Lecture-14

Consider the experiment of rolling two dice.

Here, we are interested in the sum of two dice, than the actual outcomes.

It frequently occurs that in performing an experiment we are mainly interested in some functions of the outcome as opposed to the outcome itself. These quantities of interest or more formally these real valued functions defined on the sample space are known as random variable.

Random Variable:- Let (S, F, P) be a probability model. Then a function

$$X : S \longrightarrow \mathbb{R}$$

is said to be a random variable if it must satisfy the following 2 conditions:

(i) The set $\{X \le x\}$ is an event for every x.

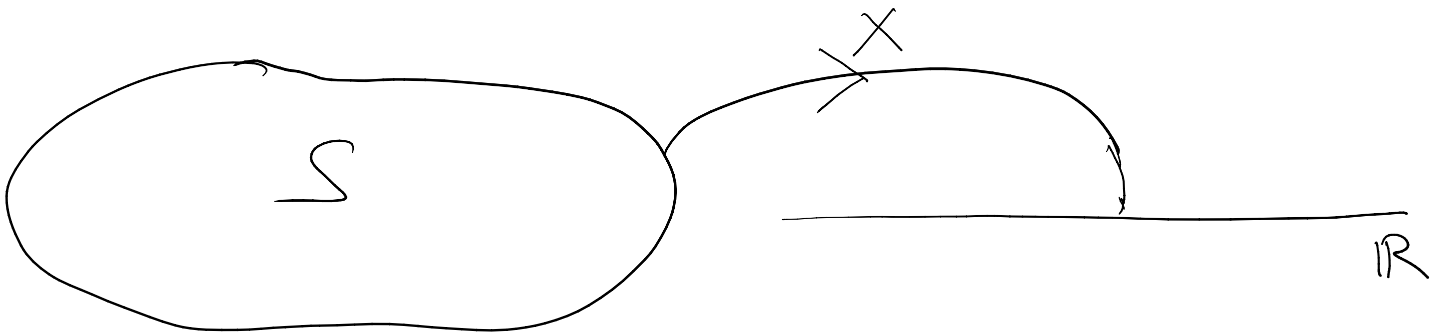
(ii) The probabilities of the event $\{X = \infty\}$ and $\{X = -\infty\}$ be 0. i.e.

$$P(X = -\infty) = P(X = \infty) = 0$$

This condition does not prevent X from

$P(X = -\infty) = P(X = \infty) = 0$

This condition does not prevent X from being either $-\infty$ or ∞ for some value of $s \in S$, it only requires that prob. of the set of those s be 0.



Ex.1 Experiment of rolling two dice.

$S = \{(1,1), (1,2), \text{———} (6,5), (6,6)\}$

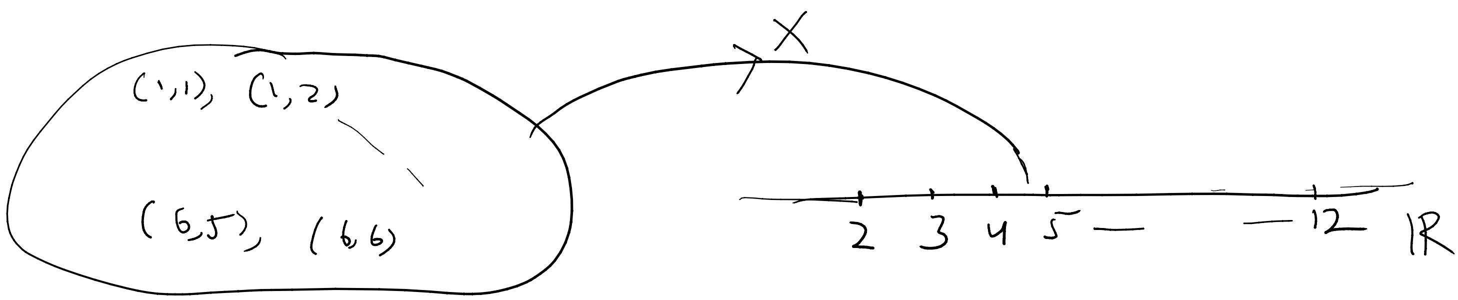
S is finite with 36 points.

$P\{X \leq 3\} =$

$= (X=2) \cup \{X=3\}$

X : Sum of two dice.

R_X : 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12



$s \in S, \quad s = (1,1),$

$X(s) = 2$

$P(X=2) = P(\{1,1\}) = \frac{1}{36}$

$P(X=3) = P\{(1,2), (2,1)\} = 2$

$$P(X=3) = P\{(1,2),(2,1)\} = \frac{2}{36}$$

$$P(X=12) = P(\{6,6\}) = \frac{1}{36}$$

$$P\left\{\bigcup_{i=2}^{12}(X=i)\right\} = \sum_{i=2}^{2} P(X=i)$$

$$= 1.$$

Thus one can notice that given an experiment defined by a sample space S with element s, we assign to every s a real number according to some rule and call $X(s)$ a random variable.

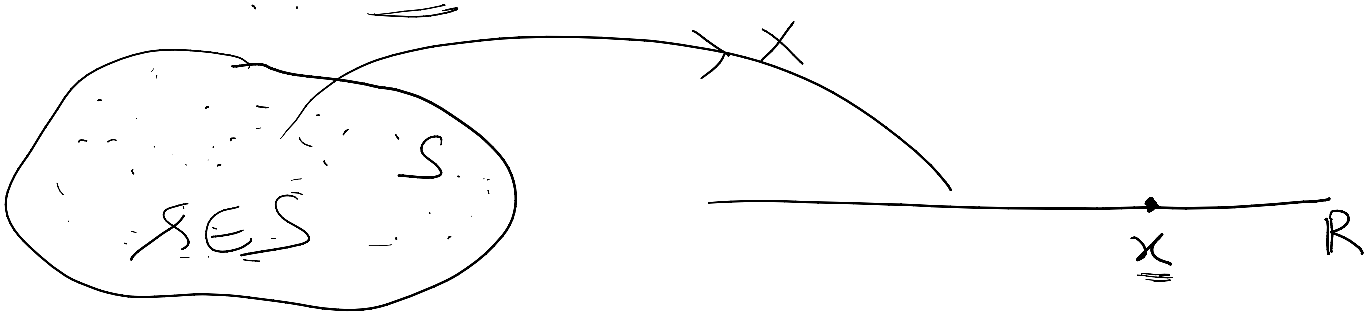
Since the value of a random variable is determined by the outcome of the experiment, we may assign probabilities to the possible values of the random variable.

__Events generated by random variable:-__ We start with the meaning of notation meaning of notation

$\{X \leq x\}$ $\quad X$: random variable.

$$\{X \leq x\} = \{s \in S \mid X(s) \leq x\} \in F$$
$$\forall x \in \mathbb{R}$$

$$X : S \longrightarrow \mathbb{R}$$



$$\{X \leq x\} = \{X(s) \leq x\}$$
$$= \{s \in S \mid X(s) \leq x\} \in F$$

i.e., given for any arbitrary real number x, we find all numbers $X(s_i)$ that do not exceed x. These correspon--ding elements s_i for the set $\{X \leq x\}$. Thus $\{X \leq x\}$ is not a set of numbers but a set of experimental outcomes.

Ex: $\{X \leq 3\} = \{(1,1), (1,2), (2,3)\}$

$\{X \le x\}$

Similarly,

$$\{x_1 \le X \le x_2\} = \{s \in S \mid x_1 \le X(s) \le x_2\}$$

The notation

$\{X = x\}$ = subset of S consisting of all elementary outcomes s s.t. $X(s) = x$.

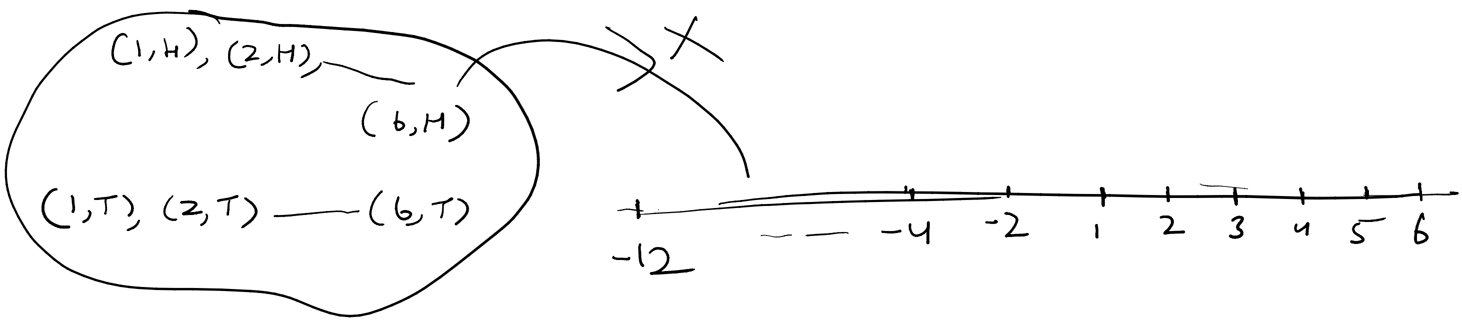
Finally,

$\{X \in \mathbb{R}\}$ represent the subset of S consisting of all outcomes s s.t. $X(s) \in \mathbb{R}$.

Ex:- Consider an experiment of rolling a die and flipping a coin.

$$X = \begin{cases} \text{Choose the number that show up on die with H.} \\ \text{Choose the -ive of twice of the number that show up on die with T.} \end{cases}$$

choose a -ive of twice of the number that show up on die with T.



$$P(X=-12)=\frac{1}{12}$$

$$P(X=-10)=P(X=-8) \text{———} =P(X=6)$$
$$=\frac{1}{12}.$$

$$P(X=-3)=0$$

Ex:- Suppose that we toss a coin having a prob. p of coming up H's until the first H appears. Let N denote the number of flips required then assuming that outcome of successive flips are independent, N is a random variable taking on one of the values 1, 2, 3 — with respective probabilities.

N: Number of flips required to get first H.

R_N : 1, 2, 3, 4, —— —

$P(N=1) = p, \quad P(N=2) = \underline{(1-p)p}$

$P(N=3) = P(TTH) = (1-p)^2 p$

$P(N=n) = P(\underbrace{TTT \text{———} T}_{n-1 \text{ times}} H) = (1-p)^{n-1} p$

$n \geq 1.$

$$P\Big(\underline{\bigcup_{n=1}^{\infty} \{N=n\}}\Big) = \sum_{n=1}^{\infty} P(N=n)$$

$$= p \sum_{n=1}^{\infty} (1-p)^{n-1}$$

$$= p \cdot \frac{1}{1-(1-p)} \qquad 0<1-p<1$$

$$= p \cdot \frac{1}{p} = \underline{1}.$$