The LNM Institute of Information Technology
Jaipur, Rajsthan

P&S

Quiz-I FEBRUARY 04, 2020

Time: 40 Minutes Maximum Marks: 10

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1. A coin is selected from two coins, where the probability of selecting each coin is $\dfrac{1}{2}$. The probability of obtaining head for one of them is $\dfrac{1}{3}$ and for other it is $\dfrac{1}{2}$. If the selected coin is tossed and the head shows up, what is the probability that it is a fair coin?

Solution: Let F denotes the event that fair coin is selected (i.e. $P(H|F) = 1/2$) and B denotes the event that the biased coin is selected (i.e. $P(H|B) = 1/3$)). It is given that $P(F) = P(B) = \frac{1}{2}$. Want to compute $P(F|H)$. Baye's theorem

$$P(F|H) = \frac{P(H|F)P(F)}{P(H|F)P(F) + P(H|B)P(B)} = \frac{1/4}{1/4 + 1/6} = \frac{1/4}{5/12} = 3/5$$

2. Two coins with probability of heads u and v, respectively, are tossed independently. If $P($ both coins shows up tails$) = P($ both coins shows up heads$)$, then $u + v$ equals

   (a) $\frac{1}{4}$
   (b) $\frac{1}{2}$
   (c) * 1
   (d) $\frac{3}{4}$

Solution: Using independence, we get

$$P(HH) = uv, P(TT) = (1 - u)(1 - v)$$

Hence $uv = 1 - v - u - uv \implies u + v = 1$.

3. Let $\Omega = (0, 1]$ be the sample space and let $P(\cdot)$ be a probability function defined by

$$P((0, x]) = \begin{cases} \dfrac{x}{2} & \text{if} \quad 0 \leq x < \dfrac{1}{2} \\ x & \text{if} \quad \dfrac{1}{2} \leq x \leq 1 \end{cases}$$

Then

(a) * $P\left(\left\{\frac{1}{2}\right\}\right) = \frac{1}{4}$ and $P\left(\left\{\frac{3}{4}\right\}\right) = 0$.

(b) If $P\left(\left\{\frac{1}{2}\right\}\right) = \frac{1}{4}$ then $P\left(\left\{\frac{3}{4}\right\}\right) = \frac{1}{4}$.

(c) * $P\left(\left\{\frac{1}{2}\right\}\right) = \frac{1}{4}$ and $P\left(\left\{\frac{1}{4}\right\}\right) = 0$.

(d) $P\left(\left\{\frac{1}{2}\right\}\right) = 0 = P\left(\left\{\frac{3}{4}\right\}\right)$.

Solution: Define $I_n := \left(\frac{1}{2} - \frac{1}{n}, \frac{1}{2} + \frac{1}{n}\right]$. Note that, for $n \geq 2$, $I_n \subseteq \Omega$ and $\bigcap_{n=2}^{\infty} I_n = \left\{\frac{1}{2}\right\}$. Hence by continuity property of probability function we have

$$
\begin{aligned}
P\left(\left\{\frac{1}{2}\right\}\right) &= \lim_{n\to\infty} P(I_n) \\
&= \lim_{n\to\infty} \left(P\left(0, \frac{1}{2} + \frac{1}{n}\right] - P\left(0, \frac{1}{2} - \frac{1}{n}\right]\right) \\
&= \lim_{n\to\infty} \left[\frac{1}{2} + \frac{1}{n} - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n}\right)\right] \\
&= \frac{1}{4}
\end{aligned}
$$

Define $I_n := \left(\frac{1}{4} - \frac{1}{n}, \frac{1}{4} + \frac{1}{n}\right]$. Note that, for $n \geq 5$, $I_n \subset (0, 1/2)$ and $\bigcap_{n=5}^{\infty} I_n = \left\{\frac{1}{4}\right\}$. Hence by continuity property of probability function we have

$$
\begin{aligned}
P\left(\left\{\frac{1}{4}\right\}\right) &= \lim_{n\to\infty} P(I_n) \\
&= \lim_{n\to\infty} \left(P\left(0, \frac{1}{4} + \frac{1}{n}\right] - P\left(0, \frac{1}{4} - \frac{1}{n}\right]\right) \\
&= \lim_{n\to\infty} \left[\frac{1}{8} + \frac{1}{2n} - \frac{1}{2}\left(\frac{1}{4} - \frac{1}{n}\right)\right] \\
&= \lim_{n\to\infty} \frac{1}{4n} = 0
\end{aligned}
$$

Define $I_n := \left(\frac{3}{4} - \frac{1}{n}, \frac{3}{4} + \frac{1}{n}\right]$. Note that, for $n \geq 4$, $I_n \subset [1/2, 1]$ and

$\bigcap_{n=4}^{\infty} I_n = \left\{\frac{3}{4}\right\}$. Hence by continuity property of probability function we have

$$\begin{aligned}
P\left(\left\{\frac{3}{4}\right\}\right) &= \lim_{n\to\infty} P(I_n) \\
&= \lim_{n\to\infty} \left(P\left(0, \frac{3}{4} + \frac{1}{n}\right] - P\left(0, \frac{3}{4} - \frac{1}{n}\right]\right) \\
&= \lim_{n\to\infty} \left[\frac{3}{4} + \frac{1}{n} - \left(\frac{3}{4} - \frac{1}{n}\right)\right] \\
&= \lim_{n\to\infty} \frac{1}{2n} = 0
\end{aligned}$$

4. Let E and F be two events defined on an experiment with $0 < P(E), P(F) < 1$. We have following three statements related to the conditional probabilites:

   (1) $P(E|F) + P(E^c|F^c) = 1$.

   (2) $P(E|F) + P(E|F^c) = 1$.

   (3) $P(E|F) + P(E^c|F) = 1$.

   Then out of the following statements, which are true and which are false:

   (a) (1) and (3) are true and (2) is false.

   (b) (1) is true and (2) and (3) are false.

   (c) All $(1), (2)$ and (3) are false.

   (d) * (3) is true and (1) and (2) are false

5. Choose integers x and y, independently at random, from amongst the integers 1 to 9 (inclusive). If $x + y > 12$, determine the conditional probability that at least one of the integers x and/or y is greater than 7.

6. Out of 100 coins one has heads on both sides. One coin is chosen at random (i.e., each coin has equal probability of getting picked up) and flipped two times. What is the probability to get two heads?