Lecture-11:B

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$AB = A \cap B$

Lecture-11

Ex:- Consider a probability model (S, F, P). Let $A \in F$ and $B \in F$ and both are mutually exclusive i.e. $AB = \phi$. Also $P(A) > 0$, $P(B) > 0$. What is the prob. that A occurs before B if the experiment is repeated independently indefinitely.

D: A occurs before B if the experiment is repeated independently indefinitely.

$$\boxed{P(D) = \frac{P(A)}{P(A)+P(B)}}$$

A

B

C A

C B

C C A

C C B

C C C A

$C = \overline{A \cup B}$

(S, F, P)

$(S \times S, F \times F, P')$

$S = \{H, T\}$

$S \times S = \{HH, HT, TH, TT\}$

D :

Event D can occur in the following mutually exclusive ways.

$A, \quad CA, \quad CCA, \quad - \quad \text{———}$

$$D = A \cup (CA) \cup (CCA) \cup (CCCA) \cup \text{——}$$

$$\Rightarrow P(D) = P(A) + P(CA) + P(CCA) + \text{——}$$

$$= P(A) + P(C)P(A) + P(C)P(C)P(A) + \text{——}$$

Since experiment is repeated independently indefinitely.

$$\Rightarrow P(D) = P(A)\left[1 + P(C) + P(C)P(C) + \text{——}\right]$$

$0 < P(C) < 1$

$$= P(A) \cdot \frac{1}{1 - P(C)}$$

$$= P(A) \cdot \frac{1}{1 - P(\overline{A \cup B})}$$

$$= \frac{P(A)}{1 - [1 - P(A \cup B)]}$$

$S = (A \cup B) \cup (\overline{A \cup B})$

$$= \frac{P(A)}{P(A \cup B)}$$

$$= \boxed{\frac{P(A)}{P(A) + P(B)}}$$

$\because AB = \phi$.

Game of Craps :-

A pair of dice is rolled on every play and the player wins at once if the sum for the first throw is 7 or 11 and player lose at once if the sum for the first throw is 2, 3, or 12.

Player wins in first throw: sum is 7 or 11

" loses in first throw: 2, 3 or 12

Game Carryover: If sum is 4, 5, 6, 8, 9, 10 in first throw.

If ~~b~~ the first throw is a carryover, then the player throws the dice repeatedly until he wins by throwing the same carryover again or losses by throwing 7.

What is the prob. of winning the game?

Sol^n :-

Sum:	2	3	4	5	6	7	8	9	10	11	12
Prob:	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The game can be won by throwing a 7 or 11 on the first throw or by throwing the carry-over on a later throw. Let P_1 and P_2 denote the prob. of these two mutually exclusive ~~wa~~ event.

Thus Prob. that player wins in first throw

$$= P_1 = \frac{6}{36} + \frac{2}{36} = \boxed{\frac{2}{9}}$$

Now suppose first throw $\in \{4, 5, 6, 8, 9, 10\}$

Suppose first throw $= 4$

Prob. of winning = P(First throw is 4 and sum 4 occurs before 7 if we throwing the dice repeatedly independently infinitely)

$$= \frac{3}{36} \times \frac{3/36}{3/36 + \frac{6}{36}} = \frac{3}{36} \times \frac{3}{3+6}$$

Suppose first throw is 5.

$$P(\text{winning}) = \frac{4}{36} \times \frac{\frac{4}{36}}{\frac{4}{36} + \frac{6}{36}} = \frac{4}{36} \times \frac{4}{4+6}$$

first throw is 6 then

$$P(\text{winning}) = \frac{5}{36} \times \frac{5}{5+6}$$

Suppose first throw is 10

Suppose first throw is 10

$P(\text{win}) = \frac{3}{36} \times \frac{3}{3\cancel{+}6}$

$\Rightarrow P(\text{Player wins}) = P_1 + P_2$

$= \frac{2}{9} + 2\left[\frac{3}{36}\times\frac{3}{3+6} + \frac{4}{36}\times\frac{4}{4+6} + \frac{5}{36}\times\frac{5}{5+6}\right]$

$= \frac{2}{9} + 2\left[\frac{3}{9}\times\frac{3}{36} + \frac{4}{10}\times\frac{4}{36} + \frac{5}{11}\times\frac{5}{36}\right]$

$= \frac{244}{495} \simeq \boxed{0.493}$

P(Winning in case of carryover) $= P_2 =$

$= P$(winning if first throw is 4) $+ P($ - - $)$

$+$