Lecture-20B

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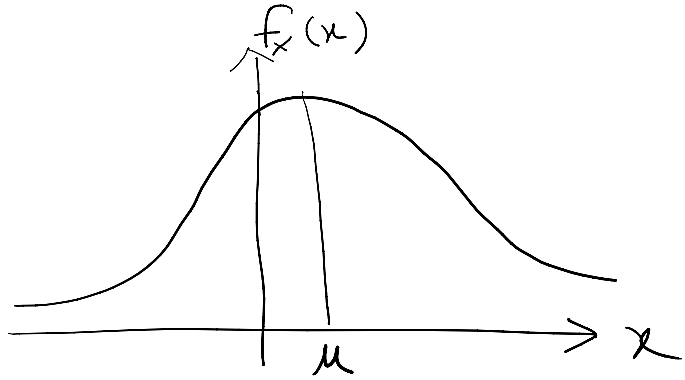
Lecture-19

Normal random Variable

$$X \sim N(\mu, \sigma^2)$$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$



$$F_X(x) = P(X \leq x) = P(-\infty < X \leq x)$$

$$= \int_{-\infty}^{x} f_X(y)\, dy$$

$$= \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dy$$

$$= G\left(\frac{x-\mu}{\sigma}\right) = \phi\left(\frac{x-\mu}{\sigma}\right)$$

where the value of the function

$$G(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

is available in tabular form.

$$X \sim N(0, 1) \quad \mu = 0$$

$$\sigma^2 =$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-0}{1}\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

$\Downarrow$

$G_X(x)$

$$= G(x)$$

Ex:- $X \sim N(3, 9)$

$\mu = 3,$

$\sigma^2 = 9.$

$$P\{2 < X < 5\}$$

$$= P\left\{\frac{2-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{5-\mu}{\sigma}\right\}$$

$$= P\left\{\frac{2-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{5-\mu}{\sigma}\right\}$$

$$= P\left\{\frac{2-3}{3} < \frac{X-3}{3} < \frac{5-3}{3}\right\}$$

$$= P\left\{-\frac{1}{3} < Z < \frac{2}{3}\right\}$$

$$Z \sim N(0,1)$$

$$= P\left(-\infty < Z < \frac{2}{3}\right) - P\left(-\infty < Z < -\frac{1}{3}\right)$$

$$= G\left(\frac{2}{3}\right) - G\left(-\frac{1}{3}\right)$$

$$= G(0.66) - G(-0.33)$$

$$= P(Z < 0.66) - P(Z < -0.33)$$

$$= 0.74537 - \left[P(Z > 0.33)\right]$$

$$= 0.74537 - \left[1 - P(Z < 0.33)\right]$$

$$= 0.74537 - \left[1 - 0.62930\right]$$

$$= 1.37467 - 1 = \boxed{0.37469}$$

$$X \sim N(\mu, \sigma^2)$$

$$X \sim N(\mu, \sigma^2)$$

$$Z = \frac{X - \mu}{\sigma}$$

then $Z \sim N(0,1)$

Ex: On May 5, in a certain city, temperature have been found to be normally distributed with parameters $\mu = 24^\circ C$ and $\sigma^2 = 9$.

The recorded temperature on that day is $27^\circ C$.

(a) What is the prob. that the record of $27^\circ C$ will be broken ~~in~~ on next May 5?

(b) What is the prob. that the record of $27^\circ C$ will be broken at least 3 times during next 5 years on May 5? (Assume that temperatures during the next 5 years on May 5 is independent.)

(c) How high must the temperature

(C) How high must the temperature be to place it among the top 5% of all temperatures recorded on May 5?

Ans (a) ⊢

X : temperature on May 5.

$$X \sim N(24, 9)$$

$X \sim N(\mu, \sigma^2)$
$Z = \frac{X-\mu}{\sigma}$
$\sim N(0,1)$

$$P(X > 27)$$
$$= P\left(\frac{X-24}{3} > \frac{27-24}{3}\right)$$
$$= P(Z > 1)$$
$$= 1 - P(Z \leq 1)$$
$$= 1 - G(1)$$
$$= 1 - 0.84134$$
$$= \boxed{0.1587}$$

(b) Y : Number of times records broken during next 5 years on May 5.

$$Y \sim B(5, 0.1587)$$

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$$\quad\ \ n \qquad\ p$$

$$P(Y \geq 3) = P(Y=3) + P(Y=4) + P(Y=5)$$

$$= {}^5C_3 (0.1587)^3 (0.8413)^2 + {}^5C_4 (0.1587)^4 (0.8413) + {}^5C_5 (0.1587)^5 (0.8413)^0$$

(C) Let x be the desired temper -ature.

$$P(X > x) = 0.05$$

$$P(X \leq x) = 0.95$$

$$\Rightarrow P\left(\frac{X-24}{3} < \frac{x-24}{3}\right) = 0.95$$

$$P\left(Z < \frac{x-24}{3}\right) = 0.95$$

$$P(Z < 1.65) = 0.95$$

$$\Rightarrow \frac{x-24}{3} = 1.65$$

$$\Rightarrow \boxed{x = 28.95^\circ C}$$

$$X \sim N(\mu, \sigma^2)$$

$$\underline{Y = \alpha X + \beta}$$

$$Y \sim N(\alpha\mu + \beta, \alpha^2\sigma^2)$$

$$X \sim N(\mu, \sigma^2)$$

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

$$\alpha = \frac{1}{\sigma}, \quad \beta = -\frac{\mu}{\sigma}$$

$$\alpha\mu + \beta = \frac{\mu}{\sigma} - \frac{\mu}{\sigma} = 0,$$