

The LNM Institute of Information Technology
Jaipur, Rajasthan
MTH 222 Probability and Statistics
Tutorial-2

1. Consider a sequence of independent Bernoulli trials each of which is a success with probability p . Let X_1 be the number of failures preceding the first success and let X_2 be the number of failures between the first two successes. Find the joint mass function of X_1 and X_2 .

2. The joint probability density of X and Y is given by

$$f(x, y) = c(y^2 - x^2)e^{-y}, \quad -y \leq x \leq y, 0 < y < \infty.$$

- (a) Find c .
(b) Find the marginal densities of X and Y .
(c) Find $E[X]$.
3. The joint density function of X and Y is given by

$$f(x, y) = \frac{6}{5}(x + y^2), 0 < x < 1, 0 < y < 1.$$

- (a) Verify that $f(x, y)$ is a valid PDF.
(b) Find the marginal distributions of X and Y .
(c) Find $P\{0 < X < \frac{1}{2}, 0 < Y < \frac{1}{2}\}$.
(d) Find $E[X]$ and $E[Y]$.
4. Find the joint probability density of the two random variables X and Y whose joint distribution function is given by

$$F(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y}, & \text{for } x > 0, y > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Use the joint probability density obtained to find $P(X + Y > 3)$. Are X and Y independent?

5. Let $f(x, y, z) = kxyz^2, 0 < x < 1, 0 < y < 1, 0 < z < 2$ be the joint density function of three random variables X, Y and Z . Find $P(Z > X + Y)$.
6. Suppose that A, B, C are independent random variables, each being uniformly distributed over $(0, 1)$.
(a) What is the joint distribution function of A, B, C ?
(b) What is the probability that all of the roots of the equation $Ax^2 + Bx + C$ are real?
7. If X and Y are jointly continuous with joint density function $f_{X,Y}(x, y)$, show that $X + Y$ is continuous with density function

$$f_{X+Y}(t) = \int_{-\infty}^{\infty} f_{X,Y}(x, t-x).$$

8. The trivariate probability density of X_1, X_2 and X_3 is given by:

$$f(x_1, x_2, x_3) = \begin{cases} (x_1 + x_2)e^{-x_3}, & \text{for } 0 < x_1 < 1, 0 < x_2 < 1, x_3 > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Find $P\{(X_1, X_2, X_3) \in A\}$, where A is the region

$$\{(x_1, x_2, x_3) \mid 0 < x_1 < \frac{1}{2}, \frac{1}{2} < x_2 < 1, x_3 < 1\}.$$

- (b) Find the joint marginal density of X_1 and X_3 .
(c) Find the marginal density of X_1 alone.
(d) Verify that X_1, X_2 and X_3 are not independent, but that the two random variables X_1 and X_3 and also the two random variables X_2 and X_3 are **pairwise independent**.

9. The joint density of X and Y is

$$f(x, y) = c(x^2 - y^2)e^{-x}, 0 \leq x < \infty, -x < y < x.$$

Find the conditional distribution of Y given $X = x$.

10. The joint density function of X and Y is

$$f(x, y) = xe^{-x(y+1)}, \quad x > 0, y > 0.$$

- (a) Find the conditional density of X , given $Y = y$, and that of Y , given $X = x$.
- (b) Find the density function of $Z = XY$.