Lecture-17B

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Lecture-17

Bernoulli Random Variables:-

$$(S, F, P)$$

$A \in F$ $\quad p(A) = p$

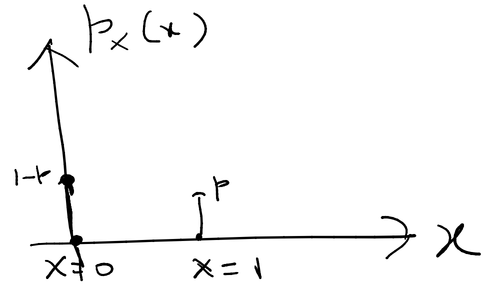
either A occurs or $\bar{A}$ occurs. $\quad p(\bar{A}) = 1-p$

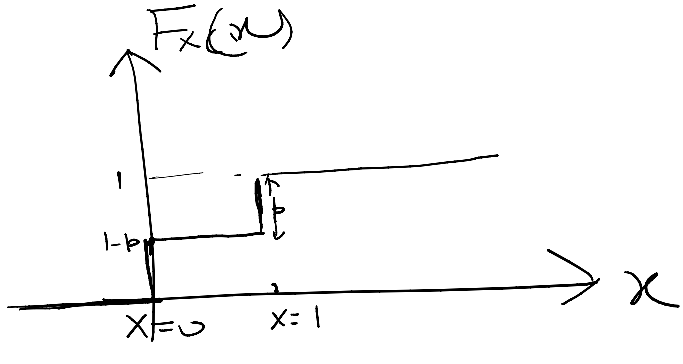
$$X = \begin{cases} 1 & A \text{ occurs} \\ 0 & \bar{A} \text{ occurs} \end{cases}$$

$R_X : 0, 1$

$p_X(1) = P(X=1) = p$

$p_X(0) = P(X=0) = 1-p$





$$F_X(x) = P(X \le x)$$

Binomial Random Variable:- Suppose above experiment is repeated n-times independently.

X: # of times A occurs i.e. # of successes that occur in the repeations of above experiment n-times independently

of above experiment n - times independently
i.e. in the repeation of Bernoulli trials
n - times independently.

$R_X : 0, 1, 2, 3, \text{———}, n$

X is said to be Binomial random variable with parameter (n, p).

$$X \sim B(n, p)$$

$P(X = 2)$

$$= {}^3C_2 \, p^2 (1-p)$$

3 - times

$p^2(1-p)$ $(AA\bar{A})$, $p^2(1-p)$ $(A\bar{A}A)$

$(\bar{A}AA)$ $p^2(1-p)$

${}^3C_2$

The prob. mass function of Binomial random variable with parameter (n, p) is given by

$$\boxed{p_X(i) = P(X = i) = {}^nC_i \, p^i (1-p)^{n-i}}$$

$i = 0, 1, 2, \text{—} n.$

$$\sum_{i=0}^{n} p_X(i) = \sum_{i=0}^{n} P(X = i)$$

$$= \sum^{n} {}^nC \; p^i \; (1-p)^{n-i} \qquad (\quad)^n$$

$$= \sum_{i=0}^{n} {}^nC_i \, p^i (1-p)^{n-i} = (p + (1-p))^n$$
$$= 1$$

Geometric Random Variable :-

Consider (S, F, P)

Suppose that independent trials, each having prob. p of being a success, are performed until the first success occurs.

X : Number of trials required until the first success.

$R_X : 1, 2, 3, 4, 5, 6, \text{———} \ldots \ldots$

$$p_X(n) = P(X = n) = (1-p)^{n-1} p, \quad n = 1, 2, 3 \text{—}$$

$$X \sim G(p)$$

Here, X is known as Geometric random Variable.

Also note that

$$P(X > n) = \sum_{k=n+1}^{\infty} P(X = k)$$

$$= \sum_{k=n+1}^{\infty} (1-p)^{k-1} \cdot p$$

$$= p(1-p)^n \left[1 + (1-p) + (1-p)^2 + \cdots \right]$$

$0 < p < 1$

$0 < 1-p < 1$

$$= p(1-p)^n \cdot \frac{1}{1-(1-p)}$$

$$= (1-p)^n.$$

Thus for integer $m, n > 1$,

$$\underline{P(X > m+n \mid X > m)} = \frac{P\{X > m+n\}}{P\{X > m\}}$$

$$= \frac{(1-p)^{m+n}}{(1-p)^m}$$

$$= (1-p)^n \qquad (*)$$

$\{X > m+n\} \cap \{X > m\}$

$m \qquad m+n$

$P(A|B) = \frac{P(AB)}{P(B)}$

$\{X > m+n\} \subset \{X > m\}$

Equation (*) states that given the first m trials had no success, the conditional prob. that the first success will appear after an additional n trials depends only on n and not on previous m-trials (not on the past)

This property is known as

This property is known as Memoryless property.

$$X \sim G(p)$$

& $m, n > 1$

$$\boxed{P\{X > m+n \mid X > m\} = (1-p)^n}$$

✳ This is the only discrete random variable, which satisfy Memoryless property.

The Geometric dist^n plays an important role in particular in the theory of queues or waiting lines.

For example, suppose a line of customers waits for a service at a counter. It is often assume that in each small time unit, either 0 or 1 new customers arrives at the counter. The prob. that a custome arrives is p and that no customers arrives is $1-p$.

X : Time elapsed until the arrival of first customer.

$$X \sim G(p)$$

Thus X: A server waits for the customer

$$P(X > m) = (1-p)^m$$

This prob. can also be found by noting that we are asking no successes (i.e. arrivals) in a sequence of m consecutive time units, where the prob. of a success in any one time unit is p.

$$P(X > m+n \mid X > m) = (1-p)^n$$

Thus, the prob. that the waiting time takes n more time units is independent of the length of the m-time units that the server has already been wait.

$X \sim G(p)$

$$P(X > 40 \mid X > 30) = (1-p)^{10}$$
$$= P(X > 10)$$