Lecture-9:A2

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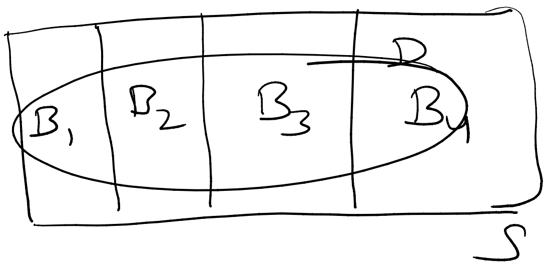
Lecture-9

Ex.:- We have four box.

Box 1 :	1900 g,	100 d	95 %	5 %
Box 2 :	300 g,	200 d	60 %	40 %
Box 3 :	900 g,	100 d	90 %	10 %
Box 4 :	900 g,	100 d	90 %	10 %

We select at random one of boxes and we remove at random a single component. What is the prob. that the selected component is defective?

B_i : event consisting of all equipments in the i-th box.



D : event consisting of all defective components.

$$P(B_i) = \frac{1}{4}, \quad i = 1, 2, 3, 4.$$

$$P(D|B_1) = \frac{100}{2000} = 0.05$$

$$P(D|B_2) = \frac{200}{500} = 0.4$$

$$P(D|B_3) = P(D|B_4) = \frac{100}{1000} = 0.1$$

By Total Prob. thm, we have

$$P(D) = \sum_{i=1}^{4} P(D|B_i)\,P(B_i)$$

$$= 0.05 \times \frac{1}{4} + 0.4 \times \frac{1}{4} + 0.1 \times \frac{1}{4} + 0.1 \times \frac{1}{4}$$

$= 0.05 \times \frac{1}{4} + 0.4 \times \frac{1}{4} + 0.1 \times \frac{1}{4} + 0.1 \times \frac{1}{4}$

$= \boxed{0.1625}$

i.e. 16.25% chance that chosen component is defective.

Ex:- Suppose we have 3 coin. A coin chosen "at random" shows H.

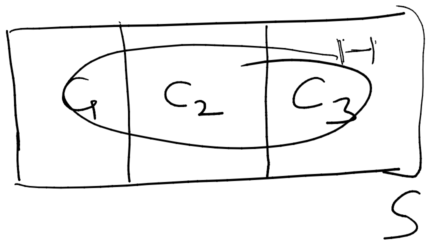
	H	T
Coin 1 :	1	0
Coin 2 :	$\frac{1}{2}$	$\frac{1}{2}$
Coin 3 :	$\frac{3}{4}$	$\frac{1}{4}$

What is the prob. that the coin chosen was coin 1. $P(\text{Coin1}/H) = ?$

C_i : i-th coin is chosen

H : Head shown

T : Tail shown



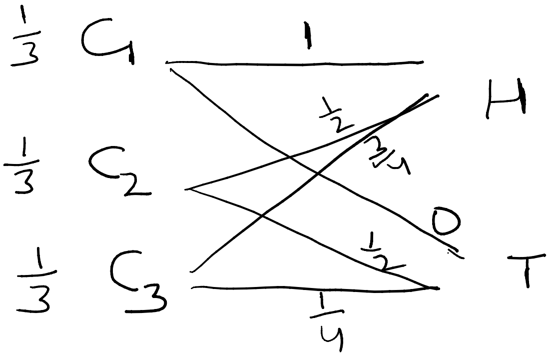
$P(C_1) = P(C_2) = P(C_3) = \frac{1}{3}$

$P(H|C_1) = 1$

$P(T/C_1) = 0$

$P(H|C_2) = P(T/C_2) = \frac{1}{2}$

$P(H/C_3) = \frac{3}{4}, P(T/C_3) = \frac{1}{4}$.



$$P\left(\frac{C_1}{H}\right) = \frac{P(H|C_1)\,P(C_1)}{P(H/C_1)P(C_1) + P(H|C_2)P(C_2) + P(H/C_3)P(C_3)}$$

$$= \frac{1 \times \frac{1}{3}}{1\times\frac{1}{3} + \frac{1}{2}\times\frac{1}{3} + \frac{3}{4}\times\frac{1}{3}}$$

$$= \boxed{\frac{4}{9}}$$

$$P(C_2|H) = \frac{2}{9}, \quad P\left(\frac{C_3}{H}\right) = \frac{1}{3}$$

A is independent of B if

$$P(A|B) = P(A)$$

i.e. A is independent of B if occurrence of B does not to change the prob. of occurrence of A.

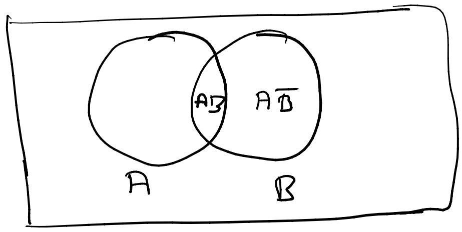
We also shown that A is independent of B if

$$\boxed{P(AB) = P(A)\,P(B)}$$ or vice-versa.

Few Properties

* A and B are independent $\Longrightarrow$ A & $\overline{B}$ are also independent.

are also independent.



$B = AB \cup \bar{A}B$, $(AB) \cap (\bar{A}B) = \phi$

$\Rightarrow P(B) = P(AB) + P(\bar{A}B)$

$\Rightarrow P(\bar{A}B) = P(B) - P(AB)$

$= P(B) - P(A)P(B)$

$= P(B)[1 - P(A)]$

$= P(B)P(\bar{A})$

$\Rightarrow P(\bar{A}B) = P(\bar{A})P(B)$

$\Rightarrow$ $\bar{A}$ & B are independent

Similarly, we can show that if A & B are independent then A & $\bar{B}$ are also independent.

(*) A and B are independent $\Rightarrow$ $\bar{A}$ and $\bar{B}$ are also independent.

$S = (A \cup B) \cup (\overline{A \cup B})$ $\qquad \overline{A \cup B} = \bar{A} \cap \bar{B}$

$= (A \cup B) \cup (\bar{A}\bar{B})$

$\Rightarrow P(S) = P(A \cup B) + P(\bar{A}\bar{B})$

$\Rightarrow P(\bar{A}\bar{B}) = P(S) - P(A \cup B)$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(AB)]$

$= 1 - P(A) - P(B) + P(A)P(B)$

$= 1 - P(A) - P(B)[1 - P(A)]$

$= (1 - P(A))(1 - P(B))$

$= P(\bar{A})P(\bar{B})$

$\Rightarrow P(\bar{A}\bar{B}) = P(\bar{A})P(\bar{B})$

Ex.1 We are given two experiments

Ex:- We are given two experiments:
The first experiment is the rolling of a fair die.

$S_1 = \{f_1, f_2, \text{———}, f_6\}, \quad P_1(f_i) = \frac{1}{6}.$

The second experiment is the tossing of a fair coin.

$S_2 = \{h, t\}, \quad P_2(h) = P_2(t) = \frac{1}{2}$

(S_1, F_1, P_1) and (S_2, F_2, P_2)

We ~~per~~form both experiments and we want to find the prob. that we get "two" on the die and "head" on the coin.
If we make reasonable assumption that the outcome of the first experiment are independent of the outcomes of the second, we conclude that the desired prob. $= \frac{1}{6} \times \frac{1}{2}$.

This conclusion is reasonable, however, the notion of independence does not agree with the def^n of independent events. In the def^n, the events A and B were subsets of the same sample space.

So in order to fit this conclusion into our theory, we must therefore construct a space S having as subsets H.&

… , we must … , construct a space S having as subsets the events "two" and "head".

$$S = S_1 \times S_2 = \{f_1h, f_2h, \ldots, f_6h, f_1t, f_2t, \ldots, f_6t\}$$

In this space, {two} is not an elementary event but a subset consisting of two elements

$$\{two\} = \{f_2h, f_2t\} \in S$$

$$A = \{two\} \in S_1 \ , \quad \{f_2h, f_2t\} = A \times S_2$$

Similarly {heads} is an event with six elements.

$$\{heads\} = \underbrace{\{f_1h, f_2h, \ldots, f_6h\}}_{\substack{\| \\ \cancel{A} \; S_1 \times B}}$$

$$B = \{heads\} \in S_2$$