

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY  
 DEPARTMENT OF MATHEMATICS  
 PROBABILITY AND STATISTICS: MTH221  
 QUIZ-1: SOLUTION

1. Choose integers  $x$  and  $y$ , independently at random, from amongst the integers 1 to 9 (inclusive). If  $x + y > 12$ , determine the conditional probability that at least one of the integers  $x$  and/or  $y$  is greater than 7.

Sol. Let  $A = \{x + y \geq 13\}$ ,  $B = \{x \geq 8\}$ ,  $C = \{y \geq 8\}$ . Then

$$\begin{aligned} P[(B \cup C)|A] &= \frac{P[(B \cup C)A]}{P(A)} = \frac{P(BA \cup CA)}{P(A)} \\ &= \frac{P(BA) + P(CA) - P(ABC)}{P(A)} \\ &= \frac{11 + 11 - 4}{21} = \boxed{0.8095}. \end{aligned}$$

2. Out of 100 coins one has heads on both sides. One coin is chosen at random and flipped two times. What is the probability to get two heads?

Sol. Let

$A$  : be the event that two heads are obtained.

$H_1$  be the event that a fair coin was chosen, and

$H_2$  : be the event that the two-headed coin was chosen.

Its clear that  $H_2 = H_1^c$ . Thus by Total probability theorem, we have

$$P(A) = P(A|H_1)P(H_1) + P(A|H_2)P(H_2) = \frac{1}{4} \times \frac{99}{100} + 1 \times \frac{1}{100} = \boxed{\frac{103}{400} = 0.2575}.$$