

$$P(x \ge \alpha n) \le \frac{E(x)}{\alpha n} = \frac{pn}{\alpha n} = \frac{p}{\alpha}$$



Chabysher's inequality gives estimate for

P (1X - E(x) > xn). So we have to rewrite the event {x > xn}, so that we can we the Chelogaher's inequality.

 $P_{X}^{2} \times 7_{\alpha} n_{y}^{2} = P_{X}^{2} \times -n_{p}^{2} \times$

= { Y < - d } U { Y > a }

 $\frac{\langle Var(x) \rangle}{(\alpha n - np)^2} = \frac{np(1-p)}{n^2(\alpha - p)^2}$ $= \frac{p(1-p)}{n(\alpha - p)^2}$

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By Markov's inequality for $p > \frac{1}{2}$ and $\alpha > \frac{3}{4}$, we have $P(X > \frac{3}{4}n) \leq \frac{2}{3}$

By Chebysher's inequality for p=1 and x=3, we have

 $P(x) \frac{3n}{4} \leq \frac{4}{n}$

If n>6, then estimate given by Chebysher's are sharpen than the estimates provided by Markor's.

Also as n increases, estimates given by Chebysher's inequality decreases, i.e., gives much information serias the estimates provided by Markov inequality remains constant as n varies.

Law of Large Numbers

Theo: (Weak law of large numbers). Let x_1, x_2, \ldots be a sequence of independent and identically distributed random variables, each having finite mean μ . Then for every 8 70,

 $\lim_{n\to\infty} P\left\{ \left| \frac{Sn}{n} - \mu \right| \right\} \leq \int_{-\infty}^{\infty} 0^n equivalently$

where $S_n = X_1 + X_2 + - - + X_n$

Remark: The weak law of large numbers states that

for large n, the bulk of the distribution of Sn
is concentrated man u. That is, if we consider
a positive length interval [M-8, M+8] around u,
then there is a high probability that Sn will fall in
that interval; as n-sa, two probability converges to 1.

Of course, if 8 is very small we may have to wait
larger (i.e., need a larger value of n) before we can
assert that Sn is highly likely to fall in that interval.

2	To understand the convergence in weak law, think in terms of PMF (if Xi's on discrete transform variables) or PDF (if Xi's have pdf then we know that Sn will possess a pdf or well) of random variable Sn.
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	Weak law states that "almost all" of the point or pdf
	Weak law states that "almost all" of the prof or pdf of Sy is concentrated within S-neighborhood of le

(3) The limit in Remark() means: 78, 270, 7 no (2.8) such that for all 17 no (2.8), we have 9515n - 1128 71-2.

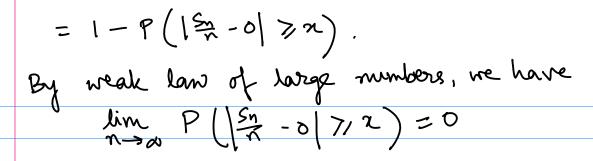
for large value of n.

If we refer to 8 on the accuracy level and & or the confidence level, the weak law takes the following intuitive form:

for any given level of accuracy and confidence, Son will be equal to μ , within these weeks of accuracy and confidence, provided n is large enough.

Example (2): Let $X_1, X_2, -\cdot$ be independent and identically distributed random variables with $E(X_i) = 0$ and $Van(X_i) = 1$ $\forall i \cdot Let Sn = X_1 + X_2 + \cdot \cdot \cdot + X_n$. Then for any $a \ge 0$, compute $\lim_{n \to \infty} P(-nx < Sn < nx)$.

Soln: For any n > 0, we have $P(-n \times \langle Sn \langle n \times \rangle) = P(-x \langle \frac{Sn}{n} \langle x \rangle) = P(\frac{|Sn}{n} - 0|ce)$



Random Sampling

Let $X_1, X_2, ..., X_n$ be n independent random variables having the same distribution. These random variables may be thought of as n independent measurements of some quantity that is distributed according to their common distribution. (E.g., height of stylents in LMMIIT). In this server, we sometimes speak of the random sociables $X_1, ..., X_n$ as constituting a random sample of size n from this distribution.

Suppose that the common distribution of these transform variables have finite mean u. then for a sufficiently large, we would expect that the sample mean

Sn = X12 - + xn should be done to the time

n mean h.

The weak law of large numbers ascerts that the sample mean of a large number of independent identically distributed random variables is very close to the true mean with high probability.

Wede law of lung numbers says that for my 570, lin PS 15m-u (7,8) =0 — 2

We may interpret egn (2) in the following way: The number & can be thought of in the derived accuracy in the approximation of u by in. Egr (2) assures us that no matter his small & many be chosen, the probability that In approximates in to within this accuracy, that i, PS/57 -4/48/ convergente la tre no. of observations gets large. Example (4): Probabilities and fraguencies: Consider an event A defined in the context of some probability of the event A. We consider n independent repetitions of this experiment and let him be the "traction" of time that event A occurs in this context. My is often called the empirical frequency of A. Note that Mn = X1e - · + ×n , where x; is I shenever A occurs and O o.w. In particular, E(Xi) = p. The weak law of large numbers applies and shows that when n is large, the empirical frequency is most likely to be within E of p. Loosely speaking, this allows us to conclude that

empirical frequencies are faithful estimates of p.
Alternatively, two is a step towards interpreting
the probability p is the frequency of the occurrence of A.
of A.