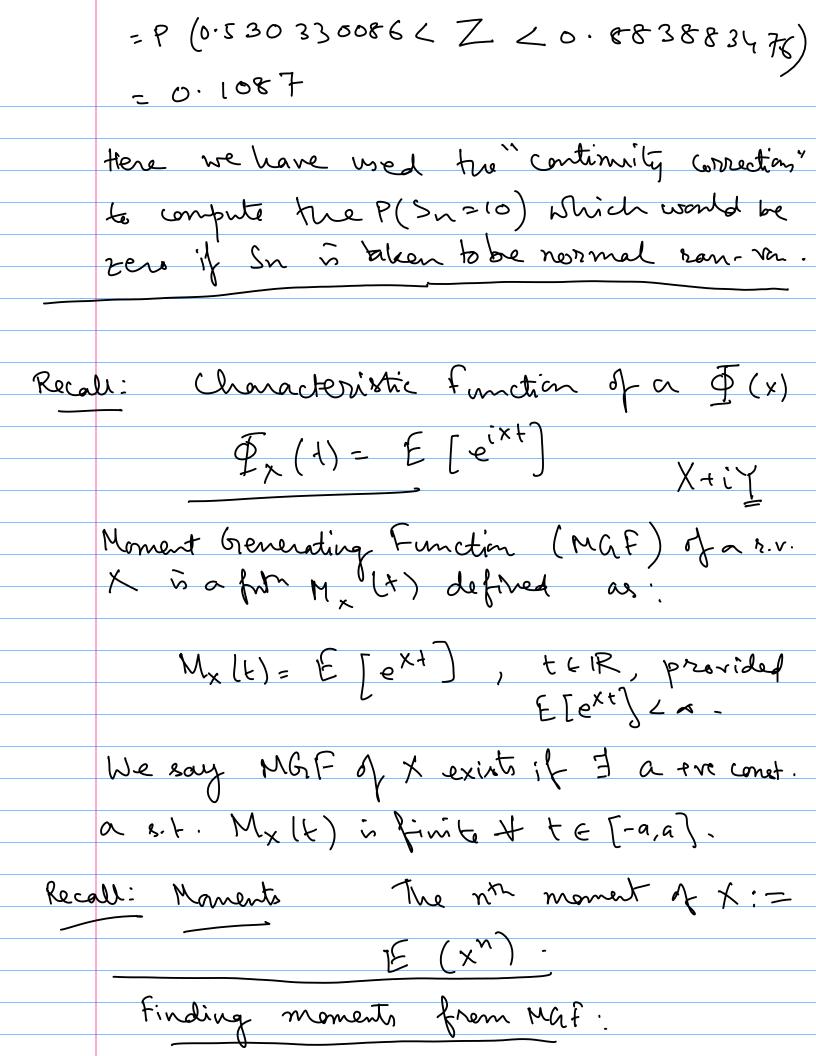
Lec-44 Convolution: Exg > pdf g x < \ (x, \ \ ane ind.). (8xq)(1)= | f(t)q(t-x)ds (f*g)+h)-f*(g*h). Example (1): Let X_1, X_2, \dots be i'd Poisson (X) CLT. RYS. Then we know $E(X_1) = Van(X_1) = \lambda$. Hence by CLT, Sn= X, exze- + xn has approximately an $N(n\lambda, n\lambda)$ distribution for large n: Let $n = (4, \lambda = 0.125 \Rightarrow n\lambda = 8)$ the sum of independent Poisson is again Poisson Hence exact detribution of Sour Poisson (64 x 0.125) I from Poisson distribution taldes $P(S_{4} = 10) = 8^{10} = 0.099261534$ Veing normal approximation,

 $P(S_{n}=10) = P(9.5 < S_{n} < 10.5)$ $= P(9.5 < S_{n} < 10.5)$



Coeff of the : Lei : E(Xk) = Lei

Gamma Function

X-> ran-var. S.t. its pdf is:

$$f(\pi) = \begin{cases} k\pi^{\alpha-1} - \mu\beta \\ 0 & 0. \text{ W} \end{cases}$$

where d>0, \$>0 and k must be e.t. the total area under the curve is equal to 1.

To evaluate k, we first substitute $y = \frac{x}{6}$

The integral thus obtained depends on a alone and it defines the well-known gamma function

$$\Gamma(\alpha) = \int y^{\alpha-1} - y \, dy$$
, for $\alpha > 0$.

$$\Gamma(\alpha) = (\alpha - 1) \Gamma(\alpha - 1)$$

Integrating by parts:

$$\Gamma(\alpha) = (\alpha - 1) \Gamma(\alpha - 1)$$
.

For $\alpha > 1$, and since $\Gamma(1) = \int_{0}^{\alpha} e^{-y} dy = 1$

It follows by trapetition that $\Gamma(\alpha) = (\alpha - 1)!$ Whenever α in a pre integer.

So me get
$$(\frac{1}{2}) = \sqrt{\pi}$$
.

So me get $(\frac{1}{2}) = \sqrt{\pi}$.

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 $(\frac{1}{2}) = \sqrt{\pi}$.

Gamma distribution.

A scan var. $(\frac{1}{2}) = \sqrt{\pi}$.

 $(\frac{1}{2}) = \sqrt{\pi}$.

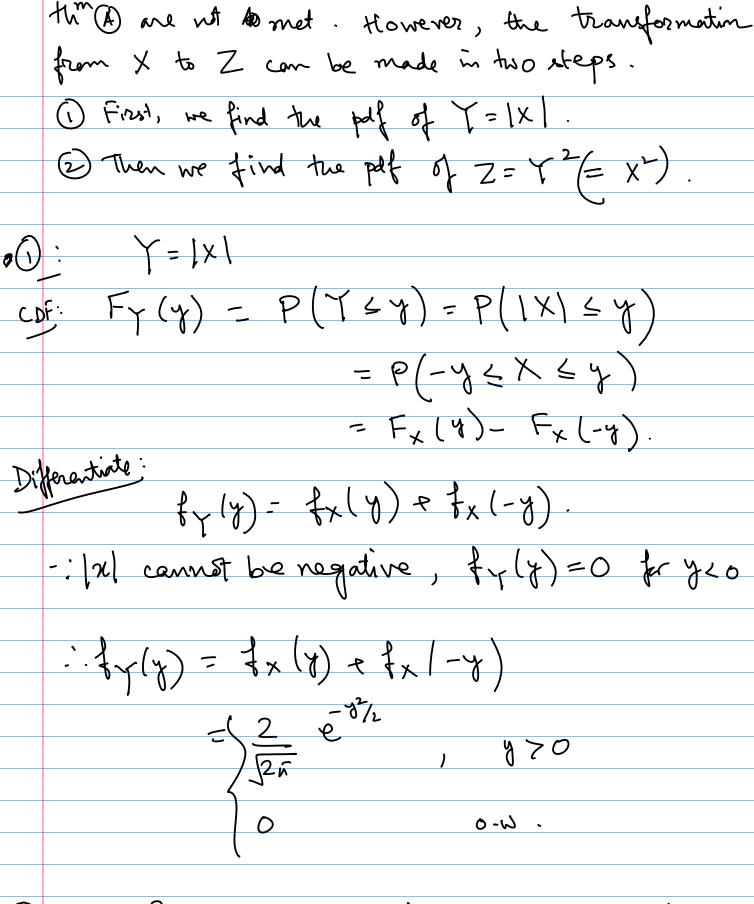
 $(\frac{1}{2}) = \sqrt{\pi}$.

A transform variable $(\frac{1}{2}) = \sqrt{\pi}$.

Defu.

the parameter I is referred to as the number of degrees of freedom or simply the degreer of freedom. Note: M'-diet. na special care of the gamma-dist. with $\alpha = \frac{2}{2}$, $\beta = 2$. Importance (Thm:) If XNN(0,1), then Z=X2
follows X2-dist. with 7=1. Before going to the proof of the above the, let is understand the following result: the fire paf of the cont. ran. var. X at x. If the full given by y = u(x) is differentiable and either increasing or decreasing for all values within the range of x for which f(x) + 0, then for there values of x, the equation y=u(x) can be uniquely edved for a togive x = w(y) and for the corresponding values of y the pdf of Y = u(X) is given by $g(y) = \frac{1}{2} \left[\omega(y) \right] \cdot \left[\omega'(y) \right]$ provided u'(x) \$0

Example: If X has the exponential distr. f(n)= \ e^x, n>0 Find pdf of Y = JX. SSM: The egn $y = \sqrt{2}$ has the unique inverse $\chi = \frac{y^2}{x = \omega(y)} \qquad \qquad \omega'(y) = \frac{dx}{dy} = \frac{2y}{2y}.$ i. g(y) - f(w(y) w'(v)) = ey 12y1 = 2y e^{y2}, 470 $P = \frac{1}{9} = \frac{1}{2} = \frac{1}{9} = \frac{1}{2} = \frac{1}{9} =$ Y= \X CDF & Y = P(Y < y) = P(TX = y) this = P(x < y) Back to the proof of our main important tim: Proof: The function given $Z = n^2$ is decreasing for -ve values of x and increasing for the radius of x, the conditions of



2) Z=y2 is increasing for y70, that is, for all values of Y for which ty(y) \$0.

:. We can we then (A). Since
$$\frac{dx}{dz} = \frac{1}{2}z^{\frac{-1}{2}}$$
, we get $h(z) = \frac{2}{\sqrt{2\pi}}e^{-\frac{-2}{2}}\left|\frac{1}{2}z^{-\frac{1}{2}}\right|$

$$= \frac{1}{\sqrt{2\pi}}e^{-\frac{-2}{2}}\left|\frac{1}{2}z^{-\frac{1}{2}}\right|$$

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$$\frac{1}{\sqrt{2\pi}} = \frac{1}{2^{-\frac{1}{2}}} = \frac{-\frac{1}{2}}{2} = \frac{\frac$$

The distr. of Z:
$$h(z) = \begin{cases} \frac{1}{2^{\frac{1}{2}}} & \frac{1}{2^{\frac{1}{2}}}$$