Lecture-2

Ex:- Show that set of all rational number is countable infinite.

Ans:- first, we will show that set of all +ive rational numbers is countable infinite.

	1	2	3	4	5	6	7
1	1 $\frac{1}{1}$	2 $\frac{1}{2}$	$\frac{1}{3}$ 4	$\frac{1}{4}$ 9	$\frac{1}{5}$ 10 —	—	— →
2	2 $\frac{2}{1}$	$\cancel{\frac{2}{2}}$	8 $\frac{2}{3}$	$\cancel{\frac{2}{4}}$	$\frac{2}{5}$		→
3	5 $\frac{3}{1}$	7 $\frac{3}{2}$	$\cancel{\frac{3}{3}}$	$\frac{3}{4}$	$\frac{3}{5}$		→
4	6 $\frac{4}{1}$	$\cancel{\frac{4}{2}}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$		
5	11 $\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$		
6	$\frac{6}{1}$	$\frac{6}{2}$	$\frac{6}{3}$	$\frac{6}{4}$	$\frac{6}{5}$		
7	$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$		

$$
\begin{array}{cccccccccc}
\mathbb{N}: & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & --- \\
 & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\
\mathbb{Q}^+: & \frac{1}{1} & \frac{2}{1} & \frac{1}{2} & \frac{1}{3} & \frac{3}{1} & \frac{4}{1} & \frac{3}{2} & \frac{2}{3} & ---
\end{array}
$$

1-1 onto

$\Longrightarrow$ $\mathbb{Q}^+$ is countable infinite

$$
\begin{array}{ccccccccccc}
\mathbb{N}: & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & --- \\
 & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\
\mathbb{Q}: & 0 & \frac{1}{1} & -\frac{1}{1} & \frac{2}{1} & -\frac{2}{1} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{3} & -\frac{1}{3} & ---
\end{array}
$$

there is a 1-1 onto mapping between

there is a 1-1 onto mapping between $\mathbb{N}$ and $\mathbb{Q}$.

$\Rightarrow$ Set of rational number $\mathbb{Q}$ is Countable infinite.

An infinite set which is not Countable infinite is called uncountable.

$$I = [0, 1]$$

Ex:- Set of irrational number ($\mathbb{R}/\mathbb{Q}$) is uncountable so $\mathbb{R} = \mathbb{Q} \cup \mathbb{R}/\mathbb{Q}$ is also uncountable.

(a, b), $(a, b]$, $[a, b)$, $[a, b]$
$a < b$ are uncountable.

Events:- Any subset E of the sample space S is known as event.

Ex:- flipping of a coin

$$S = \{H, T\}$$

$$E_1 = \{H\}, \quad E_2 = \{T\}, \quad E_3 = \{H, T\}$$

$E_1 = \{H\}$, $E_2 = \{T\}$, $E_3 = \{H, T\}$

Ex:- Rolling of a die

$$E = \{1\}$$

E is the event that 1 appears on the role of the die.

Probability:- Probability is the measure of the occurence of an event.

Relative Frequency defⁿ:- A widely used concept is the experimental probability, which uses the relative frequency of an event E and is defined as follows:

Let $n(E)$ denote the number of times in the first n repeations of the experiment that the event E occurs. Then prob. $P(E)$ of an event E is defined by the limit

$$\boxed{P(E) = \lim_{n \to \infty} \frac{n(E)}{n}} \qquad \frac{n(E)}{n} = f(n)$$

Drawbacks:- (1) How do we know that $\frac{n(E)}{n}$ will converge that $\frac{n(E)}{n}$ will converge to some constant limiting value?

(2) How do we know that if $\frac{n(E)}{n}$ will converge to some constant limiting value, that will be same for each possible sequence of repetitions of the experiment?

Classical Def^n:- According to the classical def^n, the probability $P(E)$ of an event E is determined a priori without actual experimentation. It is given by the ratio:

$$\boxed{P(E) = \frac{n(E)}{n}}$$

where,

n : Number of possible outcomes

$n(E)$: Number of outcomes that are favorable to the event E.

Ex.:- In the die experiment, the possible outcomes are 6 and the outcomes favorable to the event even are favorable to the event even are 3. Hence

$$P(E) = P(\text{Even}) = \frac{3}{6} = \frac{1}{2}.$$

Drawback:- The significance of the number n and $n(E)$ is not always clear, which is demonstrated in the example given below:

Ex:- 5