Lecture-1

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What do you mean by Probability theory?

Probability theory deals with the study of random phenomena, which under repeated experiments yields different outcomes that have certain underlying patterns about them.

Sample space:- Consider an experiment whose outcome is not predictable in advance but the set of all possible outcome is known. This set of all possible outcomes of an experiment is known as the sample space of the experiment and is denoted by S.

Ex1 tossing of a coin

Ex:- tossing of a coin

$S = \{H, T\}$

Ex:- Rolling of a die

$S = \{1, 2, 3, 4, 5, 6\}$

Ex:- flipping of two coin

$S = \{(H,H), (H,T), (T,H), (T,T)\}$

Ex:- If the outcome of an experiment is order of finish in a race among the 7 horses having post positions 1, 2, 3, 4, 5, 6, 7 then

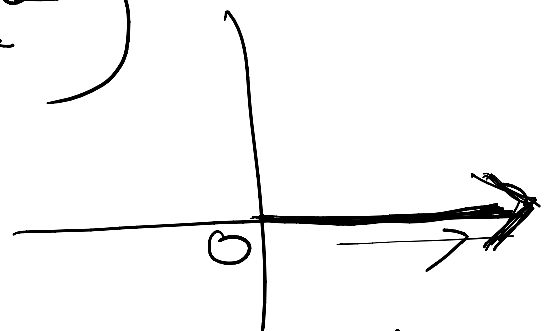
$S = \{\text{all } \lfloor 7 \text{ permutations of } \{1, 2, 3, 4, 5, 6, 7\}\}$

$\lfloor 7$ elements

Ex:- If the experiments

Ex:- If the experiments consists of measuring the life time (in hours) of a transister, then

$$S = \{x : 0 \le x < \infty\}$$



So far, we observed that there are 3 types of possible sample space:

(1) finite $\{1, 2, 3, \ldots n\}$

(2) Countable infinite $\{1, 2, \ldots\}$

(3) Uncountable infinite $\{\Rightarrow$ interval$\}$

$$S = [0, \infty)$$

Ex:- Consider an experiment consists of flipping a

Consists of flipping of coin till tail appears.

$S = \{T, HT, HHT, HHHT, \text{---} \}$

$N: \quad 1 \quad 2 \quad 3 \quad 4 \quad \text{---}$

$f: N \to S$ 1-1 onto

Countable infinite

Countable and Uncountable set:

A set S is said to be finite if there is an $n \in \mathbb{N}$ and a bijection (1-1 onto) from S onto $\{1, 2, \text{---} n\}$.

Countable Infinite: An infinite set S is said to be countable if there is a bijection from $\mathbb{N}$ to S.

If S is an infinite countable set, then w.l.o.g. any bijection $f : \mathbb{N} \to S$, we can list the elements of S as a sequence $f(1)$, $f(2)$, $f(3)$, ——— - .

$f(1) \longrightarrow T$

$f(2) \longrightarrow HT$

$f(3) \longrightarrow HHT$

$S = \{f(1), f(2), f(3), —\}$.

Uncountable Infinite: If we are not able to find any bijection able to find any bijection $f: \mathbb{N} \to S$, then S is said to be <u>uncountable</u>.

<u>Ex</u>:- Set of integers

$\mathbb{Z}$.

$\mathbb{Z} = \{ \cdots -3, -2, -1, 0, 1, 2, 3, \cdots \}$

$f: \mathbb{N} \to \mathbb{Z}$

$$f(n) = \begin{cases} \frac{1}{2} n & \text{if } n \text{ is even} \\ -\frac{1}{2}(n-1) & \text{if } n \text{ is odd} \end{cases}$$

$f(1) = 0$

$f(2) = 1$

$f(3) = -1$

$f(4) = 2$

$f(5) = -2$

$\vdots$

1-1 onto

$x, y \in \mathbb{N}$

& $x \neq y$

$\implies f(x) \neq f(y)$

$\Rightarrow f(x) \neq f(y)$

$\mathbb{Z}$ is countable

$$\mathbb{Z} = \{ \overset{f(1)}{0}, \overset{f(2)}{1}, \overset{f(3)}{-1}, \overset{f(4)}{2}, -2, \text{———} \}$$