Experiment No.: 02

1 Aim

To compute and plot the Fourier spectra for the aperiodic signals.

2 Software Used

1. MATLAB

3 Theory

Aperiodic Signals: The signals which may be repetitive but only over a finite interval. These signals are analyzed by means of the Fourier Transform.
Fourier series: A Fourier series is an expansion of a periodic function terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical. For a periodic signal, one that repeats exactly every, say, T seconds, there is a decomposition that we can use, Fourier series decomposition, to put the signal in this form. If the signals are not periodic we can extend the Fourier series approach and do another type of spectral decomposition of a signal called a Fourier Transform.
Fourier Transform: The Fourier transform expresses a function of time (a signal) as a function of frequency. This is similar to the way in which a musical chord can be expressed as the amplitude (or loudness) of its constituent notes. The Fourier transform of a function of time itself is a complex-valued function of frequency, whose complex modulus represents the amount of that frequency present in the original function, and whose complex argument is the phase offset of the basic sinusoid in that frequency. The Fourier transform is called the frequency domain representation of the original signal. The term Fourier transforms refers to both the frequency domain representation and the mathematical operation that associates the frequency domain representation to a function of time. The Fourier transform is not limited to functions of time, but in order to have a unified language, the domain of the original function is commonly referred to as the time domain. For many functions of practical interest one can define an operation that reverses this: the inverse Fourier transformation, also called Fourier synthesis, of a frequency domain representation combines the contributions of all the different frequencies to recover the original function of time. Functions that are localized in the time domain have Fourier transforms that are spread out across the frequency domain and vice versa. The critical case is the Gaussian function, of substantial importance in probability theory and statistics as well as in the study of physical phenomena exhibiting normal distribution (e.g., diffusion), which with appropriate normalizations goes to itself under the Fourier transform. Joseph Fourier introduced the transform in his study of heat transfer, where Gaussian functions appear as solutions of the heat equation.

4 Procedure

Note: Students have to do both magnitude plot & Phase Plot.

Create your own function $X = mydft(x, t_o, t_s)$ which will generate DFT of x.

Exercise:1 The Fourier transform (FT) of an aperiodic continuous-time signal x(t) is given by

$$X(\omega) = \int_{-\infty}^{\infty} \mathrm{x}(t)e^{-j\omega t}\,\mathrm{d}t \tag{1}$$

In numerical computations, the data must be finite. Let us consider the signal $x(t)$ of finite duration T_0 . We approximate the FT of the finite duration signal $x(t)$. The Fourier transform (FT) of an aperiodic continuous-time signal $x(t)$ is given by

$$\begin{aligned} X(\omega) &= \int_{0}^{T_0} \mathrm{x}(t)e^{-j\omega t}\,\mathrm{d}t \\ &= \lim_{T_s\to\infty} \sum_{k=0}^{N_0-1} x(kT_s)e^{-j\omega T_s}T_s; \end{aligned} \tag{2}$$

Where Ts denotes the sampling interval of the signal x(t) and $N = T_0/T_s$ is the total number of samples. Let us consider the samples of $X()$ at regular interval of ω_0. If X_r is the r^{th} sample, then from Equation 2, we obtain

$$\begin{aligned} X_r &= \sum_{k=0}^{N_0-1} T_s x(kT_s)e^{-jr\omega_0 kT_s} \\ &= \sum_{k=0}^{N_0-1} x_k e^{-jr\Omega_0 k}; \end{aligned} \tag{3}$$

where $x_k = T_s\ x(kT_s)$, $X_r = X(r\omega_0)$ and $\Omega_0 = \omega_0 T_s$. Use MATLAB to compute the FT of the following signal:

$$x_1(t) = e^{-2t}\ u(t) \tag{4}$$

where $u(t)$ denotes the continuous-time unit step function.

5 Observation

Plot DFT for these two cases alongside FT using the inbuild command $X = fft(x)$ with observation Table.

1. $T_0 = 4$ sec, $T_s = 1/64$ sec.
2. $T_0 = 8$ sec, $T_s = 1/32$ sec.

6 Analysis of Results

Write Your own.

7 Conclusions

Write Your Own.

Precautions

Observation should be taken properly.