Experiment No.: 04

1 Aim

1. To generate two periodic signals $x_1(t)$ and $x_2(t)$.
2. To compute and plot the Fourier spectra for the aforementioned periodic signals.
3. To illustrate the Gibb's phenomenon.

2 Software Used

1. MATLAB

3 Theory

Fourier series is a way to represent a wave-like function as the sum of simple sine waves. More formally, it decomposes any periodic function or periodic signal into the sum of a (possibly infinite) set of simple oscillating functions, namely sines and cosines (or, equivalently, complex exponentials). Fourier series make use of the orthogonality relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical.In particular, since the superposition principle holds for solutions of a linear homogeneous ordinary differential equation, if such an equation can be solved in the case of a single sinusoid, the solution for an arbitrary function is immediately available by expressing the original function as a Fourier series and then plugging in the solution for each sinusoidal component. In some special cases where the Fourier series can be summed in closed form, this technique can even yield analytic solutions.

The Fourier Series of a periodic signal $x(t)$ with period T is given by

$$x(t) = \sum_{k=-\infty}^{+\infty} D_k \, e^{j\,k\,\omega_0\,t}; \qquad \omega_0 = \frac{2\pi}{T} \tag{1}$$

where the Fourier Series coefficient D_k is calculated by

$$D_k = \frac{1}{T}\int_T x(t)\, e^{-j\,k\,\omega_0\,t} dt. \tag{2}$$

In order to compute the D_k discretely, we approximate the aforementioned finite integral.

$$D_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n\,T_s)\, e^{-j\,k\,\Omega_0\,n}; \qquad \Omega_0 = \omega_o\,T_s \tag{3}$$

where T_s denotes the sampling interval and $N = \frac{T}{T_s}$ is the number of samples in one period T.

Gibb's phenomenon: The peculiar manner in which the Fourier series of a piecewise continuously differentiable periodic function behaves at a jump discontinuity: the partial sum of the Fourier series has large oscillations near the jump, which might increase the maximum of the partial sum above that of the function itself. The overshoot does not die out as the frequency increases, but approaches a finite limit. The Gibb's phenomenon involves both the fact that Fourier sums overshoot at a jump discontinuity, and that this overshoot does not die out as the frequency increases.

4 Procedure

Exercise 1 *For this exercise we consider the periodic signals* $x_1(t)$ *and* $x_2(t)$ *defined , respectively as*

$$x_1(t) = e^{-t/2}; \qquad 0 \le t \le 1 \tag{4}$$

and

$$x_2(t) = \begin{cases} 1; & 0 \le t \le T/2 \\ -1; & T/2 < t \le T \end{cases} \tag{5}$$

where $T = 2$*.*
For the numerical computation of D_k*, we use* $N = 256$*. Compute* 10 *coefficients of the periodic signals* $x_1(t)$ *and* $x_2(t)$ *and also plot the Fourier spectra.*

Exercise 2 *Let* $x_M(t)$ *be the approximation of the original periodic signal* $x(t)$*. It is defined as*

$$x_M(t) = \sum_{k=-M}^{M} D_k \, e^{j\,k\,\omega_0\,t}. \tag{6}$$

Plot $x_M(t)$ *as a function of time for* $M = 19$ *and* 99*.*

5 Observation

Write/ Plot Your Own With Observation Table (If Required).

6 Analysis of Results

Write Your own.

7 Conclusions

Write Your Own.

Precautions

Observation should be taken properly.