THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
PROBABILITY AND STATISTICS: MTH221
QUIZ-1: SOLUTION

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1. Let X be a Binomial random variable with parameters $(5, p)$. The value of p for which $P(|X - E(X)| \leq 3 = 1)$ are given by

Sol. $P(|X - E(X)| \leq 3 = 1) \Rightarrow P(5p - 3 \leq X \leq 5p + 3) = 1$ as $E(X) = np = 5p$.
Range of X i.e., R_X : 0,1,2,3,4,5
$\Rightarrow 5p - 3 \leq 0$ and $5p + 3 \geq 5 \Rightarrow \boxed{\dfrac{2}{5} \leq p \leq \dfrac{3}{5}}$

2. Let X be a Poisson random variable with $P(X = 1) + 2P(X = 0) = 12P(X = 2)$. Then $P(X = 0)$ is

Sol. Let X be Poissonly distributed with some parameter $\lambda > 0$. Thus
$P(X = 1) + 2P(X = 0) = 12P(X = 2) \Rightarrow \dfrac{e^{-\lambda}\lambda^1}{!1} + 2\dfrac{e^{-\lambda}\lambda^0}{!0} = 12\dfrac{e^{-\lambda}\lambda^2}{!2} \Rightarrow 6\lambda^2 - \lambda - 2 = 0 \Rightarrow (3\lambda - 2)(2\lambda + 1) = 0 \Rightarrow \lambda = \dfrac{2}{3}, \Rightarrow P(X = 0) = \boxed{e^{-2/3}}$.

3. In a factory, instruments are tested one at a time until a good instrument is found. Let X denote the number of instruments that need to be tested in order to find a good one. Given that $P(X > 1) = \frac{1}{2}$, then $E(X) =$

Soln. Here X be geometric random variable with parameter p. We know that $P(x > n) = (1 - p)^n$, $n \in \mathbb{N}$, and $E(X) = 1/p$. Given that $P(X > 1) = \dfrac{1}{2} \Rightarrow (1 - p)^1 = \dfrac{1}{2} \Rightarrow p = \dfrac{1}{2} \Rightarrow E(X) = \dfrac{1}{p} = \boxed{2}$.

4. For what value of c, the following function is a probability density function:

$$f(x) = \begin{cases} \frac{15}{64} + \frac{x}{64}, & -2 \leq x \leq 0, \\ \frac{3}{8} + cx, & 0 \leq x \leq 3, \\ 0, & \text{elsewhere} \end{cases}$$

Sol. An easy calculation. $\displaystyle\int_{\infty}^{\infty} f(x)dx = 1 \Rightarrow \boxed{c = -\frac{1}{8}}$.

5. Let X be the time (in hours) required to repair a car, which is exponentially distributed with an average of 4 hours. Then $P(X > 10|X > 8) =$

Sol. Let X is exponentially distributed with parameter λ. Given that $E(x) = \dfrac{1}{\lambda} = 4 \Rightarrow \lambda = \dfrac{1}{4}$. By Memoryless property, we have

$$\begin{aligned} P(X > 10|X > 8) &= P(X > 2) \\ &= 1 - P(X \leq 2) \\ &= 1 - F_X(2) \\ &= 1 - (1 - e^{-1/2}) = \boxed{e^{-1/2}}. \end{aligned}$$