Lecture-12:B

22 January 2021 16:53

Lecture - 12

Independence of Three events :-

$$(S, F, P)$$

$$A_1, A_2, A_3 \in F$$

then A_1, A_2 and A_3 are said to be independent if

(i) $P(A_i A_j) = P(A_i) P(A_j)$, $i \neq j$ $\quad \boxed{AB = A \cap B}$

(ii) $P(A_1 A_2 A_3) = P(A_1) P(A_2) P(A_3)$

$$^3C_2 + {}^3C_3 = 2^3 - (3+1) = 4$$

we required four equations to establish the independence of 3 events.

Ex. :- We rolled two fair dice.

A : The sum of points appear on the dice is 7.

B : Point 3 came up on the die 1

C : Point 4 appears on the die 2.

$$P(A) = \frac{6}{36}, \quad P(B) = \frac{1}{6}, \quad P(C) = \frac{1}{6}$$

$$A \cap B = \{3, 4\}, \quad P(A \cap B) = \frac{1}{36}$$

$$P(A)P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$P(A \cap B) = P(AB) = P(A)P(B)$$

$$B \cap C = \{3,4\}, \quad P(B \cap C) = \frac{1}{36}$$

$$P(B)P(C) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$P(BC) = P(B)P(C)$$

$$AC = \{3,4\}, \quad P(AC) = P(A)P(C)$$

$$ABC = BC = \{3,4\}$$

$$\Longrightarrow P(ABC) = \frac{1}{36}$$

$$P(A)P(B)P(C) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$$P(ABC) \neq P(A)P(B)P(C)$$

So we can say that events A, B, C are mutually pairwise independent, but not ~~m~~ independent.

<u>Note</u>:- (1) Pairwise independence only does not imply the independence of three events.

(2) If three events are independent then any one is independent of intersection

any one is independent of intersection of the remaining two.

<u>For example</u>:- A_1, A_2, A_3 are independent.

$$P[A_1(A_2A_3)] = P[A_1A_2A_3]$$
$$= P(A_1)\,P(A_2)\,P(A_3)$$
$$= P(A_1)\,[P(A_2A_3)]$$

$\Rightarrow$ A_1 and (A_2A_3) are independent.

(3) If three events are independent then any triple of events obtained by replacing one or more by their complements are also independent, i.e. if

A_1, A_2, A_3 are independent then B_1, B_2, B_3 are also independent, where $B_i = A_i$ or $\bar{A}_i$

$i = 1, 2, 3$ —

$A_1A_2\bar{A}_3$, $A_1\bar{A}_2A_3$, $\bar{A}_1A_2A_3$, $A_1\bar{A}_2\bar{A}_3$, $\bar{A}_1\bar{A}_2A_3$, $\bar{A}_1A_2\bar{A}_3$, $\bar{A}_1\bar{A}_2\bar{A}_3$,

<u>Eg</u>:- A_1, A_2, A_3 are independent $\Rightarrow$ A_1, A_2 and $\bar{A}_3$ are independent.

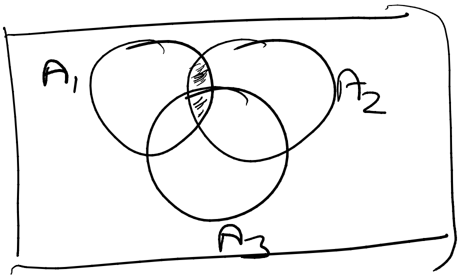
i.e. I want to show that

$$P(A_1A_2) = P(A_1)\,P(A_2)$$
$$P(A_1\bar{A}_3) = P(A_1)\,P(\bar{A}_3)$$
$$P(A\ \bar{A}\ ) = P(A\ )\,P(\bar{A}\ )$$

$$P(A_1 A_3) = P(A_1)\, P(A_3)$$
$$P(A_2 \bar{A}_3) = P(A_2)\, P(\bar{A}_3)$$
$$P(A_1 A_2 \bar{A}_3) = P(A_1)\, P(A_2)\, P(\bar{A}_3)$$

$$A_1 A_2 = (A_1 A_2 A_3) \cup (A_1 A_2 \bar{A}_3)$$

$$\Rightarrow P(A_1 A_2) = P(A_1 A_2 A_3) + P(A_1 A_2 \bar{A}_3)$$



$$\Rightarrow P(A_1 A_2 \bar{A}_3) = P(A_1 A_2) - P(A_1 A_2 A_3)$$
$$= P(A_1 A_2) - P(A_1 A_2)\, P(A_3)$$
$$= P(A_1 A_2)\,[1 - P(A_3)]$$
$$= P(A_1 A_2)\, P(\bar{A}_3)$$
$$= P(A_1)\, P(A_2)\, P(\bar{A}_3)$$

(4) If three events are independent then any one event is independent of the union of the other two.

Eg. Let A_1, A_2, A_3 are independent.

We want to show A_1 and $(A_2 \cup A_3)$ are independent.

$$P[A_1 (A_2 \cup A_3)] = P[A_1 A_2 \cup A_1 A_3]$$
$$= P[A_1 A_2) + P(A_1 A_3) - P(A_1 A_2 A_3)$$

$= P(A_1 A_2) + P(A_1 A_3) - P(A_1 A_2 A_3)$

$= P(A_1) P(A_2) + P(A_1) P(A_3) - P(A_1) P(A_2 A_3)$

$= P(A_1) \left[\underline{P(A_2) + P(A_3) - P(A_2 A_3)} \right]$

$= P(A_1) P(A_2 \cup A_3)$

$\Longrightarrow$ A_1 and $A_2 \cup A_3$ are Independent.

<u>Independence of n-events:-</u>

<u>Inductive def^n</u>:- $A_1, A_2 \text{——} A_n$ are independent

(i) if any $(n-1)$ events out of these n-events are independent and

(ii) $P(A_1 A_2 A_3 \text{——} A_n) = P(A_1) P(A_2) \text{——} P(A_n)$

<u>Note</u>:- $2^n - (n+1)$ equations are required to establish the independence of n-events.

$${}^nC_2 + {}^nC_3 + {}^nC_4 + \text{——} + {}^nC_n$$

$$= 2^n - (n+1) \qquad 2^4 - (4+1)$$

$$= 16 - 5 = 11$$

$= 16 - 5 = 11$

Definition: We say that a countably infinite collection of events is independent if any finite sub-collection of events is independent.

$$\{A_i\}, \quad i = 1, 2, 3, \ldots$$

$$\{A_{i_1}, A_{i_2}, \ldots, A_{i_n}\}$$