The LNM Institute of Information Technology
Jaipur, Rajasthan

MTH 222 Probability and Statistics

Tutorial-3

Even Semester, 2021.

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1. (Buffon's needle problem) A table is ruled with equidistant parallel lines a distance D apart. A needle of length L, where $L \leq D$, is randomly thrown on the table. What is the probability that the needle will intesect one of the lines (the other possibility being that the needle will be completely contained in the strip between two lines) ?

2. Let X, Y, Z be independent and uniformly distributed over $(0, 1)$. Compute $P(X > YZ)$.

3. If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2, calculate the conditional distribution of X given that $X + Y = n$.

4. Consider $n + m$ trials having a common probability of success. Suppose, however, that this success probability is not fixed in advance but is chosen from a uniform $(0, 1)$ population. What is the conditional distribution of the success probability given that $n + m$ trials result in n successes?

5. Let U be uniform $(0, 1)$ and V be uniform $(0, U)$.
   (a) Find $E[V|U = u]$.
   (b) Find $Var[V|U = u]$.
   (c) Find $E[V]$.
   (d) Find $Var[V]$.

6. Let X and Y be two random variables with $EX = 1, Var(X) = 4$, and $EY = 2, Var(Y) = 1$. Find the maximum possible value for $E[XY]$.

7. Let (X, Y) be uniformly distriuted random point on the quadrilateral D with vertices $(0, 0), (2, 0), (1, 1), (0, 1)$. Calculate the covariance of X and Y.

8. Let X be uniformly distributed on $[-a, a]$ for $a > 0$ and $Y = X^2$. Show that X and Y are uncorrelated even though Y is a function of X.

9. Recall that we say that I_A is an indicator variable for the event A if

$$I = \begin{cases} 1, & \text{if } A \text{ occurs} \\ 0 & \text{if } A^c \text{ occurs.} \end{cases}$$

   Show that for any A, B we have

$$\rho(I_A, I_B) = \rho(I_{A^c}, I_{B^c}).$$

10. Let X be a positive random variable with $EX = 10$. What can you say about the following quantities?

    a. $E[\frac{1}{X+1}]$
    b.$E[e^{\frac{1}{X+1}}]$
    c. $E[ln\sqrt{X}]$.

11. Let $X_1, \ldots, X_{20}$ be independent Poisson random variables with mean 1.
    (a) Use the Markov inequality to obtain a bound on

$$P\Big\{\sum_{i=1}^{20} X_i > 15\Big\}.$$

    (b) Use the central limit theorem to approximate $P\Big\{\sum_{i=1}^{20} X_i > 15\Big\}$.

12. A die is continually rolled untill the total sum of all rolls exceeds 300. What is the probability that at least 80 rolls are necessary?