Lecture-15B

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# Lecture-15

## Cummulative dist$^n$ f$^n$ :-

$$\boxed{F_X(x) = P\{X \leq x\}} \quad -\infty < x < \infty$$

$F_X(x)$ is defined for every $x$ from $-\infty$ to $\infty$.

In other words, $F_X(x)$ denotes the probability that the r.v. $X$ take on a value that is less than or equal to $x$.

**Properties of $F_X(x)$** :- Dist$^n$ f$^n$ $F_X(x)$ has some specific properties, which are

(i) $F_X(x)$ is a non-decreasing function of $x$.

(2) $\lim_{x \to \infty} F_X(x) = F_X(\infty) = 1$

(3) $\lim_{x \to -\infty} F_X(x) = F_X(-\infty) = 0$

(4) $0 \leq F_X(x) \leq 1$

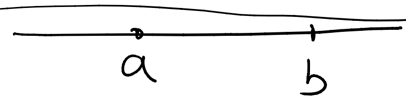
(5) $P(x_1 < X \leq x_2) = F_X(x_2) - F_X(x_1)$

(6) $F_X(x)$ is right continuous, i.e

(b) $F_X(x)$ is right continuous, i.e.

$$F_X(x^+) = F_X(x). \qquad \boxed{F_X(x^+) = \lim_{h \to 0} F_X(x+h) = F_X(x)}$$

<u>Proof</u> (1) $a < b$




for $a \leq b$, the event $\{X \leq a\}$ is contained in the event $\{X \leq b\}$. and hence $\{X \leq a\}$ must have smaller probability then of $P\{X \leq b\}$.

$$\{X \leq a\} \subseteq \{X \leq b\}$$

$$\Longrightarrow P\{X \leq a\} \leq P\{X \leq b\}$$

$$\Longrightarrow F_X(a) \leq F_X(b)$$

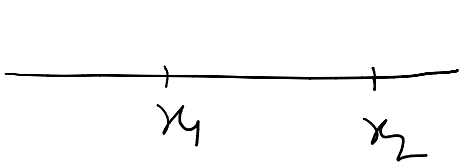
$\Longrightarrow$ $F_X(x)$ is a non-decreasing $f^n$ of $x$.

(2) $$\lim_{x \to \infty} F_X(x) = \lim_{x \to \infty} P(X \leq x)$$

$$= P(X \leq \infty) = P(S) = 1$$

(5) <u>$P(x_1 < X \leq x_2)$</u> $\qquad (x_1, x_2]$

$\{X \leq x_1\}$ and $\{x_1 < X \leq x_2\}$ are mutually exclusive



$x_1$ $x_2$

are mutually exclusive and union of them is $\{X \le x_2\}$.

$$\{X \le x_1\} \cap \{x_1 < X \le x_2\} = \phi$$

and $\{X \le x_2\} = \{X \le x_1\} \cup \{x_1 < X \le x_2\}$

$$\Rightarrow P\{X \le x_2\} = P\{X \le x_1\} + P\{x_1 < X \le x_2\}$$

$$\Rightarrow P\{x_1 < X \le x_2\} = P\{X \le x_2\} - P\{X \le x_1\}$$

$$= F_X(x_2) - F_X(x_1)$$

<u>Ex</u>:- In the coin-tossing experiment

Let $P(H) = p$, $P(T) = 1-p$

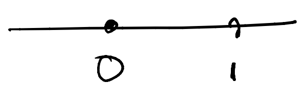
$0 < p < 1$.

Let us define a r.v. $X$ as

$$X = \begin{cases} 1 & H \text{ appears} \\ 0 & T \text{ appears} \end{cases}$$

$P(X=0) = 1-p$, $P(X=1) = p$.

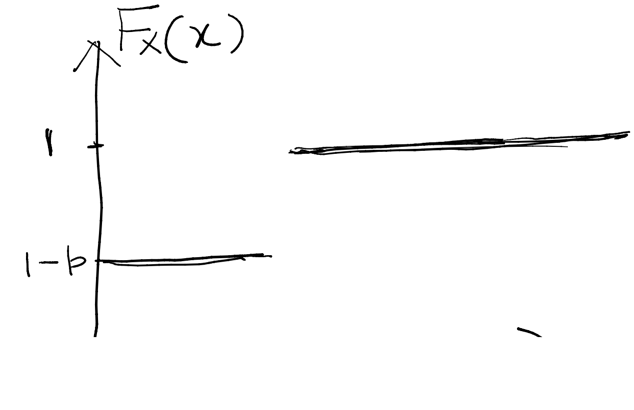
We are interested in the



c.d.f. $F_X(x)$, $-\infty < x < \infty$.

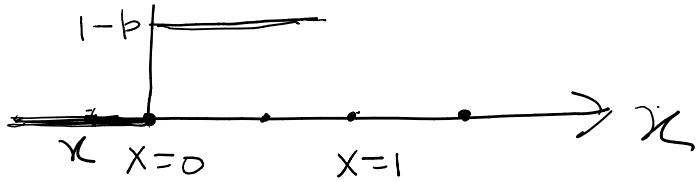
If $x \ge 1$, then

$F_X(x) = P(X \le x)$



$= P(X = 0) + P(X \ldots$

$= P(X=0) + P(X=1)$




$= (1-p) + p = 1$

If $0 < x < 1$, then

$$F_X(x) = P(X \le x)$$
$$= P(X=0)$$
$$= 1-p$$

If $x < 0$, then

$$F_X(x) = P(X \le x)$$
$$= P(\phi)$$
$$= 0$$

$$F_X(x) = \begin{cases} 0, & x<0 \\ 1-p, & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$

$P(X=0)$

$P(X=1)$

$$P(X=x_i) = F_X(x_i) - F_X(x_i^-)$$

$$P(X=0) = F_X(0) - F_X(0^-)$$
$$= 1-p-0 = 1-p$$

$$P(X=1) = F_X(1) - F_X(1^-)$$
$$= 1-(1-p) = p$$

Discrete and Continuous r.v.s!

S

**Countable**

Range of X is countable

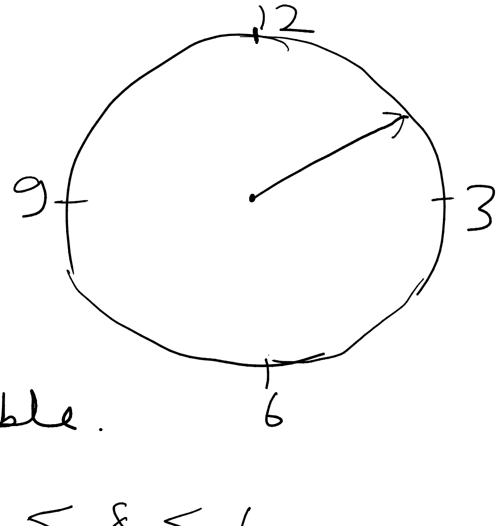
**Uncountable**

Range of X may or may not be countable.

**Discrete r.v.** :- A r.v. X ~~that~~ can take on at most a countable number of possible values on real line, is said to be discrete random variable.

Can you give an example of discrete random variable defined on uncountable sample space?

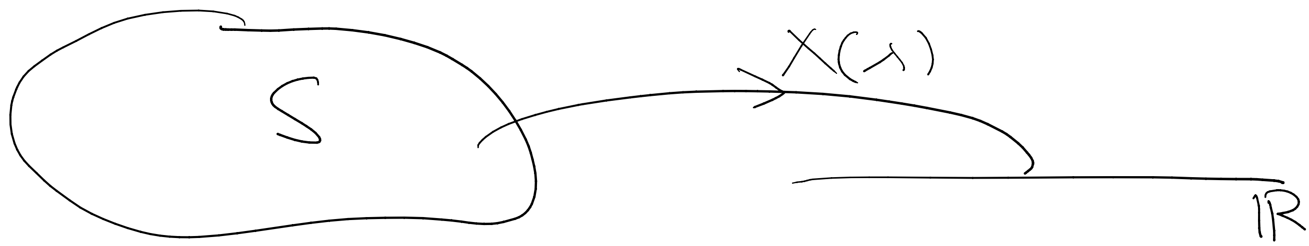
Consider Pointer on wheel of chance is spun. The possible outcomes are the numbers from 0 to 12 marked on the wheel.




Sample space is uncountable.

$$X = \begin{cases} 1 & 0 < s \leq 6 \end{cases}$$

$$X = \begin{cases} & \\ 0 & 6 < s \le 12 \end{cases}$$




X is discrete.

## Probability Mass function (PMF, pmf):-

For a discrete random variable X, we define the prob. mass $f^n$ (p.m.f.) of X by

$$\boxed{p_X(x) = P\{X = x\}}$$

**Properties of pmf:-** Let $p_X$ be the pmf of a discrete random variable X. Then

(1) $p_X(x) \geq 0 \quad \forall\, x \in \mathbb{R}$

(2) $p_X(x)$ is +ive for at most a countable number of values of $x$, i.e.

$X : x_1, x_2, x_3, \text{——} \ \text{——}$, then i.e. $x_1, x_2, x_3$, —— ——, then

$p_X(x_i) > 0 \quad i = 1, 2, 3$ ——

$p_X(x) = 0$ for other values of $x$.

(3) $\sum_{i=1}^{\infty} p_X(x_i) = 1$

The distribution function $F_X(x)$ of a discrete r.v. $X$ can be expressed in terms of $p_X(x)$ as

$$\boxed{F_X(x) = P(X \leq x) = \sum_{\text{all } x_i \leq x} p_X(x_i)}$$

It is clear that $F_X(x)$ for discrete r.v. $X$ is constant except for a finite number of jump discont-inuities.

Also note that

$$\boxed{p_X(x_i) = P(X = x_i) = F_X(x_i) - F_X(x_i^-)}$$