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Conditional Distributions

[When ran. var's are not independent]

We first look at discrete 2 vs.

Conditional PMF

Defin: Let X and Y be two discrete r.V. associated with

the same random experiment. Then the conditional

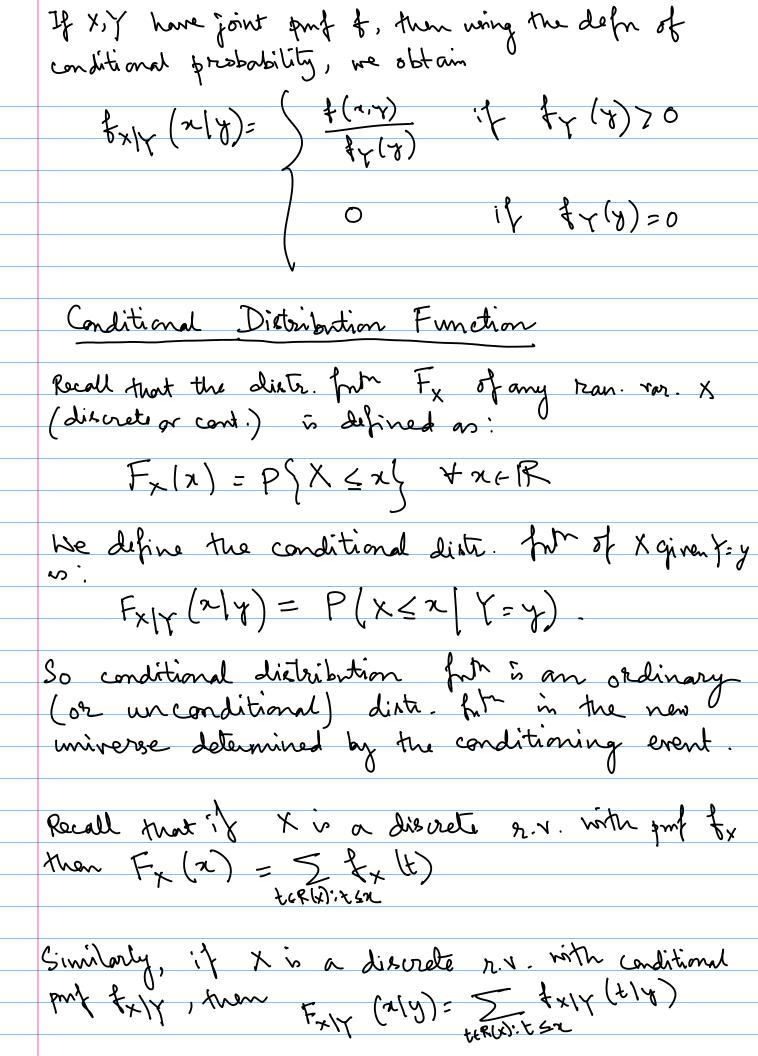
put fxy of X given Y=y is defined as $f_{XY}(x|y) = \int P(X=x|Y=y)$ if P(Y=y) > 0if P(Y=y) = 0

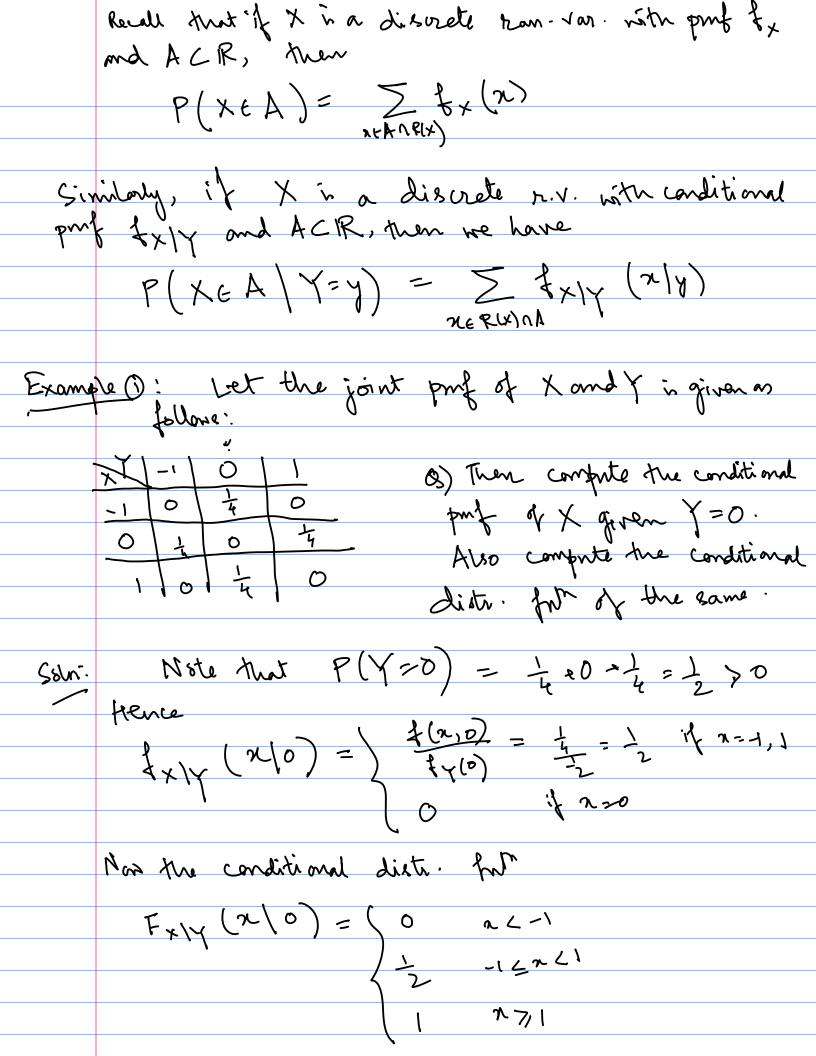
A conditional point can be thought of as an ordinary point over a new universe determined by the conditioning event.

For this, note that for fixed y, fx/ (x/y) 7,0 +xeR(x).

Also if P(Y=y) 70 then

 $\sum_{x \in R(x)} f_{x}(x) = \sum_{x \in R(x)} P(x=x) Y=y = P(\bigcup_{x \in R(x)} \{x=x\} Y=y)$





Remark: The have raid that the conditional port is a port in the new universe determined by the conditiving event In the previous example, the prob. distr. of x is $P(x=-1)=P(x=1)=\frac{1}{4}, P(x=0)=\frac{1}{2}$ Shereas in the new universe determined by the event } x = oby, the presb. dist. of X is revised as: P(x=-1 Y=0)= P(x=1 Y=0)= 1 Silve , the dort. from of X is Fx(x)=(0 if x2-1

1 if -1 < x < 0

2 if 0 < x < 1

1 if x>, Fx got revised on Fx/y (x)0) in the new universe determined by the event { Y-0}. Also note that FXIY (n10) satisfies all the properties of a dist. ful. 1) lin Fx/y(x/0)=0, lin Fx/y(x/0)=1 (2) Fx/x(n/o) is non-decreasing on R.

(3) Fx/y (2/0) is right continuous on R.

lement 2) The conditional part can also be used to calculate the marginal profs. In particular, we have by using the define $f_{\chi}(\chi) = \sum_{y \in R(Y)} f(x,y) = \sum_{y} f_{\chi|Y}(x)y f_{\gamma}(y)$ Example (2): Suppose fy (y) = \(\frac{5}{6} \) if $y = 10^2$ $\int \frac{1}{6} i \left(-\frac{y}{2} = 10^4 \right)$ $f_{X|Y}(x|10^2) = \int \frac{1}{2} i \int x = \frac{1}{10^2}$ $\frac{1}{3} i \int x = \frac{1}{10}$ $\frac{1}{6} i \int x = 1$ $\frac{1}{6} i \int x = 1$ $\frac{1}{6} (0) \text{ Then } f$ (Os) Then find the prof of X. $\frac{1}{2} \times |Y(x|10^4) = (\frac{1}{2} \text{ if } x = 10)$ $\frac{1}{6} \text{ if } x = 10^2$ Som: First of all by looking at conditional puf fx/y we see that X takes 5 values: $\frac{1}{16^2}$, $\frac{1}{10}$, $\frac{1}{10}$, $\frac{10}{10}$. Now, $f_{\chi}(\frac{1}{10^2}) = \frac{1}{2} \times \frac{5}{5} = \frac{5}{12}$, $f_{\chi}(\frac{1}{10}) = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$

$$f_{x}(1) = \frac{1}{6} \times \frac{5}{6} + \frac{1}{2} \times \frac{1}{6} = \frac{8}{36} = \frac{2}{9}$$
 $f_{x}(10) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
 $f_{x}(10^{2}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Conditional Deneity.

Defn: Let X and Y be two ran. var. with pdf f.

The conditional density of X given Y=y is

defined so:

$$f_{X|Y}(x|y) = \begin{cases} \frac{f(x,y)}{f_{Y}(y)} & \text{if } f_{Y}(y) > 0 \\ 0 & \text{if } f_{Y}(y) = 0 \end{cases}$$

As the case of conditional pmf, conditional ptf can be thought of an an ordinary (or unconditional) pdf over a new universe. The fixed y, fxfx (n(y) 70 + nc.18. Hos if T(1)>0, then

Recall that if X is a continuous &cun. var. with pdf fx and Bir any Borel subset of R, km $P(X+B) = \int f_X(x) dx$ The above motivated he following defin. Detribet X, Y be jointly continuous ran. vas, with fx/y densting the conditional density of x given y. Then for any Bord subset B of IR, $P(X \in B)Y = \int_{\mathcal{B}} f_{X|Y}(x|y) dx$ Remark: Conditional probability P(XCB|Y=y) were Left undefined if the P{Y=y}=0. But the above formula provider a natural way of défining such conditional prebabilities in the present context. In addition, it allows into view the conditional part fx/y (an a find of x) as a description of the presbability law of X, given that the event } Y= y \ has occurred. In view of eq. 10, the conditional cdf of X given Y=y in

We have $\frac{d}{dx} F_{x|y}(x|y) = f_{x|y}(x|y), \text{ where}$ equality holds at points (x,y) at which joint $\frac{d}{dx} F_{x|y}(x|y) = f_{x|y}(x|y), \text{ where}$ $\frac{d}{dx} F$