Lecture-3:A2

06 January 2021 07:46

Lecture-3

$$\boxed{P(E) = \frac{n(E)}{n}}$$

Significance of the number n and $n(E)$ is not always clear.

Ex 1 - Suppose in the experiment of rolling of two dice, we want to find out the probability p that the sum of numbers that appear on the dice is equal to 7.

Ans (A) - We could consider as total possible outcomes the 11 sums 2, 3, —— 11, 12. Of these, only one, namely the 7 is favorable. Hence

$$\boxed{p = P(7) = \frac{1}{11}}$$ X Wrong

(B) We could think outcomes all pairs of numbers not distinguishing between the first and second die.

We have total 21 outcomes

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)}

Out of which favorable outcomes are

{(1, 6), (2, 5), (3, 4)}

$\{(1,6), (2,5), (3,4)\}$

$\Rightarrow p = P(7) = \frac{3}{21} = \frac{n(E)}{n}$ ✗ wrong

The above two solutions are wrong because the outcomes in solution (A) & (B) are not equally likely.

To solve it correctly, we must count all pairs of numbers distinguishing between the first and second die.

$p = P(7) = \frac{n(E)}{n} = \frac{6}{36} = \frac{1}{6}$ ✓ $\begin{Bmatrix} (3,4), (4,3) \\ (5,2), (2,5) \\ (6,1), (1,6) \end{Bmatrix}$

Equally Likely:- When the outcome of an experiment is just as likely as another, the outcomes are said to be equally likely.

Ex:- Tossing of a fair coin

Ex:- Rolling of a fair die. $P(i) = \frac{1}{6}, i = 1, 2, \ldots 6.$

Improved version of Classical definition:-

The probability of an event E equals the ratio of its favorable outcomes $n(E)$ to the total numbers of outcomes n, provided all outcomes are equally likely.

Drawback:- (1) The term equally -

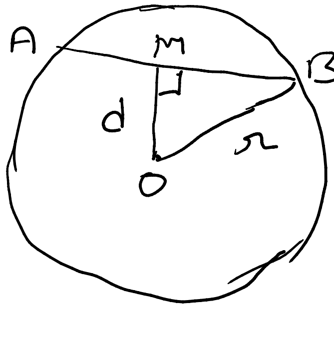
(1) The term equally-
-likely means actually equally probable.
Thus in the def^n, use is made of the concept to be defined.

(2) The def^n is applicable to only limited class of problems.

(*) If the number of possible outcome is ~~ie~~ infinite, then to apply the classical def^n we must use length, area or some other measure of infinity for determining the ratio $\frac{n(E)}{n}$.

1889

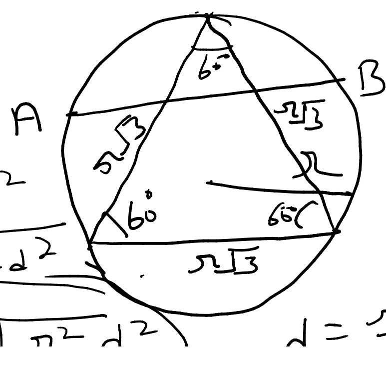
Bertrand Paradox :- We are given a circle C of radius r and we wish to find out the probability p that the length of a randomly selected card AB is greater than the length $r\sqrt{3}$ of the inscribed equilateral triangle.



$r^2 = (BM)^2 + d^2$

$\Rightarrow BM = \sqrt{r^2 - d^2}$

$\Rightarrow AB = 2\sqrt{r^2 - d^2}$



$d = \frac{r}{2}$

$\Rightarrow BM = \sqrt{r^2 - d^2}$ $\quad r\sqrt{3}$

$\Rightarrow$ $AB = 2\sqrt{r^2 - d^2}$

$d = \frac{r}{2}$

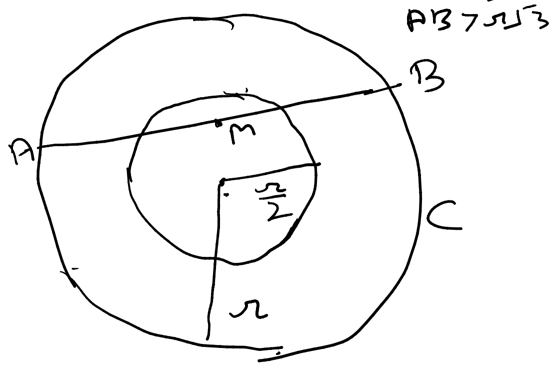
$AB = r\sqrt{3}$

$d \leq \frac{r}{2}$

$AB > r\sqrt{3}$

First Answer :-

If the center M of the cord AB lies inside the circle C_1 of radius $\frac{r}{2}$ then $l > r\sqrt{3}$

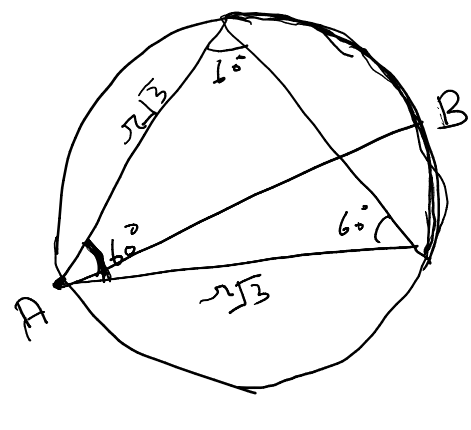


Thus it is reasonable, therefore by classical def^n

$$p = \frac{n(E)}{n} = \frac{\frac{\pi r^2}{4}}{\pi r^2} = \boxed{\frac{1}{4}}$$

Here, we consider as favorable outcomes all points inside the circle C_1 and as possible outcomes insider the circle C.

Answer 2 :- In order that the cord AB is longer than $r\sqrt{3}$, the line has to lie within a sector of $60°$ with a range of $180°$.



$$p = \frac{60}{180} = \boxed{\frac{1}{3}}$$

$$\text{or } p = \frac{2\pi r/3}{2\pi r} = \boxed{\frac{1}{3}}$$

Ans 3 :-

Ans 3 L

$$p = \frac{r}{2r} = \boxed{\frac{1}{2}}$$