

avg. power, probability density function, cdf

RANDOM VARIABLE

It is a variable whose possible values are the numerical outcomes of random phenomenon.

* Experiment ⇒ Events ⇒ Probability of Events
  (Possible outcomes)

* Sample point = Each possible outcome

* Sample space = Combinely all sample points

Ex)

Throw of dice

$S = 1, 2, 3 - -$

$\{S\} = \{1, 2, 3, 4, 5, 6\}$

R.V.

discrete R.V.	continuous R.V
can take only finite countable no. of discrete values	can take infinite number if distinct values.

Cummulative distribution function (CDF)

"$F_X(x)$"

$$\boxed{F_X(x) = P(X \leq x)}$$

R.V. = X

Properties of C.D.F:

(1) It is bounded b/w 0 & 1.

(2) $F_X(x)$ is non-decreasing function

$$F_X(x_1) \leq F_X(x_2) \text{ if } x_2 \geq x_1$$

(3) $F_X(-\infty) = 0$ (Initial value)

$F_X(\infty) = 1$ (Final value)

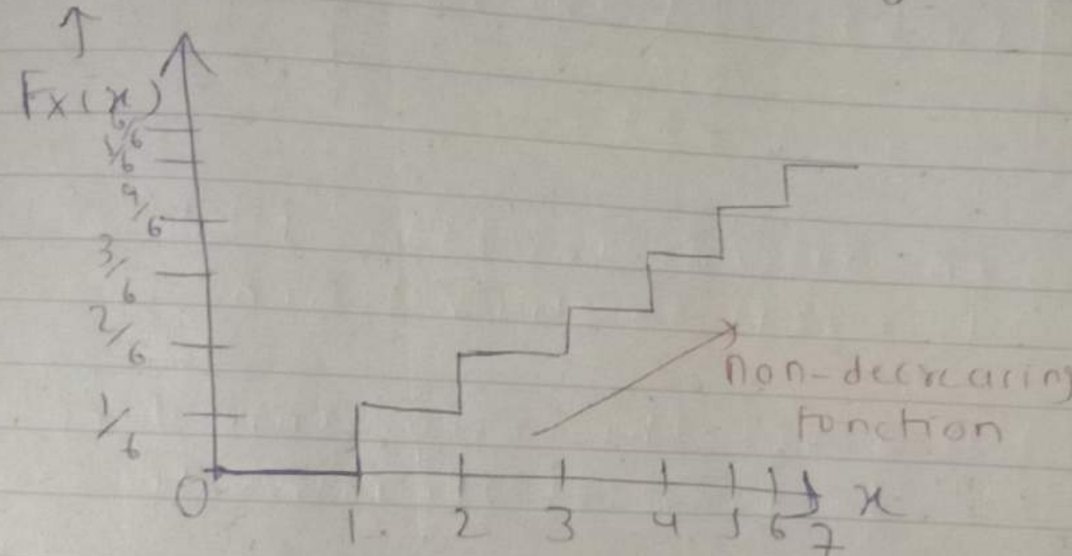
Ex 1) Throw of a dice:

Soln:

X = throw of dice

$X = \{1, 2, 3, 4, 5, 6\}$

$P(1) = P(2) = \cdots P(6) = 1/6$



$F_X(0.5) = P(X \leq 0.5) = 0$

$F_X(1) = P(X \leq 1) = 1/6$

$F_X(1.5) = P(X \leq 1.5) = 1/6$

$F_X(2) = P(X \leq 2) = P(X=1) + P(X=2)$
$= 2/6$

derivative of C.D.F.

Probability density function (PDF)

$f_X(x)$

$$\boxed{f_X(x) = \frac{d}{dx} F_X(x)}$$

$$\boxed{F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx}$$

⇒ An alternative descriptive of probability distribution

⇒ It gives the occurrence density in the range of the intervals.

$x_1 \le X \le x_2$

$$
\begin{aligned}
& P(x_1 \le X \le x_2) \\
&= P(X \le x_2) - P(X \le x_1) \\
&= \cancel{CDF}\ F_X(x_2) - F_X(x_1) \\
&= \int_{x_1}^{x_2} f_X(x)\, dx
\end{aligned}
$$

Properties of PDF:

① $f_X(x) \geq 0$ for all 'x'

Since $F_X(x)$ is non-decreasing, its ~~PDF is not~~ derivative (PDF) is non-negative

② $\boxed{\int_{-\infty}^{\infty} f_X(x)\, dx = 1}$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = \cancel{F_X(x)}\ F_X(\infty) - F_X(-\infty)$$

$$= 1 - 0 = 1$$

③ $F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$

Q1) R.V. X has p.d.f

$$f_X(x) = a e^{-b|x|}$$

$$-\infty < x < \infty$$

a = 3 ?

Find (i) relation b/w a & b

(ii) CDF of R.V. X

(iii) Prob. that X lies between -1 & 2 ?

Soln: $f_X(x) = a e^{-b|x|}$

$$= \begin{cases} a e^{bx}, & -\infty < x < 0 \\ a e^{-bx}, & 0 < x < \infty \end{cases}$$

(i) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$ (Property)

$$= \int_{-\infty}^{0} a e^{bx} + \int_{0}^{\infty} a e^{-bx} = 1$$

$\frac{a}{b}$

$=$

$=$

$a =$

(ii)

Case 1

x

F_X

$$\frac{a}{b} e^{bx}\Big|_{-\infty}^{0} + \left(\frac{a}{-b}\right) e^{-bx}\Big|_{0}^{\infty} = 1$$

$$= \frac{a}{b}\left[e^{0} - e^{-\infty}\right] - \frac{a}{b}\left[e^{-\infty} - e^{0}\right] = 1$$

$$= \frac{a}{b}(1) - \frac{a}{b}(0-1) = 1$$

$$\frac{2a}{b} = 1$$

$$2a = b$$

$$\boxed{b = 2a}$$

$a = 3, \quad b = 6$

(ii) C.D.F. of RV 'x'

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$$

Case 1:

$x < 0$

$$F_X(x) = \int_{-\infty}^{x} a e^{bx}\, dx$$

$$= \frac{a}{b}\left[e^{bx}\right]_{-\infty}^{x} = \frac{a}{b}\left[e^{bx}\right]$$

$$= \frac{1}{2} e^{bx}, \; x < 0$$

Case 2 : $x > 0$

$$F_X(x) = \int_{-\infty}^{0} f_X(x)\,dx + \int_{0}^{x} a e^{-bx}\,dx \quad \text{(iii)}$$

$$= \frac{1}{2} e^{bx}\Big|_{-\infty}^{0} + \left(\frac{a}{-b}\right) e^{-bx}\Big|_{0}^{x}$$

$$= \cancel{\frac{1}{2}(1) - 0 - \frac{1}{2}\left[e^{-bx}\right]}$$

$$= \frac{1}{2} - \frac{1}{2}\left[e^{-bx} - 1\right]$$

$$= \frac{1}{2}\left[1 - e^{-bx} + 1\right]$$

$$= \frac{1}{2}\left[2 - e^{-bx}\right], \; x > 0$$

$$= 1 - \frac{1}{2} e^{-bx}, \; x > 0$$

$$F_X(x) = \begin{cases} \frac{1}{2} e^{bx} & x < 0 \\ 1 - \frac{1}{2} e^{-bx}, & x > 0 \end{cases}$$

(iii) $P(-1 \leq X \leq 2)$

$$= \cancel{F_X(2)} \int_{-1}^{2} f_X(x)\,dx$$

$$= \int_{-1}^{0} a e^{bx}\,dx + \int_{0}^{2} a e^{-bx}\,dx$$

$$= \frac{a}{b}\left[e^{bx}\right]_{-1}^{0} - \frac{a}{b}\left[e^{-bx}\right]_{0}^{2}$$

$$= \frac{1}{2}\left[1 - e^{-6}\right] - \frac{1}{2}\left[e^{-12} - 1\right]$$

$$= \frac{1}{2}\left[1 - e^{-6} - e^{-12} + 1\right]$$

$$\frac{1}{2}\left[2 - e^{-6} - e^{-12}\right]$$

$$= 1 - \frac{1}{2} e^{-6}\left[1 + e^{-6}\right]$$

Ans

ECE14

STATISTICAL AVERAGES

To determine the average behaviour of the outcomes of random experiment

MEAN VALUE / EXPECTED VALUE OF RV X

also called first moment of X

$$m_X = E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

$E[\,] =$ Expectation operator

PROPERTIES:

* $E[cX] = cE[X]$, c = constant
* $E[X + c] = E[X] + c$
* $E[c] = c$

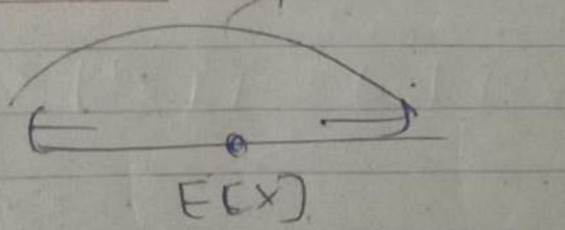
n^{th} moment of prob distribution of R.V. X

$$\boxed{E[X^n] = \int_{-\infty}^{\infty} x^n f_X(x)\,dx}$$

Mean square value of X : (average power)

$$\boxed{E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx}$$

VARIANCE OF R.V. X



✓ spread of R.V. (X) around mean value E[X]

$$Var(X) = E[(X - m_X)^2]$$

✓ mean / E[X]

$$\boxed{\operatorname{var}(x) = \int_{-\infty}^{\infty} (x - m_x)^2 f_x(x)\, dx}$$

$$\operatorname{Var}[X] = E[(X - m_X)^2]$$

$$= E[X^2 + m_X^2 - 2X m_X]$$

$$= E[X^2] + m_X^2 - 2m_X E[X]$$

$$(E[C] = C, \; E[m_X^2] = m_X^2)$$

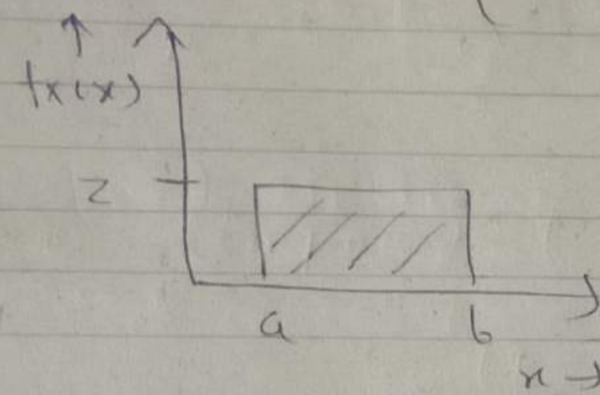
$$= E[X^2] + m_X^2 - 2m_X^2$$

$$\boxed{\operatorname{Var}[X] = E[X^2] - m_X^2}$$

Q1) Find mean & variance of R.V. X which is uniformly distributed between a & b, b > a ?

Soln: $X \sim U(a, b)$

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \end{cases}$$



$$z(b-a) = 1$$

$$z = \frac{1}{b-a}$$

$$m_x = E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

$$= \int_a^b x \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \times (b-a) \times \frac{1}{2}(b+a)$$

$$E[x] = \frac{b+a}{2}$$

$$var[x] = E[X^2] - m_x^2$$

$$E[x^2] = \int_a^b x^2 \cdot \frac{1}{b-a} \, dx - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{1}{b-a} \left(\frac{b^3 - a^3}{3}\right) - \frac{(a+b)^2}{4}$$

$$= \frac{(b-a)(b^2+a^2+ab)}{3(b-a)} - \frac{(a+b)^2}{4}$$

$$= \cancel{\frac{b^2+a^2+ab}{12}}$$

$$= \frac{b^2+a^2+ab}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{4b^2+4a^2+4ab-3a^2-3b^2-6ab}{12}$$

$$a^3 - b^3 = (a-b)(a^2+b^2+ab)$$
$$= a^3 + \cancel{ab^2} - a^2b - ba^2$$
$$- b^3 + \cancel{ab^2}$$
$$a^3 + \cancel{ab^2} + \cancel{a^2b} - \cancel{ba^2} - b^3 - \cancel{ab^2}$$

$$= \frac{a^2 + b^2 - 2ab}{12}$$

$$= \frac{(a-b)^2}{12}$$

$$\boxed{\mathrm{var}(X) = \frac{(a-b)^2}{12} =} \frac{(b-a)^2}{12}$$

$$\to \sigma_x^2$$

MEAN & VARIANCE OF SUM OF R.V.

Let 'x' & 'y' be two independent R.V. with m_x & m_y

let $z = m_x + m_y$

find m_z in terms of m_x & m_y?

$$\boxed{m_z = m_x + m_y}$$

$$z = x + y$$

$$m_z = E[(x+y)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y)\, f_{xy}(x,y)\, dx\, dy$$

x & y are independent R.V.

$$\therefore \boxed{f_{xy}(x,y) = f_x(x) \cdot f_y(y)}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f_{xy}(x,y)\, dx\, dy$$

$$+ \int_{-\infty}^{\infty}\int_{\infty}^{\infty} y\, f_{xy}(x y)\, dx\, dy$$

$$\boxed{m_z = m_x + m_y}$$

Var[Z] :

$$E[Z^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y)^2 \underline{F_{xy}(x,y)}\, dx\, dy$$

$\hookrightarrow f_x(x)\, f_y(y)$

$$1 \quad = \int_{-\infty}^{\infty} x^2 f_x(x)\, dx \int_{-\infty}^{\infty} f_y(y)\, dy$$

$$+ \int_{-\infty}^{\infty} y^2 f_x(y)\, dy \cdot \int_{-\infty}^{\infty} f_x(x)\, dx$$

$$+ 2\int_{-\infty}^{\infty} x f_x(x)\, dx \int_{-\infty}^{\infty} y f_y(y)\, dy$$

$$= E[x^2] + E[y^2] + 2 m_x m_y$$

$$\text{Var}(Z) = E[Z^2] - m_z^2$$

$= $ [illegible]

$$E[x^2] = \sigma_x^2 + m_x^2$$
$$E[y^2] = \sigma_y^2 + m_y^2$$
$$E[Z^2] = \sigma_x^2 + m_x^2 + \sigma_y^2 + m_y^2 + 2 m_x m_y$$

$$E[Z^2] = \sigma_x^2 + \sigma_y^2 + (m_x + m_y)^2$$

$$E[Z^2] = \sigma_x^2 + \sigma_y^2 + m_z^2$$

$$E[Z^2] - m_z^2 = \sigma_x^2 + \sigma_y^2$$

$$\Downarrow$$

$$\sigma_z^2$$

$$\boxed{\sigma_z^2 = \sigma_x^2 + \sigma_y^2}$$

lec 15

* PDF of sum of RV's

X, Y = 2 independent R.V.

$Z = X + Y$

* $m_Z = m_X + m_Y$

* $\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$

$$\boxed{\begin{aligned} f_X(x) &= \text{pdf of } X \\ f_Y(y) &= \text{pdf of } Y \end{aligned}}$$

find PDF of Z ?, $Z = X + Y$

$$\boxed{f_Z(z) = f_X(x) * f_Y(y)}$$

convolution

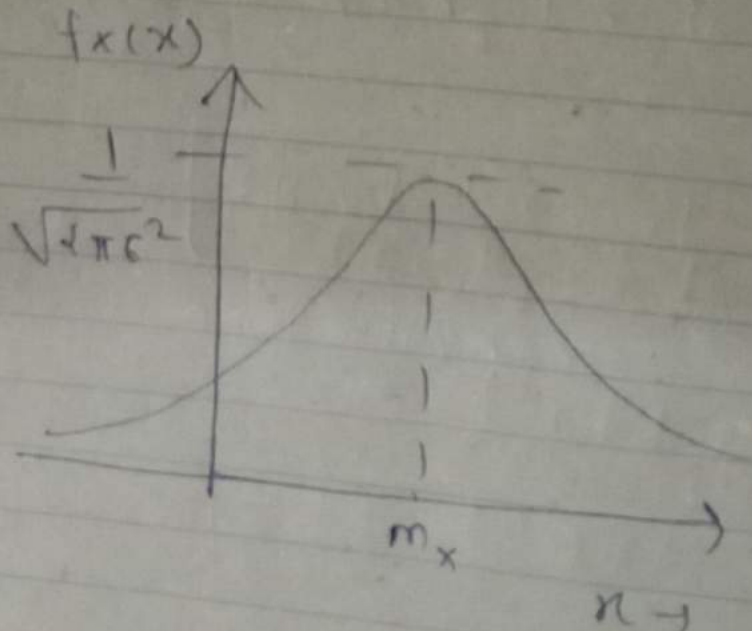
(Normal distribution)

GAUSSIAN DISTRIBUTION

Gaussian PDF is of great importance in communication scenario because many naturally occurring experiments are characterized by R.V. with gaussian density.

$$\boxed{f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-m_x)^2/2\sigma^2}}$$

σ^2 = Variance
m_x = mean



$$\boxed{X \sim N(\mu, \sigma^2)}$$

$$\boxed{X \sim N(m_x, \sigma_x^2)}$$

Notation

TRANSFORMATION OF R.V.

$$Y = f(X)$$

* Pdf of Y given Pdf of X:

- $h(y) = f^{-1}(y)$

$$\boxed{f_Y(y) = f_X(h(y)) \times \left| \frac{dh}{dy} \right|}$$

Q1) $X \sim N(0, 1)$

mean ↙ ↳ variance

$$Y = e^X$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Find $f_Y(y)$?

Soln:

$$Y = e^X$$

$$h(y) = f^{-1}(y) = \ln y = X$$

$$\frac{dh}{dy} = \frac{1}{y}$$

$$\boxed{f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln y)^2}{2}} \cdot \frac{1}{y}}$$

gives variation in R.V.

COVARIANCE B/W 2 R.V. (μ)

x & y

$$\mu = E\{(x - m_x)(y - m_y)\}$$

m_x = mean of 'x'

m_y = mean of 'y'

(1) When 'x' & 'y' are independent R.V.

$$= \int_{-\infty}^{\infty} (x - m_x) f_x(x)\, dx \int_{-\infty}^{\infty} (y - m_y) f_y(y)\, dy$$

$$\mu = (m_x - m_x)(m_y - m_y) = 0$$

(2) 'x' & 'y' are dependent R.V.

$X = Y$ OR $X = -Y$

$m_x = m_y = 0$

$$\mu = E[XY] = E[X^2] = E[Y^2]$$

$$= \sigma_x^2 = \sigma_y^2 = \sigma_x \sigma_y$$

extent to which x & y
dependent

* ρ = correlation coefficient

$$\boxed{\rho = \frac{\mu}{\sigma_x \sigma_y}}$$

$$\boxed{-1 \leq \rho \leq 1}$$

If $\rho = 0$, x & y = independent

when x & y are independent in ~~nature~~ ($\mu = 0$), x & y are uncorrelated but vice-versa not true.

✓

cannot say that when x & y = uncorrelated, they may not be independent

Q1) Let Z = RV with pdf

$$f_Z(z) = \frac{1}{2}, \quad -1 \leq z \leq 1.$$

Let $X = Z$ & $Y = Z^2$

obviously X & Y not independent, since $X^2 = Y$. Show that X & Y are uncorrelated.

Soln:

$$E[Z] = \int_{-1}^{1} \frac{1}{2} \cdot z = \frac{1}{2}[z^2]_{-1}^{1} = 0$$

$$E[X] = E[Z] = 0$$

$$E[Y] = E[Z^2] = \int_{-1}^{1} \frac{1}{2} z^2 \, dz$$

$$= \frac{1}{3}$$

$$\mu = E[(X - m_x)(Y - m_x)]$$

$$= E[X(Y - 1/3)]$$

$$= E[XY - 1/3 X)$$

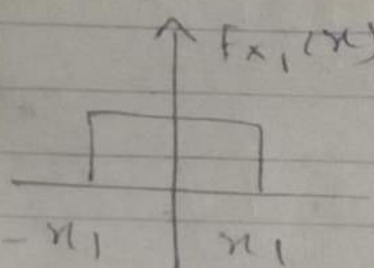
$$= E[Z^3 - Z/3]$$

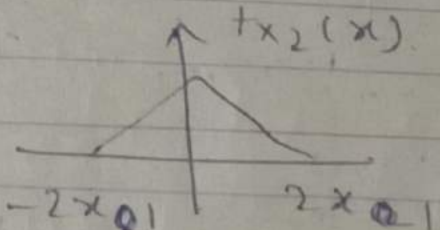
lec 16

CENTRAL LIMIT THEOREM

→ Based on concept of pdf of sum of R.v.

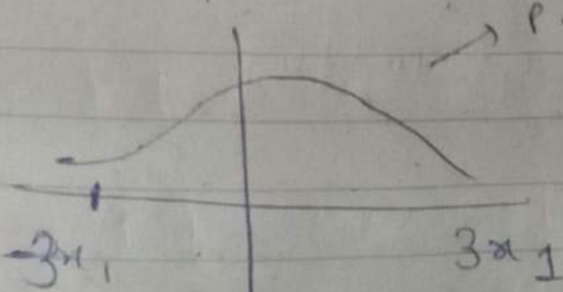
→ states that PDF of sum of 'n' independent RV tends to approach gaussian distribution as 'n' increases





$$z = x_1 + x_2$$

⇓



pdf approach to gaussian distribution

Random var = outcome of each RV is number

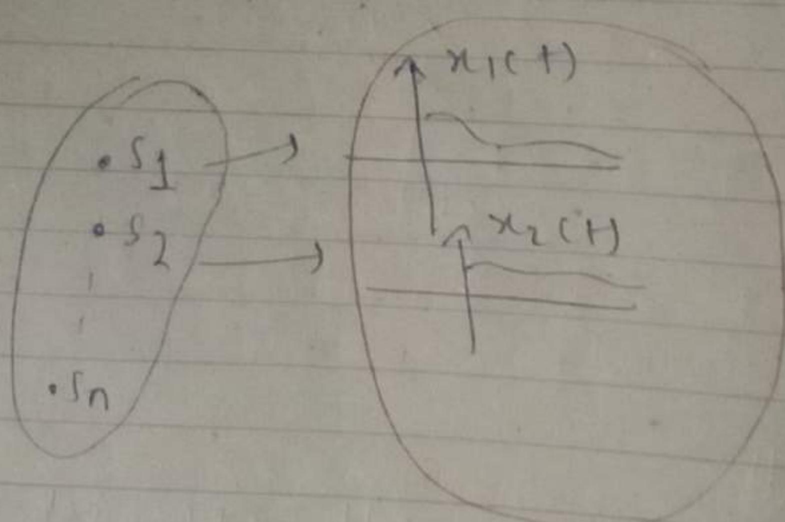
outcome of each process = waveform

RANDOM PROCESS

* Time domain representation of RV

* It is a RV which is function of time

* In describing Random signals, each sample point in our sample space = function of time



Ensemble

R.P = Random process

ENSEMBLE AVERAGE

$$= E[n^2(t_1)] = \overline{n^2(t_1)}$$

TIME AVERAGE $= \langle n^2(t) \rangle$

In general time average & ensemble average different

STATIONARITY = If the statistical characteristics does not change with time, Random process is stationary

ERGODIC R.P. = ensemble average & time average same

& stationary in nature

"An ergodic R.P. is stationary but vice-versa not true"

PSD = Power spectral density

ACF = Autocorrelation function

MEAN OF R.P.

$$\boxed{\bar{X}(t) = \int_{-\infty}^{\infty} x \, f_X(x, t) \, dx}$$

ACF & PSD of RP

$$R_X(\tau) = E[X(t+\tau)X(t)]$$

$$\boxed{R_X(0) = E[X^2(t)]}$$

∴ mean square value of the random process can be obtained from ACF at $\tau = 0$

* $R_X(\tau)$ is even function of τ

$$\boxed{R_X(\tau) = R_X(-\tau)}$$

* $R_X(\tau)$ has maximum magnitude at $\tau = 0$

$$\boxed{|R_x(\tau)| \leq R_x(0)}$$

Wiener-Khintchin theorem

PSD = FT(ACF)

, FT = fourier transform

$$\boxed{R_x(\tau) \overset{F.T.}{\longleftrightarrow} PSD}$$

Classification of Random Process

① Stationary R.P.

* A random process whose statistical characteristics does not change with time

* ACF of stationary R.P must depend on the time difference

$$\boxed{R_x(t_1, t_2) = R_x(t_2 - t_1)}$$

(2) Wide-sense (weakly) stationary R.P.

* process that is not stationary
* ~~only mean~~
in strict sense & may have mean & ACF that are independent of shift of time origin

* mean $(\bar{x}(t))$ = constant

* $R_X(\tau) = R_X(t_1, t_2)$

↳ depends on time difference