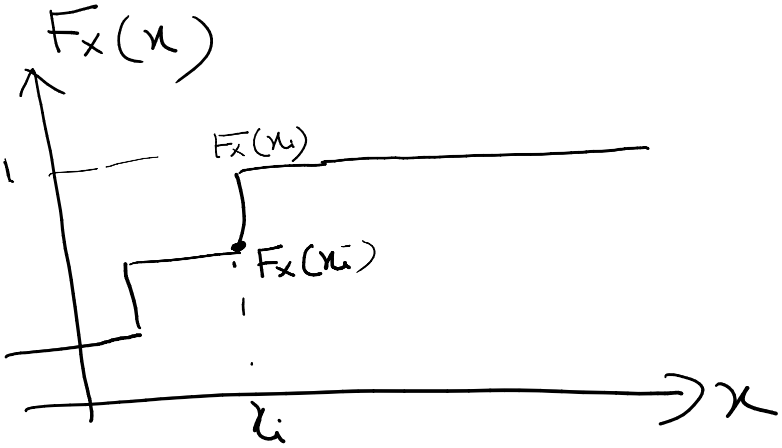
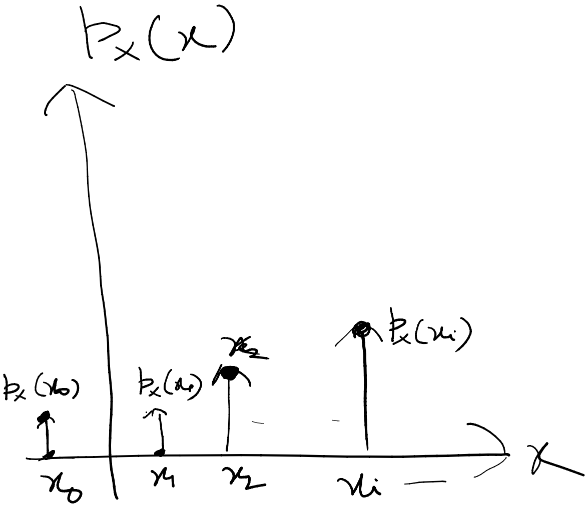
Lecture-16B

01 February 2021 13:57

Lecture-16





$$p_X(x_i) = F_X(x_i) - F_X(x_i)$$

<u>Continuous Random Variable</u> :- A random variable X, that can take uncountable number of possible values on the real line, is said to <u>Continuous r.v.</u>

A continuous r.v. can not be produced from countable sample space.

A continuous r.v. will have a <u>continuous dist^n f^n $F_X(x)$.</u>

If $F_X(x)$ is differentiable, then

$\frac{d}{dx} F_X(x) = f_X(x)$ is known as <u>probability density function</u> (pdf)

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Since $\frac{d}{dx} F_X(x) = \lim_{\Delta x \to 0} \frac{F_X(x+\Delta x) - F_X(x)}{\Delta x} \geq 0$

$\Rightarrow f_X(x) \geq 0 \;\forall\; x$

$\Rightarrow$ For a continuous random variable X, pdf $f_X(x)$ is a non-negative continuous function.

$$\boxed{F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(x)\,dx}$$

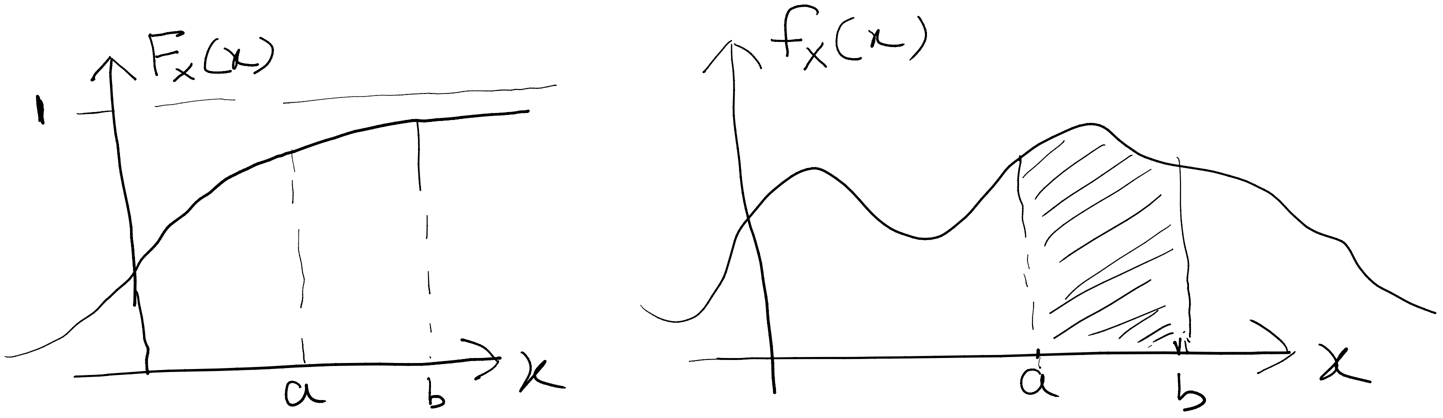
Since $F_X(\infty) = 1$

$$\Rightarrow \int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$P(a < X \leq b) = F_X(b) - F_X(a)$$

$$= \int_{a}^{b} f_X(x)\,dx$$

Thus the area under the p.d.f. $f_X(x)$ in the interval (a, b) represents the probability that the random variable X lies in the interval (a,b).



Also for continuous random variable X, $F_X(x)$ is a continuous function, so

$$P\{X(s)=a\} = F_X(a) - F_X(a^-) = 0$$

$$\Rightarrow P\{a \le X \le b\} = P\{a < X \le b\} = P\{a \le X < b\} = P\{a < X < b\}$$

$$= \int_a^b f_X(x)\,dx$$

Also note that

$$P\left\{a-\frac{\epsilon}{2} \le X \le a+\frac{\epsilon}{2}\right\} = \int_{a-\frac{\epsilon}{2}}^{a+\frac{\epsilon}{2}} f_X(x)\,dx$$

$$\simeq \underline{\underline{\epsilon f_X(a)}}$$ (by Mean Value thm of Integral calculus)

$f_X(x)$ is continuous

Mean Value thm of

$f_X(x)$ is continuous over $[a-\frac{\epsilon}{2}, a+\frac{\epsilon}{2}]$

over $(a-\frac{\epsilon}{2}, a+\frac{\epsilon}{2})$

$f_X(x)$ is having an antider

$F_X(x)$. ~~$\forall x \in$~~

calculus)

Mean Value thm of Integral Calculus

$f(x) \in C[a,b]$

and $F'(x) = f(x)$
$\forall x \in (a,b)$

then by Mean Value of differ we have

$$\frac{F(b)-F(a)}{b-a} = F'(c)(b-a)$$
$c \in (a,b)$

$$\Rightarrow F(b) - F(a) = f(c)(b-a)$$

$$\Rightarrow \boxed{\int_a^b f(t)\,dt = f(c)(b-a)}$$

i.e. $\int_a^x f(t)\,dt = F(x)$
$\forall x \in (a,b)$

When ϵ is small.

In other words, the prob. that X will be contained in an interval of small length ϵ around the point a is approximately $\epsilon f(a)$.

$f(a)$ is the measure of how likely the random variable X will be near to a.

We now consider some specific

We now consider, some specific random Variables.

Bernoulli Random Variable :- The simplest among the set of discrete random variable is Bernoulli r.v.

Consider (S, F, P)

$A \in F$

either A occurs or $\bar{A}$ occurs.

i.e. this is an experiment whose outcomes are either "success" or "failure"

$$X = \begin{cases} 1 & \text{success or } A \text{ occurs} \\ 0 & \text{failure or } \bar{A} \text{ occurs.} \end{cases}$$

$p_X(x)$

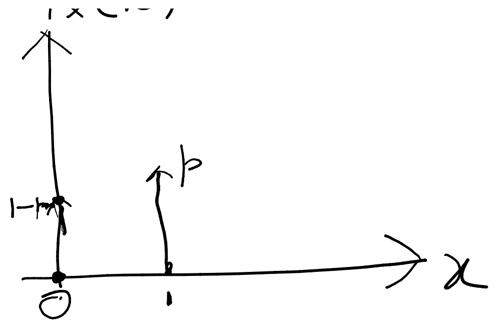
$$p_X(1) = P(X=1) = p$$

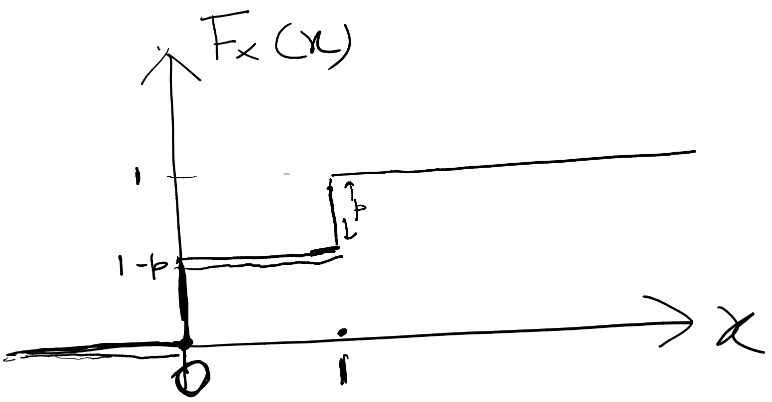
$$p_X(0) = P(X=0) = 1-p$$

$$F_X(x) = \begin{cases} 0 & , \quad x < 0 \\ 1-p & , \quad 0 \leq x < 1 \\ 1 & , \quad x \geq 1 \end{cases}$$

$p_X(x)$

$F_X(x)$





$F_X(x) = P(X \leq x)$

<u>Binomial Random Variable</u> : Suppose above experiment repeated n-times independent-ly.

X : Count the number of times A occurs i.e. number of successes that occur in the n-trials.

$R_X : 0, 1, 2, \text{———}, n$

X is said to be binomial random variable with parameter (n, p).

$$\underline{X \sim B(n, p)}$$

The probability mass function of a Binomial r.v. is given by

$$\boxed{p_X(i) = P(X = i) = {}^nC_i \, p^i (1-p)^{n-i}}$$