MATH-221: Probability and Statistics

Tutorial # 1 (Countable & uncountable sets, Properties of Probability Measure, Conditional Probability, Total Probability Theorem, Baye's Theorem)

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1. Consider the random experiment of tossing a coin indefinitely. Show that the corresponding sample space is uncountable.

2. Let A, B, C be events such that $P(A) = 0.7, P(B) = 0.6, P(C) = 0.5, P(A \cap B) = 0.4, P(A \cap C) = 0.3, P(C \cap B) = 0.2$ and $P(A \cap B \cap C) = 0.1$. Find $P(A \cup B \cup C), P(A^c \cap C)$ and $P(A^c \cap B^c \cap C^c)$.

3. Prove or disprove: If $P(A \cap B) = 0$ then A and B are mutually exclusive events.

4. Does there exists a probability measure (or function) P such that the events A, B, C satisfies $P(A) = 0.6, P(B) = 0.8, P(C) = 0.7, P(A \cap B) = 0.5, P(A \cap C) = 0.4, P(C \cap B) = 0.5$ and $P(A \cap B \cap C) = 0.1$?

5. For any events A and B, show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Hence conclude

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

6. Let $\Omega = \mathbb{N}$. Define a set function P as follows: For $A \subset \Omega$,

$$P(A) = \begin{cases} 0 & \text{if} \quad A \text{ is finite} \\ 1 & \text{if} \quad A \text{ is infinite} \end{cases} .$$

   Is P a probability measure (or function)?

7. **(Continuity of Probability Measure)** Let $A_n, n \geq 1$ be a sequence of events. Then prove the following:

   (a) If $A_1 \subset A_2 \subset \cdots$ Then $P\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k \to \infty} P(A_k)$.

   (b) If $A_1 \supset A_2 \supset \cdots$ Then $P\left(\bigcap_{k=1}^{\infty} A_k\right) = \lim_{k \to \infty} P(A_k)$.

8. Let $A_1, A_2, \cdots$ be a sequence of events then show that

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} P(A_n).$$

9. Let Ω be a nonempty set and P be a function from set of subsets of Ω to $[0, 1]$ such that

   (a) $P(\Omega) = 1$.

   (b) For A and B disjoint, $P(A \cup B) = P(A) + P(B)$.

(c) If (A_n) is a decreasing sequence of events such that $\bigcap_{n=1}^{\infty} A_n = \emptyset$, then

$$\lim_{n\to\infty} P(A_n) = 0.$$

Show that P is a probability measure.

10. Three switches connected in parallel operate independently. Each switches remains closed with probability p. Then (a) Find the probability of receiving an input signal at the output. (b) Find the probability that switch S_i is open given that an input signal is received at the output.

11. Suppose an individual applying to a college determines that he has an 80% chance of being accepted, and he knows that dormitory housing will only be provided for 60% of all of the accepted students. What is the probability that a student will be accepted and will receive dormitory housing?

12. An electronic assembly consists of two subsystems, say A and B. From previous testing procedures, the following probabilities assumed to be known: $P(A \text{ fails}) = 0.20, P(A \text{ and } B \text{ both fail}) = 0.15, P(B \text{ fails alone}) = 0.15$. Evaluate the following conditional probabilities (a) $P(A \text{ fails } | B \text{ has failed })$ (b) $P(A \text{ fails alone } | A \text{ or } B \text{ fail})$.

13. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let p be the probability that the student knows the answer and $1 - p$ the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly?

14. The completion of a construction job may be delayed because of a strike. The probabilities are 0.60 that there will be a strike, 0.85 that the construction job will be completed on time if there is no strike, and 0.35 that the construction will be completed on time if there is a strike. What is the probability that the construction job will be completed on time?